



MONASH University

Seeing is not enough for believing: Building
mathematical knowledge for teaching through
observing, deconstructing, and enacting particular
pedagogies

Louise Hodgson

Dip. Teach. University of Tasmania

BEd (Early Childhood), University of Tasmania

MEd (Mathematics Education), Australian Catholic University

A thesis submitted for the degree of Doctor of Philosophy

at Monash University in 2019

Faculty of Education

Copyright Notice

© Louise Hodgson (2019)

Notice 1

Under the Copyright Act of 1968, this thesis must be used only under the normal conditions of scholarly fair dealing. In particular, no results or conclusions should be extracted from it, not should it be copied or closely paraphrased in whole or in part without the written consent of the author. Proper written acknowledgement should be made for any assistance obtained from this thesis.

Notice 2

I certify that I have made all reasonable efforts to secure copyright permissions for third-party content included in this thesis and have not knowingly added copyright content to my work without the owner's permission.

Abstract

This thesis tells the story of the experience of 18 teachers as they engaged in observing, deconstructing, and enacting pedagogies associated with developing students' reasoning and problem-solving. The topic in focus was modelling particular pedagogies that were different from the teachers' usual practice. In Australia, teachers are encouraged to teach mathematics in ways that foster the development of increasingly sophisticated student reasoning, problem-solving, mathematical understanding, and fluency. These are the actions students are expected to undertake as they engage effectively with the content of the Australian Curriculum: Mathematics (AC:M).

Despite this clear positioning in the curriculum, there seems to be infrequent teaching emphasis on problem-solving and a disproportionate focus on repetitious procedural tasks with low levels of complexity. Moreover, teacher education research has shown it is a challenge for teachers to understand pedagogies that promote student thinking and problem-solving and to create images of what those pedagogies look like in practice. It seems feasible that seeing particular pedagogies modelled and deconstructing those pedagogies might support teachers to create visions of teaching through the proficiency strands of the AC:M.

Drawing on both constructivist and situated perspectives, the current research project sought to explore the phenomenon of teacher learning through the observation of classroom modelling and the subsequent deconstruction of pedagogies and enactments. In this regard, the inquiry focused on the experiences of the participating teachers as they took part in the intervention. As part of this experience, I also focused the inquiry on myself in learning to be a teacher of the teachers. Using a variety of data sources, the study utilised qualitative methods to describe the phenomenon of teacher learning events and multiple case studies to elaborate the complexity of the experiences of observing teachers. The experiences of the teachers were analysed from a phenomenological perspective.

Preliminary data collection occurred over a 12-month period and included a survey designed to evaluate the teacher's experience of the observation. Data collection for the actual intervention occurred over a subsequent four-month period and included surveys and audio recorded conversations and interviews. Initially, data were collected from 162 teachers across schools and grades following their observation of a modelled lesson. The intention was to ascertain what teacher actions were observed and which of those actions the teachers intended to implement in their classrooms because of their observation. These insights guided and informed the development and design of an educational intervention.

The educational intervention consisted of a scaffolded and collaborative four-stage inquiry that consisted of two episodes of modelling particular pedagogies associated with developing students' problem-solving and reasoning. These pedagogies were chosen because, in my experience as a coach, I found that reasoning and problem-solving approaches were relatively unfamiliar to several teachers. These episodes of modelling were followed by co-teaching and enactment. Each stage was two hours' duration and included a 60-minute modelled lesson and pre- and post-lesson briefings with groups of teachers. Three to seven teachers of lower primary classes (Prep to Year 3) participated in each school. There was also a 60-minute planning meeting prior to co-teaching in Stage 3. Four schools were involved in the intervention with one iteration in each school.

The findings confirmed the potential and merit of modelling particular pedagogies to enhance mathematics teaching and learning by confronting teacher assumptions about teaching and learning and in doing so providing opportunities for teachers to learn, developing teachers' pedagogical reasoning, and building teachers' mathematical knowledge for teaching. In these ways, the educational intervention prepares teachers to teach mathematics through problem-solving, facilitate students' reasoning, and adjust mathematical tasks to cater for the diverse range of learners. However, the findings also highlighted the challenges and complexities for teachers associated with learning to anticipate students' responses to tasks and orchestrating mathematical discussions. There is no such thing as a "one size fits all" professional learning intervention for teachers, so the extent to which professional learning caters for differentiation for teachers can be pivotal.

Two recommendations to improve modelling as an approach to teacher learning are presented. The first is that the pre-modelled-lesson briefings include structured processes whereby teachers solve the modelled tasks themselves, anticipate students' strategies, and plan adjustments to the enabling and extending prompts. The second recommendation is that the post-lesson briefing includes an opportunity for the modeller to explain her pedagogical reasoning regarding the decisions she made during the modelling to respond to students' unanticipated reactions. The thesis closes by identifying opportunities for further research.

Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma at any university or equivalent institution and that, to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

Signature:

A large black rectangular redaction box covering the signature area.

Louise Hodgson

11 January 2019

Publications During Enrolment

Refereed Conference Papers

Hodgson, L. (2013). What teachers see when watching others teach. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics Education: Yesterday, Today and Tomorrow: Proceedings of the 36th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 386–393). Melbourne, Australia.

Hodgson, L. (2016). Collaboration around observation of teaching: Powerful professional learning. In B. White, M. Chinnappan, & S. Trenholm (Eds.), *Opening Up Mathematics Education Research: Proceedings of the 39th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 295–302). Adelaide, Australia.

Acknowledgements

I acknowledge the services of Pam Firth (Detail Devil Editing Services) who provided professional copyediting and formatting services according to the guidelines laid out in the university-endorsed national *Guidelines for Editing Research Theses* (Institute of Professional Editors, 2010).

I am particularly grateful to the guidance, extensive patience, and incisive insights of my supervisors Karina Wilkie and Peter Sullivan. In particular, I would like to thank Karina for her consistent and gentle encouragement, which prompted me to keep persisting when I had all but given up.

I would also like to sincerely thank the teachers and children that I worked with in both Catholic Education and the Department of Education Tasmania. Without them, this study would not have been possible.

I acknowledge my husband Charley who supported me through it all by reading my drafts and encouraging me to keep persevering during the many times when I felt like giving up and did give up for periods of time because I did not have faith in myself.

To my friends who listened and encouraged me to keep persisting, I thank you sincerely.

To the Salesian fathers who demonstrated loving kindness and caring to all children during my schooling, I thank you for inspiring in me a sense of social justice and a desire to improve educational outcomes for all children and particularly the disadvantaged so that they have opportunities to reach their full potential.

Finally, I thank my family for their patience, support, and encouragement. Now we can all relax and smell the roses!

Our greatest glory is not in never falling, but in rising each time we fall. (Emerson)

TABLE OF CONTENTS

Copyright Notice	i
Abstract	ii
Declaration	iv
Publications During Enrolment	v
Acknowledgements	vi
Table of Contents	vii
List of Tables	xv
List of Figures	xv
List of Abbreviations	xvi
Chapter 1: Introduction	1
1.1 Setting the Scene.....	2
1.2 Formulating the Research Questions	5
1.3 Reasons for the Research	8
1.3.1 Fulfilment of Policy.....	8
1.3.2 Contribution to Research and Literature	8
1.4 Scope and Assumptions of This Study	9
1.5 Overview of the Study	10
Chapter 2: Literature Review	12
2.1 Part 1: Perspectives on Mathematics as a Domain	12
2.1.1 Differing Views of Mathematics	13
2.1.2 Perspectives on School Mathematics in Australia.....	14
2.1.3 Five Strands of Mathematical Proficiency	16
2.2 Part 2: Teaching Mathematics	19
2.2.1 Mathematical Knowledge for Teaching.....	19
2.2.2 Principles for Effective Teaching of Mathematics.....	21

2.2.2.1 An ethic of care.....	21
2.2.2.2 Arranging for learning.....	22
2.2.2.3 Building on students' thinking.....	24
2.2.2.4 Worthwhile mathematical tasks.....	25
2.2.2.5 Making connections.....	26
2.2.2.6 Assessment for learning.....	26
2.2.2.7 Mathematical communication.....	26
2.2.2.8 Mathematical language.....	27
2.2.2.9 Tools and representations.....	27
2.2.2.10 Teacher knowledge.....	27
2.3 Part 3: Learning to Teach Mathematics.....	28
2.3.1 Findings on Learning to Teach Mathematics.....	28
2.3.1.1 Apprenticeship of observation.....	28
2.3.1.2 Problem of enactment.....	29
2.3.1.3 Problem of complexity.....	30
2.3.2 Pedagogical Approach for Learning to Teach.....	31
2.3.2.1 Representations of practice.....	31
2.3.2.2 Decomposing practice.....	33
2.3.2.3 Approximating practice.....	34
2.3.3 Processes for Teacher Change.....	34
2.3.4 Situated Learning.....	36
2.3.5 Lesson Study.....	37
2.3.6 Modelling.....	39
2.3.6.1 Conflicting views on modelling for teacher learning.....	43
2.3.6.2 Processes for assisting teachers to interpret modelled teaching practice for enactment.....	44

2.4 Building on the Research Literature	47
Chapter 3: Preliminary Study on What Teachers See When Watching Others Teach ..	49
3.1 Research Aim and Questions for Preliminary Study	50
3.1.1 Research Context and Design.....	50
3.1.1.1 Modelling and observation processes	50
3.2 Analysis of Responses	51
3.3 Findings	52
3.4 Linking Findings to the Research Question.....	57
3.5 Moving Forward With the Research.....	58
Chapter 4: Methodology and Methods: Perspectives, Tools, and Techniques of the	59
Research.....	59
4.1 Methodology	59
4.1.1 Philosophical Worldview Proposed in This Study.....	59
4.1.2 Case Study Methodology	61
4.1.3 Choosing Multiple Case Study.....	63
4.1.4 Characteristics of Case Study Research	63
4.1.5 Steps Taken to Ensure Validity and Reliability	65
4.2 Research Methods.....	67
4.2.1 Research Participants and Contexts.....	67
4.2.1.1 The selection and characteristics of the research participants	68
4.2.1.2 Research Context	69
4.2.1.3 Differences between the school contexts.....	70
4.2.2 Data Collection Procedures	72
4.2.3 Modelling Processes in Each Intervention	76
4.2.3.1 Intervention’s focus on pedagogies for developing student reasoning and problem-solving	76
4.2.3.2 Four stages of each intervention	77

4.2.3.3 Stage 1: Modelling.....	78
4.2.3.4 Stage 2: Modelling in another class.....	79
4.2.3.5 Stage 3: Co-planning and co-teaching.....	80
4.2.3.6 Stage 4: Enactment	80
4.2.4 Analysis of Responses.....	81
4.2.5 My Processes of Reflexive Practice	82
4.2.5.1 Sources of data.....	83
4.2.5.2 Journal overview.....	83
4.2.5.3 Audio recorded professional conversations.....	84
4.2.5.4 Email correspondence.....	84
4.2.5.5 Field notes overview	85
4.2.5.6 Data analysis	85
4.2.6 Chapter Summary.....	86
Chapter 5: From Observation to Enactment	87
5.1 Research Aim and Questions for This Chapter.....	87
5.1.1 Research Context.....	87
5.1.2 Intervention.....	88
5.1.3 Preliminary Data Collection.....	88
5.2 Formulation of Meanings and Identification of Themes	89
5.3 Category 1: Challenging Teacher Assumptions	95
5.3.1 Theme: Challenging Ideas About Teaching.....	96
5.3.2 Theme: Uncertainty About Anticipated Student-Generated Responses to Tasks	98
5.3.3 Theme: Confronting Ideas About Catering for Diversity.....	99
5.3.4 Theme: Perceiving Disengagement of Some Students.....	104
5.4 Category 2: Teacher Learning From Observing	106

5.5 Category 3: Teacher Learning From Deconstructing Pedagogies	106
5.5.1 Theme: Choosing Cognitively Complex Tasks.....	106
5.5.2 Theme: Building Mathematical Knowledge for Teaching.....	107
5.5.3 Theme: Building Trust and Learning From Each Other	111
5.6 Category 4: Teacher Learning From Enactment.....	113
5.6.1 Theme: Trialling an Observed Lesson	113
5.6.2 Theme: Shifting Towards Student Centred Approaches to Teaching.....	115
5.7 Category 5: Stakeholder Reactions to the Intervention	118
5.8 Chapter Summary and Conclusion	118
5.8.1 Linking the Findings to the Research Questions.....	118
5.8.2 Moving Forward With the Research	121
Chapter 6: Seeing, Reflecting On, Talking About, and Enacting Pedagogies	123
6.1 Relevant Research Aim and Questions.....	123
6.1.1 Teachers' Perspectives on Effective Pedagogical Approaches.....	124
6.2 Formulation of Meaning and Identification of Themes.....	126
6.2.1 Common Themes From All Four Schools	127
6.2.2 Themes in This Chapter.....	128
6.3 Category 1: Challenging Teacher Assumptions	129
6.3.1 Theme: Confronting Expectations of Student Achievement.....	129
6.4 Category 2: Teacher Learning From Observing	131
6.4.1 Theme: Noticing Particular Pedagogies.....	131
6.4.2 Theme: Noticing and Uncovering the Complexity of Teaching	141
6.5 Category 3: Teacher Learning From Deconstructing Pedagogies.....	143
6.5.1 Theme: Corridor Conversations Engaging in Practice.....	143
6.5.2 Theme: Learning to Be Purposeful With Teaching.....	144
6.6 Category 4: Teacher Learning From Enactment.....	146

6.6.1	Theme: Enacting Particular Pedagogies	146
6.6.1.1	Complexity of enactment (a)	146
6.6.1.2	Assimilating particular pedagogies (b)	149
6.6.1.3	Enacting new pedagogies	150
6.7	Category 5: Stakeholder Reactions to the Intervention	155
6.7.1	Theme: Implementing the Intervention	155
6.7.2	Theme: Seeing, Deconstructing and Enacting Pedagogies	158
6.7.3	Theme: Engaging Students	160
6.7.4	Theme: Differentiating the Experience for the Teachers	161
6.7.5	Theme: Accountability	162
6.8	Chapter Summary and Conclusion	163
6.8.1	Linking Findings to the Research Questions	163
6.8.1.1	Teacher assumptions challenged	163
6.8.1.2	Reactions to intervention	165
	Chapter 7: Learning to Become a Teacher of Teachers.....	167
7.1	Research Aims and Question	167
7.1.1	Research Context	167
7.2	Analysis and Identification of Themes	167
7.3	Theme 1: Articulating a Focus for My Modelling	168
7.4	Theme 2: Balancing My Desire to Tell With Enabling Teachers to Think for Themselves	171
7.4.1	Learning to Engage Teachers in Anticipating Student Responses to Tasks	172
7.4.2	Learning to Refrain From Telling	176
7.5	Theme 3: Exposing My Practice Warts and All	179
7.5.1	Learning to Let Go of the Desire to Teach a Perfect Lesson	179
7.5.2	Providing Commentary Whilst Modelling	182

7.6 Theme 4: Mismatches Between Interpretations of Pedagogies	184
7.7 Theme 5: Building Trusting Relationships With the Teachers.....	185
7.8 Chapter Summary and Conclusion	189
7.8.1 Linking Findings to the Research Question	189
Chapter 8: Implications and Conclusions	191
8.1 Overall Findings and Implications for Teacher Professional Learning	191
8.1.1 Challenging Teacher Assumptions.....	191
8.1.2 Teacher Learning From Observing Modelling of Particular Pedagogies.....	193
8.1.3 Deconstructing Pedagogies	195
8.1.3.1 Anticipating student responses to tasks and adjusting prompts.....	195
8.1.3.2 Modelling messy practice	199
8.1.3.3 Collaborative Conversations.....	204
8.1.4 Enacting Pedagogies.....	205
8.1.5 Stakeholder Reactions to the Intervention.....	207
8.1.6 My Learning From the Experience.....	208
8.1.7 Limitations and Recommendations for Future Research	209
8.2 Conclusion	211
References.....	212
Appendices.....	222
Appendix 1: Modelled Lesson Request Proforma	223
Appendix 2: Pre-Modelled-Lesson Teacher Survey.....	224
Appendix 3: Mathematics Lesson Observation Tool.....	227
Appendix 4: Modelled Lesson Reflection Cycle 2	229
Appendix 5: Observation Focus for Enactment.....	230
Appendix 6: Modelled Lesson Intervention Exit Survey	231
Appendix 7: Themes: Raven Primary School.....	233

Appendix 8: Lesson Plan for Raven Primary School Grade 3.....	251
Appendix 9: Lesson Plan for Raven Primary School Grade Prep/1	256
Appendix 10: Lesson Plan for Raven Primary School Grade Prep/1	260
Appendix 11: Themes: Heron Primary School.....	265
Appendix 12: Themes: Magpie Primary School.....	287
Appendix 13: Themes: Swift Parrot Primary School	308
Appendix 14: Synthesised Themes All Schools	327
Appendix 15: Lesson Reflection Stage 1 Swift Parrot Primary School	355
Appendix 16: Reflections on my Modelling at Raven Primary Cycle One.....	359
Appendix 17: Reflections on Listening to Audio Recordings at HPS.....	360
Appendix 18: Lesson Plan for Swift Parrot Primary School Grade 6	361
Appendix 19: Lesson Reflection Swift Parrot Primary School	366
Appendix 20: Anticipating Strategies Proforma.....	367
Appendix 21: Reflection School B	368
Appendix 22: Lesson Plan for Swift Parrot Primary School Grade 2/3	369
Appendix 23: Lesson Reflection Stage 4 HPS	373
Appendix 24: Reflection of the Stage 3 Co-Planning at Heron Primary School.....	374
Appendix 25: Lesson Plan for Heron Primary School Grade Prep/1	377

LIST OF TABLES

Table 3.1 Categories of Responses With Illustrative Examples of Teacher Responses	53
Table 4.1 Overview of Participants, Schools, and Teaching Experience	69
Table 4.2 My School Index of Socio-Educational Advantage (ICSEA) Profile of the Four Project Primary Schools (2014)	70
Table 4.3 Summary of Correlation Between the Research Questions and the Collected Data (X Indicates the Type of Data Collected)	75
Table 5.1 Selected Theme Clusters With Their Related Formulated Meanings	91
Table 5.2 Categories Identified and Their Associated Themes	95
Table 6.1 Findings of the Pre-Intervention Survey	124
Table 6.2 School Requests for Pedagogical Focus of the Modelling	125
Table 6.3 Themes From Chapter 5 and Corresponding Confirmation From SPPS, MPS, and HPS	127
Table 6.4 New Themes That Emerged From the Cross-Case Analysis and Their Corresponding Categories	129
Table 6.5 Observed Pedagogies	133
Table 6.6 Reported Changes to Teaching Practice on Exit Surveys	151
Table 7.1 Themes That Emerged From My Self-Reflexive Analysis	168

LIST OF FIGURES

<i>Figure 8.1.</i> Pre-modelled-lesson briefing protocol	198
<i>Figure 8.2.</i> Post-modelled-lesson briefing protocol	203

LIST OF ABBREVIATIONS

ABS	Australian Bureau of Statistics
ACARA	Australian Curriculum and Assessment Reporting Authority
AP	assistant principal
CCK	common content knowledge
CTLM	Contemporary Teaching and Learning of Mathematics
DoE	Department of Education
HPS	Heron Primary School
ICSEA	Index of Socio-Educational Advantage
IMPG	interconnected model of professional growth
KCS	knowledge of content and students
KCT	knowledge of content and teaching
KH	knowledge at the horizon
MKT	mathematical knowledge for teaching
MPS	Magpie Primary School
NAPLAN	National Assessment Program—Literacy and Numeracy
NZNDP	New Zealand Numeracy Development Project
PCK	pedagogical content knowledge
RPS	Raven Primary School
SCK	specialised content knowledge
SMK	subject matter knowledge
SPPS	Swift Parrot Primary School

CHAPTER 1: INTRODUCTION

A lot of teachers rely on some sort of weekly drill and practice session and they very rarely have whole group sessions so to do this sort of maths . . . it is quite different . . . [We use] booklets made up with worksheets for each different day of the week and automatic response. (Ruth, Raven Primary School, April 2014)

This comment relates to a primary school teacher's reaction to her observation of a mathematics lesson that aimed to raise awareness of pedagogies associated with developing students' problem-solving and reasoning (Australian Curriculum and Assessment Reporting Authority [ACARA], 2016). The observation prompted the teacher to reflect on her current teaching practice, which seemed to be focused on student fluency and following rules to get answers quickly and accurately. This vignette highlights an issue in Australian classrooms where there seems to be little teaching emphasis on problem-solving and a disproportionate focus on procedures (Australian Academy of Science, 2015). These procedures have low levels of complexity, little connection to the real world, rarely involve reasoning, are repetitious, and focus on one answer (Hollingsworth, Lokan, & McCrae, 2003). This is having a detrimental effect nationally, as many students are disengaged from mathematics and fewer students are going into higher education in mathematics-related fields (Australian Academy of Science, 2015; Australian Mathematical Sciences Institute [AMSI], 2014; Sullivan, 2011). Moreover, the Australian Government Chief Scientist (2018) reported, "Australia risks a future without the specialised mathematical skills and the population-wide mathematical literacy that the nation requires" (p. 2).

One factor contributing to the focus on procedures is that many teachers rely on visions of practice from their own schooling in traditional classroom settings (Chapman 2012; Feiman-Nemser, 2001; Hammerness et al., 2005), which has a profound effect on what they bring to the task of teaching. This is because conventional and widespread teacher-led

instruction, which “tells” students what to do and how to think, is very different to instruction that supports student problem-solving and reasoning (Kisa & Stein, 2015). It suggests many teachers may not be aware of pedagogies that are focused on eliciting student problem-solving and reasoning. Related to this, classroom teaching that engages students in the active construction of knowledge is complex (Ball & Forzani, 2009; Hammerness et al., 2005). Each episode of instruction requires many moves and decisions requiring high levels of coordination (Ball, 2017). Consequently, it is a challenge to know and understand pedagogies that promote student thinking and problem-solving and to create images of what it could mean to teach them (Ball, 2017; Bass & Ball, 2014). This suggests providing visible opportunities for teachers to see and reflect on teaching in more interactional ways might raise levels of student thinking and reasoning in classrooms.

This study addressed this issue by exploring an approach to teacher education that involved modelling particular pedagogies that may illustrate to teachers what is possible for effective student learning, prompt them to reflect on their practice, and raise their awareness of ways that mathematics can be taught in their classrooms. It was explored in the context of a major challenge facing teachers of mathematics: addressing the diversity of student readiness within each class. This chapter begins with a brief overview of the context in which the study took place, previous research addressing the impact of classroom modelling on teacher learning, and the background to this study. The central research questions are presented and reasons and scope for the study considered. The chapter closes with an overview of the thesis.

1.1 Setting the Scene

A major goal of the Australian Curriculum: Mathematics (AC:M) is to prepare students to be lifelong learners who have enhanced employment prospects and can engage fully in modern life and democratic processes (Australian Academy of Science, 2015; ACARA,

2016). This suggests classroom experiences that foster the kind of problem-solving and thinking required in daily life are necessary to achieve that goal (Putnam & Borko, 2000). The assumption is that an improvement in teaching practice will improve students' mathematics learning. However, changes to teaching practice require much effort and can be difficult without guidance and support (Ball & Forzani, 2009; Borko, 2004).

The recent emphasis on practice-based teacher learning (Ball, Ben-Peretz, & Cohen, 2014; Ball & Forzani, 2009; Bass & Ball, 2014; Borko, Jacobs, Eiteljorg, & Pittman, 2008; Naik & Ball, 2014; Timperley, 2015) suggests that professional learning situated in teachers' classroom contexts may support them to enact new pedagogies. Given the popularity of Japanese lesson study (Ebaegu & Stephens, 2014; Groves & Doig, 2014; Lewis & Perry, 2014; Takahashi & McDougal, 2015; Widjaja, Vale, Groves, & Doig, 2017), it seems possible that classroom-based professional learning may be manageable and productive in other contexts, but further research is needed. The notion of *lesson study* is centred on the examination of teaching practice and involves a group of teachers planning a lesson, with one of them teaching the lesson and then all of them reviewing the lesson and planning the next cycle (Fernandez, 2005; Lewis, Perry, Hurd, & O'Connell, 2006).

There have been suggestions that classroom-based professional learning could include experienced practitioners observing less-experienced teachers and giving feedback. For example, the Australian Institute for Teaching and School Leadership (AITSL, 2012) mandated observation of teaching practice and feedback for each teacher as part of performance reviews. However, Sullivan (2011) highlighted that observation of teaching practice runs counter to our Australian teaching culture where many teachers consider their classroom private.

A number of studies have shown that video representations of practice may be helpful in supporting teachers to learn from others (e.g., Borko et al., 2008; Putnam & Borko, 2000).

The claim is that teachers will learn from authentic representations of practice that they can reflect on (Borko, 2004). However, video representations seem to be limited to interactions between the teacher and students (Naik & Ball, 2012). Collaborations with colleagues and curriculum materials that shaped the planning may not be observed (Grossman, 2011). Furthermore, Lefstein and Snell (2014) cautioned the use of video recordings of “best practice”, which can distort the complexity of teaching and make it look easy.

Whilst observing a modelled lesson in real time is much less common (Naik & Ball, 2014), a small number of studies have suggested that observing pedagogies modelled by a more experienced teacher might be a first step in assisting teachers to visualise teaching in more interactional ways (Clarke et al., 2013; Grierson & Gallagher, 2009; Higgins & Parsons, 2011). A modelled lesson is a professional learning strategy designed to support teachers to improve instructional practice and outcomes for diverse learners. Teachers gather together to observe instructional practice, which is usually modelled by a more experienced teacher and situated within a real classroom. The aim is to make new knowledge, skills, and pedagogy explicit to teachers and manageable within their practice contexts. The focus of the observation may include lesson structure, pedagogy, teacher actions, and student learning (Bruce, Ross, Flynn, & McPherson, 2009).

The literature focusing on modelled lessons in mathematics is limited. Few studies have been described, and the body of literature that exists seems to focus on disciplines other than mathematics. However, a small number of recent studies have suggested modelled lessons support teachers to see how effective teachers enact particular teaching actions and learning principles in mathematics (e.g., Clarke et al. 2013; Higgins & Parsons, 2011). Some studies (see Bruce et al., 2009; Grierson & Gallagher, 2009) have suggested this contextualised experience for teachers is a catalyst to enable them to reflect upon their own practice and make changes. Moreover, Casey (2011) suggested that, as teachers try out new pedagogical

approaches to teaching, they crave modelled lessons so that they can see the contextualised approach in action and how their students might respond. Casey also argued for the importance of modelling to a group of teachers, rather than to just one teacher, as an effective way of building capacity and collaboration among them. Nevertheless, there is little empirical evidence that modelling is likely to support teachers to change their practice in a sustainable way.

There have been suggestions in preservice teacher education that seeing representations of practice, such as a modelled lesson, is insufficient to guarantee learning (see, for example, Grossman et al., 2009). It seems important to deconstruct the representation to enable it to be studied and provide opportunities for novices to practise, and not just observe, new pedagogies (Boerst, Sleep, Ball, & Bass, 2011; Grossman et al., 2009). It seems feasible that this may also apply to practising teachers as novices in learning about particular pedagogical approaches to teaching that support students' problem-solving and reasoning.

Some studies (e.g., Polly & Hannafin, 2011; Resnick, Spillane, Goldman, & Rangel, 2010) have suggested that classroom practice may be improved by implementing a scaffolded approach to teacher learning that gradually releases responsibility from a more experienced teacher to a less-experienced teacher. Similarly, in a study focused on preparing preservice teachers to lead mathematical discussions, Boerst et al. (2011) noted that the complexity of teaching was reduced by the amount of scaffolding that was provided.

This suggests observing, deconstructing, and approximating practice when executed together through a scaffolded approach may deliberately focus the professional learning on the enactment of high-quality teaching practice. Here lies the imperative for further research.

1.2 Formulating the Research Questions

The impetus for this research stemmed from my work of over 30 years as a teacher and teacher leader across organisations, geographical regions, schools, and grades (Kindergarten

to Year 10) in Tasmania. My experience of working with teachers resonates with this research, in that classroom experiences that foster the kind of problem-solving and thinking required in daily life are not commonplace for many students currently taught in mainstream schools.

My role as Numeracy Education Officer in Catholic Education Tasmania from 2005 to 2013 involved designing and implementing curriculum support and professional learning initiatives for teachers in the 37 schools across the state. In this role, I provided in-school support through coaching and collaborative planning. As part of the coaching role, I conducted lesson observations, but it seemed to me that the teachers found the observations stressful. The observations seemed to alienate teachers rather than build a climate of trust, which is necessary to promote teacher learning. As a way of building trust, trying to support teachers, and in response to their requests, I began to model pedagogies in their classrooms. The teachers seemed to value this and reported the experience gave them confidence to try new ideas and affirm their practice. Despite the positive reports made by teachers, I was left wondering what teachers enacted because of my modelling, whether they changed their practice, and what longer term changes they made, if any. This study aimed to explore these issues.

Essentially, this study sought to explore the processes of modelling and deconstructing pedagogies as a means of professional learning for teachers and to gain insights into the experiences of teachers who participated in the modelling and any subsequent impact on their knowledge and practice.

The aim of this research was to explore the relationship between modelling pedagogies and teacher learning.

The subsidiary aims were as follows:

1. To explore the elements of modelling likely to lead to changes in teacher practice;
2. To explore how modelling pedagogies, deconstruction of those pedagogies, and enactment may help primary school teachers build their mathematical knowledge for teaching;
3. To scrutinise my pedagogy in working with the teachers to
 - a. improve my practice;
 - b. work effectively with teachers; and
 - c. develop new understandings to contribute to the knowledge base of educating teachers.

The overarching research question that informed the study was “What is the relationship between modelling pedagogies and teacher learning about mathematics teaching?”

The subsidiary questions were as follows:

1. What teacher assumptions are challenged through lesson observation?
2. What is the nature of teacher learning from lesson observation?
3. Can an educational intervention involving modelling pedagogies, deconstruction of those pedagogies, and enactment improve mathematics teaching and learning? If so, in what ways?
4. What do teachers and principals say about modelling pedagogies, deconstruction of those pedagogies, and enactment as a professional learning strategy? What explanations do they give?
5. What is the nature of my learning from the experience as a teacher of teachers?

1.3 Reasons for the Research

1.3.1 Fulfilment of Policy

The Australian Academy of Science (2015) and the National Council of Teachers of Mathematics (NCTM, 2014) in the United States identified that strengthening teachers' mathematics pedagogical content knowledge is critical for teacher learning. Furthermore, the Australian Government Department of Education and Training (2018) report of the review to achieve educational excellence in schools recommended that the growth of contemporary pedagogy be accelerated "through the use of collaboration, mentoring, observation and feedback . . . by incorporating these practices into the core role of teachers and creating the conditions to enable teachers to engage in them" (p. xiii). This study sought to contribute to contemporary research that is focused on improving teacher pedagogy in teaching mathematics using a collaborative approach to teacher professional learning.

The Department of Education (DoE) Tasmania's (2018) strategic plan (2018 – 2021) goal for numeracy is for learners to "have the skills and confidence in . . . Numeracy to successfully participate in learning, life and work" (p. 1). This study sought to articulate how the modelling of pedagogies might contribute to improved practice. In this way, teachers may enact pedagogies consistent with enabling students to develop the problem-solving and thinking required for daily life.

1.3.2 Contribution to Research and Literature

This study sought to contribute to the emerging research on modelling as an approach to teacher learning. It focused particularly on researching the actions teachers chose to take and did take after observing and deconstructing the modelled pedagogies, as well as pursuing the challenges teachers faced in enacting pedagogies that were different from their usual practice. Insights on important elements of pre- and post-modelled-lesson discussions will contribute

to the literature related to the stance and role of the modelling teacher in pre- and post-lesson discussions and during the modelling itself.

1.4 Scope and Assumptions of This Study

One of the key underpinnings of the research was the model describing different types of mathematical knowledge for teaching that was developed by Ball, Thames, and Phelps (2008). This is elaborated in Chapter 2. This research was also underpinned by social constructivist and situated perspectives of teaching and learning, also elaborated in Chapter 2.

Although the findings of this study may be relevant to other teacher learning contexts, its focus was the experiences of 18 Tasmanian lower primary teachers (Prep to Year 3, students aged 5 to 9) as they observed the modelling and participated in the subsequent deconstruction, planning, and enactment of new pedagogies. The data collection was situated in four (government) primary schools in a rural and regional community in North West Tasmania. Prior to the intervention, the pedagogical approaches used by at least half the group of teachers were found to consist of demonstration and practice (as illustrated in the vignette on the first page of this thesis), whilst just under half the teachers reported they taught in ability groupings (see Table 6.1). The subsequent experiences beyond three months and the longer term outcomes of the participating teachers were outside the scope of this study.

Whilst this study's modelling approach to teacher learning shares some similarities with lesson study (Fernandez, 2005; Lewis et al., 2006), a direct comparison of these two approaches to professional learning was not made within the scope of this research.

The study explored qualitatively the experiences of teachers as they collaborated to observe, deconstruct, and enact particular pedagogies. Comparative evaluation of teacher effectiveness was not a key aspect of this research as it may be in quantitative studies.

Whilst there may have been other unanticipated outcomes to the study, such as changes in the social dynamics of the participants' teaching teams, these were not the focus of the research.

Four primary schools were selected by the DoE's regional director in North West Tasmania to be part of the project. The criterion for selection was that a large percentage of students at each school were assessed as below the benchmarks identified in the National Assessment Program—Literacy and Numeracy (NAPLAN) testing at Grades 3 and 5. This test is an annual assessment of all students in Australia in Years 3, 5, 7, and 9. The numeracy test assesses skills necessary for every child to progress through school and life. The students were therefore considered as needing intervention.

The following assumptions were applicable to the research undertaken:

1. The participating teachers were willing participants.
2. The teachers understood my confidentiality agreement with them, and they had the freedom to present their perspectives honestly and openly without fear of reprisal or inappropriate triangulation.
3. The teachers communicated their thoughts, experiences, and perceptions as accurately as possible.

1.5 Overview of the Study

To achieve the stated purpose of the study, in Chapter 2, I examine the related literature on the findings of international and local research studies about the nature of mathematics as it relates to problem-solving and reasoning and its role and place in the AC:M. Next, I examine research into learning to teach mathematics, with an emphasis on approaches to professional learning for teachers situated in authentic contexts. I overview the studies found in the literature related to modelling then draw together the insights of the literature review and explain how they informed the approach to the current research project.

In Chapter 3, I discuss findings from preliminary research conducted in several schools in the Tasmanian Catholic sector in 2012, which describe teacher observations of modelled lessons. The findings contributed to the design of this study by suggesting that classroom modelling may be a powerful form of teacher learning.

In Chapter 4, I present the methodology and methods utilised in this study. Using a variety of data sources, the study utilised qualitative methods to describe the phenomenon of teacher learning events and multiple case studies to elaborate the complexity of the experiences of observing teachers. This chapter expands on the idea of this research as being interpretive in nature, detailing the research processes. It describes the steps taken to enhance the validity and trustworthiness of the research findings.

To capture the essence of the data analysis process, Chapter 5 contains an in-depth description and analysis of the first case (Raven Primary School) and includes the story of Rose (pseudonym), an experienced teacher with more than 15 years' teaching experience. During the intervention, Rose moved through the phases of being confident to losing confidence then regaining her confidence by transforming her teaching practice to enact new pedagogies.

In Chapter 6, I describe the cross-case analysis of all cases involved in the study. The cross-case analysis confirms findings that were presented in Chapter 5 and includes new themes that emerged.

In Chapter 7, I present my story in learning to become a teacher of teachers through inquiring into my experiences of modelling during collaboration with colleague teachers in this study.

In Chapter 8, I conclude the study with a summary of key findings and matching implications, limitations of the study, and suggestions for further research.

CHAPTER 2: LITERATURE REVIEW

This chapter outlines theoretical perspectives on the nature of mathematics, learning mathematics, teaching mathematics, and learning to teach mathematics. Part 1 explores perspectives on mathematics as a domain and includes differing views of mathematics and Australian perspectives of school mathematics. Next, it considers the five strands of mathematical proficiency. Part 2 starts with a discussion of how mathematics is learned and follows with the mathematical knowledge that teachers require for teaching mathematics. Part 3 starts with findings on learning to teach mathematics and then discusses a pedagogical approach for learning to teach. It follows with a consideration of the processes for teacher change. Next, it considers situated learning and follows with a discussion of lesson study. The discussion then centres on modelling as an approach to teacher learning. The chapter closes by drawing together the insights of the literature review and explaining how they informed the approach to the current research project.

2.1 Part 1: Perspectives on Mathematics as a Domain

This section establishes the perspective on the nature of mathematics chosen for this study. It begins with a discussion of differing views of mathematics and follows with a review of Australian perspectives on school mathematics. Next it considers the proficiency strands as defined in the AC:M and based on Kilpatrick, Swafford, and Findell's (2001) five strands of mathematical proficiency. Kilpatrick and his colleagues (2001) adopted the term *mathematical proficiency* to encapsulate what they considered essential for people to learn mathematics successfully. This follows with a discussion of the mathematical knowledge required for teaching. It concludes with a discussion of principles for effective teaching of mathematics.

2.1.1 Differing Views of Mathematics

Numerous scholars have argued that conceptions of the nature of mathematics held by individuals and society have a major effect on the development of school mathematics curriculum and instruction (e.g., Dossey, 1992; Ernest, 1991). Such views form the basis of a specific philosophy of mathematics and can be explicitly articulated or implicitly held (Ernest, 1991). One of the central issues is that these philosophies “contain—often in an implicit way—ideas, orientations or gems for theories on the teaching and learning of mathematics” (Ernest, 2004, p. 8). Ernest (1991) described two contrasting philosophical views of mathematics, both encompassing a range of perspectives. The first is the *absolutist* view. In this interpretation, mathematics is viewed as an objective fixed body of knowledge, which rests on logic and certainty and is inhuman. The alternative view is the *fallibilist* philosophy. Fallibilism rejects the idea that mathematics is fixed. Instead, mathematics is viewed as the result of social processes. It is regarded as a dynamic and expanding field of human enquiry, which is open to review.

The different philosophical views of mathematics have implications for how teachers teach mathematics (Ernest, 2004). For example, the absolutist view may lead to students being given routine mathematical tasks that require use of learnt procedures. In contrast, a fallibilist view may lead to teacher acceptance of students’ ideas and approaches to tasks. Associated with the fallibilist view is the social constructivist perspective (Ernest, 2008) on teaching and learning.

Social constructivists believe that mathematics is a human construction. In this view, learning is active, individual, and personal. It is based on previously constructed knowledge and takes place through interactions with others (Ernest, 2010a). Emphasis is placed on social context and interpersonal relationships, especially teacher–learner and learner–learner interactions including “negotiation, collaboration and discussion” (Ernest, 2010a, p. 46). This

view implies teaching mathematics cannot be about transmission where teachers fill students' heads with knowledge, but is rather an interactive event between the students, teacher, and task (Kisa & Stein, 2015). This view informed the content of this literature review. It has been argued, however, that the types of teacher–learner and learner–learner interactions highlighted are not typically employed in Australian schools where the emphasis seems to be on transmitting procedural knowledge (Australian Academy of Science, 2015).

2.1.2 Perspectives on School Mathematics in Australia

There are essentially two views about school mathematics in Australia (Sullivan, 2011): the practical and usable approach that prepares students for work and enhances their ability to function effectively in society and the specialist approach needed for those who will go on to study advanced mathematics (Ernest, 2010b). Included in the practical approach is the goal that students learn the types of calculations needed for everyday life, including budgeting, home repairs, travel routes, data interpretation in the media, time management, and so on. In Australia, the term “numeracy” is commonly used to encapsulate this perspective, whereas internationally it is commonly termed “mathematical literacy” (Stacey & Turner, 2015). Importantly, numeracy does not involve following rules to answer random exercises quickly, but rather has a greater emphasis on problem-solving and reasoning—elements that enable students to transfer their knowledge to other situations (Kilpatrick et al., 2001). The specialist perspective, which forms the foundation for advanced study in the engineering, science, and technology disciplines, includes the goals of posing and solving problems, valuing mathematics in culture, and the appreciation of mathematical ideas such as pattern, symmetry, and proof (Ernest, 2010b).

Whilst there is broad consensus among stakeholders that mathematics is an important component of the school curriculum, there is ongoing debate about which aspects of mathematics are important in the compulsory years of schooling (Sullivan, 2011). Part of the

reason for the debate is that many schools continue to be challenged by disengaged students (Grootenboer & Marshman, 2016). This has led to recommendations for reform including emphasising the need for more engaging and relevant mathematics tasks to prepare students for employment and their needs as citizens (Russell, Mackay, & Jane, 2003).

Additionally, there has been a decline in the number of students completing mathematics courses at university, and this threatens Australia's capacity for innovation and international competitiveness (Australian Academy of Science, 2015; Australian Government Chief Scientist, 2018; Rubenstein, 2009). Indeed, AMSI (2014) highlighted the impact that the deficit has on society, responding to global issues such as climate change and population health.

It has been suggested that more attention needs to be placed on practical and usable mathematics in schools. However, this must not be at the expense of introducing students to aspects of formal mathematics that lay the foundation for later study (Sullivan, 2011). Getting the balance right is a challenge for teachers in this age of uncertainty, where rapid technological change is increasingly blurring the distinction between the functional and specialist domains of mathematics (Askew, 2011). The AC:M (ACARA, 2016) sought to incorporate both perspectives. Whilst it is clear that appropriate priority be given to practical mathematics in the compulsory years, it seems the primary goal of school mathematics is to move beyond encouraging students to learn the mathematics that we think they will need in the future to convincing students that they *can* learn mathematics (Askew, 2011). It is hoped that this will motivate them to continue to learn and be equipped to solve mathematical problems as they arise throughout life (Askew, 2011), which may not have been envisaged.

Importantly, neither the practical nor the specialist approach seems to be taught well in Australian schools (Australian Academy of Science, 2015). For example, Hollingsworth et al. (2003) reported that the mathematical tasks provided for students in Australian schools had

low levels of complexity, little connection to the real world, rarely involved reasoning, were repetitious, and focused on one answer. Similarly, the Council of Australian Governments (COAG, 2008) argued,

That from the earliest years, greater emphasis be given to providing students with frequent exposure to higher-level mathematical problems rather than routine procedural tasks, in contexts of relevance to them, with increased opportunities for students to discuss alternative solutions and explain their thinking. (p. 31)

To connect both the practical and specialist approaches to teaching and learning mathematics, it seems that the focus should be on mathematical actions students undertake during learning (Australian Academy of Science, 2015). Kilpatrick et al.'s (2001) five strands of mathematical proficiency described the actions students demonstrate as they engage effectively in mathematical tasks.

2.1.3 Five Strands of Mathematical Proficiency

The five strands of mathematical proficiency capture the goals of mathematics learning (Kilpatrick et al., 2001). Kilpatrick and his colleagues (2001) drew on their experiences as learners and teachers of mathematics, as well as their analysis of cognitive psychology and mathematics education literature, to adopt a comprehensive view of what successful mathematics learning entails. Within the five strands, they described the kinds of cognitive changes that they wish to promote in children to become successful learners of mathematics. “The five strands are interwoven and interdependent in the development of proficiency in mathematics” (p. 116). These strands are summarised below.

1. *Conceptual understanding* involves “mental connections among mathematical facts, procedures and ideas” (Hiebert & Grouws, 2006, p. 382). It includes the ability to comprehend, apply, and adapt ideas flexibly to new situations and entails sense making within a community of learners (Hiebert & Grouws, 2006).

2. *Procedural fluency* includes knowledge of and the skill to carry out procedures accurately, efficiently, and flexibly.
3. *Strategic competence* includes the ability to pose, represent, and solve problems. Students with strategic competence can approach nonroutine problems flexibly and choose between methods for solving the problem to meet the demands of the problem and the context in which the problem was posed.
4. *Adaptive reasoning* includes the capacity to think logically, explain, and justify mathematical thinking and make it clear to others. It is “the glue that holds everything together, the lodestar that guides learning” (Kilpatrick et al., 2001, p. 129). Students demonstrate reasoning ability when they have an adequate knowledge base, the task is appealing and easily understood, and the context is familiar (Kilpatrick et al., 2001). Kilpatrick and his colleagues (2001) maintained that adaptive reasoning interconnects with the other strands of proficiency, mainly during problem-solving. Students utilise their strategic competence to represent problems by thinking for themselves, but adaptive reasoning takes over when they justify their strategy. Conceptual understanding provides mental images that can provide a source for adaptive reasoning. This enables students to determine if their solution is justifiable. Procedural fluency with calculations may be required, but adaptive reasoning is used to decide if the solution is suitable. Whilst solving the problem, students use strategic competence to monitor progress and to adjust if necessary. This approach depends on students having a productive disposition (Kilpatrick et al., 2001).
5. *Productive disposition* includes the inclination to see mathematics as useful and worthwhile and a belief that steady effort pays off. Whilst the teacher plays a critical role in fostering productive dispositions in their students, a student’s disposition towards learning mathematics is key to their educational achievement (Kilpatrick et al.,

2001). Moreover, students who view their mathematical ability as fixed are likely to avoid challenging tasks and become easily discouraged by failure, whereas students who see ability as expandable are more likely to seek out challenging learning opportunities (Kilpatrick et al., 2001).

The first four strands of mathematical proficiency (Kilpatrick et al., 2001) form the proficiencies of the AC:M (ACARA, 2016). The terms understanding, fluency, problem-solving, and reasoning are simplified for ease of communication, but they include the range of processes described in the five proficiencies, exemplifying the actions students undertake as they engage effectively with the content of the AC:M. These proficiency terms are used in the content descriptions and the achievement standards that are stipulated for students at each year level. The intention is to strengthen the importance of working mathematically within the content and explain how the content is explored (ACARA, 2016). In this study, the proficiency strands of reasoning and problem-solving provided a framework for analysing the participant teachers' observations during modelling. The participating teachers recorded the modeller's actions that they considered facilitated student reasoning and problem-solving.

As indicated previously, the five strands are interwoven and each needs to be developed simultaneously with the others (Kilpatrick et al., 2001). Yet, as this review has illustrated, the teaching emphasis on fluency seems to be disproportionate to the other proficiency strands in the Australian context.

In summary, this section of the review examined two different views on the nature of mathematics and of mathematics learning in Australia. Drawing on a social constructivist perspective, the argument is that the focus of school mathematics in the compulsory years should be on practical application to prepare students for work and life with some introduction to specialist mathematics. To experience such a curriculum requires a shift from the current emphasis on procedural knowledge (Australian Academy of Science, 2015;

COAG, 2008). This highlights the need to explore the mathematical knowledge that teachers need to have for the effective teaching of mathematics to address the gap between the desired and existing situation. This is reviewed in Part 2.

2.2 Part 2: Teaching Mathematics

This section presents perspectives on the nature and role of knowledge necessary for the effective teaching of mathematics and follows with a discussion of six principles that can guide effective teaching practice.

2.2.1 Mathematical Knowledge for Teaching

One of the key underpinnings of this review was a perspective on knowledge for mathematics teaching, which builds on Shulman's (1986) well-known notion of pedagogical content knowledge (PCK) and which can assist by providing a detailed description of the knowledge needed for effective teaching. Ball et al. (2008) proposed a model of mathematical knowledge for teaching (MKT) in which they described two categories: subject matter knowledge (SMK) and PCK.

Within the category of PCK, there are also three subsets: knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of curriculum. KCS is focused on teachers' understanding of how students think about and learn mathematical concepts. Knowledge of common preconceptions, partial conceptions, and misconceptions related to mathematical concepts is central to KCS. Effective teachers can "anticipate what students are likely to do with it [a task] and whether they will find it easy or hard" (Ball et al., 2008, p. 401). They have the skill to modify mathematical tasks or questions often "on the fly" to scaffold learning. This demands knowledge at the intersection of content and students. Teachers cannot effectively cater for the diverse range of students in their classes without this knowledge. This suggests it may be important for teachers to have

opportunities to anticipate potential explanations of students' thinking when planning for teaching.

It has been argued that the domain of KCS remains under conceptualised (Hill, Ball, & Schilling, 2008), and what constitutes such knowledge is not yet clear and needs further development. Hill et al. (2008) stressed the importance of classroom-based research, contributing ideas and measures of good teaching practice to this field of study.

KCT is the combined knowledge of effective teaching pedagogies and mathematics. This includes choice of representational models, sequence of tasks, and choice of questions to ask students to probe thinking and learning. For example, effective teachers know when to pose a new task to further students' learning.

Within the category of SMK, there are three subsets: *common content knowledge* (CCK), *knowledge at the horizon* (KH), and *specialised content knowledge* (SCK). The first subset (CCK) is described as the mathematical knowledge used in teaching and other professions. The second subset (KH) involves understanding how mathematical concepts are related and connected across the curriculum. The third subset (SCK) is delineated as the mathematical knowledge exclusive to teaching. This includes understanding and endorsement of diverse and intuitive approaches to solving a specific problem. Effective teaching necessitates knowledge of the principles underpinning the various approaches students take to solve specific problems.

The Ball et al. (2008) model of MKT was considered a useful conceptualisation of teacher knowledge on which to base this present study as it conceptualises several categories of knowledge required for effective mathematics teaching. In this study, I drew on these categories (Ball et al., 2008) when analysing different aspects of the teachers' learning as they experienced the intervention.

Whilst this model was considered useful for examining teacher learning in developing knowledge for teaching mathematics, further perspectives were sought in the literature about what effective day-to-day teaching practice might look like.

2.2.2 Principles for Effective Teaching of Mathematics

Anthony and Walshaw (2009) drew on a wide range of research into effective mathematics teaching to suggest 10 principles for effective teaching of mathematics that can guide everyday classroom practice. These principles are informed by beliefs that mathematics pedagogy must be inclusive of all students; acknowledge that all students can develop productive dispositions and become successful mathematics learners; be responsive to cultural heritages, students' thinking processes, and the realities of the classroom; be focused on developing students' mathematical proficiency; and be committed to improving social outcomes within the mathematics classroom that will lead to fruitful citizenship (Anthony & Walshaw, 2009). Such principles guided the modelling of the mathematics lessons analysed in this study. These are discussed in the following sections.

2.2.2.1 An ethic of care

Effective teachers develop classroom cultures with a fervent mathematical focus, clear learning goals and high expectations and levels of trust. A climate of safety ensures students can take intellectual risks and think for themselves (Anthony & Walshaw, 2009).

Importantly, the teacher needs clarity about the lesson's goal and communicate that goal to the students. Through communicating goals, students become clear about how they can participate in the lesson, the purpose of their participation, and the direction of the learning. This purposefully prepares students for their learning (Anthony & Walshaw, 2009; Edwards-Groves, Anstey, & Bull, 2014). However, teachers tend to overlook the mathematical goal of a lesson or task and can have difficulty maintaining mathematical focus once the lesson is underway (Sleep, 2012).

Smith and Stein (2011) argued that having clear learning goals enables teachers to recognise what counts as evidence of students' learning, how students' learning can be linked to specific instructional tasks, and how to revise instruction to enable students to learn more effectively. However, Askew (2011) cautioned teachers about communicating general learning goals that do not explicitly engage students in thinking about what they are learning (e.g., "Today we are learning about problem-solving").

2.2.2.2 Arranging for learning

Effective teachers ensure students have opportunities to engage with mathematical tasks both independently and in collaboration with others to develop sense making. Independent thinking time is important for students to work on problems quietly by themselves, free of the distraction of occasional conflicting viewpoints of others (Anthony & Walshaw, 2009). Conversely, working in small groups can provide responsive and concrete support to enhance student engagement, clarify ideas, encourage higher order thinking, and increase understandings (Anthony & Walshaw, 2009).

Importantly, whole-class discussions are critical for students to develop mathematical reasoning (Anthony & Walshaw, 2009). To promote substantial class discussions about mathematics, a three-phase lesson format is recommended (Smith & Stein, 2011) to enable the teacher to launch the problem, provide time for the students to solve the problem by thinking for themselves, and engage in a whole-class discussion and summary of the problem. The three phases are summarised as follows.

Launch: The teacher presents the problem to the students, the materials that are available for working on it, and the expectations of how students will participate in solving it.

Explore: Students work on the problem, individually at first and then in pairs or small groups. As students work on the problem, they are encouraged to think for themselves, solve the problem in any way that makes sense to them, and be prepared to explain their approach

to other class members. The teacher circulates, probes student thinking, and selects specific students to present their solutions. The selection is guided by the contribution the solution will make towards the mathematical goal of the lesson (Smith & Stein, 2011).

Summarise: Student-generated approaches to solving the task are presented. The work is ordered in such a way as to create a coherent storyline and make the mathematics accessible to all students (Smith & Stein, 2011). A whole-class discussion about the mathematics ensues. The teacher crafts questions to make the mathematics visible, focusing on mathematical meaning and relationships and making links between student solutions and representations (Smith & Stein, 2011).

Studies have found that the summary phase of a lesson is difficult for teachers to orchestrate (Ball, 2017; Smith & Stein, 2011). Australian teachers infrequently practise the summary phase of the three-part lesson structure (Hollingsworth et al., 2003). This is possibly because it is hard for teachers to align the different approaches that students generate in response to challenging tasks with the learning goals of the lesson (Smith & Stein, 2011). A key decision is choosing whose work will launch the discussion (Ball, 2017). The teacher's decision involves thinking about the mathematics that the children are working on and whose examples are key to the instructional goal of the lesson and will advance the thinking of the class (Ball, 2017). If the teacher does not do this, classroom discussions can become a series of "show and tell" demonstrations that are treated equally and consequently can become disconnected from the mathematical ideas that are the goal of the lesson (Smith & Stein, 2011). Another key aspect of the lesson summary is connecting students' solutions with the goal of the lesson (Smith & Stein, 2011). This highlights the importance of providing opportunities for teachers to learn how to draw connections between students' solutions and summarise the key mathematical ideas of a lesson.

2.2.2.3 Building on students' thinking

Effective teachers plan purposeful lessons that build on students' existing knowledge and differentiate instruction according to individual learning needs. This includes anticipating student solutions to problems and choosing appropriate representations or models to support student understanding. Teachers indicate their high but reasonable expectations to the students by providing appropriate challenge (Anthony & Walshaw, 2009). This makes it possible for students to engage in productive struggle to make sense of the mathematical ideas and construct new connections. The term "productive struggle" does not imply needless frustration. It refers to students working on tasks just beyond their current level of knowledge and understanding (Hiebert & Grouws, 2006).

Students are more likely to feel included and successful if teachers have enabling prompts on hand to allow those who have trouble to access a simpler task related to the goal task (Sullivan et al. 2015; Sullivan, Mousley, & Jorgensen, 2009; Sullivan, Zevenbergen, & Mousley, 2006). The intent is that the students will proceed to the learning tasks after they have experienced success with the enabling prompt (Sullivan, Borcek, Walker, & Rennie, 2016). Enabling prompts can involve slightly varying an aspect of the task demand, such as the form of representation, the size of the numbers, or the number of steps (Sullivan et al., 2006). Teachers benefit because they have an alternative strategy to telling these students what to do (Sullivan et al., 2016).

Likewise, those who complete the learning task quickly can proceed to an *extending prompt* to challenge their thinking and if possible elicit generalisation of the solution. The intention is that the prompts extend students' thinking on an aspect of the learning task (Sullivan et al., 2015; Sullivan et al., 2006). The basis for enabling and extending prompts is that the class progresses together as a community of learners, contributing to the sense making of the whole class (Sullivan et al., 2006).

2.2.2.4 Worthwhile mathematical tasks

Student opportunities to engage in problem-solving and reasoning in cognitively complex ways are embedded in challenging tasks (Anthony & Walshaw, 2009; Smith & Stein, 2011; Sullivan et al., 2016). Furthermore, Smith and Stein (2011) maintained that productive discussions highlighting key mathematical ideas are unlikely to occur if the task on which students are working requires limited thinking and reasoning. Similarly, Sullivan et al. (2016) argued for a “challenging tasks” approach to mathematics classroom pedagogy in which students are encouraged to find solutions to problems by thinking for themselves before instruction from the teacher. They maintained this approach is essential for students to develop problem-solving and reasoning.

The selection of tasks, however, does not guarantee that students will engage in high-level thinking and reasoning (Kisa & Stein, 2015). Maintaining high levels of cognitive demand requires teachers to listen actively with the intent of understanding the mathematical sense that students are making and then steer the lesson in response. Kisa and Stein (2015) argued that maintaining high-level thinking connected with challenging tasks requires

teachers to weave a lesson cloth comprised of three threads: the cognitive demands of the task, how students are responding to and thinking about the task, and their own actions with and responses to students . . . It is this interaction of the task, teacher and students that determines the nature of the opportunities students have to think and reason in the classroom. (p. 107)

Similarly, Smith and Stein (2011) explained that teacher actions that assist in maintaining the challenge of tasks include providing extended thinking time, circulating around the classroom and probing thinking, expecting students to justify their solutions, and connecting students’ solutions with key mathematical ideas.

2.2.2.5 Making connections

This stresses the importance of building on and connecting with students' prior knowledge and using engaging and contextualised tasks to foster connections between mathematical ideas, thereby building conceptual understanding (Anthony & Walshaw, 2009; Hiebert & Grouws, 2006). Moreover, making mathematical connections explicit and public promotes conceptual understanding for example, asking questions that focus on the mathematical meaning and relationships and links between student solutions and their representations (Anthony & Walshaw, 2009; Hiebert & Grouws, 2006).

2.2.2.6 Assessment for learning

Effective teachers monitor their students' learning and continuously assess student progress both informally and formally to make mindful teaching decisions (Anthony & Walshaw, 2009). As they circulate during the lesson, such teachers probe student thinking and understanding by asking questions to assess student knowledge and explore their thinking (Anthony & Walshaw, 2009). By assessing students moment by moment, effective teachers make decisions about how to steer the lesson, what questions to ask, and what to focus on in the class discussion (Anthony & Walshaw, 2009; Sleep, 2012; Smith & Stein, 2011).

2.2.2.7 Mathematical communication

Effective teachers press for student explanations. They expect students to explain their reasoning and justify their solutions (Anthony & Walshaw, 2009; Sleep, 2012; Smith & Stein, 2011). Student attempts to make conjectures, voice disagreements, and suggest counterarguments are scaffolded. Specific teacher actions that support this scaffolding include revoicing (Askew, 2012; Smith & Stein, 2011) and interjecting at the point when student explanations are unclear (Askew, 2012), prompting students to add to the discussion, for example, "Would someone like to add to that", and having students restate another's reasoning (Smith & Stein, 2011).

2.2.2.8 Mathematical language

Effective teachers model mathematical language and explain terms in ways that make sense to students (Anthony & Walshaw, 2009; Sleep, 2012). Sleep (2012) maintained the intentional repetition or overuse of mathematical vocabulary is crucial for emphasising and developing key mathematical ideas being focused on in a lesson. Further to this, expecting students to use mathematical language increases student reasoning (Sleep, 2012).

2.2.2.9 Tools and representations

Tools and representations are carefully selected by effective teachers to support students' thinking (Anthony & Walshaw, 2009). One important tool for making students' thinking easily visible in a classroom context is a document camera. For example, Sleep (2012) maintained it is important for all class members to see student examples chosen to bring the mathematics into the open for examination; otherwise, meaning can be lost.

2.2.2.10 Teacher knowledge

The planning and organisation of a lesson is determined by teacher knowledge of effective pedagogy and knowledge of content and how to put that knowledge into practice (Anthony & Walshaw, 2009; Ball et al., 2008).

In this study, the 10 principles described informed my modelling and the recording of teacher actions, which facilitated student reasoning. In the design of my modelled lessons, I included the launch, explore, and summarise lesson structure described in section 2.2.2.2 and modelled pedagogies including posing challenging tasks with enabling and extending prompts, providing extended thinking time, having high expectations, and expecting students to explain and justify their reasoning. In Part 3 of this review, the literature on learning to teach mathematics is examined.

2.3 Part 3: Learning to Teach Mathematics

This section starts with a discussion of the research findings on challenges in learning to teach mathematics and follows with a pedagogical approach for learning to teach. Next, it considers the context in which learning takes place. This follows with a discussion of lesson study as an approach for teacher learning. Next is an examination of the literature on teacher learning through modelled lessons. This section concludes by outlining the implications for the thesis.

2.3.1 Findings on Learning to Teach Mathematics

One of the ongoing themes in mathematics teacher education literature is the challenges teachers have in learning to teach. For example, in a discussion of how teachers learn and develop, Hammerness et al. (2005) highlighted three commonly recognised problems in learning to teach: the apprenticeship of observation, the problem of enactment, and the complexity of teaching. Each of these is examined below.

2.3.1.1 Apprenticeship of observation

Learning to teach requires teachers to think about teaching in new ways, which perhaps are different from their preconceptions about teaching and learning that they may have experienced as a student in traditional classroom settings (Chapman, 2012; Feiman-Nemser, 2001; Hammerness et al., 2005; Kisa & Stein, 2015; Smith, 2001). Lortie (1975) called this experience of being a student “the apprenticeship of observation”, which has a profound effect on what teachers bring to the task of teaching. Similarly, Erickson (2011) argued that what teachers observe or attend to is determined by their “pedagogical commitments”, which are highly influenced by their own prior experience of being taught. For example, teachers may think classroom order must be in place before learning can occur.

Importantly, instruction that supports student thinking and reasoning is different from conventional and widespread teacher-led instruction, which “tells” students what to do and how to think:

Teachers must shift their vision of teaching from a solo endeavour to an interactional event among their own teaching actions, students’ thinking and the nature of the task they selected . . . In the absence of such a shift in how teaching is viewed, teachers’ practice will continue to consist of telling and their students will continue to repeat and practice what they hear. (Kisa & Stein, 2015, p. 108)

This suggests supporting teachers to see teaching in more interactional ways is critical for raising levels of student thinking and reasoning in classrooms.

Similarly, Hammerness et al. (2015) maintained that developing a vision is the first step in addressing the apprenticeship of observation: “Teachers need to have a sense of where they are going and how they are going to get students there” (p. 385). Such a vision incorporates strong images of good practice can assist teachers to plan and reflect on their work and guide their learning. This was an important finding for this study as it suggests seeing effective practice modelled might potentially support teachers to enact new pedagogies. First, teachers need to know how to put what they know into action (Hammerness et al., 2005).

2.3.1.2 Problem of enactment

Hammerness et al. (2005) argued that teachers not only have to learn new knowledge, but they also have to know how to put their new knowledge into practice. This is known as “the problem of enactment” (Kennedy, 1999). Similarly, Bronkhorst, Meijer, Koster, and Vermunt (2011) argued,

Successful enactment requires the formulation of teaching intentions based on teaching knowledge coupled with procedural knowledge of how to use that knowledge in action and the chance to do so in practice. (p. 1120)

Whilst Bronkhorst et al.'s (2011) study was focused on teacher educator modelling to preservice teachers, it seems feasible that this may also be applied to practising teachers whilst they are learning new pedagogical approaches to teaching. To illustrate the problem of enactment, Polly and Hannafin (2011) examined the extent to which two primary school teachers enacted new pedagogies after participating in a 12-month professional development project, consisting of 48 hours of workshops, which focused on communication, rich mathematical tasks, and higher order questioning. The two selected teachers had indicated before the study began their intent to apply learned pedagogies in their classrooms. During workshops, project staff modelled instructional practices whilst the teachers participated as learners. Data were collected and analysed through observation and video clips of the two teachers engaging in workshops and teaching in their classrooms. Additional data were gained through teacher interviews. Polly and Hannafin (2011) found the two teachers believed that they were enacting their newly espoused pedagogies, but the analysis revealed this was not the case. For example, posing higher order questions was found to be of greatest difficulty for the teachers, even when project staff had modelled techniques in workshops repeatedly. One teacher reported, "Seeing the project staff model . . . allowed me to pose good questions to my students" (Polly & Hannafin, 2011, p. 127). This confirms that some approaches to teacher professional learning may not necessarily lead to enactment (Hammerness et al., 2005).

2.3.1.3 Problem of complexity

Another issue is the "problem of complexity" (Ball, 2017; Ball & Forzani, 2009; Hammerness et al., 2005). Teachers are required to juggle a multitude of student learning needs and academic goals that are impacted by social issues and unexpected events from day to day and moment to moment (Hammerness et al., 2005). As Ball and Forzani (2009) argued, the work of effective classroom teaching is unnatural in that it goes beyond showing

and telling. Such teaching requires a suspension of self with an orientation towards others focused on enabling others to think, learn, and do. Each episode of instruction requires many moves and decisions requiring high levels of coordination (Ball & Forzani, 2009). This illustrates the complexity of teaching.

Grossman and her colleagues (2009) developed a pedagogical approach for describing the pedagogy of learning to teach comprising of representations, decomposition, and approximations, which is discussed in Section 2.3.2. They argued novice teachers not only need to see images of effective practice, but they also need to deconstruct practice and attempt to try out new practices to capture new ideas.

2.3.2 Pedagogical Approach for Learning to Teach

Drawing on sociocultural theory and the literature on learning from experience, Grossman et al. (2009) used a comparative method to explore the teaching of practice in specific professions outside the education sector. The methodology included qualitative case studies and interviews. Data analysis focused on commonalities and features pertinent to each profession that helped novices prepare for practice in their own field. These features were then studied to determine their purpose in professional education. The notions of *representation*, *decomposition*, and *approximation* were conceptualised as underpinning common pathways to teaching practice across the professions that were studied. Grossman et al.'s (2009) pedagogical approach and how it pertains to learning to teach mathematics is discussed next.

2.3.2.1 Representations of practice

Representations involve all the different ways practice is made visible to teachers and includes artefacts such as observation of practice, videos of practice, lesson plans, student work samples, and case studies of practice (Grossman, 2011). Importantly, Grossman (2011) claimed, the specific features of representations have consequences for what novice teachers

can see and learn about practice. For example, video representations may enable novices to see interactions between the teacher and students, but they do not highlight the planning that occurs before the lesson, including the discussions with teachers that influenced the planning (Grossman, 2011). Furthermore, in a discussion of a prevalent “best practice” mentality regarding teacher professional learning, Lefstein and Snell (2014) maintained that “videos wash the dynamic complexity out of teaching” (p. 6). Importantly, Ghousseini and Sleep (2011) argued that representations of practice, such as video footage of classroom instruction, are not guaranteed to lead to preservice teacher learning without deliberate support by teacher educators to facilitate learning. Grossman (2011) highlighted the importance of considering which aspects of practice are visible and invisible when thinking about the nature, range, and use of representations in teacher learning. The challenge for teachers is making sense of representations that they see (Feiman-Nemser, 2012).

Erickson (2011) argued that what they “see” can deceive teachers. This is because *noticing* is selective. It involves attending to some phenomena and not others. Noticing is concerned with what teachers attend to in the moment of teaching and how they make sense of their observations (van Es, 2012). Interestingly, Dewey (1904, as cited in Erickson, 2011) distinguished between two types of attention behaviour by students that could be observed by teachers: outer and inner attention. Outer attention is the overt appearance of attending that is quite easy to see, for example, sitting still. In contrast, inner attention is the child’s genuine interest, which may or may not be displayed overtly to the teacher in their behaviour, for example, a child rolling on the floor whilst the teacher is talking but is genuinely interested in what is being discussed. Teachers may mistake this for inattention.

Erickson (2011) argued that noticing what we hope teachers will see requires reflection in action. Furthermore, Casey (2011), in a discussion of “modelling” as a representation of practice, illustrated the importance of thinking aloud whilst modelling, so that the invisible

decision-making becomes visible to observers (see also Feiman-Nemser, 2012). For example, when a student makes a novel conjecture, the demonstrating teacher steers the lesson into a new line of inquiry that reflects the conjecture and then explains this move to observers. Likewise, Higgins and Parsons (2011) noted that in providing commentary whilst they modelled, facilitators reported being able to draw teachers' attention to the underlying mathematical principles of their pedagogical practices.

2.3.2.2 Decomposing practice

Grossman and colleagues (2009) argued that engaging in complex practice requires novices to separate the different components or decompose the practice. Decomposition involves breaking down practice into parts for the purposes of teaching and learning. Grossman (2011) argued that decomposition is an effective way to enable novices to observe and enact teaching. However, successful decomposition is dependent on a language and structure for describing practice. Grossman (2011) argued for instructors to possess a set of instructional practices for describing teaching, and during instruction, focus attention on those practices. Interestingly, in a study that was focused on preparing preservice teachers to lead mathematical discussions, Boerst et al. (2011) claimed, "Decomposing teaching into nested practices of varying grain sizes simultaneously attends to the how and why of practice" (p. 2855). This resonates with Loughran, Keast, and Cooper's (2016) notion of pedagogical reasoning, which they maintained can be developed by uncovering the complex reality of teaching. Boerst et al.'s (2011) approach involved specifying component parts of effective mathematics discussions. For example, describing the nature of that work, explaining the mathematics that may be discussed, and describing the nature of the interaction between the students and the teacher (Boerst et al., 2011).

Whilst this study was focused on preservice teachers, it may also apply to practising teachers who are learning new pedagogical approaches to teaching in this study.

2.3.2.3 Approximating practice

Approximations involve opportunities to enact practice that is associated with the work of practising professionals. For example, in teacher education, a novice teaches part of a lesson, is observed by a more experienced teacher, and is provided with feedback. Ericsson (2002) argued that approximations enable teachers to engage in intentional practise of difficult components of practice. Boerst et al. (2011) scaffolded opportunities for preservice teachers to engage in increasingly complex approximations of practice to enable maximum success. For example, initially they focused on teacher questioning, followed by getting preservice teachers to teach a “mini problem”, and finally teaching a whole-class mathematics lesson. Boerst et al. (2011) found the complexity of teaching was reduced by the amount of scaffolding provided. This suggests a scaffolded model of teacher learning may be helpful in supporting teachers to implement new pedagogies in this study.

In summary, Grossman et al. (2009) maintained representations and approximations rarely encapsulate the entirety of practice and stressed the importance of decomposing practice in planning for enactment. This suggests all three notions are interconnected and might be important to include in the design of a model focused on teacher professional learning in this study. Whilst the Grossman et al. (2009) study was carried out with “novice” and preservice teachers in mind, this study sought to incorporate the notion of representations, decomposition, and approximations of practice to show they can equally apply to classroom teacher learning.

Associated with the Grossman et al. (2009) pedagogical approach is the process through which teacher change occurs.

2.3.3 Processes for Teacher Change

Clarke and Hollingsworth (1994) identified six viewpoints in the literature regarding teacher change: as training, as adaption, as personal development, as local reform, as

systematic restructuring, and as growth or learning. Clarke and Hollingsworth (2002) argued that current professional learning efforts most closely align with the “change as growth or learning” perspective. Within this perspective, teachers are perceived as active participants in their learning and change is expected as a natural process of their professional endeavour.

Clarke and Hollingsworth’s (2002) interconnected model of professional growth (IMPG) has been utilised in recent studies that were focused on the professional growth of teachers (e.g., Groves & Doig, 2017; Widjaja et al., 2017; Wilkie, 2017). The IMPG builds on Guskey’s (1986) linear model of teacher change in which teacher beliefs and attitudes shift after they experienced change in student learning outcomes. In the IMPG model, change is conceptualised as occurring in a nonlinear fashion.

The IMPG model features four change domains. The *external domain* provides a wide range of external sources of information or stimulus for change, for example, professional reading or structured opportunities to engage in professional conversations. The *personal change domain* includes new knowledge and change in beliefs and attitudes. The *change domain of practice* includes professional experimentation in a range of situations including approximating with innovative teaching strategies. A teacher’s changed perceptions of salient outcomes are inherent in the *change domain of consequence*. In other words, this domain refers to outcomes that are salient to individual teachers, depending on the inferences they draw from their individual experiences.

Change in one domain is connected to another through the mediating processes of *enactment* and *reflection* (Clarke & Hollingsworth, 2002), which teachers may or may not choose to engage in. However, change in one domain does not necessarily lead to change in another. The term enactment represents the enactment of something a teacher has experienced, understands, and trusts. The IMPG highlights ways in which context and an individual teacher’s inclinations towards learning shapes their professional growth.

This study drew on Clarke and Hollingsworth's (2002) IMPG to identify aspects of the changes in the different domains of the teachers as they participated in the modelling.

Related to this is the context in which the professional learning took place.

2.3.4 Situated Learning

Situated learning theorists posit that learning is constructed through social interaction, takes place in meaningful contexts, and is distributed across people and artefacts (Borko et al., 2008; Putnam & Borko, 2000), hence the current focus on practice-based professional learning (Bass & Ball, 2014; Naik & Ball, 2014; Timperley, 2015). Through social interaction, individuals learn the ways of thinking and behaving that are valued by the community of practice (Borko et al., 2008; Lave & Wenger, 1991; Putnam & Borko, 2000). Examples of meaningful contexts for teachers include classrooms and group settings with a focus on teacher practice. The various contexts enable different types of knowing; the most appropriate way depends on the goal for teacher learning (Putnam & Borko, 2000). For example, "experiences situated in the teachers' own classrooms may be better suited to facilitating teachers' enactment of specific instructional practices" (Putnam & Borko, 2000, p. 7). This suggests modelling pedagogies in teachers' classrooms in this study may have supported the teachers to enact pedagogies associated with problem-solving and reasoning.

Lave and Wenger (1991) proposed a way of understanding learning as *legitimate peripheral participation* that characterises the process by which people learn as they participate in a community of practice, beginning as a newcomer and progressing to a full participant. They illustrate their theory with observations of different apprenticeships (midwives, tailors, meat-cutters, and nondrinking alcoholics in Alcoholics Anonymous). Initially when people join communities, they learn at the periphery. As they become more competent, they move more towards the "centre" of the specific community. This social process incorporates the learning of knowledgeable skills. The nature of the situation impacts

significantly on the process, and the context in which this takes place is an important component (Lave & Wenger, 1991).

Related to this is the importance of professional learning that is built on social interaction and collaboration in professional communities (Borko et al., 2008; Little, 2002). For teachers, collaboration involves learning together in communities of practice where new ideas can be tried out and reflected upon, and new knowledge about teaching and learning is co-constructed within the context of classroom experiences (Butler, Lauscher, Jarvis-Selinger, & Beckingham, 2004). Van Driel and Berry (2012), in discussing the development of PCK, argued that it is important to focus on approaches that develop teaching and learning in such communities. An example of collaboration is “lesson study” (Ebaegu & Stephens, 2014).

2.3.5 Lesson Study

The notion of lesson study is centred on the examination of teaching practice and involves a group of teachers planning a lesson, one of them teaching the lesson, and then all of them reviewing the lesson and planning the next cycle (Fernandez, 2005; Lewis et al., 2006).

Lesson study is an effective and universal form of professional learning in Japan, practised by over 98% of public elementary and junior high schools and more than 94% of public high schools (Lewis & Perry, 2014). Over the last decade, there has been a remarkable growth in lesson study as a model for teacher professional learning in many countries including Australia (Groves & Doig, 2014; Widjaja et al., 2017). The purpose is to replicate the success of Japanese lesson study in transforming teacher-centred teaching to student-centred instruction focused on mathematical thinking and problem-solving (Takahashi & McDougal, 2015). For example, Widjaja et al. (2017) found that teachers’ professional growth was enhanced through investing in in-depth planning and enactment of student-

centred problem-solving lessons. However, effectiveness outside Japan is uneven and not clear. Few studies have been documented in which there is strong evidence of success (Takahashi & McDougal, 2015).

Those who have implemented lesson study in contexts outside Japan have usually had no experience with doing lesson study themselves, and consequently, important elements of lesson study have been misinterpreted or left out (Takahashi & McDougal, 2015). Numerous projects leave out the initial important phase of lesson study, *kyouzai kenkyuu*, [an investigation of resources for teaching] that assists teachers to increase their knowledge and understanding of student thinking. Moreover, the purpose of lesson study, which is to gain new knowledge for teaching and learning, is often misunderstood (Takahashi & McDougal, 2015). Some educators attempt to squeeze lesson study into one day, but the typical duration in Japan is more than 5 weeks. Others omit the support of a knowledgeable other, which is a critical component for improving teacher learning (Takahashi & McDougal, 2015). A knowledgeable other is someone external to the planning team with deep expertise in the content and pedagogy and much experience with lesson study. Importantly, all elements need to be included for lesson study to be effective.

Lesson study in Japan is a highly structured process involving a whole-school community aimed at addressing a problem of practice. The main elements of lesson study, according to Takahashi and McDougal (2015), are summarised as follows:

1. *Kyouzai kenkyuu*: A group of Japanese teachers begin the study by reading, examining, and discussing relevant research articles, teacher texts, curriculum, and other resources.
2. Centred on their *kyouzai kenkyuu*, the group plans a lesson focused on a topic that also addresses the broader problem of practice.
3. One member of the planning team teaches the research lesson, and the other members (including other educators not on the planning team) observe.

4. Members of the planning team and other observers discuss the lesson by focusing on how students responded to the lesson. This develops around student data collected during the observation (Takahashi & Yoshida, 2004). The purpose is to increase insights into how the teaching and learning should best be implemented based on student understandings.

It is arguable that lesson study represents effective use of the Grossman et al. (2009) pedagogical approach, which defines three important pedagogies of practice in professional education: representations, deconstruction of practice, and approximations. However, one of the issues with lesson study in an Australian context is the culture of privacy associated with teaching and teacher reluctance to being observed (Groves & Doig, 2010). Sullivan (2011) argued that observation of teaching practice is counter to our Australian teaching culture; therefore, teachers may feel challenged with the prospect of having to teach a research lesson. A different approach might be to have the knowledgeable other *model* lessons, rather than observe lessons, as a first step in building teacher trust and confidence (Higgins & Parsons, 2009). In comparison with lesson study, the research on modelled lessons is limited (Clarke et al., 2013). The present study investigated this type of modelling. The literature on modelled lessons is discussed in Section 2.3.6.

2.3.6 Modelling

Teachers directly observing modelled lessons taught by a knowledgeable other is uncommon (Naik & Ball, 2014). However, a small number of recent studies have suggested that modelled lessons have the potential to prompt teachers to consider new approaches to teaching and learning (see, for example, Clarke et al., 2013; Grierson & Gallagher, 2009).

Modelled lessons are based in classroom practice and provide a professional learning strategy for teachers to reflect on and enhance their practice (Clarke et al., 2013; Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003). The focus of the observation may relate to

lesson structure, pedagogy, teacher actions, and student learning (Bruce, Ross, Flynn & McPherson, 2009). Some studies (see Bruce et al. 2009; Grierson & Gallagher, 2009) have suggested that teachers value the contextualised experience as they can see how lessons work with their own students. This is a catalyst to enable teachers to reflect upon their own practice and make changes (Grierson & Gallagher, 2009).

The purpose of the modelling is to use pre- and post-lesson briefings to provide an opportunity for reflecting on the observed practice. Loucks-Horsley et al. (2003) stated these briefings are fundamental for raising teacher awareness of mathematical ideas and curriculum content, as well as for giving teachers explicit teaching actions. Bruce et al. (2009) noted that during the prebrief, teachers would talk about the lesson plan, the lesson objectives, anticipate responses from the students, and other pertinent issues, whereas during the post brief, teachers would share their observations and discuss implications. This study adopted these processes and sought to contribute to the literature by investigating in depth the key elements of these discussions.

Bruce et al. (2009) compared the impact of two approaches to professional learning in schools in Ontario. One approach was lesson study and the other approach was modelling lessons. They found that modelled lessons were particularly helpful for beginning teachers and suggested they may be helpful for teachers who have difficulty imagining what new approaches to teaching might look like in practice. This suggests that as experienced teachers learn to implement the AC:M proficiency strands, modelled lessons might support them to create new visions of teaching for themselves. This suggestion resonates with Bass and Ball (2014), who maintained that without opportunities to see teaching in action, teachers may have difficulties in assigning tangible meaning to practices, such as mathematical discourse, and to develop distinct images of what it could mean to teach and learn them. This implies

that observation of teaching might be more meaningful to teachers than other forms of professional learning that focus on less visible aspects of teaching.

Grierson and Gallagher (2009) investigated the effects of a classroom initiative in which an exemplary teacher was chosen to model teaching practice with her own students and have teachers from other schools visit and observe. The study sought to meet teachers' learning needs and promote curriculum reform in literacy. The qualitative case study took place over a nine-month period and involved eight teachers, a modeller, and a consultant teacher external to the school. Groups of three or four teachers visited the modeller's classroom for a series of three spaced, full-day sessions, which included the observation of three modelled lessons and debriefing. The intent of the modelling was to observe explicit instruction with sufficient time between each session to reflect on and enact learned practices. The observing teachers were interviewed regarding their observations and experiences in modifying their teaching. Specific factors identified in supporting the teachers to change their practice included seeing contextualised teaching practice in action, the organisation of the program, the exemplary teaching, interpersonal, and mentoring skills of the modelling teacher, and ongoing support for teachers after the experience. Grierson & Gallagher (2009) noted that to promote change, professional learning opportunities must be nonthreatening and supportive of teachers to be open and honest about the challenges they face and increase their comfort with taking risks in enacting new pedagogies.

An alternate approach that has been used in various places involves modelled lessons conducted by expert teachers or knowledgeable others from outside the school, which may or may not be followed by observation of the teachers' later implementation. Knowledgeable others support teachers to reflect on teaching and learning (Takahashi, 2014; Higgins & Parsons, 2011) and to interpret teaching practice for enactment (Timperley, 2015). For example, as part of the Contemporary Teaching and Learning of Mathematics project

involving 82 Victorian primary schools between 2008 and 2012, several university teacher educators modelled teaching practice in classrooms (Clarke et al., 2013). The modelled lesson structure included a 20-minute prebrief where the teacher educators outlined their planned lesson including the mathematical focus and tasks. It also included a 15-minute debrief where the observing teachers reported on their observations. During the modelled lesson, the observing teachers were discouraged from teaching their students and encouraged only to observe. The teacher educators visited each school from two to four times during the school year.

The teachers chose their own observation foci, and this appeared to contribute to their ownership of the professional learning. They completed brief questionnaires in advance of the modelled lesson, during the lesson, after the debrief, and several weeks later. These questionnaires focused on the teachers' intended and actual observation foci and anticipated changes to their practices. Common themes in the teachers' intended observations included a focus on catering for diversity, questioning, and student engagement. Clarke et al. (2013) noted the teachers tended to focus more on teacher actions (made by the modeller) than on student thinking. They called for further research both into the actions teachers chose to take and did take because of their observation, and the nature of support needed to assist teachers to implement new learning. This study sought to investigate this further.

Another example is the New Zealand Numeracy Development Project (NZNDP) (Higgins & Parsons, 2011), where eight facilitators modelled complex pedagogies and then observed teachers as they attempted to enact what they had seen. The facilitators were interviewed regarding their observations of the teachers' enactments. Their responses drew upon their experiences of up to four years' working with teachers. Aspects of facilitator practice that were reported as having an impact on teacher learning in the NZNDP included a focus on complex pedagogies, students' thinking, and classroom discourse. The facilitators

reported that putting modelling before observation was an important step in building trust, which was necessary to encourage teacher improvement. These findings suggest there may be advantages in having knowledgeable others model teaching practice. It seems they can support teachers to make knowledge, skills, and pedagogies meaningful and accessible within their contexts and develop their capacity to inquire into their practice to inform next steps in improving their teaching.

One of the issues regarding modelling practice is the lengthy debate regarding its merits (Feiman-Nemser, 2012). This is discussed next.

2.3.6.1 Conflicting views on modelling for teacher learning

Ball et al. (2014) suggested important questions remain about what can be learned from direct observation of teaching practice. They argued there is insufficient knowledge of imitation in teacher learning:

Little is known about the role of imitation in teacher learning. Other practices not only assume that professionals may imitate one another, they even design to ensure it . . . Work on developing collective professional knowledge through the study of practice can help to break through this ideology [that imitating is not professional] and ask important questions about imitation—about what forms it might take in developing teaching and what people might learn from these. (Ball et al., p. 332)

Similarly, in a discussion of the merits and challenges of modelling teaching, Feiman-Nemser (2012) argued that many critics see modelling as an exercise in learning to imitate rather than learning to understand. For example, Loucks-Horsley et al. (2003) noted the modelled lessons are often taught by an experienced teacher presenting an “exemplary” model of teaching for others to watch. This suggests Loucks-Horsley et al.’s (2003) focus was on “best practice”, which appears to be at odds with Lefstein and Snell (2014), who maintained that a focus on best practice through demonstration and imitation “shuts down possibilities for critical discussions on the complexities of teaching” (p. 3). Furthermore,

Buchmann (1993) argued that imitation is not professional because teachers need to find their own style of teaching.

Loughran (2006) argued modelling in preservice teacher education is a medium for deconstructing practice to understand the purpose of the teaching approach and illuminate the modeller's pedagogical reasoning. In this approach, it is hoped that the tacit practices of teaching may be challenged and actively made explicit so that observers can develop understanding of the complexity of teaching (Loughran, 2006). This resonates with Lefstein and Snell (2014), who argued for an approach to classroom teaching pedagogy that is "sensitive of and appreciative of the tensions and dilemmas inherent to teaching and learning in classrooms" (p. 3).

Modelling in this way involves risk because the actions of the modeller, perceptions of learning, and assumptions about teaching are under examination (Loughran, 2006). Confidence for all participants, including the modeller and the observers, is "dependent upon laying out the practice for critique and being involved in the learning whilst maintaining the integrity of the individual" (Loughran, 2006, p. 42). For this to occur, it is important that ground rules, norms, and protocols for safety are made clear to all participants. I drew upon these principles in designing the intervention in this present study by articulating norms and protocols prior to modelling.

It seems that teacher learning from modelling requires supports to enable the practice to be studied (Ghousseini & Sleep, 2011). This is discussed next.

2.3.6.2 Processes for assisting teachers to interpret modelled teaching practice for enactment

Ghousseini and Sleep (2011) argued that focused processes are necessary to enable representations of practice such as modelling to be studied. Ghousseini and Sleep's (2011) study involved a case of professional learning, which was part of the activities of the Center

for Proficiency in Teaching Mathematics in the United States. The designers deliberately sought to mediate preservice teacher learning, which was more likely to result in enactment. A teacher educator worked with a group of preservice teachers for one week. Sixty-eight observers (teacher educators, mathematicians, and school-based educators) nominated an observational focus for the beginning of the week. As the week progressed, the observers were provided with more lenses for observing lessons modelled by teacher educators and encouraged to attend to more subtle aspects of practice.

Ghousseini and Sleep's (2011) analysis identified the following five processes that can assist in making representations of practice such as a modelled lesson "studyable":

1. *Engaging the content* reflects work that is done to familiarise preservice teachers with the mathematical content of the practice being studied. This agrees with Takahashi and McDougal (2015), who began lesson study with a group of teachers by engaging them in reading, examining, and discussing relevant research articles, teacher texts, curriculum, and other resources. Ghousseini and Sleep (2011) noted that engaging the content can also take place during enactment. For example, in their study, the teacher educator explicitly identified important features of the work that arose during the observation. This echoes Casey (2011), who asserted the importance of thinking aloud whilst modelling so that the invisible decision-making becomes visible to observers (see also Bronkhorst et al., 2011; Feiman-Nemser, 2012). For example, when a student makes a novel conjecture, the modeller steers the lesson with a new line of inquiry that reflects the conjecture and then explains this move to observers.
2. *Providing insight into student thinking* supports preservice teachers to gain insights into the thinking of students in the representation of the practice being studied. For example, if the representation is a modelled lesson, then the preservice teachers might consider

student responses to tasks. This may also apply to teachers as they are learning new approaches to teaching.

3. *Orienting to the instructional context* supports preservice teachers to understand and engage with the context, structure, or flow of the representation being studied. For example, in a modelled lesson, this might involve discussing the goals of the lesson or the anticipated flow of the lesson. It suggests it may be helpful if teachers are given a lesson plan in the modelled lesson prebrief with clearly articulated goals and lesson structure.
4. *Providing lenses for viewing* involves scaffolding preservice teachers to focus their attention. These can take many forms, for example, a question such as asking teachers to identify teacher moves (Boerst et al., 2011). Another lens may be a lesson plan, which contains caution points or common misconceptions to watch for. This agrees with Casey (2011) and Grierson and Gallagher (2009) who noted that a deliberate focus to enable teachers to narrow their observation is important. Otherwise, teachers can feel overwhelmed with new ideas.
5. *Developing a disposition for inquiry* helps preservice teachers develop the skills and attitudes to take an investigative stance towards the representation of practice being studied. It requires preservice teachers to take risks, raise questions, and engage in critical reflection. This suggests that focus questions for observation of a modelled lesson may support teachers to gain a disposition of inquiry.

Ghousseini and Sleep (2011) argued that preservice teachers are more likely to enact new learning if they engage in supportive processes such as the ones described. Their research is significant because it also suggests potential processes for helping practising teachers to study modelled teaching practice with the intent of their later enactment in the

classroom. It provides a useful approach to draw upon in this thesis in supporting teachers to learn mathematical knowledge for teaching in and from modelled lessons.

2.4 Building on the Research Literature

Findings of the studies in this literature review highlight that the necessary practical and specialist approaches to teaching mathematics in Australian classrooms are in need of improvement. There is a necessity to focus more on teaching through problem-solving and facilitating students' mathematical proficiency. The current emphasis on procedural approaches to teaching is having a detrimental effect nationally as many students are disengaged in mathematics and fewer students are going into higher education in mathematics-related fields. One unfavourable flow-on effect is the potential to diminish Australia's capacity to deal with issues where high-level mathematics is required, for example, climate change.

The review also illustrates that learning to teach is complex. Teachers find it difficult to assign tangible meaning to practices that promote student thinking and problem-solving. Moreover, changes to teaching practice require much effort and can be difficult without guidance and support from a knowledgeable other.

Research has found that professional learning situated in teachers' classroom contexts may support them to enact different pedagogies. There have been suggestions that lesson study might be a helpful form of teacher learning, but in an Australian context, teachers are reluctant to be observed. Others have suggested video representations of practice may be helpful; however, it seems these may be problematic, as teachers see interactions between the teacher and students, but not the interactions with colleagues and the curriculum materials that shaped the planning.

A small number of studies have suggested that modelling by a knowledgeable other might be a first step in assisting teachers to visualise new pedagogies. However, seeing a

representation of practice on its own is unlikely to guarantee teacher learning. Studies in preservice teacher education have shown it may be important to deconstruct practice with teachers to enable the representation to be studied. It also seems valuable to provide teachers with scaffolded opportunities to enact new pedagogies that they have seen and deconstructed. These elements, when executed together, may deliberately focus the professional learning on the enactment of high-quality teaching practice. This review highlighted the need for research that focuses on bringing these elements together with practising teachers. This present study sought to investigate in depth the potential of modelling pedagogies when situated in teachers' own classroom contexts and including deconstruction, scaffolded approximations, and collaboration for promoting professional learning.

In Chapter 3, I present, as an initial step towards this goal, findings from preliminary research I conducted in several schools in the Tasmanian Catholic sector in 2012. This research describes teacher observations of modelled lessons that informed the design of the interventions in this study.

CHAPTER 3: PRELIMINARY STUDY ON WHAT TEACHERS SEE WHEN WATCHING OTHERS TEACH

This chapter presents findings from a preliminary study I conducted in 2012 exploring classroom modelling as an approach to teacher education. In particular, it presents analysis of responses via an observation proforma completed by primary school teachers in several Tasmanian Catholic schools to indicate what teacher actions they observed when watching modelled lessons and which of those actions they intended to implement in their classrooms because of the observation. I have included this preliminary study in the thesis because the findings highlight that the teachers were able to notice desirable pedagogies when observing modelled lessons. However, it was unclear to what extent these observed actions translated into changed classroom practice. The findings of this preliminary study therefore provided impetus for further research on the role of modelling in changing teacher practice, and also informed the design and data analysis for the main study in this thesis (See Hodgson, 2013). This included research on whether the teachers could enact observed pedagogies and whether the observation focus could be narrowed (Casey, 2011; Feiman-Nemser, 2012) to support teachers to notice more detailed pedagogies associated with developing students' reasoning and problem-solving.

The chapter is presented in three sections. First, the research aim, questions, and research context for the preliminary study are presented. Next, the process for the analysis of responses is explained. Following this, the findings emerging from the observation of teaching practice are presented.

3.1 Research Aim and Questions for Preliminary Study

This chapter explores the following research aim and question:

Aim	Question
To explore what pedagogies the teachers' notice when they observe a modelled lesson	When given the opportunity to observe teachers modelling, which aspects of pedagogy and mathematics do teachers notice?

3.1.1 Research Context and Design

This preliminary research was undertaken as part of my work as Education Officer: Numeracy in the Tasmanian Catholic sector in 2012. A total of 162 primary school teachers across the state participated in observing at least one modelled lesson modelled by me in a variety of classroom contexts over a 12-month period. The modelling process was as follows.

3.1.1.1 Modelling and observation processes

The process for enactment of the classroom modelling associated with the data collection was as follows. One or more teachers in a Catholic primary school context initiated the process by contacting me and inviting me to model a lesson in their school. Following this, I emailed them a proforma to complete and return. The proforma contained the following questions:

1. What would you like the mathematical focus of the lesson to be? (Please feel free to use content descriptors from the Australian Mathematics Curriculum.)
2. What would you like me to focus on with respect to pedagogy? (For example, questioning, differentiation, open-ended tasks, structure of the lesson, materials used.)
3. What would you like to observe about student learning during the lesson?

Based on teacher responses to these questions, I wrote a detailed lesson plan. The lesson plan aimed to provide teachers with an exemplar from which they could see how the lesson objectives and tasks were connected to the content of the AC:M. There was usually an interval of two to seven days between receiving teacher responses and the modelled lesson. I

met with observing teachers for a 30-minute prebrief prior to the lesson on that day. During the prebrief, observing teachers were given a copy of the lesson plan, and it was explained. Initially, I described how the lesson connected to the request from the class teacher. I attached a copy of the lesson request proforma to the lesson plan. I outlined the tasks I had planned, enabling prompts and extending prompts (see Sullivan et al., 2006), connections to the AC:M, and questions I intended to ask the students. During this meeting, I handed teachers a survey, which I invited them to complete either during or after the lesson. The survey was designed to evaluate the experience of the observation. The survey contained the following prompt:

What teacher action/s did you observe today that are different from what you usually do that you will try to implement in your classroom?

I taught each lesson observed by 2 to 10 school-based personnel. The observing teachers included the teacher of the class where the modelling was taking place and teachers of similar grades. For example, if the lesson was in Grade 4, observing teachers tended to be teaching Grades 3, 4, or 5. Other observers sometimes included the principal, teacher assistants, and relief teachers.

After the lesson, all observing teachers and I met for a 30-minute debrief. The first 10 minutes were set aside for observers to complete their survey. During the next 20 minutes, we discussed their responses, with observers having the opportunity to share anything that was pertinent to the discussion.

3.2 Analysis of Responses

Ball et al.'s (2008) classification of MKT guided the data analysis and interpretation of responses. First, written responses were read and grouped into categories. Most teachers wrote more than one response; hence, the breakdown of responses is greater than 100%. Next, the responses were inspected and the initial coding revised. The teacher responses were sorted

into the categories of Ball et al.'s (2008) MKT model. Finally, the grouped responses were sorted into subcategories.

3.3 Findings

The categories emerging from the data analysis of the teacher responses in each survey are presented in Table 3.1. The categories are presented in order of decreasing frequency. Some representative teacher responses are presented as illustrative examples of each subcategory. They are direct quotes from teacher surveys.

Table 3.1

Categories of Responses With Illustrative Examples of Teacher Responses

Category	Subcategory	% of teachers (<i>n</i> = 162)	Illustrative Examples of Teacher Responses
KCT	Opportunities for students to justify thinking and explain mathematical reasoning	41	<p>“Prove it as opposed to explain”</p> <p>“Allow the students to explain in their own words”</p> <p>“Reasoning and thinking questions”</p> <p>“Tell me about . . .”</p> <p>“Convince me”; “More talk”; “More student interaction”</p> <p>“Really encourage math thinking and exploration”</p> <p>“Children share strategies”</p> <p>“I have learned to support and encourage questioning by students”; “Keep thinking rather than no”</p> <p>“No wrong answers”; “By the end of the lesson, I saw four weaker students putting up their hand to explain”</p> <p>“Positive referral to mathematics and mathematicians and students being thinkers”</p>
	Lesson structure and features	39	<p>“Different starting numbers”; “Enabling prompts”</p> <p>“Use of ‘cliff-hanger’ to challenge students”</p> <p>“Encouraging feedback to build climate of trust”</p> <p>“Flexible groupings”</p> <p>“Relate lessons to real-life experiences”</p> <p>“Ending the lesson with an open-ended question to take the learning from the lesson further”</p> <p>“Keep it simple”</p>

Category	Subcategory	% of teachers (<i>n</i> = 162)	Illustrative Examples of Teacher Responses
			<p>“Allowing the lesson to flow when it’s all connected”; “One task, multiple entry points”</p> <p>“Find out what they know. Don’t make assumptions about what they know based on previous work”</p> <p>“Videoing children’s explanations for future sharing”</p> <p>“Combined many concepts into one lesson”</p> <p>“Highlighting connections with other areas of maths, prompted by student contribution”</p>
	Allowing the children to struggle and allowing time for them to come up with solutions	37	<p>“Letting the children struggle more and allow more time for them to come up with solutions”; “Giving time for processing allows all children to experience success”</p> <p>“Thinking time”; “Lots of time for all students to think and find solutions”</p> <p>“Opportunities for students to modify their thinking, even when they seem to have no idea”</p> <p>“The not telling stance”</p> <p>“By leaving the students to work through the problems, they came to the solutions themselves”</p> <p>“Making mistakes and retrying”</p> <p>“Don’t ever imagine the low attainers can’t enter the task . . . they can”</p> <p>“Encourage persistence”</p> <p>“Differentiation is not something we’re used to with our streamed maths classes”</p>
	Materials and representations	18	<p>“Making maths come alive through props, e.g. taking a map and recreating it as a real-life model, where the children can physically put themselves in it and solve problems”</p> <p>“The number line made it very clear to students how the numbers were connected”</p> <p>“Connect ideas to visual models”</p> <p>“Teaching multiplication and division through arrays is much easier for them to grasp”</p> <p>“The story engaged the children”; “Use of a book to stimulate a mathematical problem”</p>

Category	Subcategory	% of teachers (<i>n</i> = 162)	Illustrative Examples of Teacher Responses
	Tasks	16	<p>“A reminder not to give students neat problems” “I will work on developing problem-solving skills”</p> <p>“Division with remainders task stretched their thinking and allowed for clarification of misconceptions”</p> <p>“Made them really think”</p> <p>“Children learned so much by being challenged”</p>
	Explicit use of mathematical language	15	<p>“Very explicit reiteration of the children’s strategies”; “Repetition of keys words, e.g. dividing and sharing”; “Extends their [students] vocabulary”</p> <p>“Clarifying terminology, clarifying understandings”</p>
KCS	Modification of probing questions to scaffold or challenge thinking	19	<p>“Layered questions”; “Very much involve the low-attaining children. Using their existing knowledge to build higher order thinking”</p> <p>“Probing questions scaffolded learning and thinking”; “Inquiry rather than telling”</p>
CCK	Knowledge of mathematics	2	<p>“The task even helped me to understand area and perimeter”</p>

Three categories from Ball et al.'s (2008) model describing different types of mathematical knowledge for teaching emerged. The most frequent category was KCT, followed by KCS and CCK. Recognising some overlap between the subcategories, several teachers referred to “opportunities for students to justify thinking and explain mathematical reasoning” and “opportunities for thinking time” as their intended change. This suggests that modelling pedagogies might be a helpful way for teachers to notice and implement pedagogies that support the development of student mathematical reasoning. Quite a few teachers referred to their intended change to lesson structure and features, which included strategies for differentiation such as flexible groupings, enabling and extending prompts (Sullivan et al., 2006, 2009), and making connections. A number of teachers also indicated change to “Tasks” and “Explicit use of mathematical language”.

Interestingly, one fifth of the teachers mentioned “modification of probing questions”, which is associated with a knowledge of likely student learning trajectories and misconceptions. This falls into the category of KCS. A small number of teachers referred to aspects of CCK. It is possible that their focus was on the pedagogies rather than the mathematical content, given the wording of the survey. This suggests that teachers may need specific prompts if part of the goal in the observations is to see specific aspects of mathematical content. Importantly, these data suggest that the clear majority of teachers, unprompted, saw the aspects of practice that it was hoped they would.

The preliminary study's findings are consistent with previous studies of modelling in other learning areas (e.g., Grierson & Gallagher, 2009), in that observing teachers could identify important pedagogies and how they catered for students within the context of a real-time classroom. Clarke and his colleagues (2013) asked a similar question of observing teachers in their study of modelled lessons. The 200 teachers were asked to describe intended changes to their practice after seeing a modelled lesson. Clarke et al. (2013) found

“questioning” was the largest category for subsequent action, with 35.5% of teachers indicating their intended change.

Whilst the categories are somewhat different to Clarke et al.’s (2013) analysis, the data are clear that in this present study, the observing teachers noticed important aspects of KCT and KCS (Ball et al., 2008) that they intended to implement in their classrooms. Of course, the real test is whether teachers transferred observed practices to their own classroom practice.

3.4 Linking Findings to the Research Question

This chapter sought to answer the question “When given the opportunity to observe teachers modelling, which aspects of pedagogy and mathematics do teachers notice?” The findings indicate that when given the opportunity to observe modelled lessons, teachers noticed important aspects of MKT. The findings of the surveys revealed that teachers focused on desirable pedagogies when observing modelled lessons, suggesting that classroom modelling may be a powerful form of teacher learning. However, further research needs to be undertaken to determine to what extent these observed actions translate into classroom practice. It would also be interesting to find out whether the observation focus could be narrowed (Casey, 2011; Feiman-Nemser, 2012) to support teachers to notice more detailed pedagogies associated with developing students’ reasoning and problem-solving.

In relation to the design of modelled lessons, Higgins and Parsons (2011) argued that the role of experts is critical in supporting teachers to change their practice. In this study, the modelling of pedagogies by an experienced teacher did seem to focus the teachers’ attention on the pedagogies incorporated into the lesson.

3.5 Moving Forward With the Research

Drawing on the data analysis and research evidence presented in this chapter, I fine-tuned the observation prompt for my doctoral research to ask the participant teachers to specifically attend to any teacher actions that facilitated student reasoning and problem-solving.

In Chapter 4, I provide a synopsis of the research design and approaches, including the theoretical framework, methodology, and methods that guided this present study.

CHAPTER 4: METHODOLOGY AND METHODS: PERSPECTIVES, TOOLS, AND TECHNIQUES OF THE RESEARCH

This chapter details the methodology and methods utilised in this study. Essentially, the study sought to explore the phenomenon of teacher learning through their observation of classroom modelling and the subsequent deconstruction of pedagogies and enactments, and gain insights into the experiences of teachers who also participated in modelling and any subsequent impact on their knowledge and practice. As part of this experience, I also focused the inquiry on myself through reflexive practice, not only to inform my teaching but also to contribute to collective understandings of teacher educator learning.

4.1 Methodology

This study adopted a qualitative phenomenological multiple case study research approach. The discussion of research methodology explains my rationale for choosing multiple case study and begins with a detailed description of my social constructivist worldview that informed the research design. This is followed by a discussion of case study design and my choice of multiple case study methodology. Next is a discussion of strategies for enhancing the validity and trustworthiness of the study, along with ethical considerations.

4.1.1 Philosophical Worldview Proposed in This Study

A philosophical worldview is a set of beliefs that drive actions (Guba, 1990). It is a “philosophical orientation about the world and the nature of research that a researcher brings to a study” (Creswell, 2014, p. 7). In qualitative research, these beliefs have been known as *paradigms* (Lincoln, Lynham, & Guba, 2011) *epistemologies* and *ontologies* (Crotty, 1998), or *broadly conceived research methodologies* (Neuman, 2006). Embedded in these beliefs are four assumptions about the world: epistemology, ontology, axiology, and methodology (Creswell, 2013). Epistemology refers to the theory of knowledge, its nature and how it is developed, what is counted and valued as knowledge, and the relationship between the

researcher and the known and unknown. Ontology addresses the nature and characteristics of reality. Axiology is concerned with ensuring that the researcher declares her position and makes her values known. Methodology then focuses on how the researcher can acquire knowledge about the world with integrity (Denzin & Lincoln, 2011).

This multiple case study embraced a social constructivist worldview, a philosophical position that believes people seek to interpret the world in which they exist (Creswell, 2013; Creswell, 2014). It assumes co-construction of meaning through a subjectivist epistemology (Denzin & Lincoln, 2011). In this view, people construct multiple, varied, and often intangible meanings of their experiences in the world. This approach to research focuses on the “views, values, beliefs, feelings, assumptions and ideologies of individuals” (Creswell, 2012, p. 439). A social constructivist worldview was therefore well suited to this study’s exploration of teachers’ experiences of modelling through their values, beliefs, epistemological assumptions, and ideologies. Using a constructivist lens, the narrative in this study was written to be “more explanatory, more discursive and more probing of the assumptions and meanings for participants in the study” (Creswell, 2012, p. 430).

My epistemological view is that knowledge is constructed by individuals through interactions with others within their social and cultural contexts. This in turn shapes their reality. Implied in this is an understanding that although two individuals may experience the same situation, the subjective meanings they interpret from it may vary according to their cultural perspectives and prior experiences. I sought to interpret the realities of individual teachers as they experienced the modelling, deconstruction, and enactment of pedagogies. My interpretive stance was shaped and supported by my experience as a primary school teacher, teacher leader, and coach. My findings, conclusions, and interpretations were shaped by me, and this was one way of interpreting the experiences of the teachers. The “processes of production of the study required ongoing reflexive attention” (Yates, 2003 p. 224), which

assisted me to represent my interpretations responsibly (Creswell, 2013). My epistemological view seems to be well aligned with the social constructivist paradigm in which this study was situated.

The social constructivist epistemology adopted in this study prepared the ground for a relativist ontology, which acknowledges that multiple truths and multiple realities are encountered that are dependent on social and individual experiences (Guba & Lincoln, 2005). Kincheloe and McLaren (2005) stated that the researcher's task is shaped by the complex connection between ontology and epistemology. In this, ontology positions the researcher's worldview and frames their pursuit of new knowledge (Creswell, 2013). A relativist ontology was demonstrated in this study through using multiple forms of evidence in themes, including the actual words of participants. This provided evidence of the teachers' different perspectives of the professional learning experience. I checked my interpretations with participants, ensuring that my findings were reflective of the teachers' realities (Lincoln et al., 2011).

Case study methodology was chosen to allow an in-depth understanding of how the teachers experienced the phenomenon of modelling and the subsequent deconstruction and enactment of pedagogies.

4.1.2 Case Study Methodology

Case study research is a qualitative approach by which a researcher conducts "an in-depth exploration of a bounded system (e.g., activity, event, process or individuals) based on extensive data collection involving multiple sources of information. *Bounded* means it is separated out for research in terms of time, place or physical boundaries" (Creswell, 2012, p. 465, emphasis in original). It is something that can be "fenced in" (Merriam, 2009).

There have been various philosophical variations of case study research in the literature (Creswell, 2013). For example, Robert Stake and Robert Yin are two well-known researchers

in the field of case study research with contrasting philosophical orientations. Yin's (2014) approach to case study research aligns with post-positivist perspectives that take a scientific slant to research, where inquiry is considered a sequence of logically related steps, and emphasis is on rigorous data collection and analysis (Creswell, 2013).

Stake's (1995, 2006) approach supports the worldview of this present study as it is aligned with a constructivist and interpretive orientation. Stake (1995) stated, "The aim of the research is not to discover an external reality, for that is impossible, but to construct a clearer [experiential] reality and a more sophisticated [integrated] reality" (p. 101). Underpinning Stake's approach is a fervent drive to discover meaning and understanding of contextualised real-life experiences. Stake (2006) emphasised the critical role of the researcher in capturing her interpreted reality. She needs to understand the case, which "requires experiencing the activity of the case as it occurs in context and in its particular situation" (p. 2). Case complexities are detailed through "thick description" (Stake, 2003, p. 140) to convey findings. I found Stake's constructivist approach to case study an appropriate fit for this study, facilitating ways to develop in-depth understanding of teachers' experiences as they participated in the modelling, deconstruction, and enactment of pedagogies.

This multiple case study explored the phenomenon of modelling pedagogies to gain insights into the experiences of teachers who participated in the intervention and any subsequent impact on their knowledge and practice. The intent of the study was to understand teachers' experiences of the phenomenon of modelling pedagogies. The four cases were chosen as the best means to collect in-depth data about teachers' experiences of the modelling and how the phenomenon was experienced at different school sites. Therefore, according to Stake (2006), this study was an instrumental case study because the cases played a supporting role.

4.1.3 Choosing Multiple Case Study

Four cases were chosen to create a stronger study to allow for rich data collection and greater understanding of the phenomenon (Stake, 2006). Multiple case study can provide trustworthy and respected knowledge about how a phenomenon is experienced in different contexts (Stake, 2006). The interpretation of a study is likely to be more convincing with several cases included (Merriam, 2009; Miles & Huberman, 1994; Stake, 2006). In multicase research, the cases share a common characteristic (Stake, 2006). In this present study, the cases (school sites) were bound together through the phenomenon of modelling, deconstruction, and enactment of pedagogies. To capture the essence of the implementation of the modelling, one site is reported in detail. The experience of the other teachers was considered as part of a cross-case analysis. Cross-case analysis is a process of identifying issues within each case and aggregating findings across the cases to illustrate similar themes and predictable differences (Creswell, 2014).

In choosing multiple case study, it was important to define what each case was as “not everything is a case” (Stake, 1995, p. 2). Implied in this is that a case is a thing, not an action; therefore, the phenomenon of modelling pedagogies is not a case. The cases are the participants at each school site. A key component of a quality case study is defining the boundaries of each case. Commonsense boundaries include school sites (Merriam, 2009). The boundaries in the current multiple case study relate to the teachers working in four diverse school settings. Data collection was bounded in each setting by a period of 12 weeks and included observation proformas, audio recorded conversations, interviews, and surveys.

4.1.4 Characteristics of Case Study Research

In a review of case study literature, Merriam (2009) concluded that case studies that are qualitative in nature have four essential properties: *particularistic*, *descriptive*, *heuristic*, and *inductive*. This research is particularistic in that it focused on a phenomenon: how the

teachers experienced the modelling. The reporting of the research is descriptive as it provides a detailed account drawing on detail from a range of sources including interviews, artefacts, and recorded conversations. These data provide precise descriptions from the research participants, and the research discussion of these data provides interpretation of meaning. The researcher uncovered themes to study in each case, and the description includes the themes the researcher unearthed (Creswell, 2013). I analysed the themes across the four cases for similarities and differences. The consequence was a rich thick description (Merriam, 2009; Stake, 2014) of the phenomenon of modelling pedagogies.

The reporting of the research is heuristic as it illuminates teachers' experiences at each site (Merriam, 2009), which has the potential to inform teacher learning. It is also inductive as it relies on inductive reasoning: generalisations about teacher learning emerged from the data. I built my themes from the "bottom up" as I worked inductively, backward and forward between the themes and data, until I ascertained a comprehensive collection of themes (Creswell, 2013). For example, the complexity of the journey of Rose, a participant in this study, who moved through the phases of being confident to losing confidence then regaining a new confidence, demonstrates the appropriateness of an inductive approach. Inferences expressed within the emergent themes enable the reader to gain insight into Rose's lived experiences and realities.

Walker (1983) discussed three common risks associated with conducting case study research. First, case studies are an intrusive intervention in the lives of others. People can feel threatened when being interviewed or observed. The questions that are asked can undermine the facades that people construct to make the job of teaching easier.

Second, case studies may provide biased views of the way things are. There is a danger of overreliance on interview data. The challenge is to balance data from interviews with what is observed. Finally, case studies are conservative in that they capture an instant reality in

time and space. Once fixed, realities do not change, but the people involved may change because of the intervention. Case study methodology enabled me to analyse the diverse experiences of individual teachers as they engaged with the intervention, which assisted to illuminate its essence. This reporting sought to elaborate the complexity and connectedness of the elements of the modelling process.

In any scientific research, it is important that the researcher can demonstrate trustworthiness of their findings (Creswell, 2013). For findings to be authentic, it is important that a study be both credible and trustworthy.

4.1.5 Steps Taken to Ensure Validity and Reliability

Creswell (2014) recommended eight strategies to strengthen validity, which are equally useful to aiding reliability, including prolonged time in the field, triangulation, peer review, negative case analysis, clarifying researcher bias, member checking, rich thick description, and external audits. Creswell (2013) suggested that qualitative researchers engage with at least two of them to increase trustworthiness of a study. In this current study, I utilised six of the recommended validation strategies: prolonged time in the field, triangulation, peer debriefing, clarifying researcher bias, member checking, and rich thick description.

1. *Prolonged time in the field.* This involved building trust and contact over time with participants in the study. My goal in this study was to be accepted by the participating teachers as a knowledgeable other and peer, and for them to feel comfortable during discussions and interviews. I did this by deliberately immersing myself in the teachers' contexts through the modelling itself.

To lead professional conversations and conduct interviews, I entered each of the four school sites on seven occasions over a 12-week period. During each visit, I led professional conversations with each group. Approximately 60 minutes of each visit was allocated to these. Interviews were conducted with the four principals and the four

teachers who modelled pedagogies during the seventh visit. My 20 years' experience in teaching primary school children and my 10 years' experience as a teacher leader provided me with knowledge that supported my development of rapport and empathy with participants during these conversations.

2. *Triangulation*. This involved corroborating different sources of data to check assurances, to clarify key meanings, and to put the whole data set into perspective (Creswell, 2013; Stake, 2006). This study involved establishing themes based on the perspectives of the participating teachers and converging multiple data sources to increase the credibility of my interpretations (Creswell, 2014).
3. *Peer debriefing*. This refers to an external check on the research process. Peer review was actively sought, and deeply appreciated, from my supervisors who asked hard questions about my analysis, interpretations, and conclusions so that the story could resonate with others. School principals also commented on the data and my interpretation of the data. The publication of findings, (for example, Hodgson, 2016) and the associated conference presentations also constituted opportunities for peer review.
4. *Clarifying researcher bias*. I aimed to conduct this research by actively reflecting on my actions, beliefs, and biases and how they influenced my approach. I acknowledge my previous experience as a teacher, teacher leader, and coach and how this influenced my beliefs and assumptions. I sought to demonstrate a reflexive awareness of my interpretations and of the voices of the teachers in the stories I wrote by being clear in communicating my views whilst attempting to avoid “heavy-handed intrusive analysis” (Wolcott, 2009, p. 33).
5. *Member checking*. This assists in triangulating the researcher's observations and interpretations (Stake, 2014). Where possible, participants were asked to read what was

written about the stage of the study they were involved in to find out whether the depiction of the data was accurate. For example, Ruth and Rose (pseudonyms) provided feedback on drafts of Chapter 5 of this thesis.

6. *Rich thick description.* I aimed to “present a body of incontestable description” (Stake, 1995, p. 110) by presenting multiple perceptions and multiple realities (Stake, 2014) of the participating teachers. For example, my inclusion of teachers’ actual words in quotations in Chapters 5 and 6 offers the reader a rich thick description and allows for an in-depth understanding of the phenomenon of modelling pedagogies.

Qualitative research strives to be credible and trustworthy. Many measures were taken throughout this study to enhance these outcomes, including the six discussed, to increase the chances of mutual researcher and reader interpretations.

A fundamental measure was the data collection and processes that occurred.

4.2 Research Methods

This section describes the research methods for this multiple case study, including the research participants and contexts, data collection procedures, the modelling processes, and analysis of responses.

4.2.1 Research Participants and Contexts

Four primary schools were selected by the DoE regional director in North West Tasmania to be part of the project. As described in Chapter 1, the selection was determined by the large percentage of students at each school assessed as being below the benchmarks identified in NAPLAN testing at Grades 3 and 5. Each school had its own contextual complexities and each teacher had a range of teaching experience. These environmental and personal aspects played an influential role in how the teachers responded to each stage of the intervention at each school. Consequently, I considered that to understand the impact of the modelling on teacher practice, I needed to collect data from the four sites and attend to the

context of each school for each intervention. The following discussion details criteria and procedures for the selection of the teachers, as well as background information about each of the school contexts.

4.2.1.1 The selection and characteristics of the research participants

The DoE Tasmania regional director who managed schools in the north west of Tasmania met with principals in her region at the beginning of the school year in 2014 and invited them to participate. Following agreement from the principals, interested teams of teachers in each school were encouraged to participate. Typically, each of the four schools had a team of four to seven teachers involved, with teaching experience ranging from four years to several decades. The teams comprised teachers of lower primary classes (Prep to Year 3) and me (as researcher and modeller). In three schools, assistant principals also participated. In one school, the principal also participated in the team meetings. The principal of each school contributed to the research by participating in exit interviews. The characteristics of the participants are presented in Table 4.1. All names are pseudonyms throughout.

Table 4.1

Overview of Participants, Schools, and Teaching Experience

School	Teacher (Pseudonym)	Years of Teaching
Raven Primary School (RPS)	Rose	15
	Rachel	6
	Rebecca	5
	Ruth	25+
Swift Parrot Primary School (SPPS)	Sheila (assistant principal)	31
	Sophie	5
	Sally	14
	Sabrina	30
	Stella	10
	Suri	4
	Sue	14
Magpie Primary School (MPS)	Maude (assistant principal)	25
	Maggie	0 (preservice teacher)
	Melissa	6
	Molly	36
	Megan	8
	Marita	4
Heron Primary School (HPS)	Heather	30+
	Hannah (assistant principal)	9
	Holly	4
	Henrietta	30+

The research context is next described.

4.2.1.2 Research Context

This research was undertaken in four primary schools in a small regional city in North West Tasmania with a population of approximately 25,000 people. The schools were given the pseudonyms Raven Primary School (RPS), Swift Parrot Primary School (SPPS), Magpie Primary School (MPS), and Heron Primary School (HPS). The Index of Community Socio-Educational Advantage (ICSEA), created by ACARA describes the socioeconomic profile of each school community. The schools' enrolment and ICSEA data are presented in Table 4.2.

Table 4.2

My School Index of Socio-Educational Advantage (ICSEA) Profile of the Four Project Primary Schools (2014)

	Enrolments	ICSEA Value	Lowest Quartile	Middle Quartile		Highest Quartile
School RPS (Government)	159	892	65 %	23%	8%	3%
School SPPS	400	885	64 %	26%	9%	1%
School MPS	318	891	68%	22%	9%	1%
School HPS	140	909	52%	32%	12%	4%
Australian distribution	–	1,000	25%	25%	25%	25%

An ICSEA value brings together family background information (including parental education and occupation) provided to schools directly by families and/or sourced from the Australian Bureau of Statistics and census data for the districts where students live. The ICSEA scores for the four schools ranged from 885 to 909, respectively. These scores are below the Australian average of 1,000, indicating that students in these school communities are considered somewhat disadvantaged. Whilst the percentage of students in the bottom quartile ranged from 52% to 65% and the percentage of Indigenous students ranged from 11% to 21%, the average attendance rate across the four schools was 94%. This indicates a high level of commitment to and engagement with education. The next section discusses the differences between each school context.

4.2.1.3 Differences between the school contexts

RPS is a small school with 159 students enrolled in 2014. The school draws from a wide geographical area including from remote mining and farming communities. There were three teachers from the preparatory (Foundation) to Year 3 team who participated. A fourth external colleague teacher, Ruth, who was on leave from another primary school nearby, asked if she could also participate in the intervention at RPS, and this was agreed to by the

principal. The teachers in this school usually worked in isolation and did not have collaborative planning teams.

SPPS was built in 2011 as an amalgamation of four schools in the geographical area. There were 382 students enrolled in 2014. Seven participants from the Year 2/3 learning team participated, including the assistant principal (AP), Sheila, who also taught mathematics education at the University of Tasmania. The school had structured professional learning teams involving two or three year levels that met for two hours each fortnight during school time. The principal had determined this was a suitable time for the intervention. Sheila (the AP) was the leader of the Year 2/3 team. She reported she was hoping that Sophie would enact the lesson in Stage 4 because she intimated that Sophie required support with pedagogical approaches to teaching. It seemed that the teachers at this school were somewhat apprehensive about the intervention because of time taken from them for usual planning activities.

MPS had five participants from the Foundation to Year 2 team who met regularly for collaborative planning led by the AP, Maude. This school had a new principal who had commenced a short time before the onset of the intervention. Her involvement in the intervention was peripheral. The five teachers reported they met regularly to plan collaboratively with Maude. The group generally seemed cohesive in their collaborations. Whilst the teachers expressed their views that the intervention would be beneficial, they suggested that for them, the timing was inopportune. For example, in her exit interview Marita reported, “To begin with we thought it would be beneficial but we also thought it was *another thing* that we had to do and the timing wasn’t good as it was in our most crucial part of the day”. She placed emphasis on the words “another thing” during the interview. Marita was encouraged by her colleagues to volunteer to enact the lesson in Stage 4 as she was perceived to be an exemplary teacher.

HPS had five participants from the Foundation to Year 2 team, including the principal who also taught the Year 6 class: Hannah, the AP and numeracy coach with nine years' teaching experience; Heather, a late-career teacher with 30 plus years of teaching; Henrietta, a late-career teacher with 40 plus years' teaching experience who retired shortly after the intervention concluded; and Holly, an early career teacher with four years' teaching experience. Heather and Hannah were part of the leadership team. A fifth participant, Jackie, was a literacy lead teacher for the region who also observed the modelling in Stages 1 and 2 in the hope of emulating the intervention with a focus on literacy in other schools. The principal of this school reported he had been looking for a way of encouraging teachers to be involved in each other's classrooms, as they all worked in isolation. Heather and Hannah reported they were excited by the intervention. Hannah (AP and numeracy coach) volunteered to enact the lesson in Stage 4 even though she did not have a class. She reported that this was because of the reluctance of other participants.

In the next section, details regarding the data collection procedures are presented.

4.2.2 Data Collection Procedures

Data collection occurred over a four-month period and included a collection of pre- and post-intervention surveys, a collection of modelled lesson request forms from participating teachers, audio recorded pre- and post-lesson professional conversations between participating teachers, exit interviews with four teachers and four principals, and observation of teaching practice proformas. Each approach is outlined in the following paragraphs.

1. *Collection of pre- and post-intervention surveys:* Brewer (2009) asserted that surveys are useful for gathering information about the characteristics, attitudes, thoughts, and beliefs of groups of people. In this study, pre- and post-intervention surveys were conducted to ascertain teacher attitudes towards teaching mathematics and contemporary pedagogical practices and whether any change in teacher attitudes,

perceptions, or practice occurred over the course of the intervention. The 18 teachers completed the pre-intervention surveys (Appendix 2) during a team meeting. They completed the exit surveys (Appendix 6) 12 weeks after the intervention, also during a team meeting.

2. *Collection of modelled lesson request proformas from participating teachers:* The rationale for this proforma (Appendix 1) was to ensure that the modelled lessons met perceived teacher needs, that the teachers felt they were participants in the planning process, and to determine the priority they allocated to the content and proficiency strands of the AC:M (ACARA, 2016).
3. *Exit interviews with four teachers and four principals:* As states of mind, such as feelings, moods, or emotions, are crucial to be able to describe a phenomenon in consideration (van Manen, 1990), data gathering from interviews to elicit teachers' beliefs, perceptions, evaluations, and suggestions was an important component of the study. Brewer (2009) argued personal interviews provide "in-depth" data because interviewers can clarify questions and develop rapport with the respondents, increasing trust and expecting more honest answers.
4. *Audio recorded pre- and post-lesson professional conversations between participating teachers:* Professional conversations between teachers and observers following an observation provide the best opportunity for teachers to reflect on and think through how they could improve their practice (Danielson, 2012). Professional conversations were recorded immediately before and immediately after each modelled lesson. The conversations began with a focus on what teachers had written on their observation proformas. Each teacher was given the opportunity to share her observations in a round-robin fashion.

5. *Observation of teaching practice:* Teacher observation proformas (Appendix 3) were designed to evaluate the experience of the modelled lesson observation with respect to teachers' attention to pedagogical actions associated with facilitating student reasoning. Classroom observation can promote teacher learning if observers are clear about what to look for (Danielson, 2012). For this reason, teachers were given a narrow focus. The focus was "Teacher actions to facilitate student reasoning".

Table 4.3 provides a picture of how the research questions aligned with the data collected. It was helpful for guiding the process of data analysis.

Table 4.3

Summary of Correlation Between the Research Questions and the Collected Data (X Indicates the Type of Data Collected)

Research Questions	Pre- and Post-Modelled-Lesson Teacher Surveys	Pre-Modelled-Lesson Request Form	Lesson Observation Proforma	Personal Journal	Emails	Audio Recorded Pre- and Post-Lesson Professional Conversations	Exit Interviews	Self-Reflection
1. What teacher assumptions are challenged through observing a modelled lesson?			X		X	X		
2. What is the nature of teacher learning from the observation?			X		X	X	X	
3. What is the nature of my learning from the experience?				X	X			X
4. Can an educational intervention involving modelling pedagogies, deconstruction of those pedagogies, and enactment improve mathematics teaching and learning? If so, in what ways?	X	X	X		X	X	X	
5. What do teachers and principals say about modelling pedagogies, deconstruction of those pedagogies, and enactment as a professional learning strategy? What explanations do they give?					X	X	X	

The next section describes the modelling processes used in each intervention. The rationale for a focus on reasoning and problem-solving is explicated. Next, the stages of the implementation of the modelling are described.

4.2.3 Modelling Processes in Each Intervention

At each school site, the modelling process involved implementing a planned, scaffolded sequence of teacher learning that gradually released responsibility from me as a knowledgeable other to the participants. This was intended to support the teachers in developing the confidence over time to implement changes to their practice. I modelled two lessons as a first step in building trust. The intention of the modelling was to build on and extend teachers' views of pedagogical approaches to develop students' problem-solving and reasoning and to encourage collaboration and reflection.

4.2.3.1 Intervention's focus on pedagogies for developing student reasoning and problem-solving

In considering pedagogies to model that were particular to mathematics teaching and relevant to the teachers' curriculum context, I chose to focus on two AC:M proficiency strands (ACARA, 2016). The four proficiencies—understanding, fluency, reasoning, and problem-solving—describe the processes students undertake to engage effectively with the content of the AC:M. The reason for the emphasis on reasoning and problem-solving in the modelling and subsequent teacher learning is that, in my experience as a coach, I have found reasoning and problem-solving approaches to be relatively unfamiliar to several teachers. Consequently, I assumed that the teachers in this study may have had few opportunities to engage in detailed consideration of related pedagogies. These proficiencies require particular pedagogies in the mathematics classroom. As Fraivillig (2004) suggested, students are more likely to reason if they have developed their own strategies and justifications for solving problems, which requires thinking for themselves. This takes thinking time and comes only when students are unable to easily solve tasks they are working on (Smith & Stein, 2011).

This implied that the modelled lessons should include challenging tasks, which would allow problem-solving and thinking time as a first step in facilitating student reasoning (Sullivan et al., 2016). Given that this is a different structure from conventional teaching, it was considered a beneficial vehicle for researching teacher learning through modelling.

The next section describes the stages of the implementation of the modelling.

4.2.3.2 Four stages of each intervention

The four stages of implementation of the modelling at each school site were chosen to facilitate a scaffolded approach between the observing teachers and me. The stages were spaced two weeks apart to allow participating teachers time to reflect on their experiences. The stages are now described.

School principals initiated the communication by inviting me to meet with teachers in their school. This meeting involved outlining the project and familiarising the teachers with the proficiency strands of the AC:M (ACARA, 2016) and pedagogies for facilitating student reasoning and problem-solving. During this meeting, the participants completed a pre-intervention survey (see Appendix 2).

Following the initial meeting, I emailed participants a lesson request proforma (Appendix 2) to have them complete and send back. The proforma contained the following questions:

1. What would you like the mathematical focus of the lesson to be? (Please feel free to use content descriptors from the Australian Mathematics Curriculum.)
2. What would you like me to focus on with respect to pedagogy? (For example, questioning, differentiation, open-ended tasks, structure of the lesson, materials used.)
3. What would you like to observe during the lesson with regard to student learning?

The participating teachers nominated a specific mathematical and pedagogical focus for the first modelled lesson. Attending to the teachers' own preferences was intended to

maximise the potential for them to see themselves as partners in the intervention. Their choice was connected to their classroom contexts and their perceived needs.

Next, I planned a lesson based on the teacher responses to the questions in the proforma. This aimed to provide teachers with an exemplar in which they could see how the lesson objectives and tasks connected with the AC:M (ACARA, 2016).

4.2.3.3 Stage 1: Modelling

Stage 1 involved presenting a modelled lesson, supported by preliminary meetings with each team of teachers and a subsequent debrief of their observations. The intention was to enact specific pedagogical actions that were different from the likely usual practice of at least some of the teachers. A narrow focus for the observation was chosen deliberately to direct the attention of the teachers to the pedagogical actions being modelled, which were associated with facilitating student reasoning through problem-solving.

I met with observing teachers for a 30-minute prebrief prior to the modelled lesson. During this meeting, teachers were given a copy of the lesson plan, and it was discussed. I outlined the tasks I had planned, the prepared enabling and extending prompts (Sullivan et al., 2006), connections to the AC:M (ACARA, 2016), and questions I intended to ask the students. At the end of the meeting, I handed teachers an observation proforma, which I invited them to complete during the lesson. The observation proforma (see Appendix 3) contained the following prompt:

Write down everything you saw the modelling teacher say and do to facilitate the students' reasoning.

The observing teachers included the teacher of the class where the modelling took place and the members of that teacher's professional learning team or teachers of similar grade levels who were participating in the study. During the modelled lesson, the observers also were encouraged to make notes on anything of interest to them.

After the lesson, all the observing teachers and I met for a 30-minute debrief. Their responses to the modelling were discussed, with the observers having the opportunity to share their experiences and perceptions of the lesson and particularly the modelled teacher actions they observed that focused on student reasoning. Each discussion was audio recorded. At the conclusion, I handed participants a proforma, which they were asked to then complete. The proforma contained the following prompt:

What teacher action/s did you observe today that are different from what you usually do that you will set as a goal for your future classroom practice?

4.2.3.4 Stage 2: Modelling in another class

Stage 2 entailed presenting a second modelled lesson two weeks after the first modelled lesson. The intention of this stage was to explore whether observing a second modelled lesson would help teachers notice that pedagogies to facilitate reasoning and problem-solving could be transferred to other lessons, grades, and domains of mathematics. The processes of the 30-minute prebrief and the 30-minute debrief were repeated as per Stage 1. Again, the teachers requested the lesson content focus and the lesson was planned by me in response. The team met beforehand to discuss the lesson, and I also modelled this lesson. Participants were given an observation proforma (see Appendix 3).

During the post-lesson debrief, I handed teachers a proforma with the following prompts (Appendix 4):

1. Are there any teacher action/s that you have implemented in your classroom practice as a result of observing the modelled lesson?
2. What teacher action/s did you observe today that are different from what you usually do that you will set as a goal for your future classroom practice?

4.2.3.5 Stage 3: Co-planning and co-teaching

Following the processes in Stages 1 and 2, Stage 3 took place two weeks later with the variation that the same group of teachers now assisted in co-planning a lesson. The goal of Stage 3 was to support the teachers to plan and co-teach a lesson with a focus on facilitating student reasoning and problem-solving. The purpose was to scaffold the transition from observation of a modelled lesson and learning about new pedagogies to planning for implementation and enactment by teachers in their own classrooms. The co-planning session was 90 minutes in length and took place at least one day prior to the Stage 3 co-teaching. One of the team members self-selected to co-teach the lesson with me after the co-planning session. Prior to the lesson, the co-teacher chose which parts of the lesson she felt comfortable modelling. The remaining participants then observed the co-taught lesson. A 30-minute post-lesson meeting enabled a subsequent debrief of their observations.

4.2.3.6 Stage 4: Enactment

Stage 4 took place two weeks after Stage 3 was completed. The purpose of this stage was to ascertain any actions the teachers might choose to implement themselves after observing the modelled lessons. It involved one participating teacher in each school self-selecting to present a lesson. This was supported by co-planning the lesson with the presenting teacher. Co-planning was negotiated in terms of the perceived needs by the presenting teacher. For example, it could include my support and the support from the other participating teachers.

The presenting teacher met with participating colleagues and me for a 30-minute prebrief prior to the lesson. During this meeting, she discussed her lesson plan. The presenting teacher chose her own observation focus that was connected to the students' reasoning and problem-solving (see Appendix 5). This supported her in gaining feedback for an action she wished to trial. Participants from her school site and I observed the lesson. A

30-minute post-lesson debrief enabled participating teachers to talk about their observations and give feedback to the presenting teachers. The lesson was filmed, and afterwards, the presenting teacher was encouraged to view the footage. A week later, she was interviewed about her reflections on the lesson.

This presenting teacher was later interviewed 12 weeks after her enactment to determine what, if any, changes to her practice might have occurred. At this time, an exit survey was also given to all participants to complete (see Appendix 6). The four principals were interviewed to ascertain their perceived impact of the intervention at their schools. The interviews were audio recorded and the data were analysed.

The next section describes the method for the data analysis.

4.2.4 Analysis of Responses

In this study, Ball et al.'s (2008) MKT framework guided the data analysis and interpretation of teacher responses from the teacher observation proformas.

The following process of analysis was undertaken for these data:

1. Written responses were read and inductively coded line by line to form categories (Creswell, 2013, 2014).
2. The responses were inspected and the initial coding revised (Creswell, 2013, 2014).
3. The teacher responses were sorted into the categories of Ball et al.'s (2008) MKT framework.
4. The grouped responses were sorted into subcategories of the MKT framework.

The categories and inferences were also discussed with my doctoral supervisors.

In addition to the use of Ball et al.'s (2008) MKT framework, the analysis of data involved inductive coding for exploring any themes related to teacher professional learning that emerged from the data (Guba & Lincoln, 1989).

In analysing all other responses, regularities and themes that came from the data across the four interventions were sought to establish coding categories. A phenomenological lens was employed in analysing data (van Manen, 1990) from the surveys, conversations, and pertinent artefacts, such as lesson plans. Qualitative coding approaches suggested by Hycner (1985) in analysing these data were adopted. These included

1. having audio recordings transcribed;
2. bracketing of interview data by suspending meanings and interpretations;
3. listening to transcriptions to get a sense of the whole;
4. delineating units of meaning from the transcriptions;
5. delineating units of meaning relevant to the research questions;
6. clustering units of relevant meaning to determine themes.

Significant statements and utterances from the interviews and conversations and vignettes from the observations were highlighted to provide an understanding of how participants experienced the modelled lessons. From these, clusters of meanings were formulated into emergent themes.

The following section describes my processes of reflexive practice.

4.2.5 My Processes of Reflexive Practice

An unanticipated aspect of the research was the impact it had on me as a teacher of the teachers. I commenced the modelling process presupposing that if teachers saw examples of alternative pedagogical approaches that could be effective, they would be inspired to enact those practices for themselves. I found that the process of exploring alternate ways of educating teachers is complex, and that my role as a teacher of teachers was similarly multidimensional and nuanced. As part of the reporting of the research, I drew upon aspects of self-study as a way to make sense of my own experiences and for describing my processes of reflexive practice.

Reflexivity is fundamentally concerned with questioning one's own assumptions and ways of being, relating, and acting (Cunliffe 2009). However, it is much more than simply reflecting on what has taken place; "it involves actively considering the implications of what has been observed for the observer's own practice" (Easterby-Smith & Malina, 1999, p. 77). I sought through reflection to improve my practice and contribute to collective understandings of teacher educator learning. Data were drawn from multiple sources, including formal and informal discussions with the teachers, a researcher's journal, and my observations of practice. The different data sources are presented next.

4.2.5.1 Sources of data

The data sources for my reflexive practice were designed to create opportunities for me to consider on my practice as a teacher of the teachers from different perspectives. They included

1. a researcher's journal of my experiences at the four sites;
2. audio recordings of each pre- and post-lesson meeting with the participating teachers;
3. email correspondence with participating teachers, in which ideas about modelled lessons and teacher education were explored, and email correspondence with my supervisors, in which I raised issues to clarify my thoughts; and
4. field notes from the teacher observation proformas and lesson plans.

This range of data sources enabled me to take a stance from both within and outside myself (Brookfield, 1995, as cited in Berry, 2007) to enhance my reflexivity as a teacher educator. The choices of these data sources are now explained in more detail.

4.2.5.2 Journal overview

Teachers use journaling to reflect on their teaching practice. As Adler (1993) stated, journal writing serves as "a vehicle for reflection which then allows us to return to practice more thoughtfully, with, we hope, greater wisdom" (p. 163). I documented anything that I

perceived to be important during each intervention and wrote brief comments from listening to the recordings of teacher discussions in pre- and post-lesson meetings. I repeatedly engaged in reflexive practice to consider my own intended pedagogical practices and my enacted practices, teacher reactions to the modelling, and how I modified myself—my beliefs, attitudes, practice—because of the experiences.

4.2.5.3 Audio recorded professional conversations

Talking collaboratively about teaching can provide new insights into experiences. This is not possible when working in isolation (Brookfield, 1995, as cited in Berry, 2007). I met with participating teachers at each stage of the intervention for 30-minute post-lesson discussions. In these discussions, teachers shared what they had observed and anything else pertinent to the discussion. The discussions were audio recorded. Listening to the recordings after the events gave me great insights into the teachers' experiences and provided the opportunity to view myself "from the outside" and to compare myself with how I saw myself internally during the conversations. In this way, an important function of the audio recordings was for me to hear what I could not otherwise perceive in my dialogue with teachers.

4.2.5.4 Email correspondence

Email offers a form of information technology that enables reflection on and the reframing of practice (Berry, 2007). In this study, email provided an avenue for discussing practice. I had regular email conversations with Ruth (RPS), Heather (HPS), and Sabrina (SPPS), participating teachers in the intervention. The three teachers initiated these conversations after I had invited them to email me with any questions or concerns they might have had. They asked questions regarding the intervention. This led to a regular exchange of reflections and thoughts about teaching and learning. For example, Heather pushed me to consider the processes I was using in the intervention and how I was building trust with

participating teachers. Although this was not always easy, I found it valuable for my own professional growth.

4.2.5.5 Field notes overview

Field notes provide a written account of observations, situational details, and interactions during qualitative research (Berry, 2007). The lesson plans I prepared gave me a frame by which to make each lesson's goals and ideas explicit to the teachers. Over the course of the study, the lesson plans were refined as I reflected on the meanings the teachers had communicated about them. Having participants complete observation proformas during the modelled lessons enabled me to reflect on what they saw and refine my practice so that the pedagogies I intended to model became more explicit. The observation proformas also provided a frame for discussion after the lessons.

In summary, my reflections drew on a range of data sources to help me understand my practice from the perspectives of those who experienced it. My findings related to my reflexive practice are presented in Chapter 7.

4.2.5.6 Data analysis

In analysing the data for the substantive study, I also reflected on my role in the process of presenting the modelled lessons and gathering the data. Conversations with other educators, including my supervisors, assisted in challenging me and refining my practice. I adopted Berry's (2007) suggestions for identifying what might have been problematic in my own practice. Similarly, I identified incidents during my practice that were challenging or confronting for me during the research. I found that I needed to question held assumptions about teaching and learning. I also explored the differences between my intended pedagogical approach with teachers and my actual approach. Utilising Berry's (2007) framework, I identified certain tensions as I attempted to manage conflicting pedagogical demands within the research as a teacher of teachers and as a teacher of children.

4.2.6 Chapter Summary

This chapter outlined the multiple, instrumental case study research design for this current study. The research methodology section focused on justification and explanation of the case study approach, and steps taken to enhance the validity and trustworthiness of the study, along with ethical considerations. The research methods then detailed the research participants and contexts, data collection procedures, the modelling processes, and analysis of responses.

In Chapter 5, I present an in-depth description and analysis of the first case in the current research project. The chapter includes the story of Rose (pseudonym), an experienced teacher with more than 15 years' teaching experience. She moved through the phases of being confident to losing confidence then regaining her confidence by transforming her teaching to enact new pedagogies.

CHAPTER 5: FROM OBSERVATION TO ENACTMENT

This chapter presents the data and interpretation of findings of the study at one site (RPS) and includes the story of Rose, an experienced teacher with more than 15 years' teaching experience, who moved through the phases of being confident to losing confidence then regaining her confidence by transforming her teaching practice to enact new pedagogies. The chapter is presented in three sections. First, the research aim, questions, and research context are presented. Next, the formulation of meanings and identification of themes are explained. Following this, the themes are explicated in detail to provide answers to the three research questions for this chapter. The ways in which Rose appeared to respond to and make sense of the intervention are discussed in a detailed exposition reflecting the phenomenon of teacher professional learning in consideration.

5.1 Research Aim and Questions for This Chapter

This chapter explores the following research aim and questions:

Aim	Questions
1. To explore the elements of modelling that were likely to lead to changes in teacher practice.	1. What teacher assumptions are challenged through observing a modelled lesson? 2. What is the nature of teacher learning from the observation? 3. Can an educational intervention involving modelling pedagogies, deconstruction of those pedagogies, and enactment improve mathematics teaching and learning? If so, in what ways?

5.1.1 Research Context

This cycle of the project was undertaken at RPS, a small rural and remote primary school in North West Tasmania. In an email to the principal, the intervention was explained. Having read the plan of the four stages (articulated in Chapter 4), the principal invited three teachers from Grades Prep/1, 1/2, and 3, respectively, to participate. After obtaining agreement from the teachers (Rose, Rachel, and Rebecca), the principal emailed me,

explaining who would be involved. A fourth external colleague teacher, Ruth (pseudonym), also participated. Rebecca was absent for the Stage 3 planning meeting and co-teaching observation and the Stage 4 enactment. Prior to commencement of the intervention, the teachers were invited by their principal to volunteer to enact the pedagogies that were the focus of the modelling in Stage 4. Whilst the principal reported later that Rose was an experienced and confident teacher who would be the best one to present the Stage 4 lesson, Rose mentioned to me that she was the only one who volunteered. This suggests that Rose felt confident about her teaching practice. From my perspective, it seemed the other three teachers may have felt less confident than Rose. For example, during the initial pre-intervention meeting, Rebecca and Rachel appeared to be quite reticent to talk about the proposed intervention, but Rose seemed confident and stated, “I would prefer that you would just come and watch me teach rather than go through this process. I have been involved in Count Me in Too [A professional learning program to support the development of students understanding of number] and I’m aware of effective teaching strategies”.

5.1.2 Intervention

The intervention consisted of four stages, each involving a prebrief, modelled lesson, and debrief as described in Chapter 4. There were three opportunities for teacher learning in each stage described as follows.

5.1.3 Preliminary Data Collection

To initiate Stage 1 of the intervention, I emailed a lesson request proforma (Appendix 1) to observing teachers for them to complete and return. The completed proformas were returned, and the teachers indicated that they would like the Stage 1 modelled lesson to be in Grade 3 with a content focus on problem-solving and a pedagogical focus on explicit teaching (instructional model), questioning, differentiation, and open-ended tasks. It was interesting that teachers requested problem-solving as a content focus. Part of the rationale for inviting

them to request content and pedagogical foci was to determine the priority they allocated to the content and proficiency strands of the AC:M (ACARA, 2016). In response, one of the goals of the intervention in this school, therefore, was to demonstrate that problem-solving was a student learning process rather than a content focus. To do this, the teacher requests were woven into a discussion about the proficiency strands in an introductory meeting.

In the meeting, I surveyed participants by a written survey (see Appendix 2) to establish their perspectives of mathematics teaching and learning. The survey contained the following prompt:

Effective teachers use the following pedagogical approaches when catering for a diverse range of learning needs in a mathematics lesson:

The teachers responded with the following:

Explain learning intentions and success criteria; focused teaching of specific skill or knowledge which is modelled; children work in small groups on differentiated learning tasks; work with teaching group; reflect with whole class. (Rose)

Small group work targeted to abilities. (Rachel)

Groupings, group and individual instruction and whole-class instruction. (Rebecca)

Open-ended questioning. (Ruth)

It seemed from these statements that the teachers valued group work and questioning to differentiate learning experiences, but overall, their responses were worded in general terms. Rose's more detailed comments seemed to indicate her preference for focused teaching in groups. The next section describes the formulation of meanings and identification of themes.

5.2 Formulation of Meanings and Identification of Themes

I interpreted the meanings from the data from my perspective using an interpretive approach with a phenomenological lens (van Manen, 1990). Regularities, patterns, and topics that emerged from the data were sought to establish coding categories. Units of data were

mostly spoken or written sentences. First, meanings were formulated and clustered into emerging themes. Following this, themes were collapsed into five main categories: (1) challenging teacher assumptions, (2) teacher learning from observing, (3) teacher learning from deconstructing pedagogies, (4) teacher learning from enactment, and (5) reactions to the intervention.

A table consisting of two columns was created (see Appendix 7). The first column comprised significant literal quotations from the teachers from the body of data. The second column consisted of my formulated meanings. This illustrated how statements from the teachers were formulated into meanings that would provide answers to the research questions.

Subsequently, meanings were clustered to identify emerging themes. There were 14 themes identified as follows: (1) challenging ideas about teaching, (2) confronting ideas about catering for diversity, (3) fearing student-generated responses to tasks, (4) perceiving disengagement of some students, (5) noticing different pedagogies, (6) choosing cognitively complex tasks, (7) building mathematical knowledge for teaching, (8) Building trust and learning from each other, (9) trialling an observed lesson, (10) shifting towards student-centred approaches to teaching, (11) seeing, deconstructing, and enacting pedagogies, (12) engaging students, (13) differentiating the experience for teachers, and (14) accountability.

Table 5.1 shows how the meanings were clustered to form the themes that emerged from the analysis.

Table 5.1

Selected Theme Clusters With Their Related Formulated Meanings

Themes	Clustered Meanings	Illustrative Examples of Direct Quotes from the Teachers
Challenging ideas about teaching	Student difficulties in articulating mathematical reasoning	<p>“They [students] didn’t have the language to explain . . . We would have explained things to the nth degree where perhaps it’s better not to”. (Rachel, Debrief, Stage 1)</p> <p>“Some kids found it difficult to explain . . . I noticed that they need more practice on this kind of pedagogy”. (Rebecca, Debrief, Stage 1)</p>
	Students thinking for themselves before instruction from the teacher	<p>“In the past, we have taught strategies—for example, the jump strategy, the bridge strategy, whereas this [new approach] is asking for students to come up with the ideas themselves. That is a huge turnaround”. (Ruth, Debrief, Stage 2)</p>
Fearing student-generated responses to tasks	Responding to students thinking in the moment	<p>“I am used to lessons where I know where I am going, but in this approach, it is more about the response from the children, so it could go anywhere and I’m not confident to know where to take it or what response to give”. (Ruth, Debrief, Stage 2)</p>
Confronting ideas about catering for diversity	Uncertainty about allowing students to grapple with tasks where the solution was unknown	<p>“The fundamental difference with what you did was that you did not teach in ability-based groups. It is a whole swing for me to think differently”. (Ruth, Debrief, Stage 1)</p>
		<p>“The enabling questions allow them [students] to enter the learning but do they need more explicit instructions, more teacher focus time?” (Rachel, Debrief, Stage 2)</p>
Perceiving disengagement of some students	Students seemingly “off task”	<p>“I got distracted when I stepped back and thought, that one needs more talking to, that one has no idea, that one hasn’t got pen to paper yet”. (Rose, Debrief, Stage 2)</p>
Noticing different pedagogies	Students thinking for themselves	<p>“There seems to be so much more thinking and reasoning for the students rather than just going through the process [of telling them what to do]”. (Rebecca, Debrief, Stage 1)</p>
		<p>“[You] reinforced the expectation that the children would do the thinking and reinforcing that it’s okay to be confused”. (Rose, Debrief, Stage 1)</p>

Themes	Clustered Meanings	Illustrative Examples of Direct Quotes from the Teachers
	Students justifying, analysing, and generalising	<p>“You asked [the students] questions, ‘What were you thinking, can you explain that’. You kept prompting, ‘What did you notice about this strategy?’ (Rose, Observation Proforma, Debrief, Stage 1)</p> <p>“What advice would you give to others Jake [pseudonym]?” (Rachel, Observation Proforma, Stage 1)</p>
	Repeating and rephrasing students’ explanations to build on ideas	<p>“You rephrased and reiterated student strategies”. (Rachel, Observation Proforma, Debrief, Stage 1)</p>
	Modelling mathematical language	<p>“Named up [the student’s strategies] building to the next 10, sometimes called bridging 10”. (Rachel, Observation Proforma, Debrief, Stage 1)</p>
	Thinking time	<p>“You asked Tony [Pseudonym] to explain . . . but he was struggling and you said I’ll come back to you . . . then you came back [pause] that gave him the time to think about how he was going to explain”. (Rachel, Debrief , Observation Proforma, Stage 1)</p> <p>“You gave the students a lot of thinking time”. (Ruth, Observation Proforma, Debrief, Stage 1)</p>
	Summarising learning	<p>“When you regrouped to the mat you said, ‘so you learnt you can break up numbers to help you with your adding up’”. (Rachel, Observation Proforma, Debrief, Stage 1)</p>
	Use of document camera to make student thinking and reasoning visible	<p>“Used a document camera. Showed some good strategies on IWB [Interactive white board] from kids. Tom [pseudonym] came up with the fastest strategy. [Louise] got him to explain and [put his example] on the IWB”. (Rebecca, Observation Proforma)</p>
	Materials	<p>“Used real concrete materials connected to the task”. (Rose, Observation Proforma, Stage 2)</p>
	Scaffolding students in reaction to their responses in the moment	<p>“Stopped . . . changed lesson, reduced from $298 + 35$ to $28 + 7$. . . Build up $28 + 7$, $98 + 7$, $198 + 7$, scaffold”. (Rose, Observation Proforma, Stage 1)</p>
	Enabling and extending prompts	<p>“Teacher circulating giving enabling questions . . . Gave Tom [pseudonym] and others extending prompts . . . and asking questions to push them further”. (Rebecca, Observation Proforma, stage 1)</p>

Themes	Clustered Meanings	Illustrative Examples of Direct Quotes from the Teachers
	Surprise at engagement of perceived low-attaining students	“I liked it when Lily [pseudonym] said ‘I don’t understand’. Then later in the lesson, she really surprised me by her understanding. She solved the problem with 24 apples and could explain her thinking. Normally she . . . does not engage” (Rachel, Debrief, Stage 3)
	Encouragement to persist	“Zone of confusion, ‘It’s okay to be confused’”. (Rose, Observation Proforma, Stage 1) “Keep thinking”. (Rachel, Observation Proforma, Stage 1)
Choosing cognitively complex tasks	Recognising the importance of choosing tasks that are challenging and meaningful to students	“Having a good task in the first place drives you . . . and giving children a real-life problem to solve so that they can see a purpose for solving it”. (Ruth, Planning, Stage 3) “I’m struggling to create open-ended questions for all areas of the mathematics curriculum”. (Rebecca, Debrief, Stage 2)
Building mathematical knowledge for teaching	Recognising the importance of being clear about the purpose of their teaching	I did not know about the proficiencies. I was focused on content”. (Ruth, Planning, Stage 3) “We are looking at the curriculum . . . but I don’t think we are unpacking the rationale . . . What is the deeper understanding we want them [the students] to know?” (Rachel, Planning, Stage 3)
Building trust and learning from each other	Shifting from working in isolation to working together	“The opportunity to have that professional dialogue with your colleagues and having someone else with expertise in mathematics that I could draw on was the most valuable for me”. (Rose, Exit Interview)
Trialling an observed lesson	Lesson plan exemplar	“I want to learn to teach like that but there is so much I need to change. I want to copy your lesson to practise first . . . I need to get a feel for the language and questioning”. (Ruth, Debrief, Stage 2)
Shifting towards student-centred approaches to teaching	Purposeful planning, problematic tasks, high expectations for all students	“I thought much more about this lesson before I planned it”. (Rose, Debrief, Stage 4) “I am now conscious of the types of questions that I ask . . . probing questions and adapting those to suit children’s learning needs. I’m also having much higher expectations of the children as they may surprise me by their responses”. (Rose, Exit Interview)
Seeing, deconstructing and enacting pedagogies	Experiencing the intervention	“You are experiencing the whole package—you are seeing, you are getting involved, it is multi modal. You’re not just hearing it or reading it . . . you are getting the full picture”. (Rebecca, Exit Interview)

Themes	Clustered Meanings	Illustrative Examples of Direct Quotes from the Teachers
Engaging students	Student persistence and confidence	<p>“I observed deeper understanding of the Australian Curriculum; differentiation of tasks; questioning—enablers and extenders, to support and extend learners; acceptance of a range of answers and thinking; planning of better tasks; and the development of mathematical language through questioning”. (Principal, Exit Interview)</p> <p>“I am getting a better result from the students . . . The kids all pipe up when I say, ‘zone of confusion’ . . . They have the confidence and ammunition to have a go”. (Rebecca, Exit Interview)</p>
Differentiating the experience for teachers		<p>“[Offer] more than one co-teaching session to build confidence if needed. Another watched session to ‘improve/tweak’ practice further after initial feedback [would improve the intervention]”. (Principal, Exit Interview)</p>
Accountability	Structured inquiry and accountability	<p>“This form of modelling and coaching was very effective professional learning—it gave teachers support to practice new learning. When teachers go to external professional learning, they come back full of new ideas but most rarely implement them, there is no accountability, there is no feedback . . . it feels like wasted opportunities, and costly”. (Principal, Exit Interview)</p>

Finally, the themes were collapsed into five main categories, which were drawn from my theoretical framework outlined in Chapter 2. This shows how previous research (in particular, Ball et al.'s (2008) model describing different types of MKT, Anthony and Walshaw's (2009) principles of effective teaching of mathematics, Grossman et al.'s (2009) pedagogical approach to learning to teach, and Clarke and Hollingsworth's (2002) interconnected model of teacher growth) formed the foundation for this study. In particular, the categories of teacher learning from observing, teacher learning from deconstructing pedagogies, and teacher learning from enactment were drawn from Grossman et al.'s (2009) approach. Table 5.2 shows the themes that fall into the five main categories.

Table 5.2

Categories Identified and Their Associated Themes

Categories	Themes
(1) Challenging teacher assumptions	Challenging ideas about teaching Uncertainty about anticipated student-generated responses to tasks Confronting ideas about catering for diversity Perceiving disengagement of some students
(2) Teacher learning from observing	Noticing particular pedagogies
(3) Teacher learning from deconstructing pedagogies	Choosing cognitively complex tasks Building mathematical knowledge for teaching Building trust and learning from each other
(4) Teacher learning from enactment	Trialling an observed lesson Shifting towards student-centred approaches to teaching
(5) Reactions to the intervention	Seeing, deconstructing, and enacting pedagogies Engaging students Differentiating the experience for teachers Accountability

5.3 Category 1: Challenging Teacher Assumptions

In this section, four themes that emerged from the data within the category of challenging teacher assumptions are presented. These themes are (1) challenging ideas about teaching, (2) confronting ideas about catering for diversity, (3) fearing student-generated

responses to tasks, and (4) perceiving disengagement of some students. The first is challenging teacher orientation to teaching as “telling”.

5.3.1 Theme: Challenging Ideas About Teaching

One of the primary findings of this study was that the observation of students’ attempts to articulate mathematical reasoning prompted the four teachers to reflect on their apparent orientation to teaching. I interpreted this to mean it was seeing their role as telling students what to do. If, for the purposes of this discussion, teaching is viewed as telling students what to do, and if learning is described as students constructing their own knowledge based on experiences prompted by teachers, then teachers at this site seemed to be more oriented to teaching as telling.

The children didn’t have the language to explain [pause] We would have explained things [to the children] to the nth degree where perhaps it’s better not to [pause] I wasn’t even aware of it until I saw it happening. (Rachel)

Some kids found it difficult to explain . . . I noticed that they need more practice on this kind of pedagogy because a lot of the stuff I do in the class is very linear, for example, this is the process, this is how you solve the problem, this is what I’m wanting you to do. (Rebecca)

These comments seem to indicate the students were unfamiliar with explaining their reasoning. Unsurprisingly, in the literature, students’ limited capacity to explain reasoning has been associated with teaching that is focused on demonstration and practice (Hiebert & Grouws, 2006; Sleep, 2012). Furthermore, the four teachers in this present study articulated that their orientation towards demonstration and practice may have impeded their students’ capacity to reason. It seems they attributed students’ difficulties to their teaching. This resonates with Wilson, Edgington, Sztajn, & DeCuir-Gunby (2014), who found that attributing students’ difficulties to teaching is one of eight possible attributions that teachers ascribe to. Rachel implied she was unaware of the impact of “telling” until she saw the

students struggling to explain. Interestingly, this finding is at odds with Bragg and Vale, (2014), who found that teachers in their study attributed students' limited mathematical understandings to students' ability. A possible reason for this difference could be that in Bragg and Vale's (2014) study, teachers were asked to describe examples of student reasoning that they observed. A subtle difference in this present study was that the teachers were asked to comment on *teacher actions that facilitated student reasoning*. This suggests that the focus for the observation may have supported the teachers in prompting them to reflect on their practice.

From my perspective, Ruth and Rebecca seemed challenged by the observed pedagogical approach as they wrestled with new ideas that confronted their teacher-directed view of learning.

How will they [the students] get strategies unless teachers tell them to begin with? We have the *Walt* and the *Wilf* at our school. We tell the kids what they're learning, we tell them what we're looking for and we tell them how to get there. (Ruth)

Can this approach replace "explicit lessons" [where we] show them [the students] the process of how to get to the answer, send them off to do it and bring them back and checking or run side by side those lessons? Does this approach really have better outcomes? (Rebecca)

In the past, we have taught strategies—e.g., the jump strategy, the bridge strategy—whereas this [new approach] is asking for students to come up with the ideas themselves. That is a huge turnaround from where we have come from. If children don't have strategies they don't know where to start. If they don't have anything to bring, where do you start? (Ruth)

Further to this, it seemed that giving the students exercises for practice and marking them after school was comforting for the teachers as it provided accountability and a record of student written work regardless of the thinking and meaning behind it.

In this approach, I'm not sure how to assess kids. If we're giving lots of examples, we would be able to mark them, whereas in this approach, if we don't see it, there's nothing to mark after school. (Ruth)

These comments provide evidence that the four teachers undoubtedly recognised that the modelled approaches were different from the conventional focus of teaching as telling. Whilst the teachers seemed to grapple with the idea of allowing students to generate their own ideas, it seemed they were unsure about how to respond effectively to student-generated responses to tasks.

5.3.2 Theme: Uncertainty About Anticipated Student-Generated Responses to Tasks

From my interpretation, it seems that the teachers were uncertain about how to react to student-generated responses to tasks. My contention is supported by the Stage 2 post-lesson conversation.

I am not confident at thinking of something on the hop. I am used to lessons where I know where I am going, but in this approach, it is more about the response from the children, so it could go anywhere and I'm not confident to know where to take it or what response to give. (Ruth)

[In your approach] they [the students] don't have any strategies for figuring out problems. It's like, if I send them off to do a problem without teaching a strategy for solving it, they have no "ammo" for getting it done. I feel like I've given them no strategies to figure out the problems. (Rebecca)

From my perspective, Ruth was struggling with confidence to allow student-generated responses to tasks, and it appears that this may have conflicted with the messages she had heard about allowing students to think for themselves, which she could see was desirable. This echoes Cobb, Wood, & Yackel (1990), who studied the experience of a teacher as she engaged with a teaching experiment led by mathematics education researchers who used constructivist approaches to teaching in her classroom. The teacher appeared to become conflicted because the observed approach was very different to her traditional form of

teaching practice, but she also found it desirable. Her attempts to resolve the conflicts in her mind within the context of seeing her students responding successfully to the constructivist approaches provided her with opportunities to learn (Cobb et al., 1990).

Ruth and Rebecca both expressed concern about what to do or say if students came up with a solution that they had not anticipated because they did not have the knowledge that is required to modify lessons in the moment. Ball et al. (2008) described this as “knowledge of content and students” (p. 401) and argued that this requires both mathematical knowledge and an understanding of students’ mathematical thinking.

Rose’s level of confidence seemed lower at this stage of the intervention compared with how she presented at the beginning. Her comment seems to suggest she was finding the modelling confronting: “I’m struggling. I’m feeling overwhelmed in the Prep/1[class]. It is the beginning of the year and I’m trying to implement a play based curriculum and this as well”. This was confirmed much later in her exit interview when she said, “Initially the modelled lessons scared me because I don’t think we had enough background information”. This comment suggests Rose felt uncertainty about the modelled approach to teaching because it was very different to her established teaching approach.

Connected with this theme were teachers’ views about catering for diversity because it seemed the teachers’ usual approach to catering for the range of abilities in their classes was to group students by perceived ability and give each group different tasks to complete.

5.3.3 Theme: Confronting Ideas About Catering for Diversity

Whilst the four teachers requested the modelled lessons to address differentiation, they appeared to be confronted by the practice of posing a mathematical problem to the whole class with adjustments to the task by way of enabling and extending prompts rather than relying on their usual practice of using ability groupings. The intention of the modelled approach was to ensure that all students were part of the class learning community to enable

learning through social interaction and engagement in whole-class discussions, which are critical for the students to develop mathematical reasoning (Anthony & Walshaw, 2009; Smith & Stein, 2011). In this way, differentiated experiences were offered with the provision of enabling and extending prompts (Sullivan et al., 2009). This seemed to be in sharp contrast to the teachers' previous experience of teaching mathematics.

The fundamental difference with what you did was that you did not teach in ability-based groups. It is a whole swing for me to think differently. (Ruth)

How do you work with 25 kids all at different levels, all working on the same problem and have conversations with all of them during the lesson? (Rose)

Rachel questioned whether ability groupings might be a more suitable option with more explicit instructions to the students in each group about how to solve the problems.

Yes, we are catering for a diverse range of learners, but are we giving them the explicit teaching needed for the level they are at? . . . The enabling questions allow them to enter the learning, but do they need more explicit instructions, more teacher focus time? Could this work with ability groupings to allow a more personalised approach? (Rachel)

Rachel's comment seems to suggest that she was uncertain that her perceived low-attaining students would be able to think for themselves to solve the task. This resonates with Clarke and Clarke (2008), who found teachers often have lower expectations of what low-attaining students can do. She also seemed worried about how to respond to students in the moment if they did not understand the enabling prompt: "What do you do if low-attaining students do not understand the enabling prompts?" (Rachel).

Similarly, Ruth seemed to perceive that the enabling prompts were challenging for some students during the modelled lessons. Ruth and Rachel both intimated that they thought they would not have sufficient knowledge to adjust the enabling prompts during their teaching if a student could not proceed with the planned prompt. For example, Ruth said: "The enabling prompts are not low enough and if you say, "think of lower ones" because this

is a new [approach to teaching], we haven't got a bank of ideas that we can tap into on the spur of the moment". Ruth's comment refers to the knowledge that is required to evaluate rapidly how to adjust an enabling prompt "on the fly" in response to a student's difficulties. This is skilful work and requires unpacking of the concept being taught to enable a student to grasp the idea without reducing the mathematical demands of the task. It seems to necessitate an intersection between SCK and KCS (Ball et al., 2008). This suggests it may be important for the teachers to learn to plan a range of enabling prompts as part of a professional learning strategy involving modelling, deconstruction, and enactment of pedagogies. It seems that by having a greater focus on learning to understand the mathematical demands of the task in the prebrief, the teachers may develop a greater awareness of potential adjustments to the enabling prompts and reduce their concerns about supporting all learners. This may support enactment of the pedagogical approach.

Interestingly, whilst the teachers expressed uncertainty about the appropriateness of the enabling prompts, in Stages 2 and 3 they also expressed surprise at seeing their perceived low-attaining children responding positively to the learning experience. This seemed to create an opportunity to learn from seeing their own students experience a new teaching approach. This finding resonates with Cobb et al. (1990).

It was so nice to see all children even low ones participating in the thinking. (Rachel)

There seems to be so much more thinking and reasoning for the students rather than just going through the process. (Rebecca)

You couldn't distinguish between Grade 1 and Prep responses to the task in that lesson. (Rose)

By having all students in the class working on the same problem with variations, the teachers seemed to become aware of a range of students who found the tasks challenging, and others for whom the tasks were less challenging irrespective of grade level. This appeared to

prompt Rebecca, Rachel, and Rose to reflect on their views of teaching. It seems they could imagine the possibilities of catering for all students through variations to the one task. This echoes Cobb et al. (1990), who maintained challenging teachers' approaches by engaging effectively with students during mathematics instruction could effectively promote change.

During the Stage 3 co-teaching episode, Rose articulated that the enabling prompts were helpful for her low-attaining students to access the learning experience. Furthermore, this prompted her to gather assessment data from her Prep students and link it to the counting principles.

It was obvious children like Rosie, Tilly, Josie, Angus [pseudonyms] and quite a few others needed that manipulative [enabling prompt] [exploring putting plastic teddies on the bed and taking some off repeatedly] [pause] and then reducing it [adjusted enabling prompt] back to five . . . It goes back to some of those children not having those counting ideas that you were talking about . . . the one-to-one correspondence, the counting. (Rose)

It seems that in the process of resolving her inner conflict regarding the observed pedagogical approach, which conflicted with her traditional form of teaching practice, Rose developed pedagogical reasoning, as described by Loughran et al. (2016).

Evidence of this shift in Rose's thinking occurred in Stage 4 after her own enactment of a lesson.

In my planning, I wrote down children I needed to look out for so today I focused on those kids. Next time, I'll still have my list and I'll be able to get to those that need support. I think I am shifting my way of thinking about being able to reach all the children [in the one lesson]. (Rose)

Importantly, Rose articulated strategies that would help her address her intent to cater for all learning needs, such as making a list of students in need of extra support. Three months later in a post-lesson interview, she reported her sense of resolution by stating,

In the modelled lessons, I was challenged by the idea that you might not reach every child during that lesson, but that is okay [pause] they will be picked up in the process. As I become more experienced I will be able to pinpoint the children which may need some more assistance during that problem-solving time and give prompts, either enabling or extending. (Rose)

Furthermore, Rose reported an important shift in her reflective awareness as described by Chapman (2015). It seems she recognised that in her usual teaching approach, she did not talk to all her students during a mathematics lesson because she would be working with a group.

I normally would have whole–small–whole . . . There would be the whole group and there would be small groups. I would be working with a group and the others would be doing games or activities, but I was not there asking questions or probing thinking because I was working with a different group. (Rose)

It seems that Rose’s ongoing questioning and resolution of conflict between her existing teaching practice and the observed pedagogies led her to gain new knowledge to enhance her teaching. This resonates with Wood, Cobb, and Yackel (1991), who argued that teacher attitudes and teacher practice are interdependent and develop together. This suggests that barriers or surprises experienced in practise can lead to opportunities to reassess attitudes.

Equally, Ruth became aware that whilst she taught in ability groups, she did not meet back with her class to discuss the learning that had occurred in the lesson, which had been modelled in the intervention. She stated: “I used to teach in levelled groups. We never got together”. Similarly, Rebecca and Rachel reported shifts in their awareness related to teachers encouraging students to think for themselves and asking probing questions to facilitate student reasoning.

Instead of just teaching kids how to get the answer, I [now] realise the importance of getting kids to think about how they are getting the answer. (Rebecca)

In the past, I've probably asked one question, but now we are asking deeper questions of the children for example, how did you work that out and what were you thinking? [Previously] I would have just said yes, that's great [in response to an answer] but now I'm thinking "what are they thinking"? (Rachel)

This new awareness suggests the modelling of different approaches and the subsequent collaborative discussion of them, which enabled the teachers to compare with their usual practice, played a role in facilitating the teachers' development of pedagogical reasoning.

A fourth theme that arose in the category of challenging teacher assumptions was the issue of children perceived by their teachers to be off task during the modelled lessons.

5.3.4 Theme: Perceiving Disengagement of Some Students

Evidence that emerged from Stages 1 and 2 indicate three of the four teachers evaluated aspects of the modelled lessons negatively. The teachers focused on those students they perceived to be "off task".

I got distracted by the management. (Ruth)

Some of the children were off task. (Rachel)

I got distracted when I stepped back and thought, that one needs more talking to, that one has no idea, that one hasn't got pen to paper yet. (Rose)

This finding appears to agree with Erickson (2011), who argued that what teachers notice is dependent upon their own "pedagogical commitments" (p. 30). For example, if teachers value orderliness in the classroom, they will focus on management aspects in a lesson. Likewise, if teachers value neatly completed worksheets that are submitted in a timely manner, they may presume that the student understands what has been taught, when the student may have got the right answers but perhaps for the "wrong" reasons. In contrast, Rebecca reported that she thought the same students were not so much disengaged as "trying to get their heads around the thinking". I did not notice the students were off task. It seemed

to me that the students were grappling with the unfamiliarity of the new ways of working, including thinking for themselves.

This finding suggests it may be important to make a disclaimer in the prebrief that the modelling is not intended to be “perfect” across all aspects of classroom management and request that teachers refrain from evaluating the lesson.

Interestingly, by Stage 4 of the intervention, I found that the teachers seemed to have shifted their “pedagogical commitments” and no longer evaluated classroom management issues during the modelled lessons. It appeared that they began to consider alternative perspectives of student behaviours. For example, those students who had not immediately “put pen to paper” may have been thinking about the mathematics in the task. This seems to be at odds with Yook-Kin Loong, Vale, Herbert, Bragg, and Widjaja (2017), who found teachers’ attention to classroom management issues (e.g., students “off task”) impeded their capacity to identify teacher actions for facilitating student reasoning. The teachers did not appear to shift in their understanding of the reasoning proficiency. Whilst Yook-Kin Loong et al.’s (2017) study consisted of two modelled lessons spaced four months apart, in this present study, the two modelled lessons were spaced two weeks apart followed by co-teaching and enactment (spaced a further two weeks apart). I surmise the shorter spacing of the four episodes of modelling co-teaching and enactment was the catalyst for the teachers’ shift in this study. It is possible that this was because the teachers were able to remain focused on the pedagogical approach that was emphasised during the modelling.

It seems that four opportunities, spaced closely together, to observe, critically reflect on, and discuss the impact of observed teacher actions on student learning supported teachers to shift their pedagogical commitments and think about teaching in new ways.

In the next section, findings from the category of teacher learning from observing are presented.

5.4 Category 2: Teacher Learning From Observing

One theme emerged from the category of teacher learning from observing. This theme is (5) noticing particular pedagogies. Within this theme, the teachers' observations of my actions in the modelled lessons to facilitate student reasoning were categorised into four subthemes: (a) pressing for student explanations, (b) steering the lesson towards the goal of the lesson, (c) fostering a productive disposition, and (d) enabling differentiation through enabling and extending prompts. These subthemes, which were all categorised as relating to knowledge of content and teaching (Ball et al., 2008), are discussed in Chapter 6 as part of the cross-case analysis.

5.5 Category 3: Teacher Learning From Deconstructing Pedagogies

In this section, three themes that emerged from the data within the category of teacher learning from deconstructing pedagogies are presented. These themes are (6) choosing cognitively complex tasks, (7) building MKT, and (8) building trust and learning from each other.

5.5.1 Theme: Choosing Cognitively Complex Tasks

In post-lesson discussions in Stages 2 and 3, Rebecca and Ruth reported they learned the importance of choosing tasks that provide opportunities for students to reason mathematically as well as being engaging and meaningful to their students.

You have to find the task that is going to produce the outcome you are looking for . . . and giving children a real-life problem to solve so that they can see a purpose for solving it. (Ruth)

I have begun to do more whole-class problem-solving to get students to reason mathematically. (Rebecca)

Ruth's comment is significant in that it seems to illustrate a shift in her MKT, as explained by Ball et al. (2008), in recognising the importance of purposeful, real-life

problematic tasks for creating the potential for student learning through reasoning (Anthony & Walshaw, 2009; Sullivan et al., 2016). Importantly, Rose claimed she could see the possibility of using a problem-based approach. Yet the three teachers also noted that finding or creating tasks themselves would be difficult.

Problem-based questions linked to the curriculum . . . how to come up with these for early prep? I've changed my thinking about activity based . . . I need time to work out how this will work for me in early prep. (Rose)

I'm struggling to create open-ended questions for all areas of the mathematics curriculum. (Rebecca)

I will attempt more whole-class problem-solving . . . I need to build up resources. (Ruth)

This finding implies that providing exemplars of different challenging tasks to teachers might support them to implement them in their classroom practice. As well as recognising the importance of cognitively complex tasks, the teachers appeared to develop an increased knowledge and understanding of the AC:M (ACARA, 2016), particularly in terms of the need to develop student reasoning across their mathematics lessons.

5.5.2 Theme: Building Mathematical Knowledge for Teaching

All four teachers initially expressed minimal understanding of the proficiency strands of the AC:M and a perceived lack of support to implement them. However, the findings indicate that the modelling and subsequent discussions seemed to prompt the teachers to reflect on and deepen their knowledge and understanding of pedagogies associated with the curriculum proficiency strands.

We need to look at the Australian Mathematics Curriculum. We have just looked at the content. We haven't had any guidance. I did not know about the proficiencies. (Ruth)

I think we are looking at the curriculum, "This is what we've got to teach" but we're not unpacking the rationale. I don't think we are unpacking enough. We are teaching place value, but why? What is the deeper understanding we want them to know? (Rachel)

I haven't focused on reasoning and problem-solving before. I did not know that you could expand the section on the AC:M website and find out about the proficiencies. I was focused on content. (Rose)

In terms of understanding the content strands of the AC:M, during the co-planning meeting in Stage 3 (for the lesson focus on developing number relationships to 10), Ruth and Rachel mentioned that they had not heard of the terms *partitioning* or *part-part-whole*. The idea of partitioning in terms of student learning was clarified and the term was highlighted in the content descriptors in the AC:M. The teachers' comments suggest that they also appeared to be unfamiliar with the meaning of at least some of the important content of the curriculum. This suggests that co-planning is an ideal prompt for teacher conversations about the notions of CCK and SCK, which were first explained by Ball et al. (2008).

To illustrate my support for the teachers in developing their SCK, Rose asked if I had suggestions for the learning task and after giving her an example, she said she would like to use the task but personalise it by changing the context. The example was Ten teddies went to bed; some fell out. Please draw me a picture to show how many fell out and how many were left in the bed. In response to the question "What difficulties do you think the students might have with the experience of the task?" Rachel replied, "some children might struggle with getting 10 monkeys". This shows Rachel had a particular interpretation of the difficulties students might encounter with the task. Rose and Ruth asked what other anticipated difficulties there might be. I suggested the following:

Students may have difficulty with counting, one-to-one correspondence and cardinality

Students may have difficulty thinking of 10 in parts.

Students may have difficulty using mathematical language (makes 10, pattern) and explaining their thinking.

Rachel and Ruth also asked me to explain one-to-one correspondence and cardinality, which also belong in the domain of CCK. The teachers were asked if they could suggest

enabling and extending prompts, but they seemed unsure of what to suggest. I explained that five was a very important benchmark for children and so a helpful enabling prompt might be:

Five cheeky monkeys were jumping on the bed. Some fell out and bumped their heads.
Please draw me a picture to show how many fell out and how many were left in the bed.

I clarified that ultimately, we would like to see students presenting their responses in a systematic way, and so possible extending prompts might be:

Can you find all the ways for the monkeys to be in and out of bed? Is there a pattern?

What if there were 11 monkeys? What combinations might there be?

These comments illustrate the difficulties the teachers experienced when attempting to plan prompts to tasks that were unfamiliar. This seems to add more weight to the argument of encouraging teachers to solve the tasks themselves prior to modelling, anticipate student responses, and suggest prompts. This supportive and collaborative environment may support them to develop their MKT.

Rose nodded in acceptance of the ideas, but she was quiet throughout the discussion. It seemed that she was reflecting on her established ways of teaching and beginning to connect with the ideas. This was confirmed prior to the co-teaching episode when she informed me that the co-planning had been an “a-ha” moment for her in that she could now see how it would work. It suggests my encouragement in prompting the teachers supported Rose to build her MKT. This was confirmed by Rose in her exit interview: “the opportunity to have that professional dialogue with your colleagues and having someone else with expertise in mathematics that I could draw on was the most valuable for me”.

Rose’s reflection seems to suggest she appreciated my role in working alongside her at the elbow to support her to connect aspects of MKT, which informed her teaching in ways that she found helpful. This resonates with Timperley, Wilson, Barrar, and Fung (2008), who

maintained that knowledgeable others make knowledge and skills meaningful to teachers when they need it.

To support the teachers' understanding of the pedagogical approach during the modelling, I suggested they read the chapter "Foundations of Student Centered Instruction" (van de Walle & Lovin, 2006). This text was selected as I had used it often in working with teachers. My suggestion prompted Rose to buy the book and read the theory.

A goal for me was to do some more research and find out. I bought the van de Walle text [van de Walle, 2006] and I read all the theory. Often, you'll get a text and skip all the theory and go straight to the activities. I enjoyed reading about setting the context, differentiation, matching to the curriculum and I enjoyed that background information. (Rose)

This appears to be powerful professional learning. It also suggests Rose saw the possibility of transferring the pedagogical approach to other lessons.

The discussions seemed to prompt Rose to recognise she had not laid the foundations for developing student understanding about number relationships. This indicates that the observation and co-planning enhanced her SCK. She reported: "I have never taught friends of 10 in that way before . . . I went straight to the equations. I can see now it didn't lay the foundations for the understanding. I was too quick racing to get to the equations". Rose also conveyed she valued the opportunities to unpack the curriculum and build her knowledge of mathematical reasoning. "I think the opportunities to unpack the curriculum [in this intervention] are really valuable . . . the discussions of what [student mathematical reasoning] means and what it looks like in the classroom". These findings were supported by the school principal who reported in an interview after the intervention that she noticed that the teachers had a deeper understanding of the curriculum:

I observed deeper understanding of the Australian Curriculum; differentiation of tasks; questioning—enablers and extenders, to support and extend learners; acceptance of a

range of answers and thinking; planning of better tasks; development of language through questioning. (Principal)

Rose also felt that a scaffolded approach supported her to feel confident with taking a leading role herself in enacting the pedagogical approach.

I think the co-planning part was really valuable and then me taking the lesson . . . I think the four stages worked together because it was scaffolded. By the time it was my turn, I felt quite confident to model a lesson myself with the exposure I had to that point. (Rose)

This finding resonates with Boerst et al. (2011), who scaffolded opportunities for preservice teachers to engage in increasingly complex approximations of practice to enable maximum success. Boerst et al. (2011) found the complexity of teaching was reduced by the amount of scaffolding that was provided. From my perspective, Rose's confidence to model a lesson for her colleagues was increased through the opportunities she had to observe, deconstruct, plan, and enact particular pedagogies. Through this scaffolded process, she seemed to enhance her MKT and develop her pedagogical reasoning.

5.5.3 Theme: Building Trust and Learning From Each Other

The pre- and post-modelled lesson conversations between me and colleague teachers seemed to enable the four teachers to learn from each other's perspectives. In this, teachers were encouraged to express their thoughts aloud. The process of having teachers "write down everything the modelling teacher did and said to facilitate student reasoning" also provided a platform for teachers to communicate their observations. This was a springboard for discussion where the teachers were prompted to express their ideas, thoughts, and assumptions about what they had observed. My role was paramount in providing teachers with questions to provoke inquiry about mathematics teaching and learning such as "What did you notice about student learning in that lesson?" From my perspective, this seemed to lead to critical reflection and collaborative conversations about student learning. Furthermore, as the intervention progressed, the teachers seemed to begin to provide helpful feedback to their

colleagues. For example, during the post-lesson discussion of the Stage 3 lesson Rose co-taught with me, Rachel said to Rose, “You could have asked questions like how many more”. This seems to illustrate that Rachel not only reflected on Rose’s teaching, but she was also able to offer a suggestion for extending it. It also suggests Rachel felt comfortable providing constructive feedback to Rose, and Rose appeared to feel safe with the feedback. For example, in Stage 4, the two teachers reported that the teachers could now provide support for one another.

[This has been a catalyst to enable us to] work together now rather than sitting by ourselves. We can support each other. (Rachel)

In the past we worked in isolation, now we can plan one problem together and teach it across grades, Prep, 1, 2. We discuss how we are going, run things by each other and provide support for each other. (Rose)

This finding seems to be at odds with Sullivan (2011), who argued that Australian teachers are reluctant to be observed. There are several possible reasons why Rose might have felt comfortable receiving feedback. First, at the beginning of the intervention, she reported she would prefer to be observed (see Section 5.1.1), then she seemed to lose her confidence, but by Stage 3, she appeared to regain her confidence. This seemed to coincide with her enhanced MKT and pedagogical reasoning. Second, it seems that the teachers’ progression of change in participation was an indicator of their learning from each other and the team. This resonates with Lave & Wenger (1991). Third, it seems the cyclical episodes of modelling, deconstruction, and enactment of pedagogies supported the teachers to build trust and collaboration, which was centred on instruction within their school community where they usually worked in isolation. Whilst Bruce et al. (2009) argued that modelling offered the teachers in their study a way of opening classroom doors and working collaboratively together, the finding in this present study suggests that this process may not be immediate. It seems that trust and collaboration was built over the cyclical episodes of the intervention.

From my interpretation, the structured processes, such as the observation prompt and my focus questions to the teachers, facilitated opportunities for the teachers to take risks, raise questions, and engage in critical reflection about the mathematics. This finding echoes Ghouseini and Sleep (2011), who argued that preservice teachers are more likely to enact new learning if they engage in structured processes that support the study of practice. I found this also appeared to apply to in-service teachers. These processes seemed to help the teachers develop ways of inquiring, hearing, seeing, and noticing.

Of course, the real test was whether teachers transferred observed and discussed pedagogies to their own classroom practice. In the next section, findings are presented regarding the teachers' enactment.

5.6 Category 4: Teacher Learning From Enactment

In this section, two themes that emerged from the data within the category of teacher learning from enactment are presented. These themes are (9) trialling an observed lesson and (10) learning to facilitate student reasoning and problem-solving.

5.6.1 Theme: Trialling an Observed Lesson

The teachers reported they valued copies of the observed lesson plans as a starting point for their enactment of new pedagogies. This is consistent with Davidson, Herbert, and Bragg (2018). Furthermore, the teachers conveyed they wanted to replicate the lessons so that they could get a feel for the pedagogical approach first before tweaking it with their own nuances when applying to other lessons. This agrees with Yook-Kin Loong et al. (2017).

I want to learn to teach like that but there is so much I need to change. I want to copy your lesson to practise first . . . I need to get a feel for the language and questioning.
(Ruth)

We do need a huge bank of exemplars that we can practise and feel the sense of teaching this way until it becomes almost second nature. (Rebecca)

[My] goal was to implement it [the pedagogical approach] and plan lessons myself . . . Initially I stuck with the same concepts that you modelled in my classroom, but now I've moved on to simple addition using the same problem-solving model. (Rose)

Importantly, as can be seen from the quotation, Rose reported that initially, she taught the same concepts that I had modelled, and then, as she developed confidence, she used the pedagogical approach to plan and enact lessons around other mathematical concepts. It seems the lesson plan was an important scaffold to enable the teachers to feel confident to enact pedagogies for themselves. Whilst Sullivan et al. (2015) argued it is common for teachers to ask for lessons of this type because of the positive reactions of students in their classes, another possibility is that teachers have opportunities to practise the pedagogical approach with exemplars without the added pressure of coming up with their own ideas.

Ruth reported difficulties she experienced when trialling one of the observed lessons. She described her attempt to trial an observed lesson with her class as being thwarted by the fact that she did not know how to react to her students' unanticipated responses. She explained that whilst she thought she wrote down teacher actions to facilitate student reasoning during the modelling, she realised that she did not have the underpinning understandings of how to modify a lesson in response to the unanticipated. She indicated that this impeded her enactment:

We are writing things down from what we are seeing you do [pause] that are obvious, but it's the white space of the things that aren't obvious that we are not writing down that become the important things when we are conducting a lesson . . . It's not until we are doing a lesson that we realise we've missed something and we don't know how to bridge a gap because we don't have those underpinning understandings . . . They are just in your head and we cannot see them. [For example], if an answer comes back from somebody [that I hadn't anticipated] then I might not know how to respond . . . or how to make a connection . . . Yours was going along the right path because you had the right questioning and it was not something we could write about because it was "on the hop" questioning . . . if it's hard then we'll just opt out [and go back to our old ways of working]. (Ruth)

Importantly, Ruth made the point that unless teachers understand how to modify lesson plan exemplars to cater for their students' unanticipated responses, they will return to more traditional approaches to teaching. This finding echoes Chapman (2012), who maintained that teachers' perceived barriers towards implementation of new curricula pose difficulties for enactment. Sullivan et al. (2015) argued that teachers make decisions about implementation based on their knowledge of the mathematics. It appears that my provision of detailed lesson planning documentation (see, for example, Appendix 8) and my explanation of the lesson in the prebrief were insufficient for the teachers to understand fully all the mathematics in the task and potential student responses. It adds to the suggestion that prior to the observation, the teachers may need to solve the task for themselves, anticipate a range of student responses collaboratively, and consider appropriate pedagogies. This may support them to not only understand the lesson more deeply and build their MKT (Ball et al., 2008), but also increase their likelihood of persisting with trialling new approaches.

5.6.2 Theme: Shifting Towards Student Centred Approaches to Teaching

The task Rose chose for her enactment was closely aligned with the co-planned task in Stage 3, in that it was focused on partitioning 10. She modified the Stage 3 lesson plan exemplar (Appendix 9) for her lesson (Appendix 10). This suggests that previous co-planning was a useful scaffold for her. Rose demonstrated her commitment to students thinking for themselves during her enactment when she posed a problem, "How many ways can you show 10?" Next, she followed with a question to the class to check they understood what the problem was asking them to do before sending them off to think for themselves to find a solution. I noted evidence of this by recording that "Rose stated explicitly [to the class]: [I'm] really interested in your [students'] thinking and the mathematics". This is a noteworthy finding, indicating a shift in practice because prior to the intervention, Rose had expressed her value for direct teaching of specific skills. She confirmed her orientation to this new approach

during the post-lesson discussion when she acknowledged the importance of extended thinking time, particularly for vulnerable children to enable them to contribute to the mathematical discussion.

I think we need to step back and give the children more thinking time than we've given them before . . . In the past, I wouldn't have picked Sarah [pseudonym] to share, but she had something to contribute. The children are beginning to realise we are expecting them to explain their thinking and that everyone's thinking is different and we can gain from that. (Rose)

I interpreted this to suggest the intervention was successful in shifting Rose's views of teaching. Importantly, it seems that Rose could see that all children could be part of the community of learners and engage with the problem, not just the capable students. During her exit interview, Rose reported that she had learned to have a clear lesson focus on one idea with different entry points. This was different from her usual approach to teaching, which consisted of multiple disconnected tasks.

The biggest thing I learnt was to step back in how much content to put into a lesson. I learned to have one idea, one concept and to stretch those that need stretching and enabling those that need support in developing those concepts . . . instead of having all these learning centres around with lots of different things. (Rose)

Rose expressed surprise at her students' increased understanding of the mathematical concepts.

From [my enacted] lesson, which was focused on partitioning ten, I extended the concept with different contexts. I am really surprised now how much more knowledge the children have about 10 from the sequence of those lessons. Most of the children can now partition 10. In the past, I would have just had games out and given a few activities, but not had that problem-solving context with the whole class. (Rose)

She also described becoming aware of the types of questions she posed to her students. She increased her expectations of students' learning based on her surprise at their favourable responses to the challenging tasks.

I am now conscious of the types of questions that I ask [pause] probing questions and adapt those to suit children's learning needs. I'm also having much higher expectations of the children as they may surprise me by their responses. (Rose)

Eliciting reasoning through effective prompting is an important teacher action to facilitate student reasoning (Clarke, Clarke, & Sullivan, 2012), and having high expectations of students increases opportunity to learn (Sleep, 2012).

It appears that Rose's enhanced MKT supported her to plan and enact a sequence of lessons around the concept of partitioning 10 for her Prep class. She reported she was surprised at the increased understanding of her students. It seems she also recognised her choice of tasks was effective in promoting student understanding. This finding resonates with Bronkhorst et al. (2011), who claimed that for successful enactment, preservice teachers need to formulate intentions based on their knowledge of teaching and their knowledge of how to put that into action, along with the opportunity to do so in practice. I found this also applied to Rose.

I interpreted Rose's change sequence to correspond with a change pathway in Clarke and Hollingsworth's (2002) interconnected model of teacher learning. For example, whilst it appears Rose was initially confronted by the modelling of pedagogies (external domain), it seems her students' responses to the learning experiences prompted her to reflect on her teaching practice. This seemed to lead to her shifting her attitudes and gaining new knowledge (personal domain) to draw new conclusions (domain of consequence). It seems this change sequence supported her to enact and reflect on new pedagogies (professional experimentation). The modelling of "different" pedagogies, the conversations with colleagues, planning, and the professional reading all seemed to provide external stimuli for Rose. It seems the shift in Rose's attitude was evident in the increasing importance she placed on her new teaching strategies, for example, presenting students with one mathematical task by having multiple entry points. This represented new pedagogical knowledge for Rose. It

seems Rose's shift in the domain of consequence was firmly bound to her values and to the inferences she drew from the teaching practices she enacted. It appears the mechanisms by which Rose's shift was triggered from one domain to the other were *enaction* and *reflection* as described in Clarke & Hollingsworth, (2002). This suggests the multiple spaced learning opportunities to enable Rose to reflect and make shifts in her practice were important.

5.7 Category 5: Stakeholder Reactions to the Intervention

Four themes emerged from the category of reactions to the intervention. These themes are seeing, deconstructing, and enacting pedagogies; engaging students; differentiating the experience for teachers; and accountability. These themes are discussed in Chapter 6 as part of the cross-case analysis.

5.8 Chapter Summary and Conclusion

This chapter explored the elements of modelling that appeared to support teachers to reflect on their current teaching practice and enact changes in their later experimentation with the modelled pedagogies. It presented 10 themes that emerged from the data analysis of RPS teachers' participation in this study. The findings were organised according to the three research questions.

5.8.1 Linking the Findings to the Research Questions

The findings presented in this chapter provide evidence that the modelling challenged the four teachers' assumptions about teaching mathematics. This initially emanated from the teachers' observations of their students struggling to articulate mathematical reasoning. They attributed the students' difficulties to their teaching practice. From my perspective, it seemed that the focus for the observation on teacher actions to facilitate student reasoning prompted the teachers to reflect on their practice. Rachel implied she was not aware of the impact of her actions until she saw the students struggling to explain. This prompting seemed to provide the

teachers with an opportunity to learn. Nevertheless, the teachers were challenged by the pedagogical approach. They wondered how students would learn strategies if they did not tell them and seemed concerned that they would be unable to assess student learning in the modelled approach.

A recurring theme from my perspective was teachers' apparent uncertainty about how to react to student-generated responses to tasks they had not anticipated. Moreover, Rose and Ruth seemed to lack confidence in anticipating student responses to the co-planned task in Stage 3, and all three teachers seemed uncertain about suggesting enabling and extending prompts. Furthermore, the teachers seemed worried that they would not be able to adjust the prompts in response to students' thinking on the spur of the moment during a lesson. Moreover, Ruth reported that the teachers would go back to their old ways of working if they did not learn how to anticipate student responses to tasks. This suggests it may be important for the teachers to learn to plan a range of enabling prompts as part of a professional learning strategy involving modelling, deconstruction, and enactment of pedagogies. It seems that by having a greater focus on learning to understand the mathematical demands of the task in the prebrief, the teachers may develop a greater awareness of potential adjustments to the enabling prompts and reduce their concerns about supporting all learners. This may support enactment of the pedagogical approach.

Whilst the teachers requested modelling to address differentiation, they appeared to be confronted by seeing all students working on the same task with variations by way of enabling and extending prompts, rather than grouping students by ability, which was their usual practice. However, the teachers seemed surprised when they saw their students respond successfully to the challenging tasks. Their observations seemed to conflict with their existing teaching approaches and created an opportunity for them to reflect on their practice. It raises the question that if teachers do not have opportunities in their classrooms to see how their

students respond to alternative pedagogical approaches, how will they shift their current paradigms?

Three of the teachers initially evaluated aspects of the modelled lessons negatively. They seemed uncomfortable with some of their students' perceived struggle with the tasks. However, they appeared to shift their views over time as they observed and reflected on how their students responded positively to the pedagogical approach. Nevertheless, a disclaimer may need to be made prior to the modelling indicating that the lesson is not intended to be perfect. This might discourage the teachers from evaluating the modelling and focus on teacher actions to facilitate student reasoning.

From my perspective, the four teachers came to recognise the importance of choosing cognitively complex tasks not only to promote reasoning and problem-solving but also to enable students to engage with and learn concepts more deeply. Moreover, teachers in this study reported they would find planning and searching for suitable tasks difficult and time consuming. An implication is for these types of tasks to be made readily available for teachers. Importantly, the modelling and subsequent discussions seemed to prompt the teachers to reflect on and deepen their understanding of the AC:M proficiency strands of reasoning and problem-solving. A professional reading seemed to augment this new knowledge for Rose.

The findings confirm the positive effect my role as a knowledgeable other provided. This included supporting the teachers to build their MKT as they asked questions about the mathematics, including how to cater for diversity. The opportunity to work alongside me and work together to unpack the curriculum seemed to be a catalyst for the teachers to build trust and learn from each other. Moreover, the structured processes, such as the observation prompt and my focus questions to the teachers, seemed to facilitate opportunities for the teachers to take risks, raise questions, and engage in critical reflection about the mathematics.

The teachers reported they valued the detailed lesson plans as a starting point for enactment. This was confirmed when Rose modified the Stage 3 lesson plan for her modelled enactment by changing the context. She reported the scaffolding supported her enactment.

From my perspective, Rose's enactment illustrated a shift in her practice from direct teaching of specific skills to allowing students to think for themselves to solve a problematic task. Importantly, Rose seemed to recognise that a carefully planned lesson sequence focused around an important mathematical concept facilitated student understanding more effectively than her usual teaching practice of disjointed activity stations. It seems that the modelling and subsequent deconstruction of pedagogies prompted Rose to construct new intentions for her teaching. Further to this, the opportunity for Rose to enact pedagogies, reflect on her practice, and gain feedback from her colleagues seemed to strengthen this new learning.

5.8.2 Moving Forward With the Research

Drawing on the data analysis and research evidence presented in this chapter, I incorporated the following strategies in the interventions for the next stages of the research, as in each of the findings I found implications that could enhance, or had the potential to enhance, the teachers' experience of modelling:

- Encourage the teachers not to evaluate the lesson. Make a disclaimer up front that the lesson will not be perfect. Teachers can misinterpret the purpose of their observations if this is not made explicit.
- Encourage teachers to anticipate how students might solve the tasks prior to the lesson so that they can anticipate any difficulties students might have and think about how enabling prompts might be modified in the moment during a lesson.
- Provide lesson plans from the modelled lessons as exemplars for teachers to modify.
- Ensure that teaching moves made in the moment during modelled lessons are made explicit to the observing teachers in post-lesson discussions so that they can develop an

understanding of the modeller's thinking and the underpinnings of the pedagogical approach.

Chapter 6 focuses on synthesising and comparing the data from all four interventions at the four school sites in this study. A discussion of the observed pedagogies from the four schools is included.

CHAPTER 6: SEEING, REFLECTING ON, TALKING ABOUT, AND ENACTING PEDAGOGIES

This chapter presents a synthesis of the research findings from the four schools involved in the study. The cross-case analysis confirms findings that were presented in Chapter 5 and includes new themes that emerged. It considers the experiences of the 18 teachers in all four schools as they observed the modelling, including teacher actions for facilitating student reasoning and problem-solving. It also considers the importance of reflecting on and discussing the modelled pedagogies and the principals' and teachers' affective reactions to the intervention across the four schools. The chapter is presented in three sections. First, the research aim, four of the research questions, and initial data collection on teachers' perspectives about effective teaching approaches are presented. Next, the formulation of meanings and identification of themes are explained. Following this, the themes are explicated to provide answers to the research questions set for this chapter.

6.1 Relevant Research Aim and Questions

This chapter explores the following research aim and questions:

Aim	Questions
To explore how modelling pedagogies, deconstruction of those pedagogies, and enactment may help primary school teachers build their mathematical knowledge for teaching	<ol style="list-style-type: none">1. What teacher assumptions are challenged through lesson observation?2. What is the nature of teacher learning from lesson observation?3. What do teachers and principals say about modelling pedagogies, deconstruction of those pedagogies, and enactment as a professional learning strategy? What explanations do they give?4. Can an educational intervention involving modelling pedagogies, deconstruction of those pedagogies and enactment improve mathematics teaching and learning? If so, in what ways?

6.1.1 Teachers' Perspectives on Effective Pedagogical Approaches

In the introductory meeting with each school, I surveyed participants to establish their perspectives on mathematics teaching and learning. The survey contained the following prompt:

Effective teachers use the following pedagogical approaches (teaching actions) when catering for the diverse range of learning needs in mathematics.

The findings are presented in Table 6.1.

Table 6.1

Findings of the Pre-Intervention Survey

Pedagogies Deemed Effective By the Teachers	Total Number of Teachers $n = 20$			
	Raven Primary School	Magpie Primary School	Heron Primary School	Swift Parrot Primary School
Demonstration to students; independent practice time	2	1		6
Focus ability grouping of students	1	3	1	1
Open-ended tasks	1	1	1	
Whole–small–whole lesson structure with mixed ability groups			2	

Table 6.1 indicates that almost half the teachers valued demonstration to students in their teaching, whilst just under half the teachers valued ability grouping, but overall, their responses were worded in general terms. ACARA (2016) stated that the goal of mathematics instruction is to develop mathematical proficiency in all students. Hiebert and Grouws (2006) noted teaching that leads to mathematical proficiency should be focused on students' thinking for themselves to solve tasks where the solution is not immediately apparent. These findings suggest teacher enactment of the proficiencies for at least half the teachers might be

infrequent. This is because teaching that is focused on demonstration of procedures is very different to allowing students to think for themselves (Hiebert & Grouws, 2006; Sleep, 2012).

During the initial meeting at each of the three subsequent intervention school sites, I explained to the teachers that the purpose of the observations was to focus on teacher actions that facilitate student reasoning. I made the disclaimer that the lessons were not aiming to be “perfect”, particularly as the students were not being taught by their own class teacher. The teachers were encouraged to focus only on pedagogies for facilitating student reasoning. After the meeting, a lesson request proforma (Appendix 1) was emailed to each school. The returned proformas indicated that the school teams at HPS and MPS had requested problem-solving and reasoning as a pedagogical focus for the modelling in Stage 1, and SPPS requested a focus on questioning. These responses were not surprising given that the focus of the preliminary meeting was reasoning and problem-solving. Consistent with findings in Chapter 5, the teachers at all three schools requested the pedagogical focus of the modelling to be differentiation. Notably, this request came after the observation of the Stage 1 modelling (see Table 6.2 and Appendices 28 to 33). This suggests that experiencing the modelling in Stage 1 prompted teachers to reflect upon their current approaches to cater for student diversity. It also suggests that catering for differentiation was of concern to teachers in this study.

Table 6.2

School Requests for Pedagogical Focus of the Modelling

School	Stage 1	Stage 2
RPS	Explicit teaching (instructional model); differentiation; questioning; open-ended tasks	Differentiation
SPPS	Questioning	Differentiation; materials
MPS	Problem-solving and reasoning	Differentiation
HPS	Reasoning; lesson structure	Differentiation; reasoning; problem-solving

The next section describes the formulation of meanings and identification of themes from the cross-case analysis of data from all four schools.

6.2 Formulation of Meaning and Identification of Themes

Consistent with Chapter 5, I interpreted the meanings from the data from my perspective, using an interpretive approach with a phenomenological lens (van Manen, 1990). Regularities, patterns, and topics that emerged from the data were sought to establish coding categories. Units of data were mostly spoken or written sentences or phrases. First, meanings from each school were formulated and clustered into emerging themes (see Appendices 11, 12, and 13). A table consisting of three columns was created to show how the meanings were clustered to form the themes that emerged from the analysis (see Appendix 14). Next, a cross-case analysis was conducted to see which categories and themes from RPS (presented in Chapter 5) had commonalities with the data from the other three schools in the study. The purpose of this cross-case analysis was to compare the experiences of all participants and identify themes that were common among them. I chose to include the theme *noticing particular pedagogies* in this chapter because I wanted to discuss the observed pedagogies from the perspective of multiple observers. Similarly, I chose to include the category, *stakeholder reactions to the intervention* in this chapter so I could illustrate the reactions to the intervention from multiple perspectives. Table 6.3 presents the themes from Chapter 5 and corresponding themes from SPPS, MPS, and HPS.

Table 6.3

Themes From Chapter 5 and Corresponding Confirmation From SPPS, MPS, and HPS

Categories	Themes	RPS	MPS	HPS	SPPS
(1) Challenging teacher assumptions	Challenging ideas about teaching	✓	✓	✓	✓
	Confronting ideas about catering for diversity	✓	✓	✓	✓
	Fearing anticipated student-generated responses to tasks	✓	✓	–	✓
	Perceiving disengagement of some students	✓	✓	–	–
(2) Teacher learning from observing	Noticing particular pedagogies	✓	✓	✓	✓
(3) Teacher learning from deconstructing pedagogies	Choosing cognitively complex tasks	✓	✓	✓	–
	Building mathematical knowledge for teaching	✓	✓	✓	✓
	Trialling an observed lesson	✓	✓	✓	✓
	Building trust and learning from each other	✓	–	✓	–
(4) Teacher learning from enactment	Shifting towards student centred approaches to teaching	✓	✓	✓	✓
(5) Stakeholder reactions to the intervention	Seeing, deconstructing, and enacting pedagogies	✓	✓	✓	✓
	Engaging students	✓	✓	✓	✓
	Differentiating the experience for teachers	✓	–	–	✓
	Accountability	✓	✓	✓	✓

6.2.1 Common Themes From All Four Schools

Table 6.3 shows nine themes emerging from all four schools.

The theme *fearing anticipated student-generated responses to tasks* appeared across three schools but did not emerge at HPS. It is possible that the teachers at HPS feared loss of control if they enabled students to think for themselves, but it was not raised by the teachers

in their discussions. The theme *choosing cognitively complex tasks* also arose across three schools but did not emerge at SPPS. It is possible this was because I co-planned with only one of the teachers (Sophie) in Stages 3 and 4. The group of teachers as a whole did not experience planning together, which included choosing tasks for lessons. I found that in the other three schools, this theme emerged during co-planning.

The theme *Building trust and learning from each other* emerged in the data from two schools only. It seems that this is because SPPS and MPS already had established collaborative planning teams in place, whereas the teachers at RPS and HPS had been used to working in isolation prior to the intervention. The theme *perceiving disengagement of some students* was also apparent at MPS where two teachers raised their concerns. In Stage 1 at HPS, Jackie commented in favour of the disclaimer that was made to the observing teachers discouraging them from evaluating the modelling in terms of classroom management. Similarly, the AP at SPPS mentioned that the collaborative post-lesson discussion was powerful in supporting the teachers to understand why the times when students appear to not be doing much might be interpreted as students thinking rather than as being time off task: “Making mathematics a group activity is a powerful technique—and that allowing discussion of concepts/thinking is to be encouraged rather than feared as an indicator of time off task”. (Sheila, SPPS)

Because of the differences and nuances of each site, as explained in section 3.2.1.3, new themes emerged from the cross-case analysis. These themes are presented next.

6.2.2 Themes in This Chapter

A table was created to show the themes that emerged from the cross-case analysis (see Table 6.4). The themes were collapsed into five main categories. These categories were (1) challenging teacher assumptions, (2) teacher learning from observing, (3) teacher learning

from deconstructing pedagogies, (4) teacher learning from enactment, and (5) stakeholder reactions to the intervention.

Table 6.4 shows the themes that fall into the five main categories.

Table 6.4

New Themes That Emerged From the Cross-Case Analysis and Their Corresponding Categories

Categories	New themes	RPS	MPS	HPS	SPPS
1. Challenging teacher assumptions	Confronting expectations of student achievement	–	✓	✓	✓
2. Teacher learning from observing	Noticing and uncovering the complexity of teaching	–	✓	✓	✓
3. Teacher learning from deconstructing pedagogies	Engaging in corridor conversations about practice	–	✓	✓	–
	Learning to be purposeful with teaching	–	✓	✓	✓
4. Teacher learning from enactment	Enacting particular pedagogies	–	✓	✓	✓
5. Stakeholder reactions to the intervention	Implementing the intervention	–	✓	✓	✓

6.3 Category 1: Challenging Teacher Assumptions

In this section, one new dimension to the theme in Chapter 5, *confronting ideas about catering for diversity*, emerged from the cross-case analysis. This new theme is *confronting expectation of student achievement*.

6.3.1 Theme: Confronting Expectations of Student Achievement

Four teachers from SPPS, HPS, and MPS reported that they tended not to give all students the opportunity to solve problems unless they were perceived to be “ready”. Sophie (SPPS) reported she catered for the average to below-average students in her class and spent most of her time with low-attaining children.

I was catering for the average or below average and spending most of my time with them. (Sophie, SPPS)

I would have just taken a group that was ready for the idea and others would have been doing other concepts. (Henrietta, HPS)

I would not have had the whole group working on the one task; it was a challenge to think about how that would happen in my own class room. (Heather, HPS)

Furthermore, Sabrina (SPPS) raised the issue of “struggle” and what that meant for low-attaining students and her teaching.

One of the things that stood out to me was the amount of time you let them struggle. It’s knowing how long to let them struggle and when to move on . . . We are still not clear about the “zone of confusion”. If one of our goals is for children to learn to use efficient strategies for problem-solving, it does not seem helpful to leave them confused for long periods . . . At what point would you intervene with children like Talia [pseudonym] who was hung up with the colours of the counters? (Sabrina, SPPS)

This suggests the notion of productive *struggle* may need to be unpacked with teachers prior to modelling pedagogies. That is, having students grapple with mathematical ideas that are within reach but just beyond their current level of knowledge (Hiebert & Grouws, 2006). However, in the same way that teachers at RPS were surprised to see their low-attaining students engaging with challenging tasks (see Section 5.3.3), eight teachers in the other three schools also implied they were confronted by seeing all their students respond successfully to the tasks. It seems that this created an opportunity to learn which prompted them to reflect on and question their expectations of student achievement and their current teaching practice. This resonates with Wood et al. (1991). Such was the power of the observation that Marita noted her observation challenged her beliefs about teaching.

I’m thinking maybe I’m under pitching way too often. Maybe I’m not setting [expectations] high enough. [This modelled lesson] has got me thinking about changing my beliefs [about teaching]. (Marita, MPS)

Initially I thought my goodness, my children would not be able to do this [pause] [Their responses to the task] surprised me . . . The children were very much getting the concept

at the end and wanted to continue working, even when they heard the bell, which was surprising. (Stella, SPPS)

I reflected on what I have been doing and I don't think I let my students struggle enough. I think I pounce too quickly to do the teaching and make things easier for them. I need to let them struggle more. (Sabrina, SPPS)

Consequently, by Stage 3, 11 teachers from these three schools subsequently set goals for their future practice related to allowing their students to grapple with problem-solving.

[My goal is to] give more time for children to work problems out rather than rushing in to provide teacher assistance. (Sue, SPPS)

I feel I probably go astray; I probably offer up strategies a bit much rather than let them [have a go] . . . that is something I will attempt to do. (Melissa, MPS)

I have held back on my modelled explicit teaching and allowed the children to have more thinking time. [I saw] the children thriving on the challenge. (Megan, MPS)

This builds on findings from Chapter 5 and illustrates that whilst the teachers were initially confronted by my high expectations of student achievement during the modelling, they also observed how their students rose to the challenge. This echoes Cobb et al. (1990), who maintained that “cognitive conflict” in teachers’ minds effectively promotes change.

In the next section, findings from the second category of teacher learning from observing are presented.

6.4 Category 2: Teacher Learning From Observing

In this section, two themes within the category of teacher learning from observing are presented. These themes are *noticing particular pedagogies* and *noticing and uncovering the complexity of teaching*.

6.4.1 Theme: Noticing Particular Pedagogies

The teachers from the four schools were provided with a focus for their observations in Stages 1 and 2 (see Appendix 3) with the following prompt:

Write down everything you saw the modelling teacher do and say to facilitate student reasoning.

The pedagogies that emerged from the data analysis in Stages 1 and 2 are shown in Table 6.5. In each instance, a description of each strategy is given as are one or two illustrative examples of the types of responses from the teachers. The pedagogies are presented in order of decreasing frequency.

Table 6.5

Observed Pedagogies

Pedagogy: Pressing for Student Explanations				
Strategy:		Providing extended thinking time for students to think for themselves		
Description:		Increasing wait time; holding back from telling		
RPS	MPS	HPS	SPPS	Illustrative Examples of Teacher Responses
<i>n</i> = 4	<i>n</i> = 5	<i>n</i> = 5	<i>n</i> = 6	
4	3	4	6	<p>“‘Keep thinking and I will come back to you’ and you did go back to them”. (Sabrina, SPPS)</p> <p>“The power is in the time you gave children to reflect . . . Oliver [pseudonym] was a real surprise to me because at the start of the year he couldn’t recognise any numbers; he couldn’t count with one-to-one correspondence. He came up with 5 and 5, when normally he would not. Because he had the time to work it out, he could do that”. (Megan, MPS)</p>
Strategy:		Expecting students to explain and justify their reasoning		
Description:		Probing students to explain and justify their thinking		
4	4	4	4	<p>“You did not accept an answer without [probing] them just a little bit more . . . Even right to the very end . . . you expected them to reason [mathematically]”. (Heather, HPS)</p> <p>“We tend to accept a right or wrong answer instead of saying ‘Okay, you got it right but why is it right?’” (Sheila, SPPS)</p>
Strategy:		Revoicing; prompting students for further participation		
Description:		Asking students to repeat or add to what had been said; asking students to generalise		
3	2	2	6	<p>“You rephrased to assist them get to where you wanted them to go, by saying things like ‘Are you saying?’ or ‘Is that the same as?’” (Stella, SPPS)</p> <p>“Who agrees with . . .” (Sheila, SPPS)</p>

“The phrases that you use: Convince me . . . has somebody else got something they would like to add. It was [student] directed”. (Marita, MPS)

“What advice would you give to others Jake [pseudonym]?” (Rachel, RPS)

Strategy: Scaffolding students to reason

Description: Interjecting; asking questions to support students to analyse the problem

2 1 3 5 “I was noticing how you were scaffolding them by breaking it down into smaller steps—‘tell me about this, what did you notice about this side’ . . . you were helping break that thinking into little chunks so that [the children] were able to explain it logically”. (Sue, SPPS)

“Your questions were clarifying their [mathematical] ideas”. (Megan, MPS)

Strategy: Modelling mathematical language

Description: Repetition of language supporting students to think, explain, and justify

2 2 4 5 “I was conscious of you [modelling] the language and then they [children] started using that language that you were modelling and you . . . stretched their thinking”. (Heather, HPS)

Pedagogy: Steering the Lesson Towards the Learning Goal

Strategy: Using student explanations to exemplify the mathematical ideas

Description: Selecting students to explain their thinking to the class

2 4 5 6 “You drew out patterns in student thinking”. (Stella, SPPS)

“Students continued to scaffold each other’s ideas . . . you could see their lights go on”. (Marita, MPS)

“As soon as Rick [pseudonym] shared his idea [of using a number line] there were quite a few that went back and [used] that number line that Rick had explained. The children learned, that they could use an idea that [other students] had explained”. (Melissa, MPS)

Strategy: Making student thinking visible

Description: Projecting student examples, thereby making their explanations and meaning visible to other students

2	1	3	5	<p>“You used children’s examples on the smartboard and got children to come up and explain it”. (Sabrina, SPPS)</p> <p>“We’ve since used money to buy two document projectors . . . [Teachers now have the] desire to be able to display children’s work during lessons for student explanation or group discussion”. (Sheila, SPPS)</p>
---	---	---	---	--

Strategy: Connecting with students’ prior knowledge

Description: Checking for understanding of mathematical terms

0	1	2	5	<p>“You explained the terms prior to getting into the actual teaching . . . making sure the children understood what was going on . . . We tend to rush into the content rather than making sure they understand”. (Sheila, SPPS)</p> <p>“[My goal is] when introducing new concepts [terminology] making connections to their [students’] prior understandings”. (Suri, SPPS)</p>
---	---	---	---	--

Strategy: Clarifying the task

Description: Getting children to repeat the question that was asked and allowing children to ask any clarifying questions

0	0	3	5	<p>“You got a child to repeat back the question so that you could clarify what it was that they had heard”. (Holly, HPS)</p> <p>“Giving them [students] the opportunity to unpack the question and understand what is being asked of them before beginning is a critical step”. (Hannah, HPS)</p>
---	---	---	---	---

Strategy: Posing challenging tasks

Description: Tasks that enable “productive struggle”

3	3	1	2	<p>“I’m really thinking about why it is I am choosing [tasks]. For example, I might have had some ideas of doing numbers to 10 [with Prep/1s], but now I’m looking at the proficiencies and [asking myself] what is the foundation that I’m building the maths on?” (Heather, HPS)</p>
---	---	---	---	--

Strategy: Roving and probing

Description: Paying attention to the students’ thinking and solution strategies

1	1	0	4	<p>“Louise roves and has one-on-one conversations with the students. She asks them to explain their reasoning. [By doing that] she can see how the whole class is going and whether to move on or not”. (Sabrina, SPPS)</p>
---	---	---	---	---

Strategy: Summarising learning

Description: Connecting the mathematical ideas and students' representations

1 1 3 1 "I run out of time in bringing them [children] back together at the end and you did that so well". (Marita MPS)

Strategy: Making the Learning intention clear

Description: Students become clear about how they can participate in the lesson, the purpose and direction of their learning

0 1 0 1 "[When the teacher is really clear about the learning intention] it makes you disregard [irrelevant student] comments and draw back to what it is we are working on". (Heather, HPS)

Pedagogy: Encouraging Persistence

Strategy: High expectations; encouragement; time

Description: High expectations; acknowledging difficulties

2 5 3 6 "I noticed the way you didn't give up on [the students]. You [expected] that they would keep going . . . you acknowledged that it was going to be difficult . . . towards the end you gave them the cliff-hanger [you gave them the message] we are going to come back . . . this is important". (Heather, HPS).

"Jack [pseudonym] was distracted at the start, but you trusted him to do it and he achieved much more than he ever has. I was surprised at how much you could get out of him". (Megan, MPS)

Pedagogy: Enabling Differentiation

Strategy: Differentiating the task

Description: Fostering differentiation through enabling and extending prompts

3 5 1 4 "After he did the [enabling prompt], he did the next one in under a minute. Just having those enabling prompts really helped him to get started". (Melissa, MPS)

"The enablers and extenders mean you can cater for a greater range of children". (Hannah, HPS)

As shown in Table 6.5, all 18 teachers expressed their attention to teacher actions during the modelled lessons associated with pressing for student explanations including providing thinking time, scaffolding students to reason, interjecting (Askew, 2012), revoicing, asking students to repeat or add to what had been said, prompting students for further participation (Smith & Stein, 2011), and modelling mathematical language. Interestingly, 17 of the 18 teachers observed that I provided additional thinking time for the students who had been chosen to explain their thinking to the class but were not yet ready. This had the desirable effect of encouraging those students to persist and then succeed. This finding is consistent with Chapin, O'Connor, and Anderson (2003), who argued teachers should provide additional thinking time, if necessary, to students who have been asked to explain, so that they can clarify their thoughts. Further to this, explaining and justifying thinking is a critical component of developing mathematical proficiency (Kilpatrick et al., 2001; Sleep, 2012; Sullivan, 2009), and pressing for complete explanations increases student reasoning (Sleep, 2012).

A teacher's intentional repetition of mathematical vocabulary is crucial for emphasising and developing key mathematical ideas being focused on in a lesson (Sleep, 2012). Importantly, many teachers noticed that their students began to use the mathematical language that I modelled in their explanations to other students.

Overall, the teachers at SPPS seemed to notice pedagogies in more detail compared to the other schools. I surmised this was because the first lesson I modelled at that school seemed exemplary to me in the sense that I made many successful moves to modify my teaching in response to students' reactions. This seemed to enable pedagogies to facilitate reasoning and problem-solving to perhaps be more salient than my other initial modelled lessons (see my reflection on this lesson in Appendix 15).

Eighteen teachers noticed various important features of the lessons they observed, which steered the lesson towards the learning goal (Sleep, 2012) and supported students to develop mathematical proficiency (Sleep, 2012). These features included posing challenging tasks, explaining definitions of new mathematical terminology, clarifying the task, circulating and paying attention to students' thinking, solution strategies as they worked on the task (Smith & Stein, 2011), and using students' explanations to exemplify the mathematical ideas (Smith & Stein, 2011). Further to this, 12 teachers from the four schools noticed the document camera was a powerful tool for facilitating sense making and effective classroom discussions about the mathematics. This finding concurs with Sleep (2012), who maintained it is important for all class members to see the chosen student examples. This brings the mathematics into the open for examination. If the student examples are not visible to all, meaning can be lost. This implies document cameras are an important tool for making students' thinking easily visible in a classroom context. Consequently, at SPPS, teachers reported they purchased document cameras.

Six teachers indicated that the summary phase of the modelled lesson was a challenge for them in their previous practice because of time constraints and the difficulty in getting students focused on others' thinking. This finding is consistent with Sullivan (2011) and Smith and Stein (2011), who maintained the summary phase is difficult to orchestrate. To support teachers' understanding of the summary phase of a lesson, I offered an article for them to read (Smith, Hughes, Engle, & Stein, 2009). This text was selected as it exemplifies the pedagogical approach.

Heather (HPS) inferred that having clarity about the goal of the lesson supported teachers to remain focused on the mathematical intent during the summary phase and to disregard irrelevant student comments. This finding is consistent with Sleep (2012), who

maintained teachers need clarity about the goals of the lesson to steer the lesson towards that goal.

Sixteen teachers observed my positive stance towards the students, encouraging them and promoting persistence. They noticed students did persist with the prompt “keep thinking”. A couple of teachers reported this was the most important idea they took from the modelling.

Thirteen teachers recognised the power of the enabling and extending prompts in catering for the diverse range of learners. The prompts provided the teachers with an alternative strategy (Sullivan et al., 2015) to working separately with different groups. This led to reported changes in teachers’ practice, including implementing challenging tasks with enabling and extending prompts and allowing students to solve problems by thinking for themselves (see Table 6.6).

Nine teachers noticed student actions and seemed to be able to interpret their thinking. For example, under the heading of *using student explanations to exemplify the mathematical ideas*, Melissa noticed that a student’s explanation of his number line to the class prompted some of the other students to use a number line to solve the problem. Similarly, in the subcategory of *providing extended thinking time for students to think for themselves*, Megan was surprised when her student experienced success and attributed this to the extended thinking time provided. Arguably, these findings fall into the category of KCS (Ball et al. 2008) because they relate to how students learn a concept. This finding seems to be at odds with recent studies of modelling that have shown that without observation prompts, teachers tend to focus on teacher actions rather than student actions (see Clarke et al., 2013). In this study, the teachers seemed to notice student actions because of teacher actions. This may have been because of the specific wording of the prompt for the observation augmented by the pre-lesson discussions of the observation.

These four subthemes described support the findings of the previous work of Hodgson (2013), who found that when given the opportunity to observe modelled lessons, teachers noticed important aspects of MKT (Ball et al., 2008). These findings also build on the findings from the preliminary study discussed in Chapter 4, which showed that when given the opportunity to observe a modelled lesson, teachers noticed important pedagogies. Apart from the students' actions noted in section 6.4.1, the pedagogies identified appear to be consistent with KCT (Ball et al., 2008), which is the combined knowledge of effective teaching pedagogies and mathematics. Interestingly, the teachers seemed to observe more detailed pedagogies in this intervention than they did in the preliminary study. For example, several teachers noticed actions such as asking students to analyse, justify, and generalise, which are important reasoning behaviours described in (Yook-Kin Loong et al., 2017). This may possibly be because the teachers became aware of teacher actions to facilitate student reasoning in the pre-intervention meeting and in post-lesson conversations in Stage 1. It may also have been because I was more aware of effective pedagogies and made these explicit to the teachers during the discussions. It may also be because of the narrow focus (Casey, 2011; Feiman-Nemser, 2012) on the observation of teacher actions to facilitate student reasoning. For example, one AP (Hannah, HPS) reported that this supported the teachers at her school to restrict their focus and exclude unimportant things: “[Focusing the observation] on reasoning . . . having that one thing to look at made it much more [narrow] so that we didn't worry about or notice all the other things. Having that sole focus made it beneficial”. Moreover, another AP indicated that modelling teacher actions to develop students' proficiencies in mathematics was powerful because the teachers were able to transfer the pedagogies they observed across all mathematics lessons.

It was a valuable intervention for our school. Just the fact that you [Louise] were modelling the proficiencies rather than looking at content. You were helping the

classroom teachers with their practice of mathematics in any content area because you were helping them to develop children's proficiencies. (Maude, MPS)

Importantly, discussing the complexity of teaching after the observations seemed to support the teachers to begin to develop new ways of thinking about teaching.

6.4.2 Theme: Noticing and Uncovering the Complexity of Teaching

Two principals and two APs reported that teacher learning came from teachers seeing me persevere, rephrase questions, give thinking time, and modify the lesson "on the fly" in response to students' unanticipated reactions.

The second lesson showed them [the teachers] that if you persevere and rephrase your questions and give the children time to think it through then eventually they will get there. I think that for those people who are not that keen on this approach they saw that you don't give up, you keep going, you give time, you challenge, you let the kids stew . . . a little bit in that zone of confusion. (Sheila, SPPS)

They [teachers] liked to see you work hard . . . it wasn't a process that was a breeze for you . . . you did have to adjust and modify as you were teaching and have different parts that didn't go quite right and then how you adjusted. (Principal, SPPS)

There was so much rephrasing and scaffolding and questioning and then you were using enabling prompts and then went to concrete aids, there was a whole range of things that were going on to facilitate their [students] reasoning. (Maude, MPS)

These comments indicate the teachers valued the experience of observing me work hard in adjusting and modifying the teaching in reaction to students' unanticipated responses. This finding resonates with Loughran (2006), who argued that preservice teachers can learn to recognise, react, and respond to teachable moments by having their attention drawn to instances where teaching is adjusted in response to the unanticipated. I found this applied to the teachers in this study who observed many adjustments to my teaching in response to students' unanticipated reactions to the learning experience. This noticing prompted five teachers to ask clarifying questions during the post-lesson discussions; for example,

Why did you stop yourself from writing that number sentence? All those critical decisions that you are making . . . You made a decision about where not to go because it was best to go somewhere else; none of those things are in the plan. (Jackie, HPS)

That whole notion of within 10 is an interesting thing e.g. understanding the 6 and 4 are contained within 10. What would you have done otherwise to get the children to notice a pattern? (Heather, HPS)

Furthermore, I openly articulated my reflections on my practice, including my challenges and dilemmas and why I made specific teaching moves, in the post-lesson discussions (also see Sections 7.3 and 7.5.1). My explanations seemed to support the teachers to develop an understanding of my pedagogical decision-making. For example, four teachers reported that they valued the opportunities to talk about the uncertainties in their thinking regarding the teaching they observed.

I think it takes someone to be there as a “critical friend” with a special interest/knowledge in the subject so that the knots in the thinking can be heard. (Heather, HPS)

Gathering together and reflecting on why [teaching] choices were made was crucial. (Maggie, MPS)

This seems to indicate that “messy” elements of my modelling created possibilities for the teachers to uncover and make sense of some of the complexities of the teaching. It implies that those who model with the aspiration to teaching “best practice” may not be helpful in supporting teachers to develop an understanding of the underlying principles of the pedagogical approach. This is because when practice is perfect, the complexity of teaching is not easily noticed (Lefstein & Snell, 2014). Furthermore, “A pervasive best practice mentality shuts down possibilities for critical discussion of the complexities of teaching” (Lefstein & Snell, 2014, p. 3). This confirms that those who model have an important role in ensuring that the thinking, judgements, or decisions associated with unanticipated teaching moves are made explicit to observing teachers (see, for example, Casey, 2011; Feiman-Nemser, 2012) so that

they can be discussed, interrogated, and understood (Loughran, 2006). This may support teachers to learn to recognise and react to teachable moments, build their KCS (Ball et al., 2008), and develop their pedagogical reasoning (Loughran et al., 2016).

In the next section, findings from the category *teacher learning from deconstructing pedagogies* are presented.

6.5 Category 3: Teacher Learning From Deconstructing Pedagogies

In this section, two themes within the category of *teacher learning from deconstructing pedagogies* are presented. These themes are (4) corridor conversations engaging in practice and (6) learning to be purposeful with teaching. These themes are presented next.

6.5.1 Theme: Corridor Conversations Engaging in Practice

Following pre- and post-lesson meetings to deconstruct the pedagogies being modelled, the teachers at HPS and MPS reported that they continued the conversations “into the corridors”. This is a new dimension to the finding in Chapter 5, that the teachers valued collaborative discussions to talk about the mathematical ideas. Time to talk and reflect informally about the impact of the pedagogical approach on student learning was recognised by the teachers from these two schools as important for their learning.

The post-lesson meetings were most beneficial [in Stages 1 and 2] . . . the conversations that were happening in that room and beyond that day. For example, with the focus on reasoning, having that opportunity after the lesson to debrief and think about how we could build on it. After that we would just have conversations in the corridors . . . this was a critical phase. (Hannah, HPS)

The discussions pre and post, seeing it work and being able to see children’s thinking [prompted my learning]. (Melissa, MPS)

The opportunity to chat . . . to reflect [on our own] and a bit of distance [have been] an important part of our development. We talked about what might be the next step for some of those children that haven’t quite got the concept . . . and about times for direct teaching outside the problem-solving. (Heather, HPS)

The AP of HPS reported that the discussions prompted the teachers at HPS to focus more on the mathematical concept of difference and share their new knowledge with the primary team in their school.

I have noticed that Holly and Heather are doing a lot more work on the notion of difference. We have really picked up on that concept and about using the different vocabulary. We shared the concept of difference with the primary team and what that means. (Hannah, HPS)

This comment suggests both the formal and informal talk was critical for the teachers learning to enhance their SCK (Ball et al. 2008). This theme is consistent with Lefstein and Snell (2014), who maintained, “Some of the most important learning occurs informally, as a natural outcome of teacher interaction around the work itself” (p. 176).

As well as talking about practice, co-planning for enactment was also an important component of teachers’ learning.

6.5.2 Theme: Learning to Be Purposeful With Teaching

During co-planning for enactment, Sophie from SPPS reported she found it a challenge to think about the rationale for her modelled lesson because this practice was not usual for her: “The rationale for the lesson was the hardest. Trying to think of the theory behind something is not something we do. We just know what we have to do and we don’t go looking for the theory behind it”. However, the findings indicate that co-planning for enactment seemed to prompt Sophie and at least three other teachers to critically reflect on the purpose of a lesson. They seemed to recognise the importance of choosing tasks that align with their learning goals. Sophie also reported the planning for enactment prompted her to reflect on and anticipate any difficulties students would have.

[The planning prompted me to] think about the purpose of the lesson and the challenges for [students] before the lesson. (Sophie, SPPS)

[The planning meeting] and the questioning [by Louise] [prompted us] to look at the tasks we are setting . . . Are we setting tasks that align with goals of a lesson? . . . When we were doing the planning [Stage 3] we had our curriculum there, we thought this is what we want the kids to learn . . . but when we set the task, [we realised] a word could change the meaning of the whole task . . . we had to keep referring to our outcome to get those words correct. That challenged us. (Hannah, HPS)

I've realised you can choose tasks from different books, but if you can't unpack the maths that's in underneath the task, then you're not developing the children's maths skills; you're just choosing tasks. (Heather, HPS)

This finding builds on the finding from Section 5.5.1 where the teachers at RPS recognised their choice of task was important for providing opportunities for student reasoning.

The teachers at HPS identified the importance of the collaborative discussions in planning for enactment. They could support each other in tossing ideas around to plan challenging mathematics tasks aligned with student need and lesson goals.

Co-planning meant you had your colleagues to really sort out the problem and make sure it was going to do what you were aiming for . . . And all the toing and froing about the task and the talk that we have had [and thinking] "Is this task really going to do what we intend". (Heather, HPS)

[I have recognised] the importance of our discussions in planning for the tasks. (Hannah, HPS)

Having a plan like this helps us be more focused on the maths we are trying to draw out. (Holly, HPS)

It is important to note that at HPS, the teachers eagerly embraced the idea of collaboratively planning for enactment in Stages 3 and 4 with limited input from me due to time constraints. I questioned the teachers about their initial planned task for Stage 3 (see Section 7.7) and that seemed to instigate critical reflection. What is more, the three school leaders at HPS planned collaboratively with the other two teachers. This appeared to reinforce

the value of the work and seemed to engage one teacher (Henrietta) who was about to retire. Importantly, these findings illustrate that teacher learning took place through interactions with colleagues (Borko et al., 2008; Lave & Wenger, 1991; Putnam & Borko, 2000), where each teacher seemed to contribute to advance the MKT (Ball et al., 2008) of the group.

These findings indicate that planning for enactment was an important component of the teachers' learning to be purposeful with their teaching. The next section discusses the challenges the teachers encountered with enactment and what they reported they enacted.

6.6 Category 4: Teacher Learning From Enactment

In this section, a new theme that emerged from the cross-case analysis is presented. This theme is (6) *enacting particular pedagogies*. This theme adds a new dimension to the theme in this category in Chapter 5, which focused on Rose's learning to facilitate student problem-solving and reasoning.

6.6.1 Theme: Enacting Particular Pedagogies

Within this theme, there are three subthemes. These are (a) the complexity of enactment, (b) assimilating particular pedagogies, (c) enacting particular pedagogies.

6.6.1.1 Complexity of enactment (a)

The four teachers who modelled lessons in Stage 4—Rose (RPS), Marita (MPS), Hannah (HPS), and Sophie (SPPS)—reported that their enactment both challenged and extended them. For example, Marita recognised the difficulties she had in selecting which student work to share and in asking questions to probe thinking and connections between the mathematical ideas. She expressed she needed more practice with that.

The hardest thing I found was looking over students' work to find examples to use to model for other students and bringing the lesson back in . . . It's something I need more practice at. It highlights the importance of thinking about the questions we ask children.
(Marita, MPS)

Selecting examples to exemplify the mathematics is difficult (Smith & Stein, 2011). This is because the teacher's decision involves thinking about the mathematics that the children are working on, assessing whose examples are key to the instructional goal of the lesson, and at the same time assessing how each example will contribute to that goal (Ball, 2017; Smith & Stein, 2011). It suggests that this might need to be unpacked further in planning meetings by decomposing the important component parts (Boerst et al., 2011) described. Furthermore, Marita's difficulty in asking questions to facilitate her students' explanations is consistent with findings from Polly and Hannafin (2011), who examined the extent to which two primary school teachers enacted learner-centred pedagogical practices that they observed and participated in over a 12-month period. Polly and Hannafin (2011) found posing higher order questions to be of greatest difficulty for teachers, even when project staff had modelled techniques repeatedly. However, the level of support provided in co-planning influenced the extent to which teachers could pose such questions. This suggests that by considering questions to ask in planning for enactment and in the modelled lesson prebrief as part of anticipating student strategies and planning adjustments to the prompts, the teachers might be supported to enact purposeful questions (see also Section 7.4.1).

Marita also conveyed that whilst she thought she was enacting new pedagogies in her practice, this was not necessarily the case. For example, prior to her enacted lesson she asserted: "I have already implemented things that I've learned from you [Louise] into my classroom and I've already trialled the lessons with the kids". However, after viewing video footage of her enacted practice, she reported she did not say the encouraging phrases she thought she was using and found it difficult in refraining from telling:

When I watched myself [on video], I was saying things like "no, not quite" and I was like argh! I didn't say things that I have been saying in my classroom a lot like "keep thinking" [pause] I need to focus on the thinking . . . It is so hard not to tell them what to do. (Marita, MPS)

Importantly, Marita recognised her challenges after viewing the video and set herself a goal to improve her practice. Sophie reported she learned to challenge all children and set a goal for herself around using open-ended challenging tasks.

I learned that it's okay to challenge all the kids . . . My goal is to use more open-ended real-life problems. Even though I knew that was what I was meant to be doing, I don't think I was doing that very well. (Sophie, SPPS)

Similarly, Hannah reported that by viewing video footage of her own practice after the modelling she could critically reflect on her teaching.

It was difficult being filmed but it was very beneficial [pause] I [asked myself] what questions am I asking the students? Is my teaching affecting the learning? I realised I missed this question, I cut that off too soon, I did that well . . . It was beneficial being able to take that [insight] back into the classroom. (Hannah, HPS)

This indicates that viewing the video footage of their teaching prompted the teachers to reflect on their teaching, recognise their challenges, and set themselves goals to improve.

It seems the processes involving observing, uncovering, and reflecting on the complexity of practice seemed to support the teachers in developing new conceptions of teaching that they sought to enact. Further evidence of this is illustrated in Megan's response to observing Marita's enactment:

There was a lot of thinking time. Because you said that very clearly [to the children] how they need to explain their thinking, I could see that Tory [pseudonym] was sitting here thinking about how she was going to solve the problem and Cory [pseudonym] as well "How am I going to draw that because I sort of know . . . How am I going to explain that?" Just the way you set that up enabled it to happen. (Megan, MPS)

This suggests Megan's noticing of Marita's provision for extended thinking time and student explanations was based on her reconceptualised knowledge gained from observing, reflecting on, and discussing the modelled pedagogies. Likewise, it seems Marita's knowledge enabled her to focus on student explanations and extended thinking time for her

students. This finding supports the view that enactment increases when teachers engage in processes to understand the underlying meaning of observed pedagogies. It is consistent with others whose work has focused on preservice teachers (e.g., Bronkhorst et al., 2011; Loughran, 2006).

Marita concluded that in enacting pedagogies, she needed to work out how to assimilate aspirational pedagogies into her existing practice.

6.6.1.2 Assimilating particular pedagogies (b)

Marita recognised that rather than “imitating” me, she had to be herself and make decisions about which teacher actions she found important and how she might incorporate those into her teaching practice with her own nuances.

From my [enactment], I learnt it's tricky to try to be somebody else. I tried to model my lesson from what I saw [Louise] do and I found that was difficult . . . Sometimes we need to be just who we are but pull in the pieces of other people's teaching that we would like to aspire to. (Marita, MPS)

This finding agrees with Buchmann (1986), who argued teachers must find their own style of teaching. It also agrees with Anthony, Hunter, and Hunter (2017), who argued, “Enacting reform is not a matter of wholesale adoption of advocated practices but rather a matter of figuring out whether, when and how to incorporate new ideas” (p. 9). It resonates with Loughran (2006), who maintained the idea is for observers to understand the underlying intention of the teaching approach, which is “in stark contrast to the misconception that modelling is a mock teaching demonstration or a tacit call to ‘teach like me’” (p. 95).

In the next subtheme, Table 6.6 shows the 18 teachers' expressed changes to their own teaching practice after being involved in this study. This was reported in exit surveys three months after the conclusion of the intervention.

6.6.1.3 Enacting new pedagogies

As reported previously, the teachers in all four schools completed an exit survey three months after the completion of the intervention, which was designed to ascertain any changes to their teaching practice. The survey contained the following prompt:

Please describe any changes to your teaching practice as a result of being involved in this study.

The categories that emerged from the data analysis are shown in Table 6.6. In each instance, a description of each category is given, as are one or two illustrative examples of the types of responses in the category. The categories are presented in order of decreasing frequency.

Table 6.6

Reported Changes to Teaching Practice on Exit Surveys

Category	RPS	MPS	HPS	SPPS	Illustrative Examples of Teacher Responses
Tasks (Using problematic open-ended, real-life tasks)	3	5	1	6	“I’m more aware of the tasks I set and how they are linked to the outcome of the lesson” “Making the learning more challenging”
Time (Providing thinking time, refraining from telling students what to do, changing the pace of the lesson)	1	4	2	5	“Not telling as often as possible” “Embracing the right to be in the zone of confusion”
Differentiation (Catering for the range of abilities, challenging students, and peer teaching)	2	4	1	2	“Having enabling and extending prompts on hand for those who need them” “Planning for enablers/ extenders/ considerations/ problems which may be encountered”
Communication (Encouraging students to explain and share their thinking and reasoning)		1		5	“In the past, I tended to explain a strategy before the students came up with it” “More productive and valuable sharing of students’ work at the end using photos on smartboard”
Specific content (Greater focus on planning in advance)	1	1	3		“Greater focus on purposeful planning—what I want my students to know and do” “Designing the problem-solving tasks has enabled me to be clearer about the maths I am focusing on”

Category	RPS	MPS	HPS	SPPS	Illustrative Examples of Teacher Responses
Materials and representations (Using resources, models, and representations)				3	“Allowing students to explain their learning and reasoning using the air-server”
Questions (Improving questioning, asking students how and why)			1		“Stronger awareness of the questions that I am asking students which will draw out their thinking”
Mismatch or unclear (Where the response does not relate to the question)		2			“A greater confidence and awareness of the importance of every maths lesson” “Now I have a ‘zone of confusion’ during literacy”

Eighteen teachers completed the survey, with some teachers indicating more than one change to their practice. There were 41 comments in all, an average of two comments per teacher, indicating that on the whole, the teachers perceived that they had learned from the experience and had made changes to their practice. Two teachers did not clearly indicate an area of change (relevant to developing student reasoning in mathematics) as indicated in the last category of Table 6.5.

More than three quarters of the teachers reported that they had implemented problematic, open-ended tasks, and two thirds of the teachers stated they now provided time for the students to think for themselves. These findings are at odds with Clarke et al. (2013), who found that only one tenth of participants in their study reported their intent to choose engaging open-ended tasks, and one quarter reported issues about aspects of time. There are several possible reasons for this. One reason might be that the teachers in this present study participated in four episodes of modelling, deconstruction, and enactment of pedagogies over an eight-week period. Each episode was set two weeks apart, whereas Clarke et al. (2013) visited each school from two to four times over 12 months. The fact that the episodes in this present study were close together seemed to build momentum. One example of this was the reported corridor conversations to discuss the pedagogical approach (see Section 6.5.1). Another possibility for the difference may have been the opportunities for teachers to build their understanding of the underpinnings of the pedagogical approach in post-lesson discussions. In Clarke et al.'s (2013) study, the post-lesson debrief was 15 minutes long. In this present study, the post-lesson debrief was 30 minutes long, and the teachers also participated in a 60-minute planning session for the Stage 3 co-teaching episode. The additional length of time to talk about the pedagogical approach and the student responses to the approach may have been a contributing factor.

Interestingly, half the teachers expressed that they now catered for the diverse range of abilities with enabling and extending prompts. This finding also differs from findings by Clarke et al. (2013), who found approximately one eighth of participants in their study reported their intent to change aspects of differentiation after observing modelled lessons. The difference in findings may have been because the proposed prompts for each task were reflected upon and discussed in post-lesson briefings.

One third of the teachers conveyed they now explicitly encouraged students to reason and explain their thinking. Again, there are several possible reasons for this relatively low proportion. One reason might be that the teachers experienced difficulties in drawing out students' thinking and reasoning. This resonates with Marita's comment (see Section 6.6.1.1) regarding her difficulties in eliciting student thinking. Importantly, Smith and Stein (2011) argued that anticipating student responses is an important step in supporting students as they engage with and discuss their solutions to challenging tasks, but I surmise that I may not have facilitated this as effectively as hoped in the modelling (see Section 7.4.1). Consequently, it does not seem surprising that a low proportion of teachers in this study reported they had implemented encouraging student reasoning. Another possible reason might be the challenges the students faced with the change of pedagogical approach and teachers' expectations of them. For example, they may have assumed they were expected to respond to tasks in the way the teacher previously considered appropriate rather than express their own thinking. Another possible reason might be the challenge for the teachers to renegotiate classroom norms so that the students understood the expectation that they would think for themselves and explain their thinking.

Five teachers across the four schools stated they had a greater focus on planning. However, this included three of the four teachers at HPS. One possible reason for the high proportion of teachers at HPS may be that the principal planned collaboratively with the other

two teachers in contrast to the other schools where only the APs participated. As mentioned previously (see Section 6.5.2), this appeared to reinforce the value of the work. Another possible reason is my questioning about the tasks the teachers co-planned for Stage 3, which they reported prompted them to critically reflect on choosing tasks that aligned with the goal of the lesson.

These findings show that, on the whole, the majority of teachers reported they were able to enact pedagogies associated with facilitating student reasoning and problem-solving.

In the next section, findings from the category of *reactions to the intervention* are presented.

6.7 Category 5: Stakeholder Reactions to the Intervention

In this section, five themes that emerged from the cross-case analysis from the four schools are presented. These themes are implementing the intervention; seeing, deconstructing, and enacting pedagogies; engaging students; differentiating the experience for the teachers; and accountability. The themes are presented below.

6.7.1 Theme: Implementing the Intervention

Whilst the school leaders at all four schools raised the issues of the cost of the intervention in terms of paying emergency teachers to cover classes, they all thought it was money well spent. One principal indicated that spending more time (and money) by extending the model might be too expensive. He suggested that the four stages were optimal in terms of financial cost and teacher learning.

It was so tricky trying to get six relief teachers. We had to pay relief teachers but it was all so valuable. (Maude, MPS)

It was hard for us to get the two hours . . . we did all that because we had the back-up. (Principal, SPPS)

I personally think you can't really spend a lot more time on it [the intervention] because of the cost. (Principal, HPS)

Three teachers reported their colleagues were not happy with the timing of the intervention for various reasons, including missing planning meetings, extra pressure on time during the report-writing period, and disruptions to the timetable of the school day. One AP suggested that the timing of the intervention should be prioritised to suit the school's needs rather than the researcher's time.

We would have got a lot more value out of this [intervention] if it was not concurrent with our reporting period. (Sabrina, SPPS)

The duration of the sessions did not match the segments of the school day here and hence was disruptive (eating in to teacher lunch break). If the timing came from within the school rather than the [researcher's] timetable it might have been even more productive. (Sheila, SPPS)

I think teachers could have a say in when it is held. I think it would be nice to have an initial meeting with teachers to see what would fit in with their timetable. (Marita, MPS)

These comments suggest some of the teachers did not feel ownership of the intervention in terms of scheduling. Furthermore, one teacher indicated that some of the pre- and post-lesson meetings seemed rushed due to time constraints.

Timing is difficult . . . I thought that only having a half hour session to [debrief the last lesson] and not having a chance to plan together what we wanted to get out of today was really difficult. (Megan, MPS)

The principal of HPS proposed that his staff would come up with a problem of practice to develop greater teacher ownership.

What we would do is pick something that we're really struggling with or something new that we are really challenged by and that would hook them [the teachers] in. (Principal, HPS)

This finding highlights the importance of involving teachers in the decision-making processes so that they feel ownership of the intervention and partners in the change process (Clarke, 1994).

From my perspective, it seemed that the teachers at HPS were initially a little unsure of the processes in the initial stages of the intervention because they did not have a relationship with me.

There was the trust factor . . . trepidation [from teachers] around how am I going to go, I'm not sure I feel comfortable being watched, I'm not sure I feel comfortable talking into audio recorders . . . Their [teachers'] Achilles heel was that they didn't know [the researcher] . . . At each stage the tenseness eased. (Principal, HPS)

Initially some teachers were daunted by having an extra person in the room when they are usually just in the classroom by themselves with all the kids. (Hannah, HPS)

However, this seemed to ease as the intervention progressed. For example, in her exit interview, Heather reported, "I moved from being uncertain and wary to really seeing the benefits through the whole process at the end". This seems to indicate a shift in Heather's perception of the intervention from beginning to end. This finding, which was not apparent in other schools, is consistent with Timperley (2015), who argued that relationships are developed through conversations and do not necessarily exist prior to them.

One teacher reported that there was a risk to the modeller that her practice may be judged negatively.

There are a lot of teachers who are stuck in their ways and they might see [a modelled lesson] as personal . . . they might judge the modelling teacher as academic and not so good in the classroom, but I think it is always good to see other people's ideas and experience different ways of thinking. (Rebecca, RPS)

This corroborates with Loughran (2006), who argued modelling has risk associated with it because "assumptions, teaching behaviours and perceptions of learning are under scrutiny" (p. 42). It highlights the need for a disclaimer to be made about the modelling that the

modelling is not intended to be perfect across all aspects of classroom management and request that teachers refrain from evaluating the lesson.

6.7.2 Theme: Seeing, Deconstructing and Enacting Pedagogies

Four teachers reported that seeing the pedagogies modelled contextualised for them the theory about developing student reasoning and problem-solving. They valued the collaborative nature of the intervention and the fact that it enabled them to inquire into the teaching.

[The modelling] put theory into context. It made learning the strategies more engaging and relevant. It made [the pedagogical approach] seem achievable. (Suri, SPSS)

Everyone involved gets something out of it that they can straight away take back to their own practice because you're involved in it. We're all there. You feel inspired by what you've seen. It's not like somebody telling you what to do; it's somebody modelling a way of doing things. (Marita, MPS)

You are experiencing the whole package—you are seeing, you are getting involved, it is multimodal. You're not just hearing it or reading it . . . you are getting the full picture. (Rebecca, RPS)

Consistent with the findings of others such as Clarke et al. (2013), Bruce et al. (2009), and Yook-Kin Loong et al. (2017), it seems the contextualised experience for teachers was a catalyst to enable them to reflect upon their own practice and make changes. Marita reported she felt inspired by what she saw, and it seems the collaborative nature of the intervention also supported the learning for the teachers.

The four principals indicated that the intervention involving seeing practice modelled, deconstructing practice, and enacting practice was powerful professional learning because it was situated in classrooms and enabled teachers to see, discuss, and enact new pedagogies in their own contexts. They noted that this was different to other forms of professional learning that involve talking about teaching but may not be situated in classroom practice and therefore may not ever be enacted.

I think [modelling] is critical because it's where we get to the coalface . . . It is teaching . . . and then going and talking about teaching as opposed to other forms of PL [professional learning] that are just talking about teaching and may never actually make it back to the classroom. (Principal, HPS)

I think it bridges that gap around, "Well what does it look like in my classroom?" . . . It's the link between talking about practice, and doing practice, changing practice. We've got so many teachers in the teaching profession that can do the rhetoric, but the practice in the classroom doesn't match the rhetoric. (Principal, SPPS)

Classroom modelling is a much better way of helping teachers improve their practice [than other forms of professional learning] because you have the real-life context and you can see the [modelling teacher] making the adjustments and deal with a range of needs. It's not just talk and chalk. You can see and then go away and develop that in your own classroom. (AP, MPS)

Importantly, the four principals maintained that although some teachers were initially reluctant, they made shifts in their teaching practice, including having higher expectations of their students, posing challenging tasks, and changing the way they differentiate practice.

I've now seen in most classrooms the teachers using the same lesson structure as [Louise] did and [having the] whole class [working on the task] using the enablers and extenders. There has been more evidence of change in [teachers'] teaching practice in having high expectations and making sure they were challenging everyone. (AP, MPS)

[Observing teachers] learned that our kids could actually do the things that you were talking about. They actually saw it in practice . . . So, I'd definitely say that there has been an uptake in teaching practice . . . and there's been far more high-level discussion in teams and groups around problem-solving in maths. (Principal, SPPS)

Teachers are now allowing students more time to think through mathematics, a shift towards more "problem-solving" rather than worksheet-driven lessons. (Sheila, SPPS)

Moreover, the principal from HPS claimed the intervention supported the move towards teachers observing each other's classroom practice, which is something he had been seeking.

I have been looking for ways to get teachers to be in each other's classrooms. With the journey we've been on with both literacy and numeracy we have been trialling lots of

new ways of teaching and specific strategies and they meet regularly but the one factor they were missing was the opportunity to be in each other's classrooms. (Principal, HPS)

One principal also saw the potential of exploring the model in other curriculum areas: "Exploring the model through other curriculum areas would be interesting . . . embedding the model, probably in your whole year of team meetings and learning group meetings" (Principal, SPPS). This suggests this intervention could be replicated in other areas, such as literacy, with a different pedagogical focus.

These are encouraging findings and are harmonious with situated learning perspectives (Borko et al., 2008; Putnam & Borko, 2000). Consistent with Grossman and her colleagues (2009), it seems that observing deconstructing and enacting pedagogies supported the teachers to shift their views of teaching to include a focus on problem-solving rather than worksheets, high-level discussions in teams about the mathematics, and collaboration with their colleagues.

6.7.3 Theme: Engaging Students

In their exit interviews, the four principals and four modellers reported that the students in their schools had benefited from the intervention in the sense that they had improved their dispositions for learning mathematics. They were now willing to have a go at challenging tasks and were more engaged in learning:

The kids have really benefited. They talk about the "zone of confusion" and that it's okay to be there. (Sophie, SPPS)

From the very first lesson that we observed there were lots of children sitting there that had no idea, just didn't know how to start but now because the teachers have been continuing the work I've noticed that the children now have a go . . . The [children] are feeling that sense of achievement from it. (Maude, MPS)

Kids can take that risk and have a go and not be afraid of failure and know that they are going to be supported. If we are using modelled lessons in maths to get that happening then that is awesome. (Principal, MPS)

Importantly, these comments provide evidence that the teachers continued to enact the desired pedagogical approaches after the intervention ceased. Consequently, the students were also over time showing evidence of improving their dispositions for mathematics learning. *Productive disposition* is one of the five strands of mathematical proficiency in the United States (Kilpatrick, et al., 2001). Having a productive disposition is a major contributing factor to successful student learning (Kilpatrick et al., 2001).

6.7.4 Theme: Differentiating the Experience for the Teachers

The principal from RPS suggested that the project could be extended beyond the four stages with further co-teaching opportunities to cater for teachers' individual learning needs. This finding was supported by the AP from HPS who suggested the project be differentiated because some teachers were able to shift their views of teaching more quickly than others.

[The intervention could be improved by] more than one co-teaching session to build confidence if needed . . . another watched session to “improve/tweak” practice further after initial feedback. (Principal, RPS)

Some people changed practice within four stages and for some it would need longer; it really depends on the person and what it is you are trying to change. (Hannah, HPS)

Furthermore, the principal from SPPS proposed an extension to the project so each observing team member had an opportunity to co-teach and enact pedagogies, which she believed would assist everyone to maximise their potential for learning.

I definitely believe that the two modelled lessons at the start were really good to see you modelling . . . after that if we'd had an extended model where every teacher in the team then had a turn at doing the co-planning and co-teaching with you so that everybody got exposure. (Principal, SPPS)

Furthermore, two principals reported the scaffolded nature of the intervention was powerful. This supports Rose's comment in Chapter 5.

I just think seeing someone else [model practice] is a powerful thing, then seeing one of their colleagues practise as well. It's a powerful step into having a go, rather than just going to professional learning and talking about something. (Principal, SPPS)

These findings suggest the intervention could be improved by being flexible enough to recognise and cater for individual teacher learning needs. This agrees with Clarke and Hollingsworth (2002) and Resnick et al. (2010), who argued professional learning must offer participants every opportunity to learn in ways that each participant finds helpful. One approach might be to anticipate the possibility of multiple cycles of modelling, deconstructing, and enacting pedagogies at the start that does not inhibit teachers' learning by being too prescriptive. This would make possible an approach that would allow different pathways to be taken depending on individual teachers' learning needs.

6.7.5 Theme: Accountability

The four principals indicated that the structure and tight processes of the intervention made it robust.

I think you've got to have a tight model around it and that's what made it work so well. Our Watch Others Work stuff [professional learning for teachers] that we did last year was looser and still far more invitational, but we have some staff here that would take forever to get on board. I think that the four-session structure was the power of it. (Principal, SPPS)

They seemed to value the implied accountability measures, which expected all the teachers to contribute to pre- and post-lesson discussions to deconstruct observed pedagogies and collaboratively plan lessons.

There was some accountability to being involved "up close and personal" that is often missing . . . the ongoing nature made it more meaningful rather than "one off" professional development approaches. (Sheila, SPPS)

This [intervention] can bring a greater level of accountability planning after school. You can't fluff something up if your colleagues are in there supporting you do it. (Principal, HPS)

This form of modelling and coaching was very effective PL [professional learning]—it gave teachers support to practise new learning. When teachers go to external PL they come back full of new ideas but most rarely implement them; there is no accountability, there is no feedback . . . it feels like wasted opportunities, and costly. (Principal, RPS)

This finding resonates with Resnick et al. (2010), who argued that specified and structured opportunities for teachers to understand new forms of teaching can serve as powerful tools for transforming school practice.

6.8 Chapter Summary and Conclusion

This chapter presented 11 themes including six new themes that emerged from the cross-case analysis of RPS, SPPS, MPS, and HPS teachers' and principals' participation in this study. The findings were organised according to the four research questions. Extensive samples of quotations were included in the report to build confidence that the reality of the participants in the study has been accurately represented and interpreted reasonably.

6.8.1 Linking Findings to the Research Questions

Findings from this chapter are summarised as follows.

6.8.1.1 Teacher assumptions challenged

At least eight teachers were confronted by the high expectations of their students and how their students rose to the challenge. This seemed to create cognitive dissonance in their minds and prompted more than half of them to set goals for shifting their practice. It suggests that modelling *different* pedagogies associated with problem-solving and reasoning with the teachers' own students is a powerful form of professional learning to challenge traditional views of teaching.

The educational intervention seemed to enhance teaching and learning mathematics in the following three ways.

1. Modelling different pedagogies associated with reasoning and problem-solving (ACARA, 2016) seemed to support a considerable number of teachers to notice teacher

actions associated with developing their students' reasoning and problem-solving. Building on the findings from Chapter 3 (Hodgson, 2013), teachers' noticing my actions to facilitate student reasoning seem to be more detailed in this study. It seems the prompt for the observation of teacher actions to facilitate student reasoning supported the teachers to focus their observations. Some of the teachers seemed to notice student actions as a consequence of teacher actions. Furthermore, one AP indicated that modelling teacher actions to develop students' proficiencies in mathematics was powerful because it supported the transfer of pedagogies across all mathematics lessons.

2. Two principals reported their teachers valued seeing me work hard in adjusting and modifying the lesson "in the moment" to support student learning. It seems the "messy" elements of my modelling created possibilities for the teachers to uncover and make sense of some of the complexities of the teaching. These complexities were brought into the open through my open reflections on my practice and questions from some of the teachers. This enabled tacit elements of my practice to be discussed, interrogated, critiqued, and understood. The discussions provided opportunities for the teachers to reason through my pedagogical decisions, which seemed to support them to develop their pedagogical reasoning. Moreover, time to talk and reflect informally about the impact of the pedagogical approach on student learning appeared to be an important component of the teachers learning.
3. In co-planning for enactment, several teachers reported they learned to be purposeful with their teaching. For example, Sophie reported that she had never considered the rationale for a lesson prior to the intervention. The four modellers reported that their enactment both challenged and extended them. They conveyed they recognised difficulties they encountered with aspects of the lesson. Difficulties included selecting

which student's work to share, asking questions to probe student thinking, refraining from telling, and connecting the mathematical ideas. After watching video footage of their enactment, the four modellers were prompted to reflect on their teaching, recognise their challenges, and set themselves goals to improve. Importantly, it seems the processes involving seeing, uncovering, and reflecting on the complexity of practice supported the four teachers in developing new conceptions of teaching that they sought to enact. Furthermore, it seems the "modelling" offered the teachers opportunities to deepen their understanding of effective pedagogical moves so that they could then make choices about which pedagogies they would assimilate into their own practice.

Exit surveys revealed that more than three quarters of the teachers indicated they had implemented problematic, open-ended tasks, and two thirds of the teachers stated they now provided time for the students to think for themselves. Half the teachers claimed they catered for the diverse range of abilities with enabling and extending prompts. One third of the teachers conveyed they encouraged students to reason and explain their thinking. More than one quarter of the teachers stated they had a greater focus on planning. The majority of these teachers came from HPS. There were varying possible reasons for these findings.

6.8.1.2 Reactions to intervention

The principals reported that they thought the intervention was money well spent in terms of paying for their teachers to be released from class. They reported this was because they saw the intervention as being valuable professional learning. For example, they conveyed that their teachers made shifts in their teaching practice, including having higher expectations of their students, posing challenging tasks, and changing the way they differentiated practice. The four principals noted favourably that the intervention was different to other forms of professional learning that involve talking about teaching but may not be situated in classroom practice and therefore may never be enacted. They seemed to

value the implied accountability measures that supported teacher engagement. Importantly, the principals reported higher levels of engagement from the students who were now willing to persist with challenging tasks.

One principal suggested the intervention could be improved by providing an opportunity for all the teachers to enact pedagogies, rather than just one. Suggestions from the other principals included differentiating the experience for each teacher according to the teacher's perceived needs and extending the intervention into other learning areas.

The teachers reported that it would have been beneficial if they had experienced greater consultation about the timing of the intervention. One principal suggested that having the teachers identify a problem of practice in their school may support teacher "buy in". Whilst there appeared to be some initial apprehension from the teachers at HPS regarding working with me, this seemed to shift quickly as the intervention progressed.

The findings in this study indicate that when given a focused lens for viewing (Ghousseini & Sleep, 2011) and opportunities to deconstruct their observations (Grossman et al., 2009), the teachers could identify important pedagogies associated with developing student reasoning and problem-solving and develop images of what it could mean to teach them.

Chapter 7 focuses on my learning to be a teacher of teachers in this study.

CHAPTER 7: LEARNING TO BECOME A TEACHER OF TEACHERS

This chapter presents my story of learning to become a teacher of teachers through my reflexive practice whilst collaborating with colleague teachers. Underpinning this is a situated perspective of learning in which learning is constructed through social interaction; it takes place in meaningful contexts and is distributed across others and artefacts (Borko et al., 2008; Lave & Wenger, 1991). The chapter provides an account of my experience of teaching teachers to implement the proficiency strands of the AC:M through modelling pedagogies and the deconstruction of them. I drew on elements of self-study methods to help me uncover my assumptions, challenge my beliefs, and reframe my practice as I gained new insights into teacher learning. The chapter is presented in three sections. First, the research aim, question, and research context are presented. Next, the identification of themes is explained. Following this, the themes are explicated to provide answers to the research question set for this chapter.

7.1 Research Aims and Question

This chapter explores the following research aims and question:

Aim	Question
To scrutinise my pedagogy in working with the teachers to 1. improve my practice; 2. work effectively with teachers; and 3. develop new understandings to contribute to the knowledge base of educating teachers.	1. What is the nature of my learning from the experience of becoming a teacher of teachers?

7.1.1 Research Context

The themes in this chapter are drawn from my reflections during this study.

7.2 Analysis and Identification of Themes

My reflexive analysis consisted of cycles of sifting through video and audio transcripts, emails, notes, and journal entries stored on my laptop to construct themes that illustrated the range of my reflections, dialogue with the teachers, and electronic conversations between me,

participants in the study, and my supervisors. Regularities, patterns, and topics that emerged from the data were sought to establish coding categories. A table consisting of two columns was created to illustrate the themes that emerged from the analysis (see Table 7.1). The first column comprises themes that emerged from the body of data. The second column consists of subthemes, illustrated in Table 7.1.

Table 7.1

Themes That Emerged From My Self-Reflexive Analysis

Theme	Subthemes
1. Articulating a focus for my modelling	
2. Balancing my desire to tell with enabling teachers to think for themselves	<ul style="list-style-type: none"> a. Learning to engage teachers in anticipating student responses to tasks b. Learning to refrain from telling
3. Exposing my practice “warts and all”	<ul style="list-style-type: none"> a. Learning to let go of the desire to teach a perfect lesson b. Providing commentary whilst modelling
4. Mismatches between interpretations of pedagogies	
5. Building trusting relationships with the teachers	

7.3 Theme 1: Articulating a Focus for My Modelling

When I commenced modelling (prior to this research), I do not think that I had clarity around what I hoped the teachers would learn. Instead, I had a broad vision for them to see and implement pedagogies associated with the AC:M (ACARA, 2016). In my mind at the time, these included open-ended tasks, extended thinking time, and opportunities for mathematical talk. Little was known in the literature at the time about specific teacher actions to facilitate students’ reasoning and problem solving. Consequently, I did not formulate a clear vision of the pedagogies I hoped to enact, and I did not articulate a specific focus for the observations. Naively, I assumed that if teachers saw lessons that were engaging for their students, they would add the ideas they had observed as being helpful to their own repertoire of teaching strategies, implement them in their classrooms, and transfer them to other lessons.

For example, in 2013, during a conversation with a colleague, I lamented, “The teachers did not notice the pedagogies that I’d hoped they’d notice. For example, they did not notice wait time even though I did give wait time . . . Maybe I can’t expect them [the teachers] to notice everything” (Modelled lesson reflection, 4 November 2013). This comment illustrates my initial view, by which I was not yet aware of the complex nature of what teachers pay attention to in a “modelled lesson”. A considerable turning point for me was in late 2013 during a modelled lesson debrief with a team of teachers who had observed a lesson focused on area and perimeter that I had modelled in a Grade 5/6 class. One of the teachers remarked, “Well I learned how to teach that [area and perimeter], but it’s just one lesson; what about everything else I have to teach in maths?” This comment was significant because it jolted me into questioning my assumptions. I asked myself, “Why can’t the teachers see and transfer pedagogies? What do I need to change?” Evidence of this was recorded in an email: “Clearly that [AP] didn’t see pedagogies that he could transfer, so I’m probably at fault for not making them visible enough”. (Email to Supervisor, 7 February 2014)

I think that I may not have made the pedagogical approach visible to the teachers because I did not yet have clarity in my own mind about the pedagogy I was modelling and specifically what it was I wanted the teachers to notice. Similarly, when I commenced my doctoral research at RPS in February 2014, I was not yet convinced about posing challenging tasks before instruction to facilitate student reasoning (Sullivan et al., 2016), even though I had read about it. Evidence is provided in my journal entry (Appendix 16):

The children did struggle with the initial task that I posed, so I posed a simpler version. I am not convinced the idea of posing a challenging task as a first step is the right way to go. I think it might be more effective to provide a simpler version then make it more challenging (Journal entry, 26 February 2014)

Similarly, I grappled with the focus of teachers’ observations during modelled lessons. For example, in an email to my supervisor, I stated,

The literature around observation of classroom practice suggests giving one thing for teachers to focus on. For example, “Write down every time you saw the modelling teacher asking probing questions” . . . It is difficult and I’m still thinking . . . I’m wondering whether having multiple aspects of the lesson to focus on is disabling rather than enabling? I will keep thinking. (Email to Supervisor, 7 February 2014)

After a period of reflection, I came to the realisation that given the focus of my modelling was on pedagogies associated with the AC:M, it seemed the focus then needed to be on the proficiency strands of reasoning and problem-solving because these appeared to be the key elements missing from practice (Australian Academy of Science, 2015; Kilpatrick et al., 2001). I surmised that for teachers to make the most of their observation, it might be helpful for me to familiarise them with the proficiency strands to prepare them. This resonated with Ghouseini and Sleep (2011) and Takahashi and McDougal (2015), who argued that engaging teachers with the content of the practice being studied is an important preliminary process. In correspondence with my supervisor, I articulated this: “If I am going to focus on reasoning, then I will need to ascertain what the teachers know about it. They may not know what it means, so how will they observe teacher actions associated with it?” (Email to Supervisor, 14 February 2014).

Through a process of reflection and examining my practice over time (Hamilton & Pinnegar, 1998), I conceptualised a clearer focus for the modelling. Once I had this clarity, I think I became more confident to explain the purpose of the pedagogical actions to the teachers in the study. For example, the following excerpt is from a post-lesson conversation at MPS regarding my pedagogical decision of selecting students to present their thinking and the subsequent teacher noticing of students’ learning:

As soon as Rick [student pseudonym] shared his idea [of using a number line] there were quite a few that went back and [used] that number line that Rick had explained. The children learned, that they could use an idea that the [other students] had explained. (Melissa, MPS)

Even that little girl at the end, she used that number line in a different way by breaking it into 5s . . . That's how they learn . . . from listening to [and understanding] what the other students say. Some of them listened to Rick, because they used that notion of 10 and that [student] at the beginning who said, "I did it the same way as [the first problem] . . . That's what we want to see". (Louise)

I noticed that Rick could get the answer really quickly, but he struggled to explain his thinking and it took him a while. (Maude)

So, then we have to revoice because it was really important thinking for the other students to hear. (Louise)

This extract shows I articulated the purposes of both revoicing and having students explain their thinking as a way of making my focus on choosing student explanations more explicit and noticeable to the teachers. This relates to Loughran and Berry (2005), who argued it is important for teacher educators to unpack teaching practice to preservice teachers so that the association between knowledge and action is made more visible, "so that the relationship between knowing and doing is more accessible" (p. 194). Similarly, I found unpacking the pedagogical actions that seemed unfamiliar to the teachers was important in my attempts to support their understanding. This implies that for those who model teaching practice, it seems important to be clear about the intended focus for the modelling, so that the thoughts and actions underpinning the approach can be made clearer to teachers and therefore accessible to them for their own learning.

Theme 2 considers the tension I experienced between telling and enabling teachers to think for themselves.

7.4 Theme 2: Balancing My Desire to Tell With Enabling Teachers to Think for Themselves

Whilst I'd hoped to work with the teachers in the same way that I advocated working with students—that is, allowing them time to think for themselves—in retrospect, I was

confronted by the amount of “telling” I did, especially in pre-lesson meetings. I did not become aware of this until I listened to the audio recordings to examine the data. The two following subthemes presented explicate this. These are (1) learning to engage teachers in anticipating student responses to tasks and (2) learning to refrain from telling.

7.4.1 Learning to Engage Teachers in Anticipating Student Responses to Tasks

During the pre-lesson meetings in Stages 1 and 2,¹ I noticed there was very little if any dialogue between me and the teachers. I had explained the rationale for the lesson, how it connected to the AC:M and what I would be looking for in student responses. I had described possible misconceptions to look for, some questions I may ask students, and I had outlined what the flow of the lesson would look like. My intent had been to provide clarity for the teachers about my intended pedagogical approach with students and the rationale for this. For example, in an email to my supervisor I wrote,

I think it’s important to be clear with the observing teachers about my learning intentions for the students and the sorts of actions I intend to use. This clarity needs to happen before the lesson. (Email, Supervisor, 5 February 2014)

My assumption was that if I provided a detailed lesson plan (see, for example, Appendix 8) and explained this to the teachers in the pre-lesson meeting, then they would understand the nature of the task, the intended pedagogy, and what to look for in terms of student responses. My plan is illustrated in the following email excerpt:

I have a plan now. In keeping with what the literature says about engaging the content, having a narrow focus, orienting teachers to the instructional context, providing a lens for viewing and providing insight into student thinking, I propose to do the following: Discuss the lesson plan, the intent, the content descriptors, the task/s, enabling and

¹ Stages 1 and 2 involved me modelling 60-minute lessons with 30-minute pre- and post-lesson briefings with the group to discuss the intent and format of the lesson.

extending prompts, questions I'll ask possible responses from students and misconceptions to watch for. (Email to Supervisor, 7 February 2014)

However, I later suspected that my “telling” in the prebrief had not seemed to lead to the teachers gaining sufficient understanding of the pedagogical approach to recognise student learning in the ways that I had hoped. For example, both Ruth’s comment in Section 5.4.2 and Sabrina’s statement in Section 6.3.1 illustrate this. In another example, one of the teachers at RPS did not appear to immediately recognise my deliberate decision to incorporate key pairs of numbers—2 and 8 and 5 and 5—to target building students’ mental computation strategies in the Stage 1 modelled lesson that was focused on building to 10 and 100 (see Appendix 8). During the post-lesson meeting, she reported the following:

It didn’t even click to me till one of the little kids talked about that they’d worked out how to add two to build to 10 and they worked out how to add five to build to 10 and I was thinking I didn’t realise that you’d specifically always kept it to two or to five. I thought it was more random than that. (Rose, RPS)

A further example is from reflecting on an audio recording of the prebrief in Stage 2 at HPS when I asked the teachers how they thought their students would respond to the impending task (see Appendix 17):

I cringed at how much I talked when I went through the lesson with teachers . . . [The only thing] I asked [was] how the children would go and Holly said “Ah . . . you’ll see”. It seemed she was unsure. There was no discussion . . . In hindsight, it would have been much better to have had [the teachers] [read] the lesson [plan] and do the tasks or at least pre-empt what the kids would do. In that way, they would have had far more knowledge going in to the lesson of what to look for. (Journal, 14 June 2014)

It can be seen in this example that Holly’s response seemed ambiguous. Similarly, in a reflection of the prebrief at SPPS (Appendix 15), I wrote,

I asked the class teacher how she thought the children would [solve the task] and she said they would struggle as the tasks were very challenging. [The] other teachers said they thought the children would find the lesson too difficult. (Journal, 27 March 2014)

On reflection, it occurred to me that my “telling” was not effective in achieving my aim of engaging the teachers to think deeply about potential student responses to the task. This resonates with Berry (2007), who maintained that telling does not convey sufficient understanding to the recipient for it to be meaningful. I found myself wondering what would have happened if I had asked the teachers to anticipate student responses as described by Smith and Stein (2011) or to attempt the tasks themselves prior to observing the modelling lesson.

Consequently, I trialled a different approach to the pre-lesson meeting at SPPS in May 2014 with the Grade 5/6 team. I allocated 60 minutes to the pre-lesson discussion, which was 30 minutes longer than previous pre-lesson meetings. As a first step, I asked teachers to attempt the task called *The Dog Run* themselves (see Appendix 18). In doing so, one teacher recognised her misconception about perimeter and area. I reflected on this experience in my electronic journal (see Appendix 19).

Sarah [pseudonym] suddenly realised that her students might get 28 tiles when asked to make a dog run with a perimeter of 28 after she had demonstrated the misconception herself. I steered the conversation to get teachers to notice what we would be looking for and what the students might do. We all attempted to find all the possibilities. This process seemed to tune the teachers into the lesson . . . [The teachers] seemed to [own the] lesson. They became really clear about the maths. Initially they wanted to teach perimeter and area separately, but by doing the tasks, they came to the importance of teaching the concepts together to dispel their student’s misconceptions. (Journal, 22 May 2014)

It seemed that encouraging the teachers to try the tasks themselves before the modelling was helpful for illuminating possible existing misconceptions and providing an opportunity to develop their content knowledge. After the various solutions were shared in turn by individual teachers, they were asked how they thought their students would solve the task. Their predictions were shared, discussed, and ordered in terms of agreed levels of

sophistication, which appeared to support their attention to formative assessment for the task. One teacher asked what questions she could ask the students about the task, and we all discussed possibilities. Next, the teachers became more engaged in suggesting new ideas to refine the enabling prompt to meet the expected learning needs of the students. These processes seemed to prepare the teachers for the upcoming observation, by familiarising them with the mathematical content and what to look for in terms of student mathematical responses. It seemed they felt a sense of ownership of the lesson. Moreover, during the post-lesson discussion, the teachers made suggestions to build on their students' mathematical ideas. It seemed that their combined specialised content knowledge (Ball et al., 2008), along with my support, contributed to increased PCK of the group.

Following this described experience, I reframed my practice (Hamilton & Pinnegar, 1998) to build in enough time prior to subsequent modelled lessons to enable groups of teachers to unpack the proposed tasks together. These included encouraging teachers to try the intended tasks, anticipating student responses, categorising these in order of sophistication, collaboratively planning improvements such as modifying enabling prompts, and planning questions to ask the students. Drawing on the work of Smith and Stein (2011), a template was constructed that included a column for teacher questions for this purpose (see Appendix 20).

It seemed the process of getting the teachers to do the task themselves and talk about it before anticipating how their students would solve it (Smith & Stein, 2011) was effective in supporting them to construct mathematical knowledge for teaching (Ball et al., 2008). Importantly, this finding indicates that my providing a detailed lesson plan and explaining it to the teachers before the modelling was insufficient to guarantee the teachers' learning. This implies that, prior to the observation of a modelled lesson, it may be important for teachers to have explicit opportunities to solve the planned tasks, anticipate potential student

explanations, and plan questions to ask students so that they can learn to recognise, respond, and react successfully to the unanticipated responses of students during the lesson.

7.4.2 Learning to Refrain From Telling

Whilst I had a structured and detailed plan for the intervention, along with detailed processes described, I did not initially consider my own pedagogical approach as a teacher of the teachers. However, after reflecting on the audio recordings of the pre- and post-lesson discussions, I began to recognise that there were times when I had not made room for the teachers to think for themselves. Instead, I noticed I had often interrupted and advised the teachers without considering my responses. This is illustrated in my following journal entry:

I didn't wait long enough when they [the teachers] spoke. I needed to pause and listen and give wait time. I was jumping in far too often. It would have been more effective for me to wait until I was asked a question and then respond. (Journal, 29 May 2014)

For example, in the following excerpt from the Stage 1 post-modelled-lesson meeting at RPS, Rebecca made a comment about her students being distracted by recording their thinking in their maths workbooks. Instead of waiting to see if she had finished the conversation, I jumped in to advise:

I even thought sometimes writing in their books was a little bit distracting because they kind of weren't really thinking mentally as much then. (Rebecca, RPS)

I noticed that as well and the other thing is that it's not important for them to rule margins; it's important for them to get their thinking down on paper. We only get them to rule margins because that's the way we were taught, but we haven't thought why . . . it's not important. In maths, it's important for them to be thinking. (Me)

I think that my reaction in this example was largely unconscious and due to my genuine desire to bring the time-consuming practice of students ruling margins in maths workbooks to the teacher's attention. After retrospectively discussing my actions with a teacher from RPS much later, she reported that it had been worthwhile for me to draw her attention to the issue

of ruling margins because it was something that she always did and had never reflected on it.

Furthermore, Ruth stated,

Don't waffle around and try and get us to explore; it's just a time waster. You're the expert; just tell us what to do. We don't want to be spending twice the amount of time working it out for ourselves. (Ruth)

In Ruth's view, it was helpful that I did "tell". She saw me as an "expert" and seemed comfortable with being told what to do. I felt a sense of tension between acknowledging Ruth's position and suspecting the ineffectiveness of telling (Berry, 2007). Similarly, Berry (2007) described a tension experienced by teacher educators between balancing their desire to tell and providing opportunities for preservice teachers to think for themselves.

A further example of my telling is highlighted in the following email extracts. I was contacted by Sabrina at SPPS regarding her confusion about the meaning of "student struggle" and when to intervene. I responded with advice and attached a professional reading.

Thank you for your email. Your question is very important. First and foremost, the strategies for solving problems must come from the children. However, we have a pivotal role as teachers to develop and then build on the collective sense making of all students rather than to sanction strategies as being more efficient or demonstrate procedures ("preferred strategies") for solving tasks . . . I attach a very important professional reading for you and your colleagues. I think you will find it very helpful. (Email to Teacher, 12 May 2014)

After reflecting on my response, I wrote another email to the teacher a couple of days later and apologised for "preaching":

I have reflected on the email I sent you yesterday and think I may have come across as "preaching". If this is the case then I am very sorry. (Email to Teacher, 13 May 2014)

Interestingly, Sabrina replied to this latter email positively, by informing me that the article I had emailed her (Smith et al., 2009) was helpful in clarifying her thinking:

The reading was very helpful in that the structure of the lesson was broken down into five stages and it made sense. [From the reading, I learnt about] anticipating problems you might have and how you would address that. [For example] when you choose the students [to show their work], you are targeting those responses so that the [examples] they present to the class will help move them toward understanding. If the example that you're looking for is not there, you can create one. (Email from Sabrina, (SPPS) 13 May 2014)

Moreover, it seems the reading prompted Sabrina to notice me circulating and paying attention to the students' thinking and solution strategies as they worked on a task during the modelling in Stage 3: "Louise roves and has one-on-one conversations with the students. She asks them to explain their reasoning. [By doing that], she can see how the whole class is going and whether to move on or not" (Sabrina, SPPS). This seems to indicate that Sabrina reflected "in action" (Erickson, 2011) on the reading she was provided with, despite my telling email, which supported her to make sense of her observation. This suggests the reading was more useful than my advice! It corresponds with Loughran and Berry (2005) who argued, "We must model the use of engaging . . . teaching procedures . . . rather than 'deliver' information through the . . . transmissive approach" (p. 194). It also confirms that professional readings associated with the focus of the modelling are important tools to enable teachers to make sense of the new ideas (Clarke & Hollingsworth, 2002).

Once I recognised I needed to work with the teachers in the same way that I advocated for them to work with their students, I sought consciously to refrain from too much advising. I chose instead to ask more open-ended questions to prompt teachers to reflect on their practice. Evidence of this can be found in the following excerpt from my lesson reflection in October 2014 (see Appendix 21), which relates to a group of teachers being surprised by the success of a low-attaining student and my subsequent question to them related to streaming students.

Yesterday Sally [pseudonym] who according to her teacher was streamed into the lowest ability group was the only student to solve the *time difference* learning task. She used a number line. . . . ([The school has] changed to homogenous groups this term because of this maths project). Sally really surprised the teachers. [I asked the teachers], “What implications does that have for grouping [students]?” They replied, “Maybe we shouldn’t stream?” . . . I have realised asking invitational questions is the way to help teachers to reflect on their practice. It is a huge shift for me to become aware of this and move from telling. (Lesson Reflection, 13 October 2014)

In reflecting on my different experiences in responding to teachers with telling, I put effort into reframing my practice (Hamilton & Pinnegar, 1998) to become more deliberate in taking an exploratory stance with the teachers to aim to empower them to think for themselves. My “exploratory stance” continues to be a work in progress as I make conscious efforts not to advise but to use a pedagogical approach that includes inquiry, questioning, and probing (Loughran & Berry, 2005) to support teachers to construct their own knowledge. The next theme focuses on the challenge of exposing my teaching practice to critique.

7.5 Theme 3: Exposing My Practice Warts and All

Within this theme, there are two subthemes. These sub-themes are (1) learning to let go of the desire to teach a “perfect” lesson and (2) providing commentary whilst modelling. These subthemes are presented next.

7.5.1 Learning to Let Go of the Desire to Teach a Perfect Lesson

As an outsider, I struggled with a sense of exposing my perceived vulnerability as a teacher of the teachers’ students whilst maintaining credibility as a “teacher of the teachers”. This was partly because I did not have an established relationship with the teachers and the students and partly because of the teachers’ perceptions of me as an “expert”. Furthermore, some teachers had evidenced a negative evaluation of the modelling, as illustrated in Chapters 5 and 6. Consequently, I experienced personal pressure to teach perfect lessons so that I would gain credibility, and I noticed that I became defensive if a modelled lesson did not go

to plan. For example, at the start of the post-lesson discussion in Stage 1 at RPS, I expressed to the teachers,

I found myself having to modify the lesson on the fly because the children really struggled with the tasks . . . I decided that I would pose the enabling prompt to the whole class . . . I was hoping that they would notice that you can build to 10 . . . It was a really challenging lesson for me; it was hard to get around all the children and difficult not knowing them. If it was my own class it would've been easier because I would've known which children to go to straight away. Maybe I'm thinking that it's probably better to start with a question such as the enabling [prompt] and then scaffold it up to make it [challenging] . . . That's something I'm thinking about. (Audio Recording, 26 February 2014)

After examining this comment, I recognised I was self-defensive rather than viewing the modelling from the perspective of the teachers in terms of what they were learning from my modelling. Similarly, I felt pleased when the AP at SPPS commented favourably on my modelling in Stage 1 of this research (See Appendix 15):

I had my doubts about whether you could pull this [lesson] off . . . You are obviously an expert and I thought the language you would use would be way beyond what our teachers understand and of course you don't know these kids, but you did it, it was brilliant . . . and the kids . . . they just wanted to keep on working . . . Congratulations. (Sheila, SPPS)

I reflected that I was overly focused on either criticism or praise about my modelling. When my second lesson at SPPS did not go to plan, I reported to the teachers in the post-lesson discussion: "I found that personally it was a very difficult lesson" (Audio tape, 10 April, 2014). I think that the second lesson was difficult because I found it a challenge to modify the task in the moment in response to students' reactions to the learning task. My planned enabling prompts seemed insufficient to cater for the students' learning needs (see Appendix 22). However, I was unaware that the teachers seemed to actually find it beneficial to observe my attempts to respond to the unexpected and modify my teaching in action (see

Section 6.4.2) rather than simply watching a lesson where everything goes to plan. I was made cognisant of this when the AP reported,

The fact that [the lesson] wasn't as fluent as the previous lesson in terms of the students' responses was a blessing in disguise. It showed people things don't always go to plan, but if you stop . . . if you know the maths you are trying to teach and give the kids time to think, they can get there. (Sheila, SPPS)

Sheila's comment highlights for me the important finding from this study that the modelling drew the teachers' attention to instances where the teaching needed to be adjusted in response to the unanticipated (see Section 6.4.2). Her comment led me to recognise the power of modelling pedagogies in ways that focused the teachers' attention on the complexities of teaching (Loughran, 2006) regardless of the perceived success of the lesson. This experience made me mindful of the need to let go of my desire to teach a perfect lesson, accept the risk of being evaluated negatively by peers, and focus on reflecting aloud in the post-lesson discussion. This enabled the observing teachers to make sense of the experience from their and my perspectives.

To reduce the perceived threat to myself, I decided to lay out explicit norms and protocols for my emotional safety and the safety of the other teachers who volunteered to model. As Loughran (2006) argued, threat and risk to those who model practice can be reduced by explicitly articulating norms, protocols, and purposes so that all participants have clear expectations. I reframed my practice (Hamilton & Pinnegar, 1998) to include a disclaimer at the start that I explained to the teachers. In this, I stipulated the purpose of the modelling was for the teachers to grasp an understanding of how and why to use pedagogies (Loughran, 2006). I explained that the modelling was not intended to be perfect, nor could it be because I did not know the particularities of the students. I asked the teachers to suspend evaluative judgements and focus instead on teacher actions to facilitate student reasoning.

Evidence of this can be found in the following comment by a teacher at HPS in Stage 3 (see Appendix 11).

I liked the fact that you had a [disclaimer] about [our observation] of behaviour [in the lesson]. I was thinking it might be easy to be distracted by the students not on task. But the focus as observing teachers is for us to say what it is we're focusing on, disregarding [behaviour]. Our focus was on the reasoning. (Jackie, HPS)

I found that my introduced disclaimer as described did seem to reduce the teachers' expression of evaluative comments and allowed a greater discussion focus on my pedagogical reasoning (Loughran et al., 2016) and the underpinnings of the pedagogical approach. I think it also lessened my feelings of vulnerability and threat.

7.5.2 Providing Commentary Whilst Modelling

Several researchers have maintained that it is important to provide commentary whilst modelling—a type of “think aloud” for teachers—and make pedagogies explicit so that teachers can gain insight into the reasoning behind the teaching they perceive (Casey, 2011; Bronkhorst et al., 2011; Feiman-Nemser, 2012). However, I found it challenging to have a dual focus on the learning of both the students and the teachers during the lesson. For example, in the first post-lesson meeting at HPS, I reported to the teachers, “As the modelling teacher you forget that you've got observers”. This “forgetting” was especially apparent when I was confronted by unanticipated occurrences, such as a student misconception. This required me to listen to the student in a focused way and think on my feet to adjust the lesson.

Whilst I made initial attempts during lessons to provide commentary to the teachers about my actions in responding to student learning needs, I found that this seemed to detract from the flow of the lesson for me and the students, too. I found it much easier to explain my actions and respond to teacher questions about my actions afterwards, during the post-lesson discussions. For example, in the following excerpt, Jackie from HPS asked me a question about why I halted demonstrating a number sentence during the summary phase of a co-

taught lesson, which was focused on the concept of comparison in a composite Grade Prep/1 class.

I found it really interesting when you [started to] put up that [number sentence] and then you stopped and I could see you thinking but I was just wondering if you could explain? (Jackie, HPS)

I thought to myself, this isn't going to work . . . because the way the number sentence is recorded needs to match the semantics of the problem. So, $8 - 5 = 3$ wasn't correct . . . [I was trying to represent that Xavier saw eight zucchinis and Toula saw three; Xavier saw 5 more]. (Me)

You actually needed the answer to be 5. (Henrietta, HPS)

The difference is 5. (Holly, HPS)

Older children use subtraction to record difference . . . We must be careful how we use the language. I was conscious of that and that half the kids are preps; it's early in the year . . . I didn't want to confuse them. You can use terminology like difference but recording that [as a number sentence] is difficult. (Me)

In this situation described, I had stopped myself from making an error of judgement, and Jackie had noticed it. I do not know if the other teachers had also observed my change in direction, but they did have the opportunity to reflect on it and comment when it was raised afterwards. This implies the importance of making my hidden decisions clear to the observing teachers. Higgins and Parsons (2009) claimed that by providing commentary on the pedagogies they modelled, teacher attention is drawn to the underlying mathematical principles of their actions. I found that commentary was best situated in post-lesson discussions to promote lesson flow for the students' and my sake and to enable my teaching decisions to be deconstructed in detail with teachers and therefore more clearly understood. Connected with this finding is the importance of being aware of pedagogies that teachers may misinterpret during a modelled lesson. This is discussed next.

7.6 Theme 4: Mismatches Between Interpretations of Pedagogies

During post-lesson discussions with the teachers, I noticed instances when my chosen pedagogical approach seemed to be misinterpreted after watching me and then misapplied by the teachers in their own classes. For example, in Stage 4 at HPS, I observed the class teacher, Holly, asking her students to move to the extending prompt before they had even solved the main task. The lesson was modelled by the AP, Hannah, and Holly was assisting her. Holly's action surprised me and led me to question whether I had been clear about the purpose of the extending prompt in a lesson. It seemed Holly assumed the extending prompt was for higher attaining students as a matter of course and regardless of the appropriateness of the main task for them. I conclude this was because the teachers and I had not previously discussed the use of extending prompts during the post-lesson briefings at HPS because they had so far been infrequently needed during the lessons. This finding also illustrates that enactment is "much more than observation and imitation of expert performance" (Lefstein & Snell, 2014, p. 6). The conversations to unpack the pedagogies seem critical for helping teachers apply them in appropriate and effective ways.

In the same modelled lesson described, I had noticed several students struggling to solve the main task, but others were using efficient strategies. Hannah was working with a group of students on the floor and did not notice this. I thought it would have been great if Hannah had noticed and called the students with efficient strategies to explain their thinking to the rest of the class. This may have enabled those who were having difficulty to learn from those students. I later questioned myself whether I had been clear enough about the role of the teacher in roving and monitoring student responses to identify those that could present their solutions (Smith & Stein, 2011). I noted in my reflection, "I realised we cannot assume teachers will understand . . . unless it is made clear to them" (lesson reflection 17 April 2014).

I realised that I could not assume the teachers would receive messages about pedagogical approaches in the way I intended. This finding is consistent with Berry (2009) and adds weight to the argument that pedagogies may need to be deconstructed into fine particles (Boerst et al., 2011; Grossman et al., 2009) both before and after the modelling to enable them to be understood. It also suggests that it might be difficult to become aware of this issue unless an opportunity is provided to see teachers in action implementing particular pedagogies. Fundamentally, it appears the teachers' capacity to understand the underpinnings of these pedagogies is critical to helping them develop their own practice with them.

The final theme presented in this chapter illustrates how I sought to build trusting relationships with the teachers over a short period of time.

7.7 Theme 5: Building Trusting Relationships With the Teachers

Over the course of the intervention, the nature of professional conversations during the pre- and post-lesson meetings shifted from expert centred to teacher centred, evidenced by more frequent and active participation by teachers in discussion (as illustrated in section 5.5.3) to the teachers positioning themselves as confident co-planners by making concrete suggestions for improvements to teaching in Stage 4 (Hodgson, 2016). For example, after reflecting on the audio recording of the Stage 1 post-lesson meeting at HPS, I noted in my electronic journal that “the vibe was uncomfortable and awkward” (Audio tape, 4 March 2014). This was consistent with findings in Section 6.7.1.3 where the teachers at HPS reported that they felt untrusting at the start. However, as the intervention progressed, the teachers became more confident to discuss practice, and likewise, I became more trusting of them.

In listening to and reflecting on the audio recordings, I found that the conversations among teachers seemed to become deeper and mathematically richer, leading to more active participation and more sense making. For example, the following extract is from my

reflection on Stage 3 at HPS (see Appendix 25) “I noticed by the Stage 3 discussion it was more of a discussion between teachers . . . the conversation seemed to flow freely between all participants” (lesson reflection, 4 April, 2014). Similarly, the following dialogue after the enactment of the Stage 3 co-taught lesson at MPS illustrates this.

I’m wondering whether meaning was lost in the enabling prompt? I don’t know if it was the wording of it? Just wondering if you think it may have been more effective if you used the same context as the main problem? (Megan, MPS)

Yeah, do you think I needed to have it exactly worded the same?” (Marita, MPS)

Similarly, in another conversation, Marita stated,

I don’t think I would put the lower group in that “zone of confusion”. I am still inclined to rein it back a little. (Marita, MPS)

Putting low attainers in with those high fliers would just give them the experience. They might see what the high fliers are doing. They might not understand it but they will have the opportunity. (Maude, MPS)

These examples highlight what seemed to me to be the teachers’ increased empowerment and sense of agency in asking such questions and proposing improvements. Importantly, the progression of change in participation is an indicator of the teachers learning from the community (Lave & Wenger, 1991). Similarly, the teachers’ level of trust and confidence in me seemed to grow over a short space of time. Perhaps this relates at least in part to the dual stance I took of being both willing to be vulnerable as the modeller and at the same time comfortable in my role as a teacher leader.

I think that my vulnerability to the teachers initially felt risky to me as I struggled to model pedagogies in authentic and complex situations with students I had not taught before. Yet the teachers saw me persist with adjusting and modifying my teaching on the fly, as explained in Section 6.4.2. I surmise that this actually provided a powerful means of facilitating conversations about the complexities of enacting different pedagogies in practice,

as illustrated in Chapter 6, because of increasing the teachers' sense of solidarity with me as a fellow teacher. By placing myself in the (risky) position as a teacher of the teachers' students, I experienced a sense of empathy and understanding from the teachers. This finding resonates with Berry's (2009) study of preservice teachers. She argued that modelling "offers potential for real change as student teachers see their teacher educators struggle with authentic, complex situations" (p. 161). I found this also applied to modelling with an external "expert" and in-service teachers in a school professional learning context.

Conversely, in my role as a leader, I also sought to engender trust through my efforts to support the teachers by prompting them to reflect on their teaching and challenge their ideas. The teachers themselves also highlighted this aspect of our relationship building (see Section 5.5.3). For example, during the co-planning meeting for the co-taught lesson in Stage 3 at HPS, the class teacher raised the idea of a lesson focused on the language of comparison for her Grade Prep/1 class. The teachers decided that the task would be contextually based (the students had a school garden), and they planned the following task (see Appendix 24):

When Xavier was in the garden, he saw 8 zucchinis. But Toula saw 3. How many more did Xavier see?

Prior to the lesson, one of the teachers suggested changing the planned task to make it easier for the students. She suggested changing the task to the following:

When Xavier was in the garden, he saw 8 zucchinis. But Toula saw 3 more. How many did they see altogether?

I prompted the teachers to consider how the structure of the problem would change if they decided to use the latter task. However, the teachers seemed to have difficulty recognising this, so I explained the difference between join problems and comparison problems (see Appendix 24):

The teachers appeared somewhat confused by the maths in the task. I noted the importance of being there to assist them to see the maths. (co-planning reflection 1 April 2014)

In the post-lesson meeting, one of the teachers reported,

It feels a bit like the rug has been pulled out from under my feet . . . I had a way of working and a way of choosing tasks, but now it's turned my head around and I'm really thinking about why it is I am choosing [tasks]. For example, I might have had some ideas of doing numbers to 10 [with Prep/1], but now I'm looking at the proficiencies and [asking myself] what is the foundation that I'm building the maths on? (Heather, HPS)

Heather's comment suggests she was challenged by the experience of being prompted to examine the planned task with her colleagues; however, it seemed to be a valuable catalyst for her to reflect on her practice.

A further example of the teachers' developing trust in me was in expressing their own vulnerability as lacking in confidence to teach mathematics. For example:

I've lacked a little bit of confidence. I don't like mathematics. It's something that I feel like I must overly think about and plan for. You have given me some more ideas about where I can go . . . not just for mathematics but for all learning areas . . . The way you introduce things, reflect on things and get the children involved. (Megan, MPS)

I know that I'm not overly fantastic at maths and I need to be much better, so even though I didn't want to do that [enactment] I guess it probably stretched me in a lot of ways. (Marita, MPS)

In reflecting on my relationships with the teachers, I think that by demonstrating both my vulnerability and confidence as a leader, the teachers gained enough trust in a short period of time to be able to take risks themselves and try different ways of working in their own practice. This finding resonates with Berry (2007), who articulated the tension experienced by teacher educators between exposing one's uncertainty and maintaining preservice teachers' confidence in the teacher educator as a knowledgeable leader.

7.8 Chapter Summary and Conclusion

In this chapter, I articulate five themes that emerged from the data analysis of my inquiry into my own learning and practice as a teacher of teachers. Elements of self-study methodology supported me to illuminate and revise my assumptions and helped me to develop personally meaningful understandings of my own evolving practice.

7.8.1 Linking Findings to the Research Question

The experience of focusing the inquiry on myself enhanced my learning to be a teacher of the teachers in a number of ways. For example, by developing a clear focus for the modelling, I increased my confidence to articulate the thoughts and actions that underpinned the pedagogical approach to the teachers in the study. Importantly, I learned that explaining a detailed lesson plan was insufficient to guarantee the teachers' learning. It seems it may be important for teachers to have explicit opportunities to solve challenging tasks for themselves, anticipate potential student explanations, and plan questions to ask students. In this way, it seems they can learn to recognise, respond, and react successfully to the unanticipated and build their knowledge of content and students (Ball et al., 2008).

By reflecting on my practice, I became conscious of the need to work with the teachers in the same way that I advocated for them to work with their students; that is, in exploratory, engaging, and innovative ways. I learnt to recognise the power of modelling pedagogies in ways that focused the teachers' attention on the complexities of teaching, regardless of the perceived success of the lesson. I also became aware of the need to let go of my desire to always teach a perfect lesson and accept the risk of modelling my teaching to others. To reduce the threat to myself, I reframed my practice to include norms and protocols for my safety and the safety of the other teachers who volunteered to model.

I found that articulating the thoughts and actions that underpinned my teaching moves in the moment were best situated in post-lesson discussions to enable pedagogies to be

deconstructed and therefore more clearly understood. Moreover, I recognised that I could not assume the teachers would receive messages about pedagogical approaches in the way I intended; consequently, I realised there was a need to deconstruct pedagogies into fine particles both before and after the modelling to enable them to be understood.

Importantly, it seems that by demonstrating both my vulnerability as a fellow teacher and confidence as a leader, I facilitated the teachers in developing a sense of solidarity with me and trust, leading to their increased willingness to take risks in their practice and try different ways of working.

Chapter 8 considers the overall findings and implications of this research project. Key insights outside the parameters of the research aims and questions are identified and discussed. Limitations of the research are also discussed, as are recommendations to improve mathematics teaching practice through modelling pedagogies.

CHAPTER 8: IMPLICATIONS AND CONCLUSIONS

In this study, I sought to explore the phenomenon of teacher learning through the observation of classroom modelling and the subsequent deconstruction of pedagogies and enactments throughout structured interventions conducted in four school settings. In this regard, the inquiry focused on the experiences of the participating teachers as they took part in the intervention. As part of this experience, I also directed the inquiry to myself in learning to be a teacher of the teachers. This study employed multiple case study methodology in the conduct of this research. The experiences of the teachers were analysed through a phenomenological lens. In this chapter, I draw together the overall findings and suggest some implications. Next, I present the limitations of the research project and make suggestions for further research.

8.1 Overall Findings and Implications for Teacher Professional Learning

This section provides an overview of the overall findings and implications for teacher learning. The overall findings and implications are presented and organised in relation to the five research questions.

8.1.1 Challenging Teacher Assumptions

Recent research has highlighted that seeing practice modelled prompts teachers to consider new approaches to teaching mathematics in their classrooms (Clarke et al., 2013; Higgins & Parsons, 2009; Yook-Kin Loong et al., 2017). This present study confirms these findings. However, an emergent theme in this present study that surprised me was the sense of discomfort many of the teachers experienced when they observed the pedagogical approach that encouraged students to think for themselves to solve problems before instruction from the teacher. Moreover, the observation of students struggling to explain their reasoning prompted some teachers to recognise that their current orientation to teaching may have hampered their students' capacity to explain their thinking. It seems that this may have

been because the teachers were asked to comment on *teacher actions that facilitated student reasoning*. This finding implies that in a modelled lesson, encouraging an observation focus on teacher actions to facilitate student reasoning may be successful in prompting teachers, similar to those in this study, to reflect on their own orientations to teaching.

Another important finding that emerged from the observations related to the teachers' apparent apprehension if they allowed their students to come up with their own strategies for solving problems. From my perspective, it seemed this discomfort stemmed from the teachers' perceived lack of confidence in knowing what to do or say if students came up with a solution to a problem during enactment of an observed lesson that they had not anticipated beforehand. The knowledge required to respond to students' thinking in the moment is known as KCS (Hill et al., 2008) and is difficult to acquire because it requires knowledge of content that intersects with a student's developmental level of learning. This implies that opportunities to anticipate student responses before a modelled lesson seem critical for the enactment of teaching practice that fosters student reasoning and problem-solving.

Whilst the teachers in all four schools requested the modelling to address differentiation, many appeared to be confronted by the practice of posing a problem to the whole class with adjustments by way of enabling and extending prompts. They reported their usual practice had been to group students by ability and only give those children they perceived to be "ready" the opportunity to work on tasks that were difficult to solve. However, many teachers appeared to be surprised when they observed their perceived low-attaining students persist with and respond successfully to the challenging tasks.

Similarly, several teachers had communicated discomfort at their perceived high expectations I placed on their students during modelled lessons. However, they came to express surprise at seeing their students responding successfully to these expectations. Their observations seemed to conflict with their existing teaching approaches and prompted them to

reflect on both their expectations of student achievement and their teaching practice. It seems that in the process of resolving their discomfort regarding the observed pedagogical approach that was very different to their traditional forms of teaching practice, the teachers developed pedagogical reasoning as described by Loughran et al. (2016). This implies that modelling pedagogies associated with developing students' problem-solving and reasoning to teachers similar to those in this study may support teachers to reflect on their teaching practice and create new visions of teaching for themselves.

Many of the teachers had initially evaluated the lessons negatively by perceiving students to be "off task". However, as the intervention progressed, they began to consider alternative explanations to the students' behaviour. Further to this, in subsequent iterations of the intervention, the teachers had been discouraged from evaluating the lessons, so this was no longer an issue. Nevertheless, this finding necessitates a disclaimer being made prior to modelling pedagogies, indicating that the lesson is not intended to be "perfect", but rather an approach that supports teachers to understand the how and why of pedagogy. This might discourage the teachers from evaluating the modelling, lessen threat and risk to the modeller and focus on teacher actions to facilitate student reasoning and problem solving.

8.1.2 Teacher Learning From Observing Modelling of Particular Pedagogies

The analysis of the observations of the 18 teachers in this present study built on the findings of a preliminary study in 2012 with teachers from Catholic Education Tasmania, as described in Chapter 3 (Hodgson, 2013). In the earlier study, the observation proforma had contained a prompt asking the teachers to document observed teacher actions they would try to implement in their classrooms that were different to their usual practice. The findings showed that the teachers had noted pedagogies associated with KCT and KCS (Ball et al., 2008). The observed pedagogies included thinking time, opportunities for students to explain and justify their mathematical reasoning, allowing students to solve problems by thinking for

themselves, probing questions to scaffold thinking, and mathematical language. These observed pedagogies related to practices the teachers wished to implement in their classrooms.

Following the 2012 preliminary findings, the observation prompt was refined. Subsequently, in this present study the teachers were asked to write down teacher actions that facilitated student reasoning and problem-solving. Many teachers did indeed notice detailed pedagogies associated with reasoning and problem-solving. The findings from these two studies imply that the wording of the observation prompt was important for the outcome.

All 18 teachers expressed their attention to teacher actions during the modelled lessons associated with pressing for student explanations (Sleep, 2012), including providing thinking time, scaffolding students to reason, interjecting (Askew, 2012), revoicing, asking students to repeat or add to what had been said, prompting students for further participation (Smith & Stein, 2011), and modelling mathematical language. Importantly, the teachers noticed that the students began to use the mathematical language that I had modelled in their explanations to other students. This implies that the focus for the observation on teacher actions to facilitate student reasoning can be successful in narrowing teachers' attention.

Over three quarters of the teachers noticed detailed features of the lesson that supported students to develop mathematical proficiency (Sleep, 2012), including explaining definitions of new mathematical terminology, clarifying the task, circulating and paying attention to students' thinking and their strategies as they worked on the task (Smith & Stein, 2011), and using students' explanations to exemplify the mathematical ideas (Smith & Stein, 2011). Further to this, more than half the teachers across the schools noticed that the document camera was a powerful tool for facilitating sense making and effective classroom discussions about the mathematics. Notably, Sleep (2012) maintained it is important for all class members to see student examples chosen to bring the mathematics into the open for examination;

otherwise, meaning can be lost. This implies document cameras are a valuable and important tool for making students' thinking easily visible and fostering whole class discussions in a classroom context.

Two thirds of the teachers noticed the power of the enabling and extending prompts in catering for the diverse range of learners. This led to reported changes in teacher practice, including implementing challenging tasks with enabling and extending prompts and allowing students to solve problems by thinking for themselves. This implies the use of enabling and extending prompts have the potential to facilitate immediate change in classroom practice as long as the teachers have a full understanding of how to use them.

Almost all the teachers observed my positive stance towards the students, encouraging them and promoting persistence. Many noticed students did persist with the prompt "keep thinking". This implies modelling teacher actions to support students' productive dispositions for mathematics learning may assist teachers to notice simple ways of encouraging their students.

Many of the teachers reported they valued the detailed modelled lesson plans as a starting point for enactment. Furthermore, the teachers conveyed they wanted to replicate the modelled lessons so that they could get a feel for the pedagogical approach first before tweaking it with their own nuances when applying to other lessons. This implies teachers' lesson plan exemplars may support teachers to approximate the pedagogical approach without the added pressure of coming up with their own ideas.

8.1.3 Deconstructing Pedagogies

8.1.3.1 Anticipating student responses to tasks and adjusting prompts

During planning for enactment, the teachers seemed to find it difficult to come up with their own enabling and extending prompts for a task. Accordingly, it appears it may be important for teachers to learn to plan a range of enabling prompts as part of a professional learning

strategy involving modelling, deconstruction, and enactment of pedagogies. It seems that by having a greater focus on learning to understand the mathematical demands of the modelled task in the prebrief, teachers may develop a greater awareness of potential adjustments to the enabling prompts and reduce their concerns about supporting all learners. This may support enactment of the pedagogical approach.

I attempted to provide an opportunity for the teachers to understand potential student responses to the tasks I modelled by explaining the goals of the lesson and my anticipated student responses and prompts in the prebriefs. However, I soon realised that my explanations did not lead to the teachers gaining sufficient knowledge to predict potential misconceptions and student responses to the tasks in the ways that I had hoped. Consequently, in a subsequent iteration of the modelling, I changed my approach to include getting the teachers to do the task themselves and talk about their responses with colleagues. This supported the teachers to predict their student responses and seemed to prepare them to notice and understand my reactions and responses to the students during the modelling. It seemed to give them a sense of “knowing” what might happen next in the lesson and a sense of confidence when their students responded in ways they had predicted (see Section 7.4.1). Previous research has contributed knowledge about helpful aspects of pre-lesson briefings (e.g., Bruce et al., 2009; Clarke, 2015; Wilkie, 2017). For example, Bruce et al. (2009) noted components of a pre-lesson briefing included a discussion of learning goals, anything pertinent to planning and assessment, and anticipated student responses. However, they did not describe a process where the observing teachers could learn to anticipate different student responses beforehand. The finding of this present study builds on previous research and implies the importance of teachers having explicit and structured opportunities to solve the learning tasks themselves, discuss their responses, and additionally anticipate a range of student responses and propose adjustments to planned enabling and extending prompts before the observation. It seems this

process can increase the potential for the observing teachers to understand the teaching actions and build their KCS. In turn, this might support the teachers to feel confident to teach in more student-centred ways and sustain changes to their teaching practice. Hill et al. (2008) called for classroom-based research to contribute measures of good teaching practice to assist them to further conceptualise KCS. To support this, the following suggestion may enhance teachers' capacity to construct their knowledge of KCS:

The development of a clearly delineated and structured pre-lesson meeting protocol to support teachers to develop ways of anticipating, inquiring, hearing, and noticing

To achieve this, I propose that the pre-modelled-lesson briefing consist of the protocol as shown in Figure 8.1. The protocol includes an introduction where norms are set and the protocol outlined. Next, it includes a process for teachers to complete the learning task, discuss their responses, and anticipate how their students will solve the problem and plan questions. Following this, opportunities to adjust the enabling and extending prompts are provided. Next, the teachers are explicitly requested not to evaluate the lesson as such but rather focus on teacher actions for encouraging a particular student response (in this study, student reasoning through problem-solving). A disclaimer is made up front that the lesson is intended to be "authentic" rather than "perfect". The purpose of the observation is made explicit. An observation protocol and the lesson plan are handed to the teachers, and following this, the teachers observe the lesson. This process may support teachers to notice and understand the observed teacher's reactions and responses to the unanticipated responses of students during the modelling and in doing so develop their KCS and their pedagogical reasoning.

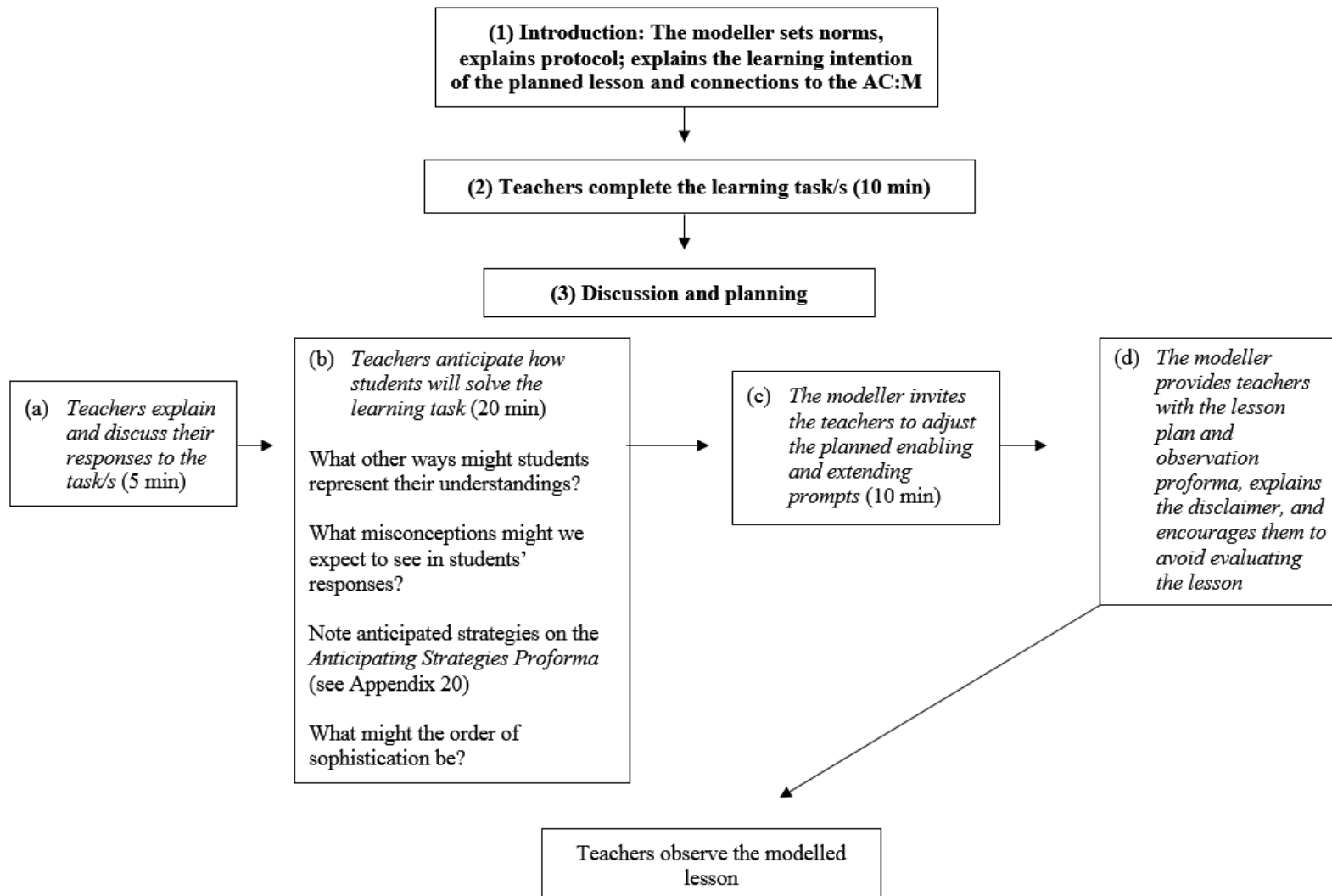


Figure 8.1. Pre-modelled-lesson briefing protocol.

8.1.3.2 Modelling messy practice

I initially assumed that modelling an exemplary lesson was essential for teachers to notice important pedagogies. Surprisingly, when my modelling did not go according to (my) plan, several teachers reported they valued seeing my challenges, dilemmas, and adjustments to support student learning. They conveyed that this made the modelling seem “real” to them. Furthermore, at least half the school leaders reported that during my modelling, the teachers’ attention had been drawn to instances where my teaching was adjusted in reaction to unanticipated student responses. On these occasions, some of the teachers queried my teaching decisions. Their noticing and questioning prompted me to discuss adjustments to my teaching, which seemed to support them to more clearly understand the thoughts and actions underpinning the pedagogical approach. This finding implies it is critical for the modeller to reflect aloud and articulate her pedagogical reasoning behind any spontaneous and unplanned teaching decisions made in response to students’ reactions. Consistent with findings from international studies, it confirms that those who model need to make the tacit overt (Bronkhorst et al., 2011; Casey, 2011; Feiman-Nemser, 2012). This would then provide the observing teachers with access to these unseen elements of practice and may generate possibilities for them to understand and interpret the underpinnings of the pedagogical approach in a similar way to the approach described by Loughran et al. (2016). The goal is to assist teachers to recognise, react, and respond to teachable moments and in doing so build their KCS (Hill et al., 2008).

Several researchers have claimed it is important to think aloud whilst modelling, so that the modeller’s invisible decision-making becomes visible to observing teachers (e.g., Bronkhorst et al., 2011; Casey, 2011; Feiman-Nemser, 2012;). However, I found that such commentary was best situated in post-lesson discussions to promote lesson flow for the

students' and my sake and to enable my pedagogical decisions during the modelling to be deconstructed in detail with teachers and therefore more clearly understood.

To support this important implication, the following suggestion may support those who model to have a clearly defined opportunity in the post-lesson meeting to articulate their thoughts and actions underpinning their teaching moves:

The development of a clearly described and structured post-lesson meeting protocol to enable modellers to articulate their pedagogical reasoning and support teachers to develop understanding of the underpinnings of the pedagogical approach

I propose that the post-modelled-lesson briefing consist of the protocol as shown in Figure 8.1. Such a process could be led by a grade team leader in a school to enable the modeller time to reflect, listen to others, and compose her/his thoughts before discussing her/his tacit teaching moves. In designing my initial debrief, I posed a question to the teachers regarding their observations, such as “What did you notice in that lesson”? The teachers were each given an opportunity to share their experiences and perceptions of the lesson, and particularly the modelled teacher actions they observed that focused on student reasoning. The open nature of my opening question seemed to generate some discussion about student responses to the learning experiences, particularly as the intervention in each school progressed. However, I did not ask deeper questions as to possible reasons for my actions or the student responses, which I think in hindsight would have made the discussion richer.

I used my findings to adapt the dialogic debrief conversation flow chart (Lefstein, Trachtenberg-Maslaton, & Pollak, 2017) by including a specific opportunity for the modeller to explain her pedagogical reasoning regarding any unplanned teaching moves she made during the lesson. For example, Lefstein et al. (2017) suggested a sequence of questions: “What happened?, Why did it happen?, What were the alternatives?, Insights?” I suggest “What happened?, What might have prompted the teacher and students?”, “The observed teacher makes explicit any spontaneous and unplanned teaching moves; What might the

observed teacher have done differently?”, “Implications?” My second question, “What might have prompted. . .?”, may seem less threatening than Lefstein et al.’s (2017) question, “why did it happen?” The protocol includes the following elements:

1. The team leader invites the teachers to share their thoughts regarding their observations of “happenings” in the lesson. Teachers are invited to share in a round-robin fashion. This is to illuminate their observations, which might be different for each teacher.
2. The team leader probes the teachers by asking, “What prompted the teacher and students?” This is to probe the teachers to reflect on the reasons for the happenings.
3. The modeller is given the opportunity to discuss her reflections of the lesson and make her spontaneous teaching decisions explicit to the observing teachers. The intent is to make the unplanned moves overt and support the teachers to understand the modeller’s pedagogical reasoning.
4. The team leader invites the observing teachers to suggest alternative teaching moves to critically reflect on their understandings of the pedagogical approach. The advantages and disadvantages of suggested teaching moves are discussed with the modeller and the observing teachers, for example, “What might have been the impact of . . .?”
5. The team leader invites the teachers to discuss implications for their practice. The teachers set a goal for their teaching, which is revisited before the start of the next cycle.

The implication for the teachers is having the opportunity to illuminate the modeller’s pedagogical reasoning, and critically reflecting collectively on the observation may support them to develop KCS. Of course, the real test is that teachers transfer this new knowledge to their own classroom practice through enactment and sustain it over time. Stakeholder reports in this study indicate that the teachers did actually continue the pedagogical approach that I modelled after the intervention ceased. For example, the four principals maintained that

although some teachers were initially reluctant, they made shifts in their teaching practice, including having higher expectations of their students, posing challenging tasks, and changing the way they differentiate practice.

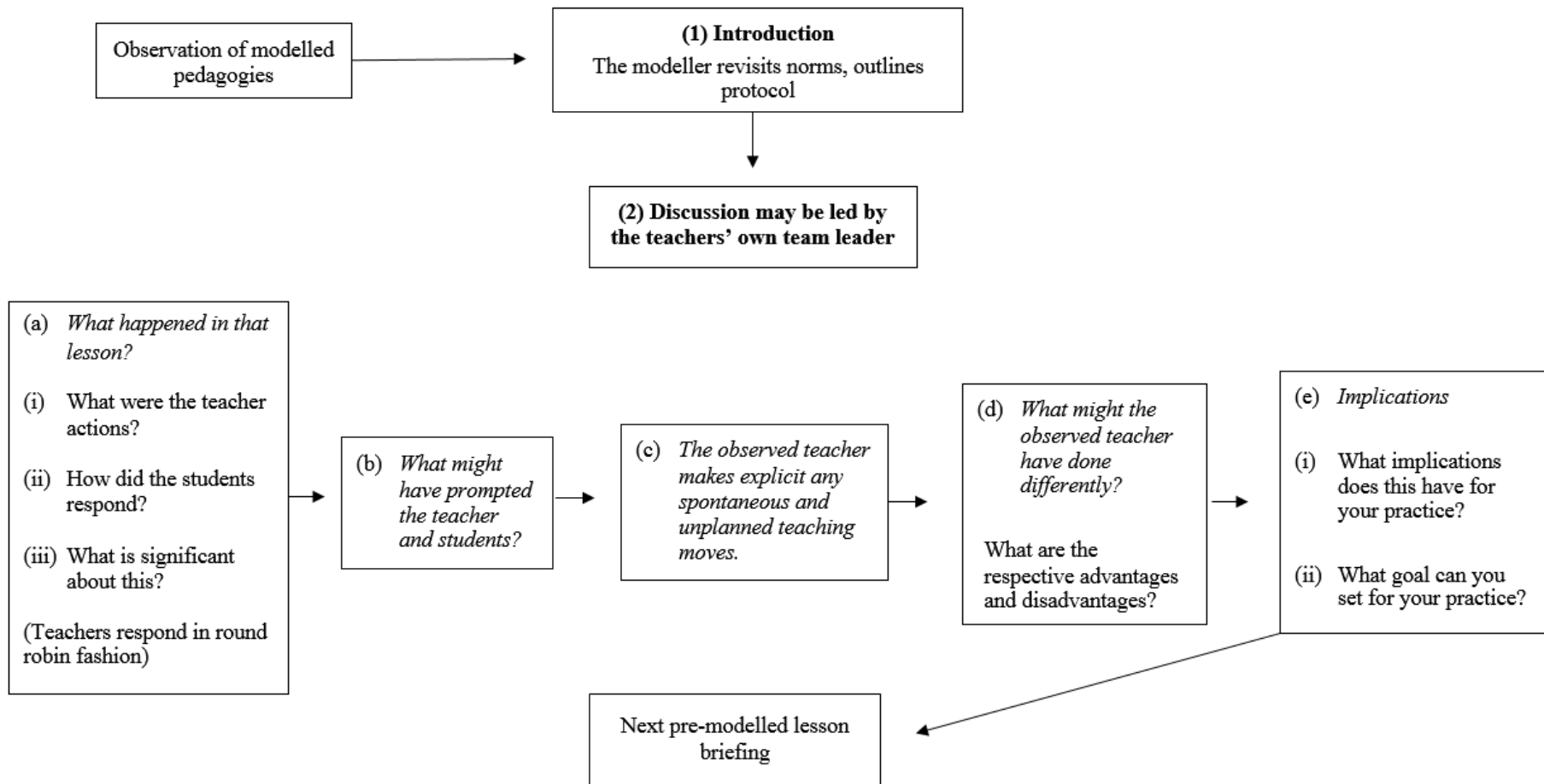


Figure 8.2. Post-modelled-lesson briefing protocol.

8.1.3.3 Collaborative Conversations

Collegial opportunities to critically reflect on and discuss the impact of observed pedagogies on their students' learning took place in pre- and post-lesson meetings and informally between stages of the intervention. Consistent with Lefstein and Snell (2014), some of the most important learning happened naturally and informally because of the collaboration around the modelling. This implies collaboration and spaced learning opportunities between stages of the modelling are helpful for teachers to engage with colleagues in reflecting on their learning.

The post-lesson discussions seemed to enable the teachers to engage in critical reflection about the observed pedagogies and in doing so develop their reflective awareness (Chapman, 2012) and pedagogical reasoning (Loughran et al., 2016). For example, by Stage 4, many teachers appeared to have shifted from being passive to engaged and confident participants in the discussions. Some had suggested adjustments to colleague teachers to cater for children having trouble. This is an important component of KCS (Ball et al., 2008). It highlights that teacher learning took place through social interaction (Borko et al., 2008; Lave & Wenger, 1991; Putman & Borko, 2000). It seems the conversations about the complexities of the modelling, the pedagogies, and student responses to the tasks gave the teachers confidence to participate in knowledge-building discussions (Lave & Wenger, 1991). This implies discussions about modelling can support teachers to build their MKT (Ball et al., 2008) and pedagogical reasoning (Loughran et al., 2016).

These processes described fostered the development of professional learning teams in schools where the teachers had usually worked in isolation. It seems the collaboration brought the teachers together and supported them to de-privatise their practice. In agreement with Bruce et al. (2009), the modelling appeared to provide an entry point into the process of opening classrooms to one another for collegial professional learning. This implies an

intervention such as this may be an important factor in bringing together teachers in a similar work environment to those in this study for a common purpose.

8.1.4 Enacting Pedagogies

One teacher from each school enacted pedagogies in Stage 4 of the intervention. They were observed by the other members of the team and me. Findings of the study showed that planning for enactment prompted the four enactors to reflect on their practice and become more purposeful and intentional with their teaching. I found that this preparation was most effective when it was planned collaboratively so that each teacher could contribute to advance the MKT (Ball et al., 2008) of the group. Moreover, I found that this collaborative planning had accountability measures built in, with each teacher implicitly expected to participate. This view was supported by the four principals. It seemed to maximise the potential for each teacher in the group to develop reflective practice and intentional teaching. In agreement with Resnick et al. (2010), it seemed that the structures and implied accountability motivated teacher participation and learning. This implies that structured and collaborative processes in planning for enactment of new pedagogies may support teacher enactment.

Not surprisingly, pedagogies that were easily seen were enacted by the four enactors. These included the provision of challenging tasks, extended thinking time, and enabling and extending prompts. These findings were consistent with the findings from the post-intervention exit surveys at three months, in which all the teachers were asked to indicate any changes they had made to their teaching practice. The fact that three quarters of the teachers reported they had implemented challenging tasks and two thirds reported they allowed the students to think for themselves is indicative of a considerable change in practice. This change is noteworthy considering that before the commencement of this intervention at least half the teachers reported their orientation towards teaching was demonstration and practice.

This indicates that the modelling was successful in prompting the majority of teachers to trial new approaches to teaching.

Interestingly, half the teachers claimed they had gone on to implement enabling and extending prompts to cater for the diverse range of learners in their classes. This finding differs from findings in Clarke et al. (2012), who maintained approximately one eighth of participants in their study reported their intent to change aspects of differentiation after observing modelled lessons. The difference in findings may have been because the proposed prompts for each task had been reflected upon and discussed in post-lesson briefings. This implies attention to issues of differentiation is valuable in both pre- and post-lesson meetings.

The four enactors reported they had difficulty probing student thinking, choosing specific student ideas to be shared to advance the mathematical thinking of the class, and summarising learning. This is also consistent with the exit survey findings where only one third of the teachers reported implementation of students' communication, and one teacher reported on improved questioning to probe student thinking. These are complex pedagogies and reflect the difficulty of connecting KCS with KCT (Ball et al., 2008). This implies that having teachers doing the tasks themselves, anticipating student responses, and planning their questions seems critical for the successful enactment of complex pedagogies. It implies that the post-lesson conversations to uncover the complexities of these pedagogies are fundamental to the success of the modelling.

Notably, one teacher reported that rather than "imitating" me, she had to be herself and make decisions about which observed teacher actions she found important and how she might incorporate those into her teaching practice with her own nuances. This finding resonates with Loughran (2006), who argued the purpose of modelling in preservice teacher education is that observers understand the underlying principles of the teaching approach rather than a call to imitate the modeller. It implies the importance of supporting teachers to understand the

pedagogical reasoning behind the teaching actions of the modeller so that they can make informed decisions about pedagogies they choose to enact and transfer those pedagogies to other lessons.

8.1.5 Stakeholder Reactions to the Intervention

In terms of the structure of the intervention, the four principals reported that it was effective professional learning because it involved seeing, deconstructing, and enacting practice in the teachers' own contexts. They compared the intervention favourably to other forms of professional learning that simply involve talking about teaching but may not be situated in classroom practice and therefore may never be enacted. The principals suggested the intervention could be improved by being flexible enough to recognise and cater for individual teacher learning needs. This agrees with Clarke and Hollingsworth (2002) and Resnick et al. (2010), who argued professional learning must offer participants every opportunity to learn in ways that each participant finds helpful. One approach might be to anticipate the possibility of multiple cycles of modelling, deconstructing, and enacting pedagogies at the start that do not inhibit teacher learning by being too prescriptive (Clarke & Hollingsworth, 2002). This would make possible an approach that would allow different pathways to be taken, depending on individual teachers' learning needs.

Importantly, the four principals and four enactors reported their perceptions that the students had improved their dispositions for learning mathematics. They stated that the students were now willing to have a go at challenging tasks and were more engaged in learning. *Productive disposition* is one of the five strands of mathematical proficiency (Kilpatrick et al., 2001) and is a major contributing factor to successful student learning (Kilpatrick et al., 2001; Sleep, 2012). This implies the intervention can support students to improve their mindsets and dispositions for mathematics learning.

In terms of cost, the principals conveyed that releasing their teachers from class to participate collaboratively in the intervention was money well spent in terms of paying emergency teachers. Essentially, the four principals suggested that given the success of the intervention, it could be expanded by giving all members of the learning teams at each grade level a chance to enact pedagogies. One principal suggested adopting the structure across learning areas. Importantly, these findings seem to imply that processes involving seeing, deconstructing, and enacting pedagogies were effective as a professional learning model for the teachers to shift their views of teaching and improve student dispositions for learning mathematics. In agreement with Grossman and her colleagues (2009), these processes when implemented together are key to the teachers' enactment of new pedagogical approaches to teaching. This does seem to suggest that the intervention could be adopted in other learning areas, particularly for teachers entrenched in traditional practices who are having difficulty understanding inquiry-based pedagogies.

8.1.6 My Learning From the Experience

I learned to work with the teachers in exploratory, engaging, and innovative ways; demonstrating reflective practice, vulnerability, and confidence; and co-constructing norms and protocols for my safety and the safety of the teachers who volunteered to model. By reflecting on my practice, I seemed to gain new knowledge about supporting the teachers to notice and interpret the underpinnings of the pedagogical approach I modelled so that they could develop pedagogical reasoning more clearly.

Drawing on the data analysis and research evidence presented in this study, I found implications that could enhance, or have the potential to enhance, the experiences of those that model teaching practice from a socioconstructivist perspective. Some helpful suggestions might be to

- make explicit the thoughts and actions underpinning the pedagogical approach in the debrief so that they can become accessible to the teachers for their own learning;
- adopt a pedagogical approach that includes inquiry, questioning, and probing to enable the teachers to construct their own knowledge;
- articulate norms, protocols, and purposes so that all participants have clear expectations, including making a disclaimer that the modelling is not intended to be perfect and encouraging teachers to suspend evaluative judgements. This may enable a greater discussion focus on the intended pedagogies;
- demonstrate both vulnerability in modelling pedagogies in complex situations and confidence in prompting teachers to reflect on their practice to gain teachers' trust in a short period of time.

8.1.7 Limitations and Recommendations for Future Research

This research project involved four interventions with 18 teachers and a sequence of modelled lessons over several months. This was a short-term study in which the teachers participated for a total of 10 hours. Whilst I collected data three months after the cessation of the intervention to ascertain longer term outcomes and impacts on teacher learning, the sustainability of teachers' changes in practice over a longer time period was outside the scope of this study. Although there is evidence of shifting in teachers' practice, the longer term impact is less conclusive. A recommendation would be to follow up with the teachers 12 months after their participation.

The design of this study involved one teacher from each school having the opportunity to enact pedagogies and be observed in Stage 4 (Week 8). This limited the data and the interpretation of findings regarding enactment. Whilst the remaining teachers in each school reported on their changed practice, the data remain limited to teachers' self-perceptions. An improvement in the design of this study would be to give all participating teachers the

opportunity to enact pedagogies. This would make the data regarding enactment more comprehensive. A recommendation that would make the intervention more financially viable in terms of teacher release would be to have the participating teachers observing the first two lessons and co-planning together for Stage 3. After that, the modeller could co-teach the collaboratively planned lesson and observe the enactments.

This study involved multiple case studies because I deliberately wanted to understand teacher learning in depth and from teachers' perspectives. Spacing each intervention and using a different methodology (such as design research) might yield different findings. A recommendation for future research would be to fine-tune the intervention to be effective but also financially viable and tested in contexts different from the current study. This could include different schools, at other year levels, and in other disciplines. One way of fine-tuning the interventions would be to have flexible time slots for the pre- and post-modelled-lesson meetings. For example, after or before school would lessen the cost of teacher release on schools.

Whilst it does seem important to have a knowledgeable other to support teachers to plan, model, and reflect on teaching practice, it would be worthwhile investigating the impact of school-based numeracy coaches leading this work. It does seem realistic that the person leading this work would ideally be situated in each school context to make it practically possible and keep the costs to a minimum.

This study was focused deliberately on teacher learning, but research into how students' development of reasoning occurs is also important. Research into student learning resulting from the intervention may provide clarity on the impact of the intervention on students' learning.

Although the teachers agreed to participate in this intervention, it was suggested that greater involvement in the decision-making processes at the start might have supported the

teachers to feel greater ownership. It would be useful to investigate the impact on teachers if they themselves identified a problem of practice within their school settings and pursued the intervention to improve teaching practice.

This research project adds to the body of knowledge around modelling as a type of teacher professional learning in teachers' own contexts and highlights the need for further educational research into ways that such an intervention may be improved. Given the success of the intervention, it seems plausible to develop, trial, study, and refine this intervention more broadly.

8.2 Conclusion

This study provides evidence that modelling pedagogies associated with developing students' mathematical proficiency in teachers' own classrooms appears to confront teacher assumptions about traditional approaches to teaching and learning mathematics. It seems the teachers surprise at their students' engagement in the challenging learning experiences conflicted with their existing views of teaching and impelled them to reflect on and set goals for improving their practice. From my perspective, the observations and subsequent deconstruction of pedagogies seemed to prompt the teachers to shift their visions of teaching from conventional to more interactive approaches and enact new ways of working. The pre- and post-lesson discussions seemed critical for the teachers to refine their interpretations of the underpinnings of the pedagogical approach and develop their pedagogical reasoning for enactment. This implies that modelling pedagogies in a sequenced and structured school-based intervention can be a productive form of professional learning for teachers. Further iterations of the approach might help teachers create new visions of teaching for themselves and promote problem-solving and reasoning for the students in their classroom.

REFERENCES

- Adler, S. A. (1993). Teacher education: Research as reflective practice. *Teaching and Teacher Education*, 9(2), 159–167.
- Anthony, G., Hunter, R., & Hunter J. (2017). Challenging teachers' perceptions of student capability through professional development: A telling case. *Professional Development in Education*, 1–13.
- Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics*. Educational series 19. Brussels: International Academy of Education; Geneva: International Bureau of Education.
- Askew, M. (2011). Forward. In P. Sullivan (Ed.), *Teaching mathematics: Using research-informed strategies* (Australian Education Review, 59) (pp. iii–vi). Camberwell, Australia: Australian Council for Educational Research.
- Askew, M. (2012). *Transforming primary mathematics: Understanding classroom tasks, tools and talk*. Oxon, England. Routledge.
- Australian Academy of Science (2015). *Desktop review of mathematics school education pedagogical approaches and learning resources*. Retrieved from <https://docs.education.gov.au/documents/desktop-review-mathematics-school-education-pedagogical-approaches-and-learning-resources>
- Australian Curriculum and Assessment Reporting Authority (ACARA). (n.d.). *Foundation to year 10 curriculum: Mathematics*. Retrieved from <https://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?layout=1>
- Australian Government Chief Scientist (n.d.). *Improving the mathematics performance of Australia's students*. Retrieved from <https://www.chiefscientist.gov.au/wp-content/uploads/Improving-the-mathematics-performance-of-Australias-students.pdf>
- Australian Government Department of Education and Training (2018). *Report of the review to achieve educational excellence in Australian schools*. Retrieved from <https://www.education.gov.au/review-achieve-educational-excellence-australian-schools>
- Australian Institute for Teaching and School Leadership (AITSL). (2012). *Australian teacher performance and development framework*. Retrieved from https://www.aitsl.edu.au/docs/default-source/default-document-library/australian_teacher_performance_and_development_framework_august_2012.pdf?sfvrsn=e7c2ec3c_0
- Australian Mathematical Sciences Institute (AMSI). (2014). *Dealing with Australia's mathematical deficit*. Melbourne, Australia: Author.
- Ball, D. L. (2017). Uncovering the special mathematical work of teaching. In G. Kaiser (Ed.), *Proceedings of the 13th International Congress on Mathematical Education* (pp. 11–34). Hamburg, Germany: Springer.

- Ball, D. L., Ben-Peretz, M., & Cohen, R. B. (2014). Records of practice and the development of collective professional knowledge. *British Journal of Educational Studies*, 62(3), 317–335.
- Ball, D. L., & Forzani, M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education*, 60(5), 497–511.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407.
- Bass, H., & Ball, D. L. (2014). Mathematics and education: Collaboration in practice. In M. N. Fried & T. Dreyfus (eds.), *Mathematics & mathematics education: Searching for common ground* (pp. 299–312). The Netherlands: Springer.
- Berry, A. (2007). *Tensions in teaching about teaching: Understanding practice as a teacher educator* (Vol. 5). Netherlands: Springer.
- Berry, A. (2009). Exploring vision in self-study. *Studying Teacher Education: A Journal of Self-Study of Teacher Education Practices*, 5(2), 159–162.
- Boerst, T., Sleep, L., Ball, D. L., & Bass, H. (2011). Preparing teachers to lead mathematics discussions. *Teachers College Record*, 113(12), 2844–2877.
- Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational researcher*, 33(8), 3–15.
- Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. E. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education*, 24(2), 417–436.
- Bragg, L., & Vale, C. (2014). Developing noticing of reasoning through demonstration lessons. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Proceedings of the 37th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 698–701). Sydney: MERGA.
- Brewer, E. W. (2009). Conducting survey research in education. In V.C. Wang (Ed.), *Handbook of Research on e-learning applications for career and technical education: Technologies for vocational training* (pp. 1–28). California, CA: IGI Global.
- Bronkhorst, L., Meijer, P., Koster, B., & Vermunt, J. (2011). Fostering meaning-oriented learning and deliberate practice in teacher education. *Teaching and Teacher Education*, 27(7), 1120–1130.
- Brookfield, S. D. (1995). *Becoming a critically reflective teacher*. San Francisco, CA: Jossey-Bass.
- Bruce, C., Ross, J., Flynn, T., & McPherson, R. (2009). Lesson study and demonstration classrooms: Examining the effects of two models of teacher professional development. Retrieved from <http://www.tmerc.ca/digitalpapers/samples/WholeResearchStory.pdf>
- Buchmann, M. (1986) Role over person: Morality and authenticity in teaching. *Teachers College Record*, 87(4), 531–543.

- Butler, D. L., Lauscher, H. N., Jarvis-Selinger, S., & Beckingham, B. (2004). Collaboration and self regulation in teachers' professional development. *Teaching and Teacher Education, 20*, 435–455.
- Casey, K. (2011). Modeling lessons. *Educational Leadership, 69*(2), 24–29.
- Chapin, S., O'Connor, C., & Anderson, N. (2003). Classroom discussions: Using math talk to help students learn. Sausalito, CA: Math Solutions Publications.
- Chapman, O. (2012). Challenges in mathematics teacher education. *Journal of Mathematics Teacher Education, 15*(4), 263–270.
- Chapman, O. (2015). Reflective awareness in mathematics teachers' learning and teaching. *Eurasia Journal of Mathematics, Science & Technology Education, 11*(2).
- Clarke, D. (1994). Ten key principles from research for the professional development of mathematics teachers. In D. B. Aichele & A. F. Coxfors (Eds.), *Professional development for teachers of mathematics* (pp. 37–48). Reston, VA: NCTM.
- Clarke, D., & Clarke, B. (2008). Is time up for ability grouping. *Curriculum Leadership, 6*(5), 31–33.
- Clarke, D. M., Clarke, D. J., & Sullivan, P. (2012). Important ideas in mathematics: What are they and where do you get them? *Australian Primary Mathematics Classroom, 17*(3), 13–18.
- Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and teacher education, 18*(8), 947–967.
- Clarke, D., Roche, A., Wilkie, K., Wright, V., Brown, J., Downton, A., . . . & Worrall, C. (2013). Demonstration lessons in mathematics education: Teachers' observation foci and intended changes in practice. *Mathematics Education Research Journal, 25*(2), 207–230.
- Cobb, P., Wood, T., & Yackel, E. (1990). Chapter 9: Classrooms as learning environments for teachers and researchers. *Journal for Research in Mathematics Education. Monograph, 4*, 125–210.
- Council of Australian Governments. Human Capital Working Group. (2008). *National numeracy review report*. Canberra, ACT: Author.
- Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (4th ed.). Boston, MA: Pearson.
- Creswell, J. W. (2013). *Qualitative inquiry & research design: Choosing among five approaches* (3rd ed.). Los Angeles, CA: SAGE.
- Creswell, J. W. (2014). *Research design: Qualitative, quantitative and mixed methods approaches* (4th ed.). Los Angeles, CA: SAGE.
- Crotty, M. (1998). *The foundations of social research: Meaning and perspective in the research process*. St Leonards, NSW, Australia: Allen & Unwin.

- Cunliffe, A. L. (2009). The philosopher leader: On relationalism, ethics and reflexivity: A critical perspective to teaching leadership. *Management Learning*, 40, 87–101
- Danielson, C. (2012). Observing classroom practice. *Educational Leadership*, 70(3), 32–37.
- Davidson, A., Herbert, S., & Bragg, L. A. (2018). Supporting elementary teachers' planning and assessing of mathematical reasoning. *International Journal of Science and Mathematics Education*, 1–21.
- Denzin, N. K., & Lincoln, Y. S. (2011). The art and practices of interpretation, evaluation, and representation. In N. K. Denzin & Y. S. Lincoln (Eds.), *The Sage handbook of qualitative research* (4th ed., pp. 563–568). Thousand Oaks, CA: Sage Publications.
- Department of Education Tasmania. (2018). *Strategic plan*. Retrieved from <https://documentcentre.education.tas.gov.au/Documents/DoE-Strategic-Plan-2018-2021.pdf>
- Dossey, J. A. (1992). The nature of mathematics: Its role and its influence. In D. A. Grouws, (Eds.). *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 39–48). New York, NY: Macmillan.
- Easterby-Smith, M., & Malina, D. (1999). Cross-cultural collaborative research: Toward reflexivity. *Academy of Management Journal*, 42, 76–86.
- Ebaegu, M., & Stephens, M. (2014). Cultural challenges in adapting lesson study to a Philippines setting. *Mathematics Teacher Education & Development*, 16(1), 43–64.
- Edwards-Groves, C., Anstey, M., Bull, G. (2014) *Classroom talk: Understanding dialogue, pedagogy and practice*. NSW, Australia: PETA.
- Erickson, F. (2011). On noticing teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 17–34). New York, NY: Routledge.
- Ericsson, K. A. (2002). Attaining excellence through deliberate practice: Insights from the study of expert performance. In M. Ferrari (Ed.), *The pursuit of excellence in education* (pp. 21–55). Hillsdale, NJ: Erlbaum.
- Ernest, P. (1991). *The philosophy of mathematics education*, London, England: Falmer Press.
- Ernest, P. (2004). What is the philosophy of mathematics education? *Philosophy of Mathematics Education Journal*, 18, 1–16 Retrieved from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.197.5854&rep=rep1&type=pdf>
- Ernest, P. (2008). Epistemology plus values equals classroom image of mathematics. *The Philosophy of Mathematics Education Journal*, 23, 1–12.
- Ernest, P. (2010a). Reflections on theories of learning. In L. English & B. Sriraman (Eds.), *Theories of mathematics education seeking new frontiers* (Advances in mathematics education) (pp. 39–47). Heidelberg, Germany: Springer.

- Ernest, P. (2010b). The social outcomes of school mathematics: Standard, unintended or visionary? In C. Glascoine & K. Hoad (Eds.), *Teaching mathematics? Make it count: What research tells us about the effective teaching and learning of mathematics*. Proceedings of the 2010 Australian Council for Educational Research Conference (pp. 21–26). Camberwell, Victoria, Australia: ACER.
- Feiman-Nemser, S. (2001). From preparation to practice: Designing a continuum to strengthen and sustain teaching. *Teachers College Record*, 103(6), 1013–1055.
- Feiman-Nemser, S. (2012). *Teachers as learners*. Cambridge, MA: Harvard.
- Fernandez, C. (2005). Lesson study: A means for elementary teachers to develop the knowledge of mathematics needed for reform-minded teaching? *Mathematical Thinking and Learning*, 7(4), 265–289.
- Fraivillig, J. (2004). Strategies for advancing children’s mathematical thinking. *Teaching Children Mathematics*, 7(8) 454–459.
- Ghousseini, H., & Sleep, L. (2011). Making practice studyable. *The International Journal of Mathematics Education*, 43(1), 147–160.
- Grierson, A. L., & Gallagher, T. L. (2009). Seeing is believing: Creating a catalyst for teacher change through a demonstration classroom professional development initiative. *Professional Development in Education*, 35(4), 567–584
- Grootenboer, P., & Marshman, M. (2016). Students’ beliefs and attitudes about mathematics and learning mathematics. In P Grootenboer & M Marshman (Eds.), *Mathematics, affect and learning* (pp. 55–74). Singapore: Springer.
- Groves, S., & Doig, B. (2010). Adapting and implementing Japanese lesson study: Some affordances and constraints. In Y. Shimizu, Y. Sekiguchi., & K. Hino (Eds.), *The proceedings of the 5th East Asia Regional Conference on Mathematics Education: In Search of Excellence of Mathematics Education* (pp. 699–706). Tokyo, Japan: Japan Society of Mathematical Education (JSME).
- Groves, S., & Doig, B. (2014). International perspectives on Japanese lesson study. *Mathematics Teacher Education & Development*, 16(1), 1–3.
- Grossman, P. (2011). Framework for teaching practice: A brief history of an idea. *Teachers College Record*, 113(12), 2836–2843.
- Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P., (2009). Teaching practice: A cross-professional perspective. *The Teachers College Record*, 111(9) 2055–2100.
- Guba, E. G. (1990). The alternative paradigm dialog. In E. G. Guba (Ed.), *The paradigm dialog* (pp. 17–30). Newbury Park, CA: Sage Publications.
- Guba, E. G., & Lincoln, Y. S. (1989). *Fourth generation evaluation*. Newbury Park, CA: Sage.

- Guba, E. G., & Lincoln, Y. S. (2005). Paradigmatic controversies, contradictions, and emerging confluences. In N. K. Denzin & Y. S. Lincoln (Eds.), *The Sage handbook of qualitative research* (3rd ed., pp. 191–215). Thousand Oaks, CA: Sage Publications.
- Guskey, T. R. (1986). Staff development and the process of teacher change. *Educational Researcher*, 15(5), 5–12.
- Hamilton, M. L., & Pinnegar, S. (1998). Introduction: Reconceptualizing teaching practice. In M. L. Hamilton (Ed.), *Reconceptualizing teaching practice: Self-study in teacher education* (pp. 1–4). London, England: Falmer Press.
- Hammerness, K., Darling-Hammond, L., Bransford, J., Berliner, D., Cochran-Smith, M., & Zeichner, K. (2005). How teachers learn and develop. In L. Darling-Hammond, & J. Bransford (Eds.), *Preparing teachers for a changing world: What teachers should learn and be able to do* (pp. 358–388). San Francisco, CA: Jossey-Bass Educational Series.
- Hiebert, J., & Grouws, D. A. (2006). The effects of classroom mathematics teaching on student learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the national council of teachers of mathematics* (pp 371- 404). Charlotte, NC: Information Age Publishing.
- Higgins, J., & Parsons, R. (2009). A successful professional development model in mathematics. *Journal of Teacher Education*, 60(3), 231–242.
- Higgins, J., & Parsons, R. (2011). Professional learning opportunities in the classroom: Implications for scaling up system level professional development in mathematics. *Mathematics Teacher Education and Development*, 13(1) 54–76.
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372–400.
- Hodgson, L. (2013). What teachers see when watching others teach. In V. Steinle, L. Ball & C. Bardini (Eds.), *Mathematics Education: Yesterday, today and tomorrow: Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia* (pp. 386–393). Melbourne, Australia: MERGA.
- Hodgson, L. (2016). Collaboration around observation of teaching: Powerful professional learning. In B. White, M. Chinnappan, & S. Trenholm (Eds.), *Opening up mathematics education research: Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia* (pp. 295–302). Adelaide, Australia: MERGA.
- Hollingsworth, H., Lokan, J., & McCrae, B. (2003), *Teaching mathematics in Australia: Results from the TIMSS 1999 Video Study* (TIMSS Australia Monograph No. 5). Melbourne, Australia: Australian Council for Educational Research.
- Hycner, R. H. (1985). Some guidelines for the phenomenological analysis of interview data. *Human Studies*, 8(3), 279–303.
- Institute of Professional Editors (2010). *Guidelines for editing research theses*. Retrieved from http://iped-editors.org/About_editing/Editing_theses.aspx

- Kennedy, M. M. (1999). The role of preservice teacher education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 54–85). San Francisco, CA: Jossey Bass.
- Kilpatrick, J., Swafford, J., Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.
- Kincheloe, J. L., & McLaren, P. (2005). Rethinking critical theory and qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *The Sage handbook of qualitative research* (3rd ed., pp. 303–343). Thousand Oaks, CA: Sage Publications.
- Kisa, M. T., & Stein, M. K. (2015). Learning to see teaching in new ways: A foundation for maintaining cognitive demand. *American Educational Research Journal*, 52(1) 105–136.
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge UK: Cambridge University Press.
- Lefstein, A., & Snell, J. (2014). *Better than best practice: Developing teaching and learning through dialogue*. Oxon, England: Routledge.
- Lefstein, A., Trachtenberg-Maslaton, R., & Pollak, I. (2017). Breaking out of the grips of dichotomous discourse in teacher post-observation debrief conversations. *Teaching and Teacher Education*, 67, 418–428.
- Lewis, C., & Perry R., (2014). Lesson study with mathematical resources. A sustainable model for locally led teacher professional learning. *Mathematics Teacher Education and Development*, 16(1), 22–40.
- Lewis, C. P., Perry, R., Hurd, J., & O’Connell, M. (2006). Lesson study comes of age in North America. *Phi Delta Kappan*, 88(4), 273–281.
- Lincoln, Y. S., Lynham, S. A., & Guba, E. G. (2011). Paradigmatic controversies, contradictions and emerging confluences revisited. In N. K. Denzin & Y. S. Lincoln, *The SAGE handbook of qualitative research* (4th ed., pp. 97–128). Thousand Oaks, CA: SAGE.
- Little, J. W. (2002). Locating learning in teachers’ communities of practice: Opening up problems of analysis in records of everyday work. *Teaching and teacher education*, 18(8), 917–946.
- Loucks-Horsley, S., Love, N., Stiles, K. E., Mundry, S., & Hewson, P. W. (2003). *Designing professional development for teachers of science and mathematics*. Thousand Oaks, CA: Sage Publications.
- Loughran, J. (2006). *Developing a pedagogy of teacher education*. Oxon, England: Routledge
- Loughran, J. & Berry, A. (2005). Modelling by teacher educators, *Teaching and Teacher Education*, 21(2), 193–203.

- Loughran, J. J., Keast, S., & Cooper, R. (2016). Pedagogical reasoning in teacher education. In J. Loughran, & M. L. Hamilton (Eds.), *International handbook of teacher education* (pp. 387–421). Singapore: Springer.
- Lortie, D. C. (1975). *Schoolteacher: A sociological study*. Chicago, IL: University of Chicago Press.
- Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. (The Jossey-Bass higher and adult education series). San Francisco, CA: Jossey-Bass.
- Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis* (2nd ed.). Thousand Oaks, CA: Sage.
- Naik, S., & Ball, D. L. (2012, 8–15 July). *Teacher learning through organized experiences*. Paper presented at the meeting of the 12th International Congress of Mathematical Education, Seoul, Korea.
- Naik, S. S., & Ball, D. L. (2014). Professional development in a laboratory setting examining evolution in teachers' questioning and participation. *Journal of Mathematics Education*, 7(2), 40–54.
- National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: NCTM.
- Neuman, W. L. (2006). *Social research methods. Qualitative and quantitative approaches*. (6th ed.). Singapore: Pearson Education.
- Polly, D., & Hannafin, M. (2011). Examining how learner centred professional development influences teachers espoused and enacted practices. *Journal of Educational Research*, 104(2), 120–130.
- Putnam, R. T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher*, 29(1), 4–15.
- Resnick, L. B., Spillane, J. P., Goldman, P., & Rangel, E. S. (2010). Implementing innovation: From visionary models to everyday practice. In H. Dumont, D. Instance, & F. Benavides (Eds.), *The nature of learning: Using research to inspire practice* (pp. 285–315). Norway: OECD Publishing.
- Rubenstein, H. (2009). A national strategy for mathematical sciences in Australia. Retrieved from <https://www.science.org.au/files/userfiles/support/reports-and-plans/2009/national-strategy-for-math-sciences-in-australia.pdf>
- Russell, V. J., Mackay, T., & Jane, G. (2003). *Messages from MYRAD (Middle Years Research and Development): Improving the middle years of schooling*. Melbourne, Australia: Independent Association of Registered Teachers of Victoria.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4–14.

- Sleep, L. (2012). The work of steering instruction toward the mathematical point: A decomposition of teaching practice. *American Educational Research Journal*, 49(5), 935–970.
- Smith, J. (2001). Modeling the social construction of knowledge in ELT teacher education. *ELT Journal*, 55(3), 221–227.
- Smith, M. S., Hughes, E. K., Engle, R. A., & Stein, M.K. (2009). *Orchestrating Discussions: Mathematics Teaching in the Middle School*, 14(9), 548–556.
- Smith, M., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA: NCTM.
- Stacey K., & Turner R. (2015). The evolution and key concepts of the PISA mathematics frameworks. In K. Stacey & R. Turner (Eds.), *Assessing mathematical literacy* (pp 5–33). Switzerland: Springer.
- Stake, R. E. (1995). *The art of case study research*. Thousand Oaks, CA: Sage Publications.
- Stake, R. E. (2003). Case studies. In N. K. Denzin & Y. S. Lincoln (Eds.), *Strategies of qualitative inquiry* (2nd ed., pp. 134–164). Thousand Oaks, CA: Sage Publications.
- Stake, R. E. (2006). *Multiple case study analysis*. New York, NY: Guilford Press.
- Sullivan, P. (2011). *Teaching mathematics: Using research-informed strategies* (Australian Education Review, 59). Camberwell, Australia: Australian Council for Educational Research.
- Sullivan, P., Askew, M., Cheeseman, J., Clarke, D., Mornane, A., Roche, A., & Walker, N. (2015). Supporting teachers in structuring mathematics lessons involving challenging tasks. *Journal of Mathematics Teacher Education*, 18(2), 123–140.
- Sullivan, P., Borcek, C., Walker, N., & Rennie, M. (2016). Exploring a structure for mathematics lessons that initiate learning by activating cognition on challenging tasks. *The Journal of Mathematical Behavior*, 41, 159–170.
- Sullivan, P., Mousley, J., & Jorgensen, R. (2009). Tasks and pedagogies that facilitate mathematical problem solving. In B. Kaur, B. Yeap, & M. Kapur (Eds.), *Mathematical problem solving: Yearbook* (pp. 17–42). Singapore: World Scientific.
- Sullivan, P., Zevenbergen, R., & Mousley, J. (2006). Teacher actions to maximize mathematics learning opportunities in heterogeneous classrooms. *International Journal for Science and Mathematics Teaching*, 4, 117–143.
- Takahashi, A. (2014). The role of the knowledgeable other in lesson study: Examining the final comments of experienced lesson study practitioners. *Mathematics Teacher Education and Development*, 16(1) 4–21.
- Takahashi, A., & McDougal, T. (2015). Collaborative lesson research: maximizing the impact of lesson study. *ZDM*, 1–14.

- Timperley, H., Wilson, A., Barrar, H., & Fung, I. (2008). *Teacher professional learning and development. Educational Series-18*. Brussels, Belgium: International Bureau of Education, Geneva.
- Timperley, H. (2015). *Professional conversations and improvement focussed feedback*. Retrieved from https://www.aitsl.edu.au/docs/default-source/default-document-library/professional-conversations-literature-review-oct-2015.pdf?sfvrsn=fc2ec3c_0
- van de Walle, J. A., & Lovin, L. A. (2006). *Teaching student centered mathematics*. United States of America: Pearson.
- van Driel, J., & Berry, A. (2012). Teacher professional development focusing on pedagogical content knowledge. *Educational Researcher*, 41(1), 26–28.
- van Es, E. A. (2012). Using video to collaborate around problems of practice. *Teacher Education Quarterly*, 39(2), 103–116.
- van Manen, M. (1990). *Researching lived experience: Human science for an action sensitive pedagogy*. London, Canada: The Athlone Press.
- Walker, R. (1983). Three good reasons for not doing case studies in curriculum research. *Journal of Curriculum Studies*, 15(2), 155–165.
- Widjaja, W., Vale, C., Groves, S., & Doig, B. (2017). Teachers' professional growth through engagement with lesson study. *Journal of Mathematics Teacher Education*, 20(4), 357–383.
- Wilson, P. H., Edgington, C., Sztajn, P., & DeCuir-Gunby, J. (2014). Teachers, attributions, and students' mathematical work. In J. Lo, K. R. Leatham, & L. R. Van Zoest (Eds.), *Research trends in mathematics teacher education* (pp. 115–132). Basel, Switzerland: Springer.
- Wilkie, K. J. (2017) The challenge of changing teaching: Investigating the interplay of external and internal influences during professional learning with secondary mathematics teachers. *Journal of Mathematics Teacher Education (advance online publication)*. DOI:10.1007/s10857-017-9376-0
- Wolcott, H. F. (2009). *Writing up qualitative research* (3rd ed.). Thousand Oaks, CA: Sage.
- Wood, T., Cobb, P., & Yackel, E. (1991). Change in teaching mathematics: A case study. *American Educational Research Journal*, 28(3), 587–616.
- Yates, L. (2003). Interpretive claims and methodological warrant in small-number qualitative, longitudinal research. *International Journal of Social Research Methodology*, 6(3), 223–232.
- Yin, R. (2014). *Case study research: Design and methods* (5th ed.). Los Angeles, CA: Sage.
- Yook-Kin Loong, E., Vale, C., Herbert, S., Bragg, L. A., & Widjaja, W. (2017). Tracking change in primary teachers' understanding of mathematical reasoning through demonstration lessons. *Mathematics Teacher Education & Development*, 19(1), 5–29.

APPENDICES

Appendix 1: Modelled Lesson Request Proforma

School name: _____ Name of Teacher: _____ Grade: _____ Date of modelled lesson: _____

What would you like the mathematical focus of the modelled lesson to be (please feel free to use content descriptors from the Australian Mathematics Curriculum)?				
What would you like to be the focus of the modelled lesson with respect to pedagogy (For example, questioning, differentiation, open ended tasks, structure of the lesson, materials used)?				
Lesson start and end time:	30 minute Pre-brief start time:	30 minute Debrief start time:	Number of students and grade level	Observers (full names)

It would be helpful if the students could wear **name tags** in the classes in which the lessons are taken. Also, a big red dot on name tags of any children who cannot be photographed or videoed.

PLEASE RETURN THIS FORM TO ME ONE WEEK BEFORE THE SCHEDULED VISIT TO ENABLE PLANNING TIME

Appendix 2: Pre-Modelled-Lesson Teacher Survey

Name: _____ School: _____

Year levels taught _____

Years of teaching _____

1. Think about the numeracy aspirations that you have for your students. What qualities do you want your students to have by the time they leave your school?

2. What gaps do you see between these aspirations and how students are actually developing in numeracy at your school?

- 3a. Suppose you were to model a mathematics lesson to some colleague teachers, what teacher actions (pedagogies) would you model?

3b. Why?

4a. What I find most difficult about teaching mathematics

is _____

4b.

Because _____

5. Effective teachers use the following pedagogical approaches (teaching actions) when catering for the diverse range of learning needs in a mathematics

lesson _____

Appendix 3: Mathematics Lesson Observation Tool

Name:

School:

stage:

Write down everything you saw the modelling teacher say and do to facilitate the students' reasoning.

Appendix 4: Modelled Lesson Reflection Cycle 2

Date:	Grade level of the class:	School:
Observing teachers name:		
Mathematical focus of the lesson:		

1. Are there any teacher action/s that you have implemented in your classroom practice as a result of observing a modelled lesson? Please describe.

2. What teacher action/s did you observe today that are different from what you usually do that you will set as a goal for your future classroom practice?

Appendix 5: Observation Focus for Enactment

Name: _____ School: _____

As you observe this lesson, please write down everything you observe with respect to the modelling teachers chosen focus.

The teacher's chosen observation focus for lesson: _____

Appendix 6: Modelled Lesson Intervention Exit Survey

Name: _____

School: _____

Year levels taught _____

Years of teaching _____

1. Suppose you were to model a lesson to some colleague teachers, what teacher actions (pedagogies) would you model?

1a. Why? _____

2. Effective teachers use the following pedagogical approaches when catering for the diverse range of learning needs _____

3. Please describe any changes to your teaching practice as a result of being involved in this project _____

Appendix 7: Themes: Raven Primary School

Table 5.2 Selected examples of significant statements and their meanings Stage 1

Selected samples of significant statements from post lesson discussions and surveys	Formulated meanings
Rose: Thinking in their heads...kids reaction, oh no!	It seems Rose observed students were afraid of this pedagogical approach, which seemed foreign to them.
Rose: [You] reinforced the expectation that the children would do the thinking and reinforcing that it's okay to be confused.	It appears that Rose observed important elements of a pedagogical approach that builds on student thinking is focused on facilitating students' reasoning and problem solving.
Rachel: I think the children were a little bit confused by language...even the word strategy...they didn't have the language to explain... We would have explained things (to the children) to the nth degree where perhaps it's better not to... I wasn't even aware of it until I saw it happening.	The observation of student attempts to articulate mathematical reasoning in the lesson seemed to prompt Rachel to reflect on how much she tells students what to do.
Rebecca: The kids really started thinking about what they were doing rather than ...doing just one answer...Some kids found it difficult to explain...I noticed that they need more practice on this kind of pedagogy because a lot of the stuff I do in the class is very linear, for example, this is the process, this is how you solve the problem, this is what I want you to do.	The modelling appeared to have the desirable effect of enabling Rebecca to see an example of alternative pedagogy and how her students responded to the approach by struggling to think for themselves. I surmise that this enabled her to recognise that more experience with the approach was needed.
Ruth: The fundamental difference with what you did was that you did not teach in ability-based groups. It is a whole swing for me to think differently.	The modelling seemed to enable Ruth to notice a different pedagogical approach to cater for diversity, which prompted her to reflect on how she teaches.
Ruth: How do they (students) get skills if we don't teach explicitly? How will they (the students) get strategies unless teachers tell them to begin with?	From my perspective Ruth was challenged by the pedagogical approach of allowing students to think for themselves before instruction from the teacher
Ruth: We have the <i>Walt</i> and the <i>Wilf</i> at our school. We tell the kids what they're learning, we tell them what we're looking for and we tell them how to get there. We are so accountable for making sure kids get stuff. We want to make sure all kids get stuff by the end of the lesson.	I interpreted this to mean that the initial modelled lesson challenged Ruth's pedagogical approach to teaching as she was wrestling with new ideas that challenged her teacher directed view of student learning.
Ruth: We need to look at the Australian Mathematics Curriculum. We have just looked at the content. We haven't had any guidance. I did not know about the proficiencies.	It seems that Ruth felt she had not had support to implement the Australian curriculum and therefore did not know about the proficiency strands.
Ruth: It didn't even click till one of the little kids talked about that they'd worked out how to add two to build to ten and they worked out how to add five to build to ten and I was thinking I didn't realise that you'd specifically always kept it to two or to five. I thought it was more random than that	Ruth, from my perspective, did not 'see' the rationale for deliberately making the tasks + 5 or plus 2

Rose: In this lesson...thinking and strategies...not answer.

It appears that Rose observed a pedagogical approach that was different from the conventional teaching approach to mathematics teaching and learning

Rose: gave enabling and extending prompts, stopped...changed lesson to $28 + 7$, Build up $28 + 7$, $98 + 7$, $198 + 7$, scaffold.

I surmise that Rose noticed the researcher gave enabling and extending prompts to those who needed them and scaffolded students in the moment.

Rose: [The researcher] named it up “building to the next ten, sometimes called bridging 10”

I interpreted this to mean that Rose noticed the researcher modelled mathematical language

Rose: Kept prompting, “how did you do it...”, “What did you notice about this strategy?” “Think about what you learned in the earlier tasks?”

It seems that Rose noticed the researcher prompted students for further participation

Rose: Used a document camera and interactive whiteboard (IWB) to show children’s work.

From my perspective, Rose noticed the researcher using the IWB to make student examples visible to all students

Rachel: “Keep thinking and I’ll come back to you”.

It appears that Rachel noticed the researcher encouraged the students to ‘keep thinking’.

Rachel: You asked Roy to explain something and he knew, but he was struggling to explain and you just said I’ll come back to you...you went onto somebody else and then you came back and that gave him the time to have a think about how he was going to explain it.

I surmise that Rachel observed my attempts at encouraging persistence

Rachel: Teacher circulating giving enabling questions

I interpreted this to mean that Rachel noticed the researcher circulating and handing out enabling and extending prompts to those who needed it.

Reduced from $298 + 35$ to $28 + 7$... Gave Roy and others extending prompts.

Rachel: Used IWB to show student examples. “you can break up numbers”

It seems that Rachel observed the researcher used the IWB to make student thinking visible.

Rachel: Rephrased and reiterated student strategies.

From my perspective Rachel observed teacher actions that included repeating and rephrasing students’ explanations to build on ideas.

Rachel: I liked the way that you asked [the students] questions, “what were you thinking in your mind, can you explain that, how did you do that then... What did you notice about this strategy?”

It appears that Rachel observed the researcher had the expectation that students would justify their reasoning

Rachel: Thinking time

I surmise that Rachel observed the researcher provided thinking time to enable students to think for themselves to solve problems.

Rachel: When you regrouped to the mat you [researcher] said “so you learnt you can break up numbers to help you with your adding up”.

I interpreted this to mean that Rachel noticed my attempts at summarising learning

Rebecca: You gave the students a lot of thinking time

It seems that Rebecca observed the researcher gave students extended thinking time.

Rebecca: Showed some good strategies on IWB from kids, patient with questioning of students, Leo came up with the fastest strategy. [The researcher] got him to explain and [put his example] on the IWB.

Rebecca, from my perspective, observed the researcher projecting students’ work through a document camera for others to see. The student then explained his thinking to the class.

Rebecca: You made the questions easier for those who needed it. I saw you giving out a couple of extending prompts [to fast

It appears that Rebecca noticed the researcher gave enabling prompts and extending prompts to those who needed them

finishers]and asking questions to push them further.

Ruth: Zone of confusion, “it’s okay to be confused”

I surmise that Ruth noticed the researcher encouraging students to persist.

Ruth: Thinking time

It seems that Ruth noticed the researcher gave the students extended thinking time

Ruth: Named up children’s strategies... “That is called partitioning”

I interpreted this to mean that Ruth observed the researcher modelling mathematical language.

Ruth: It didn’t even click to me till one of the little kids talked about how they’d worked out how to add two to build to ten and they worked out how to add five to build to ten and I was thinking I didn’t realise that you’d specifically always kept it to two or to five.

Table 5.3 Selected examples of significant statements and their meanings Stage 2

Selected samples of significant statements from post lesson discussions and surveys	Formulated meanings
Rachel: Emily really surprised me by her understanding. She solved the problem with 24 apples and could explain her thinking. Normally she has blank expressions on her face and does not engage.	It appears that Rachel saw a perceived low attaining student experience success when given the opportunity to engage in a problem-solving task.
Ruth: I got distracted by the management. Some of the kids were off task	Students seemingly ‘off task’ appeared to be a distraction for Ruth
Rose: How do you work with 25 kids all at different levels, all working on the same problem and have conversations with all of them during the lesson? I got distracted when I stepped back and thought, that one needs more talking to, that one has no idea, that one hasn’t got pen to paper yet.	Rose, from my perspective, reported she was distracted with her perceived disengagement of some students
Rachel: Yes we are catering for a diverse range of learners, but are we giving them the explicit teaching needed for the level they are at? The enabling questions allow them to enter the learning but do they need more explicit instructions, more teacher focus time? Could this work with ability groupings to allow a more personalised approach?	This suggests that Rachel could see the diverse range of learners were catered for with extending and enabling prompts but questioned whether students needed explicit instruction by the teacher in ability groups.
Ruth: How will they [students] learn strategies unless somebody shows them? What if kids don’t come up with any strategies? If they don’t have anything to bring, where do you start?	I interpreted this to mean that Ruth wrestled with the idea of allowing students to come up with strategies for themselves and what that might mean for her teaching.
Ruth: I am not really confident at thinking of something on the hop. I am used to lessons where I know where I am going, but in this approach it is more about the response from the children, so it could go anywhere and I’m not confident to know where to take it or what response to give.	I surmise that Ruth struggled with the idea of the lesson being more about students responding to the task and expressed her lack of confidence about not being in control of their responses.
Ruth: I tried the task [observed in stage one] with my grade 3 children, but they came up with different responses [compared with the children at RPS]. I didn’t know what to do. The enabler was too hard and I didn’t know what to say next.	It seems that Ruth articulated that she did not know how to respond to student responses in her grade 3 class that were different to those that she had seen in another class.

Rachel: I think we are looking at the curriculum, but not unpacking the rationale. I don't think we are unpacking enough. We are teaching place value, but why? What is the deeper understanding we want them to know?

Rose: I think the opportunities to unpack the curriculum [in this intervention] are really valuable... the discussions of what it means and what it looks like in the classroom.

Rebecca: There seems to be so much more thinking and reasoning for the students rather than just going through the process [of telling them what to do].

Rachel: I think it is easier to plan for. In the class, you have this group and this group and this group. Here you are planning just one idea

Rachel: After seeing it on my class and in another grade, I am more comfortable with the approach now, I saw that the task changed but the approach didn't.

Rebecca: Can this approach replace 'explicit lessons' where I show the kids how to get the answer, send them off to do it and bring them back to check or can it run side by side those lessons? Does this approach really have better outcomes?

Rebecca: I'm struggling to create open-ended questions for all areas of the mathematics curriculum. How does this approach teach all domains of mathematics?

Rebecca: I want to develop more confidence with your approach. I have tried it, but I tend to slip back because they (the students) don't have any strategies for figuring out problems. If I send them off to do a problem without teaching a strategy for solving it, they have no "ammo" for getting it done.

Rachel: What do you do if low attaining students do not understand the enabling prompts?

Ruth: In this approach I'm not sure how to assess kids. If we're giving lots of examples, we would be able to mark them, whereas in this approach, if we don't see it, there's nothing to mark after school.

Rebecca: Instead of just teaching kids how to get the answer, I realise the importance of getting kids to think about how they are getting the answer.

Rose: Problem based questions linked to the curriculum...how to come up with these for early prep? I've changed my thinking about activity

From my perspective, it revealed the modelled lesson and the pre- and post-lesson discussions had prompted Rachel to reflect on the purpose of her teaching. It suggested she recognised the importance of knowing why she is teaching the content she is teaching.

It appears that Rose found the pre and post lesson discussions with colleagues valuable in building her knowledge of the mathematics curriculum.

This suggests that Rebecca noticed a different pedagogical approach that was focused on student reasoning and thinking

I interpret Rachel's comment to mean she showed a growing commitment toward a pedagogical approach, which involves planning one task with multiple entry points, which seems easier than planning a multitude of tasks

I surmise that seeing the lesson in her class made it more meaningful for Rachel

It seems that Rebecca wrestled with an approach to pedagogy which builds on student thinking that was very different from his current approach.

Rebecca, from my perspective, expressed difficulty in creating open tasks for other domains of mathematics.

It appears that Rebecca articulated her desire to developing confidence with student generated approaches but was worried about letting go of telling them what they need to know

This suggests that Rachel expressed concern about how to steer the lesson if the students did not understand the enabling prompt

I interpret this to mean that marking work after school was comforting for teachers as it provided accountability and a record of student work regardless of the thinking and meaning behind it.

It seemed Rebecca recognised the importance of students thinking for themselves.

I surmise that Rose's thinking had moved towards the possibility of using a problem based approach,

based...I need time to work out how this will work for me in early prep	but that finding and coming up with tasks might be problematic
Rose: I'm struggling. I'm feeling overwhelmed in the prep one. It is the beginning of the year and I'm trying to implement a play based curriculum and this as well...	It appears that Rose's level of confidence seemed different at this stage, compared with how she came across at the beginning of the intervention. K was confronted by the modelling and this led her to reflect on her approaches to teaching.

Table 5.4 Selected examples of significant statements and their meanings Stage 3

Selected samples of significant statements from post lesson discussions and surveys	Formulated meanings
Rose: I have never taught friends of ten in that way before... I went straight to the equations. I can see now it didn't lay the foundations for the understanding. I was too quick racing to get to the equations.	This suggests that Rose recognised an alternative approach to teaching partitioning ten which was focussed on student understanding.
Rose: It was quite obvious children like Rosie, Tilly, Melissa, Angus (not their real names) and quite a few others needed that manipulative ... and then reducing it [enabling prompt] back to five... It goes back to some of those children not having those counting ideas that you were talking about... the one to one correspondence, the counting...	It seems that Rose was able to gather assessment data of her students thinking and link it to counting principles discussed in the planning meeting. Armed with this knowledge, she was able to anticipate who needed enabling prompts.
Rose: You couldn't distinguish between grade one and prep responses to the task in that lesson	By having all children in the class working on the same problem with variations, it became clear from my perspective, that within Rose's composite class there was a range of students within each grade who found the tasks challenging and there were others for whom the tasks were less challenging irrespective of grade level.
Rachel: It was so nice to see all children even low ones participating in the thinking	It appears that the modelled lesson enabled observing teachers to focus on how successfully the diverse range of students responded to the learning experience
Rose: I am now conscious of the types of questions that I ask...probing questions and adapting those to suit children's learning needs. Also I'm having much higher expectations of the children as they may surprise me by their responses.	I surmise that Rose was able to articulate her awareness and commitment to the pedagogical approach which included having high expectations and thinking more deeply about the questions she was asking students.
Ruth: Having a good task in the first place drives you because you have to find the task that is going to produce the outcome you are looking for...and giving children a real life problem to solve so that they can see a purpose for solving it.	I interpret this to mean that Ruth could see the importance of choosing tasks that were challenging and meaningful to students.

Table 5.5 Selected examples of significant statements and their meanings Stage 4

Selected samples of significant statements from post lesson discussions and surveys	Formulated meanings
Rose: I thought much more about this lesson before I planned it. I normally would have whole small whole...There would be the whole group and there	This indicates that Rose planned the lesson carefully with the support of the lesson plan exemplar. She had learned that it was possible to cater for the

would be small groups. I would be working with a group and the others would be doing games or activities but I was not there asking questions or probing thinking because I was working with a different group. Today, I could get around asking more children.	diverse range of learners in a lesson by providing the same task with variations. She recognised her previous approach which included working with a small group meant she was unable to support the rest of the class.
Rose: I haven't focussed on reasoning and problem solving before. I did not know that you could expand the section on the AC: M website and find out about the proficiencies. I was focussed on content and worked from that starting point. I did not know we could start with the proficiencies.	It seems that Rose articulated that she had not focussed on the proficiency strands in the past and that she could now use those as a starting point for her practice.
Rose: I'm still worried about those children that you don't get to, but I'm beginning to think that is okay. In my planning, I wrote down children I needed to look out for so today I focussed on those kids. Next time, I'll still have my list and I'll be able to get to those that need support. I think I am shifting my way of thinking about being able to reach all the children.	This suggests that Rose was able to articulate her shift in thinking about catering for diversity.
Rose: The children are beginning to realise we are expecting them to explain their thinking and that everyone's thinking is different and we can gain from that.	It appears that Rose expressed her new commitment to expecting children to explain their thinking.
Rachel: In the past I've probably asked one question, but now we are asking deeper questions of the children for example, "how did you work that out and what were you thinking?" In the past I would have just said yes, that's great (in response to an answer) but now I'm thinking what are they thinking?	
Rose: I think we need to step back and give the children more thinking time than we've given them before and that's okay. In the past, I wouldn't have picked Sarah (not her real name) to share, but she actually had something to contribute.	Rose, from my perspective, recognised the importance of extended thinking time which enable vulnerable children to contribute to the mathematical discussion.
Rose: We can work together now rather than sitting by ourselves. We can support each other	I interpret this to mean that Rose thought colleague teachers could collaboratively plan one problem and teach it across their grades, prep, one two. They could discuss how they were going, talk about their questioning, run things by each other and provide support for each other. K reported that in the past, teachers at RPS all worked in isolation.

Table 5.6 Selected examples of significant statements and their meanings Post Intervention Interviews

Selected samples of significant statements from post lesson interview	Formulated Meanings
---	---------------------

Rose: Initially the modelled lessons scared me because I don't think we had enough background information. As the [intervention] has gone on and as I've done my own research it's all made sense to me. The lessons that you modelled in isolation were a bit daunting, but now that I've put it all together, it all makes sense.

Rose: The biggest thing I learnt was to step back in how much content to put into a lesson. I learned to have one idea, one concept and to stretch those that need stretching and enabling those that need support in developing those concepts...instead of having all these learning centres around with lots of different things.

Rose: In the modelled lessons I was challenged by the idea that you might not reach every child during that lesson, but that is okay ...that they will be picked up in the process. As I become more experienced I will be able to pin point the children which may need some more assistance during that problem solving time and give prompts, either enabling or extending.

Rose: A goal for me was to be was to do some more research and find out. I bought the Van de Walle text and I actually read all the theory. Often you'll get a text and skip all of the theory and go straight to the activities. I actually enjoyed reading about setting the context, differentiation, matching to the curriculum and I enjoyed that background information.

Rose: Another goal was to implement it [the pedagogical approach] and plan lessons myself, but also draw on the resources that you provided. Initially I stuck with the same concepts that you modelled in my classroom, but now I've moved onto to simple addition using the same problem-solving model.

Rose: I think it was more personalised and purposeful to my own professional learning and it fitted in in with my classroom, but I think the theory needs to go alongside the modelling. After I did the research, it all made sense to me.

Rose: From the modelled lesson, which was focused on partitioning ten, I extended the concept with different contexts. For example, dinosaurs and fish. I am really surprised now how much more knowledge the children have about ten from the sequence of those lessons. Most of the children can now partition ten. In the past I would have just had games out and given a few activities, but not had that problem-solving context with the whole class.

Rose: I was happy with my questioning of the children and the language I used. I think both the tuning in and the reflection was too long in my modelled lesson (stage 4).

Rose: I think the way you showed us to photograph children's work, make it visible and use that as a discussion point would cut down on the

It seems to be important to engage teachers in context of PL

From her enactment, Rose learned to use one problematic task that was focused on one concept with multiple entry points. This was different from her usual approach to teaching which consisted of multiple disconnected tasks.

Rose articulated that she was challenged by the idea that some children may struggle with the problems presented in a maths lesson. However she articulated that with more experienced she would get better at anticipating the children that may require more assistance.

The intervention prompted Rose to seek out a recommended professional reading and read the theory, which supported the pedagogical approach. This is powerful professional learning. It also suggests K saw the possibility of transferring the pedagogical approach to other lessons.

Modelling, scaffold for enactment

Rose felt that the PL was personalised to meet her needs as it was situated in her classroom context. She noted the importance of the professional reading to understand the theory.

Rose's lesson sequence had a clear focus which enabled students to gain deeper understanding of the concept than in the past. The teacher was surprised at how much more knowledge her students had from a whole class problem based approach rather than a multitude of activities.

Rose self-reflected critically on her enacted lesson.

Rose reported that she was working towards making student thinking visible by using a document camera

<p>distractedness of the students at the end. I am working towards that.</p>	<p>to enable greater student engagement in the summary phase of the lesson</p>
<p>Rose: The opportunity to have that professional dialogue with your colleagues and having someone else with expertise in mathematics that I could draw on was the most valuable for me.</p>	<p>It seems Rose felt that having professional dialogue with colleagues as well as a knowledgeable other working alongside her enabled her to build her mathematical knowledge for teaching</p>
<p>Rose: I think the co-planning part was really valuable and then me taking the lesson...I think the four stages worked together because it was scaffolded. By the time it was my turn, I felt quite confident to model a lesson myself with the exposure I had to that point.</p>	<p>A scaffolded approach seemed to support Rose to feel confident with taking a leading role herself in enacting the pedagogical approach.</p>
<p>Rebecca: I've adopted quite a few of the methods you used. [You] showed a different way of teaching and you got through to the kids and they really enjoyed it.</p>	<p>In her exit interview Rebecca reported she had taken on board some of the teacher actions she observed.</p>
<p>Rebecca: I liked the way you catered for difference with enabling and extending [prompts]. It's simpler than a lot of techniques that we've used in the past.</p>	<p>Catering for diversity</p>
<p>Rebecca: You were more interested in getting kids understanding out by [getting them to] explain their methods rather than just the product, because a lot of maths is just product, product, product and kids can't see where it fits into life.</p>	<p>Children explaining</p>
<p>Rebecca: Initially I found some kids were getting upset before they'd even had a go because they weren't used to this method.</p>	<p>Teaching as telling</p>
<p>Rebecca: My goal was to approach tasks more openly. I was also able to use some of the teaching strategies in other learning areas</p>	<p>Rebecca's goal open ended tasks</p>
<p>Rebecca: My personal goal was to become more enthusiastic about teaching maths...and change the way I teach...Instead of [a focus on] getting the right answers. I think [the approach you modelled] allowed me to become more enthusiastic because the kids were enjoying it more. Using your approach enabled me to become more confident.</p>	<p>Seeing her children engaged led to Rebecca's enthusiasm for teaching mathematics and her change in practice</p>
<p>Rebecca: I am getting a better result from the students because kids were teaching kids and they are really enjoying it.</p>	<p>Student engagement</p>
<p>Rebecca: The kids all pipe up when I say "zone of confusion" – they are all excited about it. It's more exciting because it is real. Kids have the confidence and ammunition to have a go. At the very start they were all quite hesitant because it was new.</p>	<p>Student motivation and persistence</p>
<p>Rebecca: If they get stuck, they know they are not going to be grilled... If they've had a go and then it's a bit tricky, then I can give them that next prompt. They know they are getting success.</p>	<p>Catering for diversity</p>
<p>Rebecca: There are a lot of teachers who are stuck in their ways and they might see [a modelled lesson] as personal...they might judge the modelling teacher as academic and not so good in the classroom, but I think it is always good to see other people's ideas and experience different ways of thinking.</p>	<p>Risk to researcher</p>

Rebecca: [It would have been helpful to have] a couple more sessions to begin with to explain what you are going to see before you saw it.

Rebecca: Being there, seeing it happen it always going to be better than sitting around an office table... You are experiencing the whole package- you are seeing, you are getting involved, it is multi modal. You're not just hearing it or reading it ...you are getting the full picture.

Ruth: Getting someone in to show you how to do a particular strategy that you are unsure of is a good idea, but in this situation, we're seeing a whole new structure of how to teach our maths program and not everyone is sold on the idea to start with. Seeing someone model it helps us understand it, but doesn't make us agree with it.

Ruth: We're still having trouble with the low attaining children because we don't think they are being catered for. The enabling prompts are not low enough and if you say "think of lower ones" because this is a new structure, we haven't got a bank of ideas that we can tap into on the spur of the moment.

Ruth: A lot of teachers rely on some sort of weekly drill and practice session and they very rarely have whole group sessions so to do this sort of maths... it is quite different... [we use] booklets made up with worksheets for each different day of the week and automatic response.

Ruth: In the past, we have taught strategies – e.g. the jump strategy, the bridge strategy, whereas this [new approach] is asking for students to come up with the ideas themselves. That is a huge turnaround from where we have come from. If children don't have strategies they don't know where to start.

Ruth: We are writing things down from what we are seeing you do ...that are obvious, but it's the white space of the things that aren't obvious that we are not writing down that become the important things when we are conducting a lesson... It's not until we are doing a lesson that we realise we've missed something and we don't know how to bridge a gap because we don't have those underpinning understandings ... They are just in your head and we cannot see them. [For example] if an answer comes back from somebody [that I hadn't anticipated] then I might not know how to respond...or how to make a connection... Yours was going along the right path because you had the right questioning and it was not something we could write about because it was on the hop questioning... We're not seeing this as a must do, we're seeing it as an option so if it's hard then we'll just opt out

Ruth: [The researcher] could ["scaffold learning in the moment?"] with a class she didn't know because she knows what she's teaching, where to go and where it comes from.

Rebecca felt that more introductory professional learning about reasoning and problem solving may have been helpful to assist her understand the intervention.

Rebecca noted that seeing the model was far more powerful than reading or hearing about it.

Ruth noted that seeing a modelled lesson helps teachers understand the approach, but it does not guarantee that teachers will agree with it.

Catering for diversity

Teaching as telling

Teaching as telling

Through her attempt at enacting an observed lesson, Ruth recognised that she did not know how to respond to students thinking in the moment

Principal: [How could the intervention be improved?] More than one co-teaching session to build confidence if needed. Another watched session to “improve/tweak” practice further after initial feedback.	Reactions - The Principal recommended that the model be extended if necessary to cater for the individual learning needs of teachers
Principal: This form of modelling and coaching was very effective PL – it gave teachers support to practice new learning. When teachers go to external PL they come back full of new ideas but most rarely implement them, there is no accountability, there is no feedback ... it feels like wasted opportunities, and costly.	Reactions - The structured inquiry and accountability processes that went alongside modelling were critical to its success.
Principal: I observed deeper understanding of The Australian Curriculum; differentiation of tasks; questioning – enablers and extenders, to support and extend learners; acceptance of a range of answers and thinking; planning of better tasks; development of language through questioning.	Reactions – The Principal identified several multi-dimensional changes to teaching practice including seeing teachers taking a more conscious role in planning, differentiating tasks, understanding the Australian Curriculum and developing student language.

Table 5.6 Selected Theme Clusters with their Related formulated meanings

Themes	Sub themes	Clustered Meanings
Challenging teacher orientation towards teaching as telling		<p>The observation of student attempts to articulate mathematical reasoning in the lesson prompted C to reflect on how much she tells students what to do.</p> <p>The modelling had the desirable effect of enabling W to see an example of alternative pedagogy and how her students responded to the approach. This enabled her to recognise that more experience with the approach was needed.</p> <p>J was challenged by the pedagogical approach of allowing students to think for themselves before instruction from the teacher</p> <p>The initial modelled lesson challenged J’s pedagogical approach to teaching as she was grappling with new ideas that challenged her teacher directed view of student learning.</p> <p>K was confronted by the modelling and this led her to reflect on her approaches to teaching</p> <p>J wrestled with the idea of allowing students to come up with strategies for themselves and what that might mean for her teaching.</p> <p>J struggled with the idea of the lesson being more about students responding to the task and her lack</p>

Confronting ideas about catering for diversity

Perceived disengagement of some students

of confidence about not being in control of their responses.

W grappled with the pedagogy that was very different from her current approach. She noted difficulty in creating open tasks for other domains of mathematics.

W contended with the idea of student generated approaches rather than teacher led approaches.

C felt uncertain about allowing students to grapple with tasks where the solution was unknown. She questioned whether students needed explicit instruction in ability groupings

W articulated his desire to developing confidence with student generated approaches but was worried about letting go of telling students what they need to know.

The modelling enabled J to notice a different pedagogical approach to cater for diversity, which prompted her to reflect on how she teaches.

C felt uncertain about allowing students to grapple with tasks where the solution was unknown. She questioned whether students needed explicit instruction in ability groupings

Students seemingly off tasks were a distraction for J

K was uncomfortable with her perceived disengagement of some students

C saw a perceived low attaining student experience success when given the opportunity to engage in a problem-solving task.

By having all children in the class working on the same problem with variations, it became clear that within K's composite class there was a range of students within each grade who found the tasks challenging and there were others for whom the tasks were less challenging irrespective of grade level.

The modelled lesson enabled observing teachers to focus on how successfully the diverse range of students responded to the learning experience

Fear of student generated responses to tasks

K recognised the importance of extended thinking time which enable vulnerable children to contribute to the mathematical discussion.

K planned the lesson carefully with the support of the lesson plan exemplar. She had learned that it was possible to cater for the diverse range of learners in a lesson by providing the same task with variations. She recognised her previous approach which included working with a small group meant she was unable to support the rest of the class.

K articulated that she was challenged by the idea that some children may struggle with the problems presented in a maths lesson. However she felt that with more experience she would get better at anticipating the children that may require more assistance and be able to provide enabling and extending prompts

K recognised the importance of extended thinking time which enable vulnerable children to contribute to the mathematical discussion.

J wrestled with the idea of allowing students to come up with strategies for themselves and what that might mean for her teaching.

J struggled with the idea of the lesson being more about students responding to the task and her lack of confidence about not being in control of their responses.

W articulated his desire to developing confidence with student generated approaches but was worried about letting go of telling them what they needed to know.

It seemed marking teacher generated work was comforting for teachers as it provided accountability and a record of student work regardless of the thinking and meaning behind it.

J articulated that she did not know how to respond to student responses in her grade 3 class that were different to those that she had seen in another class.

(5) Observed pedagogies

(a) Students thinking for themselves

Rose observed important elements of a pedagogical approach that builds on student thinking is focused on facilitating students' reasoning and problem solving.

The modelling had the desirable effect of enabling Rebecca to see an example of alternative pedagogy and how her students responded to the approach by thinking for themselves. This enabled her to recognise that more experience with the approach was needed.

Rebecca noticed a different pedagogical approach that was focused on student reasoning and thinking.

(b) Posing challenging tasks

Rachel articulated a growing commitment toward a pedagogical approach, which involves planning one task with multiple entry points, which seemed easier than planning a multitude of tasks

Ruth could see the importance of choosing tasks that were challenging and meaningful to students.

Rose learned to use one problematic task that was focused on one concept with multiple entry points. This was different from her usual approach to teaching which consisted of multiple disconnected tasks.

Rose was surprised at how much more knowledge her students had from a whole class problem based approach rather than a multitude of activities.

(c) Enabling differentiation through enabling and extending prompts

Rose noticed the researcher gave enabling and extending prompts to those who needed them and scaffolded students in the moment.

Rachel noticed the researcher circulating and handing out enabling and extending prompts to those who needed it.

Rebecca noticed the researcher gave enabling prompts and extending prompts to those who needed them

(d) Providing extended thinking time

Rachel Observed the researcher provided thinking time to enable students to think for themselves to solve problems.

Ruth noticed the researcher gave the students extended thinking time

	Rebecca observed the researcher gave students extended thinking time.
(e) Expecting that students would explain and justify their thinking;	Rachel observed the researcher had the expectation that students would justify their reasoning
(f) Revoicing and prompting for further participation	Rose noticed the researcher prompted students for further participation. Rachel noticed the researcher used strategies such as revoicing
(g) Scaffolding children to explain their thinking	
(h) Modelling mathematical language	Rose noticed the researcher modelled mathematical language. Ruth observed the researcher modelling mathematical language.
(i) Connecting to prior understandings	
(j) Using student explanations	Rebecca observed the researcher selected students to explain their thinking to the class to exemplify the mathematical ideas
(k) Making students thinking visible	Rachel observed the researcher used the IWB to make student thinking visible. Rebecca observed the researcher projecting students work through a document camera for others to see. The student then explained his thinking to the class.
(l) Encouragement to persist	Rachel noticed the researcher encouraged the students to 'keep thinking'. Ruth noticed the researcher encouraging students to persist.
Knowledge and understanding of mathematics content and pedagogy	J felt she had not had support to implement the Australian Curriculum: Mathematics and therefore did not know about the proficiency strands. J struggled with the idea of the lesson being more about students responding to the task and her lack of confidence about not being in control of their responses. The modelled lesson and the pre- and post-lesson discussions prompted C to reflect on the purpose of her teaching. She recognised the importance of being

Learning through Inquiry and collaboration with colleagues

clear why she is teaching the content she is teaching.

K found the pre and post lesson discussions valuable in building her knowledge of the mathematics curriculum.

K was able to gather assessment data of her students thinking and link it to counting principles discussed in the planning meeting. She was able to see who needed enabling prompts.

K articulated that she had not focussed on the proficiency strands in the past and that she could now use those as a starting point for her practice.

K's recognised an alternative approach to teaching partitioning ten which was focussed on student understanding.

K's lesson sequence had a clear focus which enabled students to gain deeper understanding of the concept than in the past.

The intervention prompted K to seek out a recommended professional reading and read the theory, which supported the pedagogical approach. This is powerful professional learning. It also suggests K saw the possibility of transferring the pedagogical approach to other lessons.

In the post lesson survey the principal of the school identified a number of multi-dimensional changes to teaching practice including seeing teachers taking a more conscious role in planning, differentiating tasks, understanding the Australian Curriculum and developing student language.

K found the pre and post lesson discussions with colleagues valuable in building her knowledge of the mathematics curriculum.

K explained that colleague teachers could collaboratively plan one problem and teach it across their grades, prep, one two. They could discuss how they were going, talk about their questioning, run things by each other and provide support for each other. K reported that in

Modelling as a scaffold for
teacher learning

the past, teachers at RPS all worked
in isolation.

K felt that having professional
dialogue with colleagues as well as
a knowledgeable other working
alongside her enabled her to build
her mathematical knowledge for
teaching

J felt that she would need to copy
the lesson to practice the
pedagogical approach as a first step
in changing her pedagogy.

Teacher learning from
Enactment

From her enactment, Rose learned
to use one problematic task that was
focused on one concept with
multiple entry points. This was
different from her usual approach to
teaching which consisted of
multiple disconnected tasks.

Rose reflected on her enacted lesson
and decided she needed to spend
less time on tuning in and reflection
at the end.

Rose reported that she was working
towards making student thinking
visible through the use of a
document camera to enable greater
student engagement in the summary
phase of the lesson.

Through her attempt at enacting an
observed lesson, Ruth recognised
that she did not know how to
respond to students thinking in the
moment

Inquiry and accountability
processes critical to success

The Principal claimed that the
structured inquiry and
accountability processes that went
alongside modelling were critical to
its success.

Differentiation for teachers

The Principal recommended that the
model be extended if necessary to
cater for the individual learning
needs of teachers.

Engaging teachers in the
context of the professional
learning

Rebecca felt that more introductory
professional learning about
reasoning and problem solving may
have been helpful to assist her
understand the intervention.

Rose indicated that initially she was
afraid of the pedagogical approach
because it didn't make sense to her.

Experiencing the intervention

Rebecca noted that seeing the model
was far more powerful than reading
or hearing about it.

Contextualised PI

Rose felt that the PL was personalised to meet her needs as it was situated in her classroom context. She noted the importance of the professional reading to understand the theory.

The scaffolded nature of the intervention

A scaffolded approach supported Rose to feel confident with taking a leading role herself in enacting the pedagogical approach.

Themes	Research Question	Categories
1. Challenging teacher orientation towards teaching as telling	3	Challenging teacher Assumptions
2. Confronting ideas about catering for diversity	3	Challenging teacher Assumptions
3. Fear of student generated responses to tasks	3	Challenging teacher Assumptions
4. Children off task		Challenging teacher Assumptions
5. Observed pedagogies	4	Teacher learning from observing
6. Modelling as a scaffold for teacher learning	4	Teacher learning from observing
7. Creating and finding cognitively complex tasks	4	Teacher learning from deconstructing pedagogies
8. Knowledge and understanding of mathematics content and pedagogy	4	Teacher learning from deconstructing pedagogies
9. Learning through Inquiry and collaboration with colleagues	6	Teacher learning from deconstructing pedagogies
10. Inquiry and accountability processes critical to success		Reactions
11. Differentiation for teachers		Reactions to the intervention
12. Engaging teachers in the context of the professional learning		Reactions
13. Experiencing the whole package		Reactions to the intervention
14. The importance of professional reading		Reactions to the intervention

Appendix 8: Lesson Plan for Raven Primary School Grade 3

For the lesson on 26 February 2014
Modelling teacher: Louise Hodgson

Title of the Lesson: Adding numbers in your head and Footy cards

FINDING WAYS TO ADD IN YOUR HEAD

Work out how to add $298+35$ in your head
What advice would you give to someone on how
to work out answers to questions like this in their
head?

Rationale for lesson

Mental computation often involves strategies to make the process easier. The focus in this lesson is on partitioning and regrouping numbers mentally. There are also some key target numbers (such as 10, 100) and some key pairs (such as 3 and 7; 4 and 96) that students can explore.

From the Australian Curriculum

This lesson addresses the following descriptor from the AC for year 1:

Represent and solve simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts (ACMNA015)

The lesson addresses the following descriptor from AC for year 3 (year 4 is similar):

Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems (ACMNA053)

There is also potential for students to build *Understanding* of number relationships, to be more *Fluent* with the mental calculations, to find their own solution by *Problem Solving*, and to develop *Reasoning* by explaining their thinking.

Particular pedagogical considerations

The idea is that students do the tasks mentally but explain in writing how they did it. The emphasis is on students explaining their thinking. They can first explain their thinking in a pair or small group and then explain it to the class. I will ensure they have time to develop a strategy for themselves.

The learning tasks can be done in their maths journals or on mini white boards

Language like “10s family” (that is, 7 and 3, etc.) and “100s family (that is 70 and 30, etc) might be used. I will explain to the students that the focus is on finding an efficient strategy.

For the students:

You can work out efficient strategies for adding numbers in your head for yourself. You can also explain your thinking to others.

Enabling prompt(s): can be posed to students who have not been able to make progress on the learning task. The intention is that the students can complete the enabling prompt and then proceed with the learning task. Enabling prompts can involve slightly varying an aspect of the task demand, such as:

- the form of representation,
- the size of the numbers, or
- the number of steps,

If a student has success with the modified task, they can proceed with the learning task.

Work out the answer to $28 + 7$ in your head.

Work out the answer to $98 + 7$ in your head.

Work out the answer to $198 + 7$ in your head.

Extending prompt:

Some students might finish the learning task quickly. The intention is such students be posed “extending prompts” that extend their thinking on an aspect of the learning task.

Work out how to add $98 + 97 + 67$ in your head.

Work out how to add $295 + 96 + 79$ in your head.

Anticipated difficulties students may have

Students may have difficulty adding with three digit numbers

Students may have difficulty articulating strategies

Consolidating task: Most lessons propose a second task that students can complete after working on the learning task, and listening to the successful strategies. The hope is that nearly all students will be able to complete the consolidating task.

Charlotte and Harry had 295 footy cards, then I went to visit and gave them 68 more. Charlotte and Harry wanted to know how many there were altogether. What advice would you give to them on how to work out the answer in their head?

Extending prompt:

Charlotte had 98 footy cards, Harry had 97 footy cards, then I visited them and gave them 67 more. Charlotte and Harry wanted to know how many there were altogether. What advice would you give to them on how to work out the answer in their head?

Enabling Prompts:

Charlotte and Harry had 28 footy cards, then I visited and gave them 5 more. Charlotte and Harry wanted to know how many there were altogether. What advice would you give them on how to work out the answer in their head?

Charlotte and Harry had 15 footy cards, then I visited and gave them 8 more. Charlotte and Harry wanted to know how many there were altogether. What advice would you give them on how to work out the answer in their head?

Some possible student solutions:

The intention is for students to see that, for example,

$$\begin{aligned}
 &298+35 \\
 &= 298+2+33 \text{ (By partitioning the 35)} \\
 &=300+33 \\
 &= 333
 \end{aligned}$$

So it is possible to do this mentally.

Flow of the Lesson

Steps	Teacher’s Support	Points of Evaluation
<i>This column shows the major events and flow of the lesson.</i>	<i>This column shows additional moves, questions, or statements that the teacher may need to make to help students.</i>	<i>This column identifies what the teacher should look for to determine whether to proceed, and what observers should look for to determine the effectiveness of the lesson.</i>
<p>1. Introduction</p> <p><i>The Launch Phase</i></p>	<p>Let the students read the task quietly, maybe giving them time to think about what the question is asking them. If there are some students who cannot read at this level, check with them after they start working.</p> <p>Invite questions to ensure the task is clear, but do not show students how to do it.</p> <p>Set expectations for student working. Have the students work individually for at least 5 minutes on the task before working with one or more</p>	<p><i>How will we know if students understand the task?</i></p>

	<p>others (this is to give individuals time to think).</p> <p>The task has more than one pathway to solution and students can be made aware of this possibility.</p>	
<p>2. The Explore phase</p>	<p>If it looks like most students are stuck, stop them to allow a class discussion on what they have found so far, or how they are approaching the task.</p> <p>Watch what the students are doing and choose students I want to explain their thinking later. Choose a range of different strategies.</p> <p>Monitor student responses</p> <p>Select representative responses for later presentation</p> <p>Have enabling prompts ready for students who seem to be experiencing difficulty</p>	<p><i>How will we know if they are stuck?</i></p> <p><i>What strategies are they using to solve the problem?</i></p>
<p>6. Summing up <i>This section may describe how the teacher will summarize the main ideas of the lesson.</i></p>	<p>Students who have been selected to represent a range of approaches can explain their solution strategy and other insights to the class.</p> <p>Invite questions from other students, and ask them to compare student methods.</p> <p>When one student presents a solution invite other students to describe what the student has done.</p> <p>Sequence student responses and connect the students' strategies with the formal</p>	<p><i>What will indicate that students are benefiting from the discussion?</i></p>

	processes that were the intention of the task in the first place.	
--	---	--

This cycle of launch, explore, summarise will be repeated during the lesson. It may occur more than twice depending on how the students respond to the task

Appendix 9: Lesson Plan for Raven Primary School Grade Prep/1

For the lesson on 26 March 2014

Co-taught by: Louise and Rose

Title of the Lesson: Ten Cheeky Monkeys Jumping on the bed

Ten Cheeky Monkeys Jumping on the bed

Ten cheeky monkeys were jumping on the bed. Some fell out and bumped their heads. Please draw me a picture to show how many fell out and how many were left in the bed.

Rationale for lesson

To conceptualise a number as being made up of two or more parts is the most important relationship that can be developed about numbers.

Understanding part-part-whole relationships will enhance children's flexibility, enabling them to represent problems in different ways, so they can choose the most helpful.

Since 10 plays such a large role in our numeration system and because two fives make ten, it is very useful to develop the relationships for the numbers 1 to 10 to the important factors of 5 and 10.

From the Australian Curriculum

This lesson addresses the following descriptors from the AC for prep:

- Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point ([ACMNA001](#))
- Connect number names, numerals and quantities, including zero, initially up to 10 and then beyond ([ACMNA002](#))
- Subitise small collections of objects ([ACMNA003](#))
- Compare, order and make correspondences between collections, initially to 20, and explain reasoning ([ACMNA289](#))
- Represent practical situations to model addition and sharing ([ACMNA004](#))

The lesson addresses the following descriptor from AC for grade one:

- Represent and solve simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts ([ACMNA015](#))

The lesson partially addresses the following descriptor from AC for year 3 (year 4 is similar):

- Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems ([ACMNA053](#))

There is also potential for students to build *Understanding* of number relationships, to be more *Fluent* with the mental calculations, to find their own solution by *Problem Solving*, and to develop *Reasoning* by explaining their thinking.

I will endeavour to facilitate students reasoning by encouraging students to solve the problem in more than one way, allowing students to develop their own approaches, encouraging collaboration between students, and using students' explanations as the prompt to explaining the mathematical intent of the task and lesson

Particular pedagogical considerations

The emphasis is on students explaining their thinking. It will be important to encourage children to attend to the mathematical reason for the combination. I might ask questions along the lines of "How did you decide on 7 and 3"? or why did you choose 7 and 3 rather than 6 and 3 will help them to explain their thinking.

The learning task can be done on paper or on mini white boards. I will encourage children to draw quick representations (for example, heads) and use grey lead pencils. Attention to the mathematics is more important than the presentation of the drawing.

Possible questions I might ask the students

How did you know?

Tell me about your thinking.

How did you decide?

Can you convince us?

Why did you choose ...?

How many monkeys are there altogether?

How many are in bed? How many have fallen on the floor?

How could you find out?

How are you keeping track of the ways that you find?

Tell me about your thinking.

Is there a pattern?

Are there still ten?

Enabling prompt(s): can be posed to students who have not been able to make progress on the learning task. The intention is that the students can complete the enabling prompt and then proceed with the learning task. Enabling prompts can involve slightly varying an aspect of the task demand, such as:

- the form of representation,
- the size of the numbers, or
- the number of steps,

If a student has success with the modified task, they can proceed with the learning task.

Please get me five monkeys

Five monkeys went to bed, some fell out. Draw a picture to show how many monkeys were in the bed and how many fell out.

Extending prompt:

Some students might finish the learning task quickly. The intention is such students be posed "extending prompts" that extend their thinking on an aspect of the learning task.

Can you find all the ways for the monkeys to be in and out of bed? Is there a pattern?

What if there were 11 monkeys? What combinations could there be?

Anticipated difficulties students may have

Students may have difficulty with any or all of the 5 counting principles

Students may have difficulty thinking of ten in parts

Students may have difficulty providing a mathematical justification for their drawing (part-part whole, adding to 10) and using mathematical language (makes ten, fair, pattern)

Introductory task

Breaking cubes

Have a stick of ten Unifix cubes and explain to children I'm going to break my stick into two parts behind my back (each hand holding one part). Allow them to see only one of the two parts while the other part remains behind me. Ask them to figure out how many cubes are behind my back without looking at them. Discuss all the combinations that make ten

Flow of the Lesson

Steps	Teacher's Support	Points of Evaluation
<i>This column shows the major events and flow of the lesson.</i>	<i>This column shows additional moves, questions, or statements that the teacher may need to make to help students.</i>	<i>This column identifies what the teacher should look for to determine whether to proceed, and what observers should look for to determine the effectiveness of the lesson.</i>
1. Introduction <i>The Launch Phase (Rose)</i>	Role play <i>Ten Little Monkeys</i> http://www.bing.com/videos/search?q=10+cheeky+monkeys+jumping+on+the+bed&FORM=HDRSC3#view=detail&mid=4EB6A2BD475EEAA40BAB4EB6A2BD475EEAA40BAB	<i>How will we know if students understand the task?</i> Were the children able to: <ul style="list-style-type: none"> • Partition 10 monkeys in their minds? • matches numerals to the number of objects • estimate the number of objects in a group and counts to check
2. The Explore phase	Pose problem: <i>Imagine that ten monkeys went to bed. Some fell out. Please draw me a picture to show how many stayed in bed and how many fell out</i> If it looks like most students are stuck, stop them to allow a class discussion on what they have found so far, or how they are approaching the task.	<i>How will we know if they are stuck?</i> Were the children able to: <ul style="list-style-type: none"> • demonstrate one-to-one correspondence when counting • read and record numbers • partition objects • matches numerals to the number of objects • estimate the number of objects in a group and counts to check <i>What strategies are they using to solve the problem?</i>

	<p>Watch what the students are doing and choose students I want to explain their thinking later. Choose a range of different strategies.</p> <p>Monitor student responses</p> <p>Select representative responses for later presentation</p> <p>Have enabling prompts ready for students who seem to be experiencing difficulty</p>	<p>Can anyone find more than one way of representing ten monkeys?</p> <p>If so, stop the class and draw attention to it.</p>
<p>6. Summing up (Louise) <i>This section may describe how the teacher will summarize the main ideas of the lesson.</i></p>	<p>Students who have been selected to represent a range of approaches can explain their solution strategy and other insights to the class.</p> <p>When one student presents a solution invite other students to describe what the student has done.</p> <p>Sequence student responses and connect the students' strategies with the idea that ten can be made up of different parts</p> <p>Draw attention to the different combinations drawn. For example,</p> <p><i>Harry drew 5 monkeys in the bed and 5 monkeys out of bed, did anyone draw different numbers of monkeys in and out of bed?</i> <i>What other combinations are there?</i></p>	<p><i>What will indicate that students are benefiting from the discussion?</i></p>

This cycle of launch, explore, summarise will be repeated during the lesson. It may occur more than twice depending on how the students respond to the task

Appendix 10: Lesson Plan for Raven Primary School Grade Prep/1

For the lesson on Wednesday 9 April 2014

Teacher: Rose

Lesson prepared by: Rose

Title of the Lesson: Ten Little Fish

Ten Little Fish

Ten little fish were swimming around a crate. Some went in the crate. Draw a picture to show how many fish were in the crate and how many fish were out of the crate.

Rationale for lesson

To conceptualise a number as being made up of two or more parts is the most important relationship that can be developed about numbers.

Understanding part-part-whole relationships will enhance children's flexibility, enabling them to represent problems in different ways, so they can choose the most helpful.

Since 10 plays such a large role in our numeration system and because two fives make ten, it is very useful to develop the relationships for the numbers 1 to 10 to the important factors of 5 and 10.

From the Australian Curriculum

This lesson addresses the following descriptors from the AC for prep:

- Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point ([ACMNA001](#))
- Connect number names, numerals and quantities, including zero, initially up to 10 and then beyond ([ACMNA002](#))
- Subitise small collections of objects ([ACMNA003](#))
- Compare, order and make correspondences between collections, initially to 20, and explain reasoning ([ACMNA289](#))
- Represent practical situations to model addition and sharing ([ACMNA004](#))

The lesson addresses the following descriptor from AC for grade one:

- Represent and solve simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts ([ACMNA015](#))

The lesson partially addresses the following descriptor from AC for year 3 (year 4 is similar):

- Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems (ACMNA053)

There is also potential for students to build *Understanding* of number relationships, to be more *Fluent* with the mental calculations, to find their own solution by *Problem Solving*, and to develop *Reasoning* by explaining their thinking.

I will endeavour to facilitate students reasoning by encouraging students to solve the problem in more than one way, allowing students to develop their own approaches, encouraging collaboration between students, and using students' explanations as the prompt to explaining the mathematical intent of the task and lesson

Particular pedagogical considerations

The emphasis is on students explaining their thinking. It will be important to encourage children to attend to the mathematical reason for the combination. I might ask questions along the lines of "How did you decide on 7 and 3"? or why did you choose 7 and 3 rather than 6 and 3 will help them to explain their thinking.

The learning task can be done on paper or on mini white boards. I will encourage children to draw quick representations (for example, heads) and use grey lead pencils. Attention to the mathematics is more important than the presentation of the drawing.

Possible questions I might ask the students

How did you know?

Tell me about your thinking.

How did you decide?

Can you convince us?

Why did you choose ...?

How many fish are there altogether?

How many are in bed? How many have fallen on the floor?

How could you find out?

How are you keeping track of the ways that you find?

Tell me about your thinking.

Is there a pattern?

Are there still ten?

Enabling prompt(s): can be posed to students who have not been able to make progress on the learning task. The intention is that the students can complete the enabling prompt and then proceed with the learning task. Enabling prompts can involve slightly varying an aspect of the task demand, such as:

- the form of representation,
- the size of the numbers, or
- the number of steps,

If a student has success with the modified task, they can proceed with the learning task.

Please get me five fish

Five fish went to bed, some fell out. Draw a picture to show how many fish were in the bed and how many fell out.

Extending prompt:

Some students might finish the learning task quickly. The intention is such students be posed "extending prompts" that extend their thinking on an aspect of the learning task.

Can you find all the ways for the fish to be in and out of bed? Is there a pattern?

What if there were 11 fish? What combinations could there be?

Anticipated difficulties students may have

Students may have difficulty with any or all of the 5 counting principles

Students may have difficulty thinking of ten in parts

Students may have difficulty providing a mathematical justification for their drawing (part-part whole, adding to 10) and using mathematical language (makes ten, fair, pattern)

Introductory task

Sing 1, 2, 3, 4, 5 once I caught a fish alive.

Flow of the Lesson

Steps	Teacher's Support	Points of Evaluation
<i>This column shows the major events and flow of the lesson.</i>	<i>This column shows additional moves, questions, or statements that the teacher may need to make to help students.</i>	<i>This column identifies what the teacher should look for to determine whether to proceed, and what observers should look for to determine the effectiveness of the lesson.</i>
1. Introduction <i>The Launch Phase</i>	Role play <i>Ten Little Fish</i> 1. Read story of Ten Little Fish by Audrey Wood and Bruce Wood. 2. Reenact story with fish puppets and crate. Reducing the number and showing 10,9,8...as we put fish into the crate.	<i>How will we know if students understand the task?</i> Were the children able to: <ul style="list-style-type: none"> • Partition 10 fish in their minds? • matches numerals to the number of objects • estimate the number of objects in a group and counts to check
2. The Explore phase	Pose problem: <i>Imagine that ten fish were swimming around the crate. Some went into the crate. Draw me a picture to show how many fish were in the crate and how many were out of the crate.</i> Discuss how to draw simply and use grey leads. If it looks like most students are stuck, stop them to allow a class discussion on what they have found so far, or how they are approaching the task.	<i>How will we know if they are stuck?</i> Were the children able to: <ul style="list-style-type: none"> • demonstrate one-to-one correspondence when counting • read and record numbers • partition objects • matches numerals to the number of objects • estimate the number of objects in a group and counts to check <i>What strategies are they using to solve the problem?</i> Can anyone find more than one way of representing ten fish?

	<p>Watch what the students are doing and choose students I want to explain their thinking later. Choose a range of different strategies.</p> <p>Monitor student responses</p> <p>Select representative responses for later presentation</p> <p>Have enabling prompts ready for students who seem to be experiencing difficulty</p>	<p>If so, stop the class and draw attention to it.</p>
<p>6. Summing up <i>This section may describe how the teacher will summarize the main ideas of the lesson.</i></p>	<p>Students who have been selected to represent a range of approaches can explain their solution strategy and other insights to the class.</p> <p>When one student presents a solution invite other students to describe what the student has done.</p> <p>Sequence student responses and connect the students' strategies with the idea that ten can be made up of different parts</p> <p>Draw attention to the different combinations drawn. For example,</p> <p><i>Harry drew 5 fish in the crate and 5 fish out of the crate, did anyone draw different numbers of fish in and out of the crate?</i> <i>What other combinations are there?</i></p> <p>As children give a combination write up the equation and model with the fish and the crate.</p>	<p><i>What will indicate that students are benefiting from the discussion?</i></p>

--	--	--

This cycle of launch, explore, summarise will be repeated during the lesson. It may occur more than twice depending on how the students respond to the task

Appendix 11: Themes: Heron Primary School

Table 6.1 Selected examples of significant statements and their meanings Stage 1 Heron Primary School (HPS)

Selected samples of significant statements from post lesson discussions and surveys	Formulated meanings
<p>Heather: You tuned them [the children] into thinking, you gave the expectation that they were going to justify... You asked a question and gave them wait time and named up their thinking... You did not accept an answer without the expectation that you wanted to probe them just a little bit more. You rephrased their thinking. "What did you notice about what so and so said". Even right to the very end it was still very obvious that you expected them to reason [mathematically]. You set up the expectation to find more ways [to make 10] and expected them to respond.</p>	<p>Heather noticed several teacher actions to facilitate student reasoning including asking students to restate another's reasoning, revoicing, paraphrasing, waiting, having high expectations, probing. This suggests the narrow focus on teacher actions to facilitate reasoning was effective.</p>
<p>Heather: That whole notion of within 10 is an interesting thing e.g. understanding the 6 and 4 are contained within 10. What would you have done otherwise to get the children to notice a pattern?</p>	<p>Heather was interested in the mathematical idea, she posed a question to the researcher to clarify her ideas</p>
<p>Jackie: [When] you gave the instruction of doing the drawing of the ten teddies in [and out of] the bed ...I wondered if I would have done an example of what it would look like... When we talk about explicit teaching, we [think] how do we set it up so they know... Then I thought as soon as I draw 8 and 2, every single one of them will be drawing 8 and 2 because it's there [on the white board] and I would not be allowing any of this thinking.</p>	<p>Jackie reflected on the usual approach of explicit teaching and realised that telling children how to think narrows children's thinking.</p>
<p>Jackie: I liked the fact that you had a [disclaimer] earlier about [our observation] of behaviour [in the lesson]. I was thinking it might be easy to be distracted by the students not on task. But the focus as observing teachers is for us to say what it is we're focusing on, disregarding [behaviour]. Our focus was on the reasoning on the task and the enablers...</p>	<p>Jackie described the importance of teachers not getting distracted by student's behaviour</p>

Selected examples of significant statements and their meanings 18/03/14 Stage 2 Heron Primary School

Selected samples of significant statements from post lesson discussions and surveys

Stage 2 pre-lesson meeting 18 March 2014

Heather: The opportunity to chat ... to reflect [on our own] and a bit of distance [have been] an important part of our development. We talked about what might be the next step for some of those children that haven't quite got the concept... and about times for direct teaching outside the problem solving.

Holly: They [students] are very reliant on concrete [materials]. Even though they know some strategies, they keep reverting to counting by ones.

I probably haven't used that word [difference] before.

Principal: Maths is such a thing where the divide between the ones who get it quick and the ones who don't... and if the ones who get it quick are always giving the answers, I presume the ones who don't just go "oh what the ..."

Principal: [Using icy pole sticks with student's names in a container as a strategy to get all kids thinking in case they get randomly picked to answer the question] is going to get them trying at least rather than thinking well I won't even have a go coz I won't put my hand up so I won't have to give the answer. Oooh, hang on, my icy pole stick might get pulled out so I'd better try and put some thought into this.

Stage 2 post lesson meeting: 18/03/14

Holly: I noticed how many times you repeated the mathematical language...e.g. difference. [Students] were getting it in their heads that's what they were learning Summing up around difference.

Holly: ...when you put them [icy pole sticks] back in the tub ... it was a signal for those children not to stop thinking.

Holly: You got a child to repeat back the question so that you could clarify what it was that they had heard...

Formulated meanings

Heather claimed that time to talk and reflect with colleagues between stages was very important for her learning. The modelling raised questions for her about direct teaching, however she saw the possibility of teaching through problem solving.

Holly claimed her students were reliant on materials and counted by ones. After reading through the lesson plan she also claimed she had not used the word difference with her grade 2/3's

The Principal's comment reflected his view that low attaining children get left behind.

The Principal anticipated the benefits of using icy pole sticks to increase student engagement during classroom questioning.

Holly noticed the researcher modelled mathematical language.

Holly noticed the researcher had the expectation that all children would think

Holly observed the researcher asking Children to repeat back what they had heard

Heather: I noticed the way you didn't give up on [the students]. You kept expecting that they would keep going and you acknowledged for them that it was going to be difficult ... you kept encouraging them and then towards the end you gave them the cliff-hanger [you gave them the message] we are going to come back ... this is important.

Henrietta: [Children were] learning from each other. It was okay that they were all working on different numbers. Some were working with larger numbers, others were working with smaller numbers. Thinking most important. No hands up rule

Heather: [When the teacher is really clear about the learning intention] it makes you disregard the other comments and draw back to what it is we are working on. I can see that

Heather: Using students for modelling their work was a key and using the language

Holly: They found it really hard to explain what they were thinking

Heather: It really highlighted the value of the talk. I looked at one child and I thought that child was counting by tens but when I asked that child to explain their thinking I discovered they were using the ten blocks as a [representation]for one. If we hadn't had that talk time, where they were explaining what they were doing [I would not have picked that up].

Opportunity for "talk" along with goal setting for implementing elements into our practice seems in my humble opinion to be the thing that just might make a difference to long terms change in practice. If [talk] is totally reliant on the informal discussion in the staffroom it runs the danger of being just a conversation, with no accountability or sense of expectation that it might be reflected upon and action enacted.

Heather: I think it also takes someone to be there as a 'critical friend' with a special

to ensure they understood what the problem was asking them to do.

Heather noticed that the researcher had high expectations of the children and encouraged them to persist.

Henrietta noticed that the students were learning from each other even though they had enabling and extending prompts, this was because they all had the same learning goal.

Heather could see that being clear about the learning intention helps the teacher to remain focused on the mathematical intent in the summary phase of the lesson.

Heather could see that by displaying and talking about their work children could learn from each other.

Holly highlighted the difficulties children had in reasoning mathematically.

Heather saw the importance of exploring children's thinking and getting them to talk about it

Heather asserted the collaboration including opportunities for discussion and goal setting were powerful components of the model which enabled accountability and led to changes in teacher practice.

Heather claimed that the knowledgeable other was important to be part of the

interest/knowledge in the subject so that the knots in the thinking can be heard and professional learning put in place that would help tease out or challenge the ideas.

intervention to support teachers to deconstruct ideas.

Selected examples of significant statements and their meanings, Stage 3

Selected samples of significant statements from post lesson discussions and surveys

Formulated meanings

Stage 3 pre- lesson meeting

Heather: I've realised you can choose tasks from different books, but if you can't unpack the math's that's in underneath the task, then you're not developing the children's math's skills, you're just choosing tasks.

Heather recognised the importance of planning tasks that would develop children's learning in mathematics

Heather: It feels a bit like the rug has been pulled out from under my feet...I had a way of working and a way of choosing tasks, but now it's turned my head around and I'm really very conscious of how I am sequencing the maths...I'm really thinking about why it is I am choosing [tasks]. For example, I might have had some ideas of doing numbers to ten [with prep/ones], but now I'm looking at the proficiencies and [questioning] what is the foundation that I'm building the maths on? What does this really mean and what does this look like in my classroom?

Heather articulated that the modelling prompted her to reflect on her choice of tasks and think about what it means for her future classroom practice.

Heather: Starting with the problem opens up opportunities for children to demonstrate an amazing variety of thinking and ways of moving towards understanding.

Heather could see that 'not telling', provides children with opportunities to reason mathematically with a variety of responses.

Hannah: Giving them [students] the opportunity to unpack the question and understand what is being asked of them before beginning is a critical step

Hannah noted the important step of making sure children are clear about what the problem is asking them to do before sending them off to work on it.

Hannah: You used student explanations as a way of introducing concepts/tools

Hannah noticed student explanations were used as a way of introducing concepts.

Heather: I was really conscious of you [modelling] the language and then they[children] started using that language that you were modelling and you ...stretched their thinking

Heather noticed the researcher modelled mathematical language which the children began to use.

Heather: [I noticed] the importance of time [for the students] to grapple with the ideas

Heather could see that students need time to work on challenging problems

<p>Jackie: Why did you stop yourself from writing that number sentence? [Jackie noticed] All those critical decisions that you are making You made a decision about where not to go because it was best to go somewhere else, none of those things are in the plan.</p> <p>Hannah: [I have recognised] the importance of our discussions in planning for the tasks.</p> <p>Heather: And all the toing and froing about the task and the talk that we have had [and thinking] “Is this task really going to do what we intend.”</p> <p>Heather: We needed time to talk and sort out the developmental stages behind the concept of comparison so that we knew what to [plan] for...[We asked ourselves, is this task] broad enough to cater for all the developmental stages that are there.</p> <p>Heather: Now we have 50 problems, because all it is changing the numbers and changing the context. If we’ve got the language, we don’t have to dream up [new ideas].</p> <p>Jackie: [This intervention highlights the importance of] teacher pedagogy: developing questions and refining tasks.</p> <p>Heather: [Having access to a knowledgeable other] gives you the confidence to start to ask some questions around helping to sort out say maths...you’re confident that the person has more experience in that area and can share from their experience base.</p> <p>Stage 3 post lesson brief</p> <p>Hannah: The link between same amount and equal is quite an interesting concept for (them)</p>	<p>Jackie noticed important teacher moves to scaffold learning on the fly which are not part of the lesson plan.</p> <p>Hannah recognised the importance of the discussions in planning for learning.</p> <p>Heather mentioned the back and forth discussions in the planning of the stage 3 task including the focus on the question for the children to explore the concept of difference.</p> <p>Heather appreciated the time with the researcher and her colleagues to discuss and refine understanding of the concept of difference before planning a suitable task for the children with enabling and extending prompts.</p> <p>Heather recognised a task could be altered if the context and the numbers were changed.</p> <p>Jackie noted the intervention highlighted the importance of teacher pedagogy including planning and refining tasks and developing questions to ask students.</p> <p>Heather noted that having access to a knowledgeable other supported her to begin to ask questions about mathematics content and pedagogy.</p>
--	--

Table Selected examples of significant statements and their meanings Stage 4

Selected samples of significant statements from post lesson discussions and surveys	Formulated meanings
Heather: The use of the lesson plan structure was extremely helpful for being clear about the learning intention and flow of the lesson	Heather indicated the lesson plan was helpful in providing clarity for the goals and flow of the lesson.

Heather: Sometimes I don't see what they're [children] struggling with until I get the chance to see them in some problem-solving task.

Holly: Having a plan like this helps us be more focussed on the maths we are trying to draw out.

Heather: Some of the power for me was looking at the children problem solving and thinking so now where am I going, whereas I might have done it back to front in the past.

Stage 4 post lesson discussion

Heather: I think "convince us" is really powerful because [I] saw some really great wait time and then finally some thinking.

Heather: When you asked someone from the group to [repeat someone else's reasoning], they found that hard.

Hannah: Planning in this way, you know what you're looking for

Henrietta: [in the past I] would have just taken a group that was ready for the idea and others would have been doing other concepts.

Hannah: The enablers and extenders mean you can cater for a greater range of children.

Hannah: I have noticed that Holly and Heather are doing a lot more work on the notion of difference. We have really picked up on that concept and about using the different vocab. We shared the concept of difference with the Primary team and what that means

Heather indicated she had difficulty in assessing children unless she saw them working on a problem solving task.

Holly indicated that planning the lesson in stage 3 helped her to focus on the mathematics we were planning to co-teach.

The power of seeing students solving problem prompted Heather to reflect upon her assessment practices.

Heather observed the researcher asking student's to justify their answers and consequently she saw students spending time reflecting on their learning.

Heather observed students had difficulty repeating someone else's reasoning.

Hannah indicated that the detailed plan for effective in having clarity about what to look for in the lesson.

Henrietta indicated that in the past we would only have introduced the concept of difference to a small group of children that she perceived were able.

Hannah indicated that by using enabling and extending prompts, it is possible to cater for all students in one class.

In cycle 4, Hannah indicated that she had observed that her colleagues were now focussing on the concept of comparison and difference with their classes. She noted the team shared the ideas with the grade 3-6 team at her school.

Selected examples of significant statements and their meanings Post Intervention Interviews and surveys

Selected samples of significant statements from post lesson interview

Holly: I have been more purposeful with planning and writing questions and [thinking] about what the maths is about.

Formulated Meanings

Holly indicated that she had become more purposeful with her planning.

Henrietta: I am using children a little more to explain their thinking to the group

Heather: Designing the problem-solving tasks has enabled me to be clearer about the maths I'm trying to focus on.

Heather: [The modelling] affirmed [it's okay to allow] the kids to struggle a little and trust that it is okay to do that.

Heather: The most challenging thing is ... getting around everybody. That notion of being able to capture what one child is doing in one situation but also have your ear on what else is happening. In the past, I would not have had the whole group working on the one task, it was a challenge to think about how that would happen in my own class room.

Heather: My biggest challenge in the [summary phase] is how to get my children to focus on the maths, because often they are not tuned into to another child sharing if they have been through the whole process of working on problems.

Heather: Teaching in this way still feels a bit uncomfortable for me but it is exciting to watch how some children go about solving the problems.

Heather: The pre-lesson and post lesson meetings teased out some of the maths. Maths is not the subject where I feel most comfortable. It gave me a great opportunity to sort out the maths that we were focussing on.

Heather: Co-planning meant you had your colleagues to really sort out the problem and make sure it was going to do what you were aiming for.

Heather: Seeing someone model makes [the lesson] come alive... especially when you see it warts and all. Seeing the one experience together [with colleagues] is a great growth point because you are talking about the same experience.

Heather: I moved from being uncertain and wary to really seeing the benefits through the whole process at the end.

Henrietta, who was about to retire indicated that she was now getting students to explain their thinking.

Heather indicated the planning stage was important for her to become clearer about the mathematics she was focusing on.

Heather indicated that the modelling affirmed it was okay to allow students to grapple with problems.

Heather indicated the most challenging thing for her was to get around to everyone in a lesson where the whole class was working on a problem. This was different to how she normally worked.

Heather indicated the summary phase of the lesson was a challenge for her as it is difficult to get children focused on others thinking. This suggests teacher actions leading to this phase may need to be made clearer to teachers.

In her post lesson survey, Heather indicated that she still felt a little uncomfortable about teaching through problem solving but noted it was exciting to watch the children solving the problems by thinking for themselves.

Heather indicated that the pre and post lesson meetings were helpful for her to unpack the mathematical ideas that were the focus of the modelled lessons.

Working together enabled Heather to experience several heads working together

Heather saw watching the modelled lessons as a catalyst for growth with her colleagues and they could talk about the lesson and learn from that

Shift in thinking

Heather: The lesson plan structure was extremely helpful for being clear about the intention and flow of the lesson. Perhaps some mapping prior to the lesson with regards to levels of sophistication might be useful for fine tuning observations and making decisions about the maths during the [summary] phase.

Hannah: As part of the leadership team, we decided and I was excited about the intervention.

Hannah: I learned that during the lesson it is vital to allow the kids to explore the maths in their own way.

Hannah: [The planning meeting] and the questioning [by the researcher] [prompted us] to look at the [tasks] we are setting... [I asked myself] Are we setting tasks that align with goals for a lesson?

Hannah: When we were doing the planning [Stage 3] we had our curriculum there, we thought this is what we want the kids to learn ... but when we set the task, [we realised] a word could change the meaning of the whole task... we had to keep referring to our outcome to get those words correct. That challenged us.

Hannah: By being explicit in asking us to observe [teacher actions to facilitate] student reasoning it narrowed the focus for observing teachers meaning you could take it back to your own practice.

Hannah: [In my role as Assistant Principal] I set myself a goal of being able to support teachers to reflect on their goals for the lesson "what is the purpose of this lesson? have you thought of?"

Hannah: It was difficult being filmed but it was very beneficial... I watched it to begin with, then I [asked myself] what questions am I asking the students? Is my teaching effecting the learning? I realised I missed this question, I cut that off too soon, I did that well... It was beneficial being able to take that back into the classroom

Hannah: [Focusing the observation] on reasoning... having that one thing to look at

Heather's comment indicates she would have liked the opportunity to anticipate student responses to tasks in the pre lesson meeting.

As an Assistant Principal, Hannah and her leadership team decided to implement the intervention at her school and she was happy to do so.

Hannah learned the importance of students thinking for themselves.

The planning meeting with her colleagues and the researcher prompted Hannah to reflect on choice of tasks that align to the goals of the lesson.

Task focussed on comparison. Deep thinking about the goal of the lesson

Teachers recognised they needed to be clear about aligning the goals of the lesson with the task.

Hannah recognised the narrow focus of the observation on teacher actions to facilitate students reasoning supported teachers to notice these actions and in turn implement them in their own practice.

The intervention prompted Hannah to support colleague teachers to be clear about their lesson goals.

Hannah indicated that the focus of the observation on teacher actions to facilitate

made it much more [narrow] so that we didn't worry about or notice all the other things. Having that sole focus made it beneficial.

Hannah: The post-lesson meetings were most beneficial [in stages 1 and 2] ... the conversations that were happening in that room and beyond that day. For example, with the focus on reasoning, having that opportunity after the lesson to de-brief and think about how we could build on it. After that we would just have conversations in the corridors...this was a critical phase.

The planning stage [in stages 3 and 4] was probably more beneficial for me personally. As the [enacting] teacher having that opportunity to give myself some critical feedback... knowing exactly the outcomes and exactly the rationale.

Hannah: Some people changed practice within four stages and for some it would need longer, it really depends on the person and what it is you are trying to change.

Hannah: Initially some teachers were daunted by having an extra person in the room when they are usually just in the classroom by themselves with all the kids.

Hannah: The biggest change in our practice at school has been relying on each other to share the ideas and talk about how we can make things better. This has really opened [collaboration between colleagues]. We have adopted the planning, the lesson structure, the enablers, the extenders and so on.

Principal: I have been looking for ways to get teachers to be in each other's classrooms. With the journey, we've been on with both literacy and numeracy we have been trialling lots of new ways of teaching and specific strategies and they meet regularly but the one factor they were missing was the opportunity to be in each other's classrooms.

Principal: There was the trust factor... trepidation [from teachers] around how am I going to go, I'm not sure I feel comfortable

student reasoning supported teachers to narrow their focus and exclude unimportant things.

Hannah indicated that the post lesson conversations in stages 1 and 2 were critical for teacher learning.

Hannah indicated that planning and enacting a lesson were most beneficial for her. She knew exactly what the goals of the lesson were and the rationale behind it and she was able to give herself some critical feedback after watch the video of herself.

This indicates teachers are on a continuum of learning and one size does not fit all.

Hannah indicated that initially some teachers were uncertain of having the researcher into their classrooms.

Hannah pointed out that collaboration between colleagues has been the biggest change at her school since the intervention. Teachers are now relying on each other to plan for improvements to practice.

The Principal (HPS) indicated that the intervention supported the move towards teachers observing each other's classroom practice.

The Principal (HPS) revealed that teachers felt untrusting of the process in the initial stages of the intervention.

being watched, I'm not sure I feel comfortable talking into audio recorders

Principal: Teachers learned explicitness around knowing exactly what your most important purpose is in the lesson...

Principal: The second biggest thing was the importance of letting kids swim a little bit, sink a little bit – not to rescue.

Principal: I heard them talk lots about the enablers and the extenders...because we all battle with differentiation. I have seen them using them in classrooms [since the intervention]. It was a simple way of thinking I'll stick with my one task, have my enablers, have my extension rather than thinking when I go in that lesson I've got such a different thing happening for the lower ones and such a different thing happening for the upper ones.

Principal: I personally think you can't really spend a lot more time on it [the intervention] because of the cost. I think the model [stages of the intervention] is right. What we would do is pick something that we're really struggling with or something new that we are really challenged by and that would hook them [the teachers] in.

Principal: Their [teachers] Achilles heel was that they didn't know [the researcher]... At each stage the tenseness eased.

Principal: This [intervention] can bring a greater level of accountability planning after school. You can't fluff something up if your colleagues are in there supporting you do it.

Principal: We've got a group of committed teachers who had a level of trepidation at the start ...and were challenged...they have all grown through it and come out the other side as opposed to something starting well and dropping off. It [the model] has grown legs in each step along the journey... and all the snippets that they've learned along the way, even just rejigging their own practice around how they differentiate which wasn't really the primary focus.

The Principal indicated that one of the most important things teachers learned was to be clear about the learning goals of the lesson.

The Principal (HPS) indicated that teachers learned the importance of allowing the students to struggle and think for themselves.

The Principal signified that he had seen teachers both talking about and using enabling and extending prompts since the intervention, which was a change from having different groups of students doing different tasks in lessons.

The Principal highlighted the cost of the intervention but thought that the model of the intervention was about right in terms of changes in practice. He suggested that teachers could come up with the problem of practice themselves in order to get buy in.

The principal highlighted issues with trust which eased as the intervention progressed.

The principal indicated that the intervention brought a greater level of accountability to planning meetings.

The Principal of HPS indicated that although teachers were reluctant to begin with they all made shifts in their teaching practice including changing the way they differentiate practice.

Principal: I think [modelling] is critical because it's where we get to the coalface... It is teaching... and then going and talking about teaching as opposed to other forms of PL that are just talking about teaching and may never actually make it back to the classroom.	The Principal (HPS) summed up the success of the intervention at his school by implying that modelling and the collaborative discourse associated with it is a most effective form of professional learning.
--	--

Table Selected Theme Clusters with their Related formulated meanings

Themes	Illustrative examples of significant teacher statements	Clustered Meanings
Challenging teacher orientation towards teaching as telling	Jackie: [When] you gave the instruction of doing the drawing of the ten teddies in [and out of] the bed ...I wondered if I would have done an example of what it would look like... When we talk about explicit teaching, we [think] how do we set it up so they know... Then I thought as soon as I draw 8 and 2, every single one of them will be drawing 8 and 2 because it's there [on the white board] and I would not be allowing any of this thinking.	Jackie (HPS) reflected on the usual approach of explicit teaching and realised that telling children how to think narrows children's thinking.
Confronting ideas about catering for diversity	Principal: Maths is such a thing where the divide between the ones who get it quick and the ones who don't... and if the ones who get it quick are always giving the answers, I presume the ones who don't just go "oh what the ..." Principal: [Using icy pole sticks with student's names in a container as a strategy to get all kids thinking in case they get randomly picked to answer the question] is going to get them trying at least rather than thinking well I won't even have a go coz I won't put my hand up so I won't have to give the answer. Oooh, hang on, my icy pole stick might get pulled out so I'd better try and put some thought into this. Henrietta: [in the past I] would have just taken a group that was ready for the idea and others would have been doing other concepts.	The Principal's comment reflected his view that low attaining children get left behind. The Principal (HPS) anticipated the benefits of using icy pole sticks to increase student engagement during classroom questioning. Henrietta (HPS) indicated that in the past she would only have taught the concept of difference to a small group of children that she perceived were able whilst the rest of the class did other unrelated tasks.

	<p>Hannah: The enablers and extenders mean you can cater for a greater range of children.</p>	<p>Hannah indicated that by using enabling and extending prompts, it is possible to cater for all students in one class.</p>
	<p>Heather: The most challenging thing is ... getting around everybody. That notion of being able to capture what one child is doing in one situation but also have your ear on what else is happening. In the past, I would not have had the whole group working on the one task, it was a challenge to think about how that would happen in my own class room.</p>	<p>Heather indicated the most challenging thing for her was to get around to everyone in a lesson where the whole class was working on a problem. This was different to how she normally worked.</p>
	<p>Principal: I heard [teachers] talk lots about the enablers and the extenders...because we all battle with differentiation. I have seen them using them in classrooms [since the intervention]. It was a simple way of thinking I'll stick with my one task, have my enablers, have my extension rather than thinking when I go in that lesson I've got such a different thing happening for the lower ones and such a different thing happening for the upper ones.</p>	<p>The Principal (HPS) signified that he had seen teachers both talking about and using enabling and extending prompts since the intervention, which was a change from having different groups of students doing different tasks in each lesson.</p>
<p>Teacher learning through collaboration</p>	<p>Heather: The opportunity to chat ... to reflect [on our own] and a bit of distance [have been] an important part of our development. We talked about what might be the next step for some of those children that haven't quite got the concept... and about times for direct teaching outside the problem solving.</p>	<p>Heather claimed that time to talk and reflect with colleagues between stages was very important for her learning. The modelling raised questions for her about direct teaching, however she saw the possibility of teaching through problem solving.</p>
<p><i>Talk</i></p>		
	<p>Hannah: The post-lesson meetings were most beneficial [in stages 1 and 2] ... the conversations that were happening in that room and beyond that day. For example, with the focus on reasoning, having that opportunity after the lesson to de-brief and think about how we could build on it. After that we would just have conversations in the corridors...this was a critical phase.</p>	<p>Hannah indicated that the post lesson conversations in stages 1 and 2 were critical for teacher learning. She noted that deconstructing the pedagogies associated with student reasoning enabled teachers to continue the conversation into the corridors and build on the ideas that were modelled.</p>
	<p>Hannah: The biggest change in our practice at school has been relying on</p>	<p>Hannah pointed out that collaboration between</p>

each other to share the ideas and talk about how we can make things better. This has really opened [collaboration between colleagues]. We have adopted the planning, the lesson structure, the enablers, the extenders and so on.

Principal: I have been looking for ways to get teachers to be in each other's classrooms. With the journey, we've been on with both literacy and numeracy we have been trialling lots of new ways of teaching and specific strategies and they meet regularly but the one factor they were missing was the opportunity to be in each other's classrooms.

Principal: This [intervention] can bring a greater level of accountability planning after school. You can't fluff something up if your colleagues are in there supporting you do it.

Principal: We've got a group of committed teachers who had a level of trepidation at the start ...and were challenged...they have all grown through it and come out the other side as opposed to something starting well and dropping off. It [the model] has grown legs in each step along the journey... and all the snippets that they've learned along the way, even just rejigging their own practice around how they differentiate which wasn't really the primary focus.

Principal: I think [modelling] is critical because it's where we get to the coalface... It is teaching... and then going and talking about teaching as opposed to other forms of PL that are just talking about teaching and may never actually make it back to the classroom.

Hannah: [The planning meeting] and the questioning [by the researcher] [prompted us] to look at the [tasks] we are setting... [I asked myself] Are we setting tasks that align with goals for a lesson?

colleagues has been the biggest change at her school since the intervention.

Teachers are now relying on each other to plan for improvements to practice.

The Principal (HPS) indicated that the intervention supported the move towards teachers observing each other's classroom practice.

The Principal (HPS) indicated that the collaboration brought a greater level of accountability to planning meetings.

The Principal of HPS indicated that although teachers were reluctant to begin with they all made shifts in their teaching practice including changing the way they differentiate practice.

The Principal (HPS) summed up the success of the intervention at his school by implying that modelling and the collaborative discourse associated with it is a most effective form of professional learning.

The planning meeting with her colleagues and the researcher prompted Hannah to reflect on choice of tasks that align to the goals of the lesson.

Planning

Hannah: When we were doing the planning [Stage 3] we had our curriculum there, we thought this is what we want the kids to learn ... but when we set the task, [we realised] a word could change the meaning of the whole task...we had to keep referring to our outcome to get those words correct. That challenged us.

Heather: I've realised you can choose tasks from different books, but if you can't unpack the math's that's in underneath the task, then you're not developing the children's math's skills, you're just choosing tasks.

Heather: It feels a bit like the rug has been pulled out from under my feet...I had a way of working and a way of choosing tasks, but now it's turned my head around and I'm really very conscious of how I am sequencing the maths...I'm really thinking about why it is I am choosing [tasks]. For example, I might have had some ideas of doing numbers to ten [with prep/ones], but now I'm looking at the proficiencies and [questioning] what is the foundation that I'm building the maths on? What does this really mean and what does this look like in my classroom?

Heather: [When the teacher is really clear about the learning intention] it makes you disregard the other comments and draw back to what it is we are working on.

Heather: The use of the lesson plan structure was extremely helpful for being clear about the learning intention and flow of the lesson.

Holly: Having a plan like this helps us be more focussed on the maths we are trying to draw out.

Hannah recognised teachers needed to be clear about aligning the goals of the lesson with the mathematics and the task.

Heather recognised the importance of planning tasks that would develop children's learning in mathematics

Heather articulated that the modelling prompted her to reflect on her choice of tasks and think about what it means for her future classroom practice.

Heather could see that being clear about the learning intention helps the teacher to remain focused on the mathematical intent in the summary phase of the lesson.

Heather indicated the lesson plan was helpful in providing clarity for the goals and flow of the lesson.

Holly indicated that planning the lesson in stage 3 helped her to focus on the mathematics we were planning to co-teach.

Hannah: [I have recognised] the importance of our discussions in planning for the tasks.

Jackie: [This intervention highlights the importance of] teacher pedagogy: developing questions and refining tasks.

Hannah: Planning in this way, you know what you're looking for.

Holly: I have been more purposeful with planning and writing questions and [thinking] about what the maths is about.

Heather: Designing the problem-solving tasks has enabled me to be clearer about the maths I'm trying to focus on.

Heather: The lesson plan structure was extremely helpful for being clear about the intention and flow of the lesson. Perhaps some mapping prior to the lesson with regards to levels of sophistication might be useful for fine tuning observations and making decisions about the maths during the [summary] phase.

Heather: Co-planning meant you had your colleagues to really sort out the problem and make sure it was going to do what you were aiming for.

Heather: Seeing someone model makes [the lesson] come alive... especially when you see it warts and all. Seeing the one experience together [with colleagues] is a great growth point because you are talking about the same experience.

Hannah: [In my role as Assistant Principal] I set myself a goal of being able to] support teachers to reflect on their goals for the lesson "what is the purpose of this lesson? have you thought of?"

Hannah recognised the importance of the discussions in planning for learning.

Jackie noted the intervention highlighted the importance of teacher pedagogy including planning and refining tasks and developing questions to ask students.

Hannah indicated that the detailed plan for effective in having clarity about what to look for in the lesson.

In the post lesson survey, Holly indicated that she had become more purposeful with her planning.

Heather indicated the planning stage was important for her to become clearer about the mathematics she was focusing on.

Heather's comment in the post intervention interview indicates she would have liked the opportunity to anticipate student responses to tasks in the pre-lesson meeting.

Working together enabled Heather to experience several heads working together

Heather saw watching the modelled lessons as a catalyst for growth with her colleagues and they could talk about the lesson and learn from that

The intervention prompted Hannah to support colleague teachers to be clear about their lesson goals.

*Mathematical
Knowledge for
Teaching*

Heather: And all the toing and froing about the task and the talk that we have had [and thinking] “Is this task really going to do what we intend.”

The planning stage [in stages 3 and 4] was probably more beneficial for me personally. As the [enacting] teacher having that opportunity to give myself some critical feedback... knowing exactly the outcomes and exactly the rationale.

Principal: Teachers learned explicitness around knowing exactly what your most important purpose is in the lesson...

Heather: We needed time to talk and sort out the developmental stages behind the concept of comparison so that we knew what to [plan] for... [We asked ourselves, is this task] broad enough to cater for all the developmental stages that are there.

Heather: That whole notion of within 10 is an interesting thing e.g. understanding the 6 and 4 are contained within 10. What would you have done otherwise to get the children to notice a pattern?

Holly: They [students] are very reliant on concrete [materials]. Even though they know some strategies, they keep reverting to counting by ones.

I probably haven't used that word [difference] before.

Hannah: I have noticed that Holly and Heather are doing a lot more work on the notion of difference. We have really picked up on that concept and about using the different vocab. We shared the concept of difference with the primary team and what that means

Heather indicated the depth of the back and forth discussions in the planning of the stage 3 task to explore the concept of difference.

Hannah indicated that planning and enacting a lesson were most beneficial for her. This was because she thought deeply about the goals of the lesson and the rationale behind it and this enabled her to give herself some critical feedback after watching the video of herself.

The Principal indicated that one of the most important things teachers learned was to be clear about the learning goals of the lesson.

Heather appreciated the time with the researcher and her colleagues to discuss and refine understanding of the concept of difference before planning a suitable task for the children with enabling and extending prompts.

Heather was interested in the mathematical idea, she posed a question to the researcher to clarify her ideas

Holly claimed her students were reliant on materials and counted by ones. After reading through the lesson plan she also claimed she had not used the word difference with her grade 2/3's

In cycle 4, Hannah indicated that she had observed that her colleagues were now focussing on the concept of comparison and difference with their classes. She noted the team shared the ideas with

*Teacher
learning from
the researcher*

Heather: Now we have 50 problems, because all it is changing the numbers and changing the context. If we've got the language, we don't have to dream up [new ideas].

Heather (HPS): The pre-lesson and post lesson meetings teased out some of the maths. Maths is not the subject where I feel most comfortable. It gave me a great opportunity to sort out the maths that we were focussing on.

Heather: I think it also takes someone to be there as a 'critical friend' with a special interest/knowledge in the subject so that the knots in the thinking can be heard and professional learning put in place that would help tease out or challenge the ideas.

Hannah: [The planning meeting] and the questioning [by the researcher] [prompted us] to look at the tasks we are setting... Are we setting tasks that align with goals of a lesson?

Heather: We needed time to talk and sort out what are the developmental stages behind the concept of comparison so that we knew what we are looking for.

Heather: [Having access to a knowledgeable other] gives you the confidence to start to ask some questions around helping to sort out say maths...you're confident that the person has more experience in that area and can share from their experience base.

Trust

Principal: There was the trust factor... trepidation [from teachers] around how am I going to go, I'm not sure I feel comfortable being watched, I'm not sure I feel comfortable talking into audio recorders.

the grade 3-6 team at her school.

Heather recognised a task could be altered if the context and the numbers were changed.

Heather indicated that the pre and post lesson meetings were helpful for her to unpack the mathematical ideas that were the focus of the modelled lessons.

Heather claimed that the researchers' role as 'knowledgeable other' was an important part of the intervention to support teachers to deconstruct ideas.

Hannah noted that the researcher's questions prompted her to reflect upon the tasks that teachers were presenting to students.

Heather appreciated the time with the researcher and her colleagues to discuss and refine understanding of the concept of difference before planning a suitable task for the children

Heather noted that having access to a knowledgeable other supported her to begin to ask questions about mathematics content and pedagogy.

The Principal (HPS) revealed that teachers felt untrusting of the process in the initial stages of the intervention.

<i>Children learning from each other.</i>	Principal: Their [teachers] Achilles heel was that they didn't know [the researcher]... At each stage the tenseness eased.	The principal highlighted issues with trust which eased as the intervention progressed.
	Hannah: Initially some teachers were daunted by having an extra person in the room when they are usually just in the classroom by themselves with all the kids.	Hannah indicated that initially some teachers were uncertain of having the researcher into their classrooms.
	Henrietta: [Children were] learning from each other. It was okay that they were all working on different numbers. Some were working with larger numbers, others were working with smaller numbers. Thinking most important. No hands up rule.	Henrietta noticed that the students were learning from each other even though they had enabling and extending prompts, this was because they all had the same learning goal.
	Heather: Using students for modelling their work was a key and using the language.	Heather could see that by displaying and talking about their work children could learn from each other.
<i>Noticing behaviour management</i>	Heather: My biggest challenge in the [summary phase] is how to get my children to focus on the maths, because often they are not tuned into to another child sharing if they have been through the whole process of working on problems.	Heather indicated the summary phase of the lesson was a challenge for her as it is difficult to get children focused on others thinking. This suggests teacher actions leading to this phase may need to be made clearer to teachers.
	Jackie (HPS): I liked the fact that you had a [disclaimer] earlier about [our observation] of behaviour [in the lesson]. I was thinking it might be easy to be distracted by the students not on task... But the focus as observing teachers was on the reasoning and the enablers...	In stage 1, Jackie described the importance of teachers not getting distracted by student's behaviour and of having a clear focus for the observation.
<i>Modelling Mathematical Language</i>	Holly: I noticed how many times you repeated the mathematical language...e.g. difference. [Students] were getting it in their heads that's what they were learning Summing up around difference.	Holly noticed the researcher modelled mathematical language.
	Heather: I was really conscious of you [modelling] the language and then they[children] started using that language that you were modelling and you ...stretched their thinking	Heather noticed the researcher modelled mathematical language which the children began to use.

<i>Teaching decisions in the moment to steer the modelled lesson</i>	<p>Jackie: Why did you stop yourself from writing that number sentence? All those critical decisions that you are making... You made a decision about where not to go because it was best to go somewhere else, none of those things are in the plan.</p>	<p>Jackie noticed important teacher moves to scaffold learning on the fly which were not part of the lesson plan.</p>
<p><i>Teacher learning from the intervention</i> <i>Factors concerning implementation cost</i></p>	<p>Heather: I moved from being uncertain and wary to really seeing the benefits through the whole process at the end. Principal: I personally think you can't really spend a lot more time on it [the intervention] because of the cost. I think the model [stages of the intervention] is right. What we would do is pick something that we're really struggling with or something new that we are really challenged by and that would hook them [the teachers] in.</p>	<p>Shift in thinking The Principal highlighted the cost of the intervention but thought that the model of the intervention was about right in terms of changes in practice. He suggested that teachers could come up with the problem of practice themselves in order to get buy in.</p>
<p><i>Modelling high expectations, encouragement and persistence</i></p>	<p>Hannah: Some people changed practice within four stages and for some it would need longer, it really depends on the person and what it is you are trying to change. Holly: ...when you put them [icy pole sticks] back in the tub ... it was a signal for those children not to stop thinking. Heather: I noticed the way you didn't give up on [the students]. You kept expecting that they would keep going and you acknowledged for them that it was going to be difficult ... you kept encouraging them and then towards the end you gave them the cliff-hanger [you gave them the message] we are going to come back ... this is important.</p>	<p>This indicates teachers are on a continuum of learning and one size does not fit all. Holly noticed the researcher had the expectation that all children would think Heather noticed that the researcher had high expectations of the children and encouraged them to persist.</p>
<p><i>Students thinking for themselves</i></p>	<p>Heather: [I noticed] the importance of time [for the students] to grapple with the ideas. Heather: Sometimes I don't see what they're [children] struggling with until I get the chance to see them in some problem-solving task.</p>	<p>Heather could see that students need time to work on challenging problems In stage 4, Heather indicated she had difficulty in assessing children unless she saw them working on a problem solving task.</p>

	<p>Heather: Some of the power for me was looking at the children problem solving and thinking so now where am I going, whereas I might have done it back to front in the past.</p> <p>Heather: [The modelling] affirmed [it's okay to allow] the kids to struggle a little and trust that it is okay to do that.</p> <p>Hannah: I learned that during the lesson it is vital to allow the kids to explore the maths in their own way.</p> <p>Heather: Teaching in this way still feels a bit uncomfortable for me but it is exciting to watch how some children go about solving the problems.</p>	<p>The power of seeing students solving problem prompted Heather to reflect upon her assessment practices.</p> <p>Heather indicated that the modelling affirmed it was okay to allow students to grapple with problems.</p> <p>Hannah learned the importance of students thinking for themselves.</p> <p>In her post lesson survey, Heather indicated that she still felt a little uncomfortable about students solving problems by thinking for themselves but noted it was exciting to watch the children solving the problems by thinking for themselves.</p>
	<p>Heather: Starting with the problem opens up opportunities for children to demonstrate an amazing variety of thinking and ways of moving towards understanding.</p>	<p>Heather could see that by providing children with opportunities to solve problems by thinking for themselves, they came up with a variety of responses.</p>
	<p>Principal: [Teachers learned] the importance of letting kids swim a little bit, sink a little bit – not to rescue.</p>	<p>The Principal (HPS) indicated that teachers learned the importance of allowing the students to struggle and think for themselves.</p>
<i>Modelling clarifying</i>	<p>Holly: You got a child to repeat back the question so that you could clarify what it was that they had heard...</p>	<p>Holly observed the researcher asking children to repeat back what they had heard to ensure they understood what the problem was asking them to do.</p>
	<p>Hannah: Giving them [students] the opportunity to unpack the question and understand what is being asked of them before beginning is a critical step</p>	<p>Hannah noted the important step of making sure children are clear about what the problem is asking them to do before sending them off to work on it.</p>
<i>Student's explanations Difficulties</i>	<p>Holly: They found it really hard to explain what they were thinking.</p>	<p>Holly highlighted the difficulties children had in reasoning mathematically.</p>

<p>Heather: When you asked someone from the group to [repeat someone else's reasoning], they found that hard.</p>	<p>Heather observed students had difficulty repeating someone else's reasoning.</p>
<p>Heather: You tuned them [the children] into thinking, you gave the expectation that they were going to justify... You asked a question and gave them wait time and named up their thinking... You did not accept an answer without the expectation that you wanted to probe them just a little bit more. You rephrased their thinking. "What did you notice about what so and so said". Even right to the very end it was still very obvious that you expected them to reason [mathematically]. You set up the expectation to find more ways [to make 10] and expected them to respond.</p>	<p>Heather noticed several teacher actions to facilitate student reasoning including asking students to restate another's reasoning, revoicing, paraphrasing, waiting, having high expectations, probing. This suggests the narrow focus on teacher actions to facilitate reasoning was effective.</p>
<p>Heather: It really highlighted the value of the talk. I looked at one child and I thought that child was counting by tens but when I asked that child to explain their thinking I discovered they were using the ten blocks as a [representation]for one. If we hadn't had that talk time, where they were explaining what they were doing [I would not have picked that up].</p>	<p>Heather saw the importance of exploring children's thinking and getting them to talk about it</p>
<p>Hannah: You used student explanations as a way of introducing concepts/tools</p>	<p>Hannah noticed student explanations were used as a way of introducing concepts.</p>
<p>Heather: I think "convince us" is really powerful because [I] saw some really great wait time and then finally some thinking.</p>	<p>Heather observed the researcher asking student's to justify their answers and consequently she saw students spending time reflecting on their learning.</p>
<p>Henrietta: I am using children a little more to explain their thinking to the group</p>	<p>Henrietta, who was about to retire indicated on her exit survey that she was now getting students to explain their thinking.</p>
<p><i>Narrow observation focus for the modelling</i></p>	<p>Hannah: [Focusing the observation] on reasoning... having that one thing to look at made it much more [narrow] so that we didn't worry about or notice all teachers to narrow their focus</p>

the other things. Having that sole
focus made it beneficial.

and exclude unimportant
things.

Appendix 12: Themes: Magpie Primary School

Selected examples of significant statements and their meanings Magpie Primary School (MPPS) stage 1

Selected samples of significant statements from post lesson discussions and surveys	Formulated meanings
Melissa: I liked the enabling prompts where you had to bring it back for a couple of children to be able to work on a smaller [quantity]	Melissa observed the researcher used enabling prompts to enable low attaining children to access the concept by providing the same task with smaller numbers.
Megan: It was always the children that were explaining their work and even sometimes when they weren't making sense themselves you helped with the language... You didn't tell them what to say, they were doing the talking.	Megan observed the researcher revoicing the student's explanations.
Marita: I felt some kids were just lost. There were others that just didn't have a clue. I'm wondering whether you would we have a small focus group for kids that are lost?	Marita noticed children off task. She wondered whether a small focus group would benefit those children.
Marita (MPS): A huge concern is that while you've got kids that are engaged...sitting there listening ... and actually having a go, the others aren't [getting it]. They are off task...they are not thinking about learning, they have no meta cognition.	Marita was concerned about the children she perceived to be off task.
Maude: There was so much rephrasing and scaffolding and questioning and then you were using enabling prompts and then went to concrete aids, there was a whole range of things that were going on to facilitate their reasoning.	Maude observed several teacher actions which facilitated student reasoning including adjusting the lesson to suit the diverse range of learners.
Marita: Jack (pseudonym) appeared to be off task the whole time, but then when he stood up and explained this thinking.	Marita noticed a student who appeared to be off task stand up and explain his reasoning.
Marita (MPS): Students continued to scaffold each other's ideas...you could see their little lights go on and then they had the courage to have a go themselves.	Marita observed students learning from each other through their explanations of the mathematical ideas.
Megan: You paused and waited and waited ...we often step in.	Megan saw the researcher giving extended wait time which prompted her to reflect that she often steps in too quickly.
Marita: I'm thinking maybe I'm under pitching way too often? Maybe I'm not setting [expectations] high enough. [This lesson] has got me thinking about changing my beliefs [about teaching]	The modelled lesson prompted Marita to reflect on her expectations of student achievement.
Marita: Is it okay to [set high expectations and differentiate with enabling prompts]. Is that okay to do that?	Marita raised the question of whether setting high expectations for all students and them differentiating with enabling prompts was appropriate.

Maude: Did you plan the questions you were asking the children? That was an important part [of the lesson].	Maude asked if the questions that the researcher asked the children during the lesson were planned.
Megan: Jack (pseudonym) was distracted at the start, but you trusted him to do it and he achieved much more than he ever has. I was surprised at how much you could get out of him.	Megan observed a student achieving far more than he ever has.
Marita: Jack (pseudonym) felt the power of success, you encouraged him to keep thinking.	Marita observed the researcher encouraging students to keep thinking and saw a child experience success.
Melissa: “Were you trying to get them to identify 7 as being 70? Is that what you were trying to do?”	Melissa was unclear about the mathematics that the researcher was attempting to teach.
Marita: “Would you have at any time thought that it would be okay to do some counting in tens? Like ‘71’ how do you know that is 7 bags of ten? Can we count them together, 10, 20, 30, 40, 50, 60, 70?”	Marita raised the question of whether counting in tens might be helpful for the students.

Selected examples of significant statements and their meanings Stage 2

Selected samples of significant statements from post lesson discussions and surveys	Formulated meanings
Pre-lesson meeting	
Megan: Timing is difficult... I thought that only having a half hour session to [de-brief the last lesson] and not having a chance to plan together what we wanted to get out of today was really difficult.	Megan raised the issue of timing and how teachers felt they didn't have enough time to de-brief the previous lesson and co-plan for the next modelled lesson.
Megan: [That you had to steer the previous lesson in a different way] was powerful for us to see.	Megan indicated that seeing the researcher steer the lesson in a different way to cater for the unanticipated learning needs of the students was powerful for teachers to see.
Post-lesson meeting	
Megan: The power is in your questioning ... the time you gave children to reflect... Even if children did have the concept right but did not know how to articulate it, that thinking time allowed for a lot more. For example, Oliver [pseudonym] was a real surprise to me because at the start of the year he couldn't recognise any numbers, he couldn't count with one to one correspondence. He came up with 5 and 5, when normally he would not. Because he had the time to work it out, he could do that. Your questions were clarifying their ideas.	Megan observed extended student thinking time and questioning to facilitate mathematical reasoning. She indicated these teacher actions led to the success of her students which she was surprised by.
[Children felt comfortable] to have a go even if they got it wrong... “keep thinking”. Children did continue to persist with that prompt.	Megan observed elements of a pedagogical approach that fosters encouragement and persistence.
Maggie: [The researcher] said, “I like explanations like Susannah’s”. For me that was	Maggie observed a low attaining student explaining and justifying her reasoning clearly.

really really important... [I have read recently] low attaining students are not often included [in discussions] as we don't think that they grasp the concept. That's excluding them from ever having that opportunity. Susannah could vocalise so well. It was a nice example of ...reasoning and justifying.

Melissa: You could extend those that got the concept but then go back to those others who were struggling. It is normally so hard to work with low ones when you're working by yourself.

Melissa: "Keep thinking" was the most important idea that I got out of the lesson because it's horrible to say, "good try" or "try again".

Marita: "Keep thinking was so powerful" ... we didn't see those deflated little faces.

Marita: I run out of time in bringing them [children] back together at the end and you did that so well.

Marita: I liked the differentiation ..., where if we're not counting with one to one correspondence, let's go back to 5 teddies or for the extending [prompt], there's more than one way to do this and asking, "can you see any patterns."

Marita: The phrases that you use: Convince me, tell me about your thinking.... I might say "that is not quite the right answer. But let's see if we can explore that" but you say, "keep thinking...has somebody else got something they would like to add". It was [student] directed rather than you directing [the lesson].

Melissa: I think it's giving them the language to enable the children to explain their thinking. If you are always modelling that language they are able to justify.

Megan: [The modelling] clarified for me how important it is to have those three steps...the introduction, making [the task] clear; having time to [problem solve] and getting examples from [the children] and then getting them to come back and have some powerful reflection time. It brings out the importance of why each of those things is so important.

Marita: I would have given them examples before I sent them off to do a task. I was wondering how they would go [without instruction] but they did really well.

This prompted Maggie to highlight the importance of it considering her recent reading that low attaining children are often excluded from mathematical discussions.

Melissa observed that the enabling an extending prompt provided a way to cater for the diverse range of learning needs.

Melissa observed that the researcher's comment 'keep thinking' encourages children to persist. She felt it was the most important idea she took from the lesson.

Marita observed the researcher encouraging the children to persist.

Marita indicated that she often misses the lesson summary as she runs out of time.

Marita could see differentiating the task was an effective way to engage all students in the lesson.

Marita observed phrases that the researcher used in the lesson which facilitated student directed learning.

Melissa observed mathematical language being modelled by the researcher which supported students to justify their reasoning.

Megan noticed the importance of the lesson structure to facilitate student reasoning and problem solving.

Marita observed students thinking for themselves successfully before instruction from the teacher. This prompted her to reflect on her teaching practice.

Selected samples of significant statements from post lesson discussions and surveys	Formulated meanings
<p>Melissa: As soon as Rick [pseudonym] shared his idea [of using a number line] there were quite a few that went back and [used] that number line that Rick had explained. The children learned, that they could use an idea that [other students] had explained</p>	<p>Melissa observed a student sharing his number line strategy to the class which prompted other children to learn from this and use that strategy themselves.</p>
<p>Me: Even that little girl at the end, she used that number line in a different way by breaking it into 5's... So that's how they learn...from listening to what the other students say. Some of them listened to Rick, because they used that notion of 10 and that [student] at the beginning who said, "I did it the same way as [the first problem] ... That's what we want to see...</p>	<p>Maude noticed some children struggled to explain their thinking.</p>
<p>Maude: I noticed that Rick [pseudonym] could get the answer really quickly, but he struggled to explain his thinking and it took him a while.</p>	
<p>Me; So, then we have to [revoice] that because that was really important thinking for the other students to hear</p>	
<p>Melissa: I had one child who shut down because those numbers were big; he just went "Whoa! and just sat. After he did the first one [enabler], he did the next one in under a minute. Just having those enabling prompts really helped him to get started and I'm sure with a couple of more practises he could work up to the main task. Having those enabling prompts is one way to get started.</p>	<p>Melissa observed a student successfully using an enabling prompt which prompted her to see the power of enabling prompts for children to get started.</p>
<p>Me:</p>	
<p>Sabrina: What will we do with students who are unable or unwilling to attempt the enabling prompt?</p>	<p>Sabrina's question indicates she was unclear about what to do if the enabling prompts are too difficult for the children to access.</p>
<p>Melissa (MPS): It was really hard to not tell though. I had to bite my tongue a few times in there because I wanted to say, "oh could you use a ..." "I need to stop giving them the ideas about how to solve problems".</p>	<p>In her co-teaching session in stage 3, Melissa noticed herself having to hold back from telling the children what to do.</p>
<p>Maude: I think the learning intentions were very clear at the beginning as in "we have to explain our thinking, listen to each other..."</p>	<p>Maude noticed how clear the learning intentions were.</p>
<p>Marita: I think [the children] liked the zone of confusion terminology. I saw a child saying to another child, please don't tell him the answer, it is okay to be confused.</p>	<p>Marita observed children acknowledging the 'zone of confusion' as a safe space where hard thinking and struggle takes place.</p>
<p>Maggie: I saw some children with nothing on their paper. Is it okay for them to be like that?</p>	<p>Maggie noticed students with nothing on their paper. She was concerned that they were off task'.</p>
<p>Marita: We do some [tasks] together and we start off with low numbers rather than giving</p>	<p>The modelling prompted Marita to question whether she should be providing tasks that</p>

that challenging one first. I build them up, so maybe I need to [give the challenging task first] and then bring them back?

Melissa: I feel I probably go astray, I probably offer up strategies a bit much rather than let them [have a go] ...that is something I will attempt to do.

Sabrina: Sometimes you feel guilty about seeing children doing seemingly nothing.

Maggie: [the struggling students] would be the ones I would gravitate towards first. The enabling prompts were too hard for them. They weren't even in the zone of the room.

Maude: Ava struggled as well until she saw that number line and then something clicked in her head and she thought, well I can take away five and take away five.

Megan: I've lacked a little bit of confidence. I don't like mathematics. It's something that I feel like I have to overly think about and plan for. You have given me some more ideas about where I can go...not just for mathematics but for all learning areas...The way you introduce things, reflect on things and get the children involved.

children can solve easily and scaffolding up or posing challenging tasks as a first step.

The modelling of challenging tasks prompted Melissa to reflect upon her teaching where she tells the students the strategies rather than allowing them to think for themselves.

Sabrina expressed concern about students she perceived as 'seemingly doing nothing'.

Maggie indicated the enabling prompts were too challenging for some students and her preference for working with those students first.

Maude indicated that a student struggling with the mathematical ideas suddenly made a connection when she saw and heard her peer demonstrating his strategy on a number line. Maude saw the power of this student learning from another.

Megan indicated that she lacked confidence in teaching mathematics but the intervention supported her to gain pedagogical content knowledge, some of which she can transfer to other learning areas.

Selected examples of significant statements and their meanings Stage 4

Selected samples of significant statements from post lesson discussions and surveys

Pre-lesson meeting

Marita: [This is a] very similar lesson plan that I've chosen to the one we did in Melissa's class. I have kept the same title, "How many more" but I have changed the question to something that I think my kids will resonate with. I hope that I will do a similar tune in to the one [the researcher] did.

Marita: I have made up some enabling prompts and some extending prompts. I will have some that can't handle those numbers.

Marita: I think this task is quite easy. I think a few of mine will bridge to 100 and count on.

Marita: When I sit with my class, I'm going to tell them that some of the maths we are doing today is going to be tricky, but that when it is

Formulated meanings

Marita indicated her lesson plan was very like the lesson she saw co-taught in her colleague's class. She used the same concept but changed the context.

Marita made up two enabling prompts and two extending prompts for the task she had planned for her enactment in stage 4. This was in response to previous modelled lessons where the enabling prompts required further modification.

Marita anticipated that her enacted lesson would be easy for some students and could articulate an anticipated strategy that students would use for solving it

Marita was deliberate in planning to allow students to think for themselves in her enacted lesson. This shows a shift in Marita's thinking.

tricky, our brains are growing. I'm not going to mention any strategies, I want to see what they come up with themselves.

Marita: I'm not sure that I'll get anything more out of this [enactment] today than I what I've already got apart from a belly full of butterflies and feeling a bit unwell. I've already taken a lot from the sessions that I have seen. I have already implemented things that I've learned from you [researcher] into my classroom and I've already trialled the lessons with the kids.

Marita (MPPS): The biggest thing I'll take is that I have been limiting kids with their thinking in maths and not stretching them as far as I can.

Marita (MPPS): I already think I know in my mind who will get enabling prompts and who will get extending prompts. My lower kids that I think will struggle are already sitting together... I'm going to try hard not to jump in and give them the enabling prompt too early.

Marita: I've implemented some open-ended tasks that are probably more challenging than what I would previously have given... I've implemented "keep thinking" and it's okay to be the zone of confusion. You modelled that and I saw kids thinking.

Marita: In this lesson today some will use counting on, I am hoping some will use number lines and some will bridge to 100 and go from there.

Post-lesson meeting

Megan: I'm wondering whether meaning was lost in enabling prompt? I don't know if it was the wording of it? Just wondering if you think it may have been more effective if you used the same context as the main problem?

Marita: Caleb was really stuck, but he got it in the end. He really got it!

Marita: I found it really difficult to bring them back together at the end, that's where it is hard to know what to say. I need to focus on the thinking. It is so hard not to tell them what to do.

Megan: There was a lot of thinking time. Because you [Marita] said that very clearly [to the children] how they need to explain their thinking, I could see that Tory [pseudonym] was sitting here thinking about how she was going to solve the problem and Cory [pseudonym] as well "how am I going to draw that because I

Marita did not anticipate that her enactment of a maths lesson would improve her pedagogy because she felt she had already implemented new pedagogies that she observed in the modelled lessons.

Marita indicated that her greatest insight from the modelling was that she limited students in maths by not providing them with opportunities to solve challenging tasks.

Marita indicated she could anticipate who would receive enabling prompts in her enacted lesson, but she noted that she would try hard not to give the enabling prompt unless students were unnecessarily struggling.

Marita indicated she has implemented challenging tasks, extended thinking time and the notion that it's okay to be confused. She noted that seeing it modelled and how the students responded was a catalyst for her change in practice.

Marita anticipated what strategies she would see the children using in her lesson.

Megan's comment to Marita indicates she could propose an improvement to Melissa about the enabling prompt.

Marita indicated that a student in her lesson was very stuck but the enabling prompt and thinking time enabled him to experience success.

Marita indicated the difficulty she had in summarising the lesson and refraining from telling students what to do.

Megan provided feedback to Marita on how she enabled students in her class to experience success by giving them thinking time.

sought of know... How am I going to explain that? Just the way you set that up enabled it to happen.

Marita: I know that I'm not overly fantastic at math's and I need to be much better, so even though I didn't want to do that [enactment] I guess it probably stretched me in a lot of ways... I learned that to draw out kids thinking was the most difficult thing for me and remembering as I went around who needed to come back up [whose ideas would be shared]. It highlights the importance of thinking about the questions we ask children.

Maude: It's important to know what questions you are going to ask the children, and have that planned.

Marita: I don't think I would put the lower group in that zone of confusion. I am still inclined to reign it back a little.

Maude: Putting low attainers in with those high fliers would just give them the experience. They might see what the high fliers are doing. They might not understand it but they will have the opportunity.

Megan: I have held back on my modelled explicit teaching and allowed the children to have more thinking time. [I saw] the children thriving on the challenge.

Maggie: Reading about the approach isn't as successful as seeing it modelled in a real-life situation.

Maggie: I want to elicit student strategies rather than giving answers.

Marita indicated the difficulties she had in choosing which student ideas to focus on during the discussion and probing students thinking during the lesson.

Maude could give feedback to Marita about planning the questions before the lesson.

Marita's comment indicates her reluctance to place her lower attainers in the zone of confusion but Maude responded with a suggestion to support inclusion.

Megan indicated she has held back on explicitly modelling strategies for the children. She saw the children thriving on the challenge and this was the catalyst for her shift in her views of teaching.

Maggie noted the power in seeing the pedagogies modelled.

The observations prompted Maggie to desire to have students think for themselves rather than telling them what and how to think.

Selected examples of significant statements and their meanings: Post Intervention Interviews

Selected samples of significant statements from post lesson interview's and surveys

Formulated Meanings

Megan: The pre and post conversations have been powerful and ...theory put into practice in our classrooms

Megan valued the pre and post lesson conversations and seeing theory put into practice in her school.

Melissa: [In the past] I tended to explain a strategy before the [children] came up with it.

Melissa indicated that she had shifted in her views of teaching from telling students strategies to allow them to come up with solutions themselves.

Melissa: Seeing something work in a working classroom is much more beneficial than reading or PD that is totally removed from the classroom

<p>Maggie: I am applying more problem based lessons in my classroom. I am not telling as often as possible.</p>	<p>Maggie noted she had shifted from telling students what to do</p>
<p>Marita: Teachers did not have any say in what was happening until we were all informed that we would be coming along to watch the researcher model some lessons for us. To begin with we thought it would be beneficial but we also thought it was <i>another thing</i> that we had to do and the timing wasn't good as it was in our most crucial part of the day.</p>	<p>Marita indicated teachers were not consulted re the timing of the intervention.</p>
<p>Marita: The problems were far more challenging than I would have previously used so I have changed my thinking.</p>	<p>The modelling of challenging tasks prompted Marita to change her views of teaching.</p>
<p>Marita: Previously I would have started the introduction of a task by explaining and modelling and modelling again and we would have all done some together but now I find it is fine to introduce the task and let [the children] explore their understandings as they make them.</p>	<p>Marita shifted her view of teaching from telling children how to solve problems to allowing students to explore problems for themselves.</p>
<p>Marita: I now use enabling and extending prompts in the lesson as I need to. I give them [children] the opportunity to soar, but if not, I give them the opportunity to achieve success by giving enabling and extending prompts.</p>	<p>Marita indicated she now uses enabling and extending prompts.</p>
<p>Marita: I thought it was beneficial to see that some kids really did rise to the challenge but also achieve success on tasks that I thought they may have needed building up to... Rather than hold those high fliers back as I had previously been doing they could extend their thinking. It was a great way of getting kids to think outside their comfort zone.</p>	<p>The modelling prompted Marita to see students rising to the challenge of cognitively complex tasks</p>
<p>Marita: My goal was to try and incorporate challenging tasks into my classroom practice each week. In my planning, each week I've got a challenging open ended maths task with extending and enabling prompts.</p>	<p>Challenging tasks</p>
<p>Marita: Initially [low attainers in my class] would be just really stuck ...but now they are at least starting to have a go even if they're not on the right track. They are understanding that it is okay not to get the right answer.</p>	<p>Catering for diversity</p>
<p>Marita: It's a new concept for me to make the maths challenging... let's get the kids switched on thinking.</p>	<p>Marita indicated her shift in thinking to make the maths more challenging.</p>
<p>Marita From my own modelling I learnt it's really tricky to try to be somebody else. I tried to model my lesson from what I saw [the researcher] do and I found that was difficult... Sometimes we need to be just who we are but</p>	<p>Marita indicated it was too difficult to try to imitate someone else. She learned the importance of being herself and incorporating important pedagogies that she observed into her own practice</p>

pull in the pieces of other people’s teaching that we would like to aspire to.	
Marita: When I watched myself [on video], I was saying things like “no, not quite” and I was like Aaargh! I didn’t say things that I have been saying in my classroom a lot like “keep thinking”.	enactment
Marita: The hardest thing I found [from enacting the practice] was looking over students work to find examples to use to model for other students and bringing the lesson back in. It’s something I need more practice at.	enactment
Marita: I think teachers could have a say in when it is held. I think it would be nice to have an initial meeting with teachers to see what would fit in with their timetable.	Teacher voice
Marita: [Re comparing classroom modelling with other forms of PL]. Everyone involved gets something out of it that they can straight away take back to their own practice because you’re involved in it. We’re all there. You feel inspired by what you’ve seen. It’s not like somebody telling you what to do, it’s somebody modelling a way to do things.	Modelling benefits
Maude: [The researcher] started at the top end [tasks with a high ceiling, low floor] so it was challenging for all rather than with the low attainers and working up. Teachers had to think really hard about what they were doing in their own classrooms.	Maude indicated teachers were challenged by the approach of implementing tasks with a high ceiling and low floor rather than scaffolding up.
Maude: I think teachers learned that you need really clear learning intentions and use precise mathematical terminology, a structure and [the lesson] needs to be challenging for all students, to use real life contexts and to capture the children’s interests and apply it to their world.	Learning intentions
Maude: I’ve seen in most classrooms the teachers using the same lesson structure as [the researcher] did and [having the] whole class [working on the task] using the enablers and extenders. There has been more evidence of change in [teachers] teaching practice in having high expectations and making sure they were challenging everyone	Lesson structure
Maude: It was so tricky trying to get 6 relief teachers. We had to pay relief teachers but it was all so valuable.	Cost
Maude: It was a really valuable intervention for our school. Just the fact that you were modelling the proficiencies rather than looking at content... helping the classroom teachers with their practice of mathematics in any	Maude indicated that modelling teacher actions to develop student’s proficiencies in mathematics was powerful because it made the actions transferable across mathematics lessons.

content area because you are helping them to develop children's proficiencies.

Maude: Classroom modelling is a much better way of helping teachers improve their practice [than other forms of professional learning] because you have the real-life context and you can see the [modelling teacher] making the adjustments and deal with a range of needs. It's not just talk and chalk. You can see and then go away and develop that in your own classroom.

Modelling – in classrooms

Maude: From the very first lesson that we observed there were lots of children sitting there that had no idea, just didn't know how to start but now because the teachers have been continuing the work I've noticed that the children now have a go. They have a starting point.

Catering for diversity

Principal MPS: Kids can take that risk and have a go and not be afraid of failure and know that they are going to be supported. When the relationships are solid between themselves and the group or themselves and the teacher, then it augers well for the whole environment that we're asking kids to learn in. If we are using modelled lessons in maths to get that happening then that is awesome.

Encouragement and persistence

Maude: The [children] are feeling that sense of achievement from it. One of our very challenging children wasn't really interested [in learning] but when Melissa did something in her class and she ended up giving him an extending prompt and he was so excited. I have never seen him [engaged] as much.

Encouragement persistence

Selected Theme Clusters with their related formulated meanings

Themes	Illustrative examples of teacher comments	Clustered Meanings
<i>Challenging teacher orientation towards teaching as telling</i>	Marita (MPS): I would have given them examples before I sent them off to do a task. I was wondering how they would go [without instruction] but they did really well.	Marita observed students thinking for themselves successfully before instruction from the teacher. This prompted her to reflect on her teaching practice.
	Melissa (MPS): I feel I probably go astray, I probably offer up strategies a bit much rather than let them [have a go] ...that is something I will attempt to do.	The modelling of challenging tasks prompted Melissa to reflect upon her teaching where she tells the students the strategies rather than allowing them to think for themselves.
	Melissa (MPS): It was really hard to not tell though. I had to bite my tongue a few times in there	In her co-teaching session in stage 3, Melissa noticed herself having to hold

*Challenging
expectation of
student achievement*

because I wanted to say, “oh could you use a ...” “I need to stop giving them the ideas about how to solve problems”.

Megan (MPS): I have held back on my modelled explicit teaching and allowed the children to have more thinking time. [I saw] the children thriving on the challenge.

Maggie (MPPS): I want to elicit student strategies rather than giving answers.

Maggie: I am not telling as often as possible.

Marita: Previously I would have started the introduction of a task by explaining and modelling and modelling again and we would have all done some together but now I find it is fine to introduce the task and let [the children] explore their understandings as they make them.

Megan: I have held back on my modelled explicit teaching and allowed the children to have more thinking time. [I saw] the children thriving on the challenge.

Maggie: I want to elicit student strategies rather than giving answers.

Marita: I’m thinking maybe I’m under pitching way too often Maybe I’m not setting [expectations] high enough. [This modelled lesson] has got me thinking about changing my beliefs [about teaching]

Marita (MPS): We do some [tasks] together and we start off with low numbers rather than giving that challenging one first. I build them up, so maybe I need to [give the challenging task first] and then bring them back?

Marita: Is it okay to [set high expectations and differentiate

back from telling the children what to do.

The modelling prompted Megan to hold back on explicitly modelling strategies for the children. She saw the children thriving on the challenge and this was the catalyst for her shift in her views of teaching.

The observations prompted Maggie to desire to have students think for themselves rather than telling them what and how to think.

Maggie noted she had shifted from telling students what to do

Marita shifted her view of teaching from telling children how to solve problems to allowing students to explore problems for themselves.

Megan indicated she has held back on explicitly modelling strategies for the children. She saw the children thriving on the challenge and this was the catalyst for her shift in her views of teaching.

The observations prompted Maggie to desire to have students think for themselves rather than telling them what and how to think.

The modelled lesson prompted Marita to reflect on her expectations of student achievement.

The modelling prompted Marita to question whether she should solve easy problems with the children and scaffold up or pose challenging tasks for children to solve by themselves as a first step.

Marita raised the question of whether setting high expectations for all

Challenging tasks

with enabling prompts]. Is that okay to do that?

Megan: Jack (pseudonym) was distracted at the start, but you trusted him to do it and he achieved much more than he ever has. I was surprised at how much you could get out of him.

Maggie: [The researcher] said, “I like explanations like Susannah’s”. For me that was really really important... [I have read recently] low attaining students are not often included [in discussions] as we don’t think that they grasp the concept. That’s excluding them from ever having that opportunity. Susannah could vocalise so well. It was a nice example of ...reasoning and justifying.

Marita (MPPS): The biggest thing I’ll take is that I have been limiting kids with their thinking in maths and not stretching them as far as I can.

Marita: I’ve implemented some open-ended tasks that are more challenging than what I would previously have given...I’ve implemented “keep thinking” and it’s okay to be the zone of confusion. You modelled that and I saw kids thinking.

Marita: I thought it was beneficial to see that some kids really did rise to the challenge but also achieve success on tasks that I thought they may have needed building up to... It was a great way of getting kids to think outside their comfort zone.

Marita: In my planning, each week I’ve got a challenging open ended maths task with extending and enabling prompts. It’s a new concept for me to make the maths challenging... let’s get the kids switched on thinking.

Maude: [The researcher] started at the top end [tasks with a high ceiling, low floor] so it was challenging for all rather than

students and them differentiating with enabling prompts was appropriate.

Megan observed a student achieving far more than he ever has.

Maggie observed a low attaining student explaining and justifying her reasoning clearly. This prompted Maggie to highlight the importance of it considering her recent reading that low attaining children are often excluded from mathematical discussions.

Marita indicated that her greatest insight from the modelling was that she limited students in maths by not providing them with opportunities to extend their thinking.

Marita indicated she has implemented challenging tasks, extended thinking time and the notion that it’s okay to be confused. She noted that seeing it modelled and how the students responded was a catalyst for her change in practice.

The modelling prompted Marita to see students rising to the challenge of cognitively complex tasks.

Marita’s goal was to incorporate challenging tasks into her planning each week. This indicates her shift in thinking to make the maths more challenging.

Maude indicated teachers were challenged by the approach of implementing tasks with a high ceiling

*Challenging ideas
about catering for
diversity*

with the low attainers and working up. Teachers had to think really hard about what they were doing in their own classrooms.

Maude: I noticed that Rick [pseudonym] could get the answer really quickly, but he struggled to explain his thinking and it took him a while.

Melissa: I liked the enabling prompts where you had to bring it back for a couple of children to be able to work on a smaller [quantity]

Melissa: You could extend those that got the concept but then go back to those others who were struggling. It is normally so hard to work with low ones when you're working by yourself.

Marita: I liked the differentiation ... where if we're not counting with one to one correspondence, let's go back to 5 teddies or for the extending [prompt], there's more than one way to do this and asking, "can you see any patterns?"

Melissa: I had one child who shut down because those numbers were big; he just went "Whoa! and just sat. After he did the first one [enabler], he did the next one in under a minute. Just having those enabling prompts really helped him to get started and I'm sure with a couple of more practises he could work up to the main task. Having those enabling prompts is one way to get started.

Maggie: [the struggling students] would be the ones I would gravitate towards first. The enabling prompts were too hard for them. They weren't even in the zone of the room.

Marita: I have made up some enabling prompts and some extending prompts. I will have some that can't handle those numbers.

and low floor rather than scaffolding up.

Maude noticed some children struggled to explain their thinking.

Melissa observed the researcher used enabling prompts to enable low attaining children to access the concept by providing the same task with smaller numbers.

Melissa observed that the enabling an extending prompt provided a helpful way to cater for the diverse range of learning needs.

Marita could see differentiating the task was an effective way to engage all students in the lesson.

Melissa observed a student successfully using an enabling prompt which prompted her to see the power of enabling prompts for children to get started.

Maggie indicated the enabling prompts were too challenging for some students and indicated her preference for working with those students first.

Marita made up two enabling prompts and two extending prompts for the task she had planned for her enactment in stage 4. This was in response to previous modelled lessons where the enabling prompts required further modification.

Marita (MPPS): I already think I know in my mind who will get enabling prompts and who will get extending prompts... I'm going to try hard not to jump in and give them the enabling prompt too early.

Megan: I'm wondering whether meaning was lost in enabling prompt? I don't know if it was the wording of it? Just wondering if you think it may have been more effective if you used the same context as the main problem?

Marita: Caleb was really stuck, but he got it in the end. He really got it!

Marita: I don't think I would put the lower group in that zone of confusion. I am still inclined to reign it back a little.

Maude: Putting low attainers in with those high fliers would just give them the experience. They might see what the high fliers are doing. They might not understand it but they will have the opportunity.

Marita: I now use enabling and extending prompts in the lesson as I need to. I give them [children] the opportunity to soar, but if not, I give them the opportunity to achieve success by giving enabling and extending prompts.

Marita: Initially [low attainers in my class] would be just really stuck ...but now they are at least starting to have a go even if they're not on the right track. They are understanding that it is okay not to get the right answer.

Maude: From the very first lesson that we observed there were lots of children sitting there that had no idea, just didn't know how to start but now because the teachers have been continuing the work I've noticed that the children now have a go. They have a starting point.

Marita indicated she could anticipate who would receive enabling prompts in her enacted lesson, but she noted that she would try hard not to give the enabling prompt unless students were unnecessarily struggling.

Megan's comment to Marita indicates she could propose an improvement to Melissa about the enabling prompt.

Marita indicated that a student in her lesson was very stuck but the enabling prompt and thinking time enabled him to experience success.

Marita's comment in stage 4 indicates her reluctance to place her lower attainers in the zone of confusion but Maude responded with a suggestion to support inclusion.

Marita indicated she now uses enabling and extending prompts.

Marita indicated that her low attaining students are much more willing to struggle with challenging tasks now and know that it is okay if they do not get the right answer.

Catering for diversity

<i>Perceived disengagement of some students</i>	<p>Marita: I felt some kids were just lost. There were others that just didn't have a clue. I'm wondering whether you would we have a small focus group for kids that are lost?</p> <p>Marita (MPS): A huge concern is that while you've got kids that are engaged...sitting there listening ... and actually having a go, the others aren't [getting it]. They are off task...they are not thinking about learning, they have no meta cognition.</p>	<p>Marita noticed children off task. She wondered whether a small focus group would benefit those children.</p> <p>Marita was concerned about the children she perceived to be off task.</p>
<i>Fear of student generated responses to tasks</i>	<p>Marita: Jack (pseudonym) appeared to be off task the whole time, but then when he stood up and explained this thinking.</p> <p>Maggie: I saw some children with nothing on their paper. Is it okay for them to be like that?</p> <p>Sabrina: Sometimes you feel guilty about seeing children doing seemingly nothing.</p> <p>Maggie: [the struggling students] would be the ones I would gravitate towards first. The enabling prompts were too hard for them. They weren't even in the zone of the room.</p>	<p>Marita noticed a student who appeared to be off task stand up and explain his reasoning.</p> <p>Maggie noticed students with nothing on their paper. She was concerned that they were off task'.</p> <p>Sabrina expressed concern about students she perceived as 'seemingly doing nothing'.</p> <p>Maggie indicated the enabling prompts were too challenging for some students and her preference for working with those students first.</p>
<i>Students thinking for themselves</i>	<p>Sabrina: What will we do with students who are unable or unwilling to attempt the enabling prompt?</p> <p>Marita: I think [the children] liked the zone of confusion terminology. I saw a child saying to another child, please don't tell him the answer, it is okay to be confused.</p> <p>Marita: I'm going to tell [students]that some of the maths we are doing today is going to be tricky, but that when it is tricky, our brains are growing. I'm not going to mention any strategies, I want to see what they come up with themselves.</p>	<p>Sabrina's question indicates she is unclear about what to do if the enabling prompts are too difficult.</p> <p>Marita observed children acknowledging the 'zone of confusion' as a safe space where hard thinking and struggle takes place.</p> <p>Marita was deliberate in planning to allow students to think for themselves in her enacted lesson. This shows a shift in Marita's thinking</p>
<i>Learning intentions</i>	<p>Maude: I think the learning intentions were very clear at the beginning as in "we have to</p>	<p>Maude noticed how clear the learning intentions were.</p>

<i>Revoicing</i>	explain our thinking, listen to each other...	Megan observed the researcher revoicing the student's explanations.
	Megan: It was always the children that were explaining their work and even sometimes when they weren't making sense themselves you helped with the language... You didn't tell them what to say, they were doing the talking.	
	Melissa: I think it's giving them the language to enable the children to explain their thinking. If you are always modelling that language they are able to justify.	Melissa observed mathematical language being modelled by the researcher which supported students to justify their reasoning.
<i>Children learning from each other</i>	Marita (MPS): Students continued to scaffold each other's ideas...you could see their little lights go on and then they had the courage to have a go themselves.	Marita observed students learning from each other through their explanations of the mathematical ideas.
	Marita: The phrases that you use: Convince me, tell me about your thinking.... I might say "that is not quite the right answer. But let's see if we can explore that" but you say, "keep thinking...has somebody else got something they would like to add". It was [student] directed rather than you directing [the lesson].	Marita observed phrases that the researcher used in the lesson which facilitated student directed learning.
	Melissa: As soon as Rick [pseudonym] shared his idea [of using a number line] there were quite a few that went back and [used] that number line that Rick had explained.	Melissa observed a student sharing his number line strategy to the class which prompted other children to learn from this and use that strategy themselves.
	Maude: Ava struggled as well until she saw that number line and then something clicked in her head and she thought, well I can take away five and take away five.	Maude indicated that a student struggling with the mathematical ideas suddenly made a connection when she saw and heard her peer demonstrating his strategy on a number line. Maude saw the power of this student learning from another.
<i>Thinking time</i>	Megan: You paused and waited and waited ...we often step in.	Megan saw the researcher giving extended wait time which prompted her to reflect that she often steps in too quickly.
	Megan: The power is in your questioning ... the time you gave children to reflect... Even if children did have the concept right but did not know how to articulate it, that thinking time	Megan observed extended student thinking time and questioning to facilitate mathematical reasoning. She indicated these teacher actions led to the success of her students which she was surprised by.

Encouraging persistence

allowed for a lot more. For example, Oliver [pseudonym] was a real surprise to me because at the start of the year he couldn't recognise any numbers, he couldn't count with one to one correspondence. He came up with 5 and 5, when normally he would not. Because he had the time to work it out, he could do that.

Your questions were clarifying their ideas.

Megan: There was a lot of thinking time. Because you [Marita] said that very clearly [to the children] how they need to explain their thinking, I could see that Tory [pseudonym] was sitting here thinking about how she was going to solve the problem and Cory [pseudonym] as well "how am I going to draw that because I sought of know... How am I going to explain that? Just the way you set that up enabled it to happen.

Marita: Jack (pseudonym) felt the power of success, you encouraged him to keep thinking.

[Children felt comfortable] to have a go even if they got it wrong... "keep thinking". Children did continue to persist with that prompt.

Melissa: "Keep thinking" was the most important idea that I got out of the lesson because it's [deflating] to say, "good try" or "try again".

Marita: "Keep thinking" was so powerful...we didn't see those deflated little faces.

Principal MPS: Kids can take that risk and have a go and not be afraid of failure and know that they are going to be supported. If we are using modelled lessons in maths to get that happening then that is awesome.

Maude: The [children] are feeling that sense of achievement from it. One of our very challenging children wasn't really interested [in learning] but when Melissa did

Megan provided feedback to Marita on how she enabled students in her class to experience success by giving them thinking time.

Marita (MPS) observed the researcher encouraging students to keep thinking and saw a child experience success.

Megan (MPS) observed elements of a pedagogical approach that fosters encouragement and persistence.

Melissa (MPS) observed that the researcher's comment 'keep thinking' encourages children to persist. She felt it was the most important idea she took from the lesson.

Marita (MPS) observed the researcher encouraging the children to persist.

During the exit interview the Principal (MPS) indicated that students were more willing to take risks and have a go at challenging tasks.

During the exit interview Maude (MPS) indicated that even previously disengaged students are now experiencing success.

<i>Teacher learning from the researcher</i>	<p>something in her class and she ended up giving him an extending prompt and he was so excited. I have never seen him [engaged] as much.</p> <p>Maude: Did you plan the questions you were asking the children? That was an important part [of the lesson].</p>	<p>Maude asked if the questions that the researcher asked the children during the lesson were planned.</p>
<i>Knowledge and understanding of mathematics content</i>	<p>Melissa: “Were you trying to get them to identify 7 as being 70? Is that what you were trying to do?”</p> <p>Marita: “Would you have at any time thought that it would be okay to do some counting in tens? Like ‘71’ how do you know that is 7 bags of ten? Can we count them together, 10, 20, 30, 40, 50, 60, 70?”</p>	<p>Melissa asked for clarification about the unanticipated teaching move made in the modelled lesson to scaffold students thinking.</p> <p>Marita raised the question of whether counting in tens might be helpful for the students.</p>
<i>Lesson planning and structure</i>	<p>Megan: I’ve lacked a little bit of confidence. I don’t like mathematics. It’s something that I feel like I have to overly think about and plan for. You have given me some more ideas about where I can go...not just for mathematics but for all learning areas...The way you introduce things, reflect on things and get the children involved.</p> <p>Megan: [The modelling] clarified for me how important it is to have those three steps...the introduction, making [the task] clear; having time to [problem solve] and getting examples from [the children] and then getting them to come back and have some powerful reflection time. It brings out the importance of why each of those things is so important.</p> <p>Marita: I run out of time in bringing them [children] back together at the end and you did that so well.</p> <p>Marita: I found it really difficult to bring them back together at the end, that’s where it is hard to know what to say. I need to focus on the thinking. It is so hard not to tell them what to do.</p>	<p>Megan indicated that she lacked confidence in teaching mathematics but the intervention supported her to gain pedagogical content knowledge, some of which she can transfer to other learning areas.</p> <p>Megan noticed the importance of a clear lesson structure to facilitate student reasoning and problem solving.</p> <p>Marita indicated that she often misses the lesson summary as she runs out of time.</p> <p>Marita indicated the difficulty she had in summarising the lesson and refraining from telling students what to do.</p>

<i>Learning through Inquiry and collaboration with colleagues</i>	Maude: It's important to know what questions you are going to ask the children, and have that planned.	Maude could give feedback to Marita about planning the questions before the lesson.
<i>Adjusting the lesson</i>	Megan: The pre and post conversations have been powerful and ...theory put into practice in our classrooms Melissa: The discussions pre and post, seeing it work and being able to see children's thinking [prompted my learning]	Megan valued the pre and post lesson conversations and seeing theory put into practice in her school. In her exit survey, Melissa (MPS) indicated that the pre and post discussions and seeing children responding to the lessons prompted her learning.
<i>Cost factors</i>	Maude: There was so much rephrasing and scaffolding and questioning and then you were using enabling prompts and then went to concrete aids, there was a whole range of things that were going on to facilitate their reasoning. Megan: [That you had to steer the previous lesson in a different way] was powerful for us to see.	Maude observed several teacher actions which facilitated student reasoning including adjusting the lesson to suit the diverse range of learners. Megan indicated that seeing the researcher steer the lesson in a different way to cater for the unanticipated learning needs of the students was powerful for teachers to see.
<i>Timing</i>	Maude: It was so tricky trying to get 6 relief teachers. We had to pay relief teachers but it was all so valuable. Megan: Timing is difficult... I thought that only having a half hour session to [de-brief the last lesson] and not having a chance to plan together what we wanted to get out of today was really difficult.	cost Megan (MPS) raised the issue of timing and how teachers felt they didn't have enough time to de-brief the previous lesson and co-plan for the next modelled lesson.
<i>The lesson plan as a scaffold for teacher learning</i>	Marita: Teachers did not have any say in what was happening until we were all informed that we would be coming along to watch the researcher model some lessons for us. To begin with we thought it would be beneficial but we also thought it was <i>another thing</i> that we had to do and the timing wasn't good as it was in our most crucial part of the day. Marita: [This is a] very similar lesson plan that I've chosen to the one we did in Melissa's class. I have kept the same title, "How	Marita (MPS) indicated teachers were not consulted re the timing of the intervention. Marita indicated her lesson plan was very like the lesson she saw co-taught

*Teacher learning
from enactment*

many more” but I have changed the question to something that I think my kids will resonate with. I hope that I will do a similar tune in to the one [the researcher] did.

Marita: From my own modelling I learnt it’s really tricky to try to be somebody else. I tried to model my lesson from what I saw [the researcher] do and I found that was difficult... Sometimes we need to be just who we are but pull in the pieces of other people’s teaching that we would like to aspire to.

Marita: I’m not sure that I’ll get anything more out of this [enactment] today than I what I’ve already got apart from a belly full of butterflies and feeling a bit unwell. I have already implemented things that I’ve learned from you [researcher] into my classroom and I’ve already trialled the lessons with the kids.

Marita: I found it really difficult to bring them back together at the end, that’s where it is hard to know what to say. I need to focus on the thinking. It is so hard not to tell them what to do.

Marita: I know that I’m not overly fantastic at math’s and I need to be much better, so even though I didn’t want to do that [enactment] I guess it probably stretched me in a lot of ways... I learned that to draw out kids thinking was the most difficult thing for me and remembering as I went around who needed to come back up [whose ideas would be shared].

It highlights the importance of thinking about the questions we ask children.

Marita: When I watched myself [on video], I was saying things like “no, not quite” and I was like Aaargh! I didn’t say things that I have been saying in my classroom a lot like “keep thinking”.

in her colleague’s class. She used the same concept but changed the context.

Marita (MPS) indicated it was too difficult to try to imitate someone else. She learned the importance of being herself and incorporating important pedagogies that she observed into her own practice

Marita did not anticipate that her enactment of a maths lesson would improve her pedagogy because she felt she had already implemented new pedagogies that she observed in the modelled lessons.

Marita indicated the difficulty she had in summarising the lesson and refraining from telling students what to do.

Marita indicated the difficulties she had in choosing which student ideas to focus on during the discussion and probing students thinking during the lesson.

Marita realised in her enacted lesson that she did not say the encouraging phrases that she had been practising in her classroom

	<p>Marita: The hardest thing I found [from enacting the practice] was looking over students work to find examples to use to model for other students and bringing the lesson back in. It's something I need more practice at.</p> <p>Maude: I think teachers learned that you need really clear learning intentions and use precise mathematical terminology, a structure and [the lesson] needs to be challenging for all students, to use real life contexts and to capture the children's interests and apply it to their world.</p> <p>Maude: I've now seen in most classrooms the teachers using the same lesson structure as [the researcher] did and [having the] whole class [working on the task] using the enablers and extenders. There has been more evidence of change in [teachers] teaching practice in having high expectations and making sure they were challenging everyone.</p>	<p>Marita indicated the most difficult thing for her was in choosing which student ideas to focus on during the discussion</p> <p>In the post intervention interview, Maude indicated that the modelling prompted teachers to have clear learning intentions, use mathematical language, use a particular lesson structure which facilitates student reasoning and implement challenging tasks.</p> <p>In the post lesson interview Maude indicated that she had seen most teachers using the same lesson structure that the researcher used and having the whole class working on the task including enabling and extending prompts. Teachers now had higher expectations of the children and were challenging all students.</p>
<p><i>Benefits of Modelling</i></p>	<p>Marita: [Re comparing classroom modelling with other forms of PL]. Everyone involved gets something out of it that they can straight away take back to their own practice because you're involved in it. We're all there. You feel inspired by what you've seen. It's not like somebody telling you what to do, it's somebody modelling a way to do things.</p> <p>Melissa: Seeing something work in a working classroom is much more beneficial than reading or PD that is totally removed from the classroom</p>	<p>Modelling benefits</p>
<p><i>Modelling proficiencies</i></p>	<p>Maude: It was a valuable intervention for our school. Just the fact that you were modelling the proficiencies rather than looking at content. You were helping the classroom teachers with their practice of mathematics in any content area because you were helping them to develop children's proficiencies.</p>	<p>Maude indicated that modelling teacher actions to develop student's proficiencies in mathematics was powerful because it made the actions transferable across mathematics lessons.</p>

Appendix 13: Themes: Swift Parrot Primary School

Selected examples of significant statements and their meanings Stage 1 Swift Parrot Primary School

Selected samples of significant statements from post lesson discussions and surveys	Formulated meanings
Sophie: You were asking questions to prompt the children to explain their thinking. You gave thinking time and encouraged the children.	Sophie observed elements of a pedagogical approach that fosters encouragement, student thinking time and facilitates student reasoning.
Suri: Icy pole sticks made the kids accountable. Rephrasing some of the kid's responses.	Suri observed elements of a pedagogical approach that supports classroom conversations including revoicing and icy-pole sticks to encourage participation.
Suri: When introducing new concepts(terminology) making connections to their [students] prior understandings e.g., an equation can sometimes be called a sum or a number sentence.	Suri observed mathematical language being used to connect with student's prior mathematical understandings
Sally: I noticed the amount of thinking time you gave the children. A couple of us were saying we would like to get to that stage where we can give that amount of thinking time without there being lots of calling out.	Sally observed a pedagogical approach that allowed extended student thinking time. Sally and her colleagues hope they could emulate the approach.
Marita: You extended their thinking by asking questions: "tell me more about, is that the same as..." Think pair share – they had to explain their thinking to somebody else. Rephrasing the student's responses. You got them to share their reasoning to assist other students with their learning. Sometimes if they were a little off track, but you knew where they were going, you rephrased to assist them get to where you wanted them to go, by saying things like are you saying? or is that the same as?	Marita observed use of identifiable teacher actions that facilitate student reasoning and problem solving including revoicing and prompting students for further participation.
Marita: Initially I thought my goodness, my children would not be able to do this because I've got the other [grade] 2/3 class...Just seeing how you slowed the lesson right down... You spent time going through the task so that the children understood [what the task was asking them to do] ... [their responses to the task] surprised me... The children were very much getting the concept at the end and wanted to continue working, even when they heard the bell, which was surprising.	The modelling had the desirable effect of enabling Marita to see the children's engaged response to the challenging task.
Sabrina: The thinking time you gave really stood out to me and made me think how I don't give enough time. The other thing that stood out was that you gave [the students] plenty of time to think but then when you couldn't see that they had something just yet, you always went back to them "Keeping thinking and I will come back to you" and you did go back to them.	Sabrina observed the researcher provided thinking time including wait time to students after calling on them to explain. Her observation prompted her to reflect on her practice.
Suri: Uploading higher order thinking student work sample on IWB and asking that student to share reasoning and thinking	Suri observed the researcher used a tool for making children's thinking visible to other students in the class.
Sheila: I thought it was good how you explained the terms prior to getting into the actual teaching and making sure the children understood what was going	The modelling of mathematical language prompted Sheila to reflect on her practice.

on. It was probably something that we can work on. We tend to rush into the content rather than making sure they understand.

Sheila: Most of the kids were on task most of the time, so the use of things like the icy pole sticks and then thumbs up were great. Most kids most of the time were engaged in thinking.

Sheila: The error management was important... so rather than just say wrong or no, there was nothing negative about your responses to their thinking— “Keep thinking? Would you like someone to help you with that?”

Sheila: The idea of using the technology was good.

Sue: When you asked the children to talk about their answers, a lot of the kids had worked it out and had solutions on the paper but when it came to putting it into words they were struggling and I was noticing how you were scaffolding them into breaking it down into smaller steps – “tell me about this, what did you notice about this side, what did you do next?” – you were helping break that thinking into little chunks so that [the children] were able to explain it logically to other people.

Sophie: Some of the lower kids were having a really good go without giving up.

Suri: Icy-pole sticks – students all become accountable

Suri: Students sharing responses in their own reasoning for others to learn from. Using students to explain why.

Sally: Didn’t say Holly was wrong – said “keep thinking”

Marita: [Researcher] goes back to confused students “Keep thinking and I will come back to you” ...they suddenly get it!

Sheila observed student behaviour within the lesson, which was not the focus of the observation. She was comfortable with her perceived engagement of most the students.

Sheila observed the researcher took a positive stance toward the children, encouraging them and promoting persistence.

Sheila observed the researcher used a technological tool for making children’s thinking visible to other students in the class.

Sue observed the researcher supporting and scaffolding students with their explanations by revoicing or repeating what they had said.

Sophie observed the lower attaining students persisting.

Suri observed the use of icy-pole sticks ensured students were accountable and expected to participate in the discussion.

Suri observed students sharing their ideas so that others could learn from them.

Sally observed students were encouraged to keep thinking

Marita observed the researcher providing wait time to those students who had been called upon to explain their thinking and in doing so the students experienced success.

Selected examples of significant statements and their meanings Stage 2 Swift Parrot Primary school

Selected samples of significant statements from post lesson discussions and surveys

Formulated meanings

Marita: Students found it really hard to explain their reasoning [because we’ve only just started looking at maths in more of a problem solving way]. For example, Abbie knew the correct answers on her times but could not explain.

Marita observed the difficulty students had in explaining their thinking. She attributed this to her teaching.

Marita: The [students] are not used to seeing problems like that. I have only just started to introduce problems in that way... Seeing a problem seemed quite overwhelming for them.

Marita observed the students grappling with the problem which seemed foreign to them.

Suri: Even though the students can count in 5’s and 10’s, they don’t necessarily transfer that to the clocks. They haven’t quite got the concept of the passage of time.

Suri observed that the children in grade 3/4 were unable to transfer their knowledge of skip counting by 5’s to the analogue clock face.

Sheila: Many of them were quite confident with making factual statements about what the time was but explaining what that meant was where they fell over.

Marita: We're using [a program called] Stepping Stones... I follow the steps that are included... [students are required to give] either a right or wrong answer. [This program tells] us teachers [what] to do... The lesson that I just saw is very different to how I would run my maths lesson [using the program]. It is hard to find that in between way of teaching.

Marita: After seeing you last time I have adapted my way of thinking in that we are doing a lot more problem solving and giving a lot more thinking time.

Sheila: We tend to accept a right or wrong answer instead of saying "okay, you got it right but why is it right"?

Sabrina: One of the things that stood out to me was the amount of time you let them struggle. It's knowing how long to let them struggle and when to move on. I reflected on what I have been doing and I don't think I let my students struggle enough. I think I pounce too quickly to do the teaching and make things easier for them. I need to let them struggle more.

Sheila: After your first visit Marita, Suri and Sally were trying to implement strategies they saw and reproduce your lesson. They probably weren't the target for the change in pedagogy. The more reluctant the person usually correlates with the greatest need. There is some resistance.

Sheila: The second lesson showed them that if you persevere and rephrase your questions and give the children time to think it through then eventually they will get there. I think that for those people who are not that keen on this approach they saw that you don't give up, you keep going, you give time, you challenge, you let the kids stew...a little bit in that zone of confusion. The fact that it wasn't as fluent as the previous lesson in terms of the student's responses was a blessing in disguise. It showed people things don't always go to plan, but if you stop...if you know the maths you are trying to teach and give the kids time to think, they can get there.

Marita: [My goal is to] make the learning (particularly about time) more holistic – linking it to all areas of time. No concept taught in isolation

Sabrina: [My goal is to] use the question of one child in the zone of confusion to clarify information for other children that are struggling.

Sue: [My goal is to] give more time for children to work problems out rather than rushing in to provide teacher assistance.

Marita: [My goal is to] allow students to struggle more, to have a go and to use the 'zone of confusion' expression to let students know it's okay not to know things straight away.

Sheila observed the students had difficulty with explaining the duration of time.

The observation enabled Marita to see a different pedagogical approach and prompted to reflect on her current teaching approach.

Marita indicated she was now giving more opportunities for problem solving and extended thinking time.

Sheila observed the researcher probing students to articulate why their solution was correct which was different to her usual approach.

Sabrina grappled with the idea of allowing students to struggle and what that might mean for her teaching.

Sheila indicated that whilst some teachers attempted to implement strategies they observed in the modelled lesson, others were resistant to making changes to their approach to teaching and others had already made changes to their practice. She noted that the more reluctant teachers seemed to have the greatest need for change.

Sheila's comment suggests that the modelled lesson does not have to be a 'perfect' lesson for teachers to learn from it. The teacher learning comes from observers seeing the modelling teacher persevere, rephrase questions, give thinking time and modify the lesson on the fly.

The modelling and post-lesson discussion prompted Marita to reflect on her teaching and set herself a goal of teaching mathematical concepts holistically.

Sabrina set herself a goal to use children's questions to clarify information for other children that are struggling.

Sue set herself a goal to give students more thinking time.

Marita set herself a goal to allow children time to think for themselves.

Sophie: [My goal is to] Give more time to ‘keep thinking’, use a visualiser and make up more authentic problems for the children to solve	Sophie set herself a goal of giving students more thinking time, making their thinking visible by using a document camera and creating authentic problems for students to solve.
---	--

Selected examples of significant statements and their meanings Stage 3: Co-teaching with Sophie

Selected samples of significant statements from post lesson discussions and surveys	Formulated meanings
Marita: The students brainstormed what tools they would use, they displayed their thinking in different ways.	Marita observed that students chose their own tools for solving the problem.
Sabrina: The researcher roves and has one on one conversations with the students. She asks them to explain their reasoning. [By doing that] she can see how the whole class is going and whether to move on or not.	Sabrina observed the researcher circulating and paying attention to the students thinking and solutions strategies as they worked on the tasks. This is an important element of a pedagogical approach that is focussed on student reasoning and problem solving.
Sabrina: You used children’s examples on the smartboard and got children to come up and explain it.	Sabrina observed the researcher used a tool for making children’s thinking visible to other students in the class.
Sophie: It was obvious [from the lesson] that we have not done any work on arrays yet or on division but we have done some stuff on multiplication. Some of them did really well.	The student’s responses to the task prompted Sophie to reflect on the mathematics that the students need to learn.
Sabrina: We are still not clear about the zone of confusion. If one of our goals is for children to learn to use efficient strategies for problem solving, it does not seem helpful to leave them confused for long periods. I get that you want to use children to share their strategies with their peers, but what about those children who do not make those connections easily? At what point would you intervene with children like Talia (pseudonym) who was hung up with the colours of the counters?	Sabrina grappled with the idea of the ‘zone of confusion and what that meant for her teaching. She questioned when it would be appropriate to intervene for low attaining children.

Selected examples of significant statements and their meanings Stage 4

Selected samples of significant statements from post lesson discussions and surveys	Formulated meanings
Sophie: The rationale for the lesson was the hardest. Trying to think of the theory behind something is not something we do. We just know what we have to do and we don’t go looking for the theory behind it.	Sophie indicated that thinking about the rationale for a lesson was difficult because it’s not something she is used to.
Sabrina: The reading (Stein et al., 2008) was very helpful in that the structure of the lesson was broken down into 5 stages and it made sense. I thought now I know what I need to do. [For example] We’ve all been grappling with the kids being in the zone of confusion. We all felt like we were leaving them there. [I learnt] how as a responsible teacher to move them on if [I’m] not doing that explicit teaching and what [my] role in the lesson was. It gave me a lot more clarity.	The modelling prompted Sabrina to read about 5 practices for orchestrating mathematical discussions. The 5 practices answered questions she had about the pedagogical approach.
Sabrina: [From the reading I learnt about] anticipating problems you might have and how you would address that. [For example] when you choose the students [to show their work], you are targeting those responses so that the [examples] they present	Sabrina reflected upon a reading she had been given and was able to articulate an important point about purposefully selecting students and their solutions

to the class will help move them toward understanding. If the example that you're looking for is not there, you can create one.

Sabrina: We are coming from a model where we teach kids how to do things and then [we] let them practise those skills and revise them, it's been a bit hard to think [we're] giving them the challenge before [we've] taught them how to do it and what if nobody in the group knows how to do it, where do you go?

which will contribute to the mathematical goal of the lesson.

The modelling challenged Sabrina's pedagogical approach to teaching as she grappled with new ideas that confronted her teacher directed view of learning.

Selected examples of significant statements and their meanings Post Intervention Interviews

Selected samples of significant statements from post lesson interviews and surveys

Formulated Meanings

Suri: [The modelling prompted me to] recognise what my students were capable of – I observed their responses after wait time and listened to their reasoning.

The modelling had the desirable effect of enabling Suri to see how students responded to the pedagogical approach

Suri: [The modelling] put theory into context. It made learning the strategies more engaging and relevant. It made [the pedagogical approach] seem achievable.

Suri indicated the modelling enabled her to see that the pedagogical approach was achievable as it connected theory with practice.

Suri: [My goal is to] produce problem solving lessons, allow greater thinking time, allow students to share their reasoning.

Suri could articulate a goal which involved allowing students greater thinking time, problem solving and opportunities to share their reasoning.

Marita: [The modelling prompted me to] allow more thinking time and allow kids to struggle

Marita could articulate a goal for her future practice which involved extended thinking time for students where they could grapple with problems

Sally: [The main elements which prompted my learning were]: Seeing children persevere and witnessing the thinking going on and understanding (and engagement). I know I need to allow more thinking time, but seeing it is very powerful.

The main elements which prompted Sally's learning were seeing the children persevering and responding positively to the challenging tasks.

Sally: [My goal is to] implement a problem-solving lesson once or twice a week using lots of thinking time, enabling and extending prompts, zone of confusion...

Sally could articulate a goal for her practice, which included problem solving, thinking time and enabling and extending prompts

Sabrina: We would have got a lot more value out of this [intervention] if it was not concurrent with our reporting period.

Sabrina indicated that the timing of the intervention in her school was not ideal due to reporting.

Principal: I think [teachers] really got a lot from seeing the high expectations [in the classroom] and realising that our kids could rise to that expectation.

The Principal at Swift Parrot Primary School (SPPS) indicated that the observation of the modelled lesson prompted teachers to see that the students could rise to the high expectations that the researcher had of the children.

Principal: They [teachers] liked to see you work hard... it wasn't a process that was a breeze for you...you did have to adjust and modify as you were teaching and have different parts that didn't go quite right and then how you adjusted ...

The Principal (SPPS) indicated that watching the researcher work hard in adjusting and modifying the lesson was welcomed by the observing teachers as it seemed real to them.

Principal: [Observing teachers] learned that our kids could actually do the things that you were talking about. They actually saw it in practice ... so they copied your practice...

The Principal (SPPS) indicated that teachers learned from seeing their students respond favourably to the modelled lesson. Teachers copied the lesson to trial the new approach in their classrooms.

Even almost the next day teachers were actually trialling; and I've had teachers come up and share with me; we've had sharing and briefing around what kids have been doing in classes. So I'd definitely say that there has been an uptake in teaching practice.

Principal: I just think seeing someone else do it is such a powerful thing, then seeing someone else practice it as well, one of their colleagues. It's a powerful step into them having a go, rather than just going to professional learning and talking about something. I think it bridges that gap around, "Well what does it look like in my classroom?" So definitely yeah, we have seen changes in practice and there's been far more high level discussion in teams and groups around the problem solving in maths.

Principal: It was hard for us to get the two hours... we did all that because we had the back-up... I definitely believe that the two modelled lessons at the start were really good to see you modelling... after that if we'd had an extended model where every teacher in the team then had a turn at doing the co-planning and co-teaching with you. So that everybody got exposure.

Principal: I think you've got to have a tight model around it and that's what made it work so well. Our 'Watch others Work' stuff that we did last year was looser and still far more invitational, but we have some staff here that would take forever to get onboard. I think that the four session structure was the power of it. It's the link between talking about practice, and doing practice, changing practice. We've got so many teachers in the teaching profession that can do the rhetoric, but the practice in the classroom doesn't match the rhetoric.

Principal: [Modelling] it's the final link, it's the most important link. You can do the professional learning, the talking about it, then if you watch someone do the practice and then the final step of having a go yourself and evaluating and reflecting all the way through, I just think it's the final thing... what you did with us was great. It was excellent.

Principal: Exploring the model through other curriculum areas would be interesting... embedding

The Principal (SPPS) indicated that seeing the lessons modelled followed by enactment was powerful professional learning. It enabled teachers to actually have a go at the practice rather than talking about it. She noted that there has been far more discussion in teams about the problem solving.

The Principal indicated that it was difficult to find time for the intervention, but that seeing the two modelled lessons was powerful for her teachers. She suggested the possibility of an extended model where each observing team member had an opportunity to co teach and enact pedagogies would assist everyone to maximise their potential for learning.

The Principal (SPPS) indicated that the structure and tight processes of the model made it robust. The four-staged structure, which included seeing practice modelled, deconstructing that practice and opportunities to enacting the practices was powerful.

The Principal (SPPS) indicated that the model – seeing practice, deconstructing practice and enacting practice was an excellent form of professional learning for her teachers.

The Principal (SPPS) saw the potential of exploring and embedding the model in other learning areas.

the model, probably in your whole year of team meetings and learning group meetings.

Sheila: [From your modelling, teachers learned that]: Providing time for children to think through situations produces better understanding; that making mathematics “easy” is not necessarily good practice (i.e. “the zone of confusion” is not necessarily to be avoided).

Sheila indicated that from the modelling, teachers learned that extended thinking time gives children the opportunity to think deeply and develop understanding. She also indicated teachers learned that effective practice does not mean tasks are easy.

Sheila: Using technology to display children’s work and then getting them to explain their thinking was really powerful We’ve since used money to buy two document projectors. ... [Teachers now have the] desire to be able to display chn’s work during lessons for student explanation / group discussion.

Sheila indicated that the staff at SPPS had purchased document cameras because of the intervention. She noted that teachers saw that using technology to support children explaining their thinking was very powerful in that it made thinking visible to the whole class.

Sheila: Making mathematics a group activity is a powerful technique – and that allowing discussion of concepts / thinking is to be encouraged rather than feared as an indicator of time off task

Sheila indicated that teachers learned that having the whole class engaged in the task is powerful for the discussion of concepts and that thinking is to be encouraged rather than feared as time off task.

Sheila: thorough planning improves teaching

Sheila indicated that teachers recognised that thorough planning improves teaching. Teachers who had not considered the rationale for a lesson before worked together to create a lesson where they were clear about their purpose and successfully anticipated student responses.

Sheila: Teachers are now allowing students more time to think through mathematics, a shift towards more “problem solving” rather than worksheet driven lessons.

Sheila indicted evidence of changed teaching practice was that teachers had shifted their emphasis to a more problem based approach rather than worksheet driven lessons.

Sheila: Teachers copied either the exact lesson or slightly modified to suit age group soon after observing the demonstration lesson.

Sheila indicated that teachers either copied the modelled lesson or modified the lesson to suit their age groups soon after observing it. This suggests the modelled lesson may be an important scaffold for enactment.

Sheila: Planning proforma has been used by teachers for shared lessons.

Sheila indicated that the lesson plan proforma had been utilised by teachers in subsequent planning sessions. This suggests that they saw it as helpful.

Sheila: The duration of the sessions did not match the segments of the school day here and hence was disruptive (eating in to teacher lunch break). If the timing came from within the school rather than the presenter’s timetable it might have been even more productive.

Sheila indicated timing of the intervention was problematic in her school setting. The intervention was somewhat disruptive in the school as it did not align with the school day. Sheila suggested that the timing of the intervention should be a prioritised to suit the schools needs rather than researcher’s time

Sheila: Although there was consultation about the content of the modelled lessons this was often rushed

Sheila pointed out that although the researcher consulted teachers about the content of the modelled

or done via email which did not always result in lessons closely aligned to previous lessons in the class. Teachers would have like to have seen greater alignment of lessons.	lessons it was often rushed and teachers did not feel the planned lessons necessarily aligned with their current focus.
Sheila: There was some accountability to being involved “up close and personal” that is often missing...the ongoing nature made it more meaningful rather than “one off” pd approaches.	Sheila indicated that the model had accountability built in as it was situated in teachers own classrooms with the expectation that observing teachers would contribute to pre-and post-lesson discussions to deconstruct the practice they observed and collaboratively plan for the co-taught lesson.
Sophie: [The enactment prompted me to] think about the purpose of the lesson and the challenges for [students] before the lesson.	The enactment prompted Sophie to reflect on her planning to think about the purpose of the lesson and anticipating any difficulties students might encounter.
Sophie: Initially I thought the lessons may have been too difficult, but now I think more about catering for the top end and [using enabling prompts] whereas before I was catering for the average or below average and spending most of my time with them. I learned that it’s okay to challenge all the kids.	The favourable student responses to challenging tasks in the modelled lessons prompted Sophie to reflect on her teaching. She indicated that in the past she catered for the average to below average students, but now she is committed to the idea of catering for all students.
Sophie: My goal is to use more open ended real life problems. Even though I knew that was what I was meant to be doing, I don’t think I was doing that very well.	Sophie committed to the idea of using problematic maths tasks.
Sophie: I now use [enabling and extending prompts] in my maths lessons.	The modelling prompted Sophie to commit to using enabling and extending prompts to differentiate tasks in her maths lessons.
Sophie: We missed out on our planning team meetings and it felt like our valuable learning group time was being yanked away from us four times.	Sophie indicated dissatisfaction that valuable team planning time was removed so that her team could be part of the intervention
Sophie: The kids have really benefitted. They talk about the zone of confusion and that it’s okay to be there.	Sophie’s comment indicates how much the students enjoyed the pedagogical approach.

Selected Theme Clusters with their Related formulated meanings Swift Parrot Primary School

Themes	Illustrative examples of teacher comments	Clustered Meanings
Challenging teacher orientation towards teaching as telling	Sabrina (SPPS): We are coming from a model where we teach kids how to do things and then [we] let them practise those skills and revise them, it’s been a bit hard to think [we’re] giving them the challenge before [we’ve] taught them how to do it and what if nobody in the group knows how to do it, where do you go? Marita: We’re using [a program called] Stepping Stones... I follow the steps that are included... [students are required to give] either a right or wrong answer... The lesson that I just saw is very different to how I would run my maths lesson [using the program]. It is hard to find that in between way of teaching.	The modelling challenged Sabrina’s pedagogical approach to teaching as she grappled with new ideas that confronted her teacher directed view of learning. The observation enabled Marita to see a different pedagogical approach and prompted to reflect on her current teaching approach.
<i>Student difficulties in explaining their thinking</i>	Marita: Students found it really hard to explain their reasoning. For example, Abbie	Marita observed the difficulty students had in explaining their thinking.

<i>Fear of student generated responses to tasks</i>	<p>knew the correct answers on her times but could not explain.</p> <p>Sheila: Many of them were quite confident with making factual statements about what the time was but explaining what that meant was where they fell over.</p> <p>Sabrina (SPPS): We are coming from a model where we teach kids how to do things and then [we] let them practise those skills and revise them, it's been a bit hard to think [we're] giving them the challenge before [we've] taught them how to do it and what if nobody in the group knows how to do it, where do you go?</p>	<p>Sheila observed the students had difficulty with explaining the duration of time.</p> <p>The modelling challenged Sabrina's pedagogical approach to teaching as she grappled with new ideas that confronted her teacher directed view of learning.</p>
<p>Confronting ideas about catering for diversity <i>Expectation of student achievement</i></p>	<p>Sophie (SPPS): Initially I thought the lessons may have been too difficult, but now I think more about catering for the top end and [using enabling prompts] whereas before I was catering for the average or below average and spending most of my time with them.</p> <p>I learned that it's okay to challenge all the kids.</p> <p>Marita: [The modelling prompted me to] allow more thinking time and allow kids to struggle</p> <p>Principal: I think [teachers] really got a lot from seeing the high expectations [in the classroom] and realising that our kids could rise to that expectation.</p>	<p>The favourable student responses to challenging tasks in the modelled lessons prompted Sophie (SPPS) to reflect on her teaching. She indicated that in the past she catered for the average to below average students, but now she is committed to the idea of catering for all students.</p> <p>Marita could articulate a goal for her future practice which involved extended thinking time for students where they could grapple with problems</p> <p>The Principal (SPPS) indicated that the observation of the modelled lesson prompted teachers to see that the students could rise to the high expectations that the researcher had of the children.</p>
<i>Ability groupings</i>	<p>Sophie: Initially I thought the lessons may have been too difficult, but now I think more about catering for the top end and [using enabling prompts] whereas before I was catering for the average or below average and spending most of my time with them.</p> <p>I learned that it's okay to challenge all the kids.</p>	<p>The favourable student responses to challenging tasks in the modelled lessons prompted Sophie to reflect on her teaching. She indicated that in the past she catered for the average to below average students, but now she is committed to the idea of catering for all students.</p>
<i>Grappling with challenging tasks</i>	<p>Sabrina: One of the things that stood out to me was the amount of time you let them struggle. It's knowing how long to let them struggle and when to move on. I reflected on what I have been doing and I don't think I let my students struggle enough. I think I pounce too quickly to do the teaching and make things easier for them. I need to let them struggle more.</p> <p>Marita: They [students] are not used to seeing problems like that. I have only just started to introduce problems in that way. Seeing a problem seemed quite overwhelming for them.</p>	<p>Sabrina (SPPS) grappled with the idea of allowing students to struggle and what that might mean for her teaching.</p> <p>Marita observed the students grappling with the problem which seemed foreign to them.</p>

Enabling and extending prompts

Perceived disengagement of some students

Marita: After seeing you last time I have adapted my way of thinking in that we are doing a lot more problem solving and giving a lot more thinking time.

Sabrina: We are still not clear about the zone of confusion. If one of our goals is for children to learn to use efficient strategies for problem solving, it does not seem helpful to leave them confused for long periods. I get that you want to use children to share their strategies with their peers, but what about those children who do not make those connections easily? At what point would you intervene with children like Talia (pseudonym) who was hung up with the colours of the counters?

Marita: [The modelling prompted me to] allow more thinking time and allow kids to struggle

Suri: [My goal is to] produce problem solving lessons, allow greater thinking time, allow students to share their reasoning.

Sally: [The main elements which prompted my learning were]: Seeing children persevere and witnessing the thinking going on and understanding (and engagement). I know I need to allow more thinking time, but seeing it is very powerful.

Sophie: Initially I thought the lessons may have been too difficult, but now I think more about catering for the top end and [using enabling prompts] whereas before I was catering for the average or below average and spending most of my time with them.

I learned that it's okay to challenge all the kids.

Sally: [My goal is to] implement a problem-solving lesson once or twice a week using lots of thinking time, enabling and extending prompts, zone of confusion...

Sophie: I now use [enabling and extending prompts] in my maths lessons.

Sheila: Making mathematics a group activity is a powerful technique – and that allowing discussion of concepts / thinking is to be encouraged rather than feared as an indicator of time off task

Marita indicated she was now giving more opportunities for problem solving and extended thinking time.

Sabrina (SPPS) grappled with the idea of the 'zone of confusion and what that meant for her teaching. She questioned when it would be appropriate to intervene for low attaining children.

Marita could articulate a goal for her future practice which involved extended thinking time for students where they could grapple with problems

Suri could articulate a goal which involved allowing students greater thinking time, problem solving and opportunities to share their reasoning.

The main elements which prompted Sally's learning were seeing the children persevering and responding positively to the challenging tasks.

The favourable student responses to challenging tasks in the modelled lessons prompted Sophie to reflect on her teaching. She indicated that in the past she catered for the average to below average students, but now she is committed to the idea of catering for all students.

Sally could articulate a goal for her practice, which included problem solving, thinking time and enabling and extending prompts

The modelling prompted Sophie to commit to using enabling and extending prompts to differentiate tasks in her maths lessons.

Sheila indicated that teachers learned that having the whole class engaged in the task is powerful for the discussion of concepts and that thinking is to be encouraged rather than feared as time off task.

Observed pedagogies	<p>Sally: I noticed the amount of thinking time you gave the children. A couple of us were saying we would like to get to that stage where we can give that amount of thinking time without there being lots of calling out.</p>	<p>Sally observed a pedagogical approach that allowed extended student thinking time. Sally and her colleagues hope they could emulate the approach.</p>
<i>Thinking time</i>	<p>Sabrina: The thinking time you gave really stood out to me and made me think how I don't give enough time. The other thing that stood out was that you gave [the students] plenty of time to think but then when you couldn't see that they had something just yet, you always went back to them "Keeping thinking and I will come back to you" and you did go back to them.</p>	<p>Sabrina observed the researcher provided thinking time including wait time to students after calling on them to explain. Her observation prompted her to reflect on her practice.</p>
	<p>Marita: [Researcher] goes back to confused students "Keep thinking and I will come back to you" ...they suddenly get it!</p>	<p>Marita observed the researcher providing wait time to those students who had been called upon to explain their thinking and in doing so the students experienced success.</p>
	<p>Sue: [My goal is to] give more time for children to work problems out rather than rushing in to provide teacher assistance.</p>	<p>Sue (SPPS) set herself a goal to give students more thinking time.</p>
	<p>Marita: [My goal is to] allow students to struggle more, to have a go and to use the 'zone of confusion' expression to let students know it's okay not to know things straight away.</p>	<p>Marita (SPPS) set herself a goal to allow children time to think for themselves.</p>
	<p>Sophie: [My goal is to] Give more time to 'keep thinking', use a visualiser and make up more authentic problems for the children to solve.</p>	<p>Sophie (SPPS) set herself a goal of giving students more thinking time, making their thinking visible by using a document camera and creating authentic problems for students to solve.</p>
	<p>Sheila: [From your modelling, teachers learned that]:Providing time for children to think through situations produces better understanding; that making mathematics "easy" is not necessarily good practice (i.e. "the zone of confusion" is not necessarily to be avoided).</p>	<p>Sheila (SPPS) indicated that from the modelling, teachers learned that extended thinking time gives children the opportunity to think deeply and develop understanding. She also indicated teachers learned that effective practice does not mean tasks are easy.</p>
	<p>Sheila: Teachers are now allowing students more time to think through mathematics, a shift towards more "problem solving" rather than worksheet driven lessons.</p>	<p>Sheila (SPPS) indicted evidence of changed teaching practice was that teachers had shifted their emphasis to a more problem based approach rather than worksheet driven lessons.</p>
<i>High Expectations</i>	<p>Suri: [The modelling prompted me to] recognise what my students were capable of – I observed their responses after wait time and listened to their reasoning.</p>	<p>The modelling had the desirable effect of enabling Suri to see how students responded to the pedagogical approach.</p>
	<p>Sophie: You were asking questions to prompt the children to explain their</p>	<p>Sophie observed elements of a pedagogical approach that fosters encouragement, student thinking</p>

	<p>thinking. You gave thinking time and encouraged the children.</p> <p>Sally: [The main elements which prompted my learning were]: Seeing children persevere and witnessing the thinking going on and understanding (and engagement). I know I need to allow more thinking time, but seeing it is very powerful.</p> <p>Sally: [My goal is to] implement a problem-solving lesson once or twice a week using lots of thinking time, enabling and extending prompts, zone of confusion...</p>	<p>time and facilitates student reasoning.</p> <p>The main elements which prompted Sally's learning were seeing the children persevering and responding positively to the challenging tasks.</p> <p>Sally could articulate a goal for her practice, which included problem solving, thinking time and enabling and extending prompts</p>
	<p>Sheila: [From your modelling, teachers learned that]: Providing time for children to think through situations produces better understanding; that making mathematics "easy" is not necessarily good practice (i.e. "the zone of confusion" is not necessarily to be avoided).</p>	<p>Sheila indicated that from the modelling, teachers learned that extended thinking time gives children the opportunity to think deeply and develop understanding. She also indicated teachers learned that effective practice does not mean tasks are easy.</p>
	<p>Sheila: Teachers are now allowing students more time to think through mathematics, a shift towards more "problem solving" rather than worksheet driven lessons.</p>	<p>Sheila indicted evidence of changed teaching practice was that teachers had shifted their emphasis to a more problem based approach rather than worksheet driven lessons.</p>
	<p>Principal: I think [teachers] really got a lot from seeing the high expectations [in the classroom] and realising that our kids could rise to that expectation.</p>	<p>The Principal (SPPS) indicated that the observation of the modelled lesson prompted teachers to see that the students could rise to the high expectations that the researcher had of the children.</p>
	<p>Principal: [Observing teachers] learned that our kids could actually do the things that you were talking about. They actually saw it in practice ... so they copied your practice... Even almost the next day teachers were actually trialling; and I've had teachers come up and share with me; we've had sharing and briefing around what kids have been doing in classes. So I'd definitely say that there has been an uptake in teaching practice.</p>	<p>The Principal (SPPS) indicated that teachers learned from seeing their students respond favourably to the modelled lesson. Teachers copied the lesson to trial the new approach in their classrooms.</p>
<p><i>Revoicing and prompting for further participation</i></p>	<p>Marita (SPPS): You extended their thinking by asking questions: "tell me more about, is that the same as...". Sometimes if they were a little off track, but you knew where they were going, you rephrased to assist them get to where you wanted them to go, by saying things like "are you saying?" or "is that the same as?"</p>	<p>Marita observed use of identifiable teacher actions that facilitate student reasoning and problem solving including revoicing and prompting students for further participation.</p>
<p><i>Scaffolding students to explain their thinking</i></p>	<p>Sue (SPPS): When you asked the children to talk about their answers they were</p>	<p>Sue observed the researcher supporting and scaffolding students</p>

	<p>struggling and I was noticing how you were scaffolding them into breaking it down into smaller steps – “tell me about this, what did you notice about this side, what did you do next?” – you were helping break that thinking into little chunks so that [the children] were able to explain it logically to other people.</p> <p>Sabrina: The researcher roves and has one on one conversations with the students. She asks them to explain their reasoning. [By doing that] she can see how the whole class is going and whether to move on or not.</p>	<p>with their explanations by revoicing or repeating what they had said.</p> <p>Sabrina observed the researcher circulating and paying attention to the students thinking and solutions strategies as they worked on the tasks. This is an important element of a pedagogical approach that is focussed on student reasoning and problem solving.</p>
	<p>Sheila: We tend to accept a right or wrong answer instead of saying “okay, you got it right but why is it right?”</p>	<p>Sheila observed the researcher probing students to articulate why their solution was correct which was different to her usual approach.</p>
<i>Students learning from each other</i>	<p>Suri: Students sharing responses in their own reasoning for others to learn from. Using students to explain why.</p>	<p>Suri observed students sharing their ideas so that others could learn from them.</p>
<i>Connections to prior understandings</i>	<p>Suri: When introducing new concepts(terminology) making connections to their [students] prior understandings e.g., an equation can sometimes be called a sum or a number sentence.</p> <p>Sheila: I thought it was good how you explained the terms prior to getting into the actual teaching and making sure the children understood what was going on. It was probably something that we can work on. We tend to rush into the content rather than making sure they understand.</p> <p>Marita: [My goal is to] make the learning (particularly about time) more holistic – linking it to all areas of time. No concept taught in isolation</p>	<p>Suri observed mathematical language being used to connect with student’s prior mathematical understandings</p> <p>The modelling of mathematical language prompted Sheila to reflect on her practice.</p> <p>The modelling and post-lesson discussion prompted Marita to reflect on her teaching and set herself a goal of teaching mathematical concepts holistically.</p>
<i>Encouragement to persist</i>	<p>Sheila (SPPS): The error management was important... so rather than just say wrong or no, there was nothing negative about your responses to their thinking– “Keep thinking? Would you like someone to help you with that?”</p> <p>Sally: Didn’t say Holly was wrong – said “keep thinking”</p> <p>Sophie: Some of the lower kids were having a really good go without giving up.</p> <p>Marita: Goes back to confused students “Keep thinking and I will come back to you” ...they suddenly get it!</p>	<p>Sheila observed the researcher took a positive stance toward the children, encouraging them and promoting persistence.</p> <p>Sally observed students were encouraged to keep thinking</p> <p>Sophie observed the lower attaining students persisting.</p> <p>Marita observed the researcher providing wait time to those students who had been called upon to explain their thinking and in doing so the students experienced success.</p>

<i>Making student thinking visible</i>	Suri: Icy-pole sticks – students all become accountable	Suri observed the use of icy-pole sticks ensured students were accountable and expected to participate in the discussion.
	Sheila: Most of the kids were on task most of the time, so the use of things like the icy pole sticks and then thumbs up were great. Most kids most of the time were engaged in thinking.	Sheila observed student behaviour within the lesson, which was not the focus of the observation. She was comfortable with her perceived engagement of most the students.
	Sophie: The kids have really benefitted. They talk about the zone of confusion and that it's okay to be there.	Sophie's comment indicates how much the students enjoyed the pedagogical approach.
	Suri: Uploading higher order thinking student work sample on IWB and asking that student to share reasoning and thinking	Suri observed the researcher used a tool for making children's thinking visible to other students in the class.
	Sabrina: You used children's examples on the smartboard and got children to come up and explain it.	Sabrina observed the researcher used a tool for making children's thinking visible to other students in the class.
	Sheila: The idea of using the technology was good.	Sheila observed the researcher used a technological tool for making children's thinking visible to other students in the class.
	Sheila: Using technology to display children's work and then getting them to explain their thinking was really powerful. We've since used money to buy two document projectors... [Teachers now have the] desire to be able to display chn's work during lessons for student explanation / group discussion.	Sheila indicated that the staff at SPPS had purchased document cameras because of the intervention. She noted that teachers saw that using technology to support children explaining their thinking was very powerful in that it made thinking visible to the whole class.
<i>Student engagement</i>	Marita (SWPS): Initially I thought my goodness, my children would not be able to do this because I've got the other [grade] 2/3 class...Just seeing how you slowed the lesson right down... You spent time going through the task so that the children understood [what the task was asking them to do] ... [their responses to the task] surprised me... The children were very much getting the concept at the end and wanted to continue working, even when they heard the bell, which was surprising.	The modelling had the desirable effect of enabling Marita to see the children's engaged response to the challenging task.
<i>Roving and probing</i>	Sabrina: The researcher roves and has one on one conversations with the students. She asks them to explain their reasoning. [By doing that] she can see how the whole class is going and whether to move on or not.	Sabrina observed the researcher circulating and paying attention to the students thinking and solutions strategies as they worked on the tasks. This is an important element of a pedagogical approach that is focussed on student reasoning and problem solving.
	Sabrina: [From the reading I learnt about] anticipating problems you might have and how you would address that. [For example] when you choose the students [to show their work], you are targeting those responses so that the [examples] they present to the class	Sabrina reflected upon a reading she had been given and was able to articulate an important point about purposefully selecting students and their solutions which will contribute

*The modelled lesson
as a scaffold for
teacher learning*

will help move them toward understanding. If the example that you're looking for is not there, you can create one.

Sheila: After your first visit Marita, Suri and Sally were trying to implement strategies they saw and reproduce your lesson. They probably weren't the target for the change in pedagogy. The more reluctant the person usually correlates with the greatest need. There is some resistance.

Sheila: The second lesson showed them that if you persevere and rephrase your questions and give the children time to think it through then eventually they will get there. I think that for those people who are not that keen on this approach they saw that you don't give up, you keep going, you give time, you challenge, you let the kids stew... a little bit in that zone of confusion. The fact that it wasn't as fluent as the previous lesson in terms of the student's responses was a blessing in disguise. It showed people things don't always go to plan, but if you stop... if you know the maths you are trying to teach and give the kids time to think, they can get there.

Suri: [The modelling] put theory into context. It made learning the strategies more engaging and relevant. It made [the pedagogical approach] seem achievable.

Principal: They [teachers] liked to see you work hard... it wasn't a process that was a breeze for you... you did have to adjust and modify as you were teaching and have different parts that didn't go quite right and then how you adjusted ...

Sheila: Teachers copied either the exact lesson or slightly modified to suit age group soon after observing the demonstration lesson.

*Teacher Learning
from the Researcher
MKT*

Sabrina: The reading (Stein et al., 2008) was very helpful in that the structure of the lesson was broken down into 5 stages and it made sense. [For example] We've all been grappling with the kids being in the zone of confusion. We all felt like we were leaving them there. [I learnt] how to move them on if [I'm] not doing that explicit teaching and what [my] role in the lesson is.

*Knowledge and
understanding of
mathematics content
and pedagogy*

Sophie: It was obvious [from the lesson] that we have not done any work on arrays yet or on division but we have done some

to the mathematical goal of the lesson.

Sheila indicated that whilst some teachers attempted to implement strategies they observed in the modelled lesson, others were resistant to making changes to their approach to teaching and others had already made changes to their practice. She noted that the more reluctant teachers seemed to have the greatest need for change.

Sheila's comment suggests that the modelled lesson does not have to be a 'perfect' lesson for teachers to learn from it. The teacher learning comes from observers seeing the modelling teacher persevere, rephrase questions, give thinking time and modify the lesson on the fly.

Suri indicated the modelling enabled her to see that the pedagogical approach was achievable as it connected theory with practice.

The Principal (SPPS) indicated that watching the researcher work hard in adjusting and modifying the lesson was welcomed by the observing teachers as it seemed real to them.

Sheila indicated that teachers either copied the modelled lesson or modified the lesson to suit their age groups soon after observing it. This suggests the modelled lesson may be an important scaffold for enactment.

The observation prompted Sabrina to read an article which the researcher sent her called 5 Practices for Orchestrating Mathematical Discussions. Sabrina found the article helpful in answering questions she had about the pedagogical approach.

The student's responses to the task prompted Sophie to reflect on the mathematics that the students need to learn.

	<p>stuff on multiplication. Some of them did really well.</p> <p>Marita: They're [students] seeing these clocks and thinking what is the digital time that matches but they're not understanding the duration of time I've seen from this lesson.</p> <p>Suri: Even though the students can count in 5's and 10's, they don't necessarily transfer that to the clocks. They haven't quite got the concept of the passage of time.</p> <p>Sheila: Thorough planning improves teaching</p>	<p>Marita observed that the grade 3 students did not understand the duration of time.</p> <p>Suri observed that the children in grade 3/4 were unable to transfer their knowledge of skip counting by 5's to the analogue clock face.</p> <p>Sheila indicated that teachers recognised that thorough planning improves teaching. Teachers who had not considered the rationale for a lesson before worked together to create a lesson where they were clear about their purpose and successfully anticipated student responses.</p> <p>Sheila indicated that the lesson plan proforma had been utilised by teachers in subsequent planning sessions. This suggests that they saw it as helpful.</p>
<p><i>Teacher learning from Co-planning</i></p>	<p>Sheila: Planning proforma has been used by teachers for shared lessons.</p>	<p>Sheila indicated that the lesson plan proforma had been utilised by teachers in subsequent planning sessions. This suggests that they saw it as helpful.</p>
<p><i>Planning for enactment</i></p>	<p>Sophie: The rationale for the lesson was the hardest. Trying to think of the theory behind something is not something we do. We just know what we have to do and we don't go looking for the theory behind it.</p> <p>Sophie (SPPS): [The enactment prompted me to] think about the purpose of the lesson and the challenges for [students] before the lesson.</p> <p>Sophie: Initially I thought the lessons may have been too difficult, but now I think more about catering for the top end and [using enabling prompts] whereas before I was catering for the average or below average and spending most of my time with them.</p> <p>I learned that it's okay to challenge all the kids.</p> <p>Sophie: My goal is to use more open ended real life problems. Even though I knew that was what I was meant to be doing, I don't think I was doing that very well.</p> <p>Sophie: I now use [enabling and extending prompts] in my maths lessons.</p>	<p>Sophie indicated that thinking about the rationale for a lesson was difficult because it's not something she is used to.</p> <p>Enacting the lesson prompted Sophie to reflect on her planning to think about the purpose of the lesson and anticipating any difficulties students might encounter.</p> <p>The favourable student responses to challenging tasks in the modelled lessons prompted Sophie to reflect on her teaching. She indicated that in the past she catered for the average to below average students, but now she is committed to the idea of catering for all students.</p> <p>Sophie committed to the idea of using problematic maths tasks.</p> <p>The modelling prompted Sophie to commit to using enabling and extending prompts to differentiate tasks in her maths lessons.</p> <p>The Principal (SPPS) indicated that seeing the lessons modelled followed by enactment was powerful professional learning. It enabled teachers to actually have a</p>
<p><i>Teacher learning from the intervention</i></p>	<p>Principal: I just think seeing someone else [model practice] is a powerful thing, then seeing one of their colleagues practise as well. It's a powerful step into having a go, rather than just going to professional</p>	

	<p>learning and talking about something. I think it bridges that gap around, “Well what does it look like in my classroom?” ... We have seen changes in practice and there’s been far more high level discussion in teams and groups around problem solving in maths.</p>	<p>go at the practice rather than talking about it. She noted that there has been far more discussion in teams about problem solving.</p>
	<p>Principal: It was hard for us to get the two hours... we did all that because we had the back-up... I definitely believe that the two modelled lessons at the start were really good to see you modelling... after that if we’d had an extended model where every teacher in the team then had a turn at doing the co-planning and co-teaching with you. So that everybody got exposure.</p>	<p>The Principal (SPPS) indicated that whilst it was difficult to find time for the intervention, seeing the two modelled lessons was powerful for her teachers. She suggested the possibility of an extended model where each observing team member had an opportunity to co teach and enact pedagogies which would assist everyone to maximise their potential for learning.</p>
	<p>Principal: I think you’ve got to have a tight model around it and that’s what made it work so well. Our ‘Watch others Work’ stuff that we did last year was looser and still far more invitational, but we have some staff here that would take forever to get on board. I think that the four-session structure was the power of it. It’s the link between talking about practice, and doing practice, changing practice. We’ve got so many teachers in the teaching profession that can do the rhetoric, but the practice in the classroom doesn’t match the rhetoric.</p>	<p>The Principal (SPPS) indicated that the structure and tight processes of the model made it robust. The four-staged structure, which included seeing practice modelled, deconstructing that practice and opportunities to enacting the practices was powerful.</p>
	<p>Principal: [Modelling] it’s the final link, it’s the most important link. You can do the professional learning, the talking about it, then if you watch someone do the practice and then the final step of having a go yourself and evaluating and reflecting all the way through, I just think it’s the final thing...what you did with us was great. It was excellent.</p>	<p>The Principal (SPPS) indicated that the model – seeing practice, deconstructing practice and enacting practice was an excellent form of professional learning for her teachers.</p>
	<p>Sheila: There was some accountability to being involved “up close and personal” that is often missing...the ongoing nature made it more meaningful rather than “one off” pd approaches.</p>	<p>Sheila indicated that the model had accountability built in as it was situated in teachers own classrooms with the expectation that observing teachers would contribute to pre-and post-lesson discussions to deconstruct the practice they observed and collaboratively plan for the co-taught lesson.</p>
<p><i>Factors impacting implementation</i> <i>Time</i></p>	<p>Sabrina: We would have got a lot more value out of this [intervention] if it was</p>	<p>Sabrina indicated that the timing of the intervention in her school was not ideal due to reporting.</p>

not concurrent with our reporting period.

Sheila: The duration of the sessions did not match the segments of the school day here and hence was disruptive (eating in to teacher lunch break). If the timing came from within the school rather than the presenter's timetable it might have been even more productive.

Sheila indicated timing of the intervention was problematic in her school setting. The intervention was somewhat disruptive in the school as it did not align with the school day. Sheila suggested that the timing of the intervention should be a prioritised to suit the schools needs rather than researcher's time

Principal: It was hard for us to get the two hours... we did all that because we had the back-up... I definitely believe that the two modelled lessons at the start were really good to see you modelling... after that if we'd had an extended model where every teacher in the team then had a turn at doing the co-planning and co-teaching with you. So that everybody got exposure.

The Principal(SPPS) indicated that whilst it was difficult to find time for the intervention, seeing the two modelled lessons was powerful for her teachers. She suggested the possibility of an extended model where each observing team member had an opportunity to co teach and enact pedagogies which would assist everyone to maximise their potential for learning.

Sheila: Although there was consultation about the content of the modelled lessons this was often rushed or done via email which did not always result in lessons closely aligned to previous lessons in the class. Teachers would have like to have seen greater alignment of lessons.

Sheila pointed out that although the researcher consulted teachers about the content of the modelled lessons it was often rushed and teachers did not feel the planned lessons necessarily aligned with their current focus.

Sophie: We missed out on our planning team meetings and it felt like our valuable learning group time was being yanked away from us four times.

Sophie (SPPS) indicated dissatisfaction that valuable team planning time was removed so that her team could be part of the intervention

Buy in from teachers

Sheila: Although there was consultation about the content of the modelled lessons this was often rushed or done via email which did not always result in lessons closely aligned to previous lessons in the class. Teachers would have like to have seen greater alignment of lessons.

Sheila pointed out that although the researcher consulted teachers about the content of the modelled lessons it was often rushed and teachers did not feel the planned lessons necessarily aligned with their current focus.

Sophie: The Principal just told us what was happening. We did not have a choice. I felt annoyed

Sophie indicated she was angry that the Principal informed her and her colleagues about the intervention without consultation.

Implications for other curriculum areas

Principal: Exploring the model through other curriculum areas would be interesting... embedding the model,

The Principal (SPPS) saw the potential of exploring and embedding the model in other learning areas.

probably in your whole year of team
meetings and learning group meetings.

Appendix 14: Synthesised Themes All Schools

Synthesised Theme Clusters with their Related formulated meanings from three schools SPPS, HPS and MPS

Illustrative examples of teacher comments	Clustered Meanings	Themes
<p>Sabrina (SPPS): We are coming from a model where we teach kids how to do things and then [we] let them practise those skills and revise them, it's been a bit hard to think [we're] giving them the challenge before [we've] taught them how to do it and what if nobody in the group knows how to do it, where do you go?</p>	<p>The modelling challenged Sabrina's pedagogical approach to teaching as she grappled with new ideas that confronted her teacher directed view of learning.</p>	<p>Theme (1) Challenging teacher orientation towards teaching as telling</p>
<p>Marita (SPSS): We're using [a program called] Stepping Stones... I follow the steps that are included... [students are required to give] either a right or wrong answer... The lesson that I just saw is very different to how I would run my maths lesson [using the program].</p>	<p>The observation enabled Marita (SPPS) to see a different pedagogical approach and prompted her to reflect on her current teaching approach.</p>	
<p>Jackie: [When] you gave the instruction of doing the drawing of the ten teddies in [and out of] the bed ...I wondered if I would have done an example of what it would look like...When we talk about explicit teaching, we [think] how do we set it up so they know... Then I thought as soon as I draw 8 and 2, every single one of them will be drawing 8 and 2 because it's there [on the white board] and I would not be allowing any of this thinking.</p>	<p>Jackie (HPS) reflected on the usual approach of explicit teaching and realised that telling children how to think narrows children's thinking.</p>	
<p>Marita (MPS): I would have given them examples before I sent them off to do a task. I was wondering how they would go [without instruction] but they did really well.</p>	<p>Marita (MPS) observed students thinking for themselves successfully before instruction from the teacher. This prompted her to reflect on her teaching practice.</p>	
<p>Melissa (MPS): I feel I probably go astray, I probably offer up strategies a bit much rather than let them [have a go] ...that is something I will attempt to do.</p>	<p>The modelling of challenging tasks prompted Melissa (MPS) to reflect upon her teaching where she tells the students the strategies rather than allowing them to think for themselves.</p>	
<p>Melissa (MPS): It was really hard to not tell though. I had to bite my tongue a few times in there because I wanted to say, "oh could you use a ..." "I need to stop giving them the ideas about how to solve problems".</p>	<p>In her co-teaching session in stage 3, Melissa noticed herself having to hold back from telling the children what to do.</p>	
<p>Megan (MPS): I have held back on my modelled explicit teaching and allowed the children to have more thinking time.</p>	<p>The modelling prompted Megan to refrain from telling children how to think and what to do.</p>	

<p>[I saw] the children thriving on the challenge.</p>	<p>She saw the children rising to the challenge and this was the catalyst for her shift in her views of teaching.</p>	
<p>Maggie (MPS): I want to elicit student strategies rather than giving answers.</p>	<p>In stage 4, Maggie expressed her goal to have students think for themselves rather than telling them what and how to think.</p>	
<p>Maggie (MPS): I am not telling as often as possible.</p>	<p>In her exit survey Maggie noted she had shifted from telling students what to do</p>	
<p>Marita (MPS): Previously I would have started the introduction of a task by explaining and modelling and modelling again and we would have all done some together but now I find it is fine to introduce the task and let [the children] explore their understandings as they make them.</p>	<p>In her exit interview, Marita indicated she had shifted her view of teaching from telling children how to solve problems to allowing students to think for themselves.</p>	
<p>Marita (SPPS): Students found it really hard to explain their reasoning. For example, Abbie knew the correct answers on her times but could not explain.</p>	<p>Marita observed the difficulty students had in explaining their thinking about the concept of time in stage 2.</p>	<p><i>Student difficulties in explaining their thinking</i></p>
<p>Sheila: Many of them were quite confident with making factual statements about what the time was but explaining what that meant was where they fell over.</p>	<p>Sheila (SPPS) observed the students had difficulty with explaining the duration of time.</p>	
<p>Maude: I noticed that Rick [pseudonym] could get the answer really quickly, but he struggled to explain his thinking.</p>	<p>Maude (MPS) noticed some children struggled to explain their thinking. This suggests students were not familiar with this practice.</p>	
<p>Holly: They found it really hard to explain what they were thinking.</p>	<p>Holly (HPS) highlighted the difficulties children had in reasoning mathematically. This suggests students were unfamiliar with explaining their thinking.</p>	
<p>Heather: When you asked someone from the group to [repeat someone else's reasoning], they found that hard.</p>	<p>Heather (HPS) observed students had difficulty repeating someone else's reasoning. This suggests the children were not used to this approach.</p>	
<p>Marita (MPS): I'm thinking maybe I'm under pitching way too often Maybe I'm not setting [expectations] high enough. [This modelled lesson] has got me thinking about changing my beliefs [about teaching]</p>	<p>The modelled lesson in stage 1 prompted Marita (MPS) to reflect on her expectations of student achievement.</p>	<p>Theme (2) Confronting ideas about catering for diversity <i>Expectation of student achievement</i></p>
<p>Marita (MPS): We do some [tasks] together and we start off with low numbers rather than giving that challenging one first. I build them up, so maybe I need to [give the challenging task first] and then bring them back?</p>	<p>The modelling prompted Marita (MPS) to question whether she should show children how to solve easy problems and scaffold up or pose challenging tasks for</p>	

<p>Maggie (MPS): [the struggling students] would be the ones I would gravitate towards first. The enabling prompts were too hard for them. They weren't even in the zone of the room.</p>	<p>children to solve by themselves as a first step.</p>	
<p>Heather: The most challenging thing is ... getting around everybody. That notion of being able to capture what one child is doing in one situation but also have your ear on what else is happening. In the past, I would not have had the whole group working on the one task, it was a challenge to think about how that would happen in my own class room.</p>	<p>Maggie (MPS) indicated the enabling prompts were too challenging for some students and indicated her preference for working with those students first.</p>	
<p>Marita (MPS): Is it okay to [set high expectations and differentiate with enabling prompts]. Is that okay to do that?</p>	<p>Heather (HPS) indicated the most challenging thing for her was to get around to everyone in a lesson where the whole class was working on a problem. This was different to how she normally worked.</p>	
<p>Megan: Jack (pseudonym) was distracted at the start, but you trusted him to do it and he achieved much more than he ever has. I was surprised at how much you could get out of him.</p>	<p>Marita raised the question of whether setting high expectations for all students and then differentiating with enabling prompts was appropriate.</p>	
<p>Marita (MPPS): The biggest thing I'll take is that I have been limiting kids with their thinking in maths and not stretching them as far as I can.</p>	<p>Megan (MPS) observed a student achieving far more than he ever has.</p>	
<p>Henrietta: [in the past I] would have just taken a group that was ready for the idea and others would have been doing other concepts.</p>	<p>Marita indicated that her greatest insight from the modelling was that she limited students in maths by not providing them with opportunities to extend their thinking.</p>	
<p>Sabrina (MPS): What will we do with students who are unable or unwilling to attempt the enabling prompt?</p>	<p>In stage 4, Henrietta (HPS) indicated that in the past she would only have taught the concept of difference to a small group of children that she perceived were able whilst the rest of the class did other unrelated tasks.</p>	<p><i>Fear of student generated responses to tasks</i></p>
<p>Marita: I felt some kids were just lost. There were others that just didn't have a clue. I'm wondering whether you would we have a small focus group for kids that are lost?</p>	<p>Sabrina's (MPS) was unclear about what to do if the enabling prompts are too difficult.</p>	
<p>Marita (MPS): A huge concern is that while you've got kids that are engaged...sitting there listening ... and actually having a go, the others aren't [getting it]. They are off task...they are not thinking about learning, they have no meta cognition.</p>	<p>Marita (MPS) noticed children off task. She wondered whether a small focus group would benefit those children.</p>	<p><i>Perceived disengagement of some students</i></p>
<p>Marita (MPS): A huge concern is that while you've got kids that are engaged...sitting there listening ... and actually having a go, the others aren't [getting it]. They are off task...they are not thinking about learning, they have no meta cognition.</p>	<p>Marita (MPS) was concerned about the children she perceived to be off task.</p>	

Marita: Jack (pseudonym) appeared to be off task the whole time, but then when he stood up and explained this thinking.

Maggie: I saw some children with nothing on their paper. Is it okay for them to be like that?

Sabrina: Sometimes you feel guilty about seeing children doing seemingly nothing.

Sheila: Making mathematics a group activity is a powerful technique – and that allowing discussion of concepts / thinking is to be encouraged rather than feared as an indicator of time off task.

Sabrina: One of the things that stood out to me was the amount of time you let them struggle. It's knowing how long to let them struggle and when to move on. I reflected on what I have been doing and I don't think I let my students struggle enough. I think I pounce too quickly to do the teaching and make things easier for them. I need to let them struggle more.

Marita (SPPS): They [students] are not used to seeing problems like that. I have only just started to introduce problems in that way. Seeing a problem seemed quite overwhelming for them.

Sabrina: We are still not clear about the zone of confusion. If one of our goals is for children to learn to use efficient strategies for problem solving, it does not seem helpful to leave them confused for long periods. I get that you want to use children to share their strategies with their peers, but what about those children who do not make those connections easily? At what point would you intervene with children like Talia (pseudonym) who was hung up with the colours of the counters?

Marita: I think [the children] liked the zone of confusion terminology. I saw a child saying to another child, please don't tell him the answer, it is okay to be confused.

Heather: Sometimes I don't see what they're [children] struggling with until I get the chance to see them in some problem-solving task.

Marita (MPS) noticed a student who appeared to be off task stand up and explain his reasoning.

Maggie (MPS) noticed students with nothing on their paper. She was concerned that they were off task'.

Sabrina (MPS) expressed concern about students she perceived as 'seemingly doing nothing'.

Sheila (SPPS) indicated that teachers learned that having the whole class engaged in the task is powerful for the discussion of concepts and that thinking is to be encouraged rather than feared as time off task.

Sabrina (SPPS) wrestled with the idea of allowing students to struggle and what that might mean for her teaching.

Marita observed the students struggling with challenging tasks which they had not encountered before.

Sabrina (SPPS) grappled with the idea of the 'zone of confusion' and what that meant for her teaching. She questioned when it would be appropriate to intervene for low attaining children.

Marita (MPS) observed children acknowledging the 'zone of confusion' as a safe space where hard thinking and struggle takes place.

In stage 4, Heather indicated she realised she had difficulty in assessing children unless she saw them working on a problem solving task.

Grappling with tasks where the solution is unknown

Theme (3) Observed pedagogies to facilitate reasoning and problem solving

Heather: Some of the power for me was looking at the children problem solving and thinking so now where am I going, whereas I might have done it back to front in the past.

Heather: [The modelling] affirmed [it's okay to allow] the kids to struggle a little and trust that it is okay to do that.

Hannah: I learned that during the lesson it is vital to allow the kids to explore the maths in their own way.

Heather: Teaching in this way still feels a bit uncomfortable for me but it is exciting to watch how some children go about solving the problems.

Heather: Starting with the problem opens up opportunities for children to demonstrate an amazing variety of thinking and ways of moving towards understanding.

Principal: [Teachers learned] the importance of letting kids swim a little bit, sink a little bit – not to rescue.

Marita: [My goal is to] allow students to struggle more, to have a go and to use the 'zone of confusion' expression to let students know it's okay not to know things straight away.

Marita: After seeing you last time I have adapted my way of thinking in that we are doing a lot more problem solving and giving a lot more thinking time.

Marita: [The modelling prompted me to] allow more thinking time and allow kids to struggle.

Suri: [My goal is to] produce problem solving lessons, allow greater thinking time, allow students to share their reasoning.

Sally: [The main elements which prompted my learning were]: Seeing children persevere and witnessing the thinking going on and understanding

The power of seeing students solving problem prompted Heather to reflect upon her assessment practices.

Heather indicated that the modelling affirmed it was okay to allow students to grapple with problems.

Hannah learned the importance of students thinking for themselves.

In her post lesson survey, Heather indicated that she still felt a little uncomfortable about students solving problems by thinking for themselves but noted it was exciting to watch the children solving the problems by thinking for themselves.

Heather could see that by providing children with opportunities to solve problems by thinking for themselves, they came up with a variety of responses.

The Principal (HPS) indicated that teachers learned the importance of allowing the students to struggle and think for themselves.

Marita (SPPS) set herself a goal to allow children time to think for themselves.

Marita indicated she was now giving more opportunities for problem solving and extended thinking time.

Marita could articulate a goal for her future practice which involved extended thinking time for students where they could grapple with problems

Suri could articulate a goal which involved allowing students greater thinking time, problem solving and opportunities to share their reasoning.

The main elements which prompted Sally's learning were seeing the children persevering

(and engagement). I know I need to allow more thinking time, but seeing it is very powerful.

Sophie: Initially I thought the lessons may have been too difficult, but now I think more about catering for the top end and [using enabling prompts] whereas before I was catering for the average or below average and spending most of my time with them. I learned that it's okay to challenge all the kids.

Marita (MPS): I've implemented some open-ended tasks that are more challenging than what I would previously have given... I've implemented "keep thinking" and it's okay to be the zone of confusion. You modelled that and I saw kids thinking.

Marita (MPS): I thought it was beneficial to see that some kids really did rise to the challenge but also achieve success on tasks that I thought they may have needed building up to... It was a great way of getting kids to think outside their comfort zone.

Marita (MPS): In my planning, each week I've got a challenging open ended maths task with extending and enabling prompts. It's a new concept for me to make the maths challenging... let's get the kids switched on thinking.

Maude (MPS): [The researcher] started at the top end [tasks with a high ceiling, low floor] so it was challenging for all rather than with the low attainers and working up. Teachers had to think really hard about what they were doing in their own classrooms.

Sheila: Teachers are now allowing students more time to think through mathematics, a shift towards more "problem solving" rather than worksheet driven lessons.

Sophie (SPPS): I now use [enabling and extending prompts] in my maths lessons.

Hannah (HPS): The enablers and extenders mean you can cater for a greater range of children.

Melissa: I liked the enabling prompts where you had to bring it back for a

and responding positively to the challenging tasks.

The favourable student responses to challenging tasks in the modelled lessons prompted Sophie to reflect on her teaching. She indicated that in the past she catered for the average to below average students, but now she is committed to the idea of catering for all students.

Marita (MPS) indicated she has implemented challenging tasks, extended thinking time and the notion that it's okay to be confused. She noted that seeing it modelled and how the students responded was a catalyst for her change in practice.

The modelling prompted Marita to see students rising to the challenge of cognitively complex tasks.

Marita's goal was to incorporate challenging tasks into her planning each week. This indicates her shift in thinking to make the maths more challenging.

Maude indicated teachers were challenged by the approach of implementing tasks with a high ceiling and low floor rather than scaffolding up.

Sheila (SPPS) indicated evidence of changed teaching practice was that teachers had shifted their emphasis to a more problem based approach rather than worksheet driven lessons.

The modelling prompted Sophie to commit to using enabling and extending prompts to differentiate tasks in her maths lessons.

Hannah indicated that by using enabling and extending prompts, it is possible to cater for all students in one class.

Melissa (MPS) observed the researcher used enabling prompts

Differentiation of the task

couple of children to be able to work on a smaller [quantity]

Melissa: You could extend those that got the concept but then go back to those others who were struggling. It is normally so hard to work with low ones when you're working by yourself.

Marita: I liked the differentiation ... where if we're not counting with one to one correspondence, let's go back to 5 teddies or for the extending [prompt], there's more than one way to do this and asking, "can you see any patterns?"

Melissa: I had one child who shut down because those numbers were big; he just went "Whoa! and just sat. After he did the first one [enabler], he did the next one in under a minute. Just having those enabling prompts really helped him to get started and I'm sure with a couple of more practises he could work up to the main task. Having those enabling prompts is one way to get started.

Sally: [My goal is to] implement a problem-solving lesson once or twice a week using lots of thinking time, enabling and extending prompts, zone of confusion...

Marita (MPS): I have made up some enabling prompts and some extending prompts. I will have some that can't handle those numbers.

Marita (MPPS): I already think I know in my mind who will get enabling prompts and who will get extending prompts... I'm going to try hard not to jump in and give them the enabling prompt too early.

Megan (MPS): I'm wondering whether meaning was lost in enabling prompt? I don't know if it was the wording of it? Just wondering if you think it may have been more effective if you used the same context as the main problem?

Marita: Caleb was really stuck, but he got it in the end. He really got it!

to enable low attaining children to access the concept by providing the same task with smaller numbers.

Melissa observed that the enabling an extending prompt provided a helpful way to cater for the diverse range of learning needs.

Marita could see differentiating the task was an effective way to engage all students in the lesson.

Melissa observed a student successfully using an enabling prompt which prompted her to see the power of enabling prompts for children to get started.

Sally could articulate a goal for her practice, which included problem solving, thinking time and enabling and extending prompts

Marita (MPS) made up two enabling prompts and two extending prompts for the task she had planned for her enactment in stage 4. This was in response to previous modelled lessons where the enabling prompts required further modification.

For her enacted lesson, Marita (MPS) indicated she could anticipate who would receive enabling prompts. She noted that she would try hard not to give the enabling prompt unless students were unnecessarily struggling.

Megan (MPS) proposed an improvement to Melissa about the enabling prompt after the lesson she enacted.

Marita (MPS) indicated that a student in her lesson was very stuck but the enabling prompt

Marita (MPS): I don't think I would put the lower group in that zone of confusion. I am still inclined to reign it back a little.

Maude (MPS): Putting low attainers in with those high fliers would just give them the experience. They might see what the high fliers are doing. They might not understand it but they will have the opportunity.

Marita (MPS): I now use enabling and extending prompts in the lesson as I need to. I give them [children] the opportunity to soar, but if not, I give them the opportunity to achieve success by giving enabling and extending prompts.

Marita (MPS): Initially [low attainers in my class] would be just really stuck ...but now they are at least starting to have a go even if they're not on the right track. They are understanding that it is okay not to get the right answer.

Principal (HPS): I heard [teachers] talk lots about the enablers and the extenders...because we all battle with differentiation. I have seen them using them in classrooms [since the intervention]. It was a simple way of thinking I'll stick with my one task, have my enablers, have my extension rather than thinking when I go in that lesson I've got such a different thing happening for the lower ones and such a different thing happening for the upper ones.

Sally (SPPS): I noticed the amount of thinking time you gave the children. A couple of us were saying we would like to get to that stage where we can give that amount of thinking time without there being lots of calling out.

Sabrina: The thinking time you gave really stood out to me and made me think how I don't give enough time. The other thing that stood out was that you gave [the students] plenty of time to think but then when you couldn't see that they had something just yet, you always went back to them "Keeping thinking and I will come back to you" and you did go back to them.

Marita: [Researcher] goes back to confused students "Keep thinking and I will come back to you" ...they suddenly get it!

and thinking time enabled him to experience success.

Marita's comment in stage 4 indicates her reluctance to place her lower attainers in the zone of confusion but Maude responded with a suggestion to support inclusion.

During the exit interview, Marita indicated she now uses enabling and extending prompts.

During the exit interview, Marita indicated that her low attaining students are much more willing to struggle with challenging tasks now and know that it is okay if they do not get the right answer.

During his exit interview, the Principal (HPS) indicated that he had seen teachers both talking about and using enabling and extending prompts since the intervention, which was a change from having different groups of students doing different tasks in each lesson.

Sally observed a pedagogical approach that allowed extended student thinking time. Sally and her colleagues hoped they could emulate the approach.

Extended thinking time

Sabrina (SPPS) observed the researcher provided thinking time including wait time to students after calling on them to explain. Her observation prompted her to reflect on her practice.

Marita observed the researcher providing wait time to those students who had been called upon to explain their thinking

Sue: [My goal is to] give more time for children to work problems out rather than rushing in to provide teacher assistance.

Heather: I think “convince us” is really powerful because [I] saw some really great wait time and then finally some thinking.

Heather: [I noticed] the importance of time [for the students] to grapple with the ideas.

Sophie: [My goal is to] Give more time to ‘keep thinking’, use a visualiser and make up more authentic problems for the children to solve.

Sheila: [From your modelling, teachers learned that]:Providing time for children to think through situations produces better understanding; that making mathematics “easy” is not necessarily good practice (i.e. “the zone of confusion” is not necessarily to be avoided).

Megan: You paused and waited and waited ...we often step in.

Megan: The power is in your questioning ... the time you gave children to reflect... Even if children did have the concept right but did not know how to articulate it, that thinking time allowed for a lot more. For example, Oliver [pseudonym] was a real surprise to me because at the start of the year he couldn't recognise any numbers, he couldn't count with one to one correspondence. He came up with 5 and 5, when normally he would not. Because he had the time to work it out, he could do that.
Your questions were clarifying their ideas.

Megan: There was a lot of thinking time. Because you [Marita] said that very clearly [to the children] how they need to explain their thinking, I could see that Tory [pseudonym] was sitting here thinking about how she was going

and in doing so the students experienced success.

Sue (SPPS) set herself a goal to give students more thinking time rather than rescuing.

Heather (HPS) observed the researcher asking student's to justify their answers and consequently she saw students spending time reflecting on their learning.

In stage 4, Heather (HPS) could see that students need time to work on challenging problems

Sophie (SPPS) set herself a goal of giving students more thinking time, making their thinking visible by using a document camera and creating authentic problems for students to solve.

Sheila (SPPS) indicated that from the modelling, teachers learned that extended thinking time gives children the opportunity to think deeply and develop understanding. She also indicated teachers learned that effective practice does not mean tasks are easy.

Megan (MPS) observed the researcher giving extended wait time which prompted her to reflect that she often steps in too quickly.

Megan (MPS) observed extended student thinking time and questioning to facilitate mathematical reasoning. She indicated these teacher actions led to the success of her students which she was surprised by.

Megan (MPS) provided feedback to Marita on how she enabled students in her class to experience success by giving them thinking time.

to solve the problem and Cory [pseudonym] as well “how am I going to draw that because I sought of know... How am I going to explain that? Just the way you set that up enabled it to happen.

Suri (SPPS): [The modelling prompted me to] recognise what my students were capable of – I observed their responses after wait time and listened to their reasoning.

Heather: You gave the expectation that they were going to justify... You did not accept an answer without the expectation that you wanted to probe them just a little bit more. You rephrased their thinking. “What did you notice about what so and so said”. Even right to the very end it was still very obvious that you expected them to reason [mathematically]. You set up the expectation to find more ways [to make 10] and expected them to respond.

Sophie (SPPS): You were asking questions to prompt the children to explain their thinking. You gave thinking time and encouraged the children.

Sally (SPPS): [The main elements which prompted my learning were]: Seeing children persevere and witnessing the thinking going on and understanding (and engagement). I know I need to allow more thinking time, but seeing it is very powerful.

Sally: [My goal is to] implement a problem-solving lesson once or twice a week using lots of thinking time, enabling and extending prompts, zone of confusion...

Sheila (SPPS): Teachers are now allowing students more time to think through mathematics, a shift towards more “problem solving” rather than worksheet driven lessons.

Principal: I think [teachers] really got a lot from seeing the high expectations [in the classroom] and realising that our kids could rise to that expectation.

The modelling had the desirable effect of enabling Suri to see how her students responded to the challenging tasks well beyond her expectations.

In stage 1, Heather (HPS) noticed several teacher actions to facilitate student reasoning including asking students to restate another’s reasoning, revoicing, paraphrasing, waiting, having high expectations, probing. This suggests the narrow focus on teacher actions to facilitate reasoning was effective.

Sophie observed elements of a pedagogical approach that fosters encouragement, student thinking time and facilitates student reasoning.

The main elements which prompted Sally’s learning were seeing the children persevering and responding positively to the challenging tasks.

Sally could articulate a goal for her practice, which included problem solving, thinking time and enabling and extending prompts

Sheila indicated evidence of changed teaching practice was that teachers had shifted their emphasis to a more problem based approach rather than worksheet driven lessons.

The Principal (SPPS) indicated that the observation of the modelled lesson prompted teachers to see that the students could rise to the high expectations that the researcher had of the children.

High expectations

Principal: [Observing teachers] learned that our kids could actually do the things that you were talking about. They actually saw it in practice ... so they copied your practice... Even almost the next day teachers were actually trialling; and I've had teachers come up and share with me; we've had sharing and briefing around what kids have been doing in classes. So I'd definitely say that there has been an uptake in teaching practice.

The Principal (SPPS) indicated that teachers learned from seeing their students respond favourably to the modelled lesson. Teachers copied the lesson to trial the new approach in their classrooms.

Marita (SPPS): You extended their thinking by asking questions: "tell me more about, is that the same as...". Sometimes if they were a little off track, but you knew where they were going, you rephrased to assist them get to where you wanted them to go, by saying things like "are you saying?" or "is that the same as?"

Marita observed use of identifiable teacher actions that facilitate student reasoning and problem solving including revoicing and prompting students for further participation.

Revoicing and prompting for further participation

Marita: The phrases that you use: Convince me, tell me about your thinking.... I might say "that is not quite the right answer. But let's see if we can explore that" but you say, "keep thinking...has somebody else got something they would like to add". It was [student] directed rather than you directing [the lesson].

Marita (MPS) observed phrases that the researcher used in the lesson which facilitated student directed rather than teacher directed learning.

Megan (MPS): It was always the children that were explaining their work and even sometimes when they weren't making sense themselves you helped with the language... You didn't tell them what to say, they were doing the talking.

Megan (MPS) observed the researcher revoicing the student's explanations.

Holly (HPS): You got a child to repeat back the question so that you could clarify what it was that they had heard...

Holly observed the researcher asking children to repeat back what they had heard to ensure they understood what the problem was asking them to do.

Clarifying

Hannah (HPS): Giving them [students] the opportunity to unpack the question and understand what is being asked of them before beginning is a critical step

Hannah noted the important step of making sure children are clear about what the problem is asking them to do before sending them off to work on it.

Holly (HPS): I noticed how many times you repeated the mathematical language...e.g. difference. [Students] were getting it in their heads that's what they were learning.

Holly noticed the researcher modelled mathematical language extensively throughout the lesson which provided clarity for the children.

Use of mathematical language

Heather (HPS): I was really conscious of you [modelling] the language and then they[children] started using that language that you were modelling and you ...stretched their thinking

Heather noticed the researcher modelled mathematical language which the children began to use.

<p>Melissa: I think it's giving them the language to enable the children to explain their thinking. If you are always modelling that language they are able to justify.</p>	<p>Melissa (MPS) observed mathematical language being modelled by the researcher which supported students to justify their reasoning.</p>	
<p>Suri: When introducing new concepts(terminology) making connections to their [students] prior understandings e.g., an equation can sometimes be called a sum or a number sentence.</p>	<p>Suri observed mathematical language being used to connect with student's prior mathematical understandings</p>	<p><i>Connections to prior understandings</i></p>
<p>Sheila: I thought it was good how you explained the terms prior to getting into the actual teaching and making sure the children understood what was going on. It was probably something that we can work on. We tend to rush into the content rather than making sure they understand.</p>	<p>The modelling of mathematical language prompted Sheila to reflect on her practice.</p>	
<p>Sue (SPPS): When you asked the children to talk about their answers they were struggling and I was noticing how you were scaffolding them into breaking it down into smaller steps – “tell me about this, what did you notice about this side, what did you do next?” – you were helping break that thinking into little chunks so that [the children] were able to explain it logically to other people.</p>	<p>Sue observed the researcher supporting and scaffolding students with their explanations by revoicing or repeating what they had said.</p>	<p><i>Scaffolding students to explain their thinking</i></p>
<p>Sheila: We tend to accept a right or wrong answer instead of saying “okay, you got it right but why is it right”?</p>	<p>Sheila (SPPS) observed the researcher probing students to articulate why their solution was correct which was different to her usual approach.</p>	
<p>Heather: It really highlighted the value of the talk. I looked at one child and I thought that child was counting by tens but when I asked that child to explain their thinking I discovered they were using the ten blocks as a [representation]for one block. If we hadn't had that talk time, where they were explaining what they were doing [I would not have picked that up].</p>	<p>Heather (HPS) saw the importance of exploring children's thinking and getting them to talk about it.</p>	<p><i>Student explanations</i></p>
<p>Hannah: You used student explanations as a way of introducing concepts/tools</p>	<p>Hannah noticed student explanations were used as a way of introducing concepts.</p>	
<p>Suri: [The researcher] uploaded higher order thinking student work sample on IWB and asked that student to share [his]reasoning and thinking.</p>	<p>Suri (SPPS) observed the researcher used a tool for making children's thinking visible to other students in the class.</p>	<p><i>Making student thinking visible</i></p>
<p>Sabrina: You used children's examples on the smartboard and got children to come up and explain it.</p>	<p>Sabrina (SPPS) observed the researcher used a tool for making children's thinking visible to other students in the class.</p>	
<p>Sheila: Using technology to display children's work and then getting them to</p>	<p>During the exit interview, Sheila explained that the staff at SPPS</p>	

explain their thinking was really powerful We've since used money to buy two document projectors... [Teachers now have the] desire to be able to display chn's work during lessons for student explanation / group discussion.

Suri (SPPS): Students sharing responses in their own reasoning for others to learn from. Using students to explain why.

Marita (MPS): Students continued to scaffold each other's ideas... you could see their little lights go on and then they had the courage to have a go themselves.

Melissa: As soon as Rick [pseudonym] shared his idea [of using a number line] there were quite a few that went back and [used] that number line that Rick had explained.

Maude: Ava struggled as well until she saw that number line and then something clicked in her head and she thought, well I can take away five and take away five.

Henrietta: [Children were] learning from each other. It was okay that they were all working on different numbers. Some were working with larger numbers, others were working with smaller numbers. Thinking most important. No hands up rule.

Heather: Using students for modelling their work was a key and using the language.

Sheila (SPPS): The error management was important... so rather than just say wrong or no, there was nothing negative about your responses to their thinking— "Keep thinking? Would you like someone to help you with that?"

Sally (SPPS): Didn't say Holly was wrong – said "keep thinking"

Sophie: Some of the lower kids were having a really good go without giving up.

Suri: Icy-pole sticks – students all become accountable.

had purchased document cameras because of the intervention. She noted that teachers saw that using technology to support children explaining their thinking was very powerful in that it made thinking visible to the whole class.

Suri (SPPS) observed students sharing their ideas so that others could learn from them.

Marita (MPS) observed students learning from each other through their explanations of the mathematical ideas.

Melissa (HPS) observed a student sharing his number line strategy to the class which prompted other children to learn from this and use that strategy themselves.

Maude indicated that a student struggling with the mathematical ideas suddenly made a connection when she saw and heard her peer demonstrating his strategy on a number line. Maude saw the power of this student learning from another.

Henrietta noticed that the students were learning from each other even though they had enabling and extending prompts, this was because they all had the same learning goal.

Heather could see that by displaying and talking about their work children could learn from each other.

Sheila (SPSS) observed the researcher took a positive stance toward the children, encouraging them and promoting persistence.

Sally observed students were encouraged to keep thinking

Sophie (SPSS) observed the lower attaining students persisting.

Suri (SPPS) observed the use of icy-pole sticks ensured students

Students learning from each other

Encouragement to persist

Principal: [Using icy pole sticks with student's names in a container as a strategy to get all kids thinking in case they get randomly picked to answer the question] is going to get them trying at least rather than thinking well I won't even have a go coz I won't put my hand up so I won't have to give the answer. Oooh, hang on, my icy pole stick might get pulled out so I'd better try and put some thought into this.

Holly (HPS): ...when you put them [icy pole sticks] back in the tub ... it was a signal for those children not to stop thinking.

Heather (HPS): I noticed the way you didn't give up on [the students]. You kept expecting that they would keep going and you acknowledged for them that it was going to be difficult ... you kept encouraging them and then towards the end you gave them the cliff-hanger [you gave them the message] we are going to come back ... this is important.

Sophie: The kids have really benefitted. They talk about the zone of confusion and that it's okay to be there.

Maude (MPS): From the very first lesson that we observed there were lots of children sitting there that had no idea, just didn't know how to start but now because the teachers have been continuing the work I've noticed that the children now have a go. They have a starting point.

Marita: Jack (pseudonym) felt the power of success, you encouraged him to keep thinking.

Megan (MPS): [Children felt comfortable] to have a go even if they got it wrong... "keep thinking". Children did continue to persist with that prompt.

Melissa (MPS): "Keep thinking" was the most important idea that I got out of the lesson because it's [deflating] to say, "good try" or "try again".

Marita (MPS): "Keep thinking" was so powerful... we didn't see those deflated little faces.

were accountable and expected to participate in the discussion.

The Principal (HPS) anticipated the benefits of using icy pole sticks to increase student engagement during classroom questioning.

Holly (HPS) noticed the researcher had the expectation that all children would think.

Heather noticed that the researcher had high expectations of the children and continued to encourage them to persist.

During her exit interview, Sophie (SPPS) commented on how her students talk positively about the 'zone of confusion'.

During the exit interview, Maude (MPS) indicated students are now more will to persist with challenging tasks.

Marita (MPS) observed the researcher encouraging students to keep thinking and saw a child experience success.

Megan (MPS) observed elements of a pedagogical approach that fosters encouragement and persistence.

Melissa (MPS) observed that the researcher's comment 'keep thinking' encourages children to persist. She felt it was the most important idea she took from the lesson.

Marita (MPS) observed the researcher encouraging the children to persist.

<p>Principal (MPS): Kids can take that risk and have a go and not be afraid of failure and know that they are going to be supported. If we are using modelled lessons in maths to get that happening then that is awesome.</p>	<p>During the exit interview the Principal (MPS) indicated that students were more willing to take risks and have a go at challenging tasks.</p>	
<p>Maude (MPS): The [children] are feeling that sense of achievement from it. One of our very challenging children wasn't really interested [in learning] but when Melissa did something in her class and she ended up giving him an extending prompt and he was so excited. I have never seen him [engaged] as much.</p>	<p>During the exit interview Maude (MPS) indicated that even previously disengaged students were now experiencing success.</p>	
<p>Marita (SWPS): Initially I thought my goodness, my children would not be able to do this because I've got the other [grade] 2/3 class...Just seeing how you slowed the lesson right down... You spent time going through the task so that the children understood [the problem] ... [their responses to the task] surprised me... The children were very much getting the concept at the end and wanted to continue working, even when they heard the bell, which was surprising.</p>	<p>The modelling had the desirable effect of enabling Marita (SWPS) to see the children's engaged response to the challenging task.</p>	<p><i>Student engagement</i></p>
<p>Sabrina (SPSS): The researcher roves and has one on one conversations with the students. She asks them to explain their reasoning. [By doing that] she can see how the whole class is going and whether to move on or not.</p>	<p>Sabrina observed the researcher circulating and paying attention to the students thinking and solutions strategies as they worked on the tasks. This is an important element of a pedagogical approach that is focussed on student reasoning and problem solving.</p>	<p><i>Lesson structure: Roving and probing</i></p>
<p>Marita (MPS): I run out of time in bringing them [children] back together at the end and you did that so well. Heather: My biggest challenge in the [summary phase] is how to get my children to focus on the maths, because often they are not tuned into to another child sharing if they have been through the whole process of working on problems.</p>	<p>Marita (MPS) indicated that she often misses the lesson summary as she runs out of time. Heather indicated the summary phase of the lesson was a challenge for her as it is difficult to get children focused on others thinking. This suggests teacher actions leading to this phase may need to be made clearer to teachers.</p>	<p><i>Lesson summary</i></p>
<p>Sheila (SPSS): Teachers copied either the exact lesson or slightly modified to suit age group soon after observing the demonstration lesson.</p>	<p>Sheila indicated that teachers trialled and modified the lesson to suit their age groups soon after observing it. This suggests the modelled lesson plan may be an important scaffold for enactment.</p>	<p>(4) Trialling the lessons</p>
<p>Suri (SPSS): [The modelling] put theory into context. It made learning the strategies more engaging and relevant. It</p>	<p>Suri indicated the modelling enabled her to see that the pedagogical approach was</p>	

<p>made [the pedagogical approach] seem achievable.</p>	<p>achievable as it connected theory with practice.</p>	
<p>Marita: [This is a] very similar lesson that I've chosen to the one we did in Melissa's class. I have kept the same title, "How many more" but I have changed the question to something that I think my kids will resonate with. I hope that I will do a similar tune in to the one [the researcher] did.</p>	<p>Marita (MPS) indicated her lesson plan was very like the lesson she saw co-taught in her colleague's class. She used the same concept but changed the context.</p>	
<p>Sheila (SPSS): The second lesson showed them [the teachers] that if you persevere and rephrase your questions and give the children time to think it through then eventually they will get there. I think that for those people who are not that keen on this approach they saw that you don't give up, you keep going, you give time, you challenge, you let the kids stew... a little bit in that zone of confusion. The fact that it wasn't as fluent as the previous lesson in terms of the student's responses was a blessing in disguise. It showed people things don't always go to plan, but if you stop... if you know the maths you are trying to teach and give the kids time to think, they can get there.</p>	<p>Sheila's comment suggests that the modelled lesson does not have to be a 'perfect' lesson for teachers to learn from it. The teacher learning comes from observers seeing the modelling teacher persevere, rephrase questions, give thinking time and modify the lesson on the fly.</p>	<p>(5) Seeing a modelled lesson warts and all</p>
<p>Principal: They [teachers] liked to see you work hard... it wasn't a process that was a breeze for you... you did have to adjust and modify as you were teaching and have different parts that didn't go quite right and then how you adjusted ...</p>	<p>The Principal (SPPS) indicated that watching the researcher work hard in adjusting and modifying the lesson was welcomed by the observing teachers as it seemed real to them.</p>	
<p>Maude (MPS): There was so much rephrasing and scaffolding and questioning and then you were using enabling prompts and then went to concrete aids, there was a whole range of things that were going on to facilitate their reasoning.</p>	<p>Maude (MPS) observed several teacher actions which facilitated student reasoning including adjusting the lesson to suit the diverse range of learners.</p>	
<p>Melissa: Seeing something work in a working classroom is much more beneficial than reading or PD that is totally removed from the classroom.</p>	<p>Melissa (MPS) indicated the power of modelling in a working classroom where teachers could see how their students responded.</p>	
<p>Maude (MPS): I think teachers learned that you need really clear learning intentions and use precise mathematical terminology, a structure and [the lesson] needs to be challenging for all students, to use real life contexts and to capture the children's interests and apply it to their world.</p>	<p>In her exit interview, Maude (MPS) indicated that the intervention prompted teachers to have clear learning intentions, use mathematical language, use a particular lesson structure which facilitates student reasoning and implement challenging tasks.</p>	
<p>Maude (MPS): I've now seen in most classrooms the teachers using the same lesson structure as [the researcher] did</p>	<p>In her exit interview Maude (MPS) indicated that she had seen most teachers using the</p>	

and [having the] whole class [working on the task] using the enablers and extenders. There has been more evidence of change in [teachers] teaching practice in having high expectations and making sure they were challenging everyone.

Principal: [Observing teachers] learned that our kids could actually do the things that you were talking about. They actually saw it in practice ... so they copied your practice... Even almost the next day teachers were actually trialling; and I've had teachers come up and share with me; we've had sharing and briefing around what kids have been doing in classes. So I'd definitely say that there has been an uptake in teaching practice.

Heather: Seeing someone model makes [the lesson] come alive... especially when you see it 'warts and all'. Seeing the one experience together [with colleagues] is a great growth point because you are talking about the same experience.

Marita: [Re comparing classroom modelling with other forms of PL]. Everyone involved gets something out of it that they can straight away take back to their own practice because you're involved in it. We're all there. You feel inspired by what you've seen. It's not like somebody telling you what to do, it's somebody modelling a way to do things.

Hannah (HPS): [Focusing the observation] on reasoning... having that one thing to look at made it much more [narrow] so that we didn't worry about or notice all the other things. Having that sole focus made it beneficial.

Maude (MPS): It was a valuable intervention for our school. Just the fact that you were modelling the proficiencies rather than looking at content. You were helping the classroom teachers with their practice of mathematics in any content area because you were helping them to develop children's proficiencies.

Megan: [The modelling] clarified for me how important it is to have those three steps... the introduction, making [the task] clear; having time to [problem solve] and getting examples from [the children] and then getting them to come back and have some powerful reflection time. It brings out the importance of

same lesson structure that the researcher used and having the whole class working on the task including enabling and extending prompts. Teachers now had higher expectations of the children and were challenging all students.

The Principal (SPPS) indicated that teachers learned from seeing their students respond favourably to the modelled lesson. Teachers copied the lesson to trial the new approach in their classrooms.

Heather saw watching the modelled lessons as a catalyst for growth with her colleagues and they could talk about the lesson and learn from that

Marita (MPS) indicated that observing modelled lessons with her colleagues and students was powerful professional learning. Teachers could draw from what they saw were effective pedagogies to improve students learning.

Hannah indicated that the focus of the observation on teacher actions to facilitate student reasoning supported teachers to narrow their focus and exclude unimportant things.

Maude (MPS) indicated that modelling teacher actions to develop student's proficiencies in mathematics was powerful because it made the actions transferable across mathematics lessons.

Megan (HPS) noticed the importance of a clear lesson structure to facilitate student reasoning and problem solving.

Narrow focus for the observation

A focus on proficiencies

Lesson structure

why each of those things is so important.

Sabrina: The reading (Stein et al., 2008) was very helpful in that the structure of the lesson was broken down into 5 stages and it made sense. [For example] We've all been grappling with the kids being in the zone of confusion. We all felt like we were leaving them there. [I learnt] how to move them on if [I'm] not doing that explicit teaching and what [my] role in the lesson is.

Sabrina (SPPS): [From the reading I learnt about] anticipating problems you might have and how you would address that. [For example] when you choose the students [to show their work], you are targeting those responses so that the [examples] they present to the class will help move them toward understanding. If the example that you're looking for is not there, you can create one.

Jackie: Why did you stop yourself from writing that number sentence? All those critical decisions that you are making... You made a decision about where not to go because it was best to go somewhere else, none of those things are in the plan.

Maude: Did you plan the questions you were asking the children? That was an important part [of the lesson].

Melissa: "Were you trying to get them to identify 7 as being 70? Is that what you were trying to do?"

Marita: "Would you have at any time thought that it would be okay to do some counting in tens? Like '71' how do you know that is 7 bags of ten? Can we count them together, 10, 20, 30, 40, 50, 60, 70?"

Megan: I've lacked a little bit of confidence. I don't like mathematics. It's something that I feel like I have to overly think about and plan for. You have given me some more ideas about where I can go...not just for mathematics but for all learning areas...The way you [researcher] introduce things, reflect on things and get the children involved.

Heather: That whole notion of within 10 is an interesting thing e.g. understanding the 6 and 4 are contained within 10.

The observation prompted Sabrina (SPPS) to read an article which the researcher sent her called 5 Practices for Orchestrating Mathematical Discussions. Sabrina found the article helpful in answering questions she had about the pedagogical approach.

Sabrina reflected upon a reading she had been given and was able to articulate an important point about purposefully selecting students and their solutions which will contribute to the mathematical goal of the lesson.

Jackie noticed important teacher moves to scaffold learning 'on the fly' which were not part of the lesson plan.

Maude (MPS) asked if the questions that the researcher asked the children during the lesson were planned.

Melissa (MPS) asked for clarification about the unanticipated teaching move made in the modelled lesson to scaffold students thinking.

Marita (MPS) raised the question of whether counting in tens might be helpful for the students.

Megan (MPS) indicated that she lacked confidence in teaching mathematics but the researcher supported her to gain pedagogical content knowledge, some of which she can transfer to other learning areas.

Heather was interested in the mathematical idea, she posed a question to the researcher to clarify her ideas.

(6) Teacher Learning from the Researcher

Anticipating and choosing responses

Teaching decisions 'in the moment' to steer the modelled lesson

Mathematical Knowledge for teaching

What would you have done otherwise to get the children to notice a pattern?

Heather: We needed time to talk and sort out the developmental stages behind the concept of comparison so that we knew what to [plan] for...[We asked ourselves, is this task] broad enough to cater for all the developmental stages that are there.

Hannah (HPS): I have noticed that Holly and Heather are doing a lot more work on the notion of difference. We have really picked up on that concept and about using the different vocab. We shared the concept of difference with the primary team and what that means.

Heather: I think it also takes someone to be there as a 'critical friend' with a special interest/knowledge in the subject so that the knots in the thinking can be heard and professional learning put in place that would help tease out or challenge the ideas.

Hannah: [The planning meeting] and the questioning [by the researcher] [prompted us] to look at the tasks we are setting... Are we setting tasks that align with goals of a lesson?

Heather: [Having access to a knowledgeable other] gives you the confidence to start to ask some questions around helping to sort out say maths...you're confident that the person has more experience in that area and can share from their experience base.

Heather (HPS): Now we have 50 problems, because all it is changing the numbers and changing the context. If we've got the language, we don't have to dream up [new ideas].

Sophie: It was obvious [from the lesson] that we have not done any work on arrays yet or on division but we have done some stuff on multiplication. Some of them did really well.

Marita: They're [students] seeing these clocks and thinking what is the digital time that matches but they're not understanding the duration of time I've seen from this lesson.

Suri: Even though the students can count in 5's and 10's, they don't necessarily transfer that to the clocks. They haven't quite got the concept of the passage of time.

In stage 3, Heather (HPS) appreciated the time with the researcher and her colleagues to discuss and refine understanding of the concept of difference before planning a suitable task for the children with enabling and extending prompts.

In cycle 4, Hannah indicated that she had observed that her colleagues were now focussing on the concept of comparison and difference with their classes. She noted the team shared the ideas with the grade 3-6 team at her school.

Heather claimed that the researchers' role as 'knowledgeable other' was an important part of the intervention to support teachers to deconstruct ideas.

Hannah noted that the researcher's questions prompted her to reflect upon the tasks that teachers were presenting to students.

Heather noted that having access to a knowledgeable other supported her to begin to ask questions about mathematics content and pedagogy.

Heather recognised a task could be altered if the context and the numbers were changed.

The student's responses to the task prompted Sophie to reflect on the mathematics that the students need to learn.

Marita observed that the grade 3 students did not understand the duration of time.

Suri observed that the children in grade 3/4 were unable to transfer their knowledge of skip counting by 5's to the analogue clock face.

Knowledge and understanding of mathematics

<p>Holly: They [students] are very reliant on concrete [materials]. Even though they know some strategies, they keep reverting to counting by ones.</p> <p>I probably haven't used that word [difference] before.</p>	<p>Holly (HPS) claimed her students were reliant on materials and counted by ones. After reading through the lesson plan she also claimed she had not used the word difference with her grade 2/3's</p>	
<p>Megan (MPS): The pre and post conversations have been powerful and ...theory put into practice in our classrooms</p>	<p>Megan (MPS) valued the pre and post lesson conversations and seeing theory put into practice in her school.</p>	<p>(7) Collaborations engaging in practice</p>
<p>Heather (HPS): The pre-lesson and post lesson meetings teased out some of the maths. Maths is not the subject where I feel most comfortable. It gave me a great opportunity to sort out the maths that we were focussing on.</p>	<p>Heather (HPS) indicated that the pre and post lesson meetings were helpful for her to unpack the mathematical ideas that were the focus of the modelled lessons.</p>	<p><i>Unpacking the maths</i></p>
<p>Melissa: The discussions pre and post, seeing it work and being able to see children's thinking [prompted my learning]</p>	<p>In her exit survey, Melissa (MPS) indicated that the pre and post discussions and seeing children responding to the lessons prompted her learning.</p>	
<p>Heather: The opportunity to chat ... to reflect [on our own] and a bit of distance [have been] an important part of our development. We talked about what might be the next step for some of those children that haven't quite got the concept... and about times for direct teaching outside the problem solving.</p>	<p>Heather (HPS) claimed that time to talk and reflect with colleagues between stages was very important for her learning. The modelling raised questions for her about direct teaching, however she saw the possibility of teaching through problem solving.</p>	<p><i>Formal and informal opportunities to talk</i></p>
<p>Hannah: The post-lesson meetings were most beneficial [in stages 1 and 2] ... the conversations that were happening in that room and beyond that day. For example, with the focus on reasoning, having that opportunity after the lesson to de-brief and think about how we could build on it. After that we would just have conversations in the corridors...this was a critical phase.</p>	<p>Hannah (HPS) indicated that the post lesson conversations in stages 1 and 2 were critical for teacher learning. She noted that deconstructing the pedagogies associated with student reasoning enabled teachers to continue the conversation into the corridors and build on the ideas that were modelled.</p>	
<p>Hannah: The biggest change in our practice at school has been relying on each other to share the ideas and talk about how we can make things better. This has really opened [collaboration between colleagues]. We have adopted the planning, the lesson structure, the enablers, the extenders and so on.</p>	<p>Hannah (HPS) pointed out that collaboration between colleagues has been the biggest change at her school since the intervention. Teachers are now relying on each other to plan for improvements to practice.</p>	<p><i>Building collaboration between colleagues</i></p>
<p>Sheila: Thorough planning improves teaching</p>	<p>Sheila (SPPS) indicated that teachers recognised that thorough planning improves teaching. Teachers who had not considered the rationale for a lesson before worked together to create a lesson where they were clear about their purpose and</p>	<p>(8) Teacher learning from Co-planning</p>

<p>Sheila: Planning proforma has been used by teachers for shared lessons.</p>	<p>successfully anticipated student responses.</p>	<p><i>Planning proforma</i></p>
<p>Maude: It's important to know what questions you are going to ask the children, and have that planned.</p>	<p>Sheila (SPPS) indicated that the lesson plan proforma had been utilised by teachers in subsequent planning sessions. This suggests that they saw it as helpful.</p>	
<p>Hannah: [The planning meeting] and the questioning [by the researcher] [prompted us] to look at the [tasks] we are setting... [I asked myself] Are we setting tasks that align with goals for a lesson?</p>	<p>Maude gave feedback to Marita about planning the questions before the lesson in stage 4.</p>	<p><i>Planning questions</i></p>
<p>Hannah: When we were doing the planning [Stage 3] we had our curriculum there, we thought this is what we want the kids to learn ... but when we set the task, [we realised] a word could change the meaning of the whole task...we had to keep referring to our outcome to get those words correct. That challenged us.</p>	<p>The planning meeting with her colleagues and the researcher prompted Hannah to reflect on choice of tasks that align to the goals of the lesson.</p>	<p><i>Aligning learning goals with the task</i></p>
<p>Heather: I've realised you can choose tasks from different books, but if you can't unpack the math's that's in underneath the task, then you're not developing the children's math's skills, you're just choosing tasks.</p>	<p>Hannah recognised teachers needed to be clear about aligning the goals of the lesson with the mathematics and the task.</p>	<p><i>Choosing tasks</i></p>
<p>Heather: It feels a bit like the rug has been pulled out from under my feet...I had a way of working and a way of choosing tasks, but now it's turned my head around and I'm really thinking about why it is I am choosing [tasks]. For example, I might have had some ideas of doing numbers to ten [with prep/ones], but now I'm looking at the proficiencies and [asking myself] what is the foundation that I'm building the maths on?</p>	<p>Heather recognised the importance of planning tasks that would develop children's learning in mathematics</p>	
<p>Heather: [When the teacher is really clear about the learning intention] it makes you disregard the other comments and draw back to what it is we are working on.</p>	<p>Heather (HPS) articulated that the modelling prompted her to reflect on her choice of tasks and think about how it relates to the AC: M and what it means for her future classroom practice.</p>	<p><i>Learning intention</i></p>
<p>Heather: The use of the lesson plan structure was extremely helpful for being clear about the learning intention and flow of the lesson.</p>	<p>Heather (HPS) could see that being clear about the learning intention helps the teacher to remain focused on the mathematical intent in the summary phase of the lesson.</p>	
<p>Holly: Having a plan like this helps us be more focussed on the maths we are trying to draw out.</p>	<p>Heather indicated the lesson plan was helpful in providing clarity for the goals and flow of the lesson.</p>	<p>Holly (HPS) indicated that planning the lesson in stage 3 helped her to focus on the</p>

Hannah: Planning in this way, you know what you're looking for.	mathematics we were planning to co-teach.	Hannah (HPS) indicated that the detailed plan for effective in having clarity about what to look for in the lesson.	
Holly: I have been more purposeful with planning and writing questions and [thinking] about what the maths is about.	In the post lesson survey, Holly indicated that she had become more purposeful with her planning.		
Heather: Designing the problem-solving tasks has enabled me to be clearer about the maths I'm trying to focus on.	Heather indicated the planning stage was important for her to become clearer about the mathematics she was focusing on.		
Heather: The lesson plan structure was extremely helpful for being clear about the intention and flow of the lesson. Perhaps some mapping prior to the lesson with regards to levels of sophistication might be useful for fine tuning observations and making decisions about the maths during the [summary] phase.	Heather's comment in the post intervention interview indicates she would have liked the opportunity to anticipate student responses to tasks in the pre-lesson meeting.		<i>Anticipating student responses</i>
Heather: Co-planning meant you had your colleagues to really sort out the problem and make sure it was going to do what you were aiming for.	Working together enabled Heather to experience several heads working together		<i>Collaboration</i>
Hannah: [I have recognised] the importance of our discussions in planning for the tasks.	Hannah recognised the importance of the discussions in planning for learning.		
Hannah: [In my role as Assistant Principal] I set myself a goal of being able to support teachers to reflect on their goals for the lesson "what is the purpose of this lesson? have you thought of?"	The intervention prompted Hannah to support colleague teachers to be clear about their lesson goals.		<i>Clarity of purpose</i>
Heather: And all the toing and froing about the task and the talk that we have had [and thinking] "Is this task really going to do what we intend."	Heather indicated the depth of the back and forth discussions in the planning of the stage 3 task to explore the concept of difference.		
Principal: Teachers learned explicitness around knowing exactly what your most important purpose is in the lesson...	The Principal (HPS) indicated that one of the most important things teachers learned was to be clear about the learning goals of the lesson.		
Sophie: The rationale for the lesson was the hardest. Trying to think of the theory behind something is not something we do. We just know what we have to do and we don't go looking for the theory behind it.	Sophie indicated that thinking about the rationale for a lesson was difficult because it's not something she is used to.		<i>Planning for enactment</i>
Marita: I'm not sure that I'll get anything more out of this [enactment] today than I what I've already got apart from a belly full of butterflies and	Marita (MPS) did not anticipate that her enactment of a maths lesson would improve her pedagogy because she felt she		(9) Planning, enacting and reflecting

<p>feeling a bit unwell. I have already implemented things that I've learned from you [researcher] into my classroom and I've already trialled the lessons with the kids.</p>	<p>had already implemented new pedagogies that she observed in the modelled lessons.</p>	
<p>Marita: I found it really difficult to bring them back together at the end, that's where it is hard to know what to say. I need to focus on the thinking. It is so hard not to tell them what to do.</p>	<p>Marita (MPS) indicated the difficulty she had in summarising the lesson and refraining from telling students what to do.</p>	<p><i>Refraining from telling students what to do</i></p>
<p>Marita (MPS): When I watched myself [on video], I was saying things like "no, not quite" and I was like Aaargh! I didn't say things that I have been saying in my classroom a lot like "keep thinking".</p>	<p>Marita (MPS) realised from watching the video of her enacted lesson that she did not say the encouraging phrases that she had been practising in her classroom.</p>	
<p>Marita: The hardest thing I found [from enacting the practice] was looking over students work to find examples to use to model for other students and bringing the lesson back in. It's something I need more practice at.</p>	<p>Marita (MPS) indicated the most difficult thing for her in enacting the pedagogies was in choosing which student ideas to focus on during the discussion.</p>	<p><i>Summarising learning</i></p>
<p>Marita: I know that I'm not overly fantastic at maths and I need to be much better, so even though I didn't want to do that [enactment] I guess it probably stretched me in a lot of ways... I learned that to draw out kids thinking was the most difficult thing for me and remembering as I went around who needed to come back up [whose ideas would be shared]. It highlights the importance of thinking about the questions we ask children.</p>	<p>Marita indicated the enactment stretched her. It helped her recognise the difficulties she had in probing students thinking and recognise the importance of planning questions to ask students.</p>	<p><i>Planning questions</i></p>
<p>Marita: From my own modelling I learnt it's really tricky to try to be somebody else. I tried to model my lesson from what I saw [the researcher] do and I found that was difficult... Sometimes we need to be just who we are but pull in the pieces of other people's teaching that we would like to aspire to.</p>	<p>Marita (MPS) indicated it was too difficult to try to imitate someone else. She learned the importance of being herself and incorporating important pedagogies that she observed into her own practice.</p>	<p><i>Be yourself</i></p>
<p>Hannah (HPS): Planning in this way, you know what you're looking for</p>	<p>Hannah (HPS) indicated that the detailed lesson plan was helpful in providing clarity for her about what students strategies to look for in her enacted lesson.</p>	<p><i>Anticipate student responses</i></p>
<p>Hannah: The enablers and extenders mean you can cater for a greater range of children.</p>	<p>Hannah (HPS) indicated that by using enabling and extending prompts in her enacted lesson, it was possible to cater for all students in one class.</p>	<p><i>Plan enabling and extending prompts</i></p>
<p>Sophie: I now use [enabling and extending prompts] in my maths lessons.</p>	<p>The modelling prompted Sophie to commit to using enabling and extending prompts to differentiate tasks in her maths lessons.</p>	

<p>Hannah: The planning stage [in stages 3 and 4] was probably more beneficial for me personally. As the [enacting] teacher having that opportunity to give myself some critical feedback... knowing exactly the outcomes and exactly the rationale.</p>	<p>Hannah indicated that planning and enacting a lesson were most beneficial for her. She knew exactly what the goals of the lesson were and the rationale behind it and she was able to give herself some critical feedback after watching the video of herself.</p>	<p><i>Know the rationale and outcomes beforehand</i></p>
<p>Sophie (SPPS): [The planning prompted me to] think about the purpose of the lesson and the challenges for [students] before the lesson.</p>	<p>Enacting the lesson prompted Sophie to reflect on her planning to think about the purpose of the lesson and anticipating any difficulties students might encounter.</p>	<p><i>Be clear about the purpose of the lesson</i></p>
<p>Sophie: Before I was catering for the average or below average and spending most of my time with them. I learned that it's okay to challenge all the kids.</p>	<p>The favourable student responses to challenging tasks in the modelled lessons prompted Sophie to reflect on her teaching. She indicated that in the past she catered for the average to below average students, but now she is committed to the idea of catering for all students.</p>	<p><i>Challenge all the children</i></p>
<p>Sophie: My goal is to use more open ended real life problems. Even though I knew that was what I was meant to be doing, I don't think I was doing that very well.</p>	<p>Sophie committed to the idea of using problematic maths tasks.</p>	<p><i>Challenging tasks</i></p>
<p>Principal: I just think seeing someone else [model practice] is a powerful thing, then seeing one of their colleagues practise as well. It's a powerful step into having a go, rather than just going to professional learning and talking about something. I think it bridges that gap around, "Well what does it look like in my classroom?" ... We have seen changes in practice and there's been far more high level discussion in teams and groups around problem solving in maths.</p>	<p>The Principal (SPPS) indicated that seeing the lessons modelled followed by enactment was powerful professional learning. It enabled teachers to actually have a go at the practice rather than talking about it. She noted that there has been far more discussion in teams about problem solving.</p>	<p>(10) Seeing what is possible</p>
<p>Jackie: [This intervention highlights the importance of] teacher pedagogy: developing questions and refining tasks.</p>	<p>Jackie noted the intervention highlighted the importance of teacher pedagogy including planning and refining tasks and developing questions to ask students.</p>	
<p>Marita: [Re comparing classroom modelling with other forms of PL]. Everyone involved gets something out of it that they can straight away take back to their own practice because you're involved in it. We're all there. You feel inspired by what you've seen. It's not like somebody telling you what to do, it's somebody modelling a way to do things.</p>	<p>Marita (MPS) indicated that observing modelled lessons with her colleagues and students was powerful professional learning. Teachers could draw from what they saw were effective pedagogies to improve students learning.</p>	

<p>Hannah (HPS): [Focusing the observation] on reasoning... having that one thing to look at made it much more [narrow] so that we didn't worry about or notice all the other things. Having that sole focus made it beneficial.</p>	<p>Hannah indicated that the focus of the observation on teacher actions to facilitate student reasoning supported teachers to narrow their focus and exclude unimportant things.</p>	<p><i>Narrow focus for the observation</i></p>
<p>Maude (MPS): It was a valuable intervention for our school. Just the fact that you were modelling the proficiencies rather than looking at content. You were helping the classroom teachers with their practice of mathematics in any content area because you were helping them to develop children's proficiencies.</p>	<p>Maude (MPS) indicated that modelling teacher actions to develop student's proficiencies in mathematics was powerful because it made the actions transferable across mathematics lessons.</p>	<p><i>A focus on proficiencies</i></p>
<p>Principal (SPPS): It's the link between talking about practice, and doing practice, changing practice. We've got so many teachers in the teaching profession that can do the rhetoric, but the practice in the classroom doesn't match the rhetoric.</p>		<p><i>Missing Link between talking about practice and practising</i></p>
<p>Principal (HPS): I think [modelling] is critical because it's where we get to the coalface... It is teaching... and then going and talking about teaching as opposed to other forms of PL that are just talking about teaching and may never actually make it back to the classroom.</p>	<p>The Principal (HPS) summed up the success of the intervention at his school by implying that modelling and the collaborative discourse associated with it is a most effective form of professional learning.</p>	
<p>Principal (SPPS): [Modelling] it's the final link, it's the most important link. You can do the professional learning, the talking about it, then if you watch someone do the practice and then the final step of having a go yourself and evaluating and reflecting all the way through, I just think it's the final thing... what you did with us was great. It was excellent.</p>	<p>The Principal (SPPS) indicated that the model – seeing practice, deconstructing practice and enacting practice was an excellent form of professional learning for her teachers.</p>	
<p>Principal: I have been looking for ways to get teachers to be in each other's classrooms. With the journey, we've been on with both literacy and numeracy we have been trialling lots of new ways of teaching and specific strategies and they meet regularly but the one factor they were missing was the opportunity to be in each other's classrooms.</p>	<p>The Principal (HPS) indicated that the intervention supported the move towards teachers observing each other's classroom practice.</p>	<p><i>Opportunities for teachers to observe each other</i></p>
<p>Sheila (SPPS): There was some accountability to being involved "up close and personal" that is often missing...the ongoing nature made it more meaningful rather than "one off" pd approaches.</p>	<p>Sheila indicated that the model had accountability built in as it was situated in teachers own classrooms with the expectation that observing teachers would contribute to pre-and post-lesson discussions to deconstruct the practice they observed and</p>	<p><i>Accountability</i></p>

<p>Principal: This [intervention] can bring a greater level of accountability planning after school. You can't fluff something up if your colleagues are in there supporting you do it.</p>	<p>collaboratively plan for the co-taught lesson.</p> <p>The Principal (HPS) indicated that the collaboration brought a greater level of accountability to planning meetings.</p>	
<p>Principal (SPPS): It was hard for us to get the two hours... we did all that because we had the back-up... I definitely believe that the two modelled lessons at the start were really good to see you modelling... after that if we'd had an extended model where every teacher in the team then had a turn at doing the co-planning and co-teaching with you. So that everybody got exposure.</p>	<p>The Principal(SPPS) indicated that whilst it was difficult to find time for the intervention, seeing the two modelled lessons was powerful for her teachers. She suggested the possibility of an extended model where each observing team member had an opportunity to co teach and enact pedagogies which would assist everyone to maximise their potential for learning.</p>	<i>Extending the model</i>
<p>Sabrina: We would have got a lot more value out of this [intervention] if it was not concurrent with our reporting period.</p>	<p>Sabrina indicated that the timing of the intervention in her school was not ideal due to reporting.</p>	(11) Implementation Issues <i>Timing</i>
<p>Sheila: The duration of the sessions did not match the segments of the school day here and hence was disruptive (eating in to teacher lunch break). If the timing came from within the school rather than the presenter's timetable it might have been even more productive.</p>	<p>Sheila indicated timing of the intervention was problematic in her school setting. The intervention was somewhat disruptive in the school as it did not align with the school day. Sheila suggested that the timing of the intervention should be a prioritised to suit the schools needs rather than researcher's time</p>	
<p>Sophie: We missed out on our planning team meetings and it felt like our valuable learning group time was being yanked away from us four times.</p>	<p>Sophie (SPPS) indicated dissatisfaction that valuable team planning time was removed so that her team could be part of the intervention</p>	
<p>Megan: Timing is difficult... I thought that only having a half hour session to [de-brief the last lesson] and not having a chance to plan together what we wanted to get out of today was really difficult.</p>	<p>Megan (MPS) raised the issue of timing and how teachers felt they didn't have enough time to de-brief the previous lesson and co-plan for the next modelled lesson.</p>	
<p>Marita: Teachers did not have any say in what was happening until we were all informed that we would be coming along to watch the researcher model some lessons for us. To begin with we thought it would be beneficial but we also thought it was <i>another thing</i> that we had to do and the timing wasn't good as it was in our most crucial part of the day.</p>	<p>Marita (MPS) indicated teachers were not consulted re the timing of the intervention.</p>	

<p>Maude: It was so tricky trying to get 6 relief teachers. We had to pay relief teachers but it was all so valuable.</p>	<p>Maude (MPS) indicated the intervention was expensive in that the school was required to pay relief teachers to cover classes but that it was money well spent.</p>	<p><i>Cost</i></p>
<p>Principal: There was the trust factor... trepidation [from teachers] around how am I going to go, I'm not sure I feel comfortable being watched, I'm not sure I feel comfortable talking into audio recorders.</p>	<p>The Principal (HPS) revealed that teachers felt untrusting of the process in the initial stages of the intervention.</p>	<p><i>Trust</i></p>
<p>Principal: Their [teachers] Achilles heel was that they didn't know [the researcher]... At each stage the tenseness eased.</p>	<p>The principal (HPS) highlighted issues with trust which eased as the intervention progressed.</p>	
<p>Hannah: Initially some teachers were daunted by having an extra person in the room when they are usually just in the classroom by themselves with all the kids.</p>	<p>Hannah (HPS) indicated that initially some teachers were uncertain of having the researcher into their classrooms.</p>	
<p>Principal (HPS): We've got a group of committed teachers who had a level of trepidation at the start ...and were challenged...they have all grown through it and come out the other side as opposed to something starting well and dropping off. It [the model] has grown legs in each step along the journey... and all the snippets that they've learned along the way, even just rejigging their own practice around how they differentiate which wasn't really the primary focus.</p>	<p>The Principal of HPS indicated that although teachers were reluctant to begin with they all made shifts in their teaching practice including changing the way they differentiate practice.</p>	
<p>Heather: I moved from being uncertain and wary to really seeing the benefits through the whole process at the end.</p>	<p>Shift in thinking</p>	<p><i>From uncertainty to confidence</i></p>
<p>Sheila: Although there was consultation about the content of the modelled lessons this was often rushed or done via email which did not always result in lessons closely aligned to previous lessons in the class. Teachers would have like to have seen greater alignment of lessons.</p>	<p>Sheila (SPPS) pointed out that although the researcher consulted teachers about the content of the modelled lessons it was often rushed and teachers did not feel the planned lessons necessarily aligned with their current focus.</p>	<p><i>Alignment with teacher practice</i></p>
<p>Sophie: The Principal just told us what was happening. We did not have a choice. I felt annoyed.</p>	<p>Sophie (SPPS) indicated she was angry that the Principal informed her and her colleagues about the intervention without consultation.</p>	<p><i>Buy in from teachers</i></p>

<p>Sheila (SPSS): After your first visit Marita, Suri and Sally were trying to implement strategies they saw and reproduce your lesson. They probably weren't the target for the change in pedagogy. The more reluctant the person usually correlates with the greatest need. There is some resistance.</p>	<p>Sheila indicated that whilst some teachers attempted to implement strategies they observed in the modelled lesson, others were resistant to making changes to their approach to teaching and others had already made changes to their practice. She noted that the more reluctant teachers seemed to have the greatest need for change.</p>	
<p>Principal (HPS): I personally think you can't really spend a lot more time on it [the intervention] because of the cost. I think the model [stages of the intervention] is right. What we would do is pick something that we're really struggling with or something new that we are really challenged by and that would hook them [the teachers] in.</p>	<p>The Principal highlighted the cost of the intervention but thought that the model of the intervention was about right in terms of changes in practice. He suggested that teachers could come up with the problem of practice themselves in order to get buy in.</p>	<p><i>Start with a problem of practice</i></p>
<p>Hannah: Some people changed practice within four stages and for some it would need longer, it really depends on the person and what it is you are trying to change.</p>	<p>This indicates teachers are on a continuum of learning and one size does not fit all.</p>	<p><i>Differentiation for teachers</i></p>
<p>Principal: I think you've got to have a tight model around it and that's what made it work so well. Our 'Watch others Work' stuff that we did last year was looser and still far more invitational, but we have some staff here that would take forever to get on board. I think that the four-session structure was the power of it.</p>	<p>In her exit interview, the Principal (SPPS) indicated that the structure and tight processes of the model made it robust. The four-staged structure, which included seeing practice modelled, deconstructing that practice and opportunities to enacting the practices was powerful.</p>	<p><i>Structure</i></p>
<p>Principal: Exploring the model through other curriculum areas would be interesting... embedding the model, probably in your whole year of team meetings and learning group meetings.</p>	<p>The Principal (SPPS) saw the potential of exploring and embedding the model in other learning areas.</p>	<p><i>Implications for other curriculum areas</i></p>

Appendix 15: Lesson Reflection Stage 1 Swift Parrot Primary School

27 March 2014

The first modelled lesson at SPPS was requested in a grade two/three class. The assistant Principal informed me later that he suggested to the class teacher that the lesson be in her class for two reasons, one that the children were quite settled and two, because the teacher required support with pedagogical approaches to teaching. Although this teacher was mature aged, she had only five years teaching experience in a range of grades from one to six. In response to the question on a survey given to observing teachers prior to the start of the intervention asking them what they found difficult when teaching maths this teacher wrote “catering for all developmental levels because time is tight”.

I emailed the Principal with a proforma for observing teachers to complete and send back.

I received the completed lesson request proforma two days before the lesson. The teacher had requested a focus on “turn around” facts and questioning. Based on teacher responses to the questions above, I planned the lesson. I misread the lesson request and planned a lesson focused on equivalence, which was a modified version of *MAKING BOTH SIDES EQUAL (EPMC)*.

The following is the lesson outline that I planned. The main task on which the students worked was as follows:

Work out some numbers that make these equations true

$$898 + ? = 900 + ?$$

$$95 - ? = ? - 10$$

The lesson documentation also included an introductory task:

Work out some numbers that make these equations true

$$3 + ? = 6 + ?$$

$$8 + ? = 10 + ?$$

$$98 + ? = 100 + ?$$

The lesson documentation also included a consolidating task:

What might be the missing numbers? Give at least 10 possibilities.

$$224 + ? = ? + 10$$

$$330 - ? = 125 - ?$$

I met with observing teachers thirty minutes prior to the lesson. During the pre-brief, observing teachers were given a copy of the lesson plan and it was discussed. Initially I explained my misreading of the lesson request. I also assumed at least half of the children would be in grade three. I was wrong. The class teacher informed me that only three children were in grade three and they were “very weak”. I thought to myself at that point that the content and tasks might be somewhat inappropriate for the grade level of the children, but I would assess when I talked to the children.

During this meeting, I handed teachers an observation proforma, which I invited them to complete either during or after the lesson. The observation proforma was designed to focus observing teachers on reasoning and evaluate what they saw. The observation proforma contained the following prompt:

Write down everything you saw the modelling teacher say and do to facilitate the students' reasoning.

I asked the class teacher how she thought the children would go and she said they would struggle as the tasks were very challenging. Other teachers said they thought the children would find the lesson too difficult.

At the beginning of the lesson, on a slide for all students to see I had the following written:

An equation has two sides, which are equivalent.

Sometimes it is important to work things out in your head.

It is also important to explain your thinking

I asked the students if they knew what the words equation and equivalent meant. Many students seemed confused and afraid, so I decided in that moment to write the following number sentence on the white board

$$9 + 3 = 12$$

I asked the students what they called it. One child said, “a sum”. I explained that sometimes we might think of it as a sum but mathematicians call it an equation or a number sentence. I told them today we would be calling these equations. I asked what they thought equivalence meant. I gave thinking time. The children offered incorrect explanations to which I responded, “keep thinking” then I asked the children to tell me how $9 + 3$ (which was written on the white board) might be the same as 12. Children found the idea difficult to comprehend so I rubbed out 12 and asked for alternative answers to $9 + 3$. Children continued to struggle, so I asked

What is $9 + 3$ the same as?

One child could tell me $3 + 9$ and I asked others if they agreed. I asked the student how he worked it out and he replied that he used “turn arounds”. I praised the student for explaining his thinking and said we can say that $9 + 3$ is the same as $3 + 9$ because they are equivalent. They are both the same as 12.

The observers later commented that I established the vocabulary, clarified what the terms meant and linked it to the children’s known understanding.

Next, I told the students the following (I had this on a ppt slide):

In this lesson, I need you to

show how you get your answers

keep trying even if it is difficult (it is meant to be)

explain your thinking

listen to other students

I explained that it is okay to be confused. I said, “We call this thinking time the “zone of confusion” and we are all going into the zone of confusion today because this is going to be really tricky. In that moment, I decided to pose the following introductory task which was written on a slide:

Work out some numbers that make this equation true

$$3 + ? = 6 + ?$$

This decision was made because I could tell the majority of students were struggling to understand the concept of equivalence and I thought having smaller numbers might scaffold their understanding. The students sat in a group at the front of the classroom but thought about solving the task individually. I began to use icy-pole sticks with individual student names

written on each one with the intent of engaging the whole class. The observers commented that the students all became accountable. Many students seemed puzzled at first, but then they started to engage with it. Some students struggled and required longer thinking time, so I said to them: “Keep thinking and I will come back to you”. I kept going with other student responses and then went back to the students who were still thinking. Observers commented that students sharing their own reasoning or explanation provided a scaffold for the students who were still thinking. When I went back to those who had longer thinking time they most often had success with their responses.

Then I posed the next enabling prompt:

Work out some numbers that make this equation true

$$8 + ? = 10 + ?$$

This time I asked for multiple answers, to which students began to respond with enthusiasm. I posed the next task:

Work out some numbers that make this equation true

$$98 + ? = 100 + ?$$

The children became very excited and I could see many students connecting with enthusiasm and engagement. It seemed that the previous tasks provided sufficient scaffolding to enable all students to attempt this task. Some students continued to struggle but they engaged in the thinking. A girl gave a lovely explanation of why $98 + 4$ is the same as $100 + 2$ after I went back to her a second time after extended thinking. She said:

2 and 2 is 4, so $98 + 2$ is 100 and then I put the other 2 on and that was 102 so that is the same as 102

Next, I posed the main task:

Work out some numbers that make these equations true

$$898 + ? = 900 + ?$$

$$95 - ? = ? - 10$$

Give a range of responses for each equation

The students moved to their tables and floor spaces and worked individually on the main task. All students attempted the first of the two tasks eagerly. After a while I asked them to talk to the person next to them. This created positive discussions, but it seemed that the individual thinking time was important. Observers commented on the student’s persistence. After moving about the classroom to see how the students were engaging with the task, I discreetly gave some of them enabling prompts in the form of “if you are stuck cards”. One girl said to me,

That says, “if you are stuck”, oh that is so nice because I am a bit stuck

She proceeded eagerly to solve the enabling prompt then proceeded to try the main task again. I selected some students for review. A couple of students could find solutions to the second task and I chose those students to describe their thinking at the end of the lesson. I photographed their work with an iPad and projected it on to the smartboard. With just 7 minutes to go, I stopped the class and invited them to the mat. I asked the selected students to share their thinking. One of the students said.

Well, it was really hard to think and I was in the zone of confusion, but it made me really try

Then he proceeded to give a clear explanation of how he found the patterns. All students wanted to keep going even after the bell went for lunch. Observers commented on the student's persistence. An observer commented that the kids did the teaching at the end of the lesson.

I had prepared consolidating tasks, but none of the students got to that point.

In the post-lesson meeting, the assistant Principal said to me:

“I had my doubts about whether you could pull this off...you are obviously an expert and I thought the language you would use would be way beyond what our teachers understand and of course you don't know these kids, but you did it, it was brilliant...and the kids...they just wanted to keep on working...congratulations”

The class teacher commented on my encouraging stance throughout the lesson. She did not write hardly anything on her observation proforma, whereas her colleagues wrote at least one page and in some cases more than one page.

The main thing observers liked, which came through in the post lesson meeting was the thinking time and going back to the students who required more time to think before they could explain.

Appendix 16: Reflections on my Modelling at Raven Primary Cycle One

Lesson 26 February 2014

From reading through the pre-intervention teacher surveys, it is clear that teachers at Raven Primary school all have high aspirations for their students. For example, they want them to be confident, to solve problems, to generalise and to relate what they are learning to the real world. However, these teachers also wrote that their students “have limited strategies for working mathematically” and that there was a “range of abilities which are difficult to cater for”. All teachers wrote that catering for diversity was their most difficult concern in teaching mathematics.

They went on to write that in their view effective teachers explained learning intentions and success criteria. They worked with students in small groups of like ability, where teaching practice was modelled so that children could see how to complete tasks. Groups would rotate so that the teacher could have a focus group each day. This suggests a disparity between what these teachers aspire to and what they believe effective teachers do. None of the teachers mentioned the proficiencies from the Australian Curriculum.

If I’m supporting students to develop reasoning then I need to make sure my classroom is a community of inquiry where students have multiple opportunities to think, explain, refute, justify and make conjectures. This must surely mean all the students need to be working on a similar task that meets the goal of the lesson. This implies that I need to support teachers to understand the proficiencies and to make them loud and clear.

I asked Rebecca if she thought any children would struggle with the learning experience and she said “no”. Rebecca is an early career teacher with 6 year’s experience. This suggests teachers may benefit from opportunities prior to the lesson to think deeply about and discuss typical responses and possible misconceptions that children may demonstrate as they engage with the mathematics.

The children did struggle with the initial task that I posed, so I posed a simpler version. I am not convinced the idea of posing a challenging task as a first step is the right way to go. I think it might be more effective to provide a simpler version of the task I intend to pose, then make it more challenging.

Clearly, these students were not used to being challenged. They did not know how to ask a question or have a mathematical discussion. It was a mistake to suggest they record in their work books because their work books had ruled margins and were focused on neatness and correct answers. This suggestion seemed to hinder their attempts at problem solving. For example, I saw several children attempting to rule margins rather than think about how to solve the problem. I was aghast when I saw this practice.

I had to go right back to the first enabling question. We discussed it. One girl, called Issy, solved it with her fingers (as did many of the children), but then a bright spark gave me just what I was looking for (building to ten) and he explained it clearly, so then I posed the next question and Issy solved it using the building to ten strategy. It was great to see her connection. The class teacher was surprised by her response. We kept going. There was much discussion. A bright student attempted to explain his thinking for finding the answer to $295 + 35$, but described a long-winded procedure using a vertical algorithm in his head. One of the observing teachers reported that she saw him using more efficient methods after hearing other children report on their strategies.

Teachers noticed many actions that lead to facilitating student reasoning, but one of the observers said to me later “I want to learn to teach like that but there is so much I need to change. I want to copy your lesson to practice first”. I told her it wasn’t about “copying”, but she said she needs a starting point. This made me wonder if I need to support teachers to categorise teacher actions, which facilitate student reasoning to make the pedagogy more visible to teachers.

Appendix 17: Reflections on Listening to Audio Recordings at HPS

Stage 1 and 2, June 14 2014

I didn't wait long enough when they [the teachers] spoke. I needed to pause and listen and give wait time. I was jumping in far too often. It would have been more effective for me to wait until I was asked a question and then respond.

I cringed at how much I talked when I went through the lesson with teachers ... [the only thing] I asked [was] how the children would go and Holly said "Ah...you'll see". It seemed she was unsure. There was no discussion. ...In hindsight, it would have been much better to have had [the teachers] [read] the lesson [plan] and do the tasks or at least pre-empt what the kids would do. In that way, they would have had far more knowledge going in to the lesson of what to look for.

Appendix 18: Lesson Plan for Swift Parrot Primary School Grade 6

For the lesson on 22nd May 2014

Modelling teacher: Louise

Title of the Lesson: The Dog Run

The Dog Run

You have 28 metres of fencing to build a dog run in your back yard for your dog. Each fence piece is one metre, so each side of the dog run will be in whole metres. You have decided to use a rectangular shape that will stand by itself under some trees. Use any method you like to find all possible dog runs you could build.
Draw them on a piece of graph paper

Rationale for the lesson:

Middle school students should begin to develop an understanding of the relationship between area and perimeter (the distance around a region) and the units used to measure them. Area and perimeter are continually a source of confusion for students. Perhaps it is because both involve regions to be measured or because students are asked to memorise formulas for both concepts and tend to get the formulas confused. Both formulas use the same linear measurements, length (l) and width (w) and if the formulas are not understood conceptually, students can easily forget which formula to use when. An interesting idea is to contrast the two ideas as is done in this lesson.

Goals:

- To help contrast the concepts of perimeter and area
- To develop the relationships between perimeter and area of different shapes when the area is fixed
- To challenge the common misconception that a fixed perimeter yields the same area no matter how you shape it
- To compare and contrast the units used to measure perimeter and those used to measure area

Year level: Year grade 6

From the Australian Curriculum:

This lesson addresses the following descriptor from AC for year 6

- Solve problems involving the comparison of lengths and areas using appropriate units (ACMMG137)

This lesson also addresses the following descriptor from AC for year 5:

- Calculate the perimeter and area of rectangles using familiar metric units (ACMMG109)

There is also potential for students to come to *Understand* the relationship between perimeter and area, to be more *Fluent* with the mental calculations, to find their own solution by *Problem Solving*, and to develop *Reasoning* by explaining their thinking in writing.

I will endeavour to facilitate students reasoning by allowing students to develop their own approaches, encouraging collaboration between students, and using students' explanations as the prompt to explaining the mathematical intent of the task and lesson

Particular pedagogical considerations

Each student will need 28 square tiles, at least two sheets of centimetre grid paper and a recording sheet. Have extra sheets of grid paper on hand. The task can be done in pairs, but each student will need 28 tiles.

For the students:

To explore the relationship between perimeter and are of different shapes when the perimeter is fixed

Possible questions I might ask the students

Did the perimeter remain the same?

Is that what you expected?

When is the perimeter big and when is it small?

How can you be sure you have all possible rectangles?

What happens to the perimeter as the length and width change?

Enabling prompt(s) (for students experiencing difficulty):

can be posed to students who have not been able to make progress on the learning task. The intention is that the students can complete the enabling prompt and then proceed with the learning task.

Enabling prompts can involve slightly varying an aspect of the task demand, such as:

- the form of representation,
- the size of the numbers, or
- the number of steps,

If a student has success with the modified task, they can proceed with the learning task.

Make rectangles with a perimeter of 12 units.

Extending prompt (for those who finish quickly):

Some students might finish the learning task quickly. The intention is such students be posed “extending prompts” that extend their thinking on an aspect of the learning task.

Miss Sculthorpe wants to make a dog run for her dog Benny. She wants the largest area she can for Benny to run around it. She has 28 one metre panes of fencing. What's the largest area she can make with the 28 metres.

Anticipated difficulties students may have

Students may confuse perimeter and area?
 They may not notice that perimeter changes?
 They may think they have made a mistake in determining the perimeter?

Introductory task/s:

Ask the following:

If I had 8 meters of fencing to make a garden, do you think the gardens amount of space (it's area) would be the same no matter what type of rectangle I arranged the garden into?

Tell them we'll do some activities to find out if their answer is right.

**Make a rectangle with a perimeter of 8 units.
 (The side of one coloured tile has a length of one unit)
 What do you notice?**

**Make rectangles with a perimeter of 20 units.
 (The side of one coloured tile has a length of one unit)
 Record each new rectangle by sketching the outline and the dimensions on grid paper.**

Consolidating task(s):

The Dog Run

You have 28 metres of fencing to build a dog run in your back yard for your dog. Each fence piece is one metre, so each side of the dog run will be in whole metres. You have decided to use a rectangular shape that will stand by itself under some trees. Use any method you like to find all possible dog runs you could build.

Draw them on a piece of graph paper

Flow of the Lesson

Steps	Teacher's Support	Points of Evaluation
<i>This column shows the major events and flow of the lesson.</i>	<i>This column shows additional moves, questions, or statements that the teacher may need to make to help students.</i>	<i>This column identifies what the teacher should look for to determine whether to proceed, and what observers should look for to determine the effectiveness of the lesson.</i>
<p>1. Introduction</p> <p><i>The Launch Phase</i></p>	<p>Let the students read the task quietly, maybe giving them time to think about what the question is asking them. If there are some students who cannot read at this level, check with them after they start working.</p>	<p><i>How will we know if students understand the task?</i></p> <p>What strategies are they using to solve the problem?</p> <p>Are the students confusing perimeter and area? How do they react to the idea that perimeter changes?</p>

	<p>Invite questions to ensure the task is clear, but do not show students how to do it.</p> <p>Set expectations for student working. Have the students work individually for at least 5 minutes on the task before working with one or more others (this is to give individuals time to think).</p> <p>The task has more than one pathway to solution and students can be made aware of this possibility.</p>	<p>Do they think they have made a mistake in determining the perimeter?</p> <p>Did they look for patterns in how the perimeter changes?</p> <p>As they form new rectangles, are they aware that the area is not changing because they are using the same number of tiles each time?</p>
2. The Explore phase	<p>If it looks like most students are stuck, stop them to allow a class discussion on what they have found so far, or how they are approaching the task.</p> <p>Watch what the students are doing and choose students I want to explain their thinking later. Choose a range of different strategies.</p> <p>Monitor student responses</p> <p>Select representative responses for later presentation</p> <p>Have enabling prompts ready for students who seem to be experiencing difficulty</p>	<p><i>How will we know if they are stuck?</i></p> <p>How are they generating new rectangles?</p> <p>Are they using a systematic approach (e.g. changing the rectangles by one each time to ensure they have all rectangles or are they haphazard?</p> <p>How do they measure the perimeters? Do they count or measure all four sides or do they cover the sum of length and width?</p> <p>Are they aware that perimeters change?</p> <p>Select representative responses for presentation</p>
6. Summing up <i>This section may describe how the teacher will summarize the main ideas of the lesson.</i>	<p>Students who have been selected to represent a their approach can explain their solution strategy and other insights to the class.</p> <p>When one student presents a solution invite other students</p>	<p><i>What will indicate that students are benefiting from the discussion?</i></p>

	to describe what the student has done. Sequence student responses	
--	--	--

This cycle of launch, explore, summarise will be repeated during the lesson. It may occur more than twice depending on how the students respond to the task

Appendix 19: Lesson Reflection Swift Parrot Primary School

Lesson grade 5/6 – The Dog run. 22nd May 2014

It seems getting the teachers to do the task before the lesson and think about what the students might do might do was important. E.g. in grade 6 lesson de brief, Maria (pseudonym) suddenly realised that the students might get 28 tiles when asked to make a dog run with a perimeter of 28 after she had demonstrated the misconception herself.

I steered the conversation to get teachers to notice what we would be looking for and what the students might do. We all attempted to find all the possibilities. It seemed to tune the teachers into the lesson. We talked about sequencing the students thinking

It was much better to plan with the teachers. They seemed to believe it's their lesson. They became really clear about the math's. Initially they. they wanted to teach perimeter and area separately, but by doing the tasks, they came to the importance of teaching the concepts together to dispel their student's misconceptions.

To really draw out the math's at the end, it seems really important to structure the meeting so that teachers do tasks and discuss what to look for.

In lesson kids worked out difference between area and perimeter, but need much more work on this

Appendix 20: Anticipating Strategies Proforma

Student Strategy	Teacher Questions	Who and What	Order

Appendix 21: Reflection School B

13 October 2015

GRADE 4 LESSON TIME DIFFERENCE

Yesterday in Claire's (Pseudonym) class a girl, Sally (Pseudonym) who according to her teacher was streamed into the lowest ability group was the only student to solve the time difference learning task. She used a number line. School B have changed to homogenous groups this term because of the maths project. Sally surprised the teachers. The group of teachers were asked, "What implications does that have for grouping? They replied, "maybe we shouldn't stream?"

I have realised asking invitational questions is the way to help teachers to reflect on their practice. It is a huge shift for me to become aware of this and move from telling.

Appendix 22: Lesson Plan for Swift Parrot Primary School Grade 2/3

For the lesson on 10 April 2014
Modelling teacher: Louise Hodgson

Title of the Lesson: Win TV cooking shows

Win TV cooking shows

Win TV is planning to show cooking programs in the time slot between 3.30 and 5 o'clock on Thursdays. If each program lasts quarter of an hour, what times will the programs start?

Rationale for the lesson:

There are two very different concepts that children will use and must learn to measure that are associated with time. First, there is the time that something occurs (*telling time*). Second, there is the time that passes between two events, which we call *duration*.

Telling time must enable children to:

1. Develop an understanding of the size of the units of time. This includes being able to estimate and measure using units of time;
2. Read and tell the time using both analogue and digital displays.
- 3.

Learning to tell the time is best learnt in the context of real life situations in which we need to tell the time. There is extensive language associated with the concept.

This problem helps explore the idea of quarter hours

This is both an important concept in time and a useful application of the simple fraction one-quarter.

Year level: Year 2-3

From the Australian Curriculum:

This lesson addresses the following descriptor from AC for year 2:

- Tell time to the quarter-hour, using the language of 'past' and 'to' ([ACMMG039](#))

This lesson addresses the following descriptor from AC for year 3:

- Tell time to the minute and investigate the relationship between units of time (ACMMG062)

There is also potential for students to come to *Understand* the meaning of quarter hours, to be more *Fluent* with the language of time, to find their own solution by *Problem Solving*, and to develop *Reasoning* by explaining their thinking.

I will endeavour to facilitate students reasoning by encouraging students to solve the problem in more than one way, allowing students to develop their own approaches, encouraging collaboration between students, and using students' explanations as the prompt to explaining the mathematical intent of the task and lesson

Particular pedagogical considerations

The idea is that students work out their own strategies for solving the problem and explain their thinking.

It is important to have some clocks available

For the students:

We are going to be learning about quarter to and quarter past

Possible questions I might ask the students

What time is this?

How do you know?

Tell me what the time would be in quarter of an hour from then?

Can you show me that time on the clock?

Enabling prompt(s) (for students experiencing difficulty):

can be posed to students who have not been able to make progress on the learning task. The intention is that the students can complete the enabling prompt and then proceed with the learning task.

Enabling prompts can involve slightly varying an aspect of the task demand, such as:

- the form of representation,
- the size of the numbers, or
- the number of steps,

If a student has success with the modified task, they can proceed with the learning task.

Win TV is planning to show wildlife programs in the time slot between 4 o'clock and 5 o'clock on Wednesdays. If each program lasts half an hour, how many different programs will they need to buy to fill the time slot?

Extending prompt (for those who finish quickly):

Some students might finish the learning task quickly. The intention is such students be posed "extending prompts" that extend their thinking on an aspect of the learning task.

Win TV is planning to show wildlife programs in the time slot between 11 o'clock and 6.10 on Thursdays. If each program lasts quarter of an hour, how many different programs will they need to buy to fill the time slot?

Consolidating task

Win TV is planning 4 children's programs. The first program will start at quarter to 5. What time will the next 3 programs start?

Anticipated difficulties students may have

Difficulty reading the time

Difficulty understanding quarter hour

Introductory task/s:

- Draw a clock (use mini whiteboards if available)
- Show quarter past 9 on your clock.
www.visnos.com/demos/clock
- Introduce the problem by displaying half past three on the clock and asking what programmes might be on TV at that time.
- Move the hands to 4 o'clock and ask what programmes would now be on.
- Ask the students questions that focus their thinking on telling the time in 1/4 hours.

Possible student solution:

There is one and a half hours to fill in. Each hour has four quarter-hour programmes. So WinTV needs $4 + 2$ equals 6 programmes.

Flow of the Lesson

Steps	Teacher's Support	Points of Evaluation
<i>This column shows the major events and flow of the lesson.</i>	<i>This column shows additional moves, questions, or statements that the teacher may need to make to help students.</i>	<i>This column identifies what the teacher should look for to determine whether to proceed, and what observers should look for to determine the effectiveness of the lesson.</i>
1. Introduction <i>The Launch Phase</i>	Let the students read the task quietly, maybe giving them time to think about what the question is asking them. If there are some students who cannot read at this level, check with them after they start working. Invite questions to ensure the task is clear, but do not show students how to do it. Set expectations for student working. Have the students	<i>How will we know if students understand the task?</i> Does anyone solve the problem in his or her head? What strategies are they using to solve the task?

	<p>work individually for at least 5 minutes on the task before working with one or more others (this is to give individuals time to think).</p> <p>The task has more than one pathway to solution and students can be made aware of this possibility.</p>	
2. The Explore phase	<p>If it looks like most students are stuck, stop them to allow a class discussion on what they have found so far, or how they are approaching the task.</p> <p>Watch what the students are doing and choose students I want to explain their thinking later. Choose a range of different strategies.</p> <p>Monitor student responses</p> <p>Select representative responses for later presentation</p> <p>Have enabling prompts ready for students who seem to be experiencing difficulty.</p>	<p><i>How will we know if they are stuck?</i></p> <p>Select representative responses for presentation</p>
6. Summing up <i>This section may describe how the teacher will summarize the main ideas of the lesson.</i>	<p>Students who have been selected to represent their approach can explain their solution strategy and other insights to the class.</p> <p>When one student presents a solution invite other students to describe what the student has done.</p> <p>Sequence student responses</p>	<p><i>What will indicate that students are benefiting from the discussion?</i></p>

This cycle of launch, explore, summarise will be repeated during the lesson. It may occur more than twice depending on how the students respond to the task

Appendix 23: Lesson Reflection Stage 4 HPS

17 April 2014

The assistant Principal at the school taught a lesson in a class which was not her own class which I observed along with other observing teachers from the school. As I observed Hannah's lesson I noticed she did the following:

- Used an open ended problem solving task
- Planned for and used enabling and extending prompts
- Asked questions of students rather than telling
- Structured the lesson with an introductory task, main task and discussion at the end.

I noticed during the solving of the main task several students struggling, but others were using some efficient strategies. I thought to myself it would have been great if Hannah had noticed this too and had the students demonstrating success share their thinking. This may have enabled the students have difficulty to learn from the other students. Hannah was working with a low attaining group on the floor and did not notice this.

I noticed one of the other observing teachers, who happened to be the class teacher telling children to move to the extending prompt before they had solved the main task. This surprised me and led me to question whether the teacher understood what the intent of the extending prompt was; whether she understood what a high quality response to the main task would look like and why she chose to give students the extending prompt. It seemed she assumed the extending prompt was for higher attaining students regardless of success with the main task.

At one stage the Holly, the class teacher called me over to explain that she had seen one of her students using symbols to record multiplication rather than writing "groups of". I asked the student to explain his thinking he said "[the teacher] told me to". I wondered what prompted the teacher to tell the student what to do and whether she felt defensive about the students in her class.

In the post lesson discussion, I reiterated the intent of the extending prompt and we discussed the main task in terms of what we would be looking for in a high-quality student's response. I realised we cannot assume teachers will understand what to look for unless it is made clear to them.

One of the teachers, Henrietta talked about how in the past she taught in groups, now she will consider posing a problem to the whole class.

Appendix 24: Reflection of the Stage 3 Co-Planning at Heron Primary School

1 April 2014

I met with the team of five observing teachers one week before the lesson. The class teacher, Heather raised the idea of a lesson focussed on the language of comparison for her grade prep/ones. This idea arose from her and her colleague's observation of a modelled lesson in a grade 2/3 where seven grade three children struggled to understand a comparison problem. The lesson was adjusted for the children to do an enabling task with single digit numbers to enable them to access the planned enabling prompt. The discussion after that lesson challenged the teachers about teaching different structures for addition and subtraction in the early years other than "take aways". One of the observing teachers said:

I feel like the rug has been pulled out from under me...I had a way of working and a way of choosing tasks, but now I'm really thinking about why I'm choosing tasks. For example, I used to just do numbers to ten with my prep/ones, but now I'm looking at the proficiencies. I've realised you can choose tasks from different books, but if you can't unpack the math's that's in underneath the task, then you're not developing the children's maths skills, you're just choosing tasks.

Heather requested that in the co taught lesson I would teach a game called *Diffy towers*, which is focussed on the language of difference. This was a game that I had mentioned previously to the teachers which might be helpful for children to understand the language of comparison. In the planning meeting, I encouraged the teachers to focus the lesson on problem solving and perhaps have the game as well, if time permitted. We roughly planned a task, which was contextually based (the children had a school garden) and used children names from the class.

We emailed each other back and forth over the weekend with teachers making some insightful suggestions. For example, over the weekend Heather wrote the following:

I'm wondering if maybe we are complicating the lesson plan by adding in the game of diffy towers plus the problem. Maybe with just a little more discussion I could teach the diffy towers game as a follow up and talk to you as a group about the kind of questioning I might put in place to draw out the maths. I'm assuming I would just adjust the number of blocks/dice used to act as enablers and extenders. Maybe the "I wish I had." activity would be enough as a beginning task to tune them into the language of more and less than and the idea of 'difference' could be drawn out through the problem solving task. Just a thought.

It was very interesting that Heather had changed her mind and was also using the language of enabler and extenders, which she hadn't encountered before the intervention. It clearly showed she was learning from the experience.

We tentatively decided over email that the task would be the following:

When Xavier was in the garden, he saw 8 zucchinis. But Toula saw 3. How many more did Xavier see?

On the morning of the planned lesson, we met for thirty minutes to discuss the plan. One of the teachers (Hannah, the AP and Numeracy Coordinator) suggested we change the planned task to make it easier for the children. She suggested we change the task to the following:

When Xavier was in the garden, he saw 8 zucchinis. But Toula saw 3 more. How many did they see altogether.

I asked if this considered the notion of comparison in the problem. The teachers appeared somewhat confused by how the suggested wording would change the problem. I explained the difference between join and comparison problems. I recognised the importance of being there to assist them to see the maths. With prompting, the teachers could see the wording of the task was important and it was decided to keep the original task.

I mentioned that a Principal from another school in the cluster thought it best if teachers focus on what the students did in the lesson to demonstrate reasoning rather than having a focus on

the teachers. The observing teachers liked this idea and I asked them to try it. I asked them to record on their observation proforma examples of student reasoning. Some did this, but I noticed it didn't provide a record of teacher actions to facilitate reasoning (see example) below. It did not seem helpful for reflecting on pedagogy.

We began the discussion after the lesson with my question:

What did you notice about student learning in the lesson?

That seemed to promote a very rich discussion that was focussed on the students rather than me. I noticed it was more of a discussion between teachers. I asked them what they noticed about student learning in the lesson and the conversation seemed to flow freely between all participants. I wasn't so nervous. The tone of my voice was calmer.

Appendix 25: Lesson Plan for Heron Primary School Grade Prep/1

For the lesson on 4 April 2014

Modelling teachers: Hannah and Louise

Co-planned by: Heather, Hannah, Holly and Henrietta

Title of the Lesson: How many more?

HOW MANY MORE?

When Xavier was in the garden, he saw 8 zucchinis. But Toula saw 3. How many more did Xavier see?

Rationale for lesson

The concepts of “more”, “less” and ‘same are basic relationships contributing to the overall concept of number. As these particular relationships underpin the comparison structure of subtraction, it is important that teachers use the language of comparison in early childhood and provide opportunities for children to develop understanding.

Teachers of young children may focus on the experiences of the take-away interpretation of subtraction (partitioning structure) almost to the exclusion of other structures. This is extremely limiting for children. The comparison and inverse of addition structures are much more significant in the long term and require at least the same attention.

From the Australian Curriculum

This lesson addresses the following descriptors from the AC for prep:

- Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point ([ACMNA001](#))
- Connect number names, numerals and quantities, including zero, initially up to 10 and then beyond ([ACMNA002](#))
- Compare, order and make correspondences between collections, initially to 20, and explain reasoning ([ACMNA289](#))
- Represent practical situations to model addition and sharing ([ACMNA004](#))

The lesson addresses the following descriptor from AC for grade one:

- Represent and solve simple addition and **subtraction** problems using a range of strategies including counting on, partitioning and rearranging parts ([ACMNA015](#))

There is also potential for students to build *Understanding* of number relationships, to be more *Fluent* with the mental calculations, to find their own solution by *Problem Solving*, and to develop *Reasoning* by explaining their thinking.

Particular pedagogical considerations

Insert children's names from the class and use a context closely related to their lives. In this case the children had a garden where they grew pumpkins and zucchini's.

The emphasis is on students explaining their thinking.

The learning task can be done on paper or on mini white boards. Encourage children to draw quick representations (for example, circles) and use grey lead pencils. Attention to the mathematics is more important than the presentation of the drawing.

Possible questions we might ask the students

Who has more? How many more? Who has less? How many less?

Convince us

Can you explain your thinking?

Enabling prompt(s): can be posed to students who have not been able to make progress on the learning task. The intention is that the students can complete the enabling prompt and then proceed with the learning task. Enabling prompts can involve slightly varying an aspect of the task demand, such as:

- the form of representation,
- the size of the numbers, or
- the number of steps,

If a student has success with the modified task, they can proceed with the learning task.

When Xavier was in the garden, he saw 3 pumpkins, but Toula saw 1. How many more did Xavier see?

Extending prompt:

Some students might finish the learning task quickly. The intention is such students be posed "extending prompts" that extend their thinking on an aspect of the learning task.

When Xavier was in the garden, he saw 12 zucchinis. But Toula saw 3. How many more did Xavier see?

The difference between two numbers is two. What might the numbers be?

Anticipated difficulties students may have

Students may have difficulty with the language of comparison, the structure of comparison and the numbers in the problems

Introductory task

When Xavier was in the garden, he saw 5 pumpkins, but Toula saw 2. How many more did Xavier see?

Have the children act out this problem with real pumpkins.

Ask the children the following questions:

“What do we need to do?”

‘How can we find out who has more?’

“How do you know Xavier has more?”

“How many more?”

“Can you convince us?”

“What can we do to compare the pumpkins?”

Launch problem

Flow of the Lesson

Steps	Teacher’s Support	Points of Evaluation
<i>This column shows the major events and flow of the lesson.</i>	<i>This column shows additional moves, questions, or statements that the teacher may need to make to help students.</i>	<i>This column identifies what the teacher should look for to determine whether to proceed, and what observers should look for to determine the effectiveness of the lesson.</i>
1. Introduction <i>The Launch Phase</i>	Teach task and give opportunities for children to have a go in the circle	<i>How will we know if students understand the task?</i> <ul style="list-style-type: none"> • Do they understand how to compare the pumpkins? • Do they use the language of comparison
2. The Explore phase	Pose problem: If it looks like most students are stuck, stop them to allow a class discussion on what they have found so far, or how they are approaching the task. Watch what the students are doing and choose students I want to explain their thinking	<i>How will we know if they are stuck?</i> <i>What strategies are they using to solve the problem?</i>

	<p>later. Choose a range of different strategies.</p> <p>Monitor student responses</p> <p>Select representative responses for later presentation</p> <p>Have enabling prompts ready for students who seem to be experiencing difficulty</p>	
<p>6. Summing up <i>This section may describe how the teacher will summarize the main ideas of the lesson.</i></p>	<p>Students who have been selected to represent a range of approaches can explain their solution strategy and other insights to the class.</p> <p>When one student presents a solution invite other students to describe what the student has done.</p>	<p><i>What will indicate that students are benefiting from the discussion?</i></p>

This cycle of launch, explore, summarise will be repeated during the lesson. It may occur more than twice depending on how the students respond to the task