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Modeling Returns, Volatility, Volume, and Trade Durations using High Frequency Data

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Doctor of Philosophy

by

Manh Cuong Pham

B.Fin., The University of Adelaide, Australia

B.Com. (Hons), Monash University, Australia

Department of Econometrics and Business Statistics

Monash Business School

Monash University

Australia

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Abstract

The study of market microstructure has attracted increasing attention in recent years owing to the availability of high frequency data. Two important empirical predictions about how asset prices in financial markets evolve over time highlighted by market microstructure theory are: (1) trades convey information that contributes to security price movements; and (2) intensified trading activity by investors increases price volatility. This thesis provides three essays that develop new empirical models for high frequency returns, volatility, trading volumes and trade durations in order to test these two empirical predictions and provide additional insights into the interdependence between these variables.

The first essay, presented in Chapter 2, tests the first empirical prediction by investigating the information content of trades in explaining price dynamics. In this essay, we propose a nonlinear vector autoregressive model of trade durations, trade attributes (signs and volumes) and returns that incorporates the dynamic interdependence amongst these variables and relaxes the exogeneity assumption that is often imposed on durations in previous studies. The new model is applied to examine the role of durations and trade attributes in the price formation process for Australian banking stocks around interest rate announcements. The results show that durations are not only correlated but also jointly determined with trade characteristics and returns. Shorter durations are associated with an increase in the price impact and autocorrelation of trades. In addition, transactions executed within one minute around the announcements have shorter durations and larger impact on prices. Conditioning on an average before-announcement history, the cumulative price impact of an unexpected

trade tends to be higher (lower) following a negative (positive) duration shock if durations are endogenous, yet it stays unchanged if durations are treated as exogenous. Although informative about returns, shocks to durations contribute significantly less to the forecast error variance of returns than do trade attribute shocks. This result suggests that trade durations play a smaller role in explaining price dynamics than trade attributes for Australian banking stocks.

The second essay, presented in Chapter 3, focuses on the interrelationship between trading volume and price volatility implied by the second market microstructure empirical prediction. This essay proposes a joint model for volume and returns that incorporates a bivariate stochastic process for the latent conditional expected volume and instantaneous volatility. The latent process is assumed to evolve according to a first order vector autoregression which accommodates both the contemporaneous and serial cross-dependencies between the latent variables. Our proposed model is a bivariate generalization of the popular stochastic volatility (SV) and stochastic conditional duration (SCD) models in the literature, and to our knowledge it is the first time that (1) an SCD model has been used to model trading volume; and (2) the SV and SCD models have been employed to jointly model volume and volatility. We establish several statistical properties with regard to the moments and the correlation structures of the volume and volatility processes implied by our model; these properties generalize and are all consistent with those derived in previous studies for the univariate SV and SCD models. We apply the proposed model to transaction data for one big and one small market capitalization stock in the S&P/ASX200 index; the estimation is performed by quasi maximum likelihood. We show that not only does our bivariate model successfully capture the stylized positive dynamic volume-volatility relation, it also provides significant enhancements in fit relative to its single equation counterparts. Moreover, an initial positive shock to either variable increases both the trading volumes and return volatility of future transactions, with larger responses and faster convergence to the full-information equilibrium observed for the bigger and more heavily traded

stock.

The third essay, presented in Chapter 4, looks into the volume-volatility relation more deeply by investigating how the limit order book (LOB) information affects this relation. Using a high frequency transaction dataset of the constituent stocks of the S&P/ASX200 index, we find a strong positive dynamic relationship between return volatility and trading volume, which is negatively (positively) related to the market depth at the inner quotes and the LOB slope (the bid-ask spread). The impact of the LOB characteristics on the return volatility of a trade depends on a stock's liquidity, and it is conveyed via two channels; a direct channel that is mostly attributable to lagged LOB information, and an indirect channel that is transmitted through the volume-volatility relation and is mainly contributed by the prevailing LOB information right before the trade. Furthermore, there are significant asymmetries in the effects of the bid versus ask order books on return volatility and the volume-volatility relation, with the order book of the opposite side to the direction of an incoming trade being particularly informative. Amongst the LOB characteristics that we examine, the LOB slope is the key driver of return volatility as well as the volume-volatility relation. We demonstrate the negative dependence of volatility and the volume-volatility relation on the LOB slope with a simple intuitive graphical explanation.

Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university or equivalent institution, and that to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

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Manh Cuong Pham

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List of Abbreviations

The thesis makes use of mnemonics for commonly used models and terms, and a list of these is provided below in alphabetical order for the readers' reference.

Abbreviation	Explanation
ACD	Autoregressive Conditional Duration
ACF	Autocorrelation Function
AEST	Australian Eastern Standard Time
AR	Autogressive
ARCH	Autogressive Conditional Heteroskedasticity
ARMA	Autogressive Moving Average
ASX	Australian Securities Exchange
ATS	Average Trade Size
AV	Ask Volume
BV	Bid Volume
CDF	Cumulative Density Function
DGP	Data Generating Process
EST	Eastern Standard Time
FEVD	Forecast Error Variance Decomposition
FEVD	Forecast Error Variance
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
GFEVD	Generalized Forecast Error Variance Decomposition
GIR	Generalized Impulse Response
GIRF	Generalized Impulse Response Function
GMT	Greenwich Mean Time
HFT	High Frequency Trader/Trading
IRF	Impulse Response Function
LG	Log Gamma
LOB	Limit Order Book
LW	Log Weibull
MCML	Monte Carlo Maximum Likelihood
MDH	Mixture of Distribution Hypothesis
MEM	Multiplicative Error Model
MGF	Moment Generating Function
MLE	Maximum Likelihood Estimate/Estimation/Estimator
MLMSE	Minimum Linear Mean Square Estimate/Estimator

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Abbreviation	Explanation
MMSE	Minimum Mean Square Estimate/Estimator
OIB	Order Imbalance
OLS	Ordinary Least Squares
PDF	Probability Density Function
QML	Quasi Maximum Likelihood
QTT	Quote To Trade
RBA	Reserve Bank of Australia
RBAAT	Reserve Bank of Australia Announcement Time
SAIH	Sequential Arrival of Information Hypothesis
SCD	Stochastic Conditional Duration
SCV	Stochastic Conditional Volume
SEATS	Stock Exchange Automated Trading System
SIRCA	Securities Industry Research Centre of Asia-Pacific
SV	Stochastic Volatility
SVV	Stochastic Volume-Volatility
VAR	Vector Autoregression/Autoregressive
VARMA	Vector Autoregressive Moving Average
VECM	Vector Error Correction Model
VMA	Vector Moving Average
WACD	Weibull Autoregressive Conditional Duration

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Chapter 1

Introduction

1.1 Background and motivation

The increased availability of high frequency data in recent years, due to considerable improvements in storage and computational technologies, has offered an unprecedented opportunity to research financial market microstructure. With detailed records of every transaction (including the times of trades that are precisely stamped to the millisecond), the empirical study of how market microstructure variables such as returns, price volatility, trading volumes and trade durations are interrelated and determined has become feasible. Such empirical investigations are important because they provide insights into the structure and operation of financial markets by examining the arrival of information in the market, the speed with which and the channels through which information is disseminated by market traders, as well as the effects information places on security prices (Karpoff, 1987, Hasbrouck, 1991a, Nolte, 2008).[1] Furthermore, not only do investigations allow one to test the empirical predictions implied by theory, but they may also shed light on the recurring patterns or relationships that are observed in empirical data but lack theoretical explanations, providing

[1] Extensive research, both theoretical and empirical, has been carried out in the literature to investigate one or several of these aspects. See Kyle (1985), Diamond and Verrecchia (1987), Hasbrouck (1988, 1991a,b), Easley and O'Hara (1992), Dufour and Engle (2000), Foucault et al. (2005), Manganelli (2005), Xu et al. (2006), Nolte (2008), Goettler et al. (2009), Renault and Werker (2011), Renault et al. (2014), Jondeau et al. (2015), Wei and Pelletier (2015), Benos and Sagade (2016), amongst many others.

the basis for further theoretical developments. In terms of practical relevance, both theoretical and empirical studies of market microstructure help investors better evaluate their trading performance as well as create optimal trading strategies, which are different for different assets in different markets. This allows investors to optimize their specific objective functions such as maximizing profits or minimizing trading costs (e.g. Goettler et al., 2009, Duffie, 2010). More importantly, studies of market microstructure assist policy makers in designing market places that improve liquidity, attenuate market friction and manipulation, and facilitate more efficient trading (e.g. Dufour and Engle, 2000, Manganelli, 2005).

Theoretical market microstructure studies have put forward two important predictions about the process of price formation in financial markets, which are: (1) trades contain information that drives stock price adjustments (see, among others, Glosten and Milgrom, 1985, Kyle, 1985, Diamond and Verrecchia, 1987, Easley and O'Hara, 1992, Duffie, 2010); and (2) there is a positive relationship between price volatility and trading activity (e.g. Clark, 1973, Copeland, 1976, Kyle, 1985, Admati and Pfleiderer, 1988, Shalen, 1993, Andersen, 1996, Banerjee and Kremer, 2010, Banerjee, 2011).

With regard to the first prediction, Glosten and Milgrom (1985) develop a sequential trade framework in which a risk-neutral and competitive market maker sets bid and ask prices for a risky asset and trades with other participants, some of whom are informed about the true value of the risky asset while others are not. Trade occurs in a sequential manner: at any point in time, only one trader who can be either informed or uninformed is allowed to trade with the market maker at the quoted prices set by the latter. The market maker may revise his quotes before continuing to trade with the next trader. Glosten and Milgrom (1985) show that since the market maker faces an adverse selection problem and incurs a loss when trading with the informed investors, he will set a positive bid-ask spread to offset his loss, even when there are no other explicit transaction costs such as commission fees, taxes and inventory holding costs. Moreover, the market maker learns the information from trade with the informed investors; a buy may signal that the traders know good news while a sell may signal the opposite. Consequently, he may revise his prices upward (downward) upon observing a buy (sell).

Kyle (1985) also considers a model that features a risk neutral market maker and informed and uninformed traders who trade a risky asset as in Glosten and Milgrom (1985). However, in Kyle's (1985) model, there is only one informed trader, and trade is conducted via sequential auctions where, in each auction, both the single informed investor and other noise traders submit their orders to the market maker. The market maker only observes the aggregated order flow from all traders, but he does not know how much of the total order flow comes from the informed investor. After observing the total order flow, the market maker sets a single price for the asset to clear the market. Kyle (1985) shows that there is an equilibrium in this sequential auction trading model in which the informed trader strategically splits his order into a sequence of small-sized trades in order to hide his identity and gradually make use of his private information, whereas the market maker sets the equilibrium prices of the asset as a linear function of the aggregated order flow in each auction. According to Kyle (1985), asset prices are determined linearly from the aggregated trade size.

The frameworks of Glosten and Milgrom (1985) and Kyle (1985) provide the basis for numerous extensions and modifications in the theoretical literature that examines the information content of trades.[2] For example, using an extension to the sequential trade model of Glosten and Milgrom (1985) that allows for the endogeneity of trade arrival times, Easley and O'Hara (1992) demonstrate that time between trades conveys important information about price dynamics. In particular, the absence of trades or long trade duration is a signal of no news events and the absence of informed traders in the market, which leads to a narrower bid-ask spread and smaller price adjustments. Meanwhile, by extending Kyle's (1985) auction-trading framework to allow for a competition between multiple informed traders, Holden and Subrahmanyam (1992) show that the long-lived private information possessed by informed traders is incorporated quickly and immediately into prices due to aggressive trading behaviours of the in-

[2]See, amongst many others, Diamond and Verrecchia (1987), Admati and Pfleiderer (1988), Foster and Viswanathan (1990), Diamond and Verrecchia (1991), Easley and O'Hara (1992), Holden and Subrahmanyam (1992), Easley et al. (1996), Easley et al. (1997), Kaniel and Liu (2006), Li (2017). Also see textbooks of O'Hara (1995), Brunnermeier (2001), and Foucault et al. (2013), amongst others, for a review of work that extends Glosten and Milgrom (1985) and Kyle (1985), and other related market microstructure theories.

formed traders who compete with one another to capitalize on the common piece of private information. This is in contrast to the gradual revelation of private information by the monopolistic informed investor in the original Kyle's (1985) model.

There is also a rich theoretical literature that provides explanations for the positive connection between price volatility and trading activity. Clark (1973), Epps and Epps (1976), Tauchen and Pitts (1983), and Andersen (1996) develop and extend the Mixture of Distribution Hypothesis (MDH) which postulates that the arrival of new information is the key factor that drives the co-movements in both volumes and prices. As the market reacts to new information to reach a new equilibrium, there is an increase in trading activities and aggregated trading volumes, as well as upward (downward) movements in prices if the new information is "good news" ("bad news"). As a result, volume and volatility are positively related. Meanwhile, the Sequential Arrival of Information Hypothesis (SAIH) of Copeland (1976, 1977), Jennings et al. (1981), and Jennings and Barry (1983) suggests that there is a lead-lag connection between trading volume and price volatility, due to the sequential, rather than simultaneous, dissemination of new information to different market participants. Sequential reactions of market traders to the new information result in a series of intermediate equilibria, which comes with serial adjustments in trading volumes and prices, before a final equilibrium where all traders have responded to the new information is reached. Thus, the SAIH theory predicts both a contemporaneous and lagged dependence between volume and volatility.

A positive volume-volatility relation is also featured in theoretical models of Grundy and McNichols (1989), Kim and Verrecchia (1991), Shalen (1993), Harris and Raviv (1993), Wang (1994), Banerjee and Kremer (2010), Banerjee (2011), amongst others. In these models, traders differ in their information or belief sets, which may result from either information asymmetry or disagreements in the interpretation of the common public information. The over-response of one group of traders (such as speculators who receive imperfect private signals, or responsive investors who place excessive emphasis on the common information) to an observed increase in trading activity leads to a positive correlation between volume and volatility. In a related line of research, Kyle (1985), Admati and Pfleiderer (1988), Foster and Viswanathan (1990), Holden and Subrahmanyam (1992), amongst others, show that in an information-asymmetric trading environment, the strategic behaviors of the informed investors who split their large orders into a series of smaller trades can give rise to a positive volume-volatility relation.

On the empirical side, extensive effort has been devoted in the literature to test the two aforementioned predictions. Regarding the first prediction, Hasbrouck (1988, 1991a,b) confirms the informativeness of trades highlighted by theory by showing that trades result in a persistent impact on security prices in the sense that buyer-initiated transactions lift prices up while seller-initiated trades push prices down, and prices adjust more when trading volumes are larger and/or bid-ask spreads widen. A later study by Dufour and Engle (2000) modifies Hasbrouck's (1991a) framework to test the empirical predictions of Diamond and Verrecchia (1987) and Easley and O'Hara (1992) that trading frequency or the time duration between trades is informative about the evolution of prices and trading behaviors. Dufour and Engle (2000) find that higher trading intensity or shorter trade duration, which signifies the existence of news events and the increased presence of informed traders in the market, is related not only to a faster convergence of prices to a new equilibrium level but also to stronger positive autocorrelation of trades.

Although theories (e.g. Diamond and Verrecchia, 1987, Easley and O'Hara, 1992) suggest that trade durations should be determined jointly with trade attributes and prices, they are assumed to be independent of the latter in Dufour and Engle's (2000) work, as well as in many other empirical applications. Nevertheless, the importance of endogenizing trade durations has been highlighted in a small but fast growing literature (e.g. Russell and Engle, 2005, Renault and Werker, 2011, Pelletier and Zheng, 2013, Renault et al., 2014). For example, using an econometric technique that accommodates the interdependence between price changes and trade durations, Russell and Engle (2005) show that shorter durations increase the probability of a large price change in the future, while upward movements in prices and/or big price adjustments increase future trading activity and hence shorten subsequent trade durations.

There are numerous empirical models that have been developed in the literature to test the positive relationship between return volatility and trading activities. Building on the MDH, Clark (1973), Tauchen and Pitts (1983) and Andersen (1996) propose models that allow for the contemporaneous dependence of volatility on volume, and they show that the variability in prices is positively related to trading volume. Later studies (e.g. Jones et al., 1994, Chan and Fong, 2000, Ahn et al., 2001, Chan and Fong, 2006, Næs and Skjeltorp, 2006, Chevallier and Sévi, 2012, Wang and Wu, 2015, Bollerslev et al., 2018) also document a positive contemporaneous volume-volatility relation. In addition, these studies often find that the number of transactions contains significantly more explanatory power in explaining price volatility than does trade size.

Even though the assumption that return volatility and trading volume are only contemporaneously correlated has been useful in prior studies, it appears to be too restrictive. The SAIH of Copeland (1976, 1977) and Jennings et al. (1981) indicates that in a market where information is sequentially disclosed to and circulated by traders, volume and volatility exhibit a lead-lag dependence. Likewise, when investors have heterogeneous beliefs about asset prices, due to either asymmetric private information (Shalen, 1993) or differences of opinions about public information (Banerjee and Kremer, 2010, Harris and Raviv, 1993), there are both contemporaneous and lagged relationships between volume and volatility. Subsequent empirical work such as that by Manganelli (2005), Xu et al. (2006), Nolte (2008), or Carlin et al. (2014) also finds that these two variables are indeed contemporaneously and dynamically correlated, lending support to the above theories.

The main objectives of this thesis are to document and understand (i) how the information content of time between trades interacts with price and price volatility, and (ii) how trading volume and price volatility are related and what explains this relation. This thesis develops new empirical models that incorporate trade durations, trading volumes, returns and volatility that can be used to test the two aforementioned predictions implied by market microstructure theory and to provide additional insights into the interrelationships amongst these variables. This thesis consists of three separate but related essays, of which the first one, presented in Chapter 2, investigates the information content of trades about price dynamics (i.e. the first empirical prediction), whereas the latter two essays, presented in Chapters 3 and 4, examine the relationship between trading volume and return volatility (i.e. the second empirical prediction). All three papers exploit rich data sets of high frequency tick-by-tick data on equity, which record the details of every single transaction of stocks under investigation. The use of transaction data is advantageous because (i) it is supported by most theoretical studies that develop their analysis at a tick-by-tick level (e.g. Kyle, 1985, Hasbrouck, 1991a, Holden and Subrahmanyam, 1992); (ii) it enriches the understanding of how trades affect prices while avoiding a loss of information due to the aggregation of trades and prices over a fixed time interval (Engle, 2000, Manganelli, 2005, Russell and Engle, 2005); and (iii) it is also consistent with a common practice amongst studies that test the first market microstructure prediction, even though low frequency data (daily or lower) is still dominantly used for research that examines the second prediction.

1.2 Overview of the Thesis

Chapter 2 examines how trades impact security prices when they arrive at the market at endogenously random times. The motivation of this study is rooted in the theoretical work of Diamond and Verrecchia (1987) and Easley and O'Hara (1992), who find that not only do the time durations between trades convey information about prices since they signal whether there is a news release into the market, but they are also correlated with other trade information such as trading volume. A subsequent study by Dufour and Engle (2000) finds empirical evidence supportive of these theories. However, like many other studies in the literature, Dufour and Engle (2000) assume that trade durations are generated from a stochastic process that is strictly exogenous to prices and trade attributes such as signs and volumes. This chapter extends the analysis of Dufour and Engle (2000) to relax the strict exogeneity assumption of trade durations, as suggested by Diamond and Verrecchia's (1987) and Easley and O'Hara's (1992) theories. To this end, we propose a nonlinear vector autoregression (VAR) for trade durations, trade attributes (signs and volumes) and returns that allows for their dynamic interdependence. We apply the proposed model to investigate the impact of trades on the prices of Australian banking stocks around interest rate announcements. We examine the following questions: (i) are trade durations correlated with prices and trade attributes, and if so, how?; (ii) how do trade durations affect the absorption of new information into prices?; (iii) how do the announcements of monetary policy decisions influence the trading intensity and price adjustments of these stocks?; and (iv) do trade durations or trade attributes (signs and volumes) play a bigger role in explaining the variations in prices of the Australian banking stocks?

The results from Chapter 2 show that trade durations are indeed correlated with prices and trade characteristics. They are positively related to past volatility and negatively associated with past trading volumes. Shorter trade durations are associated with trades that have higher impact on prices and are more positively serially correlated. The one minute period around interest rate announcements is very active, with significantly higher trading intensity and larger price impact. Conditioning on the average history before the announcements, an unexpected transaction results in a higher (lower) cumulative impact on prices of Australian banking stocks after a negative (positive) duration shock only if trade durations are endogenously modeled; whereas, the cumulative price impact of the unanticipated trade is similar if durations are assumed to be exogenous. Finally, shocks to durations only account for a minor portion of the forecast error variance of returns of the Australian banking stocks, which is significantly smaller than that explained by trade attribute shocks. Thus, trade durations are less informative about price dynamics than trade characteristics for these stocks.

Chapter 3 proposes a new modeling methodology to investigate the interrelationship between trading volume and return volatility. This chapter is motivated by the fact that although finance and market microstructure theory advocates the endogeneity and joint determination of volume and volatility (e.g. Clark, 1973, Admati and Pfleiderer, 1988, Andersen, 1996), the empirical literature primarily studies their relationship using single-equation or univariate time series approaches that do not allow for the feedback effects between these two variables (e.g. Ahn et al., 2001, Chan and

Fong, 2006, Chevallier and Sévi, 2012, Wang and Wu, 2015). A few studies that accommodate joint modeling employ either multivariate GARCH-type or VAR-type models that impose a conditional deterministic assumption: the conditional mean of a quantity of interest (e.g. volume) is expressed as some assumed function of past information and hence is entirely known after conditioning on the past information.[3] This chapter relaxes this conditional deterministic feature by developing a joint model for volume and returns that incorporates a bivariate stochastic process for the latent conditional expected volume and instantaneous volatility, which is assumed to follow a first order VAR that accommodates the dynamic feedback effects between these variables. The proposed model is a bivariate generalization of the popular stochastic volatility (SV) and stochastic conditional duration (SCD) models in the literature. Several statistical properties of our model are established. We apply the proposed model to the transaction dataset of one big and one small stock in the Australian market, and we examine the following questions: (i) how are trading volume and return volatility interrelated?; (ii) does the proposed bivariate model outperform its univariate counterparts in terms of model fit?; and (iii) how do volume and volatility respond to a shock to either variable?

Chapter 3 finds a positive dynamic interdependence between trading volume and return volatility, which supports market microstructure theory. However, the effects of volume on volatility are much more sizable than those of the reverse. By accommodating the joint determination of volume and volatility, our bivariate model fits empirical data significantly better than its univariate counterparts. Finally, following a positive shock to either variable, both trading volumes and return volatility increase significantly, then they converge gradually to their new long run levels, with a faster convergence rate realized for the bigger and more intensively traded stock.

Chapter 4 provides a deeper investigation of the volume-volatility relation by examining whether this relationship varies with limit order book (LOB) information, and if so, how. While theoretical market microstructure studies suggest several factors

[3]A standard VAR model does not include a separate latent structure for the expected quantities as in a GARCH-type model. Nevertheless, the expectation of a variable in the VAR system, for example volume, conditioning on the past information is still a linear function of the past values of all variables in the system.

that explain and change this relation (e.g. Clark, 1973, Holden and Subrahmanyam, 1992, Shalen, 1993, Banerjee and Kremer, 2010), most empirical work documents the determinants of volatility, rather than the volume-volatility relation. This is because the effects of volume on volatility are assumed to be constant over time and do not vary with other factors. See, for example, Chan and Fong (2000), Pascual and Veredas (2010), Haugom et al. (2014), Clements and Todorova (2016). Unlike the majority of previous studies, this chapter accommodates the time-varying feature of the volume-volatility relation, and it allows the dependence of volatility on volume to vary with the LOB characteristics, which include the bid-ask spread, market depth at the inner quotes, and the slope of the LOB. The analysis is conducted for stocks listed on the S&P/ASX200 index, and we examine the following questions: (i) how does LOB information affect the volume-volatility relation and return volatility?; (ii) what are the channels through which LOB information explains volatility?; (iii) which side of the LOB is more informative about the volume-volatility relation and volatility?; and (iv) which LOB information is the driving determinant of the volume-volatility relation and volatility?

The results from Chapter 4 document a positive dynamic dependence of return volatility on trading volume, which is strongly related to the LOB information: larger market depth and/or a steeper LOB right before a transaction weakens the positive volume-volatility relation, whereas wider bid-ask spreads strengthen it. LOB characteristics affect return volatility via two channels: a direct channel that is primarily explained by the lagged LOB information, and an indirect channel that is conveyed through the volume-volatility relation and is mostly accounted for by the prevailing LOB information immediately before a trade. Moreover, two sides of the LOB convey asymmetric information about the volume-volatility relation and return volatility, and it is the opposite side order book (to the direction of an upcoming trade) that is more informative. Finally, by incorporating the information from the bid-ask spread and market depth at the best quotes, the LOB slope dominates the former in explaining return volatility and the volume-volatility relation.

1.3 Outline of the Thesis

This thesis is structured as follows. Following this introductory chapter, Chapter 2 investigates the impact of trades on the prices of Australian banking stocks around the releases of monetary policy decisions, allowing for the endogeneity of trade arrival times. Chapter 3 focuses on the interrelationship between trading volume and return volatility by developing a bivariate stochastic conditional model for these two variables. Chapter 4 analyzes the information content of the limit order book in explaining return volatility and the volume-volatility relation. Chapter 5 concludes the thesis by providing a summary of the key findings in the previous chapters and suggesting some directions for future research. A list of abbreviations for commonly used terms in this thesis is provided on page xvii.

Chapter 2

Time and the price impact of trades in Australian banking stocks around interest rate announcements

2.1 Introduction

Research into market microstructure has flourished in recent years as a consequence of increased accessibility of high frequency data and significant computational advancements. One big strand in market microstructure studies has focused on identifying factors that influence asset prices and examining how prices evolve in reaction to new information. Previous research has documented that trade attributes, such as direction and volume, and bid-ask spreads are important pieces of information that drive the price formation process (Hasbrouck, 1988, 1991a,b). In particular, unexpected trades result in a persistent impact on security prices; the larger the volume of a trade and/or the wider the bid-ask spread, the bigger the price adjustment.

Time of trade arrivals has also been shown to play an important role in explaining price dynamics. Theoretical studies by Diamond and Verrecchia (1987) and Easley and O'Hara (1992) highlight the informativeness of trade arrival times and their joint determination with the process of trade generation and price formation. Specifically, Diamond and Verrecchia (1987) hypothesize that long time intervals between trades, or, equivalently, low levels of trading activities, are signals of bad news being revealed to the market, which subsequently lead to a decrease in prices. Meanwhile, Easley and O'Hara (1992) relate long trade durations to a lack of news events and show that trading intensity is positively dependent on the proportion of informed investors in the market. Consequently, the longer the time between trades, the narrower the bid-ask spread and the smaller the price adjustment. Dufour and Engle (2000) lend support to these theories by empirically showing that more frequent trade arrivals or shorter trade durations are associated with not only stronger positive autocorrelation of trade directions but also a quicker convergence of prices to the equilibrium level. Likewise, higher trading intensity leads to higher price volatility (Engle, 2000) and is related to a stronger dependence of price volatility on trade sizes (Xu et al., 2006).

Despite the theoretical suggestion of the joint determination of trade durations and other variables such as prices and trade attributes (e.g. Diamond and Verrecchia, 1987, Easley and O'Hara, 1992), Dufour and Engle (2000), Engle (2000) and Xu et al. (2006) all assume that trade durations are strictly exogenous. That is, the time between trades is assumed to be only dependent upon previous durations but independent of past trajectories of prices and trade attributes. Nevertheless, Dufour and Engle (2000) conduct a formal test of the validity of the strict exogeneity assumption (which is often imposed on durations in previous studies) and find that it is strongly rejected. They suggest that "incorporating [the] feedback effects of returns, trades and volume on time durations may improve the in-sample performance of the model" (p. 2496). Although they do not pursue the relaxation of this assumption, these authors emphasize the importance of endogenizing trade durations since it "could ultimately provide more accurate impulse response functions" (p. 2496), i.e. it could enable a more accurate assessment of the price impact of trades. Motivated by the Dufour and Engle's suggestion, this chapter aims to build a model for returns, trade characteristics (signs and volumes) and durations that relaxes the strong exogeneity assumption of trade durations, and we use this framework to examine how trades impact prices when trade arrival times are endogenous.

Our econometric framework is built upon the general modeling approach of Engle (2000) that decomposes the joint distribution of trade durations and other variables of interest such as returns and trade attributes into the product of the marginal density of durations and the conditional density of the other variables. By incorporating the past histories of returns, trade characteristics and durations into both the marginal and conditional densities, we allow for feedback effects amongst these variables in the joint system. In particular, we follow Hasbrouck (1991a) and Dufour and Engle (2000) in modeling returns and trade characteristics with a vector autoregression (VAR) that is non-linearly related to trade durations. Meanwhile, we model durations in two ways, both of which take into account the dependence of durations on lagged returns and trade characteristics. The first way is to make durations another endogenous variable that evolves according to an autoregressive structure similar to returns and trade attributes in the VAR system, which is a natural extension of Hasbrouck's (1991a) framework to endogenize trade durations. The second way is to employ an autoregressive conditional duration (ACD) model for durations that incorporates past returns and trade characteristics.

There is a small but growing body of literature that accommodates the endogeneity of trade durations in a multivariate system (Grammig and Wellner, 2002, Manganelli, 2005, Renault and Werker, 2011, Pelletier and Zheng, 2013, Renault et al., 2014, Wei and Pelletier, 2015). Unlike these studies whose main objective is to examine the interdependence between duration and volatility (i.e. the second moment of returns), this chapter focuses on the dynamics of the first moment of returns. In addition, our proposed nonlinear VAR model incorporates trade direction, which is shown to be an important determinant of the price formation process (Hasbrouck, 1991a, Dufour and Engle, 2000, Barclay et al., 2003) but which is often excluded from the aforementioned studies due to its binary nature (i.e. trade direction can only take two values: 1 if a trade is a purchase and -1 if it is a sale). Our work also differs from another related work by Russell and Engle (2005) in that instead of studying discrete tick-size price changes with an autoregressive conditional multinomial model, we examine returns -

a widely-used and *continuous* relative measure of price changes which facilitates comparison amongst stocks of different capitalizations.

We apply the proposed model to study the role of durations and trade attributes in the process of price formation for Australian banking stocks. In addition, we investigate how the Reserve Bank of Australia (RBA) interest rate announcements affect the arrival time and the price impact of trades in these stocks. Effectively, the release of monetary policy news is treated as an exogenous event to the joint framework on which we condition our analysis. We focus on banking stocks because they are liquid and very sensitive to interest rate news. With the joint model, we examine several important issues in the microstructure literature. The first issue relates to theoretical predictions about the endogeneity of trade durations and their informativeness about price dynamics (e.g. Easley and O'Hara, 1992). Specifically, are durations correlated with prices and trade attributes, and if so, how? Also, how do trade durations affect the adjustment of security prices to new information? The second issue concerns how the occurrence of exogenous news events such as RBA announcements affect the trade generation and price formation processes. Do interest rate announcements intensify the trading frequency and the price impact of trades of Australian banking stocks? The third issue compares the relative importance of durations and trade attributes (signs and volumes) to price dynamics. Although there are theoretical justifications and empirical evidence of the informativeness of both trade arrival times and trade attributes about price adjustment, there is little guidance, either theoretical or empirical, on which of the two possess a bigger informational content. In this chapter, we empirically investigate whether durations or trade characteristics play a dominant role in explaining the behavior of prices of the Australian banking stocks under examination.

Our study contributes to the literature in several ways. First, we provide a general model to study the dynamics of returns jointly with durations and trade characteristics that relaxes the strict exogeneity of durations that is often assumed in previous studies. Using a sample of major Australian banking stocks, we find that durations are not only correlated but also jointly determined with trade attributes and returns, supporting Easley and O'Hara's (1992) theory. Specifically, while larger price adjustments tend to increase future trade durations (which is consistent with Admati and Pfleiderer (1988), Grammig and Wellner (2002)), larger past trading volumes tend to shorten the durations of incoming transactions (which supports Easley and O'Hara (1992), Manganelli (2005), Nowak and Anderson (2014)). In conformance with Dufour and Engle (2000), trades that have shorter durations are associated with stronger impact on price and more positive autocorrelation in trades.

Second, we provide evidence that monetary policy announcements affect the trading intensity and the price impact of trades in banking stocks. Our work differs from most existing studies that investigate how financial markets react to interest rate news because (i) we study how the news impacts trading frequency, in addition to how it impacts returns; and (ii) our study is conducted using tick-by-tick transaction data which helps avoid a loss of information that might bias the analysis (Engle, 2000, Russell and Engle, 2005), whereas most previous studies employ data of lower frequencies such as 5 minutes (e.g. Smales, 2012), daily (Bomfim, 2003, Gasbarro and Monroe, 2004, Kim and Nguyen, 2008), or monthly (Bernanke and Kuttner, 2005, Diggle and Brooks, 2007, Bjørnland and Leitemo, 2009). Using a dataset for major Australian banking stocks, we find that trades transacted within one minute around the RBA announcements have shorter durations and larger impacts on prices. Conditioning on an average history prior to the RBA announcements, the cumulative price impact of an unexpected trade is higher (lower) when the trade occurs faster (slower) if durations are endogenous, yet it stays unchanged if durations are treated as exogenous. The latter result highlights the importance of endogenizing trade durations, confirming Dufour and Engle's (2000) suggestion that allowing for the endogeneity of time between trades could provide a more accurate picture of how trades drive prices.

Third, to the best of our knowledge, this is the first study to compare the relative informativeness of durations and trade attributes (signs and volumes) about the price formation process. Using the generalized forecast error variance decomposition (GFEVD) proposed by Lanne and Nyberg (2016), we find that shocks to durations contribute significantly less to the forecast error variance (FEV) of returns of major Australian banking stocks than other trade attribute shocks. The relative importance of duration shocks to returns is less than 9% while that of other trade attribute innovations is typically above 50%. The contributions of both shocks to the returns' FEV are larger on days with interest rate releases, and that of duration shocks is also larger when durations are endogenously modeled. Based on a sample of major Australian banking stocks, the results suggest that although trade durations carry important informational content about prices (Easley and O'Hara, 1992, Dufour and Engle, 2000), they play a minor role in explaining the price dynamics of these stocks compared to trade attributes (signs and volumes). Given a lack of theoretical explanations, this empirical observation might provide an interesting venue for further theoretical exploration.

Our findings are potentially of interest to market participants and policy makers because they shed light on how quickly new information, for example interest rate news, is processed and how, how this news is disseminated and incorporated into security prices, as well as how policy-making influences this information dissemination process. In addition, as price impact is known to be the biggest component of trading cost (Keim and Madhavan, 1996, 1998), our findings have relevant practical implications for designing optimal strategies that minimize the cost of trading in financial markets.

The rest of this chapter is organized as follows. Section 2.2 introduces a nonlinear VAR framework for trade arrival times, trade attributes and returns advocated in this chapter. It also discusses how the information content of monetary policy announcements is incorporated into the model. Section 2.3 describes the data. Model estimates and further analyses such as impulse response and forecast error variance decomposition for Australian banking stocks are presented in Section 2.4, and Section 2.5 concludes.

2.2 A joint model of durations, trade attributes and returns

Transactions data are conventionally characterized by a sequence of their arrival times that follow a point process and the associated quantities called "marks" that are revealed to the market at those times (Engle, 2000, Manganelli, 2005, Russell and Engle, 2005). Marks are typically a vector of random variables such as the price, the direction and the volume of a transaction which, together with the trade's arrival time, are assumed to be jointly determined by some unknown data generating process (DGP). In order to estimate the true joint distribution, Engle (2000) factorizes it into the product of the conditional density of the marks and the marginal density of the arrival time, and then proposes a model for each component density. This approach has been widely utilized in various market microstructure studies such as Grammig and Wellner (2002), Manganelli (2005), and Russell and Engle (2005).

To formulate the idea of Engle (2000) statistically, let $T_t = z_t - z_{t-1}$ be the time interval, measured in seconds, between two successive transactions, where z_t denotes the time at which the t-th trade is executed. At time z_t, market participants observe a vector of marks y_t. Each pair (T_t, y_t) is assumed to follow a joint distribution $f(T_t, y_t|\mathcal{I}_{t-1}; \mu)$, where $\mathcal{I}_{t-1}$ denotes the past information and μ is a vector of parameters underlying the joint process. Engle (2000) decomposes the joint density $f(T_t, y_t|\mathcal{I}_{t-1}; \mu)$ as

$$f(T_t, y_t|\mathcal{I}_{t-1}; \mu) = g(y_t|T_t, \mathcal{I}_{t-1}; \mu_y) \times h(T_t|\mathcal{I}_{t-1}; \mu_T), \tag{2.1}$$

where $g(\cdot)$ denotes the conditional density of the marks y_t given the current trade duration T_t, and $h(\cdot)$ denotes the marginal density of T_t. Prior studies in the literature have primarily focused on modeling the $h(\cdot)$ function only (e.g. Engle and Russell, 1998, Bauwens and Giot, 2000, Knight and Ning, 2008, Xu et al., 2011). A few studies that accommodate joint modeling typically assume that trade durations are strictly exogenous (e.g. Engle, 2000, Dufour and Engle, 2000, Xu et al., 2006). In other words, these studies assume that the past information set $\mathcal{I}_{t-1}$ in the $h(\cdot)$ function only includes lagged trade durations but excludes the information from previous marks, even though these studies allow both past durations and marks to be included in the $g(\cdot)$ function.

We relax this strict exogeneity of trade durations by incorporating the past trajectories of durations and marks into both $g(\cdot)$ and $h(\cdot)$ functions, which explicitly allows for the dynamic interdependence between the variables. The decomposition (2.1) assumes that there is instantaneous Granger-causality running from T_t to y_t while the latter does not contemporaneously Granger-cause the former. This is because T_t measures the time interval between the $(t-1)$-th and the t-th transactions and thus potentially conveys relevant information that has been accumulated during the time period (Easley and O'Hara, 1992), while y_t is only realized once the t-th trade is completed. Therefore, the parameterization (2.1) appears natural and plausible.

This study aims to develop a joint modeling framework for tick-by-tick returns or quote revisions, trade characteristics (signs and volumes) and trade durations, based on which the effects of the interest rate announcements on the role of durations and trade characteristics in explaining the price dynamics will be examined. Thus, the marks of interest include (i) quote revision r_t, defined as the natural logarithmic change in the midquote price following the t-th trade and quoted in basis points (bps), i.e. $r_t = 10,000 * (\ln(q_{t+1}) - \ln(q_t))$, where q_t is the midpoint of the bid and ask quotes immediately before the t-th trade;[4] (ii) trade sign x_t^0, which equals 1 (-1) for buyer- (seller-) initiated transactions; and (iii) signed volume v_t, defined as the signed natural logarithm of the ratio of the actual share volume (V_t) of the t-th trade to the prevailing quoted depth ($depth_t$) at the best opposite-side quote immediately before that trade,[5] i.e. $v_t = x_t^0 \ln(V_t/depth_t)$. The use of the volume to depth ratio, rather than the actual share volume, is motivated by work of Chan and Fong (2000), Engle and Lange (2001), Brogaard et al. (2015), and Pham et al. (2017), who show that for a given share volume, trades have a bigger impact on prices if the prevailing depths prior to these trades are

[4]Measuring prices as the mid-point of bid and ask quotes is standard practice in the microstructure literature to circumvent the bid-ask bounce problem (e.g. Hasbrouck, 1988, Manganelli, 2005). The factor of 10,000 enables returns to be measured in basis points.

[5]If the t-th trade is a purchase (sale), $depth_t$ is defined as the number of shares available at the best ask (bid) price right before the trade.

smaller (i.e. if the market is less liquid). Thus, a trade is considered big if it has a large volume to depth ratio, and this measure not only incorporates the effects of trade sizes, but also market liquidity. We need to model the conditional density of the marks $g(\cdot)$ and the marginal density of trade durations $h(\cdot)$ to capture the joint distribution of the variables of interest, and these are discussed in the next subsections.

2.2.1 Modeling returns and trade attributes given trade durations

Building on Hasbrouck (1991a) and Dufour and Engle (2000), this chapter models the joint dynamics of quote revisions, trade signs and signed volumes, conditional on trade arrival times (i.e. $g(y_t|T_t, \mathcal{I}_{t-1}; \mu_y)$ where $y_t = (r_t, x_t^0, v_t)'$), with the following VAR framework:

$$\begin{aligned} r_t &= \alpha^r + \beta^r \text{open}_t + \sum_{i=1}^{p} b_i^r |r_{t-i}| + \left\{ \sum_{i=1}^{p} a_i^r r_{t-i} + \lambda^r \text{open}_t x_t + \sum_{i=0}^{p} \left[\gamma_i^r + \delta_i^r \ln(T_{t-i}) \right] x_{t-i} \right\} + u_t^r, \\ x_t &= \alpha^x + \beta^x \text{open}_t + \sum_{i=1}^{p} b_i^x |r_{t-i}| + \left\{ \sum_{i=1}^{p} a_i^x r_{t-i} + \lambda^x \text{open}_{t-1} x_{t-1} + \sum_{i=1}^{p} \left[\gamma_i^x + \delta_i^x \ln(T_{t-i}) \right] x_{t-i} \right\} + u_t^x, \end{aligned} \tag{2.2}$$

where $x_t = (x_t^0, v_t)'$; open_t is a dummy variable that equals 1 for trades executed within the first 30 minutes of a trading day, and 0 otherwise; α's, β's, b's, a's λ's, γ's, and δ's are conformable matrices of coefficients. This VAR framework is similar, but not identical to the original Dufour and Engle (2000) specification, which only investigates returns r_t and trade signs x_t^0 and includes components in braces of equation (2.2). The extension of the original Dufour and Engle (2000) VAR system to incorporate trading volume is motivated by the findings of Easley and O'Hara (1987), Hasbrouck (1988), O'Hara et al. (2014), among others, that there is a significant price-quantity relationship. Likewise, the inclusion of $|r_{t-i}|$ is to capture the effects of stock volatility on returns and trade attributes (e.g. Xu et al., 2006).

In conformance with Hasbrouck (1991a) and Dufour and Engle (2000), the VAR specification in (2.2) assumes that conditioning on the time of trade arrivals, there are two sources of information affecting price dynamics. One is the public or trade-unrelated information, u_t^r, and the other is the private information induced by unanticipated trades, u_t^x. These two informational innovations are assumed to have zero means and to be jointly and serially uncorrelated.[6] After observing a new trade, the market maker learns the information conveyed by the trade and then revises the quotes to take into account the new information. Thus, the trade contemporaneously affects the quote revision, but not vice versa. This fact is reflected by the inclusion of the contemporaneous value of x_t in the quote revision equation, and thus it is assumed that $\mathbb{E}(u_t^r u_t^x) = 0$.

As in Dufour and Engle (2000), the VAR setting in (2.2) allows the impact of trades on prices and future transactions to be nonlinearly dependent upon trade durations. Furthermore, trades transacted at the market open (i.e. first 30 minutes) are allowed to have different impact from those executed later in the trading day. This time-augmented structure enables one to empirically test the theoretical conjectures in the microstructure literature that the arrival times of trades possess an information content that significantly contributes to the price formation and trade generation processes (Diamond and Verrecchia, 1987, Easley and O'Hara, 1992). In the parameterization (2.2), the effects of trades on the price evolution and the autocorrelation amongst transactions that are contributed by trading intensity are measured by the δ's, while the additional impact of trades executed at the beginning of the trading day is quantified by the λ's. The joint significance of the δ's and λ's will ascertain the informativeness of trade arrival times in driving price dynamics. We also include the indicator variable open_t in each equation to account for additional opening variations that might come from other sources of information other than trades.

2.2.2 Modeling trade durations

A critical assumption that Dufour and Engle (2000) impose on their bivariate VAR framework is that trade arrivals are strongly exogenous; that is, the times of trade arrivals are not influenced by the past histories of prices and trade characteristics but only depend on previous arrival times. Although the strict exogeneity assumption of trade durations is often imposed in the duration modeling literature (e.g. Engle 2000, Xu et al. 2006, Xu et al. 2011, Knight and Ning 2008), it is too restrictive. Theoreti-

[6] That is, $\mathbb{E}(u_t^r) = 0, \mathbb{E}(u_t^x) = 0$, and $\mathbb{E}(u_t^r u_s^r) = 0, \mathbb{E}(u_t^r u_s^x) = 0, \mathbb{E}(u_t^x u_s^{x\prime}) = 0$ for $s \neq t$.

cal frameworks of Diamond and Verrecchia (1987) and Easley and O'Hara (1992) are built on the notion that time durations between trades are correlated with prices and volumes. Other empirical studies also document that returns, volume and volatility are significant predictors of trade durations (Engle and Russell, 1997, Manganelli, 2005, Russell and Engle, 2005, Nowak and Anderson, 2014). Formally testing the strict exogeneity assumption, Dufour and Engle (2000) also provide strong evidence of its rejection, and thus accentuate the importance of relaxing it, even though they do not attempt to do so.

The exogeneity test of Dufour and Engle (2000) suggests that trade durations should be treated as an endogenous variable that needs to be determined concurrently with quote revisions and trade characteristics. A natural way to endogenize durations is to extend the VAR framework in (2.2) by adding another equation for durations as below:[7]

$$\begin{aligned}
r_t &= \alpha^r + \beta^r \text{open}_t + \sum_{i=1}^{p} a_i^r r_{t-i} + \sum_{i=1}^{p} b_i^r |r_{t-i}| + \lambda^r \text{open}_t x_t + \sum_{i=0}^{p} \left[\gamma_i^r + \delta_i^r \ln(T_{t-i})\right] x_{t-i} + \sum_{i=1}^{p} c_i^r \ln(T_{t-i}) + u_t^r, \\
x_t &= \alpha^x + \beta^x \text{open}_t + \sum_{i=1}^{p} a_i^x r_{t-i} + \sum_{i=1}^{p} b_i^x |r_{t-i}| + \lambda^x \text{open}_{t-1} x_{t-1} + \sum_{i=1}^{p} \left[\gamma_i^x + \delta_i^x \ln(T_{t-i})\right] x_{t-i} + \sum_{i=1}^{p} c_i^x \ln(T_{t-i}) + u_t^x, \\
\ln(T_t) &= \alpha^T + \beta^T \text{open}_{t-1} + \sum_{i=1}^{p} a_i^T r_{t-i} + \sum_{i=1}^{p} b_i^T |r_{t-i}| + \lambda^T \text{open}_{t-1} x_{t-1} + \sum_{i=1}^{p} \left[\gamma_i^T + \delta_i^T \ln(T_{t-i})\right] x_{t-i} + \sum_{i=1}^{p} c_i^T \ln(T_{t-i}) + u_t^T,
\end{aligned} \tag{2.3}$$

where u_t^r, u_t^x and u_t^T are zero-mean, serially uncorrelated disturbances. Lags of durations, $\sum_{i=1}^{p} c_i^T \ln(T_{t-i})$, are included in the duration equation to account for the autocorrelation of durations. They are also incorporated into the quote revision and trade attribute equations to capture the additional effects of durations. In this so-called "*Endo-VAR*" specification, trade duration depends not only upon its lagged values but also upon the past histories of quote changes and trade characteristics according to an autoregressive structure. The *Endo-VAR* model, which provides a natural and nice layout built upon Hasbrouck's (1991a) framework to investigate the joint dynamics of quote revisions, trade attributes and durations, can be estimated consistently by ordinary least squares (OLS).

[7]We do not include the dummy variable open_t, but use open_{t-1} instead, in the duration equation because open_t can only be observed simultaneously with T_t, and hence it is unknown given the past information $\mathcal{I}_{t-1}$.

An alternative specification of the marginal distribution of durations, $h(T_t|\mathcal{I}_{t-1};\mu_T)$, is similar in spirit to an ACD model proposed by Engle and Russell (1998). These authors show that the ACD model works well in capturing the dynamic structure of trade durations such as duration clustering. The ACD model and a wide range of its variations have been utilized intensively in the duration modeling literature (Bauwens and Giot, 2000, Engle, 2000, Fernandes and Grammig, 2006, Pacurar, 2008). In their analysis, Dufour and Engle (2000) also model time durations with an ACD setting. However, by assuming that durations are strongly exogenous, they do not allow the past dynamics of trade attributes and quote changes to enter the conditional duration specification. This chapter relaxes this strict exogeneity assumption by allowing for the dependence of time durations on lagged values of quote revisions and trade characteristics within an ACD framework. Specifically, following Engle and Russell (1998) we firstly remove the deterministic intra-day component of durations using a cubic spline $\varphi(t)$.[8] The diurnally adjusted durations, $\widetilde{T}_t = T_t/\varphi(t)$, are then fitted with the following Weibull ACD (WACD) (p_1, p_2) model:

$$\widetilde{T}_t=[\phi_t\Gamma(1+1/\theta)]\epsilon_t, \qquad \epsilon_t \overset{iid}{\sim} \text{Weibull}\left(scale=\frac{1}{\Gamma(1+1/\theta)}, shape=\theta\right), \tag{2.4}$$

$$\mathbb{E}(\widetilde{T}_t|\mathcal{I}_{t-1},\theta)\equiv\phi_t\Gamma(1+1/\theta), \tag{2.5}$$

$$\ln(\phi_t)=\alpha^T+\sum_{i=1}^{p_1}a_i^T r_{t-i}+\sum_{j=1}^{p_1}b_i^T|r_{t-i}|+\sum_{i=1}^{p_1}\gamma_i^T x_{t-i}+\sum_{i=1}^{p_1}\rho_i\ln(\widetilde{T}_{t-i})+\sum_{i=1}^{p_2}\zeta_i\ln(\phi_{t-i})+\lambda^T\text{open}_{t-1}. \tag{2.6}$$

We incorporate the opening dummy variable into equation (2.6) to see if there remains any deterministic opening variation that cannot be fully removed by the diurnalization procedure. Equation (2.6) explicitly allows for the effects of the past quote changes and trade attributes on durations. Following Bauwens and Giot (2000) and Russell and Engle (2005), we employ a logarithmic variation of the conditional duration equation to ensure the positivity of the conditional expectation of trade durations, especially when additional explanatory variables are included. With this parameterization, the stationarity of the duration series is obtained if and only if $\sum_{j=1}^{p_1}\rho_j+\sum_{j=1}^{p_2}\zeta_j<1$.

[8] The cubic spline we employ is of the form $\varphi(t)=\beta_0+\beta_1 z_t+\beta_2 z_t^2+\beta_3 z_t^3+\sum_{j=1}^{k}\beta_{j+3}\left[(z_t-c_j)^3\times I_{z_t>c_j}\right]$, where z_t is the clock time of the t–th trade, c_j $(j=1,\cdots,k)$ are the spline knots that we set at 10:30, 11:00, 11:30, 12:00, 12:30, 13:00, 13:30, 14:00, 14:30, 15:00, 15:30, 15:45 since the trading day in our dataset runs from 10:10 to 16:00.

Replacing the duration equation of the *Endo-VAR* model with the WACD(p_1, p_2) model gives us the following *WACD-VAR* system:

$$
\begin{aligned}
r_t &= \alpha^r + \beta^r \text{open}_t + \sum_{i=1}^{p} a_i^r r_{t-i} + \sum_{i=1}^{p} b_i^r |r_{t-i}| + \lambda^r \text{open}_t x_t + \sum_{i=0}^{p} \left[\gamma_i^r + \delta_i^r \ln(T_{t-i})\right] x_{t-i} + \sum_{i=i}^{p} c_i^r \ln(T_{t-i}) + u_t^r, \\
x_t &= \alpha^x + \beta^x \text{open}_t + \sum_{i=1}^{p} a_i^x r_{t-i} + \sum_{i=1}^{p} b_i^x |r_{t-i}| + \lambda^x \text{open}_{t-1} x_{t-1} + \sum_{i=1}^{p} \left[\gamma_i^x + \delta_i^x \ln(T_{t-i})\right] x_{t-i} + \sum_{i=i}^{p} c_i^x \ln(T_{t-i}) + u_t^x, \\
\widetilde{T}_t &= T_t / \varphi(t) = [\phi_t \Gamma(1 + 1/\theta)] \epsilon_t, \qquad (2.7) \\
\ln(\phi_t) &= \alpha^T + \sum_{i=1}^{p_1} a_i^T r_{t-i} + \sum_{i=1}^{p_1} b_i^T |r_{t-i}| + \sum_{i=1}^{p_1} \gamma_i^T x_{t-i} + \sum_{i=1}^{p_1} \rho_i \ln(\widetilde{T}_{t-i}) + \sum_{i=1}^{p_2} \zeta_i \ln(\phi_{t-i}) + \lambda^T \text{open}_{t-1}.
\end{aligned}
$$

The estimation of the *WACD-VAR* model is obtained by OLS for the marks (i.e. $(r_t, x_t')'$) and by maximum likelihood for trade durations.

2.2.3 Modeling the impact of RBA interest rate announcements

Each year, there are eleven scheduled RBA board meetings on the first Tuesday of every month except in January. Since December 2007, the RBA board's decision to change or keep the interest rate has been released to the media at 14:30:00 Australian Eastern Standard Time (GMT + 10) on the same day of the meeting (Smales, 2012). In order to examine how the RBA target rate announcements influence the role of durations and trade attributes in the process of price formation for Australian banking stocks, we modify the *Endo-VAR* model in (2.3) and the *WACD-VAR* model in (2.7) to incorporate the information contained by the RBA monetary policy releases. It would be of interest to examine the effects of the *surprise* or *unexpected* component of the news, as in Balduzzi et al. (2001), Kuttner (2001), Andersen et al. (2003), Kim and Nguyen (2008), Smales (2012), among others. In these studies, the unexpected news is calculated as the difference between the actual announcement and the expected component, where the latter is either proxied by the median analyst forecasts (Balduzzi et al., 2001, Andersen et al., 2003) or inferred from interest rate futures prices (Kuttner, 2001, Kim and Nguyen, 2008, Smales, 2012). In addition, since news events are released at some particular point in (calendar) time (e.g. 14:30:00) at which there might not be any transaction being executed, prior research in the literature normally converts transaction time or tick-by-tick data into calendar time data by aggregating trades over some fixed time interval such as a day or 5 minutes to match the occurrence of the news announcements. However, such an aggregation procedure inevitably results in a loss of information and may potentially bias the analysis (Engle, 2000, Russell and Engle, 2005), because, as highlighted in the theoretical work by Easley and O'Hara (1992) and Diamond and Verrecchia (1987), the existence or absence of each individual trade is informative about the price formation process.

Our analysis is conducted in transaction time. This circumvents the information loss coming from trade aggregation, but this makes it difficult to incorporate information that is released in calendar time at which there are usually no trades (Hamilton and Jordà, 2002, Nowak and Anderson, 2014). To address this issue, we adopt a simple approach that is along a similar line to Ederington and Lee (2001) and Nowak and Anderson (2014) to match calendar time with transaction time. This accounts for the effects of the RBA announcements in our *Endo-VAR* and *WCAD-VAR* specifications by including three announcement indicator variables that record the occurrence of the news events. These dummies, denoted by bef_t, aro_t and aft_t, respectively identify transactions executed five minutes *before* (i.e. 14:24:30-14:29:30), one minute *around* (14:29:30-14:30:30), and ten minutes *after* (14:30:30-14:40:30) the RBA announcements. The length of the event windows chosen in this chapter is suggested by Simonsen (2006) and Nowak and Anderson (2014). The modified models that incorporate the effects of RBA announcements are given by

$$\begin{aligned}
r_t&=\alpha^r+\beta^r D_t+\sum_{i=1}^{p}a_i^r r_{t-i}+\sum_{i=1}^{p}b_i^r|r_{t-i}|+\lambda^r D_t\otimes x_t+\sum_{i=0}^{p}\left[\gamma_i^r+\delta_i^r\ln(T_{t-i})\right]x_{t-i}+\sum_{i=1}^{p}c_i^r\ln(T_{t-i})+u_t^r,\\
x_t&=\alpha^x+\beta^x D_t+\sum_{i=1}^{p}a_i^x r_{t-i}+\sum_{i=1}^{p}b_i^x|r_{t-i}|+\lambda^x D_{t-1}\otimes x_{t-1}+\sum_{i=1}^{p}\left[\gamma_i^x+\delta_i^x\ln(T_{t-i})\right]x_{t-i}+\sum_{i=1}^{p}c_i^x\ln(T_{t-i})+u_t^x, \qquad (2.8)\\
\ln(T_t)&=\alpha^T+\beta^T D_{t-1}+\sum_{i=1}^{p}a_i^T r_{t-i}+\sum_{i=1}^{p}b_i^T|r_{t-i}|+\lambda^T D_{t-1}\otimes x_{t-1}+\sum_{i=1}^{p}\left[\gamma_i^T+\delta_i^T\ln(T_{t-i})\right]x_{t-i}+\sum_{i=1}^{p}c_i^T\ln(T_{t-i})+u_t^T,
\end{aligned}$$

and

$$
\begin{aligned}
r_t &= \alpha^r+\beta^r D_t+\sum_{i=1}^{p} a_i^r r_{t-i}+\sum_{i=1}^{p} b_i^r |r_{t-i}|+\lambda^r D_t \otimes x_t+\sum_{i=0}^{p}\left[\gamma_i^r+\delta_i^r \ln(T_{t-i})\right]x_{t-i}+\sum_{i=1}^{p} c_i^r \ln(T_{t-i})+u_t^r,\\
x_t &= \alpha^x+\beta^x D_t+\sum_{i=1}^{p} a_i^x r_{t-i}+\sum_{i=1}^{p} b_i^x |r_{t-i}|+\lambda^x D_{t-1}\otimes x_{t-1}+\sum_{i=1}^{p}\left[\gamma_i^x+\delta_i^x \ln(T_{t-i})\right]x_{t-i}+\sum_{i=1}^{p} c_i^x \ln(T_{t-i})+u_t^x,\\
\widetilde{T}_t &= T_t/\varphi(t)=[\phi_t \Gamma(1+1/\theta)]\epsilon_t,\\
\ln(\phi_t) &= \alpha^T+\sum_{i=1}^{p_1} a_i^T r_{t-i}+\sum_{i=1}^{p_1} b_i^T |r_{t-i}|+\sum_{i=1}^{p_1}\gamma_i^T x_{t-i}+\sum_{i=1}^{p_1}\rho_i \ln(\widetilde{T}_{t-i})+\sum_{i=1}^{p_2}\zeta_i \ln(\phi_{t-i})+\lambda^T D_{t-1},
\end{aligned}
\tag{2.9}
$$

where $D_t = (\text{open}_t, \text{bef}_t, \text{aro}_t, \text{aft}_t)'$, β's and λ's are conformable matrices of associated coefficients, and $\otimes$ denotes the Kronecker product.[9]

2.3 Data

This chapter focuses on all transactions for six major Australian banking stocks, namely ANZ Banking Group (ANZ), Commonwealth Bank of Australia (CBA), National Australia Bank (NAB), Westpac Banking Corporation (WBC), Macquarie Group (MQG) and Bendigo and Adelaide Bank (BEN), in eleven weeks that contain the eleven RBA interest rate announcement days in 2013, which were Feb 5, Mar 5, Apr 2, May 7, Jun 4, Jul 2, Aug 6, Sep 3, Oct 1, Nov 5 and Dec 3. Of these eleven announcements, two reported an interest rate fall of 25 basis points (May 7, from 3% to 2.75% and Aug 6, from 2.75% to 2.5%), and nine reported no changes in the cash rate. In total, there are 54 days in the sample.[10]

Most previous empirical microstructure work uses US data. In contrast, we work with Australian data provided by the Securities Industry Research Centre of Asia-Pacific (SIRCA). We choose the Australian market for several reasons. First, unlike

[9] Note that $D_t \otimes x_t = (\text{open}_t x_t^0, \text{bef}_t x_t^0, \text{aro}_t x_t^0, \text{aft}_t x_t^0, \text{open}_t v_t, \text{bef}_t v_t, \text{aro}_t v_t, \text{aft}_t v_t)'$. The quote revision equation in models (2.8) and (2.9) has the following full expression:

$$
\begin{aligned}
r_t &= \alpha^r+\beta^r_{op}\text{open}_t+\beta^r_{be}\text{bef}_t+\beta^r_{ar}\text{aro}_t+\beta^r_{af}\text{aft}_t+\sum_{i=1}^{p} a_i^r r_{t-i}+\sum_{i=1}^{p} b_i^r |r_{t-i}|+\lambda^r_{x^0,op}\text{open}_t x_t^0+\lambda^r_{x^0,be}\text{bef}_t x_t^0+\lambda^r_{x^0,ar}\text{aro}_t x_t^0\\
&+\lambda^r_{x^0,af}\text{aft}_t x_t^0+\sum_{i=0}^{p}\left[\gamma^r_{x^0,i}+\delta^r_{x^0,i}\ln(T_{t-i})\right]x^0_{t-i}+\lambda^r_{v,op}\text{open}_t v_t+\lambda^r_{v,be}\text{bef}_t v_t+\lambda^r_{v,ar}\text{aro}_t v_t+\lambda^r_{v,af}\text{aft}_t v_t\\
&+\sum_{i=0}^{p}\left[\gamma^r_{v,i}+\delta^r_{v,i}\ln(T_{t-i})\right]v_{t-i}+\sum_{i=1}^{p} c_i^r \ln(T_{t-i})+u_t^r.
\end{aligned}
$$

Full expressions for trade attribute and time duration equations are similarly obtained.

[10] Normally, there are five trading days in a typical week, so we might expect the sample to consist of 55 days. However, Apr 1, 2013, the day before the cash rate announcement in April, was Easter Monday on which the market was closed, which consequently leaves us with a sample of 54 trading days.

the US stock market which has a high degree of market fragmentation with 11 equity exchanges and many alternative trading systems (O'Hara, 2015), the Australian stock market is much less fragmented, which enables a more complete investigation of the joint dynamics of returns, trade attributes and durations. Second, the information about trade direction (which is shown to be an important determinant of the price dynamics (Hasbrouck, 1991a, Dufour and Engle, 2000)) is directly available to traders in Australia but concealed in the US markets. This helps avoid the need to use an indirect procedure to classify buys and sells such as the widely used Lee and Ready's (1991) algorithm which has an accuracy rate of only about 85% (Odders-White, 2000, Lillo et al., 2003). Finally, since the Australian stock market is a limit order book market and so are most major financial markets around the globe (Næs and Skjeltorp, 2006, Goettler et al., 2009, Malinova and Park, 2013), our findings may provide implications for these similarly structured markets.

We collect two datasets from the SIRCA database. The first dataset records details on every order submitted to the central limit order book, including stock code, order type (order submission, order revision, order cancellation and execution), date and time, order price, order volume (number of shares), order value (dollar value), and order direction (buy or sell order). We extract information for all transactions (order executions) in the continuous trading session (from 10:10:00 to 16:00:00) and discard all trades that are performed in the opening auction (10:00:00-10:10:00). We extract buyer-initiated and seller-initiated trades based on the directions of the (marketable) orders that initiate each trade.

The second dataset contains information on the intra-day bid and ask quotes, including stock code, date, time (precise to the millisecond), and the best bid-ask quotes and volumes in the limit order book. We remove all observations with negative bid or ask quote, with zero volume, and with a bid quote higher than ask quote. We merge the transaction data with the bid-ask quote data to work out the bid-ask midpoint and the prevailing depth before each transaction. Since one large buy (sell) order can be matched against several orders on the sell (buy) side and result in multiple transactions, we aggregate trades executed at the same time and initiated by the same order into one "large" trade by summing up the volumes of the simultaneous trades. This aggregation approach, which is standard in the literature (see, amongst others, Hasbrouck, 1991a, Dufour and Engle, 2000, Nowak and Anderson, 2014), leaves us with nearly 900,000 trades for all six stocks during the sample period. All continuous variables in this chapter are winsorized at the 0.5th and 99.5th quantiles to avoid the effects of outliers.

Table 2.1 provides the market capitalization, as at the beginning of 2013, and some summary statistics for the six banking stocks in 11 RBA announcement weeks (Panel A), on 11 RBA announcement days (Panel B) and on the remaining 43 non-RBA announcement days (Panel C). For the whole sample, the averages of absolute quote revisions, share volumes, volume to prevailing depth ratios and trade durations, together with the number of transactions, for each stock are reported. For smaller subsamples (Panels B and C), the summary is further categorized into five different time intervals, namely opening 30 minutes of the trading day (10:10:00-10:40:00), 5 minutes before the RBA announcement time (14:24:30-14:29:30), one minute around the announcements (14:29:30-14:30:30), 10 minutes after the announcements (14:30:30-14:40:30), and the remaining trading period (10:40:00-14:24:30 and 14:40:30-16:00:00). An asterisk (*) signifies that the average of a quantity of interest in a time interval is statistically significantly different from that of the "Remaining" period at a 5% significance level.[11]

The big four Australian banks, namely CBA, WBC, ANZ and NAB, are much larger and more heavily traded than the other two banks, with an average trade duration between 5.2 and 6.9 seconds. MQG, despite being relatively small, is traded quite intensively at every 8.3 seconds. Trades in the smallest stock, BEN, are much more dispersed and occur once in every 24.9 seconds on average. The majority of transactions in the six banking stocks are of smaller size than the prevailing quoted depths available right before these trades (which, in theory, should not move the best bid or ask levels), since the average volume to depth ratios for all stocks are significantly less

[11] Strictly speaking, reference made to the release time of the monetary policy decisions, such as "before", "around" and "after" the announcements, is only applicable to days on which such decisions are announced. However, to obtain an overall picture of how the cash rate announcements affect trades and prices, we also examine the same time windows on the non-RBA announcement days as those on the announcement days.

than unity. This is consistent with an observation by Dufour and Engle (2000) and Pham et al. (2017) that the majority of trades in their samples do not result in any quote revisions. For all stocks, an average transaction has a volume of 78.8 to 244.8 shares and moves quotes by 0.65 to 2.63 basis points (bps) (see Panel A). In general, the summary statistics are in agreement with the conventional wisdom that trades in more liquid stocks are more frequent and have less impact on prices (e.g. Dufour and Engle, 2000, Lillo et al., 2003).

Table 2.1: Descriptive statistics for Australian banking stocks

	ANZ	CBA	NAB	WBC	MQG	BEN
Market cap ($AUD bn)	68.727	100.059	58.555	80.821	12.037	3.420
Panel A: Whole sample						
Absolute Quote Revision (bps)	1.033	0.658	1.298	1.127	1.342	2.634
Volume (shares)	207.92	105.90	233.43	244.84	78.86	150.95
Volume/depth	0.370	0.525	0.417	0.460	0.496	0.256
Duration (seconds)	6.139	5.205	6.906	6.756	8.266	24.924
Observations	178,545	211,088	159,316	162,725	132,867	43,958
Panel B: RBA announcement days						
Absolute Quote Revision (bps)						
Open (10:10:00-10:40:00)	1.615*	1.118*	1.888*	1.527*	2.035*	3.135*
Before (14:24:30-14:29:30)	0.928	0.523	1.037*	0.955	0.975	1.795*
Around (14:29:30-14:30:30)	1.657*	0.898*	2.522*	1.838*	2.000*	4.809*
After(14:30:30-14:40:30)	1.245*	0.742*	1.598*	1.270*	1.482*	2.820
Remaining	0.946	0.576	1.354	1.011	1.143	2.516
Volume (shares)						
Open (10:10:00-10:40:00)	258.06*	129.99*	307.76*	279.15*	101.19*	168.77
Before (14:24:30-14:29:30)	202.43	85.18*	231.10	148.10*	51.78*	130.54
Around (14:29:30-14:30:30)	199.43	126.07	273.02	334.26*	83.58	228.24
After(14:30:30-14:40:30)	222.40*	139.25*	248.32	273.02*	91.15*	138.72
Remaining	173.96	106.62	218.21	220.35	66.94	143.74
Volume/depth						
Open (10:10:00-10:40:00)	0.480*	0.526*	0.502*	0.429*	0.603*	0.274*
Before (14:24:30-14:29:30)	0.327	0.529	0.354*	0.371	0.386	0.241
Around (14:29:30-14:30:30)	0.496*	0.509	0.531*	0.503*	0.524	0.276
After(14:30:30-14:40:30)	0.400*	0.520	0.460	0.446*	0.470	0.275*
Remaining	0.334	0.491	0.422	0.367	0.441	0.216
Duration (seconds)						
Open (10:10:00-10:40:00)	4.710*	4.422*	5.267*	4.836*	5.507*	19.826*
Before (14:24:30-14:29:30)	7.800*	5.826	8.033	8.043	10.607	23.966
Around (14:29:30-14:30:30)	2.856*	3.984*	5.197*	3.660*	3.931*	25.136
After(14:30:30-14:40:30)	5.025*	4.855*	6.339*	5.918*	7.327	23.282
Remaining	6.111	5.594	7.593	7.202	8.157	23.720
Observations						
Open (10:10:00-10:40:00)	4,153	4,400	3,728	4,065	3,518	966
Before (14:24:30-14:29:30)	412	549	401	410	320	130
Around (14:29:30-14:30:30)	198	184	132	188	173	37
After(14:30:30-14:40:30)	1,305	1,326	1,000	1,104	882	278
Remaining	31,940	35,204	25,892	27,358	24,186	8,358
Panel C: Non RBA announcement days						
Absolute Quote Revision (bps)						
Open (10:10:00-10:40:00)	1.579*	1.132*	1.653*	1.652*	2.157*	3.845*
(14:24:30-14:29:30)	0.846*	0.523*	0.994*	0.844*	1.055*	1.897*
(14:29:30-14:30:30)	0.850	0.580	1.106	0.936	1.541	2.210
(14:30:30-14:40:30)	0.869*	0.491*	1.030*	0.906*	1.178	2.231
Remaining	0.960	0.607	1.218	1.067	1.263	2.550
Volume (shares)						
Open (10:10:00-10:40:00)	257.29*	136.14*	303.15*	304.85*	103.69*	218.12*
(14:24:30-14:29:30)	169.27*	76.75*	209.86	183.09*	75.99	149.94
(14:29:30-14:30:30)	172.16	86.03	168.93*	267.36	73.40	108.36
(14:30:30-14:40:30)	208.13	102.22	210.87	245.03	77.16	115.95*
Remaining	208.56	101.20	224.65	241.02	77.90	146.72
Volume/depth						
Open (10:10:00-10:40:00)	0.461*	0.554*	0.475*	0.469	0.616*	0.350*
(14:24:30-14:29:30)	0.324*	0.489	0.390	0.516	0.473	0.239
(14:29:30-14:30:30)	0.367	0.450*	0.432	0.331*	0.479	0.281
(14:30:30-14:40:30)	0.344	0.501	0.371*	0.626	0.449*	0.273
Remaining	0.364	0.531	0.406	0.479	0.492	0.257
Duration (seconds)						
Open (10:10:00-10:40:00)	4.418*	3.943*	4.581*	4.500*	6.241*	23.047*
(14:24:30-14:29:30)	7.965*	6.508*	7.530	8.135*	9.617	23.657
(14:29:30-14:30:30)	7.985*	6.989*	7.466	9.504*	9.727	26.785
(14:30:30-14:40:30)	7.157*	5.983*	7.449	8.303*	10.600*	25.254
Remaining	6.405	5.262	7.137	7.011	8.607	25.654
Observations						
Open (10:10:00-10:40:00)	17,350	19,320	16,725	16,985	12,073	3,110
(14:24:30-14:29:30)	1,536	1,868	1,644	1,493	1,292	521
(14:29:30-14:30:30)	322	385	346	283	261	113
(14:30:30-14:40:30)	3,387	4,121	3,317	2,979	2,342	928
Remaining	117,942	143,731	106,131	107,860	87,820	29,517

Continued on next page

Table 2.1 – *continued from previous page*

This table reports summary statistics for six Australian banks. The sample consists of trades occurring between 10:10:00 and 16:00:00 in eleven weeks that contain eleven RBA interest rate announcement days in 2013. Market capitalization for each stock is at the beginning of 2013. Panels A, B and C respectively provide the summary for the whole sample (eleven weeks), the eleven RBA announcement days and the remaining non-RBA announcement days. For the latter two subsamples (Panels B and C), descriptive statistics over different time intervals for absolute quote revision, volume, volume divided by prevailing depth, duration and number of transactions are reported. These time intervals include "Open" which covers the first 30 minutes of the trading day (10:10:00-10:40:00), "Before" which covers 5 minutes before the RBA announcement time (14:24:30-14:29:30), "Around" which covers one minute during the RBA announcement time (14:29:30-14:30:30), "After" which covers 10 minutes after the RBA announcement time (14:30:30-14:40:30), and "Remaining" which covers the remaining time of a trading day. Except for "Observations" and "Market capitalization", all other numbers are averages. Asterisks (*) denote statistically significant difference from the averages for the "Remaining" period at a 5% level.

More interesting features are observed when the whole sample is partitioned into RBA and non-RBA announcement days, as shown in Panels B and C. In both subsamples, stocks are traded much more frequently at the market open than during the reference ("Remaining") time period. Moreover, trades performed in the first 30 minutes of the day have significantly larger sizes and volume to depth ratios and result in bigger price adjustments. This observation is in conformance with Anand et al. (2005), Bloomfield et al. (2005), and Duong et al. (2009), who show that higher trading intensity observed at the market open is driven by an increased engagement of informed investors whose transactions, according to Easley and O'Hara's (1992) theory, normally have a large volume and big impact on prices.

On the eleven days when the RBA announces its monetary policy stance, trades are performed with very short durations during the one minute around the announcement time (see Panel B). With the exception of stock BEN regarding time durations, such trades have remarkably smaller durations, larger volumes, larger volume to depth ratios and bigger price impact than those transacted in the reference period, and even bigger than those executed at the market open in some cases. Similar features (although less noticeable) are also observed for transactions performed within 10 minutes after the news release in comparison to those in the base period, whereas trades that are executed during 5 minutes before the announcement often exhibit opposite characteristics. The summary statistics seem to suggest a relatively tranquil market before the announcement, while market participants are awaiting the RBA interest rate decision. Near the announcement time, the market becomes more active and is very active for one minute around the release of the decision, possibly due to the heightened activities of informed investors. The high trading intensity gradually attenuates as more time elapses after the announcement.

The above trading pattern, however, is not observed on non-RBA announcement days (see Panel C). In particular, transactions executed during the same time intervals (i.e. between 14:24:30 and 14:40:30) typically have larger durations, smaller volumes, smaller volume to depth ratios and less price impact than those performed outside these times, with statistical significance realized in many cases. Most noticeably, trades transacted within one minute around the RBA announcements typically lead to a price impact that is twice as large as that resulting from trades during the same time period on a non-RBA day, while their durations are often a half of the latter's (except for BEN). The contrasting results between RBA and non-RBA announcement samples suggest that the release of the RBA monetary policy decisions has significant effects on the dynamic behaviors of prices, trade attributes and trade arrival times of the major Australian banking stocks. Such effects are formally investigated in the next section.

2.4 Results and discussion

In this section, we empirically investigate the joint dynamics of returns, trade attributes (signs and volumes), and trade durations for 6 major Australian banking stocks around interest rate announcements in 2013. We begin with a description of the estimated results for the two joint models, namely *Endo-VAR* and *WACD-VAR*, which are proposed in Section 2.2 of this chapter (subsection 2.4.1). We then conduct an impulse response analysis to study how the prices of the banking stocks change when there are shocks to the joint system and how the reactions of prices to the shocks depend on the occurrence of an exogenous monetary policy announcement (subsection 2.4.2). We also provide detailed forecast error variance decomposition that compares the relative importance of trade durations and trade attributes to the explanation of price dynamics (subsection 2.4.3).

2.4.1 Estimation results

Endo-VAR models

We begin with the estimation of the *Endo-VAR* model (2.8) for a representative stock NAB and discuss this in detail before presenting results for the remaining five stocks. Since one of the main objectives of this chapter is to relax the strict exogeneity assumption that is often imposed on durations in previous studies (e.g. Dufour and Engle, 2000, Engle, 2000, Xu et al., 2006), we draw attention to the duration equation of the estimated *Endo-VAR* model, reported in Table 2.2 using the whole sample period of eleven RBA announcement weeks. We compute Student's t (in parentheses) and Wald statistics using the Newey and West (1994) robust standard errors, and use bold format to signify statistical significance at a 5% level. As expected, trade durations are positively and persistently autocorrelated (see coefficients on $\ln(T_{t-i})$), which implies the clustering feature inherent in the duration process: long (short) durations tend to follow long (short) durations. This stylized fact is widely seen and documented in numerous empirical studies on durations (Engle and Russell, 1998, Engle, 2000, Russell and Engle, 2005).

The observation that trade durations are correlated with price adjustments and trade attributes is of particular interest here, given the focus on the possible endogeneity of the time between trades. We find that the magnitude of price changes or the variation in prices, rather than the direction of price moves, is informative about the durations of future trades since the coefficients of absolute returns, $|r_{t-i}|$, are highly significant, whereas those of raw returns, r_{t-i}, are not. A positive coefficient sum of past absolute returns implies that larger price adjustments tend to increase future trade durations. This is in conformance with the predictions from the Admati and Pfleiderer's (1988) theoretical model and the empirical findings of Grammig and Wellner (2002) and Wei and Pelletier (2015) that show a positive feedback effect of past volatility on future durations. However, it appears to be inconsistent with Manganelli (2005) and Russell and Engle (2005), who document an opposite result. If big variation in quote midpoints is interpreted by market participants as a result of informed trades, then the presence of informed agents in the market might discourage uninformed investors from trading and reduce the likelihood of trades (Admati and Pfleiderer, 1988, Grammig and Wellner, 2002). Consequently, trade durations might be longer following large price changes.

Table 2.2: Estimated trade duration equation of the *Endo-VAR* model for stock NAB in eleven RBA announcement weeks in 2013

	Coef.	t-stat		Coef.	t-stat
const	**-0.083**	**(-7.88)**	v_{t-1}	**-0.014**	**(-4.62)**
open_{t-1}	**-0.163**	**(-8.61)**	v_{t-2}	**-0.010**	**(-3.21)**
bef_{t-1}	0.082	(0.73)	$\sum_{i=1}^{5} v_{t-i}$	**-0.023**	**(-4.85)**
aro_{t-1}	-0.410	(-1.81)	$v_{t-1}\text{open}_{t-1}$	**-0.027**	**(-3.21)**
aft_{t-1}	-0.120	(-1.55)	$v_{t-1}\text{bef}_{t-1}$	0.057	(1.02)
r_{t-1}	-0.004	(-1.53)	$v_{t-1}\text{aro}_{t-1}$	-0.047	(-0.36)
r_{t-2}	0.006	(1.50)	$v_{t-1}\text{aft}_{t-1}$	-0.016	(-0.39)
$\sum_{i=1}^{5} r_{t-i}$	-0.002	(-0.28)	$v_{t-1}\ln(T_{t-1})$	0.002	(1.77)
$\|r_{t-1}\|$	**0.281**	**(80.85)**	$v_{t-2}\ln(T_{t-2})$	**-0.002**	**(-2.23)**
$\|r_{t-2}\|$	**-0.204**	**(-53.39)**	$\sum_{i=1}^{5} v_{t-i}\ln(T_{t-i})$	0.004	(1.79)
$\sum_{i=1}^{5} \|r_{t-i}\|$	**0.029**	**(4.62)**	$\ln(T_{t-1})$	**0.174**	**(64.72)**
x_{t-1}^{0}	**0.030**	**(2.50)**	$\ln(T_{t-2})$	**0.067**	**(25.01)**
x_{t-2}^{0}	**-0.026**	**(-2.20)**	$\sum_{i=1}^{5}\ln(T_{t-i})$	**0.342**	**(70.94)**
$\sum_{i=1}^{5} x_{t-i}^{0}$	0.007	(0.39)	Adj. R^2	0.092	-
$x_{t-1}^{0}\text{open}_{t-1}$	-0.025	(-0.97)	$\text{Wald}_{\text{diur}}$	**89.8**	-
$x_{t-1}^{0}\text{bef}_{t-1}$	0.014	(0.07)	$\text{Wald}_{\text{time}}$	**7124.0**	-
$x_{t-1}^{0}\text{aro}_{t-1}$	0.056	(0.17)	$Q_{15,\text{raw}}$	**11897.6**	-
$x_{t-1}^{0}\text{aft}_{t-1}$	-0.150	(-1.26)	$Q_{15,\text{resid}}$	**670.5**	-
$x_{t-1}^{0}\ln(T_{t-1})$	**0.007**	**(2.06)**	JB_{resid}	**10373.8**	-
$x_{t-2}^{0}\ln(T_{t-2})$	0.000	(0.10)			
$\sum_{i=1}^{5} x_{t-i}^{0}\ln(T_{t-i})$	0.010	(1.36)			

This table reports the coefficient estimates and Newey and West (1994) heteroskedasticity and autocorrelation consistent t-statistics (in parentheses) for the trade duration equation of the *Endo-VAR* model specified in (2.8) for stock NAB.

$$\ln(T_t)=\alpha^T+\beta^T D_{t-1}+\sum_{i=1}^{p} a_i^T r_{t-i}+\sum_{i=1}^{p} b_i^T|r_{t-i}|+\lambda^T D_{t-1}\otimes x_{t-1}+\sum_{i=1}^{p}\left[\gamma_i^T+\delta_i^T\ln(T_{t-i})\right]x_{t-i}+\sum_{i=1}^{p} c_i^T\ln(T_{t-i})+u_t^T.$$

r_t is the logarithmic change in midquotes following the t-th trade. x_t is a column vector of trade signs (x_t^0, which equals 1 for buys and -1 for sells) and volumes (v_t, defined as the signed logarithm of the ratio of the share volume to the prevailing quoted depth) of the t-th trade. T_t is the time duration between the $(t-1)$-th and t-th trades. $D_t = (\text{open}_t, \text{bef}_t, \text{aro}_t, \text{aft}_t)'$ is a vector of four diurnal dummy variables including open_t that marks the first 30 minutes of a trading day (i.e. 10:10:00-10:40:00), and bef_t, aro_t and aft_t that respectively identify trades executed 5 minutes *before* (14:24:30-14:29:30), one minute *around* (14:29:30-14:30:30), and 10 minutes *after* (14:30:30-14:40:30) the RBA announcements. $\otimes$ denotes the Kronecker product.

The lag length p is set to $p = 5$. We only report the individual coefficients of the first two lags. $\text{Wald}_{\text{diur}}$ is the Wald test statistic associated with the null hypothesis that the coefficients on all diurnal dummies (i.e. open_t, bef_t, aro_t and aft_t) are jointly zero. $\text{Wald}_{\text{time}}$ is the Wald test statistic associated with the null hypothesis that the coefficients on all diurnal dummies and durations are jointly zero. $Q_{15,\text{raw}}$ ($Q_{15,\text{resid}}$) is the Ljung-Box statistic associated with the null hypothesis of no autocorrelation up to order 15 in the raw (residual) series. JB_{resid} is the Jarque-Bera statistic associated with the null that the residuals are normally distributed. Bold format denotes statistical significance at a 5% level.

Past trading volumes are also an important predictor of the time between trades. Negative and strongly significant coefficients and coefficient sums of previous trade sizes imply that larger transactions tend to shorten the duration of incoming trades. This lends support to Easley and O'Hara (1992), who hypothesize that large trades are more likely to be initiated by informed traders who always trade to capitalize on new information. Thus, large transactions are likely to lead to higher trading rates and, consequently, shorter durations. The negative relationship between durations and trading volumes is also found in previous empirical studies such as Bauwens and Giot (2000), Manganelli (2005) and Nowak and Anderson (2014).

The positive coefficient on trade sign, x^0_{t-1}, suggests that it likely takes longer time for a trade to occur when it is preceded by a purchase than by a sale. Moreover, the positive serial dependencies of time durations are stronger for buyer-initiated trades but weaker for seller-initiated ones, as implied by the significantly positive coefficient on $x^0_{t-1}\ln(T_{t-1})$. However, the asymmetry in the autocorrelation of trade durations between buys and sells appears short-lived, and so do the effects of trade signs on future trade durations, as suggested by the insignificance of the coefficient sums of $x^0_{t-i}\ln(T_{t-i})$ and x^0_{t-i}, respectively. While there is evidence that trading intensifies at the market open, no similar evidence is observed around the RBA announcements since the coefficients on the RBA announcement dummies and their interactions with trade characteristics are not significant, even though they are generally of expected signs and are economically meaningful in comparison with the corresponding coefficients on the open_t dummy. This result is surprising, given the clear pattern shown in Table 2.1 that durations between trades in stock NAB are significantly shorter during the one minute around the RBA interest rate releases. Perhaps, however, the unconditional pattern in trade durations around the RBA announcements simply reflects those in the marks (i.e. returns and trade attributes) and thus disappears when one conditions on the latter.

We now examine the dynamics of prices by looking at the equation for quote revisions. The results are reported in Panel A of Table 2.3 that uses the whole eleven RBA announcement week sample. Consistent with previous studies such as Hasbrouck (1991a,b) and Dufour and Engle (2000), price changes are negatively serially correlated, as indicated by a negative first lag coefficient. Meanwhile, a signed trade positively affects prices in the sense that a buy leads to an upward adjustment in prices while a sell drags prices down. The price impact of a trade in stock NAB is influenced by both the direction (x_t^0) and volume (v_t) of the trade, which is consistent with the findings of Hasbrouck (1991a,b). Immediately following a purchase with an average volume to depth ratio of 0.417 and an average duration of 6.906 seconds (i.e. an average purchase, see Table 2.1), the price of stock NAB is lifted up by 0.984 bps, assuming that the purchase is executed during the reference time period (i.e. not at the market open or during the 16-minute window around an RBA interest rate announcement).[12] Meanwhile, the coefficients on lagged trade signs and lagged volumes are generally negative but are much smaller in magnitude compared to the contemporaneous coefficients, implying that the cumulative price impact of a buy remains strongly positive.

We find a significant role for the time of trade arrivals in the process of price formation for stock NAB, lending support to Diamond and Verrecchia (1987), Easley and O'Hara (1992), Dufour and Engle (2000) and Xu et al. (2006). As implied by the significantly negative contemporaneous coefficients and coefficient sums of the interactions between trade attributes and durations, the price impact of a trade is negatively dependent on its duration, suggesting that prices tend to adjust more following a trade that has a shorter duration. The explanation is that shorter time between trades or higher trading intensity is inferred by the market maker as a signal of more private news being released to the market and the increased presence of informed traders to exploit such information (Easley and O'Hara, 1992, Dufour and Engle, 2000). Since a higher probability of informed trading discourages liquidity providers, possibly via toxic order flows that adversely select the latter (Easley et al., 2011, 2012), trades result in larger price adjustments. In addition, there is some positive direct impact of durations on price changes which suggests that prices tend to adjust upward after a long time interval from a previous trade, as indicated by a positive sum of coefficients on $\ln(T_{t-i})$. However, this positive direct dependence is relatively weak and dominated by

[12] $0.984 = \underbrace{-0.002}_{\text{const}} + \underbrace{1.227}_{x_t^0} - \underbrace{0.031 \times \ln 6.906}_{x_t^0 \ln(T_t)} + \underbrace{0.219 \times \ln 0.417}_{v_t} - \underbrace{0.006 \times \ln 0.417 \times \ln 6.906}_{v_t \ln(T_t)}$. Note that v_t is calculated as the signed logarithm of the volume to depth ratio of the t-th transaction.

the negative indirect influence of durations on prices (which is a portion of the price impact of a trade captured by the interaction terms $x_t^0 \ln(T_t)$ and $v_t \ln(T_t)$), leading to an overall negative relation between price changes and trade durations, which supports Easley and O'Hara's (1992) theory.

The price impact of a trade also exhibits a diurnal pattern. Purchases (sales) that are performed within the first 30 minutes of a trading day on average raise (reduce) prices markedly more than do those executed during the reference period, as shown by the positive coefficients on $x_t^0\text{open}_t$ and $v_t\text{open}_t$. This can be explained by higher trading intensity induced by a larger proportion of informed traders who are trying to capitalize on relatively more information that has accumulated overnight being revealed to the market in the early morning (Anand et al., 2005, Bloomfield et al., 2005, Duong et al., 2009, Pham et al., 2017). Since prices tend to move more with higher informed trading rates, the price impact of a trade is likely to be higher at the beginning of the trading day.

Table 2.3: Estimated return equation of the *Endo-VAR* model for stock NAB

Panel A: Eleven RBA announcement weeks

	Coef.	t-stat		Coef.	t-stat		Coef.	t-stat		Coef.	t-stat
const	-0.002	(-0.30)	x_t^0	**1.227**	**(126.26)**	v_t	**0.219**	**(92.44)**	$\ln(T_{t-1})$	0.000	(-0.06)
open_t	-0.016	(-0.89)	x_{t-1}^0	-0.012	(-1.21)	v_{t-1}	-0.004	(-1.69)	$\ln(T_{t-2})$	0.000	(0.21)
bef_t	-0.039	(-0.42)	x_{t-2}^0	**-0.045**	**(-4.48)**	v_{t-2}	**-0.014**	**(-5.91)**	$\sum_{i=1}^5 \ln(T_{t-i})$	**0.010**	**(2.80)**
aro_t	0.182	(0.58)	$\sum_{i=0}^5 x_{t-i}^0$	**1.099**	**(70.42)**	$\sum_{i=0}^5 v_{t-i}$	**0.185**	**(47.37)**	Adj. R^2	0.178	-
aft_t	0.004	(0.05)	$x_t^0\text{open}_t$	**0.300**	**(11.30)**	$v_t\text{open}_t$	**0.097**	**(11.27)**	$\text{Wald}_{\text{diur}}$	**169.8**	-
r_{t-1}	**-0.027**	**(-7.56)**	$x_t^0\text{bef}_t$	-0.058	(-0.35)	$v_t\text{bef}_t$	-0.034	(-0.91)	$\text{Wald}_{\text{time}}$	**316.8**	-
r_{t-2}	**0.010**	**(2.98)**	$x_t^0\text{aro}_t$	**1.310**	**(3.18)**	$v_t\text{aro}_t$	**0.327**	**(2.14)**	$Q_{15,\text{raw}}$	**4397.2**	-
$\sum_{i=1}^5 r_{t-i}$	0.006	(0.73)	$x_t^0\text{aft}_t$	**0.319**	**(2.84)**	$v_t\text{aft}_t$	**0.096**	**(2.77)**	$Q_{15,\text{resid}}$	19.9	-
$\lvert r_{t-1}\rvert$	**-0.009**	**(-2.76)**	$x_t^0 \ln(T_t)$	**-0.031**	**(-11.16)**	$v_t \ln(T_t)$	**-0.006**	**(-8.03)**	JB_{resid}	**37992.9**	-
$\lvert r_{t-2}\rvert$	-0.001	(-0.18)	$x_{t-1}^0 \ln(T_{t-1})$	0.000	(-0.13)	$v_{t-1} \ln(T_{t-1})$	-0.001	(-1.46)			
$\sum_{i=1}^5 \lvert r_{t-i}\rvert$	-0.009	(-1.58)	$x_{t-2}^0 \ln(T_{t-2})$	0.004	(1.46)	$v_{t-2} \ln(T_{t-2})$	0.000	(0.62)			
			$\sum_{i=0}^5 x_{t-i}^0 \ln(T_{t-i})$	**-0.020**	**(-3.18)**	$\sum_{i=0}^5 v_{t-i} \ln(T_{t-i})$	**-0.004**	**(-2.35)**			

Panel B: Non RBA announcement days

	Coef.	t-stat		Coef.	t-stat		Coef.	t-stat		Coef.	t-stat
const	0.003	(0.33)	x_t^0	**1.212**	**(110.53)**	v_t	**0.216**	**(81.97)**	$\ln(T_{t-1})$	0.002	(0.97)
open_t	-0.002	(-0.08)	x_{t-1}^0	-0.003	(-0.29)	v_{t-1}	-0.002	(-0.58)	$\ln(T_{t-2})$	0.000	(0.18)
bef_t	-0.059	(-1.41)	x_{t-2}^0	**-0.048**	**(-4.36)**	v_{t-2}	**-0.016**	**(-6.22)**	$\sum_{i=1}^5 \ln(T_{t-i})$	**0.009**	**(2.39)**
aro_t	0.105	(1.05)	$\sum_{i=0}^5 x_{t-i}^0$	**1.093**	**(63.18)**	$\sum_{i=0}^5 v_{t-i}$	**0.183**	**(42.57)**	Adj. R^2	0.178	-
aft_t	**0.081**	**(2.69)**	$x_t^0\text{open}_t$	**0.269**	**(9.29)**	$v_t\text{open}_t$	**0.088**	**(9.46)**	$\text{Wald}_{\text{diur}}$	**155.6**	-
r_{t-1}	**-0.031**	**(-7.81)**	$x_t^0\text{bef}_t$	**-0.203**	**(-3.13)**	$v_t\text{bef}_t$	**-0.038**	**(-2.46)**	$\text{Wald}_{\text{time}}$	**272.4**	-
r_{t-2}	**0.008**	**(2.14)**	$x_t^0\text{aro}_t$	-0.257	(-1.83)	$v_t\text{aro}_t$	-0.063	(-1.82)	$Q_{15,\text{raw}}$	**3683.3**	-
$\sum_{i=1}^5 r_{t-i}$	-0.005	(-0.57)	$x_t^0\text{aft}_t$	**-0.159**	**(-3.38)**	$v_t\text{aft}_t$	**-0.042**	**(-3.41)**	$Q_{15,\text{resid}}$	21.1	-
$\lvert r_{t-1}\rvert$	**-0.007**	**(-1.97)**	$x_t^0 \ln(T_t)$	**-0.030**	**(-9.83)**	$v_t \ln(T_t)$	**-0.006**	**(-7.00)**	JB_{resid}	**32183.3**	-
$\lvert r_{t-2}\rvert$	-0.003	(-0.68)	$x_{t-1}^0 \ln(T_{t-1})$	-0.002	(-0.57)	$v_{t-1} \ln(T_{t-1})$	-0.001	(-1.65)			
$\sum_{i=1}^5 \lvert r_{t-i}\rvert$	-0.011	(-1.79)	$x_{t-2}^0 \ln(T_{t-2})$	0.002	(0.79)	$v_{t-2} \ln(T_{t-2})$	0.000	(0.36)			
			$\sum_{i=0}^5 x_{t-i}^0 \ln(T_{t-i})$	**-0.028**	**(-3.92)**	$\sum_{i=0}^5 v_{t-i} \ln(T_{t-i})$	**-0.005**	**(-2.26)**			

This table reports the coefficient estimates and Newey and West (1994) heteroskedasticity and autocorrelation consistent t-statistics (in parentheses) for the return equation of the *Endo-VAR* model specified in (2.8) for stock NAB in eleven RBA announcement weeks (Panel A) and on non-RBA announcement days (Panel B) in 2013.

$$r_t = \alpha^r + \beta^r D_t + \sum_{i=1}^{p} a_i^r r_{t-i} + \sum_{i=1}^{p} b_i^r |r_{t-i}| + \lambda^r D_t \otimes x_t + \sum_{i=0}^{p} \left[\gamma_i^r + \delta_i^r \ln(T_{t-i})\right] x_{t-i} + \sum_{i=1}^{p} c_i^r \ln(T_{t-i}) + u_t^r.$$

See Table 2.2 notes for definitions of the variables. In Panel B, bef_t, aro_t and aft_t are respectively indicator variables identifying trades performed within 14:24:30-14:29:30, 14:29:30-14:30:30 and 14:30:30-14:40:30 time periods (i.e. corresponding time intervals to those on the RBA announcement days).

The RBA monetary policy announcements significantly affect the price impact of a trade, through both sign and size channels. While trades performed within five minutes before the release of monetary decisions result in a price impact that is statistically indistinguishable from that of those occurring during the reference time window, trades executed within one minute around the announcement and during the subsequent ten minutes affect prices significantly more. Practically, an average buy in stock NAB transacted one minute around (ten minute after) the announcements immediately raises the quote midpoint by about 1.206 bps (0.239 bps) higher than, or equivalently 2.23 times (1.24 times) as high as, does a similar purchase arriving during the reference time period.[13] Trades around the announcements are even more informative than those at the market open, suggesting a higher concentration of informed traders during the one minute around the interest rate releases than at the market open. It appears that informed investors await the interest rate decisions from the RBA and thus are relatively inactive five minutes before the announcement. As time draws closer to 14:30:00 - the scheduled release time, more information is revealed, inducing a higher likelihood of informed traders. The presence of informed market participants is highest during the one minute around the release, and gradually decreases in the next ten minutes when more information is incorporated into prices. Consequently, trades occurring within one minute around the announcements have the greatest impact on prices.

To further highlight the impact of the monetary announcements on prices of stock NAB, we re-estimate the return equation of the *Endo-VAR* model using data on the non-RBA announcement days of our sample only. The results are reported in Panel B of Table 2.3. On non-RBA announcement days, the effects of trade characteristics and time durations on quote revisions for stock NAB remain qualitatively similar to those previously discussed. Differences, however, are found when looking at the 16-minute window that corresponds to the announcement time period on the announcement days. On days with no monetary policy releases, trades performed within the

[13] $1.206 = \underbrace{0.182}_{\text{aro}_t} + \underbrace{1.310}_{x_t^0\text{aro}_t} + \underbrace{0.327 \times \ln 0.417}_{v_t\text{aro}_t}$, and $0.239 = \underbrace{0.004}_{\text{aft}_t} + \underbrace{0.319}_{x_t^0\text{aft}_t} + \underbrace{0.096 \times \ln 0.417}_{v_t\text{aft}_t}$. Note that the immediate price impact of an average buy in the reference period is 0.984 bps (see Footnote 12).

16-minute period typically have a smaller impact on prices than those executed in the reference period, reflecting a typical diurnal pattern on a normal day (see Table 2.1). This result thus confirms the important information content of the interest rate announcements that significantly affect prices of stock NAB, which is in agreement with previous findings such as Kim and Nguyen (2008) and Smales (2012).

Regarding the estimation for trade attributes which is tabulated in Table 2.4, trades exhibit a strong positive serial correlation structure, both in terms of direction and volume. This pattern is typically observed in empirical applications (e.g. Hasbrouck, 1991a, Dufour and Engle, 2000, Manganelli, 2005), and it suggests a clustering feature inherent in a trade series. In particular, purchases (sales) tend to follow purchases (sales), and large (small) transactions tend to induce large (small) transactions. Moreover, there is a significant bilateral Granger-causal relationship between trade directions and volumes, as well as strong Granger-causality running from quote revisions to trade characteristics, which is in conformance with the findings of Hasbrouck (1991a). Consistent with Dufour and Engle (2000), trade signs become more positively autocorrelated when time durations between trades get shorter, as reflected in Panel A of Table 2.4 by a significantly negative coefficient on $x^0_{t-1}\ln(T_{t-1})$ at the first lag, even though the coefficient sum is not significant. Similarly, not only is higher trading intensity or shorter trade duration associated with larger future transactions (see the coefficients on $\ln(T_{t-i})$ in Panel B - which is in harmony with Easley and O'Hara's (1992) theory that predicts a negative association between trade durations and trading size), but it also tends to increase the positive autocorrelation of trading volumes, as evidenced by the coefficients on $v_{t-i}\ln(T_{t-i})$ in Panel B.

Table 2.4: Estimated trade sign and volume equations of the *Endo-VAR* model for stock NAB in eleven RBA announcement weeks in 2013

Panel A: Trade sign equation

	Coef.	t-stat		Coef.	t-stat		Coef.	t-stat		Coef.	t-stat
const	**0.039**	**(10.46)**	x^0_{t-1}	**0.395**	**(97.69)**	v_{t-1}	**0.017**	**(14.87)**	$\ln(T_{t-1})$	0.000	(0.05)
open_t	**0.029**	**(4.10)**	x^0_{t-2}	**0.076**	**(18.78)**	v_{t-2}	-0.001	(-0.81)	$\ln(T_{t-2})$	0.001	(0.83)
bef_t	-0.015	(-0.38)	$\sum_{i=1}^{5} x^0_{t-i}$	**0.502**	**(83.68)**	$\sum_{i=1}^{5} v_{t-i}$	**-0.004**	**(-2.36)**	$\sum_{i=1}^{5} \ln(T_{t-i})$	-0.001	(-0.50)
aro_t	0.057	(0.69)	$x^0_{t-1}\text{open}_{t-1}$	**0.030**	**(3.28)**	$v_{t-1}\text{open}_{t-1}$	**0.014**	**(4.22)**	Adj. R^2	0.246	-
aft_t	-0.017	(-0.61)	$x^0_{t-1}\text{bef}_{t-1}$	0.079	(1.33)	$v_{t-1}\text{bef}_{t-1}$	0.016	(0.85)	$\text{Wald}_{\text{diur}}$	**41.3**	-
r_{t-1}	**-0.199**	**(-139.13)**	$x^0_{t-1}\text{aro}_{t-1}$	0.172	(1.73)	$v_{t-1}\text{aro}_{t-1}$	0.056	(1.30)	$\text{Wald}_{\text{time}}$	**239.4**	-
r_{t-2}	**-0.028**	**(-21.41)**	$x^0_{t-1}\text{aft}_{t-1}$	0.028	(0.74)	$v_{t-1}\text{aft}_{t-1}$	0.024	(1.64)	$Q_{15,\text{raw}}$	**19196.9**	-
$\sum_{i=1}^{5} r_{t-i}$	**-0.217**	**(-70.17)**	$x^0_{t-1}\ln(T_{t-1})$	**-0.005**	**(-4.59)**	$v_{t-1}\ln(T_{t-1})$	**0.002**	**(5.96)**	$Q_{15,\text{resid}}$	**393.8**	-
$\|r_{t-1}\|$	**-0.008**	**(-6.29)**	$x^0_{t-2}\ln(T_{t-2})$	0.001	(1.26)	$v_{t-2}\ln(T_{t-2})$	**0.002**	**(4.11)**	JB_{resid}	**4649.9**	-
$\|r_{t-2}\|$	0.000	(0.15)	$\sum_{i=1}^{5} x^0_{t-i}\ln(T_{t-i})$	-0.002	(-0.99)	$\sum_{i=1}^{5} v_{t-i}\ln(T_{t-i})$	**0.004**	**(5.61)**			
$\sum_{i=1}^{5} \|r_{t-i}\|$	**-0.008**	**(-3.72)**									

Panel B: Trade volume equation

	Coef.	t-stat		Coef.	t-stat		Coef.	t-stat		Coef.	t-stat
const	**-0.232**	**(-15.81)**	x^0_{t-1}	**-0.239**	**(-18.79)**	v_{t-1}	**0.183**	**(38.92)**	$\ln(T_{t-1})$	**-0.010**	**(-3.41)**
open_t	**-0.087**	**(-4.45)**	x^0_{t-2}	-0.022	(-1.70)	v_{t-2}	**0.080**	**(18.73)**	$\ln(T_{t-2})$	-0.003	(-0.87)
bef_t	-0.090	(-0.55)	$\sum_{i=1}^{5} x^0_{t-i}$	**-0.151**	**(-7.80)**	$\sum_{i=1}^{5} v_{t-i}$	**0.442**	**(58.38)**	$\sum_{i=1}^{5} \ln(T_{t-i})$	**-0.013**	**(-2.54)**
aro_t	-0.103	(-0.56)	$x^0_{t-1}\text{open}_{t-1}$	**-0.067**	**(-2.78)**	$v_{t-1}\text{open}_{t-1}$	**-0.054**	**(-4.57)**	Adj. R^2	0.114	-
aft_t	**0.189**	**(2.39)**	$x^0_{t-1}\text{bef}_{t-1}$	-0.332	(-1.64)	$v_{t-1}\text{bef}_{t-1}$	-0.102	(-1.24)	$\text{Wald}_{\text{diur}}$	**64.4**	-
r_{t-1}	**0.223**	**(66.27)**	$x^0_{t-1}\text{aro}_{t-1}$	**-0.452**	**(-2.35)**	$v_{t-1}\text{aro}_{t-1}$	-0.045	(-0.34)	$\text{Wald}_{\text{time}}$	**136.4**	-
r_{t-2}	**0.029**	**(7.74)**	$x^0_{t-1}\text{aft}_{t-1}$	-0.160	(-1.46)	$v_{t-1}\text{aft}_{t-1}$	**-0.122**	**(-2.94)**	$Q_{15,\text{raw}}$	**36259.6**	-
$\sum_{i=1}^{5} r_{t-i}$	**0.189**	**(23.44)**	$x^0_{t-1}\ln(T_{t-1})$	**0.014**	**(4.30)**	$v_{t-1}\ln(T_{t-1})$	0.000	(0.12)	$Q_{15,\text{resid}}$	**1180.6**	-
$\|r_{t-1}\|$	**0.018**	**(4.62)**	$x^0_{t-2}\ln(T_{t-2})$	-0.004	(-1.23)	$v_{t-2}\ln(T_{t-2})$	**-0.006**	**(-3.98)**	JB_{resid}	**5230.6**	-
$\|r_{t-2}\|$	**0.011**	**(2.86)**	$\sum_{i=1}^{5} x^0_{t-i}\ln(T_{t-i})$	0.002	(0.29)	$\sum_{i=1}^{5} v_{t-i}\ln(T_{t-i})$	**-0.011**	**(-3.41)**			
$\sum_{i=1}^{5} \|r_{t-i}\|$	**0.051**	**(6.92)**									

This table reports the coefficient estimates and Newey and West (1994) heteroskedasticity and autocorrelation consistent t-statistics (in parentheses) for the trade sign and volume equations of the *Endo-VAR* model specified in (2.8) for stock NAB.

$$x_t = \alpha^x + \beta^x D_t + \sum_{i=1}^{p} a_i^x r_{t-i} + \sum_{i=1}^{p} b_i^x |r_{t-i}| + \lambda^x D_{t-1} \otimes x_{t-1} + \sum_{i=1}^{p} \left[\gamma_i^x + \delta_i^x \ln(T_{t-i})\right] x_{t-i} + \sum_{i=1}^{p} c_i^x \ln(T_{t-i}) + u_t^x.$$

See Table 2.2 notes for definitions of the variables.

From Tables 2.2-2.4, the Ljung-Box statistics suggest that the *Endo-VAR* model appears to capture most of the dynamics of the joint system by filtering out most of the serial correlation exhibited in the raw series. However, there is still significant autocorrelation in the residuals for trade direction, size and duration. For all four equations, the residuals are not normally distributed, as shown by large Jarque-Bera statistics. Nevertheless, significant Wald test statistics at a 5% level again confirm the role of time in explaining the price and trade formation processes.

We now investigate the results for the remaining stocks. Since our main interest is on the dynamics of prices and trade durations as well as on how the interest rate announcements affect these quantities, we only report the results for the quote revision and duration equations in Tables 2.5 and 2.6 respectively, using the whole sample of eleven RBA announcement weeks. The results for these stocks' trade attributes are qualitatively similar to those for stock NAB.

The dynamic behavior of prices and trade durations for the other Australian banking stocks is qualitatively similar to that for stock NAB. For example, Table 2.6 shows that the time between trades exhibits a persistent positive dependence structure and is positively related to past volatility while negatively linked to trading volumes (except for WBC). From Table 2.5, a trade has a significant impact on prices which is contributed by both trade direction and trading volume channels and negatively related to trade durations. Trades performed within one minute around the RBA announcement have higher price impact, although statistical significance is not obtained for WBC. However, there are some important differences with regard to the duration dynamics shown in Table 2.6. First, in contrast to other stocks it takes less (similar) time, instead of more time, for a trade in stock BEN (MQG) to occur when it is preceded by a purchase rather than a sale. Second, there is some evidence that time durations for stocks ANZ, WBC and MQG are significantly shorter within one minute around the RBA monetary policy releases (see the coefficients on aro_t), which is consistent with the duration pattern observed in Table 2.1 and implies a higher concentration of informed agents around the announcements that consequently increases the probability of trades in these stocks.

Table 2.5: Estimated return equation of the *Endo-VAR* model for banking stocks in eleven RBA announcement weeks in 2013

		ANZ		CBA		WBC		MQG		BEN	
		Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
const		**-0.030**	**(-5.81)**	0.005	(1.52)	-0.012	(-1.92)	0.014	(1.64)	**-0.181**	**(-4.93)**
open_t	Lag 0	0.004	(0.26)	**0.026**	**(2.39)**	**-0.041**	**(-2.31)**	0.029	(1.19)	0.172	(1.59)
bef_t	Lag 0	0.103	(1.06)	0.083	(1.81)	0.148	(1.80)	0.046	(0.53)	-0.396	(-0.98)
aro_t	Lag 0	0.158	(1.10)	0.092	(0.70)	0.134	(0.75)	-0.262	(-1.15)	1.422	(1.22)
aft_t	Lag 0	0.027	(0.44)	0.051	(1.42)	0.058	(0.87)	0.030	(0.40)	0.035	(0.11)
r_t	Lag 1	**-0.055**	**(-15.38)**	**-0.027**	**(-7.87)**	**-0.050**	**(-14.42)**	0.000	(-0.04)	**-0.056**	**(-7.26)**
	Lag 2	0.005	(1.38)	**0.033**	**(10.05)**	0.003	(0.84)	**0.027**	**(7.15)**	-0.002	(-0.34)
	$\Sigma_{1:p}$	**-0.033**	**(-3.86)**	**0.072**	**(10.08)**	**-0.030**	**(-3.73)**	**0.076**	**(9.27)**	**-0.100**	**(-5.68)**
$\|r_t\|$	Lag 1	0.002	(0.54)	-0.006	(-1.58)	-0.004	(-1.27)	**-0.012**	**(-2.77)**	-0.003	(-0.52)
	Lag 2	0.001	(0.24)	0.000	(0.02)	0.000	(-0.01)	-0.002	(-0.47)	-0.007	(-1.13)
	$\Sigma_{1:p}$	**0.016**	**(3.36)**	-0.002	(-0.49)	-0.005	(-1.03)	-0.011	(-1.75)	-0.014	(-1.46)
x_t^0	Lag 0	**1.040**	**(111.11)**	**0.595**	**(119.95)**	**1.085**	**(106.89)**	**1.205**	**(115.19)**	**3.633**	**(59.12)**
	Lag 1	**0.048**	**(5.51)**	0.009	(1.95)	0.008	(0.88)	**-0.038**	**(-3.73)**	0.091	(1.43)
	$\Sigma_{0:p}$	**0.991**	**(67.57)**	**0.490**	**(65.33)**	**0.958**	**(61.28)**	**0.952**	**(53.76)**	**3.011**	**(36.66)**
$x_t^0\text{open}_t$	Lag 0	**0.395**	**(16.11)**	**0.427**	**(26.10)**	**0.334**	**(12.75)**	**0.716**	**(20.34)**	**0.499**	**(2.91)**
$x_t^0\text{bef}_t$	Lag 0	0.152	(1.00)	-0.071	(-1.03)	-0.042	(-0.22)	-0.127	(-0.86)	-0.452	(-0.78)
$x_t^0\text{aro}_t$	Lag 0	**0.562**	**(2.87)**	**0.410**	**(3.09)**	0.120	(0.39)	**0.761**	**(2.15)**	**2.993**	**(2.13)**
$x_t^0\text{aft}_t$	Lag 0	**0.262**	**(2.59)**	0.098	(1.52)	0.158	(1.58)	**0.269**	**(2.14)**	-0.018	(-0.04)
$x_t^0\ln(T_t)$	Lag 0	**-0.044**	**(-16.69)**	**-0.021**	**(-14.60)**	**-0.034**	**(-12.11)**	**-0.011**	**(-3.62)**	**-0.092**	**(-6.26)**
	Lag 1	**-0.007**	**(-2.72)**	0.002	(1.55)	0.005	(1.93)	0.006	(1.90)	-0.005	(-0.33)
	$\Sigma_{0:p}$	**-0.056**	**(-9.62)**	**-0.008**	**(-2.65)**	**-0.015**	**(-2.37)**	0.008	(1.22)	**-0.104**	**(-3.43)**
v_t	Lag 0	**0.197**	**(90.17)**	**0.129**	**(93.66)**	**0.192**	**(82.66)**	**0.265**	**(80.38)**	**0.566**	**(50.73)**
	Lag 1	0.001	(0.28)	-0.001	(-0.72)	-0.004	(-1.81)	**-0.007**	**(-2.24)**	0.012	(1.05)
	$\Sigma_{0:p}$	**0.161**	**(47.40)**	**0.091**	**(44.20)**	**0.152**	**(41.93)**	**0.196**	**(35.39)**	**0.462**	**(27.97)**
$v_t\text{open}_t$	Lag 0	**0.085**	**(11.12)**	**0.098**	**(17.69)**	**0.092**	**(11.84)**	**0.179**	**(12.78)**	**0.121**	**(3.22)**
$v_t\text{bef}_t$	Lag 0	0.010	(0.27)	-0.020	(-1.16)	0.005	(0.09)	-0.029	(-0.66)	-0.003	(-0.02)
$v_t\text{aro}_t$	Lag 0	**0.175**	**(2.57)**	**0.111**	**(2.67)**	0.101	(0.92)	0.132	(1.00)	0.519	(1.41)
$v_t\text{aft}_t$	Lag 0	**0.075**	**(2.80)**	0.017	(0.94)	**0.079**	**(2.75)**	**0.143**	**(2.75)**	0.010	(0.10)
$v_t\ln(T_t)$	Lag 0	**-0.011**	**(-16.24)**	**-0.007**	**(-16.21)**	**-0.007**	**(-9.96)**	**-0.009**	**(-8.13)**	**-0.015**	**(-5.11)**
	Lag 1	**-0.002**	**(-2.62)**	**0.001**	**(2.02)**	0.000	(0.47)	**0.002**	**(2.14)**	0.001	(0.49)
	$\Sigma_{0:p}$	**-0.015**	**(-9.76)**	**-0.004**	**(-3.84)**	-0.002	(-1.25)	0.000	(-0.03)	-0.011	(-1.79)
$\ln(T_t)$	Lag 1	0.001	(0.61)	**0.003**	**(2.69)**	-0.002	(-1.18)	**0.005**	**(2.20)**	0.015	(1.66)
	Lag 2	0.001	(0.65)	0.000	(0.18)	-0.001	(-0.75)	0.000	(-0.15)	0.005	(0.53)
	$\Sigma_{1:p}$	0.003	(0.91)	**0.004**	**(2.50)**	-0.003	(-0.95)	0.002	(0.58)	**0.045**	**(3.07)**
Adj. R^2		0.171		0.161		0.155		0.175		0.192	
$\text{Wald}_{\text{diur}}$		**278.7**		**702.9**		**197.5**		**429.1**		**27.7**	
$\text{Wald}_{\text{time}}$		**622.0**		**1038.0**		**370.6**		**542.5**		**88.5**	
$Q_{15,\text{raw}}$		**6221.7**		**2721.7**		**4282.3**		**922.2**		**1993.8**	
$Q_{15,\text{resid}}$		23.6		**105.2**		**36.2**		**34.6**		**31.0**	
JB_{resid}		**64575.3**		**208084.3**		**60180.5**		**101836.4**		**19233.0**	

This table reports the coefficient estimates and Newey and West (1994) heteroskedasticity and autocorrelation consistent t-statistics (in parentheses) for the return equation of the *Endo-VAR* model specified in (2.8) for stocks ANZ, CBA, WBC, MQG and BEN.

$$r_t = \alpha^r + \beta^r D_t + \sum_{i=1}^{p} a_i^r r_{t-i} + \sum_{i=1}^{p} b_i^r |r_{t-i}| + \lambda^r D_t \otimes x_t + \sum_{i=0}^{p} \left[\gamma_i^r + \delta_i^r \ln(T_{t-i})\right] x_{t-i} + \sum_{i=1}^{p} c_i^r \ln(T_{t-i}) + u_t^r.$$

See Table 2.2 notes for definitions of the variables.

Table 2.6: Estimated trade duration equation of the *Endo-VAR* model for banking stocks in eleven RBA announcement weeks in 2013

		ANZ		CBA		WBC		MQG		BEN	
		Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
const		**-0.166**	**(-18.90)**	**-0.324**	**(-41.01)**	**-0.054**	**(-5.84)**	**-0.247**	**(-21.83)**	**0.764**	**(32.47)**
open_t	Lag 1	**-0.077**	**(-4.21)**	**-0.123**	**(-6.95)**	**-0.171**	**(-9.20)**	-0.248	(-10.81)	**-0.171**	**(-3.49)**
bef_t	Lag 1	0.170	(1.30)	0.126	(1.21)	0.082	(0.60)	0.020	(0.12)	0.083	(0.29)
aro_t	Lag 1	**-0.465**	**(-2.72)**	-0.228	(-1.43)	**-0.691**	**(-4.09)**	**-0.627**	**(-2.89)**	0.504	(1.35)
aft_t	Lag 1	**-0.163**	**(-2.29)**	-0.045	(-0.67)	-0.105	(-1.36)	0.035	(0.42)	0.037	(0.24)
r_t	Lag 1	-0.004	(-1.23)	0.004	(0.80)	**-0.016**	**(-5.41)**	0.003	(0.86)	0.001	(0.53)
	Lag 2	-0.002	(-0.41)	0.000	(-0.08)	**-0.013**	**(-3.33)**	**0.011**	**(2.84)**	0.005	(1.52)
	$\Sigma_{1:p}$	-0.005	(-0.53)	-0.007	(-0.58)	**-0.038**	**(-4.41)**	**0.023**	**(2.92)**	0.004	(0.50)
$\lvert r_t \rvert$	Lag 1	**0.234**	**(62.62)**	**0.523**	**(95.31)**	**0.273**	**(77.12)**	**0.337**	**(84.90)**	**0.067**	**(21.71)**
	Lag 2	**-0.204**	**(-51.01)**	**-0.328**	**(-54.82)**	**-0.199**	**(-51.10)**	**-0.171**	**(-39.93)**	**-0.063**	**(-19.27)**
	$\Sigma_{1:p}$	**-0.014**	**(-2.33)**	**0.033**	**(3.85)**	-0.001	(-0.22)	**0.107**	**(16.07)**	0.004	(0.78)
x_t^0	Lag 1	**0.038**	**(3.12)**	**0.023**	**(2.41)**	**0.059**	**(4.72)**	-0.018	(-1.42)	**-0.142**	**(-4.49)**
	Lag 2	-0.017	(-1.49)	-0.009	(-0.89)	**0.036**	**(3.02)**	-0.001	(-0.11)	**-0.119**	**(-3.84)**
	$\Sigma_{1:p}$	0.020	(1.07)	0.027	(1.86)	**0.126**	**(6.80)**	-0.003	(-0.15)	**-0.349**	**(-7.99)**
$x_t^0\text{open}_t$	Lag 1	-0.031	(-1.18)	-0.027	(-1.13)	**-0.069**	**(-2.60)**	-0.022	(-0.76)	0.032	(0.42)
$x_t^0\text{bef}_t$	Lag 1	0.413	(1.86)	0.056	(0.30)	-0.113	(-0.47)	-0.301	(-1.53)	-0.137	(-0.30)
$x_t^0\text{aro}_t$	Lag 1	**-0.509**	**(-2.14)**	0.040	(0.16)	-0.109	(-0.42)	-0.408	(-1.88)	-0.563	(-0.88)
$x_t^0\text{aft}_t$	Lag 1	0.157	(1.32)	-0.123	(-1.22)	-0.062	(-0.54)	0.070	(0.59)	**-0.608**	**(-2.09)**
$x_t^0\ln(T_t)$	Lag 1	**0.020**	**(5.71)**	**0.006**	**(2.22)**	**0.009**	**(2.46)**	-0.001	(-0.41)	**0.017**	**(2.10)**
	Lag 2	-0.002	(-0.50)	0.003	(1.01)	-0.004	(-1.00)	0.004	(1.25)	0.005	(0.64)
	$\Sigma_{1:p}$	**0.034**	**(4.64)**	**0.022**	**(3.65)**	0.007	(0.83)	0.006	(0.75)	0.022	(1.33)
v_t	Lag 1	**-0.015**	**(-4.89)**	**-0.009**	**(-2.83)**	-0.001	(-0.38)	**-0.040**	**(-9.05)**	**-0.039**	**(-6.02)**
	Lag 2	**-0.006**	**(-1.97)**	-0.005	(-1.58)	0.003	(1.05)	0.004	(0.92)	**-0.013**	**(-2.02)**
	$\Sigma_{1:p}$	**-0.024**	**(-4.98)**	**-0.021**	**(-4.47)**	**0.013**	**(2.70)**	-0.012	(-1.82)	**-0.043**	**(-4.57)**
$v_t\text{open}_t$	Lag 1	-0.015	(-1.80)	**-0.019**	**(-2.10)**	**-0.025**	**(-3.14)**	0.000	(-0.03)	-0.023	(-1.26)
$v_t\text{bef}_t$	Lag 1	0.102	(1.82)	0.047	(0.86)	-0.023	(-0.36)	**-0.155**	**(-2.09)**	-0.011	(-0.11)
$v_t\text{aro}_t$	Lag 1	-0.015	(-0.22)	0.130	(1.38)	0.026	(0.24)	-0.092	(-1.16)	-0.199	(-1.17)
$v_t\text{aft}_t$	Lag 1	0.064	(1.73)	0.027	(0.78)	0.011	(0.24)	-0.003	(-0.06)	-0.070	(-1.01)
$v_t\ln(T_t)$	Lag 1	0.002	(1.43)	**0.003**	**(2.69)**	-0.001	(-1.10)	**-0.005**	**(-3.29)**	0.003	(1.30)
	Lag 2	0.000	(0.01)	**0.004**	**(3.32)**	-0.001	(-0.94)	**0.004**	**(2.55)**	**-0.005**	**(-2.81)**
	$\Sigma_{1:p}$	**0.008**	**(3.71)**	**0.014**	**(6.29)**	-0.001	(-0.46)	0.001	(0.23)	**-0.007**	**(-2.03)**
$\ln(T_t)$	Lag 1	**0.180**	**(70.84)**	**0.186**	**(82.23)**	**0.173**	**(65.00)**	**0.173**	**(60.50)**	**0.195**	**(35.99)**
	Lag 2	**0.068**	**(26.92)**	**0.073**	**(31.23)**	**0.076**	**(28.66)**	**0.071**	**(24.38)**	**0.099**	**(18.25)**
	$\Sigma_{1:p}$	**0.338**	**(73.39)**	**0.387**	**(97.29)**	**0.362**	**(75.05)**	**0.374**	**(73.73)**	**0.422**	**(45.54)**
Adj. R^2		0.078		0.100		0.090		0.104		0.099	
$\text{Wald}_{\text{diur}}$		**45.1**		**64.8**		**108.8**		**134.8**		**22.8**	
$\text{Wald}_{\text{time}}$		**8029.3**		**12739.5**		**7862.9**		**7442.6**		**3327.9**	
$Q_{15,\text{raw}}$		**14708.3**		**24311.0**		**15032.9**		**14738.9**		**8677.8**	
$Q_{15,\text{resid}}$		**803.3**		**1354.4**		**795.7**		**1017.0**		**335.2**	
JB_{resid}		**10445.4**		**9150.2**		**12026.2**		**5882.9**		**7632.7**	

This table reports the coefficient estimates and Newey and West (1994) heteroskedasticity and autocorrelation consistent t-statistics (in parentheses) for the trade duration equation of the *Endo-VAR* model specified in (2.8) for stocks ANZ, CBA, WBC, MQG and BEN.

$$\ln(T_t) = \alpha^T + \beta^T D_{t-1} + \sum_{i=1}^{p} a_i^T r_{t-i} + \sum_{i=1}^{p} b_i^T |r_{t-i}| + \lambda^T D_{t-1} \otimes x_{t-1} + \sum_{i=1}^{p} \left[\gamma_i^T + \delta_i^T \ln(T_{t-i})\right] x_{t-i} + \sum_{i=1}^{p} c_i^T \ln(T_{t-i}) + u_t^T.$$

See Table 2.2 notes for definitions of the variables.

Overall, we find strong evidence of the endogeneity of trade durations, supporting the theories of Diamond and Verrecchia (1987) and Easley and O'Hara (1992). In particular, larger past trading volumes and smaller past volatility tend to shorten subsequent trade durations. Further, the price impact of a signed trade is positive, contributed by both trade direction and trading volumes, and negatively related to trade durations. Higher trading intensity or shorter trade duration is associated with not only a higher price impact of a trade but also more positive autocorrelation of trade characteristics. Moreover, the release of the RBA monetary policy decisions significantly affects the joint system of trade arrival times and the associated marks of interest, with trades executed within one minute around the announcement time typically being more informative about prices (i.e. having a larger price impact) and having shorter durations.

WACD-VAR **models**

Although the VAR framework is able to capture the internal dynamics of the joint system of durations and the marks, it does not seem to find changes in the pattern of trade durations around the interest rate announcements, especially for NAB and CBA. We now examine if the WACD model, which is widely used in the duration modeling literature, finds evidence of such changes. The estimated WACD(2,1) models for six Australian banking stocks in eleven RBA announcement weeks in 2013 are reported in Table 2.7. Panel A shows the results when the strict exogeneity assumption of trade durations is imposed (as in Dufour and Engle (2000) and Xu et al. (2006)), while Panel B reports the results when this assumption is relaxed. From both panels, all autoregressive parameter estimates for durations and conditional durations are highly significant and sum up to between 0.938 and 0.994 for all stocks, suggesting that the duration process is strongly persistent, which is consistent with the results from the estimated *Endo-VAR* models. The estimate for the Weibull parameter, θ, ranges between 0.44 and 0.51 and is significantly less than one. This implies an overdispersed distribution of trade durations - a stylized fact often observed in empirical work (Engle and Russell, 1998, Bauwens and Veredas, 2004, Renault et al., 2014), and it suggests that the use of an exponential distribution (i.e. $\theta = 1$), which implies equi-dispersion, to model trade durations is deficient.

From Panel B, most the parameter estimates for trade attributes and absolute returns (a proxy for volatility) are highly statistically significant, indicating that these variables are important predictors of trade durations, which in turn invalidates the strict exogeneity assumption of durations typically imposed in the literature. Consequently, the incorporation of these additional variables into the conditional duration model improves the log likelihood of the model markedly; and it is easy to verify that likelihood ratio (LR) tests strongly support the endogenous duration model. Consistent with the *Endo-VAR* model, it is the magnitude of price changes, rather than the direction of price adjustments, that is informative about the (conditional) durations of future trades. The lag-one coefficients of absolute returns are positive, implying that conditional trade durations increase with larger price adjustments. However, they are almost offset by the negative lag-two coefficients, suggesting that conditional trade durations tend to be higher the larger the change in quote revisions (i.e. the second order difference in prices). While conditional time durations tend to be larger if the last trade is a buy than a sell for four stocks ANZ, CBA, NAB and WBC, the reverse is observed for BEN, and there appears to be no relation between the conditional durations and trade signs for MQG. However, similar to the effect of quote revisions on durations, it appears that big changes in the direction of previous trades on average lengthen future durations. Conversely, larger trading volumes and bigger volume changes for all stocks (except WBC) tend to induce higher future trading intensity, and thus reduce time durations, which is in agreement with previous studies such as Bauwens and Giot (2000), Manganelli (2005) and Nowak and Anderson (2014).

Table 2.7: Estimated W-ACD(2,1) models for banking stocks in eleven RBA announcement weeks in 2013

Panel A: Exogenous-duration W-ACD(2,1) model

Stock	θ	α^T	a_1^T	a_2^T	b_1^T	b_2^T	$\gamma_{x^0,1}^T$	$\gamma_{x^0,2}^T$	$\gamma_{v,1}^T$	$\gamma_{v,2}^T$	ρ_1	ρ_2	ζ	λ_{op}^T	λ_{be}^T	λ_{ar}^T	λ_{af}^T	Log Lik.	$Q_{15,raw}$	$Q_{15,resid}$
ANZ	**0.467**	**0.013**	-	-	-	-	-	-	-	-	**0.122**	**-0.108**	**0.979**	**-0.002**	-0.007	**-0.034**	0.003	-47118.4	**13705.3**	**2158.5**
	(0.001)	**(0.001)**	-	-	-	-	-	-	-	-	**(0.002)**	**(0.002)**	**(0.001)**	**(0.000)**	(0.004)	**(0.007)**	(0.002)	-	-	-
CBA	**0.457**	**0.017**	-	-	-	-	-	-	-	-	**0.129**	**-0.107**	**0.967**	**-0.001**	-0.001	**-0.034**	0.003	-42621.6	**14516.2**	**1560.0**
	(0.001)	**(0.001)**	-	-	-	-	-	-	-	-	**(0.002)**	**(0.003)**	**(0.003)**	**(0.001)**	(0.004)	**(0.010)**	(0.002)	-	-	-
NAB	**0.477**	**0.011**	-	-	-	-	-	-	-	-	**0.107**	**-0.093**	**0.977**	**-0.002**	0.000	**-0.024**	0.003	-52905.1	**10221.7**	**1452.8**
	(0.001)	**(0.001)**	-	-	-	-	-	-	-	-	**(0.002)**	**(0.002)**	**(0.002)**	**(0.000)**	(0.004)	**(0.010)**	(0.002)	-	-	-
WBC	**0.475**	**0.014**	-	-	-	-	-	-	-	-	**0.109**	**-0.082**	**0.949**	**-0.002**	-0.002	**-0.066**	-0.001	-53651.8	**11006.7**	**1340.3**
	(0.001)	**(0.001)**	-	-	-	-	-	-	-	-	**(0.002)**	**(0.003)**	**(0.005)**	**(0.001)**	(0.007)	**(0.011)**	(0.003)	-	-	-
MQG	**0.438**	**0.017**	-	-	-	-	-	-	-	-	**0.115**	**-0.094**	**0.968**	**-0.002**	-0.007	**-0.038**	0.003	-13480.4	**9964.4**	**1116.1**
	(0.001)	**(0.001)**	-	-	-	-	-	-	-	-	**(0.002)**	**(0.003)**	**(0.002)**	**(0.001)**	(0.007)	**(0.010)**	(0.003)	-	-	-
BEN	**0.498**	**0.012**	-	-	-	-	-	-	-	-	**0.103**	**-0.082**	**0.958**	-0.001	-0.019	-0.017	0.004	-18768.8	**4328.6**	**683.3**
	(0.002)	**(0.001)**	-	-	-	-	-	-	-	-	**(0.004)**	**(0.004)**	**(0.005)**	(0.002)	(0.014)	(0.043)	(0.008)	-	-	-

Panel B: Endogenous-duration W-ACD(2,1) model

Stock	θ	α^T	a_1^T	a_2^T	b_1^T	b_2^T	$\gamma_{x^0,1}^T$	$\gamma_{x^0,2}^T$	$\gamma_{v,1}^T$	$\gamma_{v,2}^T$	ρ_1	ρ_2	ζ	λ_{op}^T	λ_{be}^T	λ_{ar}^T	λ_{af}^T	Log Lik.	$Q_{15,raw}$	$Q_{15,resid}$
ANZ	**0.471**	**0.012**	0.002	0.000	**0.176**	**-0.175**	**0.019**	**-0.020**	**-0.011**	**0.010**	**0.125**	**-0.110**	**0.977**	**-0.004**	-0.006	**-0.041**	0.002	-45312.1	**13705.3**	**1759.4**
	(0.001)	**(0.001)**	(0.002)	(0.002)	**(0.003)**	**(0.003)**	**(0.008)**	**(0.008)**	**(0.002)**	**(0.002)**	**(0.002)**	**(0.002)**	**(0.002)**	**(0.000)**	(0.004)	**(0.008)**	(0.002)	-	-	-
CBA	**0.465**	**0.019**	**0.007**	**-0.007**	**0.411**	**-0.408**	**0.022**	**-0.020**	**-0.008**	**0.005**	**0.133**	**-0.101**	**0.949**	**-0.004**	0.001	**-0.051**	0.001	-38684.3	**14516.2**	**935.7**
	(0.001)	**(0.001)**	**(0.003)**	**(0.003)**	**(0.005)**	**(0.005)**	**(0.007)**	**(0.007)**	**(0.002)**	**(0.002)**	**(0.002)**	**(0.003)**	**(0.005)**	**(0.001)**	(0.006)	**(0.012)**	(0.003)	-	-	-
NAB	**0.484**	**0.010**	0.000	0.002	**0.208**	**-0.205**	**0.024**	**-0.026**	**-0.010**	**0.010**	**0.113**	**-0.095**	**0.971**	**-0.004**	0.000	**-0.044**	0.003	-50127.6	**10221.7**	**1171.8**
	(0.001)	**(0.001)**	(0.002)	(0.002)	**(0.003)**	**(0.003)**	**(0.008)**	**(0.008)**	**(0.002)**	**(0.002)**	**(0.002)**	**(0.003)**	**(0.003)**	**(0.001)**	(0.005)	**(0.011)**	(0.002)	-	-	-
WBC	**0.482**	**0.016**	-0.004	**0.006**	**0.209**	**-0.210**	**0.040**	**-0.035**	-0.001	0.001	**0.114**	**-0.078**	**0.929**	**-0.003**	0.000	**-0.091**	-0.001	-51071.8	**11006.7**	**928.7**
	(0.001)	**(0.001)**	(0.002)	**(0.002)**	**(0.003)**	**(0.003)**	**(0.008)**	**(0.008)**	(0.002)	(0.002)	**(0.002)**	**(0.004)**	**(0.011)**	**(0.001)**	(0.009)	**(0.017)**	(0.005)	-	-	-
MQG	**0.448**	**0.011**	**0.007**	-0.003	**0.255**	**-0.250**	-0.015	0.012	**-0.030**	**0.030**	**0.118**	**-0.092**	**0.958**	**-0.009**	-0.007	**-0.050**	0.001	-10325.2	**9964.4**	**682.5**
	(0.001)	**(0.001)**	**(0.002)**	(0.002)	**(0.003)**	**(0.003)**	(0.009)	(0.009)	**(0.003)**	**(0.003)**	**(0.002)**	**(0.003)**	**(0.004)**	**(0.001)**	(0.008)	**(0.013)**	(0.004)	-	-	-
BEN	**0.506**	**0.003**	0.001	-0.001	**0.054**	**-0.050**	**-0.080**	**0.067**	**-0.027**	**0.026**	**0.109**	**-0.085**	**0.953**	**-0.008**	-0.005	-0.077	0.008	-18276.0	**4328.6**	**425.9**
	(0.002)	**(0.001)**	(0.002)	(0.002)	**(0.002)**	**(0.002)**	**(0.018)**	**(0.018)**	**(0.004)**	**(0.004)**	**(0.004)**	**(0.005)**	**(0.006)**	**(0.002)**	(0.014)	(0.043)	(0.009)	-	-	-

This table reports the estimates and robust standard errors (in parentheses) from the W-ACD(2,1) models in eleven RBA announcement weeks in 2013. Panel A shows the results for the Dufour and Engle (2000) exogenous-duration model, in which trade durations are independent of prices and trade attributes. Panel B shows the results for the WACD-VAR model (2.9) with the following W-ACD specification

$$\widetilde{T}_t = [\phi_t \Gamma(1+1/\theta)]\epsilon_t,$$

$$\ln(\phi_t) = \alpha^T + \sum_{i=1}^{2} a_i^T r_{t-i} + \sum_{i=1}^{2} b_i^T |r_{t-i}| + \sum_{i=1}^{2} \gamma_i^T x_{t-i} + \sum_{i=1}^{2} \rho_i \ln(\widetilde{T}_{t-i}) + \zeta \ln(\phi_{t-1}) + \lambda_{op}^T \text{open}_{t-1} + \lambda_{be}^T \text{bef}_{t-1} + \lambda_{ar}^T \text{aro}_{t-1} + \lambda_{af}^T \text{aft}_{t-1}.$$

$\widetilde{T}_t$ is the cubic-spline diurnally adjusted duration of the t-th trade. $\epsilon_t \overset{iid}{\sim} \text{Weibull}\left(scale = \frac{1}{\Gamma(1+1/\theta)}, shape = \theta\right)$. $\phi_t \Gamma(1+1/\theta)]$ is the conditional duration mean of the t-th trade. open_t is a dummy variable for the first 30 minutes of the trading day. r_t and $|r_t|$ are quote revisions and *absolute* quote revisions, respectively; x_t is a column vector of trade signs (x_t^0, which equals 1 for buys and -1 for sells) and volumes (v_t, defined as the signed logarithm of the ratio of the share volume to the prevailing quoted depth) of the t-th trade. bef_t, aro_t and aft_t are indicator variables identifying trades that are executed 5 minutes *before* (14:24:30-14:29:30), one minute *around* (14:29:30-14:30:30), and 10 minutes *after* (14:30:30-14:40:30) the RBA announcements. $Q_{15,\text{raw}}$ ($Q_{15,\text{resid}}$) is the Ljung-Box statistic associated with the null hypothesis of no autocorrelation up to order 15 in the raw (residual) diurnally adjusted duration series. Bold format denotes statistical significance at a 5% level.

There is evidence that the RBA interest rate announcements have significant impact on trade durations. In particular, trades in stocks other than BEN that occur within one minute around the announcement lead to higher future trading intensity, and hence are followed by trades that have shorter durations. This is in line with the findings of Nowak and Anderson (2014) that airline stocks in the U.S. are more frequently traded around the release of macroeconomic news. Meanwhile, the insignificant coefficients of bef_{t-1} and aft_{t-1}, which respectively signify 5 minutes before and 10 minutes after the interest rate release, suggest that there appears to be no information leak prior to the announcement and the information content of the news release is quickly absorbed within one minute. There is also evidence that trades performed at the market open tend to have shorter conditional durations than those executed at other times, even though trade durations have been diurnally adjusted using a cubic spline. Thus, it seems that intraday periodicities have not been totally removed by the spline. In addition, although most of the serial autocorrelation associated with adjusted trade durations is explained by the conditional duration equation, the residuals of the model are still strongly autocorrelated. A deeper lag structure may be required.

2.4.2 Impulse response analysis

We now examine how prices evolve if there are shocks to the trade, duration, and/or return equation(s) of the system at an event time t. Conditioning on all information up to the transaction time $t-1$, $\mathcal{I}_{t-1}$, the best guess of the value of the quote revision h periods after unexpected shocks to trade attributes, trade durations, and/or returns at time t is its conditional expectation $\mathbb{E}(r_{t+h}|\varepsilon_t = \varepsilon, \mathcal{I}_{t-1})$ given the shock vector ε_t, which is $(u_t^r, u_t^{x\prime}, u_t^T)'$ if the joint system is *Endo-VAR* and $(u_t^r, u_t^{x\prime}, \epsilon_t)'$ if the joint system is *WACD-VAR*. However, if there is no shock at time t, the quote revision h periods later is expected to be $\mathbb{E}(r_{t+h}|\mathcal{I}_{t-1})$. The impact of the unanticipated trade, duration, and/or return shocks at t on quote revisions after h periods is calculated as the difference between the two conditional expectations, denoted by $I_r(\cdot)$, when all other current and future shocks (for r_t, x_t and T_t) are integrated out. That is,

$$I_r(h,\varepsilon,\mathcal{I}_{t-1}) = \mathbb{E}(r_{t+h}|\varepsilon_t = \varepsilon,\mathcal{I}_{t-1}) - \mathbb{E}(r_{t+h}|\mathcal{I}_{t-1}), \tag{2.10}$$

defines a *generalized impulse response* function (GIRF) for quote revisions r_t which was initially proposed by Koop et al. (1996). GIRFs generated by a multivariate system typically depend on the past history $\mathcal{I}_{t-1}$ before the system is shocked and the size and sign of the shocks hitting the system at time t (Koop et al., 1996, Pesaran and Shin, 1998, Lanne and Nyberg, 2016). Since quote revisions and trade attributes are nonlinearly linked to time durations via either the *Endo-VAR* or the *WACD-VAR* system, the impulse response function specified in (2.10) is also nonlinear. To calculate $I_r(\cdot)$, we follow Koop et al. (1996) and Dufour and Engle (2000) to simulate all possible trajectories for $(r_{t+k}, x_{t+k}, T_{t+k}), k = 0,1,...,h$ that share the same initial information set, $\mathcal{I}_{t-1}$, with and without the shock(s) at t. The impulse response $I_r(\cdot)$ is computed by averaging the realizations obtained from all trajectories. Steps to compute $I_r(\cdot)$ are described in more details in Appendix A.

In the subsequent analysis, we will examine how prices of each stock evolve under the following two scenarios: (1) there is an unanticipated purchase (i.e. sign shock = +1, while other shocks including return, volume and duration shocks are integrated out); and (2) there is an unanticipated purchase with a one standard deviation duration shock. We consider both positive and negative duration shocks in the latter scenario.[14,15] In order to see how the release of RBA interest rate decisions affects prices, we shock the system on days with and without the monetary policy announcements. Since the GIRFs of a nonlinear system are dependent on the state of the system at time $t-1$ before being shocked (Koop et al., 1996, Pesaran and Shin, 1998, Lanne and Nyberg, 2016) and there are many RBA and non-RBA announcement days in the current sample, we shock the joint system of quote revisions, trade attributes and trade durations conditioning on a *hypothetical* average RBA announcement time (RBAAT) and

[14]Since the *Endo-VAR* system assumes an additive error model for durations, while the *WACD-VAR* framework differs and specifies durations with a multiplicative error model, a one standard deviation positive (negative) duration shock is defined as $\widehat{\sigma}^T_{Endo-VAR}$ ($-\widehat{\sigma}^T_{Endo-VAR}$) for the former system, but as $\widehat{\sigma}^T_{WACD-VAR}$ ($1/\widehat{\sigma}^T_{WACD-VAR}$) for the latter.

[15]These two scenarios enable us to see how an unexpected buy affects prices if it arrives as quickly as expected, slower than expected, or more quickly than expected.

non-RBAAT histories. A hypothetical average RBAAT (non-RBAAT) history for each stock is defined as an equally weighted average of all histories right before the interest rate release time, 14:30:00, on the eleven RBA (forty-three non-RBA) announcement days in the current sample for that stock. Conditioning on the history and shock vectors, the simulation is conducted for $h = 300$ steps into the future with $N = 10,000$ repetitions.

The cumulative quote changes for stock NAB following an unanticipated purchase with either (i) no duration shock, (ii) a positive one standard deviation duration shock, or (iii) a negative one standard deviation duration shock are plotted in Panels (a), (b), and (c) of Figure 2.1, respectively.[16] In addition to reporting the cumulative price impact produced by the *Endo-VAR* and *WACD-VAR* systems, we also chart those for the augmented Dufour and Engle (2000) exogenous-duration VAR model (i.e. with volume incorporated) for comparison.[17] These impulse responses are pictured in both transaction time and calendar time starting from the conditioning trade that occurs immediately before 14:30:00 of the average RBAAT or non-RBAAT history. To convert the cumulative price impact from transaction time to calendar time, we follow Dufour and Engle (2000) to exploit the simulated trade duration series under the "shock" scenario (discussed in Points (A.2) and (A.3) in the Appendix A) to sample the cumulative quote changes every five seconds, and then we compute averages.

Panel (a) of Figure 2.1 reveals that, for all models, after an unexpected purchase

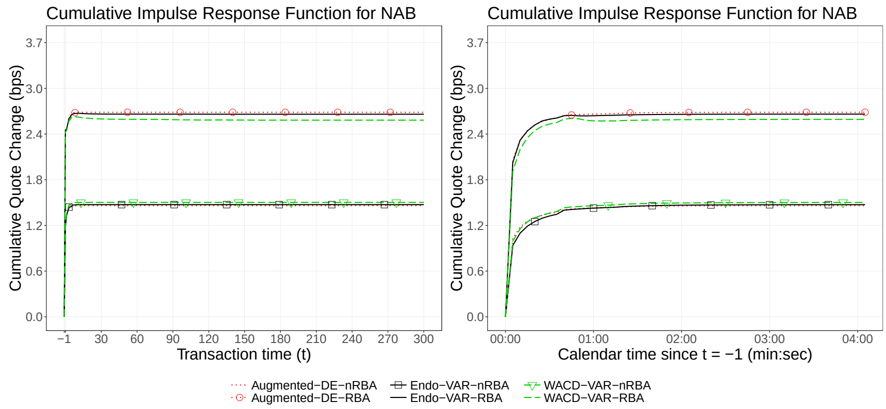
[16]We employ the estimated joint system using the whole sample of eleven RBA announcement weeks in all simulation experiments, and simply plot the mean responses in keeping with other related literature. Caution is required when interpreting these responses, given that the 5%-95% percentile bands of simulated responses often include zero.

[17]The original Dufour and Engle (2000) VAR framework contains only two equations for quote revisions and trade signs and does not include RBA dummy variables. However, an augmented Dufour and Engle (2000) framework that incorporates another equation for trading volumes as well as the RBA dummy variables is employed. By allowing all three systems to have comparable trade attribute information (i.e. sign and size), the differences amongst the cumulative price impacts obtained from these systems can be attributed to the differences in their treatments of durations and/or to the effects of RBA announcements. The augmented Dufour and Engle (2000) model is given by

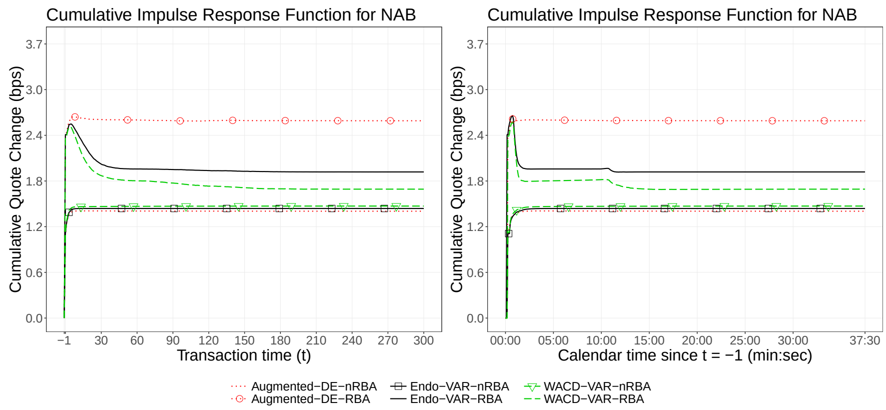
$$r_t = \alpha^r + \sum_{i=1}^{5} a_i^r r_{t-i} + \lambda^r S_t x_t + \sum_{i=0}^{5} \left[\gamma_i^r + \delta_i^r \ln(T_{t-i})\right] x_{t-i} + u_t^r,$$
$$x_t = \alpha^x + \sum_{i=1}^{5} a_i^x r_{t-i} + \lambda^x S_{t-1} x_{t-1} + \sum_{i=1}^{5} \left[\gamma_i^x + \delta_i^x \ln(T_{t-i})\right] x_{t-i} + u_t^x,$$
$$\widetilde{T}_t = T_t/\varphi(t) = [\phi_t \Gamma(1 + 1/\theta)]\epsilon_t,$$
$$\ln(\phi_t) = \alpha^T + \rho_1 \ln(\widetilde{T}_{t-1}) + \rho_2 \ln(\widetilde{T}_{t-2}) + \zeta_1 \ln(\phi_{t-1}) + \lambda^T D_{t-1}.$$

prices initially increase considerably and then taper off relatively quickly after about 10 transactions or 1 minute. As expected, prices respond more strongly to the unanticipated buy at around the announcement time on an average RBA announcement day than at the equivalent time on days when there is no interest rate release. In particular, while the unanticipated purchase performed around 14:30:00 in the average non-RBAAT history raises prices of stock NAB by about 1.5 bps in the long run, a twice-as-large permanent price increase (of about 2.7 bps) results from the same trade in the average RBAAT history, which suggests that trades at around the RBA announcement time are more informative about the price formation process. The result is consistent with the fact that there is higher trading intensity (i.e. shorter trade duration) around the release of the monetary policy news at 14:30:00 on the RBA days than around the corresponding time window on the non-RBA days (see Table 2.1). Since a higher trading rate or shorter duration implies a higher probability of informed traders in the market (Easley and O'Hara, 1992, Dufour and Engle, 2000), trades around RBA announcements have larger impact on prices. For each average history, there are negligible differences in the cumulative quote revisions produced by three models that augment the information of trade arrival times, namely the *Endo-VAR*, *WACD-VAR* and extended Dufour and Engle (2000) models. Given that there is no duration shock to the systems and the main differences amongst these three models lie in their treatment of durations, this result is not surprising.

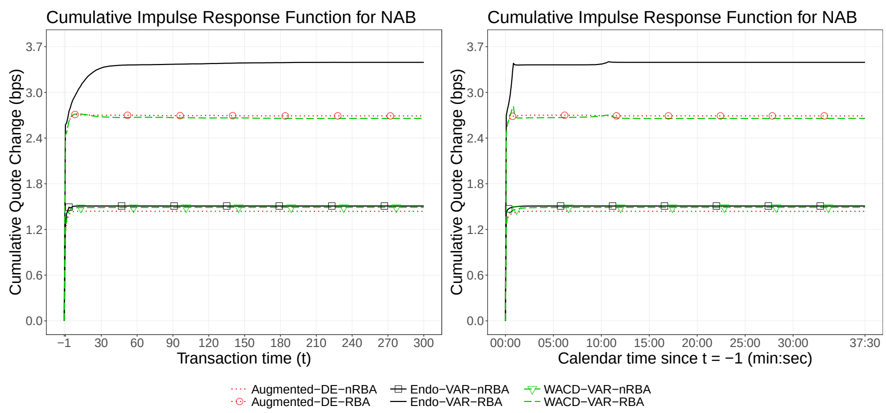
Figure 2.1: Impulse response functions for quote revisions of stock NAB



(a) No duration shock



(b) A positive one standard deviation duration shock



(c) A negative one standard deviation duration shock

Note: This figure plots the cumulative impulse response of quote revisions after an unexpected buy with either (i) no duration shock (Panel (a)), (ii) a positive one standard deviation duration shock (Panel (b)), or (iii) a negative one standard deviation duration shock (Panel (c)). Conditioning on a trade that occurred right before 14:30:00 of an **average** day on which the RBA made or did not make announcements (i.e. conditioning on the average RBAAT or non-RBAAT history), we simulate and compute 10,000 impulse response functions for 300 transactions into the future. Averages of cumulative quote changes at each step are calculated and plotted in both transaction time (left graphs) and calendar time (right graphs, for the first 4 minutes in Panel (a)) since the conditioning transaction (i.e. $t = -1$). Cumulative price impacts obtained from the *Endo-VAR* model (2.8) and *WACD-VAR* model (2.9) are pictured. We also chart the cumulative quote changes for the augmented Dufour and Engle (2000) (DE) exogenous-duration VAR model (i.e. with volume incorporated) for comparison.

The impact on prices when the aforementioned joint systems are disturbed with a duration shock from an unexpected purchase is depicted in Panels (b) and (c) of Figure 2.1. Overall, the comparison of the cumulative quote revisions for stock NAB conditioning on the average RBAAT and non-RBAAT histories remains qualitatively unchanged in the sense that an unexpected buy with a duration shock that occurs right before the monetary policy release conveys more information about prices and hence has higher price impact than does a comparable buy transacted on a no-news day. In addition, there are almost no changes to the shape and level of the cumulative GIRFs for quote revisions produced by the three time-augmented VAR systems based on the average non-RBAAT history, either with or without the duration shock. It appears that the informativeness of trade durations about prices is negligible for trades executed at around 14:30:00 on non-RBA days, during which the market is relatively tranquil (see Table 2.1). This lends support to the Easley and O'Hara (1992) theory which demonstrates that long trade durations neither imply the appearance of informed traders nor news, and hence they have little impact on prices.

Interestingly, conditioning on the average RBAAT history, the cumulative price impact functions of an unexpected purchase with a duration shock implied by the augmented Dufour and Engle (2000) model are almost the same as those under no duration shock. Although it highlights a significant difference in the response of prices to an unanticipated trade that comes from different trading histories (e.g. active versus inactive histories), the augmented Dufour and Engle (2000) model seems to suggest a minimal role for duration shocks in explaining prices, once the history before the shocks has been taken into account. This might be a consequence of the exogeneity assumption of trade durations imposed by the model.

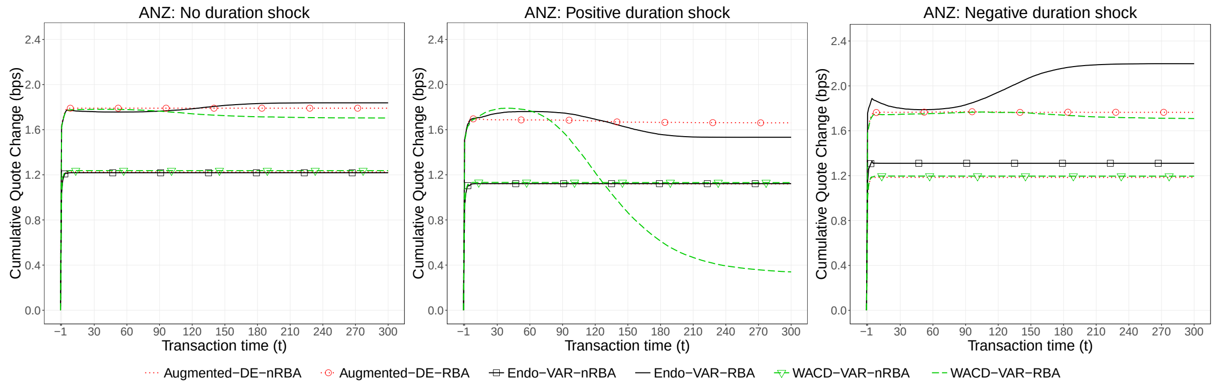
When the exogeneity of durations is relaxed, we observe some differences in the shape and/or level of the cumulative quote revisions around the RBA announcements. In particular, when the duration shock is positive and an unexpected trade arrives slower than expected, the two endogenous-duration models (i.e. *Endo-VAR* and *WACD-VAR*) show an initial surge in the cumulative price impact, followed by a gradual decline to the equilibrium level (which is about 0.9 bps lower than the steady state when there is no duration shock) after about 60 transactions or about 11 minutes. When the duration shock is negative and an unanticipated trade occurs more quickly than expected, prices adjust more strongly according to the *Endo-VAR* model, with the accumulation of quote revisions of roughly 3.4 bps in the long run (approximately 0.7 bps higher than that under no duration shock). Surprisingly, such a large price increase is not observed for the *WACD-VAR* system. Generally, the result suggests an overall negative relationship between trade durations and quote revisions, even after controlling for the history: given the average RBAAT history, trades appear to possess a richer (poorer) information content about prices when they arrive sooner (later) than expected, which is in conformance with Easley and O'Hara's (1992) theory. However, this result is obtained only when trade durations are endogenously determined.

The cumulative impulse response functions of quote revisions to different shock scenarios for the remaining banking stocks are plotted in Figure 2.2. Consistent with the conventional wisdom, Figure 2.2 shows that the more liquid a stock, the smaller the price impact of a trade (compare the scales on the vertical axis of the plots). In general, the long-run price impact functions for other banking stocks exhibit qualitatively similar features to those for stock NAB. Specifically, the cumulative quote changes of an unanticipated purchase executed around the RBA announcements is generally higher than that of a comparable trade occurring at a similar time on a no-news day (except for stock WBC). Moreover, when there is no duration shock, the differences in the long-run price impact of an unanticipated purchase produced by the time-augmented VAR models (i.e. the *Endo-VAR*, *WACD-VAR* and extended Dufour and Engle (2000) models) conditioning on the same history are negligible, except for stock BEN (see the left plots). However, when an unexpected buy is accompanied by a one standard deviation duration shock, prices typically respond less (more) strongly (i.e. the long run price impact is lower (higher)) when the shock to durations is positive (negative) than do they without the duration shock, except for stock MQG (see the middle (right) plots). This observation is only obtained when one conditions on an average history for an RBA announcement day and utilizes a joint specification that allows for the endogeneity of trade durations such as the *Endo-VAR* or *WACD-VAR* model.[18]

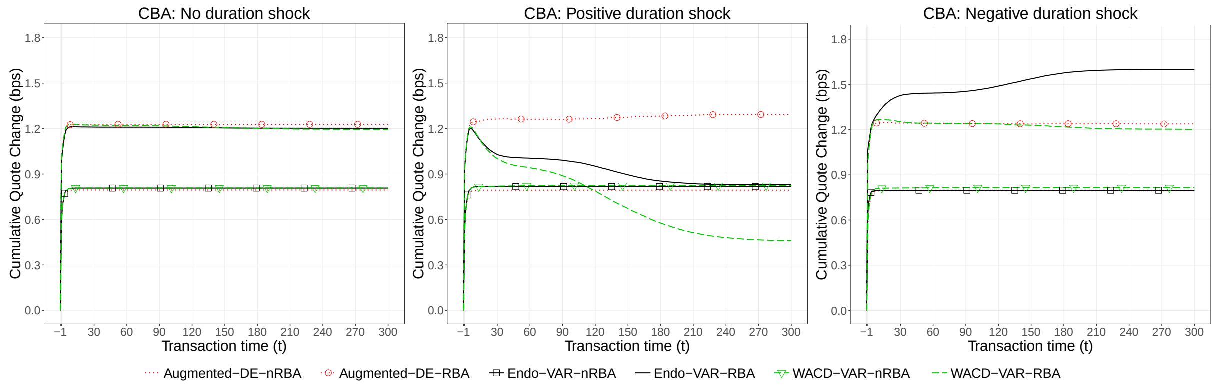
Overall, the impulse response analysis for quote revisions confirms the previous findings in the literature that the time of trade arrivals conveys important information about prices (Diamond and Verrecchia, 1987, Easley and O'Hara, 1992) and that trades have a larger price impact when the time durations between trades are shorter (Easley and O'Hara, 1992, Dufour and Engle, 2000). In addition, we find that trades transacted around the release of monetary policy news possess more important information about prices and have larger price impacts than do comparable trades on non-RBA days. If there is no duration shock to the system, the cumulative price impact of an unanticipated trade is almost the same, regardless of whether or not trade durations are endogenously modeled. However, when the unexpected trade is accompanied by a duration shock, the long-run price impact of the trade whose duration is treated as exogenous is quite different, in terms of shape and/or level, to that when its duration is endogenously determined. In particular, after controlling for the trading history prior to the interest rate announcements, the permanent price impact of a trade tends to be higher (lower) when there is a negative (positive) duration shock if trade durations are endogenous, and yet it is almost the same if durations are assumed to be exogenous.

[18] Note, however, that a negative duration shock leading to higher long-run price impact is only obtained using the *Endo-VAR* model.

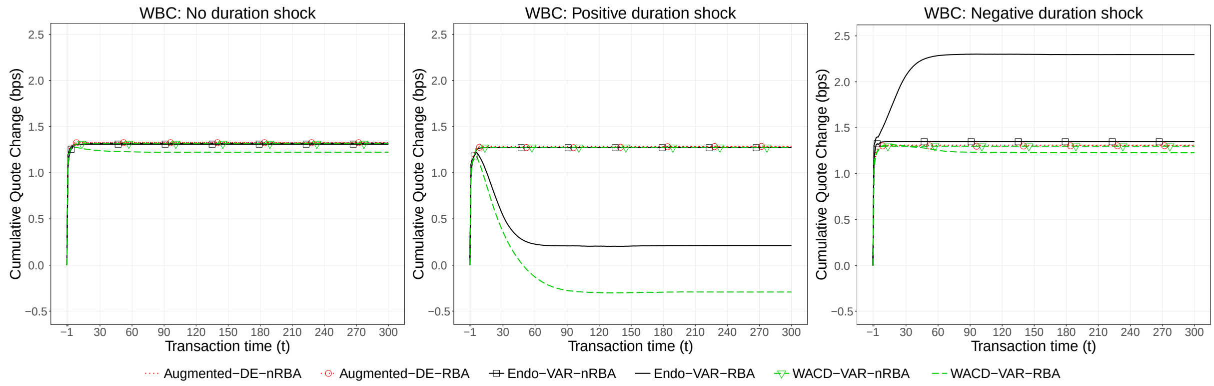
Figure 2.2: Impulse response functions for quote revisions of banking stocks



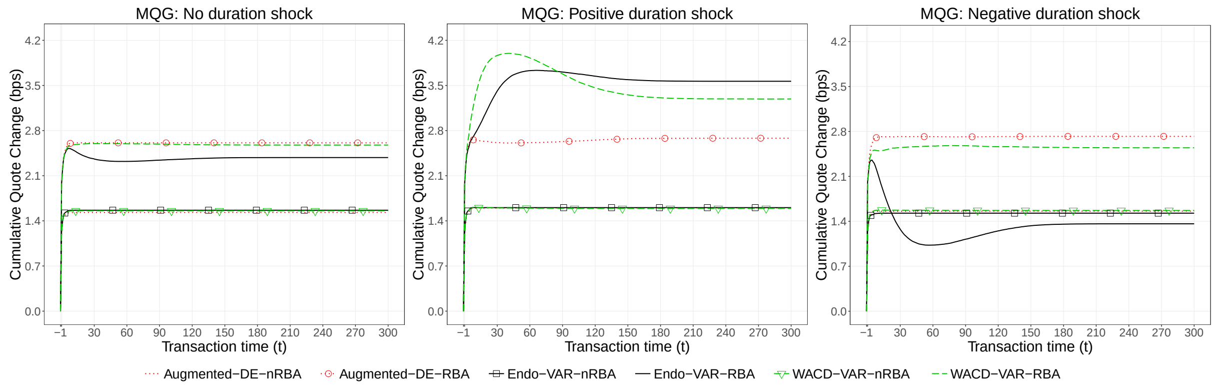
(a) ANZ



(b) CBA



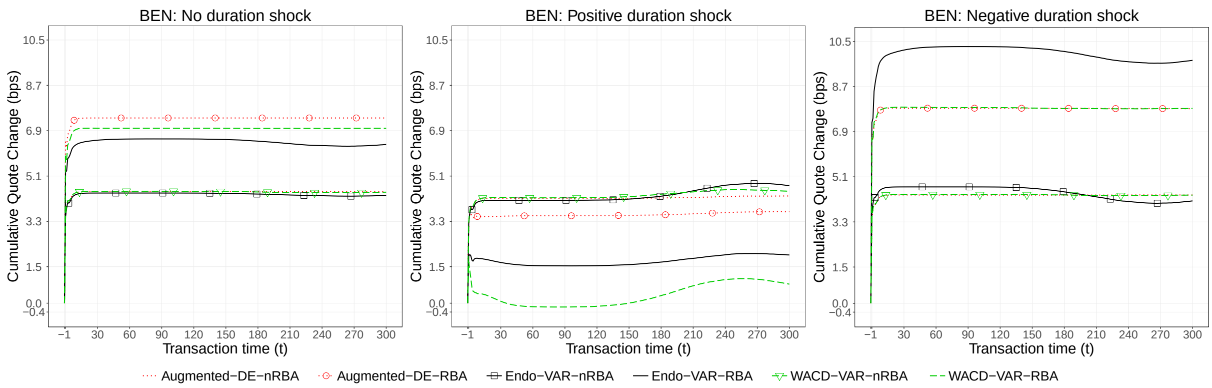
(c) WBC



(d) MQG

(Figure continued on next page)

Figure 2.2 – *continued*



(e) BEN

Note: This figure plots the cumulative impulse response of quote revisions after an unexpected buy with either (i) no duration shock (left plots), (ii) a positive one standard deviation duration shock (middle plots), or (iii) a negative one standard deviation duration shock (right plots) for 5 Australian banking stocks, namely ANZ, CBA, WBC, MQG and BEN. Conditioning on a trade that occurred right before 14:30:00 of an **average** day on which the RBA made or did not make announcements (i.e. conditioning on the average RBAAT or non-RBAAT history), we simulate and compute 10,000 impulse response functions for 300 transactions into the future. Averages of cumulative quote changes at each step are calculated and plotted in transaction time since the conditioning transaction (i.e. $t = -1$). Cumulative price impacts obtained from the *Endo-VAR* model (2.8) and *WACD-VAR* model (2.9) are pictured. We also chart the cumulative quote changes for the augmented Dufour and Engle (2000) (DE) exogenous-duration VAR model (i.e. with volume incorporated) for comparison.

2.4.3 Forecast error variance decomposition analysis

The previous impulse response analysis demonstrates that both trade durations and trade attributes convey important information about prices to the market. However, the impulse response methodology does not directly estimate the relative importance of each trade attribute in the overall price formation process. We now quantify their relative importance, which helps answer the question of whether a trade duration contributes more to the process of price formation than other trade attributes by decomposing the forecast error variance of quote revisions into portions that are accounted for by innovations in each trade characteristic, including its duration.

Forecast error variance decomposition (FEVD) of a weakly stationary linear VAR model is often computed from an infinite-order vector moving average (VMA) representation of the model with orthogonal shocks, assuming that suitable identification restrictions to recover the structural shocks from the reduced-form errors are available. However, since our *Endo-VAR* and *WACD-VAR* models, as well as the original and volume-augmented Dufour and Engle (2000) models, are nonlinear multivariate systems for which a VMA equivalent does not exist, the traditional orthogonalized FEVD cannot be applied. Instead, we employ the generalized FEVD (GFEVD) method proposed by Lanne and Nyberg (2016) that mimics the traditional orthogonalized FEVD by replacing the orthogonal impulse response functions with the GIRFs that are calculated based on the notion that only one equation of the multivariate system is shocked at a time. By construction and similar to the traditional orthogonalized FEVD, Lanne and Nyberg's (2016) GFEVD features a nice property that the proportions of the forecast error variance of the h-period forecast of a variable that are accounted for by innovations in all variables in the system always sum to unity, facilitating the economic interpretation.[19] Conditioning on a history $\mathcal{I}_{t-1}$, Lanne and Nyberg (2016) define the contribution of shocks to variable i to the forecast error variance of the h-period fore-

[19] Lanne and Nyberg (2016) modify the original GFEVD proposed by Pesaran and Shin (1998) (which was developed for a linear Gaussian VAR model) to address a shortcoming of the latter that is that the forecast error variance proportions generally do not add up to 1, as a consequence of the potential contemporaneous correlatedness amongst the reduced-formed innovations. Moreover, Lanne and Nyberg's (2016) GFEVD can be applied to any linear or nonlinear, Gaussian or non-Gaussian model for which GIRFs can be computed.

cast of variable j, denoted by $\lambda_{i\to j,\mathcal{I}_{t-1}}(h)$, in a K-dimensional multivariate system of the form $y_t = G(y_{t-1}, y_{t-2}, \cdots, y_{t-p}; \mu) + \eta_t$, where $G(\cdot)$ is some linear or nonlinear function characterized by the parameter vector μ, as

$$\lambda_{i\to j,\mathcal{I}_{t-1}}(h) = \frac{\sum_{k=0}^{h} I_j(k, \eta_{i,t} = \delta_i, \mathcal{I}_{t-1})^2}{\sum_{i=1}^{K}\sum_{k=0}^{h} I_j(k, \eta_{i,t} = \delta_i, \mathcal{I}_{t-1})^2}, \qquad i, j = 1, 2, \cdots, K, \tag{2.11}$$

and $I_j(k, \eta_{i,t} = \delta_i, \mathcal{I}_{t-1}) = \mathbb{E}\left(y_{j,t+k} | \eta_{i,t} = \delta_i, \mathcal{I}_{t-1}\right) - \mathbb{E}\left(y_{j,t+k} | \mathcal{I}_{t-1}\right), k = 0, 1, 2, \cdots, h,$ is the GIRF of the j-th variable k periods after a shock at time t of size δ_i to the i-th variable, given the past history $\mathcal{I}_{t-1}$, where all other contemporaneous and future shocks are integrated out; and K is the number of variables in the system. The GFEVD is often calculated by averaging $\lambda_{i\to j,\mathcal{I}_{t-1}}(h)$ over shocks δ_i that are bootstrapped from the residuals, and over all histories $\mathcal{I}_{t-1}$. However, if interest is drawn to a particular subset of shocks and/or histories, the conditional GFEVD can also be computed.

We note that Lanne and Nyberg's (2016) GFEVD is different from the efficient price variance decomposition (Hasbrouck, 1991b) and the information share methodology (Hasbrouck, 1995), which are widely used in the microstructure literature to compare the information contributions of different trader groups or different markets to price discovery (e.g. Barclay et al., 2003, Hendershott and Riordan, 2011, Benos and Sagade, 2016, Brogaard et al., 2018). In both Hasbrouck's methods, the observed price or midpoint is written as the sum of an unobserved random walk (which is equated with the permanent efficient price) and an unobserved stationary component (considered as transient noise). The total price discovery is defined as the variance of the efficient price innovations, whereas the transient disturbance, which might be correlated with the efficient price, is effectively ignored. Hasbrouck's methodologies rely on a critical assumption that there exists a ***linear*** stationary VAR that links price changes or returns with other trade-related information (Hasbrouck, 1991b) or a ***linear*** vector error correction model (VCEM) that connects different price series closely related to a single security (Hasbrouck, 1995). Consequently, Hasbrouck's price discovery decomposition can be straightforwardly calculated from a VMA equivalent of the linear VAR or VECM. However, if the VAR or VECM is nonlinear such that its VMA representation cannot be obtained, it is not clear how Hasbrouck's price discovery decomposition can be computed.

In contrast, GFEVD decomposes the forecast error variance (FEV) of a variable, such as returns, into portions that are accounted for by innovations in each variable of the system. Since returns are defined as changes in prices, the FEV of returns effectively captures the FEV of both the efficient price and transient noise, and is consequently different from the variance of the efficient price innovations. As Lanne and Nyberg's (2016) GFEVD can be computed for nonlinear VARs while Hasbrouck's measures are inapplicable in our context, we employ the former in the subsequent analysis. However, in order to prevent any confusion with the price discovery literature we interpret a GFEVD result as the relative informativeness or importance of a variable (e.g. durations) to another (e.g. returns), as in the traditional FEVD literature, and deliberately avoid saying "the contribution to the price discovery process".

Steps to compute the GFEVD, conditioning on the average RBAAT and non-RBAAT histories, for various multivariate systems discussed in this chapter are detailed in Appendix B. The GFEVD results of quote revisions, conditioning on the average RBAAT history (up to $h = 50$ future transactions) and the average non-RBAAT history (up to $h = 20$), for six Australian banking stocks for the *Endo-VAR* and *WACD-VAR* models are reported in Tables 2.8 and 2.9, respectively. Each entry in these tables, reported in %, is computed according to equation (2.11) by averaging over $M = 1,000$ vectors of shocks bootstrapped from the estimated residuals; for each shock vector, the GIRF $I_j(\cdot)$ in equation (2.11) is calculated from $N = 1,000$ simulated repetitions. We also compute the corresponding results for the augmented Dufour-Engle (i.e. with volume) and original Dufour-Engle (without volume) models for comparison.[20]

[20]The specification of the augmented Dufour-Engle model is shown in Footnote 17. Meanwhile, the original Dufour-Engle model, which does not include trading volumes and RBA dummy variables, is given by

$$r_t = \alpha^r + \sum_{i=1}^{5} a_i^r r_{t-i} + \lambda^r \text{open}_t x_t^0 + \sum_{i=0}^{5} \left[\gamma_i^r + \delta_i^r \ln(T_{t-i})\right] x_{t-i}^0 + u_t^r,$$
$$x_t^0 = \alpha^x + \sum_{i=1}^{5} a_i^x r_{t-i} + \lambda^x \text{open}_{t-1} x_{t-1}^0 + \sum_{i=1}^{5} \left[\gamma_i^x + \delta_i^x \ln(T_{t-i})\right] x_{t-i}^0 + u_t^x,$$
$$\widetilde{T}_t = T_t/\varphi(t) = [\phi_t \Gamma(1+1/\theta)]\epsilon_t,$$
$$\ln(\phi_t) = \alpha^T + \rho_1 \ln(\widetilde{T}_{t-1}) + \rho_2 \ln(\widetilde{T}_{t-2}) + \zeta_1 \ln(\phi_{t-1}) + \lambda^T \text{open}_{t-1}.$$

We employ a logarithmic WACD model, rather than a WACD model as in Dufour and Engle (2000), to ensure the positivity of the conditional durations.

Table 2.8: Generalized Forecast Error Variance Decomposition for **Returns** conditioning on the average **RBAAT** history

Stock	**ANZ**				**CBA**				**NAB**				**WBC**				**MQG**				**BEN**			
Response	**returns**				**returns**				**returns**				**returns**				**returns**				**returns**			
Impulse	ret	sign	vol	dur	ret	sign	vol	dur	ret	sign	vol	dur	ret	sign	vol	dur	ret	sign	vol	dur	ret	sign	vol	dur
Horizon																								
A: Endo-VAR model																								
0	33.18	43.08	23.65	0.10	30.89	49.51	19.11	0.48	27.16	46.71	25.75	0.38	40.99	35.85	22.52	0.64	31.20	52.49	15.24	1.07	30.20	40.14	25.01	4.66
1	33.93	41.49	24.48	0.11	31.07	48.46	19.90	0.57	28.82	45.05	25.58	0.55	40.69	34.34	23.84	1.14	31.14	51.57	15.48	1.82	31.01	38.25	25.70	5.04
2	33.96	41.46	24.48	0.11	31.02	48.47	19.85	0.66	28.82	44.97	25.55	0.67	40.53	34.10	23.76	1.61	30.94	51.33	15.33	2.40	30.89	38.20	25.53	5.38
3	33.96	41.46	24.47	0.11	30.99	48.47	19.82	0.73	28.79	44.93	25.52	0.77	40.38	33.92	23.68	2.01	30.80	51.02	15.22	2.96	30.86	38.11	25.49	5.54
10	33.94	41.43	24.46	0.18	30.89	48.25	19.74	1.13	28.64	44.61	25.35	1.40	39.58	32.88	23.18	4.35	30.13	49.52	14.76	5.60	30.81	38.06	25.43	5.70
20	33.93	41.41	24.45	0.21	30.84	48.12	19.70	1.34	28.55	44.44	25.26	1.75	38.91	32.01	22.73	6.35	29.67	48.51	14.44	7.38	30.80	38.05	25.43	5.71
40	33.93	41.41	24.45	0.22	30.82	48.10	19.69	1.39	28.53	44.40	25.25	1.82	38.58	31.57	22.49	7.36	29.50	48.14	14.32	8.04	30.80	38.05	25.43	5.72
45	33.93	41.41	24.45	0.22	30.82	48.10	19.69	1.39	28.53	44.40	25.25	1.82	38.58	31.55	22.48	7.39	29.50	48.13	14.32	8.05	30.80	38.05	25.43	5.72
50	33.93	41.41	24.45	0.22	30.82	48.10	19.69	1.39	28.53	44.40	25.25	1.82	38.57	31.55	22.48	7.40	29.50	48.13	14.32	8.05	30.80	38.05	25.43	5.72
B: WACD-VAR model																								
0	33.25	42.19	24.55	0.01	30.36	49.02	20.59	0.04	28.09	47.78	24.08	0.05	41.07	36.32	22.54	0.07	31.49	54.35	13.97	0.20	27.84	44.25	25.94	1.96
1	33.97	40.66	25.36	0.01	30.57	47.95	21.43	0.05	29.77	46.24	23.90	0.09	40.96	34.99	23.94	0.11	31.57	53.82	14.26	0.35	28.86	41.53	27.36	2.25
2	34.01	40.63	25.35	0.01	30.55	47.99	21.40	0.06	29.79	46.19	23.90	0.11	40.96	34.95	23.95	0.14	31.49	53.86	14.19	0.47	28.76	41.57	27.18	2.49
3	34.01	40.63	25.35	0.01	30.53	48.02	21.38	0.07	29.78	46.19	23.89	0.14	40.94	34.94	23.95	0.17	31.45	53.80	14.17	0.59	28.74	41.48	27.15	2.63
10	34.00	40.62	25.35	0.03	30.50	48.01	21.36	0.13	29.73	46.09	23.85	0.33	40.82	34.81	23.89	0.48	31.32	53.52	14.08	1.09	28.69	41.42	27.09	2.81
20	34.00	40.62	25.35	0.04	30.49	47.98	21.35	0.19	29.70	46.04	23.83	0.42	40.69	34.63	23.82	0.86	31.28	53.42	14.05	1.25	28.69	41.41	27.08	2.82
40	34.00	40.61	25.35	0.04	30.48	47.96	21.34	0.21	29.70	46.03	23.83	0.44	40.60	34.48	23.76	1.16	31.27	53.41	14.05	1.28	28.69	41.41	27.08	2.82
45	34.00	40.61	25.35	0.04	30.48	47.96	21.34	0.22	29.70	46.03	23.83	0.44	40.59	34.47	23.75	1.19	31.27	53.40	14.05	1.28	28.69	41.41	27.08	2.83
50	34.00	40.61	25.34	0.04	30.48	47.96	21.34	0.22	29.70	46.03	23.83	0.44	40.58	34.46	23.75	1.21	31.27	53.40	14.05	1.28	28.69	41.41	27.08	2.83
C: Augmented Dufour-Engle model																								
0	33.98	40.61	25.41	0.00	29.99	48.97	21.03	0.02	29.62	46.88	23.51	0.00	42.21	34.70	23.09	0.01	32.49	51.31	16.20	0.00	26.32	44.25	29.39	0.04
1	34.68	39.10	26.21	0.00	30.16	47.87	21.95	0.02	31.37	45.24	23.39	0.00	42.04	33.37	24.57	0.01	32.61	50.87	16.51	0.00	27.57	41.61	30.76	0.06
2	34.72	39.07	26.21	0.00	30.14	47.92	21.92	0.02	31.39	45.20	23.40	0.00	42.04	33.35	24.60	0.01	32.55	50.98	16.46	0.01	27.55	41.69	30.68	0.08
3	34.71	39.07	26.21	0.00	30.12	47.96	21.91	0.02	31.39	45.21	23.39	0.00	42.04	33.35	24.60	0.01	32.54	51.00	16.45	0.01	27.55	41.67	30.69	0.09
10	34.71	39.07	26.21	0.01	30.11	47.97	21.89	0.03	31.39	45.21	23.39	0.01	42.04	33.35	24.59	0.01	32.53	51.01	16.44	0.02	27.54	41.67	30.66	0.13
20	34.71	39.07	26.21	0.01	30.11	47.97	21.89	0.04	31.39	45.20	23.39	0.02	42.04	33.35	24.59	0.02	32.53	51.00	16.44	0.03	27.54	41.67	30.66	0.13
40	34.71	39.07	26.21	0.01	30.11	47.96	21.89	0.04	31.38	45.20	23.39	0.03	42.04	33.35	24.59	0.02	32.53	51.00	16.44	0.03	27.54	41.67	30.66	0.13
45	34.71	39.07	26.21	0.01	30.11	47.96	21.89	0.05	31.38	45.20	23.39	0.03	42.04	33.35	24.59	0.02	32.53	51.00	16.44	0.03	27.54	41.67	30.66	0.13
50	34.71	39.07	26.21	0.01	30.10	47.96	21.89	0.05	31.38	45.20	23.39	0.03	42.04	33.35	24.59	0.02	32.53	51.00	16.44	0.03	27.54	41.67	30.66	0.13
D: Original Dufour-Engle model																								
0	75.75	24.23	-	0.02	71.22	28.77	-	0.01	76.84	23.15	-	0.01	73.28	26.68	-	0.04	68.27	31.73	-	0.00	74.64	25.36	-	0.00
1	75.89	24.09	-	0.02	70.78	29.21	-	0.01	76.98	23.00	-	0.01	73.31	26.65	-	0.04	67.92	32.08	-	0.00	75.09	24.91	-	0.00
2	75.83	24.15	-	0.02	70.64	29.35	-	0.01	76.92	23.07	-	0.01	73.23	26.74	-	0.04	67.82	32.18	-	0.00	75.02	24.97	-	0.00
3	75.79	24.19	-	0.02	70.58	29.41	-	0.01	76.89	23.10	-	0.01	73.21	26.75	-	0.04	67.82	32.18	-	0.00	75.02	24.98	-	0.00
10	75.78	24.20	-	0.02	70.56	29.43	-	0.01	76.88	23.11	-	0.01	73.19	26.77	-	0.04	67.81	32.19	-	0.00	74.98	25.01	-	0.00
20	75.78	24.20	-	0.02	70.56	29.43	-	0.01	76.88	23.11	-	0.01	73.19	26.77	-	0.04	67.81	32.19	-	0.00	74.98	25.02	-	0.01
40	75.78	24.20	-	0.02	70.56	29.43	-	0.01	76.88	23.11	-	0.01	73.19	26.77	-	0.04	67.81	32.19	-	0.00	74.98	25.02	-	0.01
45	75.78	24.20	-	0.02	70.56	29.43	-	0.01	76.88	23.11	-	0.01	73.19	26.77	-	0.04	67.81	32.19	-	0.00	74.98	25.02	-	0.01
50	75.78	24.20	-	0.02	70.56	29.43	-	0.01	76.88	23.11	-	0.01	73.19	26.77	-	0.04	67.81	32.19	-	0.00	74.98	25.02	-	0.01

This table reports the generalized forecast error variance decomposition (GFEVD) for **returns**, conditioning on the average **RBAAT** history, for six Australian banking stocks in 2013 for 4 models, namely the *Endo-VAR*, *WACD-VAR*, Augmented Dufour-Engle (i.e. with volume), and Original Dufour-Engle (without volume) models. Following the step-by-step procedure described in Appendix B, each entry in the table, reported in %, is calculated according to Equation (2.11), by averaging over $M = 1,000$ vectors of shocks bootstrapped from the residuals of the corresponding estimated models. For each vector of shocks, the GIRF $I_j(\cdot)$ in Equation (2.11) is based on $N = 1,000$ simulated realizations. The average RBAAT history is defined as the average of all trading histories right before 14:30:00 on each of eleven RBA days in 2013.

Table 2.9: Generalized Forecast Error Variance Decomposition for **Returns** conditioning on the average **non-RBAAT** history

Stock	**ANZ**				**CBA**				**NAB**				**WBC**				**MQG**				**BEN**			
Response	**returns**				**returns**				**returns**				**returns**				**returns**				**returns**			
Impulse	ret	sign	vol	dur	ret	sign	vol	dur	ret	sign	vol	dur	ret	sign	vol	dur	ret	sign	vol	dur	ret	sign	vol	dur
Horizon																								
A: Endo-VAR model																								
0	46.94	37.64	15.42	0.00	45.89	40.39	13.71	0.01	51.09	34.87	14.04	0.00	49.68	36.28	14.03	0.00	46.16	40.66	13.18	0.00	42.68	36.56	20.76	0.00
1	47.33	37.00	15.67	0.01	45.77	40.59	13.62	0.02	51.42	34.38	14.19	0.01	49.94	35.89	14.15	0.01	46.06	40.82	13.11	0.02	43.22	35.59	21.17	0.02
2	47.34	36.98	15.67	0.01	45.70	40.69	13.59	0.02	51.43	34.37	14.18	0.01	49.93	35.92	14.14	0.01	46.00	40.91	13.08	0.02	43.22	35.61	21.16	0.02
3	47.34	36.99	15.67	0.01	45.66	40.75	13.57	0.02	51.41	34.40	14.17	0.02	49.92	35.93	14.14	0.01	46.00	40.91	13.08	0.02	43.22	35.59	21.17	0.02
10	47.33	36.97	15.66	0.04	45.65	40.76	13.57	0.02	51.40	34.40	14.17	0.03	49.91	35.93	14.13	0.03	45.99	40.90	13.08	0.03	43.19	35.60	21.15	0.06
15	47.33	36.97	15.66	0.04	45.65	40.76	13.57	0.02	51.40	34.40	14.17	0.03	49.91	35.93	14.13	0.03	45.99	40.90	13.08	0.03	43.19	35.60	21.15	0.06
20	47.33	36.97	15.66	0.04	45.65	40.76	13.57	0.02	51.40	34.40	14.17	0.03	49.91	35.93	14.13	0.03	45.99	40.90	13.08	0.03	43.19	35.60	21.15	0.06
B: WACD-VAR model																								
0	46.83	37.09	16.08	0.00	43.84	40.92	15.24	0.01	51.43	34.67	13.90	0.00	50.09	35.40	14.50	0.00	46.55	41.14	12.31	0.00	41.16	36.63	22.21	0.00
1	47.23	36.44	16.33	0.00	43.72	41.12	15.15	0.01	51.76	34.20	14.05	0.00	50.37	34.99	14.63	0.01	46.45	41.31	12.24	0.01	41.71	35.62	22.66	0.01
2	47.25	36.42	16.34	0.00	43.66	41.21	15.11	0.01	51.76	34.19	14.05	0.00	50.36	35.01	14.63	0.01	46.39	41.39	12.21	0.01	41.71	35.64	22.65	0.01
3	47.24	36.42	16.33	0.00	43.63	41.27	15.10	0.01	51.74	34.21	14.04	0.01	50.35	35.02	14.62	0.01	46.39	41.39	12.21	0.01	41.71	35.62	22.66	0.01
10	47.24	36.42	16.33	0.02	43.62	41.28	15.09	0.01	51.74	34.22	14.03	0.01	50.34	35.03	14.62	0.02	46.39	41.39	12.21	0.01	41.69	35.64	22.64	0.03
15	47.24	36.42	16.33	0.02	43.62	41.28	15.09	0.01	51.74	34.22	14.03	0.01	50.34	35.03	14.62	0.02	46.39	41.39	12.21	0.01	41.69	35.64	22.64	0.03
20	47.24	36.42	16.33	0.02	43.62	41.28	15.09	0.01	51.74	34.22	14.03	0.01	50.34	35.03	14.62	0.02	46.39	41.39	12.21	0.01	41.69	35.64	22.64	0.03
C: Augmented Dufour-Engle model																								
0	47.44	37.29	15.27	0.00	46.04	39.92	14.05	0.01	51.01	35.31	13.68	0.00	47.27	37.57	15.16	0.00	44.38	42.64	12.98	0.00	42.60	36.83	20.57	0.00
1	47.83	36.65	15.52	0.00	45.93	40.08	13.99	0.01	51.36	34.80	13.84	0.00	47.54	37.15	15.31	0.00	44.29	42.78	12.93	0.00	43.15	35.84	21.01	0.00
2	47.85	36.63	15.53	0.00	45.88	40.16	13.96	0.01	51.37	34.79	13.84	0.00	47.52	37.17	15.30	0.00	44.24	42.86	12.90	0.00	43.15	35.85	21.00	0.00
3	47.84	36.63	15.52	0.00	45.85	40.21	13.94	0.01	51.35	34.82	13.83	0.00	47.52	37.18	15.30	0.00	44.24	42.86	12.90	0.00	43.15	35.84	21.01	0.00
10	47.84	36.63	15.52	0.00	45.84	40.22	13.94	0.01	51.35	34.82	13.83	0.00	47.51	37.19	15.30	0.00	44.23	42.86	12.90	0.00	43.14	35.87	21.00	0.00
15	47.84	36.63	15.52	0.00	45.84	40.22	13.94	0.01	51.35	34.82	13.83	0.00	47.51	37.19	15.30	0.00	44.23	42.86	12.90	0.00	43.14	35.87	21.00	0.00
20	47.84	36.63	15.52	0.00	45.84	40.22	13.94	0.01	51.35	34.82	13.83	0.00	47.51	37.19	15.30	0.00	44.23	42.86	12.90	0.00	43.14	35.87	21.00	0.00
D: Original Dufour-Engle model																								
0	77.19	22.81	-	0.00	70.03	29.96	-	0.01	75.12	24.88	-	0.00	75.50	24.50	-	0.01	68.76	31.24	-	0.00	74.45	25.53	-	0.03
1	77.32	22.68	-	0.00	69.59	30.40	-	0.01	75.27	24.73	-	0.00	75.53	24.47	-	0.00	68.42	31.58	-	0.00	74.89	25.08	-	0.02
2	77.26	22.74	-	0.00	69.44	30.55	-	0.01	75.20	24.80	-	0.00	75.44	24.55	-	0.00	68.32	31.68	-	0.00	74.83	25.15	-	0.02
3	77.22	22.77	-	0.00	69.38	30.61	-	0.01	75.17	24.83	-	0.00	75.43	24.57	-	0.00	68.31	31.69	-	0.00	74.82	25.15	-	0.02
10	77.22	22.78	-	0.00	69.36	30.63	-	0.01	75.16	24.84	-	0.00	75.41	24.59	-	0.00	68.31	31.69	-	0.00	74.79	25.19	-	0.03
15	77.22	22.78	-	0.00	69.36	30.63	-	0.01	75.16	24.84	-	0.00	75.41	24.59	-	0.00	68.31	31.69	-	0.00	74.78	25.19	-	0.03
20	77.22	22.78	-	0.00	69.36	30.63	-	0.01	75.16	24.84	-	0.00	75.41	24.59	-	0.00	68.31	31.69	-	0.00	74.78	25.19	-	0.03

This table reports the generalized forecast error variance decomposition (GFEVD) for **returns**, conditioning on the average **non-RBAAT** history, for six Australian banking stocks in 2013 for 4 models, namely the *Endo-VAR*, *WACD-VAR*, Augmented Dufour-Engle (i.e. with volume), and Original Dufour-Engle (without volume) models. Following the step-by-step procedure described in Appendix B, each entry in the table, reported in %, is calculated according to Equation (2.11), by averaging over $M = 1{,}000$ vectors of shocks bootstrapped from the residuals of the corresponding estimated models. For each vector of shocks, the GIRF $I_j(\cdot)$ in Equation (2.11) is based on $N = 1{,}000$ simulated realizations. The average non-RBAAT history is defined as the average of all trading histories right before 14:30:00 on each of forty-three non-RBA days in the current sample.

From both tables a big proportion of the FEV of returns of six major Australian banking stocks is accounted for by trade-related innovations (i.e. shocks to trade attributes and durations) which are often considered private information in the market microstructure literature. Amongst these sources of private information, trade direction is found to be the most important factor to explain the price dynamics of these stocks. Its innovations account for between 22% and 33% of the FEV of returns based on the original Dufour and Engle (2000) model that does not incorporate the information from trade sizes, and for more than 35% (even above 50% in some cases) of the returns' FEV according to other models that have also included trading volumes. This result lends support to Hasbrouck (1991a), Dufour and Engle (2000), Barclay et al. (2003), and Hendershott and Riordan (2011), who show that trade sign is an important determinant of the price formation process. Likewise, consistent with the findings of Easley and O'Hara (1987), Hasbrouck (1988, 1991a), and O'Hara et al. (2014) that there is a significant price-quantity relationship, shocks to trading volume possess remarkable explanatory power for the FEV of returns, which ranges between 12% and 31%. Moreover, the inclusion of trade sizes into a joint system significantly increases the informativeness of trade direction about the dynamic behavior of prices of the banking stocks, possibly due to the correlatedness between trade signs and sizes.

Meanwhile, shocks to durations contribute much less to the FEV of returns of the Australian banking stocks than do other trade attributes' shocks. The contribution of duration innovations is less than 9% for all stocks and is typically below 1% in cases where durations are treated as exogenous and/or one conditions on an average history prior to 14:30:00 on non-RBA days during which the market is relatively tranquil (see Table 2.1). On the other hand, the contribution of other trade attributes' shocks is normally above 50%. These results suggest that the time between trades is significantly less important in explaining price dynamics of the Australian banking stocks than other trade characteristics. Despite this, the informativeness of trade durations about the price formation process of these stocks is much higher when durations are endogenously modeled than when they are treated as exogenous (which is consistent with the results in subsection 2.4.2), since the proportion of the FEV of quote revisions explained by duration shocks under the former scenario is many times as high as that under the latter case, especially when one conditions on the average RBAAT history. This finding is in agreement with theory in Easley and O'Hara (1992) which demonstrates that the informativeness of trade arrival time about security prices is related to its correlatedness and joint determination with trading volumes and prices. Duration shocks have a significantly larger relative contribution to the returns' FEV in the *Endo-VAR* model than in the *WACD-VAR* model, especially on the RBA announcement days. The reasons for this might be that durations exhibit a significant nonlinear dynamic behavior (Zhang et al., 2001, Fernandes and Grammig, 2006), and the *Endo-VAR* model, which allows for a higher degree of nonlinearity in the duration dynamics (which includes not only a deeper lag serial dependence of durations but also interactions between durations and trade attributes) than does the *WACD-VAR* model, might better capture this nonlinearity.

We find that the RBA announcements significantly affect the relative importance of durations and trade attributes to the process of price adjustments for Australian banking stocks. In particular, shocks to both trade characteristics and durations account for larger proportions of the FEV of returns of these stocks on the RBA announcement days than on days without RBA announcements. This implies that trades executed around the interest rate announcements convey more important information, through both durations and other trade attributes, about prices than trades transacted during a similar calendar time window on a non-RBA day. Consistent with the findings in previous subsections, this result suggests that trades around the RBA announcements are likely to be initiated by informed traders and thus are more informative about the price dynamics of the Australian banking stocks that we examine.

2.5 Conclusion

This chapter relaxes the strict exogeneity assumption of time between trades that is often imposed in prior studies by proposing a nonlinear VAR model for trade durations, trade characteristics (signs and volumes) and returns that allows for the feedback effects amongst these variables. Building upon the general econometric methodology developed by Engle (2000), our proposed model extends the VAR model in Hasbrouck (1991a) and Dufour and Engle (2000) to study the joint dynamics of trades and returns. We apply this model to examine the effects of trade arrival times and other trade attributes on the price dynamics of Australian banking stocks around the RBA interest rate announcements. Consistent with Dufour and Engle (2000) and Manganelli (2005), we find strong evidence to reject the exogeneity of trade durations. The time between trades is positively dependent on past absolute price changes but negatively related to previous trading volumes. We also observe that as trading intensifies or trade durations get shorter, trades become more positively autocorrelated and have a bigger impact on prices, which is in line with the findings of Dufour and Engle (2000).

Our results show the significant effects of the RBA announcements on the role that durations and trade characteristics play in explaining the price dynamics of major Australian banking stocks. Trades executed within one minute around the releases of the monetary policy news typically have shorter durations and larger price impacts. Conditioning on an average before-announcement history, when an unanticipated trade in these banking stocks arrives faster (slower) than on average, its cumulative impact on prices is higher (lower) only if durations are endogenously modeled. No similar results are found if durations are treated as exogenous. This result confirms the importance of allowing for the endogeneity of trade durations that underlies the theoretical model of Easley and O'Hara (1992).

Using Lanne and Nyberg's (2016) GFEVD methodology, we find that duration shocks account for a significantly smaller proportion of the forecast error variance of returns of the Australian banking stocks than do other trade attribute shocks. The relative importance of duration innovations to returns is, however, remarkably higher when durations are endogenously modeled. Moreover, conditioning on RBA announcements, the contributions of both duration and other trade attribute shocks to the forecast error variance of returns increase. The results indicate that the time between trades is an important determinant of banking stock prices, especially around the interest rate announcements, even though it explains the dynamics of prices significantly less than do other trade characteristics.

2.6 Appendix

A Simulation procedure to compute GIRFs

The simulation experiment explained in subsection 2.4.2 to produce the GIRFs for quote revision r_t is carried out via the following steps.

A.1 Pick a history $\mathcal{I}_{t-1}$.

A.2 For a given horizon h, generate a $4\times(1+h)$ matrix of random noise for quote revisions, trade attributes and time durations. For the *Endo-VAR* system, the noise series are bootstrapped from their respective residuals $\widehat{\varepsilon_t}$ since the usual normal assumption is too restrictive, as implied by the large Jarque-Bera statistics discussed earlier. The bootstrapping avoids the imposition of unrealistic distributional assumptions on the error terms. The error terms (of sign and volume equations in particular) are contemporaneously correlated, so we first transform the correlated $\widehat{\varepsilon_t}$ to contemporaneously uncorrelated residuals, $\widehat{\zeta_t} = P^{-1}\widehat{\varepsilon_t}$, where P is the lower Cholesky decomposition of the estimated covariance matrix of ε_t (i.e. $\widehat{Var}(\varepsilon_t) = PP'$) (see Koop et al., 1996, Pesaran and Shin, 1996). We retain the serial correlation inherent in the observed $\widehat{\varepsilon_t}$ (which is also imported to $\widehat{\zeta_t}$) by applying the stationary bootstrap procedure proposed by Politis and Romano (1994) with an average block bootstrap length set to 10 to each element of $\widehat{\zeta_t}$. We recover $\widehat{\varepsilon_t} = P\widehat{\zeta_t}$ from the draws of $\widehat{\zeta_t}$. For the *WACD-VAR* system, duration innovations are randomly drawn from the estimated Weibull distribution, while the innovations for quote changes and trade characteristics are drawn using the above bootstrap method.

A.3 Given $\mathcal{I}_{t-1}$, compute T_t, x_t, and then r_t according to their joint system, using the disturbances produced in step A.2. Simulated values for (T,x,r) at each period are augmented into the past information set to compute the next period values until the h-th future period is reached. This gives a trajectory of $(r_{t+k}, x_{t+k}, T_{t+k})$ for $k = 0, 1, \cdots, h$ under the "no shock" scenario. Special attention is given to the simulation of the *WACD-VAR* system. Since the WACD model is applied to diurnally adjusted duration $\widetilde{T}_t$, after $\widetilde{T}_{t+k}, k = 0, 1, \cdots, h$ is calculated, these $\widetilde{T}_{t+k}$ are transformed back to T_{t+k}, for use in the other equations.

A.4 Shock the joint system at transaction time t with trade, duration, and/or return shocks and repeat step A.3 using the same set of noise series generated in step A.2.[21] At each horizon k, calculate a realization of $I_r(k, \cdot)$ as $r_{t+k,\text{ shock}} - r_{t+k,\text{ no shock}}$. The simulated path of $I_r(k, \cdot)$ indexed in transaction time can be used directly, and/or converted into calendar time.

A.5 Repeat steps A.3 to A.4 N times, where N is a sufficiently large number. This gives N realizations of the impulse response $I_r^{(l)}(k, \cdot)$ for $l = 1, 2, \cdots, N$. Averaging these realizations provides an estimate of $I_r(k, \cdot)$ for $k = 0, 1, \cdots, h$.

[21] That is, the first vector of the noise series in step A.2 (i.e. at time t), or a part of it, is replaced by the relevant shocks.

B Simulation procedure to compute GFEVD

The GFEVD for a multivariate system of quote revisions, trade attributes and trade durations is calculated via the following steps.

B.1 Pick a history $\mathcal{I}_{t-1}$ (i.e. either the average RBAAT or average non-RBAAT history in our case).

B.2 Draw a shock vector ε_t from the residuals of the estimated model. This can be done similarly to step A.2 in subsection 2.4.2, but without the embedded stationary bootstrap procedure since only one shock vector is drawn.

B.3 Compute the GIRF $I_j(\cdot)$ in equation (2.11) associated with each element of the shock vector drawn in step B.2 for all variables in the multivariate system. This consists of performing steps A.2 to A.5 in subsection 2.4.2 but now for all variables. Note that in step A.4 we now only shock one equation of the system at a time using the relevant element of the shock vector, and the GIRF $I_j(\cdot)$ corresponding to each shock element is computed for h future transactions based on N repetitions.

B.4 Use the GIRFs obtained in step B.3 to compute $\lambda_{i\to j,\mathcal{I}_{t-1}}(h), h = 0, 1, 2, \cdots$ as in equation (2.11) for the particular history and shock.

B.5 Repeat steps B.2 to B.4 M times. Compute the mean of $\lambda_{i\to j,\mathcal{I}_{t-1}}(h), h = 0, 1, 2, \cdots$ to average out the effects of different shock sizes.

Chapter 3

The volume-volatility relation of trades: A bivariate stochastic conditional model

3.1 Introduction

Examination of how trading volume and price volatility are interrelated is an important research question in finance which has attracted the attention of researchers for a long time. According to Karpoff (1987), the study of the volume-volatility relation(ship) is important for several reasons. First, it is related to the existence, dissemination and absorption of information in markets, and hence it provides insights into how financial markets are structured. Second, the relationship is important for research such as event studies that need the joint distribution of prices and volumes in order to conduct inference. Third, the study of the volume-volatility relation offers ways to investigate the empirical distributions of prices and returns, which are of great interest in finance.

Theoretical studies demonstrate a positive relationship between trading volume and price volatility, and they also highlight the endogeneity and joint determination of these two variables (see, amongst others, Clark, 1973, Admati and Pfleiderer, 1988, Andersen, 1996). For example, the Mixture of Distribution Hypothesis (MDH) of Clark (1973) and Andersen (1996) posits that both volume and return volatility are jointly driven by an underlying latent information flow. The arrival of new private information generates a sequence of trading activities and price movements that move the market to a new equilibrium where all private information is fully revealed. Consequently, return volatility is positively related to trading volume.[22]

Despite the theoretical suggestions of the joint determination of volume and volatility, most existing empirical studies (see, amongst others, Ahn et al., 2001, Chan and Fong, 2006, Næs and Skjeltorp, 2006, Park, 2010, Chevallier and Sévi, 2012, Wang and Wu, 2015, Clements and Todorova, 2016, Bollerslev et al., 2018) investigate this relationship using single-equation or univariate time series models that preclude the feedback effects between these variables, rather than embrace a joint framework. Nevertheless, there is a growing literature that provides multivariate analyses of the two variables, including Manganelli (2005), Xu et al. (2006), Nolte (2008), Fleming and Kirby (2011), Rossi and Santucci de Magistris (2013), Carlin et al. (2014), and Do et al. (2014).

Manganelli (2005) and Nolte (2008) provide systems of equations for price changes, volumes and other variables such as trade durations that model the latent quantities (for example, expected volume or volatility) with an autogressive conditional duration- (ACD-) or GARCH- type structure. The application of an ACD model to a volume series is due to the distributional similarities between trade durations and trading volumes. However, an ACD or GARCH process is conditionally deterministic: the expectation of a quantity of interest (e.g. volume) is assumed to be some function of past information and hence is completely known given the past information. Meanwhile, Xu et al. (2006), Fleming and Kirby (2011), Rossi and Santucci de Magistris (2013), Carlin et al. (2014), and Do et al. (2014) employ a vector autoregression (VAR) or fractionally integrated VAR to characterize the relationship between volume, volatility and other variables in a way that does not require a separate latent structure for the expected

[22]It is noted that the MDH theory only considers a contemporaneous relationship between trading volume and return volatility.

quantities. However, the VAR model implies that both **expected** volume and volatility are known after conditioning on the past information.[23]

Motivated by the stochastic volatility (SV) and stochastic conditional duration (SCD) literatures (Harvey et al., 1994, Harvey and Shephard, 1996, Bauwens and Veredas, 2004, Strickland et al., 2006, Renault and Werker, 2011, Renault et al., 2014, Pelletier and Zheng, 2013, Wei and Pelletier, 2015), this chapter develops a bivariate stochastic conditional model to investigate the joint evolution of returns and trading volumes. In particular, returns are assumed to follow an SV model while volumes are characterized by a multiplicative error model (which is proposed by Engle (2002) for non-negative processes) that embeds a latent stochastic structure for the conditional expected volumes. The joint latent process, which consists of the conditional expected volume and instantaneous volatility, is further assumed to evolve according to a first order VAR structure that accommodates both the contemporaneous and serial cross-dependencies between the latent variables.

Our proposed model is a bivariate generalization of the popular univariate SV and SCD models in the literature, which are often known to be superior to their corresponding analogues such as GARCH and ACD models in explaining the empirical data dynamics and producing well-behaved residuals (Jacquier et al., 1994, Kim et al., 1998, Bauwens and Veredas, 2004, Carnero et al., 2004). Unlike the models put forward by Manganelli (2005), Xu et al. (2006), Nolte (2008), Fleming and Kirby (2011), Rossi and Santucci de Magistris (2013), Carlin et al. (2014), and Do et al. (2014), the conditional expected volume and instantaneous volatility in our model have their own innovations, and therefore, they are no longer conditionally deterministic. We examine a few alternative parameterizations of the joint latent process and establish several statistical properties with regard to the moments and the correlation structures of the volume and volatility processes implied by these bivariate considerations. These properties generalize and are all consistent with those derived in previous studies for univariate SV and SCD models.

[23]Since there are no latent variables in a standard VAR model, the conditional expectation of a variable in the VAR system, for example volume, is simply a linear function of the past values of all variables in the system.

Similar to the univariate SV and SCD models, it is not easy to evaluate the exact likelihood function of our bivariate stochastic model since it involves a high-dimensional integral over the space of all the latent variables which are stochastic and not fully observed given the past information. We estimate our model, which is essentially a nonlinear non-Gaussian state space model, by transforming it into a linear non-Gaussian state space representation and then employing Quasi Maximum Likelihood (QML) to produce a so-called quasi likelihood function that approximates the exact one. The quasi likelihood function is obtained by treating the non-Gaussian errors as if they were normally distributed and then applying the Kalman filter. Under correct model specifications and standard regularity conditions, the QML estimates, which maximize the quasi likelihood function, are still consistent and asymptotically normal, but they are no longer asymptotically efficient (see, for example, White, 1982, Harvey et al., 1994, Ruiz, 1994, Bauwens and Veredas, 2004).

We conduct a simulation study to check the applicability of the QML method to our model, and the results support the consistency of our QML estimators. Although more sophisticated methods that better evaluate the exact likelihood of a nonlinear non-Gaussian state space models exist (e.g. Kim et al., 1998, Durbin and Koopman, 2000, Sandmann and Koopman, 1998), they rely on complicated simulation techniques and hence are highly demanding in computing time, especially for large datasets like ours. In addition, these methods require that the true (joint) non-Gaussian density be entirely known, which may not be the case in our bivariate model unless the non-Gaussian measurement disturbances are independent (which we do not assume). Therefore, we do not consider these methods in this chapter but rely on QML, which is much less computationally demanding.

We apply the proposed bivariate stochastic model to the transaction data of one big market capitalization stock (BHP - BHP Billiton Limited) and one small market capitalization stock (CHC - Charter Hall Group) listed on the S&P/ASX200 index in August 2014. We find that there are strong positive contemporaneous and temporal feedback effects between trading volume and return volatility for both stocks, which is consistent with the empirical findings of Manganelli (2005), Xu et al. (2006), Fleming and Kirby (2011), Rossi and Santucci de Magistris (2013), Carlin et al. (2014) and Do et al. (2014), and which also supports the endogeneity and joint determination of these two variables, as highlighted in market microstructure theory (e.g. Clark, 1973, Admati and Pfleiderer, 1988, Easley and O'Hara, 1992, Andersen, 1996). Thus, our results suggest that studies that examine the relationship between volume and volatility using single-equation regression models might be subject to endogeneity or reverse causality issues, and their findings might only reflect correlation rather than causation unless estimation techniques have taken this into account. In addition, we find that there is an asymmetry in the feedback effects between volume and volatility, with the impact of volume on volatility usually being much more significant. This latter result reaffirms one fundamental prediction in market microstructure theory which is that trading is an important channel that explains price dynamics.

We obtain significant estimates for the variance of the latent expected volume and volatility innovations, especially for the latter. This observation suggests the inadequacy of the GARCH/ACD-type models in modeling conditional expected volume and instantaneous volatility, and the necessity of treating these conditional quantities as latent variables that follow a stochastic process, which is in agreement with previous studies such as Jacquier et al. (1994), Kim et al. (1998), and Carnero et al. (2004). In addition, by accommodating the joint determination and feedback effects between volume and volatility, our bivariate stochastic model provides significant enhancements in fit relative to its univariate stochastic counterparts, producing not only statistically significantly higher log likelihood values but also better-behaved smoothed residuals that are less serially autocorrelated.

We also find that a positive shock to either trading volume or return volatility increases the expectation of both quantities of the future transactions in both stocks. Such an increase is largest after the first few trades and then declines steadily to zero as both volume and volatility converge to their new full information equilibrium. The responses of both variables to the initial perturbation are remarkably larger for the bigger and more frequently traded stock (BHP), and the convergence to the steady state is also faster for BHP in both transaction and calendar time. The latter result is consistent with Manganelli (2005), in that it takes less time for more heavily traded stocks to reach their long run equilibrium after an initial shock. It also lends support to the theoretical predictions of Holden and Subrahmanyam (1992), who show that higher trading frequency hastens the speed of disseminating private information by informed traders to the market.

This chapter contributes to the literature that investigates the volume-volatility relation by developing a bivariate stochastic conditional model for trading volume and return volatility that allows for their joint determination, and for dynamic feedback effects between the two variables, as suggested by theory. This distinguishes our study from most existing studies that examine the relationship between volume and volatility using a single-equation or univariate framework, which cannot facilitate joint determination. Our proposed model is a bivariate generalization of the popular univariate SV and SCD models in the literature, and it relaxes the conditional deterministic feature imposed by a few previous studies that accommodate the joint modeling of volume and volatility using either multivariate GARCH-type approaches (Manganelli, 2005, Nolte, 2008) or a VAR model that does not allow a latent structure for the expected quantities (Xu et al., 2006, Fleming and Kirby, 2011, Rossi and Santucci de Magistris, 2013, Carlin et al., 2014, Do et al., 2014). To the best of our knowledge, it is the first time that the SCD model has been applied to the modeling of trading volumes, and it is also the first time that the SV and SCD models have been employed to jointly model volatility and volume. An empirical application confirms that not only does our model successfully capture the stylized positive dynamic relationship between trading volume and return volatility often observed in empirical data, it also provides additional insights into this relation.

The rest of the chapter is structured as follows. Section 3.2 introduces the bivariate stochastic conditional model for trading volumes and returns/volatility proposed in this chapter. It also discusses some statistical properties of the model and proposes QML for estimation. Section 3.3 describes the data. Estimated results, model diagnostics and an impulse response analysis are presented in Section 3.4. Section 3.5 concludes.

3.2 A bivariate stochastic conditional model for volume and volatility

3.2.1 Model setup

Denote the trading volume and the returns of the t^{th} transaction by v_t and $r_t = 100(\ln(q_{t+1}) - \ln(q_t))$ respectively, where q_t is the mid-point of the prevailing bid and ask prices right before the t^{th} trade. As a starting point which is typical in both the autoregressive conditional and stochastic modeling literatures, we assume that v_t can be factorized according to a multiplicative error model (MEM) as the product of a latent variable times a positive random error term (see, amongst others, Engle, 2002, Manganelli, 2005, Renault et al., 2014), while r_t, as usual, is decomposed as the sum of a drift term and a zero-mean innovation with heteroskedastic volatility as follows:

$$\begin{aligned} v_t &= \phi_t(\theta_v|\mathcal{I}_{t-1}) \times \varepsilon_t, \qquad \varepsilon_t \sim \text{i.i.d}(1, \sigma_\varepsilon^2), \text{ and } \varepsilon_t > 0, \\ r_t &= \mu_t + e_t = \mu_t + \sigma_t(\theta_r|\mathcal{I}_{t-1}) \times \zeta_t, \qquad \zeta_t \sim \text{i.i.d}(0, 1), \end{aligned} \tag{3.1}$$

where $\mathcal{I}_{t-1}$ denotes the information available at time $t-1$, and i.i.d(a, b) denotes an independent and identical distribution with a mean of a and variance of b. While the innovation ζ_t is typically assumed to be normally distributed, and sometimes Student-t distributed to better capture the fat tails commonly found in a return series, ε_t is usually assumed to follow a Weibull or Gamma distribution, of which an Exponential distribution is a special case. The normalization of the mean of ε_t to 1 implies that ϕ_t is the expected value of v_t conditioning on past information (i.e. $\mathbb{E}(v_t|\mathcal{I}_{t-1}) = \phi_t$) if the dynamics of ϕ_t, given $\mathcal{I}_{t-1}$, are uncorrelated with ε_t.[24] The use of an MEM, which is widely utilized to study the time duration between trades or events (e.g. Engle and Russell, 1997, 1998, Bauwens and Veredas, 2004, Renault et al., 2014), is introduced in this chapter to model trading volume, since volume and durations have similar distributional properties. A few studies such as Manganelli (2005) and Nolte (2008) have also modeled trading volume with an MEM; however, they assume the conditional

[24]Normalizing ε_t to have a unit mean also helps to achieve identification since the mean of ε_t and the constant in a process for ϕ_t (e.g. γ_v in Equation (3.2)) cannot be separately identified.

expected volume ϕ_t to follow a GARCH-type structure that is conditionally deterministic.

A common approach in the SV or SCD literature to modeling the latent conditional expected volume ϕ_t and instantaneous volatility σ_t^2 is to assume that their logarithmically transformed quantities (which help to ensure positivity) follow the following correlated bivariate AR(1) process[25]

$$\begin{aligned} \log \phi_t &= \gamma_v + a_v \log \phi_{t-1} + u_{v,t}, \\ \log \sigma_t^2 &= \gamma_{e^2} + a_{e^2} \log \sigma_{t-1}^2 + u_{e^2,t}, \end{aligned} \tag{3.2}$$

where $u_t = (u_{v,t}, u_{e^2,t})' \sim \text{i.i.d}(0, \Sigma_u)$. The appearance of the errors u_t in the latent process indicates that the latent quantities (i.e. expected volume and volatility) are no longer entirely known given the past information $\mathcal{I}_{t-1}$. Thus, the stochastic latent process in (3.2) relaxes the conditionally deterministic feature implied either by an autoregressive conditional model such as a GARCH or ACD model, or by a standard VAR model that does not allow a latent structure for the expected quantities. We note that it is the first time that (1) the SCD model has been employed to model trading volumes, and (2) the SV and SCD models have been used to jointly model volatility and volume.

Following the standard literature (e.g. Andersen and Sørensen, 1996, Bauwens and Veredas, 2004, Strickland et al., 2006), we assume that conditioning on the past information $\mathcal{I}_{t-1}$, the measurement innovations in (3.1) are independent of the disturbances of the latent states in (3.2) at all times, such that the contemporaneous and serial interdependencies between the observed volume and return series are only driven by (i) the contemporaneous dependence between the measurement innovations and (ii) the instantaneous and serial feedback effects between the latent quantities. That is, we assume $\eta_t := (\varepsilon_t, \zeta_t)'$ is independent of $u_s := (u_{v,s}, u_{e^2,s})'$ for all t and s, however we allow for the correlatedness between ε_t and ζ_t, as well as between $u_{v,t}$ and $u_{e^2,t}$. The independence assumption between measurement and latent errors is typically maintained in a bivariate or multivariate setting (Harvey et al., 1994, Danielsson, 1998, Wei and Pelletier, 2015, Pelletier and Wei, 2018), even though its relaxation has been explored

[25]Throughout this chapter, the log notation is used to refer to natural logarithms.

in a univariate setting (Harvey and Shephard, 1996, Feng et al., 2004, Xu et al., 2011).

The latent system (3.2) is a restricted reduced form VAR(1) process where the matrix of coefficients is diagonal. Consequently, the contemporaneous and serial cross-dependencies between the latent variables stem entirely from the correlation between the state innovations $u_{v,t}$ and $u_{e^2,t}$, and there is no **direct** influence of one latent variable on another. If the disturbances in (3.2) are Gaussian and uncorrelated, then the expected volume ϕ_t and the instantaneous volatility σ_t^2 will be independent of each other. The system in (3.1) and (3.2) presents a state space framework for the joint volume and return/volatility process, in which the measurement equations are nonlinear. A logarithmic linearization of the measurement system has the following form,

$$\begin{aligned} \log v_t &= \log \phi_t + \log \varepsilon_t = \log \phi_t + \mathrm{w}_{v,t}, \\ \log e_t^2 &= \log \sigma_t^2 + \log \zeta_t^2 = \log \sigma_t^2 + \mathrm{w}_{e^2,t}, \end{aligned} \tag{3.3}$$

where $\mathrm{w}_t := (\mathrm{w}_{v,t}, \mathrm{w}_{e^2,t})' = (\log \varepsilon_t, \log \zeta_t^2)'$ is a vector of non-Gaussian disturbances whose means are $\omega = \left(\mathbb{E}(\log \varepsilon_t), \mathbb{E}(\log \zeta_t^2)\right)'$, and $e_t = r_t - \mu_t$.[26] A compact representation of the state-space system (3.2)-(3.3) is given by

$$\begin{aligned} y_t &= \omega + \alpha_t + (\mathrm{w}_t - \omega), \\ \alpha_t &= \gamma + A\alpha_{t-1} + u_t, \end{aligned} \tag{3.4}$$

where $y_t = (\log v_t, \log e_t^2)'$, $\alpha_t = (\log \phi_t, \log \sigma_t^2)'$, $\gamma = (\gamma_v, \gamma_{e^2})'$, $A = diag(a_v, a_{e^2})$, $\mathrm{w}_t \sim$ i.i.d$(\omega, \Sigma_{\mathrm{w}})$, $u_t \sim$ i.i.d$(0, \Sigma_u)$, and w_t is independent of u_s for all t, s. It is noted that the variance covariance matrix of w_t, Σ_{w}, is not the same as that of η_t, and the transformation from one to another, in general, cannot be attained theoretically.

The state-space system (3.4) is similar to a multivariate stochastic variance model put forward by Harvey et al. (1994) and Danielsson (1998). The only difference is that while this model investigates the behaviors of the return/volatility series of different assets which, to some extent, may share many similarities, the stochastic model in our study focuses on the volume and returns/volatility of a single asset, whose behaviors might be very different. Taken individually, returns follow an SV process while trading

[26] Although r_t is observable, $e_t = r_t - \mu_t$ is arguably not. In the estimation of the model, we first fit an ARMA(1,1) to r_t and then replace e_t with the residuals from the fitted model. This is common practice in the literature (e.g. Harvey and Shephard, 1996, Manganelli, 2005), although various ARMA structures have been employed. Our results are robust to the usage of different ARMA processes.

volumes evolve according to a so-called stochastic conditional volume (SCV) model, which is an analogue to an SCD model proposed by Bauwens and Veredas (2004) for trade durations. However, these two stochastic processes are correlated, as a consequence of the correlatedness between the measurement innovations $\mathrm{w}_{v,t}$ and $\mathrm{w}_{e^2,t}$, and between the latent errors $u_{v,t}$ and $u_{e^2,t}$.

The system (3.4) offers a baseline bivariate stochastic model for trading volume and price volatility, in which the instantaneous and serial cross-relationships between the latent variables $\log\phi_t$ and $\log\sigma_t^2$ are driven by and can only be inferred **indirectly** from the correlation between their innovations, as previously discussed. To allow for some **direct** cross-dependencies between the latent quantities, a natural remedy is to relax the diagonality restriction on the reduced form coefficient matrix A of the system (3.4). The resulting unrestricted VAR(1) state process is in line with a discretization of a general continuous-time bivariate Ornstein-Uhlenbeck specification for the latent log expected volume and log instantaneous volatility. See Pelletier and Zheng (2013) and Wei and Pelletier (2015) for applications of a continuous-time bivariate Ornstein-Uhlenbeck process to the joint modeling of return volatility and trade durations.

Although the system (3.4) accommodates the contemporaneous and serial cross-dependencies between the latent expected volume and volatility of trades, either directly (when A is a full matrix) or indirectly (when A is diagonal), it precludes the impact of shocks to the observed volumes and returns on the expected quantities, as a consequence of the independence assumption between the measurement and state disturbances. In other words, the latent component α_t and the innovation component w_t of the observed quantities y_t are two independent processes at all times.[27] An alternative parameterization of the state equations that relaxes this independence between α_t and w_t while still accommodating some direct interdependence between the latent quantities α_t is of the following form:

$$
\begin{aligned}
\log\phi_t &= \gamma_v + a_v \log\phi_{t-1} + b_{v,e^2}\log e_{t-1}^2 + u_{v,t}, \\
\log\sigma_t^2 &= \gamma_{e^2} + a_{e^2}\log\sigma_{t-1}^2 + b_{e^2,v}\log v_{t-1} + u_{e^2,t}.
\end{aligned}
\tag{3.5}
$$

[27]This can be easily shown by looking at the moving average presentation of the state variable α_t, assuming that the latent process is stationary.

This parameterization of the latent equations offers several advantages. First, the effects of past **observed** values of one variable on the **expected** values of another variable in the next period are directly captured by the b coefficients, making the interpretation more intuitive. Second, shocks to the measurement equations are transmittable to the latent process, and hence will affect the future realizations of volumes and returns/volatility. The strength of the transmission depends on that of the direct cross-dependencies between the expected quantities (measured by the b parameters) and the persistence of each latent variable (quantified by the a coefficients). Thus, similar to a univariate stochastic model with a "leverage effect" proposed by Feng et al. (2004) for trade durations, this model allows for an intertemporal correlation between the observed variables and their conditional quantities, which according to Feng et al. (2004) better captures the local asymmetries or "leverage effect" in the observed variables, improving the model fit. Third, the incorporation of the past observed quantities into the current latent process makes the assumption that the measurement innovations w_t and the latent disturbances u_t are independent **at all times** (which was imposed in the system (3.2)-(3.3)) more plausible. If this independence assumption is violated, we can alternatively assume that the latent errors u_t in (3.2) can be decomposed into two uncorrelated components: one that consists of the past observed variables and hence is dependent on past measurement shocks, and the other (which is the latent innovation in (3.5)) that is independent of the measurement errors. This gives rise to the extended system (3.3)-(3.5), for which the independence assumption between measurement and latent innovations is still maintained at all times.

All three alternatives of the joint stochastic process for the volumes and returns/volatility of trades formulated above can be cast into a more general state-space framework that reads

$$\begin{aligned} y_t &= \omega + \alpha_t + (\mathrm{w}_t - \omega), \\ \alpha_t &= \gamma + A\alpha_{t-1} + By_{t-1} + u_t, \end{aligned} \tag{3.6}$$

where different forms of coefficient matrices A and B determine different parameterizations of the latent equations. When A is a diagonal matrix and B is a null matrix, we obtain the baseline stochastic volume-volatility model specified in (3.2) and (3.3) which we shall label SVV_0. When A is a full matrix and B is still a null matrix, we get an extended bivariate stochastic model with a latent system that evolves according to an unrestricted reduced form VAR(1) process. We name this state-space model SVV_1. Finally, when A is a diagonal matrix and B is a general square matrix with zeros on the main diagonal, another extended bivariate stochastic setting with the latent equations in (3.5) is realized, which shall be called SVV_2. It is noted that other combinations of (more general) A and B matrices can lead to identification problems as far as estimation is concerned.[28]

3.2.2 Distributional assumptions

We parameterize the distributions of the innovations by assuming that $u_t \overset{i.i.d}{\sim} N(0, \Sigma_u)$, which is an assumption that is typically imposed by most studies in the literature. Meanwhile, the innovation ζ_t of the return equation in (3.1) is assumed to be i.i.d $N(0,1)$, implying that $w_{e^2,t} = \log \zeta_t^2$ has a $\log \chi^2_{(1)}$ distribution. For the distribution of ε_t of the volume equation in (3.1), we consider two cases: Weibull (denoted as W) and Gamma (G) which are appropriately scaled to have a unit mean. Consequently, $w_{v,t} = \log \varepsilon_t$ are log-Weibull (LW) or log-Gamma (LG) distributed. Some basic properties of the distributions of ε_t and $w_{v,t}$ are summarized in Table 3.1. It is worth noting that both Weibull (log-Weibull) and Gamma (log-Gamma) distributions nest and collapse to an Exponential (log-Exponential) distribution when δ_j or κ_j equals 1. Similarly, the $\log \chi^2_{(1)}$ distribution for $w_{e^2,t}$ is a special case of a log-Gamma distribution with $\kappa_j = 1/2$, and thus has mean of $\psi(1/2) + \log 2 \approx -1.2704$ and variance of $\psi'(1/2) \approx 4.9348$.

[28]When using a dataset simulated from a data generating process that is similar to the system (3.6) but with more general forms of A and B matrices, the parameter estimates, which were obtained using Quasi Maximum Likelihood, varied with the starting values of the optimization and were often very different from the true values, even though the maximization process was initiated at the true values, showing a lack of identification. This issue was not resolved when the sample size increased. These simulation experiments are omitted for brevity but are available upon request. With regard to the proposed state space systems in the current study, QML works well in simulation, as will be discussed in subsection 3.2.5.

Table 3.1: Summary of error distributions

Distribution	Parameters	PDF	Mean	Variance	MGF
Panel A: Weibull & log-Weibull					
For $X = \varepsilon_t$					
$\mathrm{W}(\delta, \lambda)$	$\delta > 0;\ \lambda = \frac{1}{\Gamma(1+1/\delta)}$	$f(x) = \frac{\delta x^{\delta-1}}{\lambda^{\delta}} \exp\left(-\frac{x^{\delta}}{\lambda^{\delta}}\right)$	$\lambda\Gamma(1+1/\delta) = 1$	$\lambda^2\Gamma\left(1+\frac{2}{\delta}\right) - 1$	$\sum_{n=0}^{\infty} \frac{z^n\lambda^n}{n!}\Gamma\left(\frac{n}{\delta}+1\right)$
For $X = \mathrm{w}_{v,t} = \log\varepsilon_t$					
$\mathrm{LW}(\delta, \lambda)$	$\delta > 0;\ \lambda = \frac{1}{\Gamma(1+1/\delta)}$	$f(x) = \frac{\delta e^{x\delta}}{\lambda^{\delta}} \exp\left(-\frac{e^{x\delta}}{\lambda^{\delta}}\right)$	$\frac{\psi(1)}{\delta} + \log\lambda$	$\frac{\pi^2}{6\delta^2}$	$\lambda^z\Gamma\left(\frac{z}{\delta}+1\right)$
Panel B: Gamma & log-Gamma					
For $X = \varepsilon_t$					
$\mathrm{G}(\kappa, \lambda)$	$\kappa > 0;\ \lambda = \frac{1}{\kappa}$	$f(x) = \frac{x^{\kappa-1}}{\Gamma(\kappa)\lambda^{\kappa}} \exp\left(-\frac{x}{\lambda}\right)$	$\lambda\kappa = 1$	$\lambda^2\kappa = \frac{1}{\kappa}$	$\frac{1}{(1-\lambda z)^{\kappa}}$ for $z < \frac{1}{\lambda}$
For $X = \mathrm{w}_{v,t} = \log\varepsilon_t$					
$\mathrm{LG}(\kappa, \lambda)$	$\kappa > 0;\ \lambda = \frac{1}{\kappa}$	$f(x) = \frac{e^{x\kappa}}{\Gamma(\kappa)\lambda^{\kappa}} \exp\left(-\frac{e^{x}}{\lambda}\right)$	$\psi(\kappa) + \log\lambda$	$\psi'(\kappa)$	$\lambda^z\frac{\Gamma(\kappa+z)}{\Gamma(\kappa)}$

This table shows some distributional properties of the random variables ε_t and $\log\varepsilon_t$ with different probability density functions (PDFs). MGF stands for moment generating function, which is defined as $\mathbb{E}(\exp(zX))$ for some real number z and some random variable X. $\Gamma(x) = \int_0^{\infty} t^{x-1}e^{-t}dt$ is the gamma function. $\psi(x)$ is the digamma function, which is the logarithmic derivative of the gamma function, i.e. $\psi(x) = \frac{d\log\Gamma(x)}{dx} = \frac{\Gamma'(x)}{\Gamma(x)}$. $\psi'(x) = \frac{d\psi(x)}{dx}$ is the trigamma function. The PDF of the log-Weibull (log-Gamma) distribution is derived from that of the Weibull (Gamma) distribution using the standard Jacobian transformation. The MGF of a log-Weibull (log-Gamma) random variable X (i.e. $\mathbb{E}(\exp(zX))$) equates to the z-th moment (for some real number z) of the corresponding Weibull (Gamma) random variable $\widetilde{X} = \exp(X)$; See subsection 3.6.1 in the Appendix for the derivation of the MGF.

3.2.3 Statistical properties

In this subsection we study some statistical properties of the processes $y_t = (\log v_t, \log e_t^2)'$ and $\alpha_t = (\log \phi_t, \log \sigma_t^2)'$ of the state space system (3.6), which nests the SVV_0, SVV_1 and SVV_2 models. We also examine some properties of the original processes $\chi_t := (\phi_t, \sigma_t^2)'$ and $\tau_t := (v_t, e_t^2)'$. Proofs of these results are relegated to subsection 3.6.2 in the Appendix.

Propositions 3.1-3.4 below detail the weak stationarity condition, the first two moments, and the correlation functions of the processes α_t and y_t. Corresponding results for the level quantities χ_t and τ_t are presented in Propositions 3.5-3.7. To facilitate the exposition of these propositions, let I_m denote an $m \times m$ identity matrix and $H = A + B$.

Proposition 3.1 *The latent process α_t and the measurement process y_t in the state space system (3.6) are weakly or covariance stationary if and only if* $\det(I_2 - Hz) \neq 0$ *for all* $|z| \leq 1$.

We implicitly assume that the stationarity condition for the state space system (3.6) holds in this section. Under stationarity, the first two moments of both the measurement (y_t) and latent (α_t) processes are given below.

Proposition 3.2 *Let* $\Theta_\alpha(s) := \mathrm{Cov}(\alpha_t, \alpha_{t-s})$ *for* $s \geq 0$. *The latent process α_t of the system (3.6) has the following moments:*

$$\mathbb{E}[\alpha_t] = (I_2 - H)^{-1}(\gamma + B\omega),$$

$$\Theta_\alpha(0) = \sum_{i=0}^{\infty} H^i (\Sigma_u + B\Sigma_{\mathrm{w}} B')(H')^i$$

$$\Theta_\alpha(s) = H^s \Theta_\alpha(0) \text{ for } s \geq 0,$$

The vectorization of $\Theta_\alpha(0)$ *is* $\mathrm{vec}(\Theta_\alpha(0)) = (I_4 - H \otimes H)^{-1} \mathrm{vec}(\Sigma_u + B\Sigma_{\mathrm{w}} B')$, *where* vec *denotes the vectorization operator, and* $\otimes$ *denotes the Kronecker product.*

Proposition 3.3 *Let $\Theta_y(s) := \mathrm{Cov}(y_t, y_{t-s})$ for $s \geq 0$. The measurement process y_t of the system (3.6) has the following moments:*

$$\begin{aligned}\mathbb{E}[y_t] &= \omega + (I_2 - H)^{-1}(\gamma + B\omega),\\ \Theta_y(0) &= \Theta_\alpha(0) + \Sigma_{\mathrm{w}},\\ \Theta_y(s) &= \Theta_\alpha(s) + H^{s-1}B\Sigma_{\mathrm{w}} \text{ for } s \geq 1.\end{aligned}$$

Autocovariances are often not bounded and are dependent on the units of the variables in the system. An alternative measure of linear relationships between the variables that is unit-invariant and bounded within the [-1, 1] range is autocorrelation. Proposition 3.4 details the expressions for the autocorrelations of processes α_t and y_t.

Proposition 3.4 *Let D_α and D_y be diagonal matrices whose diagonal elements are the square roots of the diagonal elements of $\Theta_\alpha(0)$ and $\Theta_y(0)$, respectively. The autocorrelations of the processes α_t and y_t of the system (3.6), for $s \geq 0$, are given by*

$$\begin{aligned}R_\alpha(s) &= D_\alpha^{-1}\Theta_\alpha(s)D_\alpha^{-1},\\ R_y(s) &= D_y^{-1}\Theta_y(s)D_y^{-1}.\end{aligned}$$

In the special case of the SVV_0 system where B is a zero matrix and A is a diagonal matrix (i.e. $A = diag(a_v, a_{e^2})$), let $\Sigma_u = [{}_u\sigma_{j,k}]$ and $\Sigma_{\mathrm{w}} = [{}_{\mathrm{w}}\sigma_{j,k}]$ ($j,k \in \{v, e^2\}$), then the autocorrelations of the processes α_t and y_t, for $s \geq 0$, simplify to

$$\begin{aligned}R_\alpha(s) &= [{}_\alpha\rho_{j,k,s}],\\ R_y(s) &= [{}_y\rho_{j,k,s}],\end{aligned}$$

where $${}_\alpha\rho_{j,k,s} = \frac{\dfrac{a_j^s\,{}_u\sigma_{j,k}}{1 - a_j a_k}}{\sqrt{\dfrac{{}_u\sigma_{j,j}}{1 - a_j^2}}\sqrt{\dfrac{{}_u\sigma_{k,k}}{1 - a_k^2}}} \text{ and } {}_y\rho_{j,k,s} = \frac{\dfrac{a_j^s\,{}_u\sigma_{j,k}}{1 - a_j a_k} + {}_{\mathrm{w}}\sigma_{j,k} \times \mathbb{1}_{\{s=0\}}}{\sqrt{\dfrac{{}_u\sigma_{j,j}}{1 - a_j^2} + {}_{\mathrm{w}}\sigma_{j,j}}\sqrt{\dfrac{{}_u\sigma_{k,k}}{1 - a_k^2} + {}_{\mathrm{w}}\sigma_{k,k}}},$$ *where $j,k \in \{v, e^2\}$ and $\mathbb{1}_{\{C\}}$ is an indicator function that equals 1 if the event C occurs, and 0 otherwise.*

Proposition 3.4 implies that for the SVV_0 model ${}_\alpha\rho_{j,k,s}/{}_\alpha\rho_{j,k,s-1} = {}_y\rho_{j,k,s}/{}_y\rho_{j,k,s-1} = a_j$ for any $j,k \in \{v, e^2\}$ and $s \geq 2$. If the volume and price volatility of trades jointly evolve according to the SVV_0 system, they become more autocorrelated (or more persistent, when $j = k$) and more cross-correlated (when $j \neq k$) when the AR(1) coefficients,

a_j, are closer to 1. Furthermore, $|_y\rho_{j,k,s}| < |_\alpha\rho_{j,k,s}|$ for any $j,k \in \{v, e^2\}$ and $s \geq 1$, suggesting that the measurement process y_t is less auto- and cross-correlated than the latent process α_t, as a consequence of an extra source of disturbances (i.e. the measurement errors) that only affects the former process.

Even though the first and second moments of $\alpha_t = (\log\phi_t, \log\sigma_t^2)'$ (i.e. the logarithms of expected volume and instantaneous volatility) and $y_t = (\log v_t, \log e_t^2)'$ (i.e. the logarithms of volume and the square of demeaned returns) of the state space system (3.6) exist in closed forms, those of the corresponding level quantities ($\chi_t = (\phi_t, \sigma_t^2)'$ and $\tau_t = (v_t, e_t^2)'$) in general do not have closed analytical expressions if B is not a null matrix (as in the SVV_2 model). This is because the joint distribution of the measurement errors $\mathrm{w}_t = (\mathrm{w}_{v,t}, \mathrm{w}_{e^2,t})'$ is not known, while the expressions of χ_t and τ_t involve products of the powers (with non-integer exponents) of $\exp \mathrm{w}_{v,t-i}$ and $\exp \mathrm{w}_{e^2,t-i}$ when B is not a zero matrix (as implied by equation (3.22) in the Appendix). In order to obtain some analytical expressions for the moments and cross-moments of χ_t and τ_t, we further assume that the measurement errors w_t are mutually independent when B is not null. Let

$$(I_2 - H)^{-1}\gamma = \begin{bmatrix} \overline{\gamma}_v \\ \overline{\gamma}_{e^2} \end{bmatrix}; H^i = \begin{bmatrix} h'_{v,i} \\ h'_{e^2,i} \end{bmatrix}; \text{and } H^i B = \begin{bmatrix} \beta_{v,v,i} & \beta_{v,e^2,i} \\ \beta_{e^2,v,i} & \beta_{e^2,e^2,i} \end{bmatrix} \text{for } i = 0, 1, 2, \cdots, \tag{3.7}$$

and let us define the moment generating function $M_q(\cdot)$ ($q \in \{v, e^2\}$) as[29]

$$M_q(z) := \mathbb{E}(\exp(z\mathrm{w}_{q,t})) = \begin{cases} \lambda_q^z \Gamma\left(\frac{z}{\delta_q} + 1\right), & \text{when } \mathrm{w}_{q,t} \sim \mathrm{LW}(\delta_q, \lambda_q), \\ \lambda_q^z \dfrac{\Gamma(\kappa_q + z)}{\Gamma(\kappa_q)}, & \text{when } \mathrm{w}_{q,t} \sim \mathrm{LG}(\kappa_q, \lambda_q), \end{cases} \tag{3.8}$$

then various moments of the processes $\chi_t := (\chi_{v,t}, \chi_{e^2,t})' = (\phi_t, \sigma_t^2)'$ and $\tau_t := (\tau_{v,t}, \tau_{e^2,t})' = (v_t, e_t^2)'$ when B is not a zero matrix (as in the SVV_2 model) are detailed in Proposition 3.5 below.

Proposition 3.5 *Consider the case where B is **not** a zero matrix. If the measurement errors w_t of the system (3.6) are mutually independent, then for $j,k \in \{v, e^2\}$ and $m,n,s \geq 0$, the*

[29]Note that $\mathrm{w}_{e^2,t} \sim \log\chi^2_{(1)} \equiv \mathrm{LG}(1/2, 2)$, and the summary for the $\mathrm{LW}(\delta_q, \lambda_q)$ and $\mathrm{LG}(\kappa_q, \lambda_q)$ distributions is given in Table 3.1.

expressions of the cross-moments of the processes χ_t and τ_t are given by

$$\mathbb{E}\left[\chi_{j,t}^m \chi_{k,t-s}^n\right] = \exp\left(m\overline{\gamma}_j + n\overline{\gamma}_k\right) \prod_{i=0}^{s-1} \exp\left(\frac{m^2}{2} h_{j,i}' \Sigma_u h_{j,i}\right) \prod_{i=0}^{\infty} \exp\left(\frac{1}{2}(mh_{j,i+s}' + nh_{k,i}')\Sigma_u (mh_{j,i+s} + nh_{k,i})\right)$$
$$\times \prod_{i=0}^{s-1} \left(\prod_{q\in\{v,e^2\}} M_q(m\beta_{j,q,i}) \right) \prod_{i=0}^{\infty} \left(\prod_{q\in\{v,e^2\}} M_q(m\beta_{j,q,i+s} + n\beta_{k,q,i}) \right) \tag{3.9}$$

$$\mathbb{E}\left[\tau_{j,t}^m \tau_{k,t-s}^n\right] = \mathbb{E}\left[\chi_{j,t}^m \chi_{k,t-s}^n\right] \times g(j,k,m,n,s), \tag{3.10}$$

where $\overline{\gamma}, h$ and β are defined in (3.7), $M_q(\cdot)$ is defined in (3.8), and

$$g(j,k,m,n,s) = \begin{cases} M_j(m)\dfrac{M_k(m\beta_{j,k,s-1} + n)}{M_k(m\beta_{j,k,s-1})}, & \text{when } s \geq 1, \\ M_j(m)M_k(n), & \text{when } s = 0 \,\&\, j \neq k, \\ M_j(m+n), & \text{when } s = 0 \,\&\, j = k. \end{cases} \tag{3.11}$$

When B is a null matrix (i.e. one assumes there is no transmission of the measurement shocks to the latent processes) as in the SVV_0 and SVV_1 models, $H = A$ and $\beta_{j,k,i} = 0$ for all $j,k \in \{v, e^2\}$ and $i = 0, 1, 2, \cdots$. Consequently, the moving average representation of χ_t and τ_t does not depend on the lags of the measurement errors w_t, and hence the cross-moments of χ_t and τ_t, except for the contemporaneous cross-moments of τ_t (i.e. $\mathbb{E}\left[\tau_{j,t}^m \tau_{k,t}^n\right]$ when $j \neq k$), can be analytically attained without the assumption of mutual independence between the measurement errors. Proposition 3.6 details this result.

Proposition 3.6 *Consider the case where B is a zero matrix. For $j,k \in \{v, e^2\}$ and $m,n,s \geq 0$, the expressions for the cross-moments of the processes χ_t and τ_t are given by*

$$\mathbb{E}\left[\chi_{j,t}^m \chi_{k,t-s}^n\right] = \exp\left(m\overline{\gamma}_j + n\overline{\gamma}_k\right) \prod_{i=0}^{s-1} \exp\left(\frac{m^2}{2} h_{j,i}' \Sigma_u h_{j,i}\right) \prod_{i=0}^{\infty} \exp\left(\frac{1}{2}(mh_{j,i+s}' + nh_{k,i}')\Sigma_u (mh_{j,i+s} + nh_{k,i})\right)$$

$$\mathbb{E}\left[\tau_{j,t}^m \tau_{k,t-s}^n\right] = \mathbb{E}\left[\chi_{j,t}^m \chi_{k,t-s}^n\right] \times g_1(j,k,m,n,s),$$

where $\overline{\gamma}, h$ and β are defined in (3.7),

$$g_1(j,k,m,n,s) = \begin{cases} M_j(m)M_k(n), & \text{when } s \geq 1, \\ \mathbb{E}(\exp(m\mathrm{w}_{j,t})\exp(n\mathrm{w}_{k,t})), & \text{when } s = 0 \,\&\, j \neq k, \\ M_j(m+n), & \text{when } s = 0 \,\&\, j = k, \end{cases}$$

and $M_j(\cdot)$ is defined in (3.8). When $s = 0 \,\&\, j \neq k$, $g_1(j,k,m,n,s)$ simplifies to $M_j(m)M_k(n)$ if one assumes that the measurement errors w_t are mutually independent.

Proposition 4.1 of Knight and Ning (2008) derives the cross-moments of trade durations that are modeled with an SCD model proposed by Bauwens and Veredas (2004). If the SCD model of Bauwens and Veredas (2004) was used to model trading volumes (which share many distributional similarities with trade durations), then we would have an SCV model for volumes, which is essentially the volume component of the SVV_0 model. In such a case the cross-moments of trading volumes ($\mathbb{E}\left[\tau_{v,t}^{m}\tau_{v,t-s}^{n}\right]$ or $\mathbb{E}[v_t^m v_{t-s}^n]$) implied by our Proposition 3.6 when B is a zero matrix and A is a diagonal matrix (i.e. under the SVV_0 model) would coincide with those detailed in Proposition 4.1 of Knight and Ning (2008), after straightforward simplifications.

To better examine the statistical properties of each individual volume and volatility series, we state their moments in the following Corollary.

Corollary 3.1 *Let $\overline{\gamma}, h$ and β be defined in (3.7), and $M_j(\cdot)$ ($j \in \{v, e^2\}$) be defined in (3.8). The m-th moment of $\tau_{j,t}$ ($j \in \{v, e^2\}$) is given by*

$$\mathbb{E}\left[\tau_{j,t}^{m}\right] = \mathbb{E}\left[\chi_{j,t}^{m}\right] \times M_j(m),$$

where

(i) if B is a non zero matrix and the measurement errors w_t are mutually independent,

$$\mathbb{E}\left[\chi_{j,t}^{m}\right] = \exp\left(m\overline{\gamma}_j\right)\prod_{i=0}^{\infty}\exp\left(\frac{m^2}{2}h_{j,i}'\Sigma_u h_{j,i}\right)\prod_{i=0}^{\infty}\left(\prod_{q\in\{v,e^2\}} M_q(m\beta_{j,q,i})\right),$$

(ii) if B is a zero matrix,

$$\mathbb{E}\left[\chi_{j,t}^{m}\right] = \exp\left(m\overline{\gamma}_j\right)\prod_{i=0}^{\infty}\exp\left(\frac{m^2}{2}h_{j,i}'\Sigma_u h_{j,i}\right),$$

(iii) if B is a zero matrix and A is a diagonal matrix (i.e. $A = diag(a_v, a_{e^2})$),

$$\mathbb{E}\left[\chi_{j,t}^{m}\right] = \exp\left(\frac{m\gamma_j}{1-a_j}\right)\exp\left(\frac{m^2\,{}_u\sigma_{j,j}}{2(1-a_j^2)}\right),$$

where ${}_u\sigma_{j,j} = \mathrm{Var}(u_{j,t})$.

From Corollary 3.1, the m-th moment of the squared demeaned returns $\tau_{e^2,t}$ or e_t^2 of a standard univariate SV model, which is the return/volatility component of our baseline SVV_0 model, is given by

$$\mathbb{E}\left[\tau_{e^2,t}^{m}\right] = \mathbb{E}\left[e_t^{2m}\right] = \exp\left(\frac{m\gamma_{e^2}}{1-a_{e^2}}\right)\exp\left(\frac{m^2\,{}_u\sigma_{e^2,e^2}}{2(1-a_{e^2}^2)}\right) \times \frac{2^m\Gamma(m+\frac{1}{2})}{\Gamma(\frac{1}{2})},$$

which is the same as that stated in Ghysels et al. (1996, eq. 3.2.2) and Shephard (1996, eq. 1.13).

Having obtained the cross-moments of χ_t and τ_t, we can derive the autocorrelation and cross correlation functions for these processes. The correlation functions implied by the SVV_1 and SVV_2 models, albeit analytically available if one assumes the mutual independence between the measurement errors, are complicated and difficult to interpret. Instead, we report the results for the simplest bivariate model SVV_0 in which B is a zero matrix and A is a diagonal matrix (i.e. $A = diag(a_v, a_{e^2})$). Despite its simplicity, the SVV_0 model is similar to the multivariate stochastic volatility model developed by Harvey et al. (1994) and Danielsson (1998) and it generalizes the basic but popular univariate SV and SCD models in the literature (e.g. Jacquier et al., 1994, Ruiz, 1994, Bauwens and Veredas, 2004, Knight and Ning, 2008). The results are given in Proposition 3.7.

Proposition 3.7 *Consider the SVV_0 system where B is a zero matrix and A is a diagonal matrix (i.e. $A = diag(a_v, a_{e^2})$). Define by ${}_x\rho_{j,k,s} = \text{Corr}(x_{j,t}, x_{k,t-s})$ the cross correlation function between two series $x_{j,t}$ and $x_{k,t-s}$, for some $s \geq 0$. Let $\Sigma_u = [{}_u\sigma_{j,k}]$ $(j,k \in \{v, e^2\})$. The cross correlation functions of the processes χ_t and τ_t, for $j,k \in \{v, e^2\}$ and $s \geq 0$, are given by*

$$
{}_\chi\rho_{j,k,s} = \frac{\exp\left(\frac{a_j^s\, {}_u\sigma_{j,k}}{1-a_j a_k}\right) - 1}{\sqrt{\exp\left(\frac{{}_u\sigma_{j,j}}{1-a_j^2}\right) - 1}\sqrt{\exp\left(\frac{{}_u\sigma_{k,k}}{1-a_k^2}\right) - 1}},
$$

$$
{}_\tau\rho_{j,k,s} = \frac{\exp\left(\frac{a_j^s\, {}_u\sigma_{j,k}}{1-a_j a_k}\right) \times g_1(j,k,1,1,s) - M_j(1)M_k(1)}{\sqrt{M_j(2)\exp\left(\frac{{}_u\sigma_{j,j}}{1-a_j^2}\right) - [M_j(1)]^2}\sqrt{M_k(2)\exp\left(\frac{{}_u\sigma_{k,k}}{1-a_k^2}\right) - [M_k(1)]^2}},
$$

where $M_j(\cdot)$ is defined in (3.8) and $g_1(\cdot)$ in Proposition 3.6.

Proposition 3.7 implies that the cross correlation functions of the χ_t and τ_t processes of the SVV_0 system tend to 0 as s goes to infinity, since (i) $|a_j| < 1$ (assuming stationarity) so a_j^s goes to 0 when s becomes large; and (ii) $g_1(j,k,1,1,s) = M_j(1)M_k(1)$ for $s \geq 1$ (see Proposition 3.6). Moreover, for large s and/or small (i.e. close to 0) ${}_u\sigma_{j,k}$, these correlation functions decay geometrically at the rate of a_j,[30] suggesting that the χ_t and τ_t processes are more persistent the closer a_j is to unity.

The autocorrelation function (ACF) of the squared demeaned returns under the SVV_0 system implied by Proposition 3.7 is given by

$$\text{Corr}(e_t^2, e_{t-s}^2) = {}_{\tau}\rho_{e^2,e^2,s} = \frac{\exp\left(\dfrac{a_{e^2}^s \; {}_u\sigma_{e^2,e^2}}{1-a_{e^2}^2}\right) - 1}{3\exp\left(\dfrac{{}_u\sigma_{e^2,e^2}}{1-a_{e^2}^2}\right) - 1}, \; s \geq 1, \tag{3.12}$$

where we have used equation (3.8) to obtain $M_{e^2}(2)/[M_{e^2}(1)]^2 = \Gamma(\frac{1}{2}+2)\Gamma(\frac{1}{2})/\left[\Gamma(\frac{1}{2}+1)\right]^2$ $= 3$, since $\mathrm{w}_{e^2,t} \sim \log\chi^2_{(1)} \equiv \text{LG}(1/2, 2)$. Despite the difference in notation, the ACF in (3.12) coincides with the one that was previously derived for a standard univariate SV model; See Ghysels et al. (1996, eq. 3.2.3, when $c = 2$), Shephard (1996, eq. 1.14), Carnero et al. (2004, eq. 7), or Taylor (2008, eq. 3.5.10).

3.2.4 Estimation of the state-space system

The difficulty associated with the estimation of a state space model such as model (3.6) is driven by the stochastic nature of the latent state variable which is not fully observed given past information. Consequently, there are more unknowns (which include the unknown parameters of the model and the unobserved state variables) than the number of observations, rendering the likelihood function for the state space model very complex since its expression involves a high-dimensional integral over the space of all the nuisance latent variables. This is in sharp contrast to autoregressive conditional models such as ARCH, GARCH or ACD models, which assume that conditioning on the past information an expected quantity is deterministic, allowing the likelihood function to be presented in an analytic and closed form expression. Due to the curse of dimensionality, the integration of a state space likelihood function cannot be evaluated using traditional deterministic numerical methods.

[30] For large s and/or small (i.e. close to 0) ${}_u\sigma_{j,k}$ such that $(a_j^s \; {}_u\sigma_{j,k})/(1-a_ja_k)$ is close to 0, $\dfrac{{}_{\chi}\rho_{j,k,s}}{{}_{\chi}\rho_{j,k,s-1}} = \dfrac{{}_{\tau}\rho_{j,k,s}}{{}_{\tau}\rho_{j,k,s-1}} = \dfrac{\exp\left(\frac{a_j^s \; {}_u\sigma_{j,k}}{1-a_ja_k}\right)-1}{\exp\left(\frac{a_j^{s-1} \; {}_u\sigma_{j,k}}{1-a_ja_k}\right)-1} \approx \dfrac{\frac{a_j^s \; {}_u\sigma_{j,k}}{1-a_ja_k}}{\frac{a_j^{s-1} \; {}_u\sigma_{j,k}}{1-a_ja_k}} = a_j.$

Different methods have been developed in the literature to estimate state space models. The Kalman filter algorithm is probably the most popular method in the field, and it can be applied to linear and Gaussian state space models (i.e. both measurement and state equations are linear with Gaussian disturbances) to produce the minimum mean square estimates or estimators (MMSEs) of the state variables in a recursive manner via conditional expectations (Harvey, 1989). The **exact** likelihood function constructed using these estimates is then maximized to obtain the maximum likelihood estimates (MLEs) of the unknown parameters which, under usual regularity conditions, attain consistency, asymptotic normality and asymptotic efficiency, provided that the linearity and Gaussianity of the state space model has been correctly specified.

However, when the Gaussianity condition is not satisfied the Kalman filter no longer produces the MMSEs of the latent variables but only the minimum **linear** mean square estimators (MLMSEs) (Harvey, 1989, Ruiz, 1994). Also, the exact likelihood cannot be obtained. A quasi likelihood function is produced instead by treating the disturbances as if they were normally distributed, with the mean and variance of a normal approximation that correctly match those of the exact non-normal distribution. The maximization of the quasi likelihood function results in QML estimates which still attain consistency and asymptotic normality under correct model specifications and regularity conditions, but they are no longer asymptotically efficient (Harvey et al., 1994, Ruiz, 1994). Nevertheless, QML is still an attractive approach because it is relatively easy to implement and it also provides the best linear estimates of the latent variables via the Kalman filter and smoother. Examples of the application of QML estimation in financial econometrics include Harvey et al. (1994), Ruiz (1994), and Harvey and Shephard (1996) in an SV context and Bauwens and Veredas (2004) in an SCD model.

There are more sophisticated methods of estimating nonlinear and non-Gaussian state space models that better capture the exact likelihood, such as Kim et al. (1998) (for a Bayesian approach) and Durbin and Koopman (1997) (for a frequentist approach). These methods provide better ways to approximate the true non-Gaussian distribution (usually of the measurement equation), either by a mixture of normal densities (Carter and Kohn, 1994, Shephard, 1994, Kim et al., 1998) or by bias-correcting for the difference between the true non-Gaussian density and the approximating Gaussian one (Durbin and Koopman, 1997, 2000, Sandmann and Koopman, 1998, Feng et al., 2004, Strickland et al., 2006). The latter method involves (i) the approximation of the non-Gaussian density at each observation by a Gaussian distribution such that the first two derivatives of the two log densities (the true non-Gaussian and the approximating Gaussian) at each observation are the same; and (ii) the addition (to the quasi log likelihood function) of a bias correction term that is computed via simulation from the approximating Gaussian (i.e. "importance") density (see Durbin and Koopman (1997) and Sandmann and Koopman (1998) for more details). For these methods to work, the true non-Gaussian density must be entirely known. These methods are based on simulations and hence are highly computationally demanding, especially for large data sets. In our bivariate stochastic conditional volume-volatility model, the joint non-Gaussian distribution of the measurement innovations is unknown unless they are independent. Moreover, since the transaction datasets we investigate are large (see Section 3.3), the implementation of the aforementioned methods (which have been mostly applied to single-equation or univariate settings) to a bivariate or multivariate model will be computationally difficult and time consuming, even if we assume the measurement errors to be independent and hence know their joint distribution. Thus, we leave these methods for future work and rely on QML for estimating our models.

We treat $\epsilon_t = \mathrm{w}_t - \omega$ as if it were normally distributed to apply QML estimation. That is, we approximate the true distribution of ϵ_t with a bivariate normal distribution with zero means and a variance-covariance matrix that exactly matches that of ϵ_t, i.e. $N(0, \Sigma_\epsilon)$, where $\Sigma_\epsilon = \Sigma_{\mathrm{w}} := \mathrm{Var}(\mathrm{w}_t)$. Equivalently, the true distribution of w_t is approximated with a normal distribution $N(\omega, \Sigma_{\mathrm{w}})$. The matrix Σ_ϵ, or Σ_{w}, which is to be estimated, has diagonal elements that depend on the assumed distributions of ϵ_t, and these are given in the second last column of Table 3.1.[31] With this normal approximation and an assumed (bivariate) normal distribution for the latent disturbances u_t, we have a linear and Gaussian approximation of the state space system (3.6) based

[31] The log $\chi^2_{(1)}$ distribution for $\log \zeta_t^2$ is equivalent to an $\mathrm{LG}(1/2, 2)$ distribution.

on which a quasi-likelihood function can be derived via the Kalman filter and subsequently be maximized. If we denote by $\widehat{\theta}$ the QML estimator of the true unknown parameter vector θ_0, then its limiting distribution (e.g. White, 1982, Hamilton, 1994) is

$$\sqrt{n}(\widehat{\theta}-\theta_0) \xrightarrow{d} N(0, I^{-1}JI^{-1}),$$

where

$$I = -\mathbb{E}_{f_0}\left[\frac{\partial^2 \log f(Y|\theta)}{\partial\theta\partial\theta'}\right]_{\theta=\theta_0},$$

$$J = \mathbb{E}_{f_0}\left[\frac{\partial \log f(Y|\theta)}{\partial\theta}\frac{\partial \log f(Y|\theta)}{\partial\theta'}\right]_{\theta=\theta_0},$$

where $\mathbb{E}$ denotes the expectation operator, f_0 and f respectively denote the true density and the approximating quasi density, Y denotes the observed data, and n is the sample size. The matrices I and J can be consistently estimated by their sample quantities

$$\widehat{I} = -\frac{1}{n}\sum_{i=1}^{n}\left[\frac{\partial^2 \log f(y_i|\theta)}{\partial\theta\partial\theta'}\right]_{\theta=\widehat{\theta}}, \text{ and}$$

$$\widehat{J} = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{\partial \log f(y_i|\theta)}{\partial\theta}\frac{\partial \log f(y_i|\theta)}{\partial\theta'}\right]_{\theta=\widehat{\theta}}.$$

Several parameters in our bivariate stochastic models are either positive (i.e. the (log-) Weibull parameter δ and the (log-) Gamma parameter κ) or positive definite (i.e. Σ_{w} and Σ_u). We estimate the logarithmic transformation (i.e. $\log\delta$ or $\log\kappa$) to ensure the positivity of the (log-) Weibull or Gamma parameter. Meanwhile, we ensure the positive definiteness of Σ_j ($j \in \{\mathrm{w}, u\}$) by decomposing it as $\Sigma_j = D_j R_j D_j$, where D_j is a diagonal matrix of the standard deviations (i.e. the square roots of the diagonal elements of Σ_j), and R_j is the corresponding correlation matrix, which, by definition, has one on its diagonal. Since R_j is a 2×2 matrix, imposing that its symmetric off-diagonal elements are between -1 and 1 (which can be achieved by applying the following transformation $z = (1-e^x)/(1+e^x)$, where z is bounded between -1 and 1 while x is unbounded) is sufficient to ensure that R_j, and hence Σ_j ($j \in \{\mathrm{w}, u\}$), is positive definite. While the parameters of the diagonal standard deviation matrix D_{w} are deterministic functions of the (log-) Weibull or Gamma parameter (see Table 3.1) and hence they do not require separate estimation, those of D_u need to be estimated and they are logarithmically transformed to ensure positivity. We impose the stationarity restriction on our bivariate models by checking the stationarity condition presented in Proposition 3.1 at each estimation iteration throughout the maximization of the quasi log likelihood function, and we penalize the log likelihood function (i.e. make the log likelihood value very small, which in our case is set equal to -10^8 multiplied by the number of observations) at iterations for which the stationarity condition is not satisfied.

3.2.5 Simulation study

We generate samples of $n = 10,000$, $n = 100,000$, and $n = 250,000$ observations for volume and volatility from the following SVV_2 data generating process (DGP), and estimate the parameters using QML, to illustrate the performance of QML in our setting. The generated model is[32]

$$\begin{bmatrix}\log v_t\\ \log r_t^2\end{bmatrix} = \begin{bmatrix}\log\phi_t\\ \log\sigma_t^2\end{bmatrix} + \begin{bmatrix}\log\varepsilon_t\\ \log\zeta_t^2\end{bmatrix} = \begin{bmatrix}\log\phi_t\\ \log\sigma_t^2\end{bmatrix} + \begin{bmatrix}\mathrm{w}_{v,t}\\ \mathrm{w}_{e^2,t}\end{bmatrix},$$

$$\begin{bmatrix}\log\phi_t\\ \log\sigma_t^2\end{bmatrix} = \begin{bmatrix}0.2060\\ 0.1906\end{bmatrix} + \begin{bmatrix}0.90 & 0\\ 0 & 0.85\end{bmatrix}\begin{bmatrix}\log\phi_{t-1}\\ \log\sigma_{t-1}^2\end{bmatrix} + \begin{bmatrix}0 & 0.05\\ 0.15 & 0\end{bmatrix}\begin{bmatrix}\log v_{t-1}\\ \log r_{t-1}^2\end{bmatrix} + \begin{bmatrix}u_{v,t}\\ u_{e^2,t}\end{bmatrix},$$

where $\mathrm{w}_{v,t} \overset{iid}{\sim} \mathrm{LW}(\delta = 0.7, \lambda = 1/\Gamma(1+1/\delta))$, $\mathrm{w}_{e^2,t} \overset{iid}{\sim} \log\chi^2_{(1)}$, and for simplicity, $\mathrm{w}_{v,t}$ and $\mathrm{w}_{e^2,t}$ are generated independently. Thus, $\omega = \mathbb{E}\begin{bmatrix}\mathrm{w}_{v,t}\\ \mathrm{w}_{e^2,t}\end{bmatrix} = \begin{bmatrix}-1.0603\\ -1.2704\end{bmatrix}$, and $\Sigma_{\mathrm{w}} = \mathrm{Var}\begin{bmatrix}\mathrm{w}_{v,t}\\ \mathrm{w}_{e^2,t}\end{bmatrix} = \begin{bmatrix}3.3570 & 0\\ 0 & 4.9348\end{bmatrix}$. Meanwhile, $u_t = (u_{v,t}, u_{e^2,t})' \overset{iid}{\sim} \mathrm{N}(0, \Sigma_u)$ with $\Sigma_u = \begin{bmatrix}0.4922 & 0.1329\\ 0.1329 & 1.3381\end{bmatrix}$, so $\mathrm{Corr}(u) = \begin{bmatrix}1 & 0.1638\\ 0.1638 & 1\end{bmatrix}$. $\mathrm{w}_t = (\mathrm{w}_{v,t}, \mathrm{w}_{e^2,t})'$ and u_s are independent for all t and s. The true parameter values are fixed so that $\mathbb{E}(\log v_t) = \mathbb{E}(\log r_t^2) = 2$, $\mathrm{Var}(\log v_t) = 9$, $\mathrm{Var}(\log r_t^2) = 16$ and $\mathrm{Corr}(\log v_t, \log r_t^2) = 0.5$.[33]

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[32]We assume that the drift term μ_t of the return process is equal to 0 so that $r_t = e_t$.

[33]We first fix the distribution of $\mathrm{w}_{v,t}$ and the coefficient matrices A and B, then compute γ and Σ_u (using the results in Proposition 3.3) to achieve the target.

Table 3.2 presents the estimated SVV_2 models using QML estimation.[34] As the sample size increases, the estimated parameters become closer to the true values, while the standard errors become smaller. This result demonstrates the consistency of the QML estimators and illustrates the validity of the QML estimation method.[35]

Table 3.2: Estimated SVV_2 model using simulated data

	δ	γ	A		B		Σ_{w}		Σ_u	
			$\log\phi_{t-1}$	$\log\sigma^2_{t-1}$	$\log v_{t-1}$	$\log e^2_{t-1}$	v	σ^2	v	σ^2
Panel A: True DGP										
$\log\phi_t$	0.7000	0.2060	0.9000	-	-	0.0500	3.3570	-	0.4922	-
$\log\sigma^2_t$	-	0.1906	-	0.8500	0.1500	-	0.0000	4.9348	0.1638	1.3381
Panel B: $n = 10{,}000$ observations										
$\log\phi_t$	0.7199	0.2323	0.8896	-	-	0.0514	3.1738	-	0.5514	-
	(0.0087)	(0.0218)	(0.0121)	-	-	(0.0086)	-	-	(0.0443)	-
$\log\sigma^2_t$	-	0.2041	-	0.8569	0.1320	-	-0.0324	4.9348	0.2174	1.2863
	-	(0.0182)	-	(0.0111)	(0.0152)	-	(0.0245)	-	(0.0801)	(0.0869)
Panel C: $n = 100{,}000$ observations										
$\log\phi_t$	0.7011	0.2082	0.9008	-	-	0.0477	3.3465	-	0.4992	-
	(0.0026)	(0.0071)	(0.0040)	-	-	(0.0029)	-	-	(0.0134)	-
$\log\sigma^2_t$	-	0.1873	-	0.8548	0.1438	-	-0.0138	4.9348	0.2026	1.2806
	-	(0.0056)	-	(0.0038)	(0.0050)	-	(0.0080)	-	(0.0273)	(0.0296)
Panel D: $n = 250{,}000$ observations										
$\log\phi_t$	0.7008	0.2058	0.9008	-	-	0.0486	3.3494	-	0.4921	-
	(0.0016)	(0.0040)	(0.0022)	-	-	(0.0016)	-	-	(0.0075)	-
$\log\sigma^2_t$	-	0.1943	-	0.8499	0.1489	-	-0.0024	4.9348	0.1835	1.3526
	-	(0.0036)	-	(0.0020)	(0.0028)	-	(0.0048)	-	(0.0165)	(0.0175)

This table shows the estimates from a bivariate stochastic conditional volume-volatility SVV_2 model using QML estimation. Simulated data are generated from the following true DGP:

$$y_t = \alpha_t + \text{w}_t,$$
$$\alpha_t = \gamma + A\alpha_{t-1} + By_{t-1} + u_t,$$

where $y_t = (\log v_t, \log e_t^2)'$, $\alpha_t = (\log\phi_t, \log\sigma_t^2)'$, $\text{w}_t = (\log\varepsilon_t, \log\zeta_t^2)' \sim \text{iid}(\omega, \Sigma_{\text{w}})$, $u_t \overset{iid}{\sim} N(0, \Sigma_u)$, w_t is independent of u_s for all t,s, and the true values of $\gamma, A, B, \Sigma_{\text{w}}$, Σ_u are specified in Panel A. The diagonal elements of Σ_{w} and Σ_u are variances, while the lower off-diagonal elements are correlations. The **measurement error of trading volumes** (i.e. $\log\varepsilon_t$) is generated from a **log-Weibull distribution** $\text{LW}(\delta = 0.7, \lambda = 1/\Gamma(1+1/\delta))$, while that of volatility (i.e. $\log\zeta_t^2$) is $\log\chi^2_{(1)}$ distributed and independent of $\log\varepsilon_t$. Panels B, C and D report the QML estimates of the model parameters based on a simulated sample of $n = 10{,}000$, $n = 100{,}000$ and $n = 250{,}000$ observations, respectively. Robust standard errors are reported in parentheses. The diagonal elements of Σ_{w} are not free parameters in the $SDVV_2$ model, but they are functions of the free parameter δ (given in Table 3.1). We therefore do not report their standard errors, even though the latter can be computed using the Delta method.

[34] We obtained the same set of QML estimates, regardless of where the maximization of the quasi log likelihood function had been initiated.

[35] We conducted a wide range of simulation experiments using all three state space alternatives as the true DGP. Results of these simulations, which are omitted for brevity but available upon request, are qualitatively similar to those of the simulation displayed in this section, and the QML estimates closely match the true DGP.

3.3 Data

We illustrate the estimation of the bivariate stochastic models SVV_0, SVV_1 and SVV_2 using tick-by-tick data for one large cap and one small cap stock listed on the Australian Securities Exchange (ASX), which are BHP (BHP Billiton Limited) and CHC respectively. We choose BHP because it is one of the largest and most important companies in Australia (with market capitalization of about AU$115.62bn, as of 1 July 2014) and is often selected to examine by many studies in Australia, just as IBM (International Business Machines Corporation) is often selected as a well-known stock traded in the US market. In contrast, CHC is chosen because it is a small property firm (with market capitalization of about AU$1.47bn) in the Real Estate Investment Trust industry, which is quite different from the Industrial Metals & Minerals industry in which BHP belongs. The choice of these two stocks, which belong in two different market capitalization groups and two different industries, illustrates differences in the relationship between volume and volatility for stocks with different corporate characteristics.

We consider a sample time period that covers twenty-one business trading days in August 2014, during which there were no abnormal market events. We collect a time and sales dataset from the Securities Industry Research Center of Asia-Pacific (SIRCA), which records details on every transaction of a stock, such as the date and time (to millisecond precision), price, volume (number of shares), value (dollar value) and qualifiers.[36] The dataset also contains information about the best bid and ask quotes such as price, volume, spread and relative spread at any time instant when there is a change to the best bid/ask prices or volumes.

We extract all trades that are performed within the continuous trading session in the lit market (from 10:10:00 to 16:00:00) and discard all transactions executed in the opening auction (i.e. either during 10:00:00-10:10:00 or with "AC" qualifiers that define auction trades) and in dark pools. We use the information about the best quotes

[36] Each trade contains a qualifier that declares some qualitative property of the trade. For example, a "Bi" ("Si") qualifier signifies a buyer-initiated (seller-initiated) trade, an "XT" denotes a cross trade, while a "CX" is attached to trades that are executed in an Australian dark pool called Centre Point.

(after having removed all observations with a negative bid or ask quote, and any observation with a bid quote larger than ask quote) to work out the bid-ask midpoint before each transaction. Since one large buy (sell) marketable order can be matched against several limit orders queuing on the sell (buy) side of the limit order book and hence result in multiple instantaneous transactions that have zero durations, we aggregate trades executed at the same time into one "large" trade by calculating volume-weighted average prices and summing up the volumes of the small trades. After this cleaning procedure, which is standard in the literature (see, amongst others, Dufour and Engle, 2000, Nowak and Anderson, 2014, Renault et al., 2014), the final data sets consist of 117,970 trades for BHP and 15,685 trades for CHC. We winsorize the volume and return data of each stock at the 0.5th and 99.5th percentiles to avoid the effects of outliers.

Table 3.3: Summary statistics for stocks BHP and CHC in August 2014

	BHP		**CHC**	
	Volume (shares)	Return (%)	Volume (shares)	Return (%)
Min	1.0	-0.027	1.0	-0.238
25%	60.0	0.000	13.0	0.000
Mean	717.6	0.000	416.5	0.000
Median	200.0	0.000	54.0	0.000
75%	692.0	0.000	236.0	0.000
Max	10822.1	0.038	10993.1	0.238
Std. dev.	1450.1	0.011	1259.2	0.064
Skewness	4.1	0.110	5.8	0.014
Kurtosis	23.6	5.082	42.1	11.030
n	117970	117970	15685	15685

This table reports some summary statistics of trading volumes and returns for stocks BHP and CHC in August 2014. These statistics include the sample minimum (Min), 25% quantile (25%), sample average (Mean), sample median (Median), 75% quantile (75%), sample maximum (Max), sample standard deviation (Std. dev.), sample skewness (Skewness), sample kurtosis (Kurtosis), and sample size (n).

Table 3.3 provides some descriptive statistics for the two stocks in August 2014. Reflecting its higher level of liquidity, BHP trades much more frequently (117,970 vs. 15,685 trades) with a remarkably larger average trading volume (717.6 vs. 416.5 shares) but with a significantly narrower range of price adjustments ([-0.027%, 0.038%] vs. [-0.238%, 0.238%]) than does CHC. Consistent with well-documented stylized facts, trading volumes of both stocks exhibit positive skewness and excess kurtosis (e.g. Andersen, 1996, Manganelli, 2005, Menkhoff et al., 2010, Nowak and Anderson, 2014). In addition, trading volumes are over-dispersed since the standard deviation is much larger than the mean. This characteristic is often observed for trade durations (e.g. Engle and Russell, 1998, Bauwens and Veredas, 2004), but it is also noted elsewhere for trading volumes (e.g. Menkhoff et al., 2010, Nowak and Anderson, 2014). The majority of transactions for each stock do not result in any price adjustment since returns are zero within the interquantile range (25%-75%) of the distribution. This observation is typically found in many financial tick-by-tick return series, and it is reported by, for example, Dufour and Engle (2000), Pelletier and Zheng (2013), Renault et al. (2014), and Pham et al. (2017) in their studies.

It is well-known in the literature that trading volumes and returns exhibit diurnal patterns (see Engle and Russell, 1998, Manganelli, 2005, Renault et al., 2014, Pham et al., 2017). These financial data can be thought of as a multiplicative function of two components: a stochastic part that is to be analyzed by some econometric model, and a deterministic part that is driven by the systematic and cyclical pattern of market activities throughout the trading day (Engle and Russell, 1998). The deterministic component is predictable and is usually removed from the raw financial data. Following Engle and Russell (1998), we estimate the diurnal patterns of trading volumes and absolute returns by fitting a cubic spline to the raw series that has the following form

$$\varphi(z_i) = \beta_0 + \beta_1 z_i + \beta_2 z_i^2 + \beta_3 z_i^3 + \sum_{j=1}^{k} \beta_{j+3} \left[(z_i - c_j)^3 \times I_{z_i > c_j} \right],$$

where z_i is the clock time of the i-th trade, c_j ($j = 1, \cdots, k$) are the spline knots,[37] $I_{z_i > c_j}$ is an indicator function that equals 1 if $z_i > c_j$ and 0 otherwise. Diurnally adjusted data (volume and returns) are obtained by dividing the raw data by the corresponding fitted diurnal component.

Figure 3.1 shows the intradaily patterns for trading volumes and absolute returns for BHP and CHC over the sample period. Consistent with well-known stylized facts, there is an overall inverse U-shaped pattern for trading volumes, with larger transac-

[37] Since the trading day in our dataset runs from 10:10 to 16:00, we set the knots at 10:30, 11:00, 11:30, 12:00, 12:30, 13:00, 14:00, 14:30, 15:00, 15:30 and 15:45.

tions often being executed around the beginning and towards the end of the trading day during which the market is most active, and with smaller trades around lunchtime (13:00). Interestingly, trading volumes of BHP drop sharply during the last 15 minutes before the market close. Meanwhile, price adjustments, in magnitude, exhibit a downward trend as the trading day progresses, which is also observed by Renault et al. (2014) and Pham et al. (2017). Prices move more quickly around the open of the market as a result of the higher trading intensity initiated by informed investors who attempt to take advantage of new information that has accumulated overnight, whereas increased trading activities around the close of the market, which are primarily contributed by uninformed traders, have slight impact on prices, leading to small price adjustments (Anand et al., 2005, Bloomfield et al., 2005, Duong et al., 2009). However, it appears that transactions around the market close of stock CHC have a slightly higher impact on prices than those during lunchtime.

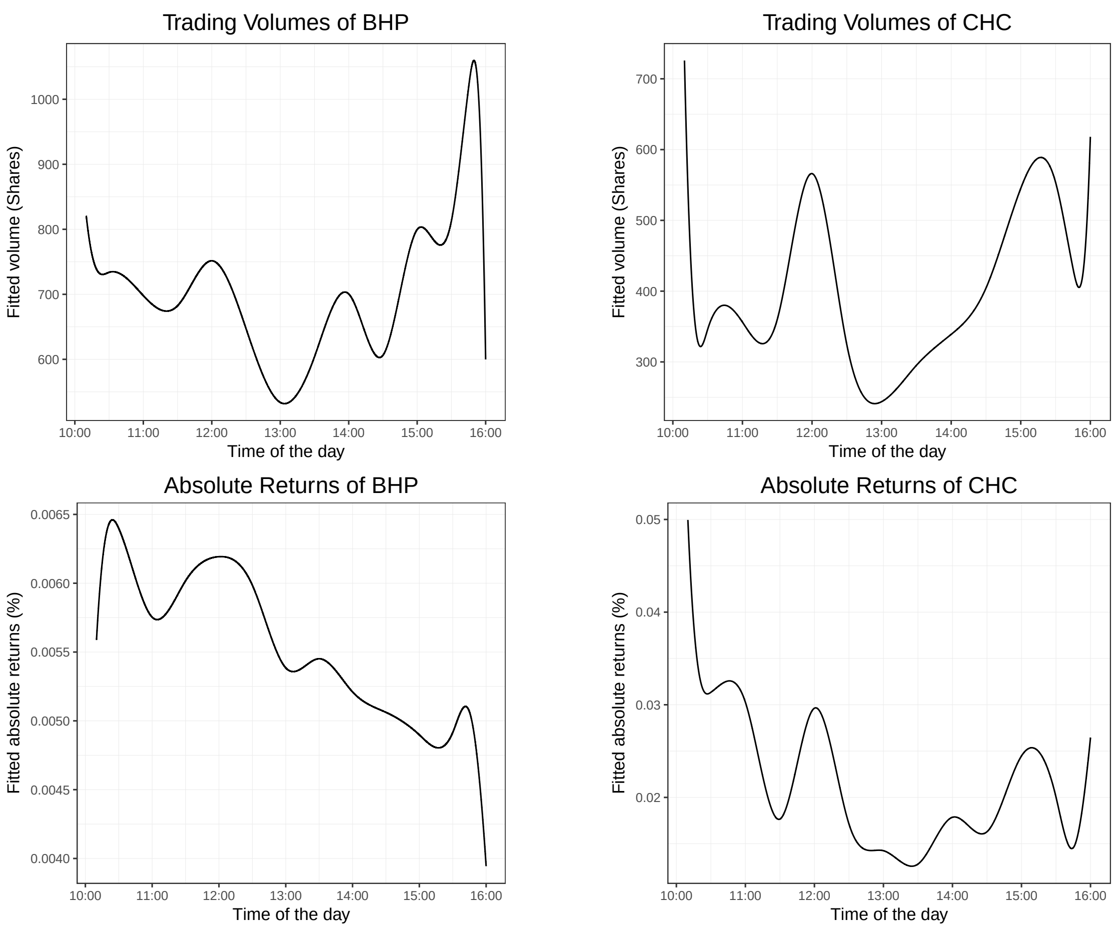


Figure 3.1: Diurnal patterns for the trading volumes and absolute returns of stocks BHP and CHC.

3.4 Results and discussion

This section discusses the estimated results for our proposed bivariate stochastic volume-volatility models of two Australian stocks, namely BHP and CHC. The estimation results for these models are presented in subsection 3.4.1, followed by a discussion of model diagnostics in subsection 3.4.2. We conduct an impulse response analysis in subsection 3.4.3 in order to answer the question of how both trading volumes and return volatility react if there is a shock to either variable.

3.4.1 Estimation results

Tables 3.4 and 3.5 present the QML estimation results of two univariate stochastic models (Panel A) and of three bivariate stochastic conditional volume-volatility models (Panels B, C, and D), imposing a **log-Weibull** distributional assumption on the measurement error of trading volumes, for BHP and CHC, respectively. The corresponding results that assume a **log-Gamma** distribution for volumes' measurement innovation are reported in Tables 3.6 and 3.7. These estimated results are insensitive to starting values for the QML estimates. We report, in the measurement and latent variance-covariance matrices (i.e. Σ_{w} and Σ_u, respectively) of the bivariate models, correlation estimates, instead of covariance estimates, in the lower off-diagonal positions, while still reporting variance estimates in the diagonal positions. This facilitates interpretation. Note that due to the use of QML estimation that approximates the true distribution of the non-Gaussian measurement errors with a Gaussian distribution with the same mean and variance, the assumption of different distributions for the measurement innovations of the state space model (3.6) does not affect the estimation and result of the structural parameters of the model (which include the coefficient matrices A and B and the variance-covariance matrices Σ_{w} and Σ_u), as well as the maximized log likelihood value. It only affects those of the parameters of the assumed distribution and the intercept γ.[38] Thus, we obtain the same log likelihood value and identical

[38] In our cases, the VARMA(1,1) representation of y_t in the state space model (3.6) derived in equation (3.21) is $y_t = (\omega + \gamma - A\omega) + (A + B)y_{t-1} + (u_t + \epsilon_t - A\epsilon_{t-1})$, where $\epsilon_t = \mathrm{w}_t - \omega$, $\omega = \mathbb{E}(\mathrm{w}_t)$, $\mathrm{w}_t \overset{iid}{\sim} (\omega, \Sigma_{\mathrm{w}})$,

estimated coefficients for the structural parameters ($A, B, \Sigma_{\mathrm{w}}$, and Σ_u) for each stock when a different error distribution for trading volume is assumed; however, we obtain different estimated coefficients for the intercept (γ) and the distributional parameter (either δ or κ) (compare Table 3.4 with Table 3.6, and Table 3.5 with Table 3.7).

We first investigate the estimated results for two separate univariate stochastic models (SCV and SV) which are reported in Panels A of Tables 3.4-3.7. Consistent with previous studies such as Harvey and Shephard (1996), Manganelli (2005) and Yu (2005), both volume and volatility processes of each stock are highly persistent but still stationary since their corresponding persistence parameters or autoregressive coefficients are large but significantly smaller than one.

The estimated shape parameter δ (κ) of the log-Weibull (log-Gamma) distribution for the trading volumes of the two stocks is significantly less than one, which, as discussed in Bauwens and Veredas (2004), is a sufficient (but not necessary) condition for an overdispersed distribution. Thus, the SCV model is able to capture the overdispersion in the empirical data (shown in Table 3.3), whereas the use of a log-Exponential distributional assumption, which implies an equidispersed distribution, would not be able to do this.[39] Meanwhile, the estimated variances of the latent innovations ($u_{v,t}$ and $u_{e^2,t}$) of each process are significantly different from zero, especially for volatility. Consistent with prior studies such as Jacquier et al. (1994), Kim et al. (1998), and Carnero et al. (2004), this result implies that the GARCH/ACD-type or VAR-type models, which assume conditional deterministicity, might be inadequate for capturing the dynamics of conditional expected volume and volatility. These quantities are better mod eled with a stochastic process.

$u_t \overset{iid}{\sim} N(0, \Sigma_u)$, and w_t is independent of u_s for all t and s. When we approximate the non-Gaussian distribution of w_t with a Gaussian distribution $N(\omega, \Sigma_{\mathrm{w}})$, we can estimate $A, B, \Sigma_{\mathrm{w}}, \Sigma_u$, and $\widetilde{\gamma} = \omega + \gamma - A\omega$, which are unaffected by the actual non-Gaussian shape of w_t. Depending on the assumption of the true non-Gaussian distribution of w_t, which can be either log-Weibull or log-Gamma in this Chapter, one can retrieve the estimates of its parameters and then derive ω from the estimate of Σ_{w} using Table 3.1, which allows γ to be computed. Therefore, the estimation of the non-Gaussian distribution parameters and the intercept γ depends on the assumption of the true distribution.

[39] Note, however, that we are not able to tell, based on an information criterion, which distribution (log-Weibull or log-Gamma, both of which have the same number of parameters) is more appropriate for the modeling of trading volume since we obtain the same log likelihood value for the two distributions, as a consequence of QML estimation.

Table 3.4: Estimated stochastic conditional volume-volatility models for stock BHP, using the **log-Weibull** distribution

	δ	γ	A		B		Σ_{w}		Σ_u		Loglik
			$\log\phi_{t-1}$	$\log\sigma^2_{t-1}$	$\log v_{t-1}$	$\log e^2_{t-1}$	v	σ^2	v	σ^2	
Panel A: Univariate stochastic models											-557994.5
Stochastic volume											
$\log\phi_t$	0.7090	-0.0270	0.9297	-	-	-	3.2721	-	0.0748	-	-243199.8
	(0.0025)	(0.0029)	(0.0065)	-	-	-	-	-	(0.0090)	-	
Stochastic volatility											
$\log\sigma^2_t$	-	-0.4505	-	0.7966	-	-	-	4.9348	-	6.0740	-314794.7
	-	(0.0062)	-	(0.0023)	-	-	-	-	-	(0.0559)	
Panel B: Bivariate stochastic model SVV_0											
$\log\phi_t$	0.7124	-0.0382	0.9025	-	-	-	3.2409	-	0.1049	-	-556482.0
	(0.0036)	(0.0059)	(0.0129)	-	-	-	-	-	(0.0182)	-	
$\log\sigma^2_t$	-	-0.4460	-	0.7986	-	-	0.2051	4.9348	0.2001	6.0643	
	-	(0.0064)	-	(0.0024)	-	-	(0.0078)	-	(0.0160)	(0.0564)	
Panel C: Bivariate stochastic model SVV_1											
$\log\phi_t$	0.7566	-0.1510	0.7261	-0.0075	-	-	2.8739	-	0.4615	-	-555857.3
	(0.0081)	(0.0226)	(0.0356)	(0.0009)	-	-	-	-	(0.0775)	-	
$\log\sigma^2_t$	-	-0.2188	0.5907	0.7702	-	-	0.2441	4.9348	0.0203	5.7161	
	-	(0.0140)	(0.0141)	(0.0026)	-	-	(0.0071)	-	(0.0163)	(0.0668)	
Panel D: Bivariate stochastic model SVV_2											
$\log\phi_t$	0.7070	0.0049	0.8649	-	-	0.0161	3.2912	-	0.1456	-	-554319.1
	(0.0027)	(0.0038)	(0.0104)	-	-	(0.0010)	-	-	(0.0150)	-	
$\log\sigma^2_t$	-	0.1293	-	0.7662	0.4552	-	0.5361	4.9348	-0.5705	6.3190	
	-	(0.0129)	-	(0.0025)	(0.0089)	-	(0.0109)	-	(0.0223)	(0.0598)	

This table shows the QML estimates from the univariate stochastic models and the bivariate stochastic conditional volume-volatility models specified in (3.6) for BHP using data in August 2014. The model is

$$y_t = \alpha_t + \mathrm{w}_t,$$
$$\alpha_t = \gamma + A\alpha_{t-1} + By_{t-1} + u_t,$$

where $y_t = (\log v_t, \log e_t^2)'$, $\alpha_t = (\log\phi_t, \log\sigma_t^2)'$, $\mathrm{w}_t = (\log\varepsilon_t, \log\zeta_t^2)' \sim \mathrm{iid}(\omega, \Sigma_{\mathrm{w}})$, $u_t \overset{iid}{\sim} N(0, \Sigma_u)$, and w_t is independent of u_s for all t, s. The **measurement error of trading volumes** (i.e. $\log\varepsilon_t$) is assumed to follow a **log-Weibull distribution**, while that of volatility (i.e. $\log\zeta_t^2$) is assumed to be $\log\chi^2_{(1)}$ distributed. Robust standard errors are reported in parentheses. The diagonal elements of Σ_{w} and Σ_u are variance estimates, while the lower off-diagonal elements are correlation estimates. The diagonal elements of Σ_{w} are not free parameters in our models, but they are functions of the free parameter δ (given in Table 3.1). We therefore do not report their standard errors, even though the latter can be computed using the Delta method.

Table 3.5: Estimated stochastic conditional volume-volatility models for stock CHC, using the **log-Weibull** distribution

	δ	γ	A		B		Σ_{w}		Σ_u		Loglik
			$\log\phi_{t-1}$	$\log\sigma^2_{t-1}$	$\log v_{t-1}$	$\log e^2_{t-1}$	v	σ^2	v	σ^2	
Panel A: Univariate stochastic models											-69709.8
Stochastic volume											
$\log\phi_t$	0.7678	-0.0794	0.9274	-	-	-	2.7902	-	0.2286	-	-32108.4
	(0.0069)	(0.0092)	(0.0068)	-	-	-	-	-	(0.0242)	-	
Stochastic volatility											
$\log\sigma^2_t$	-	-0.2420	-	0.9337	-	-	-	4.9348	-	2.3710	-37601.4
	-	(0.0131)	-	(0.0023)	-	-	-	-	-	(0.0769)	
Panel B: Bivariate stochastic model SVV_0											
$\log\phi_t$	0.7585	-0.0798	0.9258	-	-	-	2.8589	-	0.2080	-	
	(0.0071)	(0.0089)	(0.0068)	-	-	-	-	-	(0.0224)	-	-69292.0
$\log\sigma^2_t$	-	-0.2392	-	0.9345	-	-	0.1736	4.9348	0.4831	2.3663	
	-	(0.0133)	-	(0.0024)	-	-	(0.0222)	-	(0.0269)	(0.0739)	
Panel C: Bivariate stochastic model SVV_1											
$\log\phi_t$	0.7593	-0.0780	0.9379	-0.0031	-	-	2.8532	-	0.2094	-	
	(0.0075)	(0.0090)	(0.0098)	(0.0017)	-	-	-	-	(0.0275)	-	-69256.8
$\log\sigma^2_t$	-	-0.1701	0.1406	0.9119	-	-	0.2047	4.9348	0.3672	2.3474	
	-	(0.0166)	(0.0163)	(0.0038)	-	-	(0.0227)	-	(0.0412)	(0.0699)	
Panel D: Bivariate stochastic model SVV_2											
$\log\phi_t$	0.7382	-0.0475	0.9268	-	-	0.0057	3.0189	-	0.1970	-	
	(0.0073)	(0.0076)	(0.0112)	-	-	(0.0023)	-	-	(0.0258)	-	-69108.2
$\log\sigma^2_t$	-	0.0853	-	0.8959	0.2323	-	0.4197	4.9348	0.0548	2.4660	
	-	(0.0267)	-	(0.0039)	(0.0160)	-	(0.0307)	-	(0.0592)	(0.0785)	

This table shows the QML estimates from the univariate stochastic models and the bivariate stochastic conditional volume-volatility models specified in (3.6) for CHC using data in August 2014. The model is

$$y_t = \alpha_t + \mathrm{w}_t,$$
$$\alpha_t = \gamma + A\alpha_{t-1} + By_{t-1} + u_t,$$

where $y_t = (\log v_t, \log e^2_t)'$, $\alpha_t = (\log\phi_t, \log\sigma^2_t)'$, $\mathrm{w}_t = (\log\varepsilon_t, \log\zeta^2_t)' \sim \mathrm{iid}(\omega, \Sigma_{\mathrm{w}})$, $u_t \overset{iid}{\sim} N(0, \Sigma_u)$, and w_t is independent of u_s for all t, s. The **measurement error of trading volumes** (i.e. $\log\varepsilon_t$) is assumed to follow a **log-Weibull distribution**, while that of volatility (i.e. $\log\zeta^2_t$) is assumed to be $\log\chi^2_{(1)}$ distributed. Robust standard errors are reported in parentheses. The diagonal elements of Σ_{w} and Σ_u are variance estimates, while the lower off-diagonal elements are correlation estimates. The diagonal elements of Σ_{w} are not free parameters in our models, but they are functions of the free parameter δ (given in Table 3.1). We therefore do not report their standard errors, even though the latter can be computed using the Delta method.

Table 3.6: Estimated stochastic conditional volume-volatility models for stock BHP, using the **log-Gamma** distribution

	κ	γ	A		B		Σ_{w}		Σ_u		Loglik
			$\log\phi_{t-1}$	$\log\sigma^2_{t-1}$	$\log v_{t-1}$	$\log e^2_{t-1}$	v	σ^2	v	σ^2	
Panel A: Univariate stochastic models											-557994.5
Stochastic volume											
$\log\phi_t$	0.6401	-0.0327	0.9297	-	-	-	3.2721	-	0.0748	-	-243199.8
	(0.0028)	(0.0034)	(0.0065)	-	-	-	-	-	(0.0090)	-	
Stochastic volatility											
$\log\sigma^2_t$	0.5000	-0.4505	-	0.7966	-	-	-	4.9348	-	6.0740	-314794.7
	-	(0.0062)	-	(0.0023)	-	-	-	-	-	(0.0559)	
Panel B: Bivariate stochastic model SVV_0											
$\log\phi_t$	0.6439	-0.0460	0.9025	-	-	-	3.2409	-	0.1049	-	-556482.0
	(0.0040)	(0.0068)	(0.0130)	-	-	-	-	-	(0.0184)	-	
$\log\sigma^2_t$	0.5000	-0.4460	-	0.7986	-	-	0.2051	4.9348	0.2001	6.0643	
	-	(0.0066)	-	(0.0024)	-	-	(0.0078)	-	(0.0166)	(0.0551)	
Panel C: Bivariate stochastic model SVV_1											
$\log\phi_t$	0.6939	-0.1670	0.7261	-0.0075	-	-	2.8739	-	0.4615	-	-555857.3
	(0.0087)	(0.0223)	(0.0336)	(0.0009)	-	-	-	-	(0.0722)	-	
$\log\sigma^2_t$	0.5000	-0.1843	0.5907	0.7702	-	-	0.2441	4.9348	0.0203	5.7161	
	-	(0.0180)	(0.0194)	(0.0025)	-	-	(0.0079)	-	(0.0140)	(0.0620)	
Panel D: Bivariate stochastic model SVV_2											
$\log\phi_t$	0.6378	-0.0063	0.8649	-	-	0.0161	3.2912	-	0.1456	-	-554319.1
	(0.0031)	(0.0045)	(0.0110)	-	-	(0.0010)	-	-	(0.0161)	-	
$\log\sigma^2_t$	0.5000	0.1293	-	0.7662	0.4552	-	0.5361	4.9348	-0.5705	6.3190	
	-	(0.0131)	-	(0.0025)	(0.0091)	-	(0.0114)	-	(0.0230)	(0.0602)	

This table shows the QML estimates from the univariate stochastic models and the bivariate stochastic conditional volume-volatility models specified in (3.6) for BHP using data in August 2014. The model is

$$y_t = \alpha_t + \mathrm{w}_t,$$
$$\alpha_t = \gamma + A\alpha_{t-1} + By_{t-1} + u_t,$$

where $y_t = (\log v_t, \log e_t^2)'$, $\alpha_t = (\log\phi_t, \log\sigma_t^2)'$, $\mathrm{w}_t = (\log\varepsilon_t, \log\zeta_t^2)' \sim \mathrm{iid}(\omega, \Sigma_{\mathrm{w}})$, $u_t \overset{iid}{\sim} N(0, \Sigma_u)$, and w_t is independent of u_s for all t, s. The **measurement error of trading volumes** (i.e. $\log\varepsilon_t$) is assumed to follow a **log-Gamma distribution**, while that of volatility (i.e. $\log\zeta_t^2$) is assumed to be $\log\chi^2_{(1)}$, or equivalently $\mathrm{LG}(\kappa = 0.5, \lambda = 2)$, distributed. Robust standard errors are reported in parentheses. The diagonal elements of Σ_{w} and Σ_u are variance estimates, while the lower off-diagonal elements are correlation estimates. The diagonal elements of Σ_{w} are not free parameters in our models, but they are functions of the free parameter κ (given in Table 3.1). We therefore do not report their standard errors, even though the latter can be computed using the Delta method.

Table 3.7: Estimated stochastic conditional volume-volatility models for stock CHC, using the **log-Gamma** distribution

	κ	γ	A		B		Σ_{w}		Σ_u		Loglik
			$\log\phi_{t-1}$	$\log\sigma^2_{t-1}$	$\log v_{t-1}$	$\log e^2_{t-1}$	v	σ^2	v	σ^2	
Panel A: Univariate stochastic models											-69709.8
Stochastic volume											
$\log\phi_t$	0.7069	-0.0833	0.9274	-	-	-	2.7902	-	0.2286	-	-32108.4
	(0.0081)	(0.0094)	(0.0069)	-	-	-	-	-	(0.0242)	-	
Stochastic volatility											
$\log\sigma^2_t$	0.5000	-0.2420	-	0.9337	-	-	-	4.9348	-	2.3710	-37601.4
	-	(0.0131)	-	(0.0023)	-	-	-	-	-	(0.0769)	
Panel B: Bivariate stochastic model SVV_0											
$\log\phi_t$	0.6962	-0.0841	0.9258	-	-	-	2.8589	-	0.2080	-	
	(0.0082)	(0.0091)	(0.0068)	-	-	-	-	-	(0.0224)	-	-69292.0
$\log\sigma^2_t$	0.5000	-0.2392	-	0.9345	-	-	0.1736	4.9348	0.4831	2.3663	
	-	(0.0132)	-	(0.0024)	-	-	(0.0222)	-	(0.0265)	(0.0725)	
Panel C: Bivariate stochastic model SVV_1											
$\log\phi_t$	0.6970	-0.0816	0.9379	-0.0031	-	-	2.8532	-	0.2094	-	
	(0.0086)	(0.0093)	(0.0098)	(0.0017)	-	-	-	-	(0.0274)	-	-69256.8
$\log\sigma^2_t$	0.5000	-0.1621	0.1406	0.9119	-	-	0.2047	4.9348	0.3672	2.3474	
	-	(0.0171)	(0.0162)	(0.0038)	-	-	(0.0225)	-	(0.0407)	(0.0722)	
Panel D: Bivariate stochastic model SVV_2											
$\log\phi_t$	0.6728	-0.0524	0.9268	-	-	0.0057	3.0189	-	0.1970	-	
	(0.0083)	(0.0078)	(0.0113)	-	-	(0.0024)	-	-	(0.0259)	-	-69108.2
$\log\sigma^2_t$	0.5000	0.0853	-	0.8959	0.2323	-	0.4197	4.9348	0.0548	2.4660	
	-	(0.0267)	-	(0.0039)	(0.0161)	-	(0.0310)	-	(0.0605)	(0.0787)	

This table shows the QML estimates from the univariate stochastic models and the bivariate stochastic conditional volume-volatility models specified in (3.6) for CHC using data in August 2014. The model is

$$y_t = \alpha_t + \mathrm{w}_t,$$
$$\alpha_t = \gamma + A\alpha_{t-1} + By_{t-1} + u_t,$$

where $y_t = (\log v_t, \log e_t^2)'$, $\alpha_t = (\log\phi_t, \log\sigma_t^2)'$, $\mathrm{w}_t = (\log\varepsilon_t, \log\zeta_t^2)' \sim \mathrm{iid}(\omega, \Sigma_{\mathrm{w}})$, $u_t \overset{iid}{\sim} N(0, \Sigma_u)$, and w_t is independent of u_s for all t, s. The **measurement error of trading volumes** (i.e. $\log\varepsilon_t$) is assumed to follow a **log-Gamma distribution**, while that of volatility (i.e. $\log\zeta_t^2$) is assumed to be $\log\chi^2_{(1)}$, or equivalently $\mathrm{LG}(\kappa = 0.5, \lambda = 2)$, distributed. Robust standard errors are reported in parentheses. The diagonal elements of Σ_{w} and Σ_u are variance estimates, while the lower off-diagonal elements are correlation estimates. The diagonal elements of Σ_{w} are not free parameters in our models, but they are functions of the free parameter κ (given in Table 3.1). We therefore do not report their standard errors, even though the latter can be computed using the Delta method.

Although they are able to capture some stylized facts about trading volumes and price volatility, the separate univariate stochastic models preclude and are silent on the interdependence amongst the two quantities of interest which, according to theories such as Admati and Pfleiderer (1988) and Andersen (1996), are endogenously and jointly determined. Further, not only is the baseline bivariate model SVV_0 in Panels B of Tables 3.4-3.7 able to recapture the results delivered by the univariate models, but it also provides confirmation that the interrelationship between volume and volatility indeed exists and it is significant, as shown by the correlation estimates in the measurement and latent covariance matrices (i.e. Σ_{w} and Σ_u). Consistent with previous findings in the literature, there is a strong positive *contemporaneous* relationship between volume and volatility which is contributed by the interactions between both latent and measurement shocks. One plausible explanation for this positive dependence, which is in line with theoretical studies such as Kyle (1985), Admati and Pfleiderer (1988), Easley and O'Hara (1992), Holden and Subrahmanyam (1992), and Andersen (1996), is that larger transactions are potentially initiated by informed traders. Consequently, they convey more information about prices and move prices more quickly, implying a positive volume-volatility relation.

The SVV_0 model allows for the interdependence between trading volumes and volatility which is typically observed in empirical data, and therefore it fits the data much better than do the two univariate models, producing a maximized quasi log likelihood value that is substantially higher than the sum of the two univariate log likelihood values. The improvement in the log likelihood is highly significant when compared against an asymptotic χ^2 distribution with 2 degrees of freedom (i.e. $\chi^2_{(2)}$) under the usual null hypothesis of a likelihood ratio test, where 2 is the number of additional correlation parameters between the latent and measurement innovations in the SVV_0 model.

Extensions to the baseline bivariate model to better capture direct relationships between the expected volumes and instantaneous volatility are shown in Panels C and D of Tables 3.4-3.7. As expected, both SVV_1 and SVV_2 models provide a significantly better fit to the empirical data than the baseline SVV_0 model, with the largest maximized log likelihood value obtained by the SVV_2 model. This result suggests that the interdependence between trading volumes and return volatility is complicated and thus, the use of a basic bivariate stochastic model that allows for this interrelationship only indirectly via the correlations between the innovations may be insufficient.

Interestingly, the SVV_1 model indicates that there are opposite direct *dynamic* feedback effects between trading volumes and return volatility for both stocks.[40] In particular, bigger previous transactions strongly increase future volatility, which is consistent with prior empirical findings of Manganelli (2005), Xu et al. (2006), Nolte (2008), Carlin et al. (2014) and Do et al. (2014), as well as with the theoretical predictions of Shalen (1993) and Banerjee and Kremer (2010). In contrast, higher past volatility tends to reduce the expected volume of future transactions, even though such an effect is much weaker, both economically and statistically, than the impact of past volume on future volatility. These contrasting effects, although hard to explain, are also empirically observed in Manganelli (2005) and Carlin et al. (2014). Despite this, there remains a strong positive *contemporaneous* connection between trading volumes and return volatility, as evidenced by the correlation coefficients between the measurement and latent errors. This implies that large conditional volumes are often accompanied by high volatility, and thus large transactions often come with big price adjustments. Combining both contemporaneous and dynamic effects, it appears plausible that there is an overall positive relationship between trading volumes and return volatility, which is in conformance with most theoretical and empirical studies in the literature.

Unlike the SVV_1 model, the SVV_2 model shows that there is a significant positive dynamic volume-volatility relation in that larger trading volumes inflate future return volatility, and vice versa higher volatility increases the size of an incoming trade, even though the strength of the latter effect is considerably weaker. This implies an asymmetry in the dynamic feedback effects between volume and volatility. The positive dynamic relationship between volume and volatility is in agreement with the theory of Admati and Pfleiderer (1988) in which informed and discretionary liquidity

[40] It is noted that while there are numerous studies in the literature that investigate the volume-volatility relation, they mainly look at the impact of volume on volatility, and very limited research also examines the effects of volatility on volume.

traders strategically time their trades in order to maximize profits or minimize trading costs. It is also consistent with Easley and O'Hara's (1992) theory which suggests that large transactions are typically transacted by informed traders who attempt to trade as quickly as possible to capitalize on their private information and make profits. Consequently, trading activities are often concentrated during some time periods (possibly after the release of private information) that feature both large trading volumes and big price movements.

Similar to the SVV_0 model, the SVV_2 model indicates that there is a strong positive correlation between the measurement innovations (see the off-diagonal element of Σ_w), suggesting that trading volume and return volatility are contemporaneously positively correlated. By accommodating direct feedback effects between volumes and volatility, the SVV_2 model seems to better capture the interdependence between the two variables for stock CHC, which helps significantly to reduce the contemporaneous correlation between the latent volume and volatility errors (see the off-diagonal element of Σ_u). As for stock BHP, it appears that the estimates of the positive direct feedback effects between trading volumes and return volatility are so strong that they are offset by a contemporaneous correlation that is often positive but is now negative between the latent innovations.

In general, the three bivariate stochastic models proposed in this chapter, namely SVV_0, SVV_1 and SVV_2, generalize the well-known univariate stochastic conditional models for trading volumes and price volatility in the literature. Our proposed bivariate models are capable of capturing both the positive contemporaneous and dynamic interdependence between trading volumes and price volatility, in addition to improving on the fit of the univariate models (by producing substantially higher log likelihood values) and recapturing the results delivered by the latter. Our results lend support to theoretical studies such as Admati and Pfleiderer (1988) and Andersen (1996), which suggest the endogeneity and joint determination of volumes and volatility.

3.4.2 Model diagnostics

In this section, we conduct some diagnostic tests on the fitted models, both univariate and bivariate. In these models, both measurement errors ($(\varepsilon_t, \zeta_t)'$ as specified in Equation (3.1) or $(\mathrm{w}_{v,t}, \mathrm{w}_{e^2,t})'$ as in Equation (3.3)) and latent disturbances ($(u_{v,t}, u_{e^2,t})'$) are assumed to be i.i.d.. In addition, the latent errors are assumed to be normally distributed. If the models are correctly specified, the residuals from the fitted models should satisfy the independence assumption and/or the Gaussian assumption, to some extent. We explore this idea and investigate (i) if there is any serial correlation in the residuals, and (ii) if the latent residuals follow a normal distribution.

For each univariate and bivariate stochastic model, we run the Kalman filter and smoother at the QML estimates to obtain the smoothed estimates $\widehat{\alpha_t}$ of the logarithmic expected quantities α_t and the smoothed residuals $\widehat{u_t}$ of the latent innovations u_t. The logarithmic measurement residuals $\widehat{\mathrm{w}}_t$ are calculated as the difference between logarithmically transformed observed data y_t and the latent estimates $\widehat{\alpha_t}$. For the original measurement errors $(\varepsilon_t, \zeta_t)'$ specified in (3.1), we follow Bauwens and Veredas (2004) to define their corresponding residuals as

$$\widehat{\varepsilon_t} = \frac{v_t}{\exp\left(\widehat{\log\phi_t}\right)} = \exp\left(\log v_t - \widehat{\log\phi_t}\right) = \exp(\widehat{\mathrm{w}}_{v,t}), \text{ and}$$

$$\widehat{\zeta_t} = \mathrm{sign}(e_t)\left[\frac{e_t^2}{\exp\left(\widehat{\log\sigma_t^2}\right)}\right]^{1/2} = \mathrm{sign}(e_t)\left[\exp\left(\log e_t^2 - \widehat{\log\sigma_t^2}\right)\right]^{1/2} = \mathrm{sign}(e_t)\left[\exp\left(\widehat{\mathrm{w}}_{e^2,t}\right)\right]^{1/2},$$

where $\widehat{\alpha_t} = \left(\widehat{\log\phi_t}, \widehat{\log\sigma_t^2}\right)'$, e_t is estimated by the residuals from an ARMA(1,1) model fitted to r_t as discussed in footnote 26, and $\mathrm{sign}(x)$ denotes the sign function that equals 1 if $x > 0$, -1 if $x < 0$, and 0 if $x = 0$. The definition for the return residuals $\widehat{\zeta_t}$ makes use of the fact that ζ_t and e_t are of the same sign, as discussed in Harvey and Shephard (1996). Note that we obtain the same set of the smoothed estimates ($\widehat{\alpha_t}$, $\widehat{u_t}$, $\widehat{\mathrm{w}}_t$, $\widehat{\varepsilon_t}$, and $\widehat{\zeta_t}$) via the Kalman filter and smoother, regardless of the distribution of trading volume errors. This is because the calculation of these smoothed estimates is only dependent on the QML estimates of the structural parameters of the models (including $A, B, \Sigma_{\mathrm{w}}, \Sigma_u$, and $\widetilde{\gamma}$), which, as a result of QML estimation, are unaffected by the assumed non-Gaussian shape of the measurement innovations of trading volumes (see footnote 38).

Autocorrelation structures of residuals

We examine the autocorrelation functions (ACFs) of the fitted residuals to check the i.i.d assumption of the measurement and latent errors. If the models are correctly specified, there should be no systematic patterns in the residuals and the autocorrelation coefficients should be statistically indistinguishable from zero. Tables 3.8 and 3.9 report the ACFs, truncated at the first seven autocorrelation levels, of different residual series produced by various univariate and bivariate stochastic models for BHP and CHC, respectively. Under the null hypothesis that a true autocorrelation coefficient is equal to 0, the estimated sample autocorrelation coefficient converges in distribution to a standard normal distribution at the rate of $\sqrt{n}$, where n is the sample size. Thus, the 5% critical value under the null hypothesis of a zero true autocorrelation coefficient is approximately $2/\sqrt{n}$, which equals 0.0058 and 0.0160 for stock BHP and CHC, respectively.

From Tables 3.8 and 3.9, there is significant serial correlation in the residuals of all models, with the exception of the original measurement residuals $\widehat{\varepsilon_t}$ of the trading volumes for stock CHC. This indicates that all models, both univariate and bivariate, are misspecified. In particular, similar to typical findings in the stochastic literature (e.g. Bauwens and Veredas, 2004, Feng et al., 2004), the residuals of the latent equations of all models are highly serially correlated, exhibiting substantial autocorrelation coefficients at the first few lags. The result suggests that the AR(1) or VAR(1) structure assumed in the latent process may not be adequate to fully capture the dynamics inherent in the empirical data; more flexible and higher-order latent processes may be needed to attenuate this problem.

Table 3.8: Autocorrelation functions of the residuals for stock BHP

	Volume				Volatility			
	Univariate	SVV_0	SVV_1	SVV_2	Univariate	SVV_0	SVV_1	SVV_2
Panel A: Logarithmic measurement residuals $(\widehat{w}_{v,t}, \widehat{w}_{e^2,t})$								
AC1	-0.016	-0.031	-0.129	-0.009	-0.216	-0.205	-0.214	-0.088
AC2	-0.054	-0.064	-0.074	-0.044	-0.151	-0.143	-0.146	-0.063
AC3	-0.056	-0.060	-0.028	-0.047	-0.064	-0.061	-0.058	-0.042
AC4	-0.057	-0.056	-0.009	-0.049	-0.028	-0.028	-0.023	-0.036
AC5	-0.050	-0.046	0.002	-0.045	-0.039	-0.040	-0.036	-0.050
AC6	-0.041	-0.035	0.007	-0.037	-0.010	-0.014	-0.011	-0.036
AC7	-0.032	-0.025	0.011	-0.026	0.026	0.022	0.024	-0.007
Panel B: Original measurement residuals $(\widehat{\varepsilon_t}, \widehat{\zeta_t})$								
AC1	0.000	-0.008	-0.076	0.003	0.204	0.195	0.197	0.152
AC2	-0.023	-0.029	-0.043	-0.017	0.132	0.126	0.128	0.099
AC3	-0.019	-0.022	-0.009	-0.017	0.086	0.083	0.085	0.066
AC4	-0.018	-0.019	0.001	-0.018	0.056	0.054	0.056	0.043
AC5	-0.005	-0.005	0.016	-0.009	0.024	0.025	0.026	0.024
AC6	0.003	0.004	0.021	-0.003	0.002	0.002	0.003	0.004
AC7	-0.001	0.000	0.012	-0.005	0.001	0.001	0.002	0.003
Panel C: Latent residuals $(\widehat{u}_{v,t}, \widehat{u}_{e^2,t})$								
AC1	0.841	0.748	0.538	0.351	0.396	0.391	0.380	0.288
AC2	0.692	0.566	0.272	0.123	0.084	0.076	0.077	0.045
AC3	0.564	0.437	0.150	0.060	-0.032	-0.039	-0.024	-0.015
AC4	0.458	0.340	0.098	0.040	-0.070	-0.074	-0.052	-0.031
AC5	0.371	0.266	0.085	0.029	-0.076	-0.078	-0.056	-0.038
AC6	0.303	0.217	0.091	0.050	-0.038	-0.037	-0.018	-0.006
AC7	0.250	0.186	0.099	0.072	0.011	0.012	0.027	0.031

This table reports the autocorrelation functions, truncated at the first 7 autocorrelation coefficients (ACs), of the residuals obtained from different univariate and bivariate stochastic conditional models for BHP. Panel A presents the results for the logarithmic measurement residuals $(\widehat{w}_{v,t}, \widehat{w}_{e^2,t})$ specified in equation (3.3). Panel B presents the results for the original measurement residuals $(\widehat{\varepsilon_t}, \widehat{\zeta_t})$ specified in equation (3.1). Panel C presents the results for the latent residuals $(\widehat{u}_{v,t}, \widehat{u}_{e^2,t})$ specified in equation (3.6). The critical value at a 5% significance level under the null hypothesis that an autocorrelation coefficient equals 0 is $2/\sqrt{n}$ = **0.0058**, where $n = 117,970$ is the number of observations for stock BHP in our sample.

Table 3.9: Autocorrelation functions of the residuals for stock CHC

	Volume				Volatility			
	Univariate	SVV_0	SVV_1	SVV_2	Univariate	SVV_0	SVV_1	SVV_2
Panel A: Logarithmic measurement residuals $(\widehat{w}_{v,t}, \widehat{w}_{e^2,t})$								
AC1	-0.091	-0.070	-0.078	-0.061	0.029	0.030	0.027	0.035
AC2	-0.072	-0.061	-0.066	-0.055	-0.149	-0.149	-0.150	-0.117
AC3	-0.102	-0.097	-0.099	-0.094	-0.123	-0.123	-0.123	-0.099
AC4	-0.061	-0.061	-0.060	-0.061	-0.100	-0.100	-0.099	-0.090
AC5	-0.043	-0.046	-0.043	-0.047	-0.061	-0.061	-0.060	-0.060
AC6	-0.040	-0.045	-0.041	-0.045	-0.038	-0.039	-0.036	-0.035
AC7	-0.021	-0.027	-0.023	-0.028	-0.032	-0.033	-0.030	-0.027
Panel B: Original measurement residuals $(\widehat{\varepsilon}_t, \widehat{\zeta}_t)$								
AC1	-0.005	-0.004	-0.010	0.009	0.162	0.163	0.159	0.115
AC2	-0.017	-0.020	-0.023	-0.009	0.126	0.127	0.124	0.090
AC3	-0.012	-0.017	-0.019	-0.005	0.099	0.100	0.098	0.072
AC4	-0.019	-0.024	-0.025	-0.017	0.083	0.083	0.081	0.058
AC5	-0.007	-0.006	-0.005	-0.009	0.066	0.066	0.065	0.045
AC6	-0.012	-0.013	-0.014	-0.014	0.064	0.064	0.062	0.045
AC7	-0.009	-0.012	-0.013	-0.009	0.048	0.048	0.047	0.030
Panel C: Latent residuals $(\widehat{u}_{v,t}, \widehat{u}_{e^2,t})$								
AC1	0.737	0.738	0.755	0.752	0.706	0.706	0.705	0.650
AC2	0.526	0.502	0.532	0.555	0.398	0.398	0.396	0.347
AC3	0.357	0.326	0.353	0.395	0.180	0.180	0.180	0.160
AC4	0.244	0.207	0.227	0.285	0.037	0.035	0.039	0.051
AC5	0.164	0.132	0.142	0.203	-0.048	-0.049	-0.043	-0.009
AC6	0.109	0.086	0.086	0.141	-0.097	-0.098	-0.090	-0.048
AC7	0.076	0.057	0.052	0.100	-0.123	-0.124	-0.116	-0.073

This table reports the autocorrelation functions, truncated at the first 7 autocorrelation coefficients (ACs), of the residuals obtained from different univariate and bivariate stochastic conditional models for CHC. Panel A presents the results for the logarithmic measurement residuals ($\widehat{w}_{v,t}$, $\widehat{w}_{e^2,t}$) specified in equation (3.3). Panel B presents the results for the original measurement residuals ($\widehat{\varepsilon}_t$, $\widehat{\zeta}_t$) specified in equation (3.1). Panel C presents the results for the latent residuals ($\widehat{u}_{v,t}$, $\widehat{u}_{e^2,t}$) specified in equation (3.6). The critical value at a 5% significance level under the null hypothesis that an autocorrelation coefficient equals 0 is $2/\sqrt{n}$ = **0.0160**, where n = 15,685 is the number of observations for stock CHC in our sample.

Despite the inadequacy of these models in fully explaining the empirical dynamics, we note that the residuals produced by the bivariate stochastic models proposed in this chapter are often less (and considerably less in some cases) serially correlated than those created by univariate stochastic models. This observation confirms the interdependencies between trading volumes and price volatility, and it advocates the employment of a bivariate setting that can facilitate this interplay in modeling these quantities, as opposed to univariate models that preclude such interrelation. In addition, the extended bivariate models SVV_1 and SVV_2 that accommodate both *direct* and *indirect* relationships between volumes and volatility often fit the data better than the baseline model SVV_0 that only allows for indirect connections. This result indicates the joint volume and volatility distribution is highly complex, and requires the use of a more flexible model.

Q-Q plots of latent residuals

Figures 3.2 and 3.3 plot the Q-Q plots for the latent residuals of the volume equation (left plots) and volatility equation (right plots) produced by various univariate and bivariate stochastic models for BHP and CHC, respectively. Significant departures from normality are observed for the latent residuals of all models, especially for the latent residuals of the volatility equation of stock CHC. Similar to the autocorrelation test, this observation implies the mis-specification of both univariate and bivariate models. A more flexible distribution such as a Student-t distribution may be needed to model the latent errors.

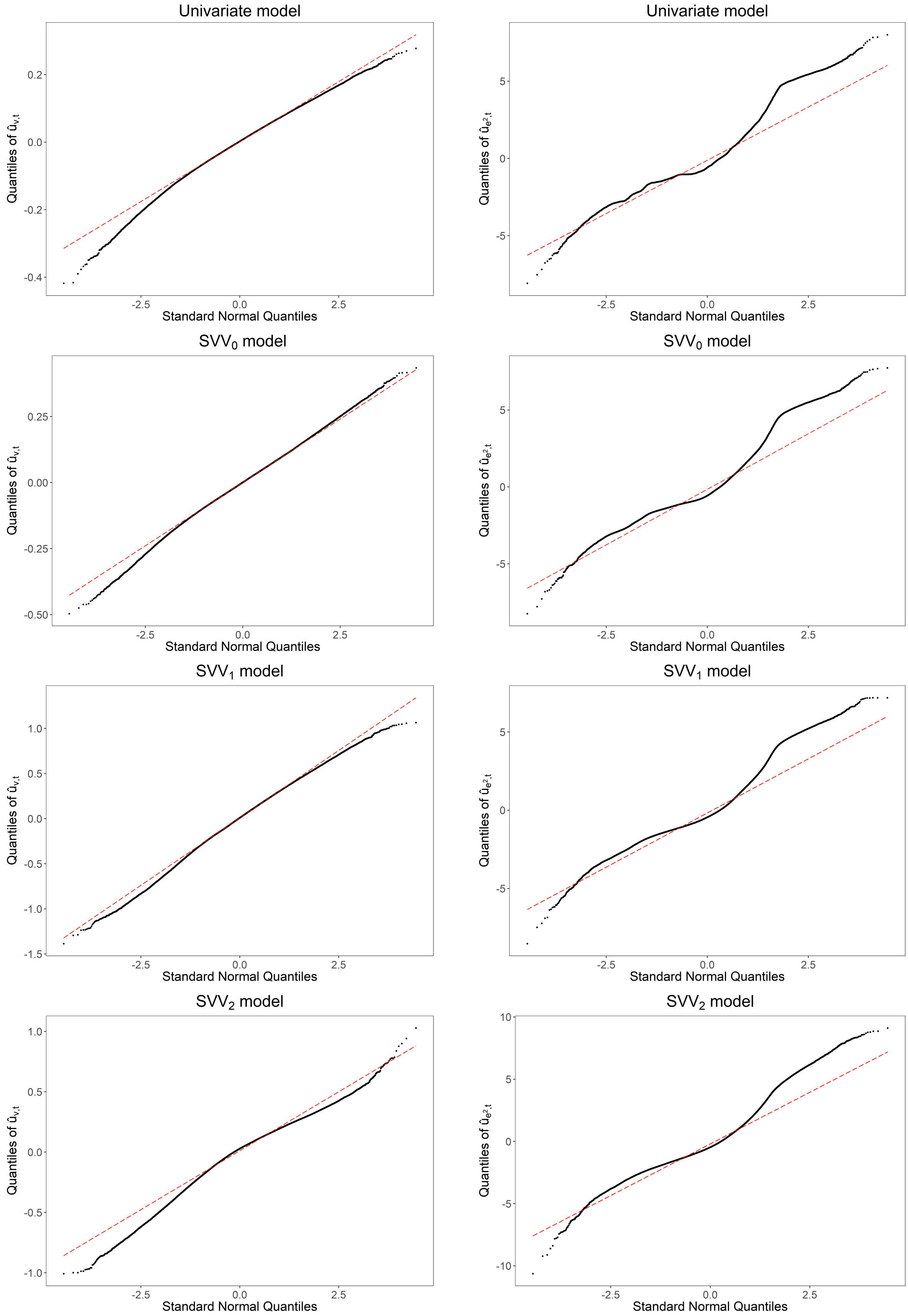


Figure 3.2: Q-Q plots of the latent residuals for stock BHP. Left plots are Q-Q plots of the latent residuals $\widehat{u}_{v,t}$ of the volume equation, while right plots are Q-Q plots of the latent residuals $\widehat{u}_{e^2,t}$ of the volatility equation.

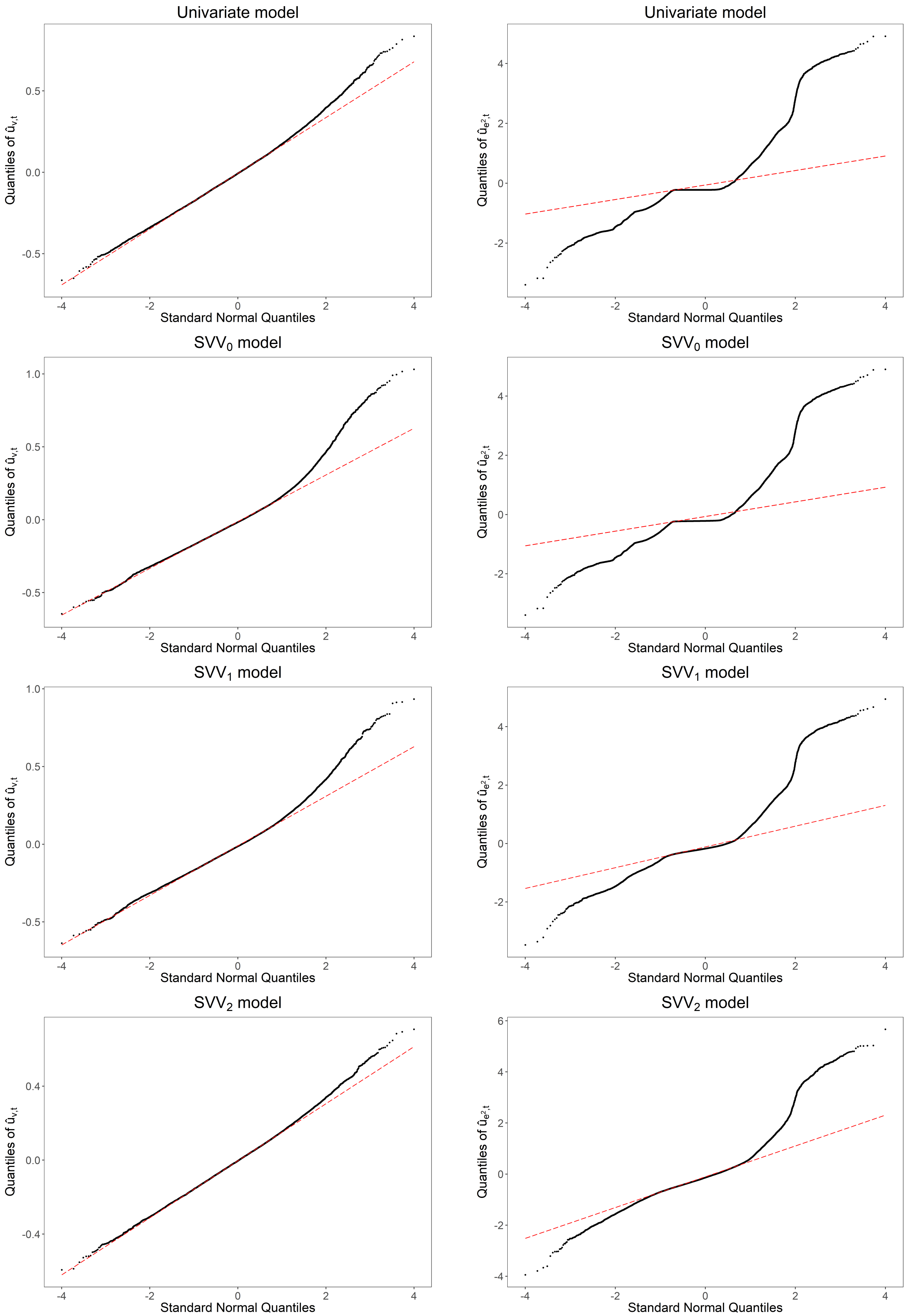


Figure 3.3: Q-Q plots of the latent residuals for stock CHC. Left plots are Q-Q plots of the latent residuals $\widehat{u}_{v,t}$ of the volume equation, while right plots are Q-Q plots of the latent residuals $\widehat{u}_{e^2,t}$ of the volatility equation.

3.4.3 Impulse response analysis

The SVV_2 model allows one to measure the response of the expected volume and volatility of future transactions to a shock to the system at time t, which provides insights into how prices and volumes react to information. Assuming that the SVV_2 system is weakly stationary, the impulse response functions (IRFs) can be conveniently computed from the moving average representation of the log expected volume and volatility, which is given by

$$\alpha_t = (I_2 - H)^{-1}\gamma + \sum_{i=0}^{\infty} H^i (u_{t-i} + B\mathrm{w}_{t-1-i}), \tag{3.13}$$

where $H = A + B$ (see equation (3.22) in the Appendix). Due to the potential correlation between the measurement errors w_t, a shock to one variable, for example trading volume, is likely to be accompanied by a shock to the other variable (volatility) in the system. To examine the effect of a shock to one variable in the system after controlling for the correlation between the errors, the correlated errors are often orthogonalized into a set of uncorrelated or structural shocks. Since there is usually no unique way to do the orthogonal transformation, one needs to impose suitable identification restrictions, which are often suggested by theories, to retrieve the structural shocks from the correlated errors.

In the current analysis, we orthogonalize the correlated measurement errors w_t by imposing the restriction that there is an instantaneous Granger-causal relationship running from trading volume to return volatility, but not vice versa. Given that trading volume is known at the execution of a trade, whereas the return and volatility of the trade can only be realized ex-post once the trade is fully transacted, this restriction is intuitive and reflects the chronological operation in a financial market which is typically highlighted in market microstructure theory. For example, according to Kyle (1985) and Hasbrouck (1991a,b), after observing a new trade, the market maker learns the information conveyed by the trade such as its volume and then revises his quotes accordingly to take into account the new information. As a result, the volume of a trade contemporaneously affects the price and volatility of that trade.[41]

[41] We obtain qualitatively similar results when the reverse contemporaneous Granger causality is

Given the setup of the SVV_2 model that places trading volume in the first equation of the latent process and volatility in the last, the orthogonalization of the measurement errors w_t based on the assumption of a causality direction from volume to volatility is equivalent to the one based on a lower Cholesky decomposition of the measurement error covariance matrix. By letting $\mathrm{w}_t = P_\mathrm{w}\mathrm{w}_t^*$, where P_w is a lower triangular matrix such that $P_\mathrm{w}P_\mathrm{w}' = \Sigma_\mathrm{w} = \mathrm{Var}(\mathrm{w}_t)$, we obtain the structural shocks w_t^* that are uncorrelated and have unit variance (i.e. $\Sigma_{\mathrm{w}^*} = I_2$). The moving average form of the latent process (3.13) then becomes

$$\alpha_t = (I_2 - H)^{-1}\gamma + \sum_{i=0}^{\infty} H^i(u_{t-i} + BP_\mathrm{w}\mathrm{w}_{t-1-i}^*), \tag{3.14}$$

which implies that the IRF of α_{t+i} (i.e. the expected volume and volatility, measured in a log scale, at transaction time $t+i$, $i \geq 1$) to a (positive) structural shock w_t^* to the system at time t is[42]

$$\frac{\partial \mathbb{E}(\alpha_{t+i}|\mathcal{I}_{t-1})}{\partial \mathrm{w}_t^{*\prime}} := \Phi_i = H^i BP_\mathrm{w}. \tag{3.15}$$

Given that α_{t+i} measures the logarithm of the expected volume and volatility at transaction time $t+i$ while w_t^* is an orthogonal transformation of the logarithm of the original innovations of the trading volume and volatility at time t (i.e. ε_t and ζ_t^2), the impulse response function Φ_i in equation (3.15) can be interpreted as the *elasticity* of the trading volume and volatility of a trade at time $t+i$ to a structural shock to either variable at time t. To compute the standard error of Φ_i, we note that if $\underset{(p\times 1)}{\widehat{\theta}}$ is the QML estimator of the true unknown parameter vector θ_0 that underlies the SVV_2 model ($p = 11$ here) and standard regularity conditions hold, then $\sqrt{n}(\widehat{\theta} - \theta_0) \xrightarrow{d} N(0, \Sigma_{\theta_0})$, where n is the sample size and the limiting distribution of $\underset{(4\times 1)}{\widehat{\nu}_i} := \nu_i(\widehat{\theta}) := \mathrm{vec}\big(\Phi_i(\widehat{\theta})\big)$ is $\sqrt{n}(\widehat{\nu}_i - \nu_{0,i}) \xrightarrow{d} N(0, G_i\Sigma_{\theta_0}G_i')$ according to the Delta method, where $\underset{(4\times p)}{G_i} = \left.\frac{\partial \nu_i}{\partial \theta'}\right|_{\theta=\theta_0}$. While Σ_{θ_0} is estimated using a sandwich-type estimator detailed at the end of subsection 3.2.4, the matrix of derivatives G_i is estimated using numerical differentiation. Specifically, the j-th column of G_i is computed as $G_i^j = \frac{\nu_i(\widehat{\theta} + I_p^j\Delta) - \nu_i(\widehat{\theta} - I_p^j\Delta)}{2\Delta}$, where

imposed. That is, our results are robust to the ordering of volume and volatility in the SVV_2 system.

[42]Meanwhile, $\partial\mathbb{E}(\alpha_t|\mathcal{I}_{t-1})/\partial\mathrm{w}_t^{*\prime} = 0$. Thus, the SVV_2 model assumes that (structural) shocks to the system at time t only affect the expected quantities (i.e. volume and volatility) of a future trade but have no effects on those of the trade at t.

I_p^j is the j-th column of the $(p \times p)$ identity matrix I_p and Δ is a small number.

Figures 3.4 and 3.5 depict the responses of the trading volume and volatility of a trade to a structural shock to either variable, implied by the SVV_2 model, for stocks BHP and CHC, respectively. The average IRFs are represented by solid curves, while the corresponding 95% confidence intervals are illustrated with dotted lines. Since the IRFs in equation (3.15) only depend on the structural parameters (including A, B and Σ_w) of the SVV_2 model, whose QML estimates are not affected by the assumed distribution of the measurement innovation for trading volumes in the model (see footnote 38), we obtain an identical set of IRFs for trading volume and volatility for each stock regardless of the distributional assumption of the volume error.

The IRFs depicted in Figures 3.4 and 3.5 bear a remarkable similarity to the ones reported in Manganelli (2005) for U.S. stocks, and they demonstrate that there are strong positive dynamic relationships between the trading volume and volatility of trades, reaffirming the results discussed in subsection 3.4.1. A positive shock to either variable in the SVV_2 system triggers an increase in the expected trading volume and return volatility of the future trades in both stocks BHP and CHC, with the biggest rise observed after the first or first few transactions. After that, the increase in return volatility and trading volume declines gradually to zero, signifying that both quantities are approaching their steady-state equilibrium, where the effects of the initial shock are fully incorporated.

Consistent with Manganelli (2005), the convergence to the long run equilibrium of both return volatility and trading volume is quicker for the bigger and more liquid stock (BHP) than for the smaller and less liquid one (CHC). In particular, while the price volatility and volume of BHP converge to their long run levels in about 60 to 70 transactions after an initial perturbation, it takes about 90 to 100 transactions for those of CHC to reach their steady states. Given that BHP is traded much more frequently than is CHC (117,970 vs. 15,685 trades in August 2014 - see Table 3.3),[43] the price volatility and trading volume of BHP converge even much more rapidly in calendar time to their long run equilibrium than do those of CHC. This result lends

[43]The average trade duration is 3.739 (28.210) seconds for BHP (CHC) in our sample.

support to the theoretical work of Holden and Subrahmanyam (1992), who show that the speed with which the private information of informed traders is revealed to the public through their strategic trading activities increases dramatically with either the trading frequency (i.e. the number of auctions or trades in a fixed time interval) or the number of informed traders in the market. This is because of the competition between the informed agents who trade strategically and aggressively to capitalize on their informational advantage. Consequently, as the number of trades or the number of informed investors increases (which strengthens the competition between the traders), the error variance of the price (which measures the remaining informational content of the private signals) declines to zero very quickly. Due to the much higher trading frequency of BHP and its likely larger number of informed followers, it is understandable that an initial perturbation such as a news event results in a quicker convergence of price volatility and trading volume to their long run levels for BHP than for CHC - a much smaller stock.

Figures 3.4 and 3.5 also reveal that a volume shock has strong effects, both statistically and economically, on the future return volatility of both stocks, whereas the impact of a volatility shock on the trading volume of future trades, although statistically significant, is weaker. This observation is also noted in Manganelli (2005), and it suggests an asymmetry in the feedback effects between volume and volatility, of which the impact of volume on volatility is much more dominant. The sizeable impact of trading volume on volatility is also consistent with one of the most important theoretical predictions in finance which is that trades contain crucial information that drives stock price movements (e.g. Kyle, 1985, Hasbrouck, 1991a, Easley and O'Hara, 1992, Duffie, 2010). In addition, the responses of the price volatility and trading volume of BHP to an initial shock are considerably stronger than those of CHC, as indicated by the scales on the vertical axes. The result implies that the positive dynamic relationships between trading volume and return volatility are more significant for the bigger and more liquid stock BHP, which is consistent with the coefficient estimates reported in Tables 3.4-3.7.

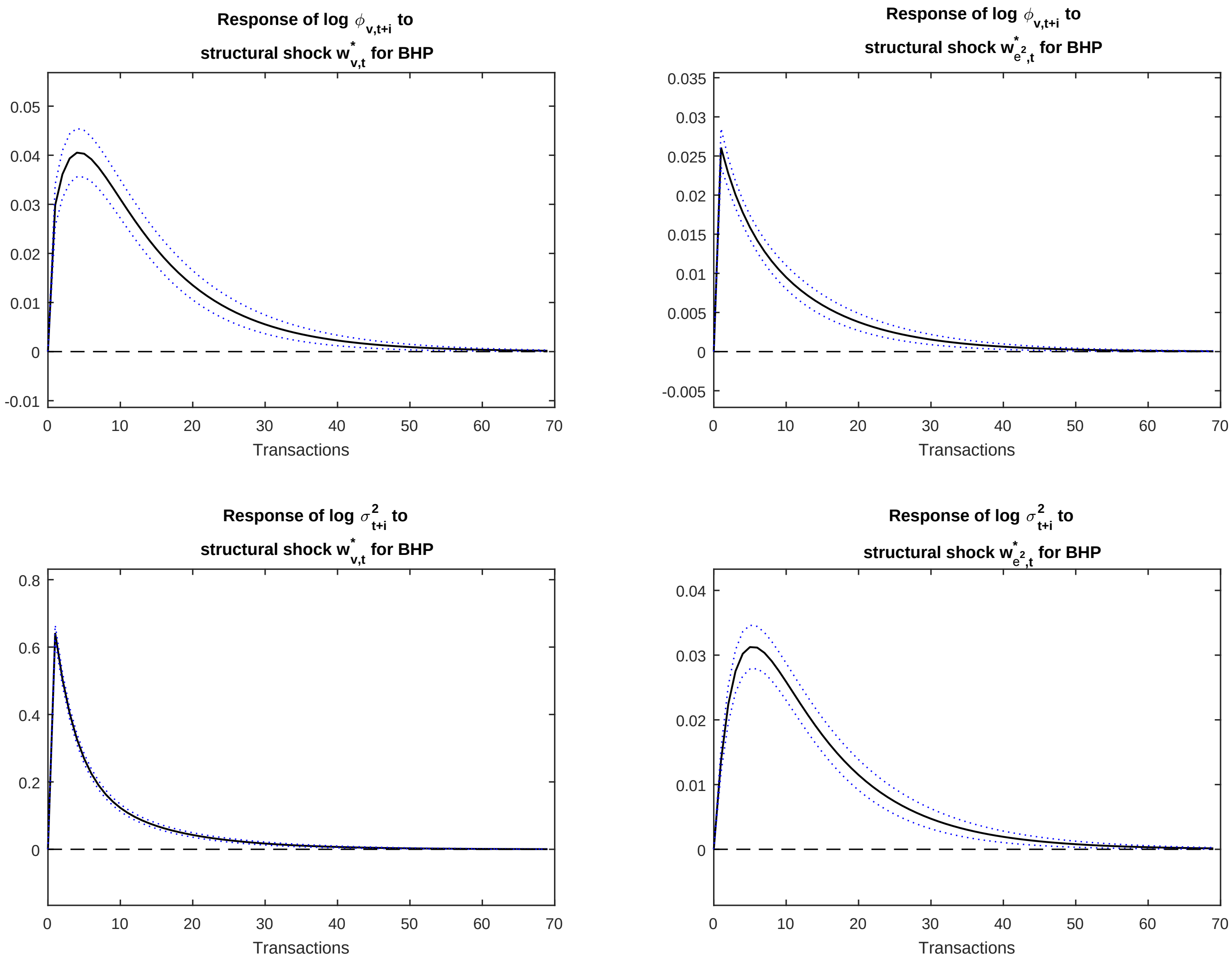


Figure 3.4: Impulse response to *structural measurement* shocks for stock BHP implied by the SVV_2 model. The solid curve represents the mean estimate, while dotted curves represent the 95% confidence interval.

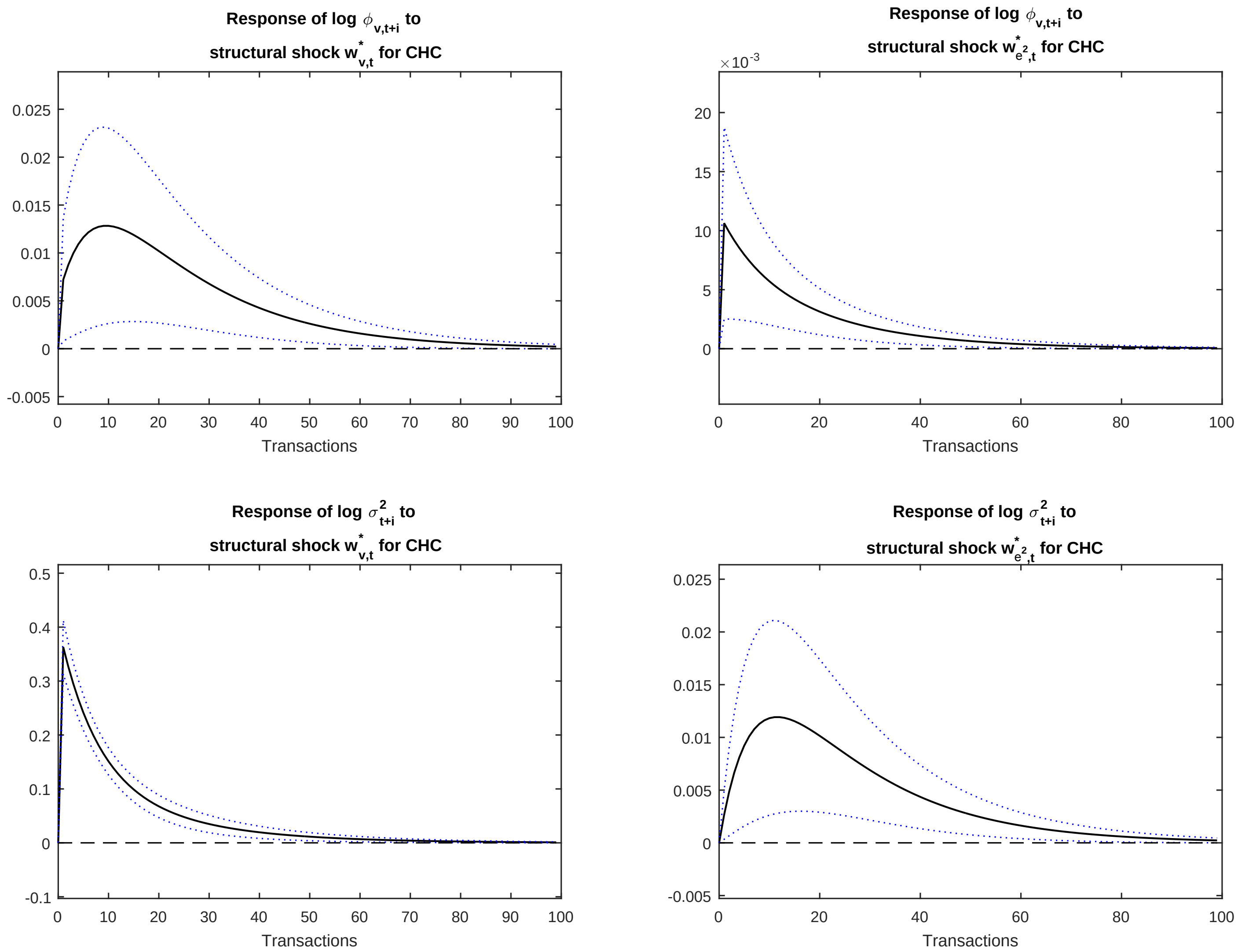


Figure 3.5: Impulse response to *structural measurement* shocks for stock CHC implied by the SVV_2 model. The solid curve represents the mean estimate, while dotted curves represent the 95% confidence interval.

Overall, the impulse response analysis re-confirms the result that the trading volume and return volatility of trades are strongly, positively and dynamically related, with the effect of volume on volatility being much more pronounced. After a positive shock to either variable, both price volatility and trading volume increase and gradually converge to their new long-run equilibrium, with a faster rate of convergence for the bigger and more liquid stock (BHP).

3.5 Conclusion

This chapter develops a bivariate stochastic conditional model to study the interaction between trading volume and return volatility. Unlike most existing studies in the literature that use univariate approaches, which cannot accommodate joint modeling to investigate the relationship between volume and volatility, we explicitly allow for the dynamic feedback effects between these two variables in our model. Our proposed model generalizes the popular univariate SV and SCD models in the literature to a bivariate setting, and it relaxes the conditional deterministic assumption imposed by a few prior studies that employ multivariate GARCH-type or VAR-type approaches to model trading volumes and return volatility. We derive several statistical properties of our bivariate model with regard to the moments and correlation functions of the volume and volatility series. We show that these properties generalize and are all consistent with those derived previously in a univariate SV and SCD context.

The bivariate stochastic conditional volume-volatility model is applied to a transaction dataset of one big and one small market capitalization stock in the Australian stock market, and we employ QML to estimate the model. Consistent with market microstructure theory (e.g. Admati and Pfleiderer, 1988, Andersen, 1996), we show that trading volume and return volatility are jointly determined, and our proposed model is capable of capturing the positive dynamic feedback effects between these variables for both stocks. However, these feedback effects are asymmetric, with the impact of volume on volatility being dominant. Significant estimates obtained for the variance of the errors of the latent conditional expected volume and volatility errors indicate that these conditional quantities cannot be fully explained by the past information, and hence the use of a GARCH-type model (which imposes the conditional deterministic assumption) to model the expected volume and volatility is inadequate. In addition, by allowing for the joint determination of volume and volatility, our bivariate stochastic model fits the empirical data significantly better than its univariate counterparts. In agreement with the theoretical prediction of Holden and Subrahmanyam (1992) and the empirical finding of Manganelli (2005), we find that after a positive shock to either variable both trading volume and return volatility increase significantly, after which they gradually converge to their full information equilibrium, with quicker convergence observed for the bigger and more frequently traded stock.

3.6 Appendix

3.6.1 Derivation of the MGF

In this subsection, we derive the MGF of a log-Weibull and a log-Gamma random variable specified in Table 3.1. The MGF of a random variable X, denoted by $M_X(z)$, is defined as $M_X(z) := \mathbb{E}(\exp(zX)) = \mathbb{E}(\widetilde{X}^z)$, where $\widetilde{X} = \exp(X)$.

Consider the case $X \sim \text{LW}(\delta, \lambda)$, it follows that $\widetilde{X} \sim \text{W}(\delta, \lambda)$. The MGF of X is given by

$$
\begin{aligned}
M_X(z) = \mathbb{E}(\widetilde{X}^z) &= \int_0^\infty \widetilde{x}^z \frac{\delta \widetilde{x}^{\delta-1}}{\lambda^\delta} \exp\left(-\frac{\widetilde{x}^\delta}{\lambda^\delta}\right) d\widetilde{x} \\
&= \frac{1}{\lambda^\delta} \int_0^\infty \widetilde{x}^z \exp\left(-\frac{\widetilde{x}^\delta}{\lambda^\delta}\right) d\widetilde{x}^\delta \\
&= \frac{1}{\lambda^\delta} \int_0^\infty t^{z/\delta} \exp\left(-\frac{t}{\lambda^\delta}\right) dt \qquad (3.16) \\
&= \frac{1}{\lambda^\delta} \times \Gamma(1 + z/\delta) \left(\lambda^\delta\right)^{1+z/\delta} \qquad (3.17) \\
&= \lambda^z \Gamma\left(\frac{z}{\delta} + 1\right),
\end{aligned}
$$

where we have replaced the Gamma kernel in equation (3.16) with its integrating constant, $\Gamma(1 + z/\delta)\left(\lambda^\delta\right)^{1+z/\delta}$, in equation (3.17).

Now consider the case $X \sim \text{LG}(\kappa, \lambda)$, it follows that $\widetilde{X} \sim \text{G}(\kappa, \lambda)$. The MGF of X is given by

$$
\begin{aligned}
M_X(z) = \mathbb{E}(\widetilde{X}^z) &= \int_0^\infty \widetilde{x}^z \frac{\widetilde{x}^{\kappa-1}}{\Gamma(\kappa)\lambda^\kappa} \exp\left(-\frac{\widetilde{x}}{\lambda}\right) d\widetilde{x} \\
&= \frac{1}{\Gamma(\kappa)\lambda^\kappa} \int_0^\infty \widetilde{x}^{\kappa+z-1} \exp\left(-\frac{\widetilde{x}}{\lambda}\right) d\widetilde{x} \qquad (3.18) \\
&= \frac{1}{\Gamma(\kappa)\lambda^\kappa} \times \Gamma(\kappa + z)\lambda^{\kappa+z} \qquad (3.19) \\
&= \lambda^z \frac{\Gamma(\kappa + z)}{\Gamma(\kappa)},
\end{aligned}
$$

where we have replaced the Gamma kernel in equation (3.18) with its integrating constant, $\Gamma(\kappa + z)\lambda^{\kappa+z}$, in equation (3.19).

We note that the MGF $M_X(z)$ derived in this study for a log-Weibull or log-Gamma random variable X is similar to that derived in Feng et al. (2004, p. 419), even though Feng et al. (2004) use a slightly different parameterization of the log-Weibull or log-Gamma distribution and a different set of notation.

3.6.2 Proofs of Propositions

Proof of Proposition 3.1: To establish the weak stationarity conditions for the state space system (3.6), we rewrite the system as follows. First, considering the latent state equation we have:

$$\begin{aligned}\alpha_t &= \gamma + A\alpha_{t-1} + B(\omega + \alpha_{t-1} + \epsilon_{t-1}) + u_t \\ &= (\gamma + B\omega) + (A+B)\alpha_{t-1} + u_t + B\epsilon_{t-1}\end{aligned}$$

where $\epsilon_t = \mathrm{w}_t - \omega \sim \text{i.i.d}(0, \Sigma_{\mathrm{w}})$. Since $B\epsilon_{t-1}$ is a white noise process which is independent of the white noise process u_t, it follows that $u_t + B\epsilon_{t-1}$ is also white noise (Granger and Morris, 1976).[44] Therefore, the state vector $\alpha_t := (\log\phi_t, \log\sigma_t^2)'$ follows a VAR(1) model, which will be covariance stationary if and only if

$$\det(I_2 - Hz) \neq 0 \text{ for all } |z| \leq 1, \tag{3.20}$$

where $H = A + B$ (Lütkepohl, 2005, p. 423).

As for the measurement equation we have

$$y_t = (\omega + \gamma - A\omega) + (A+B)y_{t-1} + (u_t + \epsilon_t - A\epsilon_{t-1}). \tag{3.21}$$

Since $\epsilon_{t-1} - A\epsilon_{t-1}$ is an MA(1) process independent of the white noise process u_t, $u_t + \epsilon_t - A\epsilon_{t-1}$ is also an MA(1) process. Consequently, y_t follows a VARMA(1,1) process. Therefore, (3.20) is also the necessary and sufficient condition for $y_t := (\log v_t, \log e_t^2)'$ to be weakly stationary. □

Proof of Proposition 3.2: When the covariance stationarity condition (3.20) holds, the latent process of the system (3.6) can be expressed as

$$\begin{aligned}\alpha_t &= \gamma + H\alpha_{t-1} + u_t + B\mathrm{w}_{t-1} \\ &= (I_2 - HL)^{-1}\gamma + (I_2 - HL)^{-1}(u_t + B\mathrm{w}_{t-1}) \\ &= (I_2 - H)^{-1}\gamma + \sum_{i=0}^{\infty} H^i(u_{t-i} + B\mathrm{w}_{t-1-i}),\end{aligned} \tag{3.22}$$

[44] Indeed, let $x_t = u_t + B\epsilon_{t-1}$, then we have: (i) $\mathbb{E}(x_t) = 0$, and (ii) $\mathbb{E}(x_t x_{t-s}') = \begin{cases}\Sigma_u + B\Sigma_{\mathrm{w}}B' & \text{if } s = 0,\\ 0 & \text{if } s \geq 1,\end{cases}$ since $\epsilon_t \sim \text{i.i.d}(0, \Sigma_{\mathrm{w}})$, $u_t \sim \text{i.i.d}(0, \Sigma_u)$, and u_t and ϵ_s are independent for all t and s.

where $H = A + B$ and L is the lag operator.

Given that $\mathrm{w}_t \sim \text{i.i.d}(\omega, \Sigma_{\mathrm{w}})$, $u_t \sim \text{i.i.d}(0, \Sigma_u)$, and u_t and w_s are independent for all t and s, we then have

$$\mathbb{E}(\alpha_t) = (I_2 - H)^{-1}\gamma + \sum_{i=0}^{\infty} H^i B\omega = (I_2 - H)^{-1}(\gamma + B\omega),$$

and

$$\Theta_\alpha(0) := \text{Var}(\alpha_t) = \text{Var}\left(\sum_{i=0}^{\infty} H^i (u_{t-i} + B\mathrm{w}_{t-1-i})\right) = \sum_{i=0}^{\infty} H^i (\Sigma_u + B\Sigma_{\mathrm{w}}B')(H')^i,$$

which implies

$$\begin{aligned}\text{vec}(\Theta_\alpha(0)) &= \sum_{i=0}^{\infty} \text{vec}\left(H^i (\Sigma_u + B\Sigma_{\mathrm{w}}B')(H')^i\right) = \sum_{i=0}^{\infty} \left(H^i \otimes H^i\right) \text{vec}(\Sigma_u + B\Sigma_{\mathrm{w}}B') \\ &= (I_4 - H \otimes H)^{-1} \text{vec}(\Sigma_u + B\Sigma_{\mathrm{w}}B').\end{aligned}$$

Now for $s \geq 1$,

$$\begin{aligned}\Theta_\alpha(s) :=& \text{Cov}(\alpha_t, \alpha_{t-s}) \\ =& \text{Cov}\left(\sum_{i=0}^{\infty} H^i(u_{t-i} + B\mathrm{w}_{t-1-i}), \sum_{i=0}^{\infty} H^i(u_{t-s-i} + B\mathrm{w}_{t-s-1-i})\right) \\ =& \text{Cov}\left(\sum_{i=0}^{s-1} H^i(u_{t-i} + B\mathrm{w}_{t-1-i}) + \sum_{i=0}^{\infty} H^{i+s}(u_{t-s-i} + B\mathrm{w}_{t-s-1-i}), \sum_{i=0}^{\infty} H^i(u_{t-s-i} + B\mathrm{w}_{t-s-1-i})\right) \\ =& \sum_{i=0}^{\infty} H^{i+s}(\Sigma_u + B\Sigma_{\mathrm{w}}B')(H')^i \\ =& H^s \sum_{i=0}^{\infty} H^i(\Sigma_u + B\Sigma_{\mathrm{w}}B')(H')^i \\ =& H^s \Theta_\alpha(0).\end{aligned}$$

This completes the proof. □

Proof of Proposition 3.3: Since $y_t = \alpha_t + \mathrm{w}_t$, from Proposition 3.2 we have that

$$\mathbb{E}(y_t) = \mathbb{E}(\alpha_t) + \mathbb{E}(\mathrm{w}_t) = (I_2 - H)^{-1}(\gamma + B\omega) + \omega,$$

and

$$\Theta_y(0) := \text{Var}(y_t) = \Theta_\alpha(0) + \Sigma_{\mathrm{w}} + \text{Cov}(\alpha_t, \mathrm{w}_t) + \text{Cov}(\mathrm{w}_t, \alpha_t) = \Theta_\alpha(0) + \Sigma_{\mathrm{w}},$$

and, for $s \geq 1$,

$$\Theta_y(s) := \text{Cov}(y_t, y_{t-s}) = \text{Cov}(\alpha_t, \alpha_{t-s}) + \text{Cov}(\alpha_t, \mathrm{w}_{t-s}) + \text{Cov}(\mathrm{w}_t, \alpha_{t-s}) + \text{Cov}(\mathrm{w}_t, \mathrm{w}_{t-s})$$

$$= \Theta_\alpha(s) + H^{s-1}B\Sigma_{\mathrm{w}}.$$

These above results are obtained using the fact that $\mathrm{Cov}(\mathrm{w}_t, \alpha_{t-s}) = 0$ for all $s \geq 0$, and $\mathrm{Cov}(\alpha_t, \mathrm{w}_{t-s}) = H^{s-1}B\Sigma_{\mathrm{w}}$ for $s \geq 1$, both of which result from the moving average representation of α_t in equation (3.22), in combination with the usual assumptions that $\mathrm{w}_t \sim \mathrm{i.i.d}(\omega, \Sigma_{\mathrm{w}})$, $u_t \sim \mathrm{i.i.d}(0, \Sigma_u)$, and u_t and w_s are independent for all t and s. □

Proof of Proposition 3.4: The first part of the Proposition is trivial and follows directly from the definition of autocorrelations (see, for example, Lütkepohl, 2005, p. 30). To prove the second part of Proposition 3.4, note that under the SVV_0 model, $B = 0$ so $H = A = diag(a_v, a_{e^2})$ and $H^i = diag(a_v^i, a_{e^2}^i)$, for $i \geq 0$. Assuming the stationarity of the SVV_0 system implies that $|a_j| < 1$, $j \in \{v, e^2\}$. From Proposition 3.2 and letting $\Sigma_u = [{}_u\sigma_{j,k}]$, $(j, k \in \{v, e^2\})$, we have

$$\Theta_\alpha(0) = \sum_{i=0}^{\infty} H^i \Sigma_u (H')^i = \sum_{i=0}^{\infty} diag(a_v^i, a_{e^2}^i) \times [{}_u\sigma_{j,k}] \times diag(a_v^i, a_{e^2}^i)$$
$$= \sum_{i=0}^{\infty} \left[a_j^i a_k^i \; {}_u\sigma_{j,k}\right] = \left[\frac{{}_u\sigma_{j,k}}{1 - a_j a_k}\right], \text{ for } j, k \in \{v, e^2\}.$$

Thus, $D_\alpha = diag\left(\sqrt{\frac{{}_u\sigma_{v,v}}{1-a_v^2}}, \sqrt{\frac{{}_u\sigma_{e^2,e^2}}{1-a_{e^2}^2}}\right)$ (see the definition for D_α in Proposition 3.4), and

$$\Theta_\alpha(s) = H^s\Theta_\alpha(0) = \begin{bmatrix} a_v^s \frac{{}_u\sigma_{v,v}}{1-a_v^2} & a_v^s \frac{{}_u\sigma_{v,e^2}}{1-a_v a_{e^2}} \\ a_{e^2}^s \frac{{}_u\sigma_{e^2,v}}{1-a_v a_{e^2}} & a_{e^2}^s \frac{{}_u\sigma_{e^2,e^2}}{1-a_{e^2}^2} \end{bmatrix} = \left[\frac{a_j^s \; {}_u\sigma_{j,k}}{1-a_j a_k}\right], \text{ for } j, k \in \{v, e^2\}.$$

Therefore, $R_\alpha(s) = D_\alpha^{-1}\Theta_\alpha(s)D_\alpha^{-1} = [{}_\alpha\rho_{j,k,s}]$, where ${}_\alpha\rho_{j,k,s} = \dfrac{\frac{a_j^s \, {}_u\sigma_{j,k}}{1-a_j a_k}}{\sqrt{\frac{{}_u\sigma_{j,j}}{1-a_j^2}}\sqrt{\frac{{}_u\sigma_{k,k}}{1-a_k^2}}}$, $j, k \in \{v, e^2\}$.

For the process y_t of the SVV_0 system, it is noted from Proposition 3.3 that $\Theta_y(0) = \Theta_\alpha(0) + \Sigma_{\mathrm{w}}$ and $\Theta_y(s) = \Theta_\alpha(s)$ for $s \geq 1$. Straightforward calculations of $R_y(s)$ give the last result of Proposition 3.4, which completes the proof. □

Proof of Proposition 3.5: We first prove equation (3.9). From equations (3.22) and

(3.7) we have

$$\alpha_t = \begin{bmatrix} \alpha_{v,t} \\ \alpha_{e^2,t} \end{bmatrix} := \begin{bmatrix} \log \phi_t \\ \log \sigma_t^2 \end{bmatrix} = \begin{bmatrix} \overline{\gamma}_v \\ \overline{\gamma}_{e^2} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} h'_{v,i} u_{t-i} + \sum_{q\in\{v,e^2\}} \beta_{v,q,i} \mathrm{w}_{q,t-1-i} \\ h'_{e^2,i} u_{t-i} + \sum_{q\in\{v,e^2\}} \beta_{\sigma^2,q,i} \mathrm{w}_{q,t-1-i} \end{bmatrix}. \tag{3.23}$$

Thus, $\alpha_{j,t} = \overline{\gamma}_j + \sum_{i=0}^{\infty} \left(h'_{j,i} u_{t-i} + \sum_{q\in\{v,e^2\}} \beta_{j,q,i} \mathrm{w}_{q,t-1-i} \right)$, for $j \in \{v, e^2\}$.

Since $\chi_t := (\chi_{v,t}, \chi_{e^2,t})' = (\phi_t, \sigma_t^2)'$, we have, for $j \in \{v, e^2\}$,

$$\chi_{j,t} = \exp(\alpha_{j,t}) = \exp\left(\overline{\gamma}_j\right) \prod_{i=0}^{\infty} \exp(h'_{j,i} u_{t-i}) \prod_{i=0}^{\infty} \left(\prod_{q\in\{v,e^2\}} \exp(\beta_{j,q,i} \mathrm{w}_{q,t-1-i}) \right). \tag{3.24}$$

Given the serial and mutual independence between u_t and w_t, the mutual independence between $\mathrm{w}_{j,t}$ ($j \in \{v, e^2\}$), and $u_t \sim N(0, \Sigma_u)$, the cross-moment between $\chi_{j,t}$ and $\chi_{k,t-s}$, for $j,k \in \{v, e^2\}$ and $m, n, s \geq 0$, is given by

$$\begin{aligned}
\mathbb{E}\left[\chi_{j,t}^m \chi_{k,t-s}^n\right] =& \mathbb{E}\Bigg[\exp\left(m\overline{\gamma}_j\right) \prod_{i=0}^{\infty} \exp(m h'_{j,i} u_{t-i}) \prod_{i=0}^{\infty} \left(\prod_{q\in\{v,e^2\}} \exp(m\beta_{j,q,i} \mathrm{w}_{q,t-1-i}) \right) \\
& \times \exp\left(n\overline{\gamma}_k\right) \prod_{i=0}^{\infty} \exp(n h'_{k,i} u_{t-s-i}) \prod_{i=0}^{\infty} \left(\prod_{q\in\{v,e^2\}} \exp(n\beta_{k,q,i} \mathrm{w}_{q,t-s-1-i}) \right)\Bigg] \\
=& \mathbb{E}\Bigg[\exp\left(m\overline{\gamma}_j + n\overline{\gamma}_k\right) \prod_{i=0}^{s-1} \exp(m h'_{j,i} u_{t-i}) \prod_{i=0}^{\infty} \exp((m h'_{j,i+s} + n h'_{k,i}) u_{t-s-i}) \\
& \times \prod_{i=0}^{s-1} \left(\prod_{q\in\{v,e^2\}} \exp(m\beta_{j,q,i} \mathrm{w}_{q,t-1-i}) \right) \prod_{i=0}^{\infty} \left(\prod_{q\in\{v,e^2\}} \exp((m\beta_{j,q,i+s} + n\beta_{k,q,i}) \mathrm{w}_{q,t-s-1-i}) \right)\Bigg] \\
=& \exp\left(m\overline{\gamma}_j + n\overline{\gamma}_k\right) \prod_{i=0}^{s-1} \mathbb{E}\left[\exp(m h'_{j,i} u_{t-i})\right] \prod_{i=0}^{\infty} \mathbb{E}\left[\exp((m h'_{j,i+s} + n h'_{k,i}) u_{t-s-i})\right] \\
& \times \prod_{i=0}^{s-1} \left(\prod_{q\in\{v,e^2\}} \mathbb{E}\left[\exp(m\beta_{j,q,i} \mathrm{w}_{q,t-1-i})\right] \right) \prod_{i=0}^{\infty} \left(\prod_{q\in\{v,e^2\}} \mathbb{E}\left[\exp((m\beta_{j,q,i+s} + n\beta_{k,q,i}) \mathrm{w}_{q,t-s-1-i})\right] \right) \\
=& \exp\left(m\overline{\gamma}_j + n\overline{\gamma}_k\right) \prod_{i=0}^{s-1} \exp\left(\frac{m^2}{2} h'_{j,i} \Sigma_u h_{j,i}\right) \prod_{i=0}^{\infty} \exp\left(\frac{1}{2} (m h'_{j,i+s} + n h'_{k,i}) \Sigma_u (m h_{j,i+s} + n h_{k,i})\right) \\
& \times \prod_{i=0}^{s-1} \left(\prod_{q\in\{v,e^2\}} M_q(m\beta_{j,q,i}) \right) \prod_{i=0}^{\infty} \left(\prod_{q\in\{v,e^2\}} M_q(m\beta_{j,q,i+s} + n\beta_{k,q,i}) \right),
\end{aligned}$$

where $M_q(\cdot)$ is defined in (3.8). The last equality is obtained using the fact that $m h'_{j,i} u_{t-i} \sim N(0, m^2 h'_{j,i} \Sigma_u h_{j,i})$, $(m h'_{j,i+s} + n h'_{k,i}) u_{t-s-i} \sim N(0, (m h'_{j,i+s} + n h'_{k,i}) \Sigma_u (m h_{j,i+s} + n h_{k,i}))$, and $\mathbb{E}(\exp(x)) = \exp(\mu_x + \frac{1}{2}\sigma_x^2)$ if $x \sim N(\mu_x, \sigma_x^2)$. Thus, equation (3.9) is proved.

We now prove equation (3.10). Since $\tau_{j,t} = \exp(\alpha_{j,t} + \mathrm{w}_{j,t}) = \chi_{j,t} \exp(\mathrm{w}_{j,t})$,

$$\tau_{j,t}^m \tau_{k,t-s}^n = \chi_{j,t}^m \chi_{k,t-s}^n \exp(m\mathrm{w}_{j,t}) \exp(n\mathrm{w}_{k,t-s}) \qquad \text{for } j,k \in \{v, e^2\} \text{ and } m, n, s \geq 0.$$

From equation (3.24), $\chi_{j,t}$ and $\mathrm{w}_{k,t}$ are independent for any $j,k \in \{v,e^2\}$, due to the serial and mutual independence between u_t and w_t.

Now, if $s=0$ and $j \neq k$, then

$$\mathbb{E}\left[\tau_{j,t}^m \tau_{k,t-s}^n\right] = \mathbb{E}\left[\tau_{j,t}^m \tau_{k,t}^n\right] = \mathbb{E}\left[\chi_{j,t}^m \chi_{k,t}^n \exp(m\mathrm{w}_{j,t})\exp(n\mathrm{w}_{k,t})\right] = \mathbb{E}\left[\chi_{j,t}^m \chi_{k,t}^n\right] M_j(m) M_k(n), \tag{3.25}$$

since $\chi_{j,t}^m \chi_{k,t}^n$, $\mathrm{w}_{j,t}$ and $\mathrm{w}_{k,t}$ are independent.

If $s=0$ and $j=k$, then

$$\mathbb{E}\left[\tau_{j,t}^m \tau_{k,t-s}^n\right] = \mathbb{E}\left[\tau_{j,t}^{m+n}\right] = \mathbb{E}\left[\chi_{j,t}^{m+n} \exp((m+n)\mathrm{w}_{j,t})\right] = \mathbb{E}\left[\chi_{j,t}^{m+n}\right] M_j(m+n). \tag{3.26}$$

If $s \geq 1$, then

$$\begin{aligned}
\mathbb{E}\left[\tau_{j,t}^m \tau_{k,t-s}^n\right] =& \mathbb{E}\left[\chi_{j,t}^m \chi_{k,t-s}^n \exp(m\mathrm{w}_{j,t})\exp(n\mathrm{w}_{k,t-s})\right] \\
=& \mathbb{E}\Bigg[\exp\left(m\overline{\gamma}_j + n\overline{\gamma}_k\right)\prod_{i=0}^{s-1}\exp(mh_{j,i}' u_{t-i})\prod_{i=0}^{\infty}\exp\left((mh_{j,i+s}' + nh_{k,i}')u_{t-s-i}\right) \\
& \times\exp(m\mathrm{w}_{j,t})\prod_{i=0}^{s-2}\left(\prod_{q\in\{v,e^2\}}\exp(m\beta_{j,q,i}\mathrm{w}_{q,t-1-i})\right)\times\left(\prod_{\substack{q\in\{v,e^2\}\\ q\neq k}}\exp(m\beta_{j,q,s-1}\mathrm{w}_{q,t-s})\right) \\
& \times\exp((m\beta_{j,k,s-1}+n)\mathrm{w}_{k,t-s})\prod_{i=0}^{\infty}\left(\prod_{q\in\{v,e^2\}}\exp((m\beta_{j,q,i+s}+n\beta_{k,q,i})\mathrm{w}_{q,t-s-1-i})\right)\Bigg] \\
=& \exp\left(m\overline{\gamma}_j + n\overline{\gamma}_k\right)\prod_{i=0}^{s-1}\exp\left(\frac{m^2}{2}h_{j,i}'\Sigma_u h_{j,i}\right)\prod_{i=0}^{\infty}\exp\left(\frac{1}{2}(mh_{j,i+s}' + nh_{k,i}')\Sigma_u(mh_{j,i+s} + nh_{k,i})\right) \\
& \times M_j(m)\prod_{i=0}^{s-2}\left(\prod_{q\in\{v,e^2\}} M_q(m\beta_{j,q,i})\right)\times\left(\prod_{\substack{q\in\{v,e^2\}\\ q\neq k}} M_q(m\beta_{j,q,s-1})\right) \\
& \times M_k(m\beta_{j,k,s-1}+n)\prod_{i=0}^{\infty}\left(\prod_{q\in\{v,e^2\}} M_q(m\beta_{j,q,i+s}+n\beta_{k,q,i})\right) \\
=& \mathbb{E}\left[\chi_{j,t}^m \chi_{k,t-s}^n\right]\times M_j(m)\frac{M_k(m\beta_{j,k,s-1}+n)}{M_k(m\beta_{j,k,s-1})}.
\end{aligned} \tag{3.27}$$

Combining the results in equations (3.9), (3.25), (3.26) and (3.27) gives equation (3.10). Thus, the proof of Proposition 3.5 is complete. □

Proof of Proposition 3.6: The proof of Proposition 3.6 is similar to that of Proposition 3.5 but it does not assume the mutual dependence amongst the measurement errors

w_t because with B being a zero matrix, $\chi_{j,t} = \exp\left(\overline{\gamma}_j\right)\prod_{i=0}^{\infty}\exp(h'_{j,i}u_{t-i})$, for $j \in \{v, e^2\}$, which does not depend on w_{t-i}, $i \geq 0$. However, we note that the contemporaneous cross-moment of τ_t, $\mathbb{E}\left[\tau_{j,t}^m \tau_{k,t}^n\right]$ $(j \neq k)$, equals $\mathbb{E}\left[\chi_{j,t}^m \chi_{k,t}^n\right]\mathbb{E}\left[\exp(m\mathrm{w}_{j,t})\exp(n\mathrm{w}_{k,t})\right]$ (see equation (3.25)), where $\mathbb{E}\left[\exp(m\mathrm{w}_{j,t})\exp(n\mathrm{w}_{k,t})\right]$ generally does not have a closed-form expression if the joint distribution of w_t is not known, but it simplifies to $M_j(m)M_k(n)$ if one assumes that the measurement errors w_t are mutually independent. □

Proof of Corollary 3.1: The main result that $\mathbb{E}\left[\tau_{j,t}^m\right] = \mathbb{E}\left[\chi_{j,t}^m\right] \times M_j(m)$, $j \in \{v, e^2\}$, follows directly from the fact that $\tau_{j,t} = \chi_{j,t}\exp(\mathrm{w}_{j,t})$ and the independence between $\chi_{j,t}$ and $\mathrm{w}_{j,t}$. Meanwhile, the expressions for $\mathbb{E}\left[\chi_{j,t}^m\right]$ in points *(i)* and *(ii)* immediately follow from Propositions 3.5 and 3.6, respectively, by letting $n = 0$. When B is a zero matrix and $A = diag(a_v, a_{e^2})$, $H = A$, $\overline{\gamma} = (I_2 - H)^{-1}\gamma = \left(\frac{\gamma_v}{1-a_v}, \frac{\gamma_{e^2}}{1-a_{e^2}}\right)'$, and $H^i = \begin{bmatrix} h'_{v,i} \\ h'_{e^2,i} \end{bmatrix} = diag(a_v^i, a_{e^2}^i)$, for $i \geq 0$. Consequently, $\overline{\gamma}_j = \frac{\gamma_j}{1-a_j}$ and $\exp\left(\frac{m^2}{2}h'_{j,i}\Sigma_u h_{j,i}\right) = \exp\left(\frac{m^2}{2}a_j^{2i}\,{}_u\sigma_{j,j}\right)$, where ${}_u\sigma_{j,j} = \mathrm{Var}(u_{j,t})$, $j \in \{v, e^2\}$. Simple simplifications of the result in Point *(ii)* give the result in Point *(iii)*. □

Proof of Proposition 3.7: For two generic time series $x_{j,t}$ and $x_{k,t}$, their cross correlation function is

$$ {}_x\rho_{j,k,s} = \mathrm{Corr}(x_{j,t}, x_{k,t-s}) = \frac{\mathrm{Cov}(x_{j,t}, x_{k,t-s})}{\sqrt{\mathrm{Var}(x_{j,t})\mathrm{Var}(x_{k,t-s})}} = \frac{\mathbb{E}(x_{j,t}x_{k,t-s}) - \mathbb{E}(x_{j,t})\mathbb{E}(x_{k,t-s})}{\sqrt{\mathbb{E}\left(x_{j,t}^2\right) - \left[\mathbb{E}(x_{j,t})\right]^2}\sqrt{\mathbb{E}\left(x_{k,t-s}^2\right) - \left[\mathbb{E}(x_{k,t-s})\right]^2}}, $$

for $s \geq 0$. We use this formula to compute the cross correlation functions of the processes χ_t and τ_t of the SVV_0 model.

Under the SVV_0 system, $\overline{\gamma} = (I_2 - H)^{-1}\gamma = \left(\frac{\gamma_v}{1-a_v}, \frac{\gamma_{e^2}}{1-a_{e^2}}\right)'$, and $H^i = \begin{bmatrix} h'_{v,i} \\ h'_{e^2,i} \end{bmatrix} = diag(a_v^i, a_{e^2}^i)$, for $i \geq 0$. Stationarity implies that $|a_j| < 1$, $j \in \{v, e^2\}$. From Proposition 3.6, for $j, k \in \{v, e^2\}$,

$$ \begin{aligned} \mathbb{E}\left[\chi_{j,t}\chi_{k,t-s}\right] &= \exp\left(\overline{\gamma}_j + \overline{\gamma}_k\right)\prod_{i=0}^{s-1}\exp\left(\frac{1}{2}h'_{j,i}\Sigma_u h_{j,i}\right)\prod_{i=0}^{\infty}\exp\left(\frac{1}{2}(h'_{j,i+s} + h'_{k,i})\Sigma_u(h_{j,i+s} + h_{k,i})\right) \\ &= \exp\left(\frac{\gamma_j}{1-a_j} + \frac{\gamma_k}{1-a_k}\right)\prod_{i=0}^{s-1}\exp\left(\frac{1}{2}a_j^{2i}\,{}_u\sigma_{j,j}\right)\prod_{i=0}^{\infty}\exp\left(\frac{1}{2}\left(a_j^{2i+2s}\,{}_u\sigma_{j,j} + 2a_j^{i+s}a_k^i\,{}_u\sigma_{j,k} + a_k^{2i}\,{}_u\sigma_{k,k}\right)\right) \end{aligned} $$

$$=\exp\left(\frac{\gamma_j}{1-a_j}+\frac{\gamma_k}{1-a_k}\right)\exp\left(\frac{(1-a_j^{2s})_u\sigma_{j,j}}{2(1-a_j^2)}\right)\exp\left(\frac{a_j^{2s}\,{}_u\sigma_{j,j}}{2(1-a_j^2)}\right)\exp\left(\frac{a_j^s\,{}_u\sigma_{j,k}}{1-a_ja_k}\right)\exp\left(\frac{{}_u\sigma_{k,k}}{2(1-a_k^2)}\right)$$
$$=\exp\left(\frac{\gamma_j}{1-a_j}+\frac{\gamma_k}{1-a_k}\right)\exp\left(\frac{{}_u\sigma_{j,j}}{2(1-a_j^2)}\right)\exp\left(\frac{a_j^s\,{}_u\sigma_{j,k}}{1-a_ja_k}\right)\exp\left(\frac{{}_u\sigma_{k,k}}{2(1-a_k^2)}\right).$$

From Corollary 3.1,

$$\mathbb{E}\left[\chi_{j,t}\right]\mathbb{E}[\chi_{k,t-s}]=\exp\left(\frac{\gamma_j}{1-a_j}+\frac{\gamma_k}{1-a_k}\right)\exp\left(\frac{{}_u\sigma_{j,j}}{2(1-a_j^2)}\right)\exp\left(\frac{{}_u\sigma_{k,k}}{2(1-a_k^2)}\right),$$
$$\mathbb{E}\left[\chi_{j,t}^2\right]=\exp\left(\frac{2\gamma_j}{1-a_j}\right)\exp\left(\frac{2\,{}_u\sigma_{j,j}}{1-a_j^2}\right),\ \text{and}$$
$$(\mathbb{E}\left[\chi_{j,t}\right])^2=\exp\left(\frac{2\gamma_j}{1-a_j}\right)\exp\left(\frac{{}_u\sigma_{j,j}}{1-a_j^2}\right).$$

Therefore,

$$_\chi\rho_{j,k,s}=\frac{\mathbb{E}\left[\chi_{j,t}\chi_{k,t-s}\right]-\mathbb{E}\left[\chi_{j,t}\right]\mathbb{E}[\chi_{k,t-s}]}{\sqrt{\mathbb{E}\left[\chi_{j,t}^2\right]-(\mathbb{E}\left[\chi_{j,t}\right])^2}\sqrt{\mathbb{E}\left[\chi_{k,t-s}^2\right]-(\mathbb{E}[\chi_{k,t-s}])^2}}=\frac{\exp\left(\frac{a_j^s\,{}_u\sigma_{j,k}}{1-a_ja_k}\right)-1}{\sqrt{\exp\left(\frac{{}_u\sigma_{j,j}}{1-a_j^2}\right)-1}\sqrt{\exp\left(\frac{{}_u\sigma_{k,k}}{1-a_k^2}\right)-1}}.$$

For the process τ_t, using Proposition 3.6 and Corollary 3.1 we have

$$_\tau\rho_{j,k,s}=\frac{\mathbb{E}\left[\tau_{j,t}\tau_{k,t-s}\right]-\mathbb{E}\left[\tau_{j,t}\right]\mathbb{E}[\tau_{k,t-s}]}{\sqrt{\mathbb{E}\left[\tau_{j,t}^2\right]-(\mathbb{E}\left[\tau_{j,t}\right])^2}\sqrt{\mathbb{E}\left[\tau_{k,t}^2\right]-(\mathbb{E}[\tau_{k,t}])^2}}$$
$$=\frac{\mathbb{E}\left[\chi_{j,t}\chi_{k,t-s}\right]\times g_1(j,k,1,1,s)-\mathbb{E}\left[\chi_{j,t}\right]\mathbb{E}[\chi_{k,t-s}]M_j(1)M_k(1)}{\sqrt{\mathbb{E}\left[\chi_{j,t}^2\right]M_j(2)-(\mathbb{E}\left[\chi_{j,t}\right]M_j(1))^2}\sqrt{\mathbb{E}\left[\chi_{k,t-s}^2\right]M_k(2)-(\mathbb{E}[\chi_{k,t-s}]M_k(1))^2}}$$
$$=\frac{\exp\left(\frac{a_j^s\,{}_u\sigma_{j,k}}{1-a_ja_k}\right)\times g_1(j,k,1,1,s)-M_j(1)M_k(1)}{\sqrt{M_j(2)\exp\left(\frac{{}_u\sigma_{j,j}}{1-a_j^2}\right)-[M_j(1)]^2}\sqrt{M_k(2)\exp\left(\frac{{}_u\sigma_{k,k}}{1-a_k^2}\right)-[M_k(1)]^2}}.$$

The proof of Proposition 3.7 is complete. □

Chapter 4

Dynamics of the limit order book and the volume-volatility relation

4.1 Introduction

Investigation of the relationship between trading volume and price volatility has been an area of active research in finance for a long time. The study of the volume-volatility relation(ship) is important because it provides evidence on how information flows into the market, how it is processed and disseminated by trading activities of market participants, and hence how it affects the price formation process (Karpoff, 1987). Prior literature often documents a positive relationship between volume and volatility, which is normally measured by the correlation between the two variables in theoretical research, or alternatively by the change in volatility that results from an increase in trading volume. Market microstructure theories provide several suggestions for factors that drive this positive connection, be it either the arrival of new information that generates both price and volume movements (e.g. Clark, 1973, Andersen, 1996), the disagreement among investors about asset values (e.g. Grundy and McNichols, 1989, Shalen, 1993, Banerjee and Kremer, 2010), or strategic trading activities by informed and uninformed traders in an asymmetric trading environment (e.g. Kyle, 1985, Holden and Subrahmanyam, 1992).

Meanwhile, empirical work on this issue is rather limited. Most empirical studies (e.g. Ahn et al., 2001, Chan and Fong, 2006, Pascual and Veredas, 2010, Chevallier and Sévi, 2012, Carlin et al., 2014, Haugom et al., 2014, Clements and Todorova, 2016) document the determinants of volatility, rather than the volume-volatility relation. These studies show that trading activity (the number of trades and average trade size), disagreements amongst traders, order flow information, and order book characteristics (bid-ask spread and book depth) all contribute to the explanation of volatility, yet they are silent on the way in which these factors affect the *volume-volatility relation*. In other words, these studies assume that the dependence of volatility on volume is constant over time and does not vary with other factors.

The study by Xu et al. (2006) sheds some empirical light on this matter. They identify trading intensity (or trade durations) as a determinant of the volume-volatility relation by showing that higher trading frequency (which shortens durations between trades) tends to strengthen the positive association between volatility and volume. Using the slope of the order book as a proxy for the heterogeneity in investors' information and beliefs, Næs and Skjeltorp (2006) find that increased disagreement amongst traders, which is associated with a more gentle order book slope, makes the positive daily correlation between volume and volatility more pronounced. Similarly, Bollerslev et al. (2018) show that larger differences in beliefs around public news announcements affect the volume-volatility relation for the S&P500 equity index and U.S. Treasury bonds. In a related study, Wang and Wu (2015) find that the contemporaneous dependence of volatility on volume varies across different corporate bond groups that are stratified according to various liquidity and credit risk measures. These authors suggest that liquidity and credit qualities are important determinants of the volume-volatility relation in a corporate bond market. With the exception of Xu et al. (2006), these studies only consider the contemporaneous relationship between volume and volatility, and ignore lagged effects.

Electronic limit order books (LOBs) have become the dominant market trading platform in recent years, replacing the traditional specialists or quote-driven trading platforms in many major financial markets around the globe (Bloomfield et al., 2005, Goettler et al., 2009, Malinova and Park, 2013). Coming with the popularity of limit order trading is increased interest in studying the properties of limit order book markets and their role in explaining price dynamics. While many theoretical and empirical studies examine the composition of the order flow in an LOB market,[45] research into the use of LOB information to predict future returns and return volatility is relatively sparse, yet they all find evidence that the LOB informs the price process.[46] For example, Foucault et al. (2007), Nolte (2008) and Pascual and Veredas (2010) find that a wider bid-ask spread leads to higher future volatility. Thicker book depths help mitigate the return volatility of incoming orders (Ahn et al., 2001, Pascual and Veredas, 2010). Further, Næs and Skjeltorp (2006) document that a larger LOB slope is associated with a decrease in daily trading activity and return volatility, and it also tends to dampen the contemporaneous volume-volatility relation.

The objective of this chapter is to extend studies in the volume-volatility literature and the LOB literature to examine the role that the LOB information plays in explaining not only return volatility but also the volume-volatility relation. Unlike most existing studies which assume that the relationship between return volatility and trading volume is constant and fully contemporaneous (e.g. Chan and Fong, 2006, Næs and Skjeltorp, 2006, Wang and Wu, 2015, Clements and Todorova, 2016), we allow for the serial dependencies of volatility on volume, as implied by theoretical work of Copeland (1976), Shalen (1993) and Banerjee and Kremer (2010) and by the empirical work of Manganelli (2005), Xu et al. (2006), Carlin et al. (2014), Do et al. (2014), as well as Chapter 3 of this thesis. We allow the dynamic volume-volatility relationship to depend on the dynamics of LOB characteristics such as the bid-ask spread, the market depth at the inner quotes, and the LOB slope. The latter variable essentially measures how the quantity of stocks supplied in the LOB changes as a function of the

[45] These studies explore possible answers to questions such as which types of orders (limit vs. market) are often used by different types of investors (informed vs. uninformed), when these orders are used, and why. See, for example, Glosten (1994), Biais et al. (1995), Ranaldo (2004), Foucault et al. (2005), Wald and Horrigan (2005), Bloomfield et al. (2005), Anand et al. (2005), Kaniel and Liu (2006), Goettler et al. (2009), Roşu (2009).

[46] Notable studies in this strand of literature include Ahn et al. (2001), Næs and Skjeltorp (2006), Foucault et al. (2007), Nolte (2008), Kalay and Wohl (2009), and Pascual and Veredas (2010).

limit price, and hence it captures the LOB information not just at but also beyond the best quotes.

We conduct our volume-volatility analysis at a transaction or tick-by-tick level, rather than at daily or lower frequencies like most previous studies. Not only does the tick-by-tick analysis fit the frameworks of most theoretical studies and enable a deeper understanding of how information from trades is incorporated into prices (e.g. Easley and O'Hara, 1987, Shalen, 1993, Holden and Subrahmanyam, 1992), but it also offers a natural remedy to the undetermined causality between volatility and its explanatory variables, which results from the fixed-time aggregation of trades and prices (Hasbrouck, 1995). It also helps to avoid an information loss that comes from aggregating trades and prices over a fixed time interval that might bias estimation results (Engle, 2000, Manganelli, 2005, Russell and Engle, 2005). Acknowledging the random nature of trade arrival times at the transaction level, we follow Engle (2000) and Xu et al. (2006) in employing time-consistent measures of volume and volatility that are adjusted for trade durations. Our analysis also accommodates potential asymmetries between the bid and ask order books, and it controls for the effects of the order flow prior to a trade.

We examine the Australian limit order book market using a high-frequency tick-by-tick dataset of the constituent stocks of the S&P/ASX200 index during July-December 2014. We find strong evidence that the LOB possesses significant information content about the volume-volatility relation and the return volatility of trades. The dependence of return volatility on trading volume is positive, dynamic, and strongly related to the LOB information. Both the return volatility and the volume-volatility relation of a trade are positively associated with the bid-ask spread but negatively correlated with the market depth at the best quotes and the slope of the LOB prior to the transaction. These results generalize and are consistent with previous findings in the literature (Ahn et al., 2001, Næs and Skjeltorp, 2006, Foucault et al., 2007, Nolte, 2008, Pascual and Veredas, 2010, Haugom et al., 2014).

The impact of LOB characteristics on the return volatility of an incoming trade is conveyed via two channels; a *direct* channel that is mostly attributable to the lagged

LOB information, and an *indirect* channel that transfers through the volume-volatility relation and is mainly contributed by the prevailing LOB information right before the trade. This is because the direct (indirect) channel captures the partial effect of the LOB on return volatility without (with) the knowledge of a trade's volume. Therefore, if the volume of a trade is not known, the recent past order book attributes, which contain information about recent past trading volumes and volatility (which in turn are correlated with the volume and volatility of the trade), should play a critical role in explaining future return volatility. However, if the volume that a trade demands is known (i.e. if one conditions on the volume of the trade), the LOB characteristics immediately before the trade, which provide the most recent information about recent supply in the market, should be more relevant to the prediction of the trade's volatility than the past LOB information. This result is consistent with work by Pham et al. (2017), who find that in conjunction with the volume of a trade, the prevailing market depth right before the trade is particularly useful for detecting whether the trade has zero immediate price impact, and the depth information significantly improves the forecast accuracy of an immediate price impact model. In addition, the effects of the LOB information on return volatility, either direct or indirect, depend on stocks' liquidity.

We also observe significant asymmetries in the effects of the bid versus ask order books on return volatility and the volume-volatility relation, as previously noted in Ahn et al. (2001), Engle and Patton (2004) and Harris and Panchapagesan (2005). In particular, it is the order book of the opposite side to the direction of an incoming trade that is particularly predictive of the return volatility of the current trade. Interestingly, the effects of the bid-ask spread and market depth on return volatility and the volume-volatility relation either switch signs or become much less significant after controlling for the LOB slope, suggesting that the LOB slope is the driving determinant of return volatility and the volume-volatility relation. This is because the LOB slope summarizes the LOB information at all quote levels, and thus, it captures and dominates the information contained in the bid-ask spread and the market depth at the best quotes alone.

In order to explain the empirically observed negative dependence of return volatility and the volume-volatility relation on the (opposite-side) LOB slope, we provide an intuitive graphical illustration that can be summarized as follows: A large LOB slope indicates a steep and concentrated LOB that has many shares queued close to the inner quotes, which, as suggested by Næs and Skjeltorp (2006), might result from a high degree of agreement amongst investors who submitted their limit orders over a narrow range of prices. This state of the LOB is very liquid and is able to accommodate trades with little (or no) price impact. Consequently, all else being equal, for a trade of a given volume, the larger the slope of the (opposite-side) LOB, the smaller the change in the price caused by the trade (i.e. the smaller the volatility of the trade), which implies a negative correlation between return volatility and the LOB slope. Similarly, for a given increase in the trading volume, the steeper the (opposite-side) LOB, the smaller the price change, resulting in a weaker volume-volatility relation. Thus, the volume-volatility relation is negatively dependent on the order book slope.

Our contributions to the literature are threefold. First, we extend research on the volume-volatility relation by showing that the dynamics of the LOB are important factors that drive a positive dynamic relationship between return volatility and trading volume at a high frequency tick-by-tick level. Existing studies primarily assume that the volume-volatility relation is either fully contemporaneous or does not vary with other characteristics that may explain volatility, or both. In addition, they mainly examine the volume-volatility relation at a low frequency such as a day (see, amongst others, Næs and Skjeltorp, 2006, Chevallier and Sévi, 2012, Carlin et al., 2014, Wang and Wu, 2015, Clements and Todorova, 2016). Complementing prior findings, our study highlights the dynamic nature of the volume-volatility relation which is strongly dependent on the LOB characteristics at a transaction level. Second, our work also contributes to the literature that examines the information content of the LOB. We provide empirical evidence that supports the informativeness of the LOB about return volatility and the volume-volatility relation of trades. We also find strong evidence of the asymmetries between the effects of the bid and ask order books, as noted elsewhere (e.g. Ahn et al., 2001, Engle and Patton, 2004, Harris and Panchapagesan, 2005). Finally, our simple but intuitive graphical justification for the negative dependence of return volatility and the volume-volatility relation on the slope of the (opposite-side) LOB complements prior interpretations of the informativeness of the LOB slope in the literature (e.g. Næs and Skjeltorp, 2006).

This chapter extends the closely related work of Næs and Skjeltorp (2006) that also investigates the informativeness of the LOB attributes about the volume-volatility relation in several aspects. First, while Næs and Skjeltorp (2006) only consider the contemporaneous dependence of volatility on volume, we allow this relationship to be dynamic, as highlighted in the theoretical studies of Shalen (1993) and Banerjee and Kremer (2010). Second, Næs and Skjeltorp (2006) do not control for the *direct* impact of LOB characteristics on volatility when estimating how the LOB information alters the volume-volatility relation. Consequently, they tend to overestimate the effects of the LOB characteristics on the volume-volatility relation. Our study, on the other hand, explicitly accounts for such direct impact, and it shows different mechanisms through which LOB information affects volatility, depending on whether the effect is direct or indirect. In addition, we allow for the asymmetric effects of the bid versus ask order books on the volume-volatility relation, which are not considered in Næs and Skjeltorp (2006). Indeed, we find strong evidence of such asymmetries. Finally, instead of using lower frequency (daily) data as in Næs and Skjeltorp (2006), we employ high-frequency tick-by-tick data to conduct our analysis. This offers several advantages that will be discussed in more detail in subsection 4.2.2.

It is worth noting that since the properties of the volume-volatility relation are likely to be different depending on the horizons at which trading volume and return volatility are sampled,[47] the determinants of this relation in general depend on which frequencies we employ. In other words, factors that affect the volume-volatility relation at short horizons are likely to differ from those at longer frequencies. This chapter focuses on the high frequency effects of volume on volatility, and it investigates how the LOB information explains return volatility and the volume-volatility relation at

[47] For example, at a tick-by-tick horizon, much of the impact of volume on volatility is temporary; whereas, this impact becomes more permanent at longer horizons.

this high frequency tick-by-tick level. Nevertheless, our findings on the informativeness of the LOB information about return volatility and the volume-volatility relation are generally consistent with the findings of previous studies that utilize longer sampling horizons. This result suggests that the effects of the LOB information on return volatility and the volume-volatility relation are genuine and robust across horizons.

The rest of this chapter is organized as follows. Section 4.2 details a general empirical framework that we employ to investigate the informativeness of the LOB characteristics about the volume-volatility relation. Section 4.3 describes the data, and discussions of the main empirical results follow in Section 4.4. In Section 4.5, we provide a graphical rationale for the informativeness of the LOB slope, and Section 4.6 offers some concluding remarks.

4.2 The volume-volatility relation

4.2.1 A general empirical framework

Let $\sigma_{i,t}$ and $v_{i,t}$ be proxies for the volatility and volume associated with stock i at time (or during the time interval) t, respectively. Previous studies (e.g. Jones et al. 1994, Chan and Fong 2000, Chan and Fong 2006, Chevallier and Sévi 2012, Clements and Todorova 2016) have typically examined the volume-volatility relation via the following regression:

$$\sigma_{i,t} = \alpha_0 + \sum_{j=1}^{q} \alpha_j \sigma_{i,t-j} + \beta v_{i,t} + \gamma' x_{i,t} + \eta_{i,t}, \tag{4.1}$$

where $\sum_{j=1}^{q} \alpha_j \sigma_{i,t-j}$ allows for the persistence in volatility, $\eta_{i,t}$ is a zero-mean disturbance term, and $x_{i,t}$ is a vector of independent variables that may explain the return volatility of stock i at time t. Numerous factors $x_{i,t}$ have been suggested by theoretical literature as important determinants of return volatility and the volume-volatility relation. For example, the Mixture of Distribution Hypothesis (MDH) of Clark (1973), Tauchen and Pitts (1983) and Andersen (1996), and the Sequential Arrival of Information Hypothesis (SAIH) of Copeland (1976), Jennings et al. (1981) and Jennings and Barry (1983) suggest that the arrival of new information is the driving force of the volume-volatility relation, since it generates the comovements in both trading volumes and asset prices. Meanwhile, studies of Kim and Verrecchia (1991), Harris and Raviv (1993), Shalen (1993), Kandel and Pearson (1995), Banerjee and Kremer (2010), Banerjee (2011), amongst others, attribute the volume-volatility relation to the investors' heterogeneity about asset values. According to these studies, a positive correlation between trading volumes and price volatility is driven by an over-reaction of one group of traders (such as speculators who observe noisy private information, or responsive traders who interpret the common public information too favorably or too unfavorably) after observing an increase in trading activities.

Factors that affect price impact of trades are also likely to explain the volume-volatility relation. For example, in the models of Kyle (1985) and Admati and Pfleiderer (1988), the market maker revises his quotes upon observing the aggregated order flow from both informed and uninformed traders. Thus, the order flow contains important information about prices, and hence the volume-volatility relation. Similarly, both adverse selection risk and inventory holding risk are important for the understanding of price dynamics and the relationship between volume and price volatility. This is because the larger a market order is submitted to the market, the further away the market maker is from his preferred inventory position and the more severely he is adversely selected if the incoming order is from an informed trader (O'Hara and Oldfield, 1986, Easley and O'Hara, 1987). Consequently, the market maker has to adjust prices more in order to compensate for his higher risk.

In equation (4.1), the volume coefficient β measures the impact of volume on volatility. Clearly, equation (4.1) does not answer the question of whether $x_{i,t}$, which contributes to the explanation of return volatility, also explains the volume-volatility relation, since the marginal effect of volume on volatility (i.e. β) does not vary with $x_{i,t}$. We investigate this question by allowing the volume coefficient β in (4.1) to be a function of $x_{i,t}$. To keep things simple, we decompose β as $\beta_0 + \delta' x_{i,t}$ to obtain the following model:[48]

$$\sigma_{i,t} = \alpha_0 + \sum_{j=1}^{q} \alpha_j \sigma_{i,t-j} + [\beta_0 + \delta' x_{i,t}] v_{i,t} + \gamma' x_{i,t} + \eta_{i,t}, \tag{4.2}$$

in which $x_{i,t}$ influences return volatility ($\sigma_{i,t}$) via two channels: by its *direct* impact on the latter (captured by γ), and by its *indirect* effect that alters the volume-volatility relation (captured by $\delta' x_{i,t}$). The decomposition in (4.2) is similar in spirit to the econometric methodology proposed by Dufour and Engle (2000) to investigate the informativeness of trade arrival times in explaining the price impact of a trade. It is also similar to the technique employed by Avramov et al. (2006), who show that selling activity is the driving factor of the asymmetric effect in daily volatility, which is often known as the "leverage effect" in individual stock returns. A recent study by Bollerslev et al. (2018) utilizes a similar decomposition and documents that the volume-volatility elasticity of the S&P500 equity index and U.S. Treasury bonds, which measures the expected percentage change in trading volume for a small percentage change in return volatility, becomes weaker around public news announcements and when there is a higher degree of disagreement in beliefs amongst investors.

Similar to most empirical studies in the literature that examine the volume-volatility relation (e.g. Chan and Fong, 2006, Næs and Skjeltorp, 2006, Shahzad et al., 2014, Wang and Wu, 2015, Bollerslev et al., 2018), both models (4.1) and (4.2) characterize the correlation between volume and volatility as fully *contemporaneous*. However, the SAIH of Copeland (1976) and Jennings et al. (1981) implies that there is a lead-lag relationship between volume and volatility, which results from sequential, rather than simultaneous, dissemination of information to market participants that creates a sequence of intermediate equilibria before the final equilibrium is reached. Similarly, theoretical models that feature heterogeneity in investors' beliefs about asset prices, due to either asymmetric private information (Shalen, 1993) or differences of opinions about public information (e.g. Harris and Raviv, 1993, Banerjee and Kremer, 2010),

[48] We only consider the case where the volume coefficient β in (4.1) varies linearly with $x_{i,t}$. It is, however, possible to allow for the nonlinear dependence of β on $x_{i,t}$, either nonparametrically by letting $\beta = f(x_{i,t})$, where $f(\cdot)$ is some unknown smooth function, or parametrically by writing $\beta = \beta_0 + \delta' z_{i,t}$, where $z_{i,t}$ contains $x_{i,t}$ and possibly its higher orders, nonlinear transformations, and/or interaction terms that allow for nonlinearities. We leave this direction for future research.

show that there are both contemporaneous and serial dependencies of volatility on volume. In addition, as highlighted in the microstructure model of Hasbrouck (1991a,b), microstructure imperfections such as price discreteness and inventory control effects might induce lagged adjustments in stock prices to a trade's information, implying that past trading volumes could be informative about future prices and volatility. Consistent with these theories, empirical work by Manganelli (2005), Xu et al. (2006), Nolte (2008), Carlin et al. (2014), and Do et al. (2014) finds significant current and lagged volume effects on return volatility. In order to accommodate this *dynamic* volume-volatility dependence, we modify equation (4.2) as follows:

$$\sigma_{i,t} = \alpha_0 + \sum_{j=1}^{q} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{p} [\beta_{0,k} + \delta_k' x_{i,t-k}] v_{i,t-k} + \sum_{k=0}^{p} \gamma_k' x_{i,t-k} + \eta_{i,t}, \tag{4.3}$$

where $\beta_{0,0} + \delta_0' x_{i,t}$ captures the contemporaneous impact of volume on volatility, $\beta_{0,k} + \delta_k' x_{i,t-k}$ $(k \geq 1)$ measures the lagged impact, and $\sum_{k=0}^{p} [\beta_{0,k} + \delta_k' x_{i,t-k}]$ gauges the cumulative impact. We add the term $\sum_{k=1}^{p} \gamma_k' x_{i,t-k}$, which measures the lagged direct effects of $x_{i,t}$ on return volatility, to (4.3) in order to ascertain whether $x_{i,t}$ is a genuine predictor of the volume-volatility relation, even after controlling for its contemporaneous and lagged direct impact on volatility.

4.2.2 Proxies for volatility and volume

Given the availability of high frequency data that detail the record of every trade in recent years, this chapter examines the dynamic volume-volatility relation at a tick-by-tick or transaction level. Although the use of transaction data is widespread in the market microstructure literature (e.g. Hasbrouck, 1991a,b, Dufour and Engle, 2000, Barclay et al., 2003) and in the duration-volatility modeling literature (e.g. Engle, 2000, Renault and Werker, 2011, Renault et al., 2014), most research investigates the volume-volatility relation at daily or lower frequencies,[49] and only a few studies, including Manganelli (2005), Xu et al. (2006) and Nolte (2008), provide an examination of the

[49] See, amongst others, Jones et al. (1994), Andersen (1996), Chan and Fong (2000), Avramov et al. (2006), Næs and Skjeltorp (2006), Giot et al. (2010), Chevallier and Sévi (2012), Carlin et al. (2014), Shahzad et al. (2014). Also see Karpoff (1987) for a detailed survey of more distant research on the topic. There are also several studies that examine the volume-volatility relation at an intradaily level such as Ahn et al. (2001) and Bollerslev et al. (2018) (15 minutes), Duong and Kalev (2008) (30 minutes and 1 hour), Pascual and Veredas (2010) (1 and 5 minutes), and Do et al. (2014) (5 minutes).

volume-volatility relation at a transaction level. This is quite surprising, given that the majority of theoretical studies on the topic develop their analysis at a tick-by-tick level (see, for example, Kyle, 1985, Easley and O'Hara, 1987, Holden and Subrahmanyam, 1992, Shalen, 1993). Despite being under-researched, possibly due to the computational difficulties associated with huge data sets needed for this, a tick-by-tick analysis of the volume-volatility relation could enable a deeper understanding of how prices adjust to absorb the information from trades because, as highlighted in the theoretical work by Diamond and Verrecchia (1987) and Easley and O'Hara (1992), the existence or absence of each individual trade is informative about price formulation. In addition, a tick-by-tick analysis helps avoid a loss of information that results from the aggregation of trades and prices over a fixed time interval that may potentially bias the analysis (Engle, 2000, Manganelli, 2005, Russell and Engle, 2005). It also provides a natural solution to the undetermined causality between volatility and its predictors often recognized in prior studies that employ lower frequency data (see subsection 4.2.4 for a more detailed discussion).

Volatility is typically measured over a fixed time interval such as an hour, a day or a week, depending on the frequency at which data are sampled. Previous studies in the volume-volatility literature often estimate fixed-interval volatility by using either (i) the absolute size of residuals from an autoregression of returns (e.g. Jones et al., 1994, Chan and Fong, 2000, Avramov et al., 2006, Næs and Skjeltorp, 2006); or (ii) a realized volatility measure computed by summing up the squared returns that are sampled intradaily (typically every 5 minutes) during the time interval over which volatility is measured (normally a day) (e.g. Chan and Fong, 2006, Giot et al., 2010, Chevallier and Sévi, 2012, Shahzad et al., 2014).[50] However, when working with tick-by-tick data, researchers face a challenging issue which is that transactions do not arrive in the market at regularly spaced time intervals but at irregular, random times.

[50]Under the assumption that asset prices follow a continuous time diffusion process without jumps, realized volatility is shown to be a consistent and more accurate estimator of the true unknown integrated volatility of the underlying price process than the absolute or squared return over the same period (Andersen et al., 2001, Barndorff-Nielsen and Shephard, 2002). When there are random jumps in prices, the latent integrated volatility can be consistently estimated by realized bi-power variation, which is proportional to, by a factor of $\pi/2$, the summation of the product of two adjacent absolute intraday returns over the time interval (Barndorff-Nielsen and Shephard, 2004, 2006).

This complicates the measurement of the volatility of a trade since the time duration of each trade is not the same. In order to obtain a meaningful and time-consistent measure of volatility, Engle (2000) and Xu et al. (2006) suggest that volatility should be adjusted for trade durations, and that *volatility per unit of time* should be a natural measure of volatility in tick-by-tick empirical analyses. In this chapter, we estimate the volatility per unit of time for a transaction by dividing the absolute size of the residual from the following regression by the duration of the trade:

$$r_{i,t} = \sum_{k=1}^{5} \psi_{i,k} Day_{k,i,t} + \sum_{k=1}^{5} \phi_{i,k} block_{k,i,t} + \sum_{k=1}^{q} \rho_{i,k} r_{i,t-k} + \varphi' x_{i,t} + \epsilon_{i,t}, \tag{4.4}$$

where $r_{i,t}$ denotes the return of the t-th trade in stock i, defined as the difference in the natural logarithms of the bid-ask midpoint following the trade and quoted in %, i.e. $r_{i,t} = 100(\ln(q_{i,t+1}) - \ln(q_{i,t}))$, where $q_{i,t}$ is the midpoint of the bid and ask quotes immediately before the t-th trade. $Day_{k,i,t}$ are day-of-week dummies, and $block_{k,i,t}$ are time-of-day dummy variables.[51] Lagged returns ($r_{i,t-k}$) are used to control for the autocorrelation in the return series. Meanwhile, the incorporation of $x_{i,t}$ (in (4.1)) into equation (4.4) allows for its possible power in explaining returns, which ensures that the effects of $x_{i,t}$ on volatility obtained from equations (4.1)-(4.3) are genuine and are not driven by the impact of $x_{i,t}$ on returns. We define $\sigma_{i,t}$ by $\sigma_{i,t} := |\widehat{\epsilon}_{i,t}|/T_{i,t}$, where $T_{i,t}$ is the duration of the t-th trade which measures the time (in seconds) between the $(t-1)$-th and t-th trades, and we use $\sigma_{i,t}$ as a proxy for volatility in models (4.2) and (4.3).[52]

Two proxies for trading activities that are popularly used in the volume-volatility literature are the number of trades and the average trade size during a fixed time interval. While both measures are found to be positively related to return volatility, prior studies often document that the number of trades, which essentially captures the trad-

[51] Each trading day in the Australian Securities Exchange (ASX) is partitioned into six hourly intervals: 10:10-11:00, 11:00-12:00, 12:00-13:00, 13:00-14:00, 14:00-15:00 and 15:00-16:00. All trades in the first 10 minutes of each trading day are excluded from the analysis to avoid the effects of the ASX opening procedure. The first five hourly dummies are included in equation (4.4), while the last trading hour serves as the base category.

[52] Our proxy for volatility is a time-consistent version of the volatility measure of Jones et al. (1994), Chan and Fong (2000), Avramov et al. (2006) and Næs and Skjeltorp (2006) that can be applied to transaction data. Note that at a tick-by-tick level, a realized volatility measure is not defined and hence cannot be computed.

ing intensity, is far more informative about return volatility than the average trade size (e.g. Chan and Fong, 2006, Næs and Skjeltorp, 2006, Chevallier and Sévi, 2012). When working with transaction data, previous studies usually measure the volume of a trade by the number of shares executed by the trade (e.g. Hasbrouck, 1991a,b, Manganelli, 2005, Nolte, 2008), which coincides with the average trade size since the number of trades at a transaction time is always one.[53] Motivated by the suggestion of Engle (2000) and Xu et al. (2006) that variables computed in transaction time should be adjusted to account for calendar time, we employ a time-consistent measure of volume $v_{i,t}$, called *volume per unit of time*, which is defined as $V_{i,t}/T_{i,t}$, where $V_{i,t}$ is the number of shares traded (times 1000) divided by the total number of shares outstanding right before the t-th trade in stock i,[54] and $T_{i,t}$ is the duration of the trade.

4.2.3 Limit order book characteristics and the volume-volatility relation

The study of the information content of the LOB has attracted the attention of researchers for a long time. Most of the efforts, however, have been devoted to explaining what types of orders (i.e. limit vs. market) are employed by various types of investors (i.e. informed vs. uninformed), when, and why.[55] For example, Glosten (1994) shows that in order to capitalize quickly on their information, informed traders, who are presumably impatient, prefer market orders to limit orders. In contrast, Wald and Horrigan (2005) suggest that the optimal trading strategy for informed investors is to submit slightly discounted limit orders, rather than market orders, since these limit orders not only have a very high execution probability but also save the traders from paying a full bid-ask spread. Meanwhile, an experimental study by Bloomfield et al. (2005) indicates that although informed traders are more likely to place market orders

[53]Although multiple instantaneous transactions can be observed, they typically result from the matching of one big market order against several opposite side limit orders and are normally aggregated into one big trade in empirical analyses.

[54]Standardizing the number of shares of a trade by the number of shares outstanding right before the trade helps facilitate comparison amongst different stocks by putting them on roughly the same footing. We obtain qualitatively similar results without this standardization.

[55]See Glosten (1994), Biais et al. (1995), Parlour (1998), Anand et al. (2005), Bloomfield et al. (2005), Wald and Horrigan (2005), Foucault et al. (2005), Goettler et al. (2009), Roşu (2009), amongst others.

at the beginning of a trading day to take advantage of their information, they tend to switch to limit orders to earn the spread as the trading day progresses and prices have converged to the equilibrium level. The order placement decisions of informed traders also depend on the expected time horizon of their private information, as suggested by Kaniel and Liu (2006). If their information is long-lived, informed traders are more likely to submit limit orders.

Parlour (1998) considers a dynamic limit order book market that has no information asymmetry and shows that the state of the LOB is an important factor that affects traders' decisions to place limit versus market orders, which in turn influence price dynamics. In an information symmetric environment, the choice of submitting a limit versus market order and at which price involves a trade-off between the execution probability of the submission and its execution price. According to Parlour (1998), traders prefer the submission of limit orders over market orders when the execution probability of the former is high, e.g. when the book depth is small. Thus, a thick limit order book encourages traders to submit more aggressive orders (i.e. either limit orders with more aggressive prices or market orders) to increase the execution probability of their orders. Asymmetry between the bid and ask sides of the LOB also plays an important role in traders' order placement decisions. For example, a thicker ask book than bid book increases the submission of more aggressive buy orders but less aggressive sell orders, since the execution probability of a limit buy (sell) order is lower (higher). The theoretical findings of Parlour (1998) are empirically supported in the work of Ranaldo (2004), Aitken et al. (2007) and Duong and Kalev (2013).

There is a small but growing literature that examines whether the information contained in the LOB is predictive of the price formation and trade generation processes. Hasbrouck (1991a) documents that a wider bid-ask spread increases the price impact or return of an incoming trade. Similarly, Foucault et al. (2007) develop a theoretical model that predicts a positive relationship between the bid-ask spread and future volatility, which is strongly supported by empirical evidence documented in Næs and Skjeltorp (2006), Foucault et al. (2007), Nolte (2008) and Pascual and Veredas (2010). Meanwhile, an increase in the market depth available in the LOB leads to a decline in the price impact and return volatility of future trades (Ahn et al., 2001, Pascual and Veredas, 2010, Brogaard et al., 2015). In addition, Pham et al. (2017) document that the use of the market depth information right before a trade significantly improves the prediction of the immediate price impact of the trade. Næs and Skjeltorp (2006) show that higher average daily slope of the LOB dampens daily trading activity and return volatility, as well as the contemporaneous volume-volatility relation. Likewise, Kalay and Wohl (2009) find that their buying pressure measure, which is calculated from the slopes of the bid and ask order books, is predictive of future returns.

This chapter aims to investigate the role played by the LOB in explaining the dynamic volume-volatility relation and ultimately the return volatility of trades at a tick-by-tick level. Following the prior literature, the LOB characteristics that we examine consist of (i) the relative bid-ask spread, $Spread_{i,t}$, defined as the quoted spread divided by the mid-quote right before a trade; (ii) the market depth available at the inner quotes, $Depth_{i,t}$, defined as the total number of shares available at the best bid and ask prices (times 1000) and standardized by the total number of shares outstanding right before a trade (see footnote 54); and (iii) the slope of the LOB that prevails immediately before a trade. The latter variable captures the steepness of the limit order book, and it essentially measures how the quantity of stocks supplied in the LOB changes as a function of the limit price. Thus, the slope measure summarizes the LOB information at all limit price levels, whereas the first two attributes (i.e. bid-ask spread and market depth) only capture the LOB information at the best quotes.

Following Næs and Skjeltorp (2006), we compute the LOB slope for stock i immediately before transaction time t or the t-th trade as

$$Slope_{i,t} = \frac{Slpb_{i,t} + Slpa_{i,t}}{2}, \tag{4.5}$$

where $Slpb_{i,t}$ and $Slpa_{i,t}$ respectively denote the slopes of the bid and ask order books and are given by

$$Slpb_{i,t} = \frac{1}{100N_B}\left\{\frac{v_1^B}{|p_1^B/p_0 - 1|} + \sum_{\tau=1}^{N_B-1}\frac{v_{\tau+1}^B/v_\tau^B - 1}{|p_{\tau+1}^B/p_\tau^B - 1|}\right\}, \quad \text{and} \tag{4.6}$$

$$Slpa_{i,t} = \frac{1}{100N_A}\left\{\frac{v_1^A}{p_1^A/p_0 - 1} + \sum_{\tau=1}^{N_A-1}\frac{v_{\tau+1}^A/v_\tau^A - 1}{p_{\tau+1}^A/p_\tau^A - 1}\right\}, \tag{4.7}$$

where N_B and N_A are the total number of bid and ask prices (tick levels) containing orders of stock i right before time t, respectively. τ denotes the tick levels that have positive share volumes, and thus, p_1^B (p_1^A), where $\tau = 1$, represents the best bid (ask) price. p_0 denotes the best bid-ask midpoint immediately prior to time t. v_τ^B and v_τ^A are the natural logarithms of the *accumulated* total share volume at each tick level τ on the bid (p_τ^B) and ask (p_τ^A) side right before time t, respectively. That is, if we denote by V_τ^B (V_τ^A) the total share volume demanded (supplied) at p_τ^B (p_τ^A), then $v_\tau^B = \ln\left(\sum_{j=1}^{\tau} V_j^B\right)$, which measures the natural logarithm of the total share volume demanded at p_τ^B or higher, and $v_\tau^A = \ln\left(\sum_{j=1}^{\tau} V_j^A\right)$, which measures the natural logarithm of the total share volume supplied at p_τ^A or lower. Intuitively, the bid (ask) slope measures the percentage change in the bid (ask) volumes relative to the percentage change in the corresponding bid (ask) prices, which is averaged across all limit price levels in the bid (ask) order book, and the LOB slope is an average of the bid and ask slopes. For each point in transaction time t, we employ the 10 best bid and ask quotes, together with the share volumes queued at these quotes right before time t, to calculate the LOB slope.[56] In addition, all undisclosed or hidden orders are removed from the calculation of the LOB slope.

Market microstructure studies highlight the importance of trade direction or trade type (i.e. buy vs. sell) in explaining price dynamics (e.g. Hasbrouck, 1991a,b, Dufour and Engle, 2000, Barclay et al., 2003). In particular, an unexpected purchase (sale) results in a significant increase (decrease) in a stock's price. Meanwhile, Ahn et al. (2001), Engle and Patton (2004) and Harris and Panchapagesan (2005) document that there are significant asymmetries between the bid and ask sides of the LOB that are important to explain price dynamics. To incorporate the trade direction information and to allow for potential asymmetric effects of the bid and ask order books, we split the $Depth_{i,t}$ and $Slope_{i,t}$ measures into the corresponding bid and ask quantities, and interact them with the trade indicator. This brings us the following set of LOB attributes: $BV_{i,t}B_{i,t}, BV_{i,t}S_{i,t}, AV_{i,t}B_{i,t}, AV_{i,t}S_{i,t}, Slpb_{i,t}B_{i,t}, Slpb_{i,t}S_{i,t}, Slpa_{i,t}B_{i,t}$, and

[56] If less than 10 levels of the best bid and ask quotes with positive volumes are available at a particular point in time for a stock, the slope is computed using all levels of quotes available. We also employ different sets of the LOB information (of 5 and 20 best bid and ask levels) to compute the slope measures. Results of this experiment are reported in subsection 4.4.4.

$Slpa_{i,t}S_{i,t}$, where $BV_{i,t}$ ($AV_{i,t}$) is the bid (ask) depth volume, $Slpb_{i,t}$ ($Slpa_{i,t}$) is the bid (ask) order book slope, and $B_{i,t}$ ($S_{i,t}$) is a buy (sell) indicator that equals 1 if the t-th trade is a buy (sell), and 0 otherwise.

Previous studies find that the information about the order flow such as order imbalance possesses some explanatory power about return volatility (e.g. Chan and Fong 2000, Chan and Fong 2006, Shahzad et al. 2014). In order to ascertain that the informativeness of the LOB about the return volatility and the volume-volatility relation of a trade is genuine and not driven by the order flow information prior to the trade, we incorporate into all models several control variables that allow for the effects of the order flow. This partly mitigates problems associated with the endogeneity and the joint determination of the LOB variables and trading volume. The vector of control variables that we employ is $y_{i,t} = (\ln(T_{i,t}), N_{i,t}, ATS_{i,t}, OIB_{i,t}, QTT_{i,t})'$, where $T_{i,t}$ is the duration of the t-th trade, $N_{i,t}$, $ATS_{i,t}$, $OIB_{i,t}$ and $QTT_{i,t}$ respectively measure the number of transactions, the average trade size (times 10^6 and divided by the total number of shares outstanding), the order imbalance (defined as the number of buys minus the number of sells), and the quote to trade ratio (defined as the total number of order submissions, revisions and cancellations divided by the number of trades) during the 5-minute interval right before the t-th trade. See Table 4.8 in the Appendix (i.e. Section 4.7) for a complete list of all variables used in this chapter.

The inclusion of the above considerations suggests modifying the dynamic volume-volatility regression in equation (4.3) as

$$\sigma_{i,t}=\alpha_0+\mu_1 Monday_{i,t}+\mu_2 block1_{i,t}+\sum_{j=1}^{q}\alpha_j\sigma_{i,t-j}+\sum_{k=0}^{p}[\beta_{0,k}+\delta_k' x_{i,t-k}]v_{i,t-k}+\sum_{k=0}^{p}\gamma_k' x_{i,t-k}+\pi' y_{i,t}+\eta_{i,t}, \tag{4.8}$$

where $\sigma_{i,t} = |\widehat{\epsilon}_{i,t}|/T_{i,t}$, and $\widehat{\epsilon}_{i,t}$ is the residual from the following autoregressive model of returns which also includes the control variables $y_{i,t}$; i.e.

$$r_{i,t} = \sum_{k=1}^{5}\psi_{i,k}Day_{k,i,t} + \sum_{k=1}^{5}\phi_{i,k}block_{k,i,t} + \sum_{k=1}^{q}\rho_{i,k}r_{i,t-k} + \varphi' x_{i,t} + \lambda' y_{i,t} + \epsilon_{i,t}. \tag{4.9}$$

We incorporate $Monday_{i,t}$ (a dummy variable for Monday) and $block1_{i,t}$ (a dummy variable for the first trading hour (10:10:00-11:00:00) of a day) into equation (4.8) to capture additional Monday and opening effects on return volatility that might not be fully removed in the first stage regression (4.9) (e.g. Jones et al., 1994, Avramov et al., 2006). Vector $x_{i,t}$ contains the LOB characteristics, consisting of either $Spread_{i,t}$, $Depth_{i,t}$ and $Slope_{i,t}$ if a combined LOB is considered, or $Spread_{i,t}$, $BV_{i,t}B_{i,t}$, $BV_{i,t}S_{i,t}$, $AV_{i,t}B_{i,t}$, $AV_{i,t}S_{i,t}$, $Slpb_{i,t}B_{i,t}$, $Slpb_{i,t}S_{i,t}$, $Slpa_{i,t}B_{i,t}$, and $Slpa_{i,t}S_{i,t}$ if we allow for a separation of the bid and ask order books. Note that the imposition of the restrictions that $p = 0$ and $\delta_0 = 0$ on equation (4.8) gives a constant contemporaneous volume-volatility relation model similar to equation (4.1), which is examined by most existing studies. The imposition of $p = 0$ on equation (4.8) gives an "*endogenous*" contemporaneous volume-volatility relation model similar to equation (4.2) and to the model estimated by Næs and Skjeltorp (2006) for daily data.[57] We use the word "endogenous" here to indicate that the volume-volatility relation is no longer constant but dependent on the LOB information. For the full model (4.8) that allows the dynamic dependence of return volatility on trading volume to be dependent on the LOB attributes, we assume that the model can be truncated at $p = 5$ lags, as is typically assumed in the literature (e.g. Hasbrouck, 1991a,b, Dufour and Engle, 2000, Xu et al., 2006). We also truncate the lags of returns and volatility in equations (4.8) and (4.9) at $q = 12$, as typically done in previous studies (e.g. Avramov et al., 2006, Chan and Fong, 2006, Chevallier and Sévi, 2012). We find that our results are negligibly affected by the choice of p and q.

4.2.4 A caveat on causality

Previous empirical studies on the volume-volatility relation often face the problem of the undetermined causal relationships between volatility and its determinants such as trading volume and the LOB information. This arises from the use of low frequency data that are aggregated over a fixed time interval such as a day, which leads to the contemporaneous correlation or bilateral relationship between the variables (Hasbrouck, 1995, Barclay et al., 2003, Benos and Sagade, 2016). For example, a large transac-

[57] It is noted that Næs and Skjeltorp (2006) do not control for the direct impact of LOB information on volatility. Specifically, they first compute the sample correlation between the daily number of trades and daily volatility in every month, then regress this monthly correlation series on the monthly averages of the LOB attributes for a panel of all stocks in their sample.

tion, which is often originated by informed traders, has a big impact on security prices and increases price volatility, which then sends signals to the market and affects the trading intensity and volume of subsequent trades (Easley and O'Hara, 1987, 1992, Dufour and Engle, 2000). If these transactions are aggregated, the combined volume and volatility should be interrelated and jointly determined, and it will be difficult to disentangle the causal relationship between the two. Similarly, a thin order book increases price volatility (Ahn et al., 2001, Pascual and Veredas, 2010), but a prediction of high future volatility may reduce the aggressiveness in quoting of market participants, which in turn affects the LOB composition (Foucault et al., 2007). Hence, a fixed-time aggregation of trading activities leads to undetermined causality between volatility and LOB information, as empirically observed in Næs and Skjeltorp (2006). Given that trading and quoting activities often arrive sequentially, Hasbrouck (1995) suggests that shortening the sampling time interval might mitigate this issue as it helps reduce the contemporaneous correlation due to time aggregation.

Motivated by Hasbrouck's (1995) suggestion, this chapter utilizes tick-by-tick data, and it assumes that at a transaction level there are Granger-causality relationships running from trading volumes and LOB characteristics to return volatility, and the LOB information Granger-causes the volume-volatility relation of trades. Given that our LOB attributes (trading volumes) are known right before (at) the execution of a trade, whereas the return and volatility of the trade can only be realized ex-post once the trade is fully transacted, these assumptions are intuitive and reflect the chronological operation of an electronic LOB market. They are also consistent with the trading and quoting procedure in a traditional quote driven market, where designated market makers or specialists revise the bid and ask quotes to take into account the information that they have observed and learned from a new trade, as described in many theoretical market microstructure studies (see, for examples, Kyle, 1985, Easley and O'Hara, 1987, Hasbrouck, 1991a,b, Easley and O'Hara, 1992).

However, the use of transaction data and the above assumptions do not completely rule out the potential endogeneity of trading volumes and LOB information. This is because these variables are correlated with traders' unobserved liquidity needs and information sets as well as the release of new information in the market, which certainly affect return volatility. Nevertheless, it is difficult to find sensible instruments for volume and LOB characteristics because (i) these instruments need to be measured at a tick-by-tick level, and (ii) they must only affect return volatility indirectly through trading volume and LOB information (i.e. they must be uncorrelated with the error of the volatility equation), which is very unlikely given the trading and price dynamics. Instead of finding possible instruments, we offer an alternative solution to this problem by incorporating into our models the lags of return volatility as well as variables that account for the order flow information prior to a trade (see discussions in the previous subsections). This is typically done in the volume-volatility literature, and we hope that these variables, which capture some information about traders' unobserved characteristics, are sufficient to minimize the endogeneity and joint determination problems and maintain the validity of our results.[58]

4.3 Data

4.3.1 The Australian stock market

The Australian Securities Exchange (ASX) is amongst the 15 largest listed exchange groups in the world by market capitalization. Prior to 31 October 2011 it was the only stock exchange where all Australian listed stocks were traded, and it has been the primary Australian equity exchange ever since, accounting for more than 80% of all trading volumes in Australia.[59] As of July 2017, almost 2,200 companies and issuers were listed on the ASX, ranging from big well established companies to small start-up firms and possessing a total market capitalization of about AUD$ 1.5 trillion (or USD$

[58] Another way to accommodate the endogeneity and joint determination of trading volumes and LOB information is to model these variables and price volatility simultaneously in a multivariate system of equations similar to the (nonlinear) VAR model examined in Chapter 2. Such a system includes equation (4.3) for volatility and other similarly defined equations for trading volumes and LOB attributes, and it can be consistently estimated (equation by equation) by OLS. We do not estimate this entire system of equations in this chapter since our main focus is on the volatility equation of the system.

[59] On 31 October 2011 a new trading platform called Chi-X Australia was launched and it became the second equity exchange in Australia. Since then, the market share of Chi-X has increased from 1.7% in April 2012 to 14.3% in September 2013 (ASIC, 2014), and to over 20% in July 2017 (see http://chi-x.com.au).

1.2 trillion).

The ASX has operated as a purely electronic order-driven market via a system called "Stock Exchange Automated Trading System" (SEATS) since 1991. Orders submitted to the ASX follow a price-time priority, as typically seen in most other electronic LOB markets. In particular, limit orders are queued and ranked in the LOB first by price priority and then in the time sequence that they arrive at the market. Meanwhile, market orders, which are orders with the highest price priority, are executed at the best available prices immediately upon their submissions. The LOB is updated instantaneously whenever an order submission, revision, cancellation, or execution occurs. The submitted price of an order must be in multiples of the minimum tick size, which is pre-specified by the exchange and is dependent on the price level of the security. The tick size is currently AUD$ 0.001, 0.005, and 0.01 for stock prices that are below AUD$ 0.1, from AUD$ 0.1 but below AUD$ 2, and from AUD$ 2, respectively. A typical trading day consists of two sessions: a pre-market session from 7:00am to 10:00am Australian Eastern Standard Time (AEST), and a normal trading session from 10:00am to 4:00pm AEST. The first 10 minutes of the normal trading session are opening auctions. There is also a closing single price auction between 4:10pm and 4:12pm during which the daily closing price for each stock is determined (see http://www.asx.com.au).

4.3.2 The data

The informativeness of the LOB about the dynamic volume-volatility relation is investigated using a sample of the constituent stocks of the S&P/ASX200 index between 1 July and 31 December 2014. Comprising of the 200 largest Australian listed stocks, the S&P/ASX200 index is the primary stock market index that serves as the main investment benchmark in Australia and it constitutes about 80% of Australia's sharemarket capitalization. We follow the ASX's classification to partition these stocks into three groups: "**Large cap**" which contains stocks in the S&P/ASX50 index, "**Mid cap**" which contains stocks in the S&P/ASX100 index but outside the S&P/ASX50 index, and "**Small cap**" which contains the remaining stocks in the S&P/ASX200 index. There were 198 stocks in our sample, consisting of 49 large cap, 50 mid cap, and 99 small cap stocks.[60]

We collect two datasets from the Securities Industry Research Centre of Asia-Pacific (SIRCA) database. The first dataset records details on every order submitted to the Australian central LOB, including the stock code, the order type (order submission, order revision, order cancellation and execution), the date and time (to millisecond precision), the order price, the order volume (number of shares), the order value (dollar value), and the order qualifiers.[61] Each new order is assigned a unique identication number (ID) so that the order can be tracked from its initial submission through any revision, cancellation or execution. We extract all trades that are performed within the continuous trading session in the lit market (from 10:10:00 to 16:00:00) and discard all transactions executed in the opening auction (i.e. either during 10:00:00-10:10:00 or with "AC" qualifiers that define auction trades) and in dark pools. We classify trades into buyer-initiated and seller-initiated trades based on the direction of the (market) orders that initiate the trade. Since one large buy (sell) market order can be matched against several limit orders queuing on the sell (buy) side of the LOB and appear as multiple instantaneous transactions that have zero durations, we follow the standard practice in the literature (e.g Dufour and Engle, 2000, Nowak and Anderson, 2014, Renault et al., 2014) and aggregate trades executed at the same time and of the same direction into one "big" trade by calculating volume-weighted average prices and summing up the volumes of small trades. We use the order book dataset to compute the control variables that allow for the effects of the order flow during a 5-minute interval immediately prior to each trade such as the number of transactions ($N_{i,t}$), the average trade size ($ATS_{i,t}$), the order imbalance ($OIB_{i,t}$), and the quote to trade ratio ($QTT_{i,t}$).

[60]We exclude two stocks, namely WES (Wesfarmers Limited - Large cap) and NWS (News Corporation - Small cap) from our analysis, since the SIRCA database did not record data for these stocks during the sample period, even though they were listed and traded throughout the period. In addition, there are 6 stocks that were delisted during the sample period. We do not remove them from our sample since our analysis is conducted on a stock-by-stock basis and we still have a large sample size for these stocks (of more than 4000 transactions). Nevertheless, excluding these stocks negligibly affects our results.

[61]Each limit or non-market order has a qualifier indicating the order direction (buy or sell order). Meanwhile, each trade or market order contains a qualifier that declares some qualitative property of the trade. For example, a "Bi" ("Si") qualifier signifies a buyer-initiated (seller-initiated) trade, an "XT" denotes a cross trade, while a "CX" is attached to trades that are executed in an Australian dark pool called Centre Point.

The second dataset contains detailed information on stock code, date, time, and the best bid/ask quotes and volumes up to 20 levels in the LOB. We remove all observations with either (i) a negative bid/ask quote or volume at any level, (ii) a bid quote higher than ask quote at any level, (iii) a positive bid or ask quote but with zero volume at any level, (iv) a zero bid or ask quote but with a positive volume at any level, or (v) a bid (ask) quote at level j lower (higher) than or equal to the bid (ask) quote at level $k > j$.[62] The transaction data are merged with the bid and ask quotes data to work out the bid-ask midpoint, the bid-ask spread, the depth volume at the best bid and ask quotes, and the LOB slope immediately before each transaction. Finally, we collect daily data on the numbers of shares outstanding for each stock from the DatAnalysis Premium database.[63] In order to avoid the effects of outliers, all variables in the study are winsorized at the 1$^{\text{st}}$ and 99$^{\text{th}}$ percentiles on a stock-by-stock basis.[64] The winsorization filters out another four stocks, leaving us with the final sample of 194 stocks, consisting of 49 large cap, 48 mid cap, and 97 small cap stocks.[65]

[62]The last filtering criterion is to ensure that best bid (ask) quotes must be in a strictly decreasing (increasing) order as one moves further away from the best, i.e. level 1, bid (ask) quote. However, it is worth noting that some stocks, especially the illiquid stocks, might not have all 20 levels of the best bid/ask quotes and volumes available at some point in time, but only 5 or 10, for example, levels instead. In such a case, entries for the bid/ask prices and volumes of the remaining levels are displayed as 0. These observations are still valid and hence will not be removed if they pass the first four aforementioned filtering criteria ((i) - (iv)).

[63]Delivered by the Morningstar company, DatAnalysis Premium database provides comprehensive and daily updated corporate data (such as company histories, announcements, reports and financial data) on all Australian companies listed and delisted on the ASX (see https://corporate.morningstar.com/au/asp/subject.aspx?xmlfile=6765.xml).

[64]Note that we estimate the return volatility of a trade as $\sigma_{i,t} = |\widehat{\epsilon}_{i,t}|/T_{i,t}$, where $\widehat{\epsilon}_{i,t}$ is the residual obtained from an autoregressive model in equation (4.9) of winsorized returns, and $T_{i,t}$ is the winsorized duration of the trade. We do not winsorize $\sigma_{i,t}$ since it would be effectively a double winsorization.

[65]Four stocks, namely DJS (David Jones Limited - Mid cap), ENV (Envestra Limited - Mid cap), AQA (Aquila Resources Limited - Small cap), and GFF (Goodman Fielder Limited - Small cap), have the return series $r_{i,t}$ of all zeros after the winsorization. This is because more than 98% of their returns were zero during the sample period. Consequently, the return volatility estimates, defined in subsection 4.2.2, for these four stocks are all zero and the subsequent volume-volatility regressions cannot be performed. Thus, we exclude these stocks from our analysis. Note that we also investigate the case where we use the unwinsorized return series, together with other variables (either winsorized or unwinsorized), of these four stocks to conduct the analyses. Results from this unreported experiment are qualitatively similar to the ones in the main text and are available upon request.

Table 4.1: Summary statistics of trading activities and the order book characteristics

	Large cap	**Mid cap**	**Small cap**	**All stocks**
Number of stocks	49	48	97	194
Market capitalization ($AUD bn)	22.707	3.266	1.016	7.051
Shares outstanding (millions)	1748.409	856.875	482.700	894.969
Return (%) (×100)	-0.003	0.007	-0.034	-0.016
Volume (thousand shares)	1.457	1.564	2.328	1.919
Duration (secs)	11.582	21.788	33.864	25.248
Absretpd	0.834	1.502	2.005	1.585
Volpd	81.681	84.777	93.033	88.123
Spread (%)	0.123	0.237	0.442	0.311
Bidvol (thousand shares)	56.436	155.741	100.329	102.953
Askvol (thousand shares)	56.861	66.036	84.114	72.758
Depth (thousand shares)	113.696	222.123	184.846	176.098
Slpb	25.268	11.456	7.030	12.732
Slpa	25.155	11.355	6.987	12.657
Slope	25.215	11.407	7.009	12.696
N	42.804	23.124	16.235	24.650
ATS (thousand shares)	1.639	1.767	2.710	2.206
OIB	0.635	0.451	0.427	0.486
QTT	13.763	13.004	12.349	12.868

This table presents summary statistics of trading activities and the order book characteristics for the constituent stocks of the S&P/ASX200 index in *July-December 2014*. These stocks are classified into three groups: "**Large cap**" which contains stocks in the S&P/ASX50 index, "**Mid cap**" which contains stocks in the S&P/ASX100 index but outside the S&P/ASX50 index, and "**Small cap**" which contains the remaining stocks in the S&P/ASX200 index. "Market capitalization" (in $AUD billion) is the market capitalization of firms as of 1 July 2014. "Shares outstanding" is the number of shares outstanding (in millions) right before a trade. "Return" (in %, and multiplied by 100) measures the change in log of the mid-quote right before a trade and the next trade. "Volume" is the number of shares (in thousands) traded in each trade. "Duration" (in seconds) is the time interval between two consecutive trades. "Absretpd" is the absolute return per unit of time, calculated as the absolute value of the return of a trade divided by its duration. "Volpd" is the share volume traded per unit of time, calculated as the volume (in thousands) of a trade divided by its duration. "Spread" (in %) is the relative spread (i.e. quoted spread as a % of the mid-quote right before a trade). "Bidvol", "Askvol" and "Depth" are respectively the total share volumes (in thousands) available at the best bid price, the best ask price, and both best bid and ask prices right before a trade. "Slpb" ("Slpa") is the slope of the bid (ask) side of the order book using 10 best bid/ask price levels right before a trade. "Slope" is the slope of the limit order book right before a trade, calculated as ("Slpb" + "Slpa")/2. "N" ("ATS") is the number of trades (the average trade size, in thousands) during a 5 minute interval right before a trade. "OIB" is the order imbalance, defined as the number of buys minus the number of sells during a 5 minute interval right before a trade. "QTT" is the quote to trade ratio during a 5 minute interval right before a trade. All variables for each stock are winsorized at the 1^{st} and 99^{th} quantiles to avoid the effects of outliers. All the statistics reported in the table (excepting those in the first line) are first computed for each individual stock and then equally averaged across all stocks.

Table 4.1 provides some cross-sectional summary statistics of trades and LOB attributes for the constituent stocks of the S&P/ASX200 index during July-December 2014. In terms of market capitalization, large cap stocks are on average about seven times as big as mid cap stocks, which in turn are about three times as big as small cap stocks. In addition, the number of shares outstanding of large cap stocks is respectively twice and four times as high as those of mid and small cap stocks. Consistent with previous empirical evidence, the transaction returns for all stock groups have a mean of almost zero percent (e.g. Renault and Werker, 2011, Renault et al., 2014, Jondeau et al., 2015). In conformance with well-documented stylized facts (e.g Manganelli, 2005, Xu et al., 2006, Jondeau et al., 2015, Pham et al., 2017), larger cap stocks trade more frequently and consequently have significantly smaller trade durations, reflecting their higher levels of liquidity. Moreover, they tend to trade in a smaller volume, either in the number of shares or per unit of time. Thus, the larger cap stocks have smaller volatility, as evidenced by the smaller absolute return per unit of time - a raw proxy for return volatility per unit of time.

Regarding the LOB characteristics, larger cap and more liquid stocks on average have a smaller relative spread, as reported elsewhere (e.g. Dufour and Engle, 2000, Næs and Skjeltorp, 2006). Interestingly, large cap stocks have significantly fewer shares supplied at the inner quotes than do mid and small cap stocks, possibly because the former are much more heavily traded so that more depth at the best quotes is absorbed. While the number of shares or depth available at the best bid and ask quotes are roughly equal for large cap stocks, significantly more shares are queued at the best bid than at the best ask for mid and small cap stocks. For all stock groups, the average amount of shares supplied at the best quotes is much larger than the average volume demanded by a trade, implying that the majority of transactions do not move the best bid or ask prices and hence have zero returns - an observation that is also documented by Dufour and Engle (2000), Renault et al. (2014), Pham et al. (2017), amongst others. The bid, ask and overall order book slopes are larger for more liquid stocks, suggesting that for these stocks more shares are queued closer to the inner bid/ask quotes, making their LOB steeper, which is consistent with the findings of Næs and Skjeltorp (2006) and Duong and Kalev (2008). Moreover, the LOB slope appears slightly higher on the bid or buy side than on the ask or sell side.

The order flow characteristics are also in conformance with the liquidity of stocks. More specifically, more liquid stocks trade more frequently in a smaller volume than less liquid stocks, as can be seen from the number of trades and the average trade size during a 5-minute interval. Larger stocks also have a slightly higher quote to trade ratio, which, coupled with higher trading intensity, suggests that bigger stocks attract more attention and more intensive quoting activities from market participants than do smaller stocks, as expected. There are on average more purchases than sales for all stock groups, and the imbalance between buying and selling activities tends to increases with the liquidity level of stocks.

4.4 Results and discussion

In this section, we empirically examine the role played by the LOB characteristics in explaining the return volatility and the volume-volatility relation of trades. We begin with an investigation of the information content of the LOB at the best bid and ask prices (subsection 4.4.1). Then, to ascertain whether the LOB information beyond the best quotes possesses additional explanatory power about prices, we study the interaction between the LOB slope, which summarizes the LOB information at all quote levels, and the volume-volatility relation (subsection 4.4.2). We also conduct a comparison of the predictive power of the LOB slope and the LOB information at the best quotes (subsection 4.4.3), along with a series of sensitivity analyses to ensure the robustness of our results (subsection 4.4.4).

4.4.1 Spread, depth and the volume-volatility relation

This subsection investigates the informativeness of the inner bid/ask quotes and depths about return volatility as well as the dynamic volume-volatility relation for the constituent stocks of the S&P/ASX200 index during July-December 2014. We estimate equations (4.1)-(4.3) separately for each individual stock in our sample, then we report the median coefficient and the proportions of coefficients that are negatively and positively significant at a 5% level for three stock groups. The results for a combined LOB are reported in Table 4.2, while those for a separation of the bid and ask sides of the LOB are shown in Table 4.3. Note that since the median operator is not additive, "Lag 0" and "$\sum_{1:p}$" median coefficients generally do not add up to that of "$\sum_{0:p}$".[66]

Panel A of Table 4.2 contains an analysis of the volume-volatility relation formulated in equation (4.1) which is fully exogenous and contemporaneous, as typically assumed in most previous studies. Consistent with prior findings, we observe a strong positive contemporaneous relation between trading volume and return volatility for all stock groups which is statistically significant at a 5% level for almost all stocks, even after controlling for the effects of the bid-ask spread, the depth at the best quotes as well as other order flow characteristics. The best level of the LOB has predictive power about future return volatility. The wider the bid-ask spread prior to a trade, the larger the volatility (per unit of time) of the trade. This positive relation between spread and volatility is significant at a 5% level for the majority of stocks in three groups and is consistent with the theoretical model of Foucault et al. (2007) and the empirical findings of Hasbrouck (1991a), Næs and Skjeltorp (2006), Foucault et al. (2007), Nolte (2008), Pascual and Veredas (2010), and Haugom et al. (2014). Meanwhile, the return volatility of a trade is negatively dependent on the prevailing quoted depth right before the trade. This result is intuitive because larger market depths available at the best bid and ask prices are better able to accommodate a trade of a given size, resulting in fewer quote revisions and consequently lower price impact and volatility of the trade (e.g. Ahn et al., 2001, Brogaard et al., 2015, Pham et al., 2017). Interestingly, the informativeness of the bid-ask spread and market depth about return volatility is most pronounced for mid cap stocks, in terms of both the magnitude of the coefficients and the proportion of significant estimates.

[66]The mean coefficients, which preserve additivity but are more prone to outliers, are generally qualitatively similar to the reported median coefficients. They are omitted for brevity but are available upon request.

Table 4.2: Spread, depth and the volume-volatility relation: Combined limit order book

		Large cap (49 stocks)			**Mid cap** (48 stocks)			**Small cap** (97 stocks)		
		Med	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$
Panel A: Constant contemporaneous volume-volatility relation										
$v_{i,t}$		0.937	0.0%	95.9%	0.847	0.0%	100.0%	1.022	0.0%	100.0%
$Spread_{i,t}$		3.607	20.4%	71.4%	5.573	10.4%	72.9%	3.029	21.6%	63.9%
$Depth_{i,t}$		-0.994	49.0%	36.7%	-2.994	58.3%	29.2%	-1.600	57.7%	24.7%
$\ln(T_{i,t})$		-0.653	100.0%	0.0%	-1.259	100.0%	0.0%	-1.487	100.0%	0.0%
$N_{i,t}$		-0.009	100.0%	0.0%	-0.036	100.0%	0.0%	-0.063	100.0%	0.0%
$ATS_{i,t}$		0.191	0.0%	95.9%	0.226	4.2%	95.8%	0.123	0.0%	99.0%
$OIBtr_{i,t}$		0.000	12.2%	34.7%	0.002	4.2%	37.5%	0.005	9.3%	37.1%
$QTT_{i,t}$		0.017	0.0%	100.0%	0.029	2.1%	97.9%	0.040	0.0%	100.0%
adj. R^2		0.155	-	-	0.171	-	-	0.179	-	-
Panel B: Endogenous contemporaneous volume-volatility relation										
$v_{i,t}$		-0.463	44.9%	38.8%	-0.908	45.8%	27.1%	0.444	19.6%	39.2%
$v_{i,t}Spread_{i,t}$		18.181	10.2%	75.5%	12.988	6.2%	79.2%	5.169	10.3%	68.0%
$v_{i,t}Depth_{i,t}$		-26.223	83.7%	0.0%	-13.381	85.4%	8.3%	-7.094	79.4%	1.0%
$Spread_{i,t}$		2.292	22.4%	67.3%	2.969	12.5%	66.7%	1.637	24.7%	54.6%
$Depth_{i,t}$		1.205	36.7%	51.0%	-1.617	56.2%	35.4%	-0.412	47.4%	35.1%
adj. R^2		0.162	-	-	0.180	-	-	0.184	-	-
Panel C: Endogenous dynamic volume-volatility relation										
$v_{i,t}$	Lag 0	-0.420	44.9%	40.8%	-0.872	45.8%	27.1%	0.460	19.6%	40.2%
	$\sum_{1:p}$	0.204	22.4%	24.5%	0.317	10.4%	12.5%	0.040	8.2%	10.3%
	$\sum_{0:p}$	-0.014	28.6%	28.6%	-0.272	22.9%	16.7%	0.392	8.2%	36.1%
$v_{i,t}Spread_{i,t}$	Lag 0	16.727	10.2%	75.5%	12.819	6.2%	75.0%	5.057	10.3%	67.0%
	$\sum_{1:p}$	8.092	0.0%	49.0%	2.885	2.1%	22.9%	0.783	3.1%	19.6%
	$\sum_{0:p}$	26.320	4.1%	85.7%	15.889	2.1%	79.2%	6.055	6.2%	68.0%
$v_{i,t}Depth_{i,t}$	Lag 0	-25.380	83.7%	0.0%	-13.103	85.4%	8.3%	-7.176	79.4%	0.0%
	$\sum_{1:p}$	-9.332	49.0%	12.2%	-3.680	43.8%	12.5%	-1.552	29.9%	1.0%
	$\sum_{0:p}$	-29.747	75.5%	4.1%	-18.056	85.4%	8.3%	-9.570	84.5%	1.0%
$Spread_{i,t}$	Lag 0	-7.793	63.3%	26.5%	-1.510	43.8%	29.2%	-4.518	54.6%	22.7%
	$\sum_{1:p}$	19.423	0.0%	71.4%	9.051	4.2%	56.2%	9.304	12.4%	66.0%
	$\sum_{0:p}$	14.797	0.0%	73.5%	7.596	0.0%	66.7%	4.718	2.1%	68.0%
$Depth_{i,t}$	Lag 0	61.806	14.3%	79.6%	25.541	27.1%	58.3%	25.687	7.2%	71.1%
	$\sum_{1:p}$	-61.657	79.6%	6.1%	-28.805	62.5%	12.5%	-32.570	74.2%	4.1%
	$\sum_{0:p}$	-13.354	59.2%	10.2%	-7.715	66.7%	6.2%	-4.852	77.3%	0.0%
adj. R^2		0.172	-	-	0.188	-	-	0.194	-	-

This table reports the results of the following model, for the constituent stocks of the S&P/ASX200 index during *Jul-Dec 2014*,

$$\sigma_{i,t} = \alpha_0 + \mu_1 Monday_{i,t} + \mu_2 block1_{i,t} + \sum_{j=1}^{12} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{p} [\beta_{0,k} + \delta'_k x_{i,t-k}] v_{i,t-k} + \sum_{k=0}^{p} \gamma'_k x_{i,t-k} + \pi' y_{i,t} + \eta_{i,t},$$

where $\sigma_{i,t}$ is a proxy for return volatility *per unit of time* of the t-th transaction in stock i, which is estimated as the absolute value of the residual $|\widehat{\epsilon}_{i,t}|$ of the following autoregressive model of returns $r_{i,t}$ divided by the duration $T_{i,t}$ of the trade (i.e. $\sigma_{i,t} = |\widehat{\epsilon}_{i,t}|/T_{i,t}$):

$$r_{i,t} = \sum_{k=1}^{5} \psi_{i,k} Day_{k,i,t} + \sum_{k=1}^{5} \phi_{i,k} block_{k,i,t} + \sum_{k=1}^{12} \rho_{i,k} r_{i,t-k} + \varphi' x_{i,t} + \lambda' y_{i,t} + \epsilon_{i,t}.$$

$x_{i,t} = (Spread_{i,t}, Depth_{i,t})'$ is a vector of potential predictors of the volume-volatility relation. $y_{i,t} = (\ln(T_{i,t}), N_{i,t}, ATS_{i,t}, OIB_{i,t}, QTT_{i,t})'$ is a vector of control variables that allow for the effects of the order flow prior to a trade. See Table 4.8 for the definitions of the variables.
Panel A reports the results for a **constant contemporaneous** volume-volatility relation model, in which $p = 0$ and the restriction that $\delta_0 = 0$ is imposed on the volatility equation.
Panel B reports the results for an **endogenous contemporaneous** volume-volatility relation model, in which $p = 0$ and no restrictions are imposed on the volatility equation.
Panel C reports the results for an **endogenous dynamic** volume-volatility relation model, in which $\boldsymbol{p = 5}$ and no restrictions are imposed on the volatility equation.
The table only reports the coefficient estimates for $v_{i,t}$ and $x_{i,t}$ from the volatility equation. The coefficient estimates for $y_{i,t}$ are only reported in Panel A. The regression is separately run for each stock, using Newey-West heteroskedasticity and autocorrelation consistent estimation. $\Sigma_{i:p}$ (in Panel C only) denotes the sum of the coefficients from lag i up to lag p. For brevity, we only report the median coefficients in "**Med**" column for each group. $\%_{-5\%}$ ($\%_{+5\%}$) indicates the proportion of estimates in each group that are significantly negative (positive) at a 5% level. "adj.R^2" denotes the adjusted R^2. Note that as the median operator is not additive, "Lag 0" and "$\sum_{1:p}$" median coefficients generally do not add up to that of "$\sum_{0:p}$".

We also find strong evidence in support of the predictability of the order flow information about future return volatility at the tick-by-tick level.[67] Consistent with the findings of Xu et al. (2006), Manganelli (2005), Russell and Engle (2005) and Nolte (2008), a shorter trade duration increases the return volatility of the trade. The result lends support to Easley and O'Hara's (1992) theory which implies that shorter time between trades or higher trading intensity is a signal of more private news and a higher fraction of informed traders present in the market. Since the increased presence of informed investors constrains liquidity traders from entering the market, possibly via toxic order flows that adversely select the latter (Easley et al., 2012), trades with shorter durations have larger impacts on prices, leading to higher volatility. The average size of trades that are executed during a 5-minute interval before a trade is found to be positively related to the return volatility of the trade, with statistical significance observed for most stocks. Not only does this result strengthen the positive relation between volume and volatility discussed previously, but it also suggests that past trading volumes are predictive of future volatility, and hence the volume-volatility relation is dynamic.

The return volatility of a trade is inversely dependent on the number of trades during a 5-minute interval prior to a trade - a proxy for the trading frequency prior to the trade, which appears to be inconsistent with the findings of most previous studies. This surprising observation can be explained as a result of both measures of trading intensity, namely trade duration $T_{i,t}$ and the number of trades $N_{i,t}$, being included in the regression. Results from an unreported experiment in which trade duration is removed from the volatility equation show a positive and significant relation between return volatility and the number of trades. While reaffirming the findings of previous work, this outcome suggests that the most recent information about trading intensity, captured by $T_{i,t}$, appears to be more relevant for the explanation of future volatility than the more distant information, proxied by $N_{i,t}$.

In conformance with Chan and Fong (2000), Chan and Fong (2006) and Shahzad et al. (2014), there is a positive link between the order imbalance of trades and volatil-

[67] We do not report the estimated coefficients for the order flow characteristics in subsequent analyses to save space but note that the results are qualitatively similar to those being discussed here.

ity which is statistically significant for a fair proportion of stocks in our sample (more than 34% for all groups), suggesting that trade order imbalance does possess some predictive power about future return volatility. Meanwhile, there is a strong positive dependence of the volatility of a trade on the quote to trade ratio which measures the quoting activities during a 5-minute interval before the trade. Quote to trade ratios have increased considerably in today's fast trading environment, as a consequence of the dominance of algorithmic and high frequency traders (HFTs) who utilize their speed advantage to split and submit many orders that subsequently get canceled very quickly (e.g. SEC, 2010, Hasbrouck and Saar, 2013, Conrad et al., 2015, O'Hara, 2015). Our result suggests that HFT activities tend to increase future return volatility, as also noted by Boehmer et al. (2015).

Panel B of Table 4.2 reports the results relating to the relaxation of the assumption of a constant contemporaneous volume-volatility relation. There is strong evidence that the contemporaneous volume-volatility relation is endogenously related to the LOB characteristics at the best level. In particular, the positive dependence of return volatility on the trading volume of a trade becomes stronger, the larger the bid-ask spread right before the trade. In contrast, a larger amount of shares being supplied at the best bid/ask prices weakens the volume-volatility relation. Both results are statistically significant at the 5% level for the majority of stocks (more than 68% and 79% respectively), and they are stronger, in terms of the magnitude of the coefficients, for more liquid stocks. Note that the dependence of the contemporaneous volume-volatility relation on the bid-ask spread and market depth is genuine since the direct impacts of the spread, depth and other order flow attributes on volatility are already controlled for. In fact, a comparison of the coefficients on $Spread_{i,t}$ and $Depth_{i,t}$ between Panel A and Panel B suggests that allowing the volume-volatility relation to vary with the bid-ask spread and market depth remarkably reduces the direct effects of these order book characteristics on return volatility, even though the direct effects are still strong and significant for a big proportion of stocks, especially with regard to the bid-ask spread. This finding implies that a more liquid order book market (which has deeper depths and/or narrower bid-ask spreads) reduces trading volatility via two channels: by its direct impact on volatility and by its indirect effect that weakens the volume-volatility relation. Interestingly, the direct impact of trading volume on volatility, captured by β in equation (4.1), changes sign from positive to negative for large and mid cap stocks and becomes much less significant for small stocks following the relaxation of the constant volume-volatility relation (comparing the coefficients on $v_{i,t}$ in Panels A and B). The result suggests that the well-documented positive association between trading volume and return volatility seems to be driven by the LOB characteristics.

We now investigate the results for the volume-volatility relation that is allowed to be dynamically dependent on the LOB information at the best level, which are reported in Panel C of Table 4.2. The results from Panel C, in which we assume that the volatility regression can be truncated at $p = 5$ lags, indicate the dynamic nature of the volume-volatility relation, which is also related to the dynamics of the bid-ask spread and market depth. Larger bid-ask spreads and smaller depths available at the best quotes strengthen the positive dependence of return volatility not only on current trading volumes, as discussed in Panel B, but also on lagged volumes. However, most of the effects are attributable to the LOB information immediately before a trade (see the "Lag 0" coefficients), while the contribution of the past order book information, albeit of expected sign, is of much smaller magnitude and of much less statistical significance. In contrast, most of the direct impact of the bid-ask spread and market depth on future return volatility comes from the lagged information (see the "$\sum_{1:p}$" coefficients), while the coefficients measuring the direct effects of the bid-ask spread and market depth right before a trade (i.e. the "Lag 0" coefficient estimates) on volatility are usually of opposite and unexpected signs. All else being equal, the influence, either direct or indirect, of the bid-ask spread and the depth available at the best quotes prior to a trade on the return volatility of the trade increases, in magnitude, with a stock's liquidity.

The contrasting results between the direct and indirect effects of the LOB attributes at the best level are interesting and can be explained as follows. In order to predict the return volatility of an incoming trade without knowing the volume of that trade, one needs to make use of all past trading information, including the past order book characteristics, to draw a likely and sensible picture of the relation between future volatility and the past information. However, if one knew the volume that the trade would demand, then the information of what is currently being supplied right before the execution of the trade contained in the LOB would be more relevant than the past order book information to the determination of how the trade would move prices. As a result, it seems reasonable that the direct impact of the bid-ask spread and market depth on return volatility, which does not take into account the information about the volume of the current trade, is mostly contributed by the lagged effects, whereas their indirect effects on volatility, which channel through the volume-volatility relation and incorporate the current volume information, are primarily driven by the current state of the LOB right before the trade. This result is in conformance with the findings of Pham et al. (2017), who show that a comparison of the volume of a trade with the prevailing market depth information right before the trade is of particular relevance to identifying whether the trade results in any immediate impact on prices. These authors show that the incorporation of the depth information into an immediate price impact model significantly enhances the forecast accuracy of the model.

We now turn to an analysis of the dynamic volume-volatility relation that allows for possible asymmetries between the bid and ask sides of the LOB. The results are reported in Table 4.3. While the effects of the bid-ask spread on return volatility and the volume-volatility relation remain qualitatively similar to those previously discussed in Table 4.2, the separation of the bid and ask order books offers some interesting insights into how market depth affects the volatility of trades, especially when one also takes the information of trade direction into consideration. First, as expected, when an incoming trade is a buy (sell), it is the opposite (i.e. ask (bid)) side of the LOB that is more important for determining the impact that the trade will place on prices. Specifically, for a given trading volume the larger the amount of shares available at the best ask (bid) quote immediately before a purchase (sale), the smaller the return volatility (see the coefficients on $AV_{i,t}B_{i,t}$ ($BV_{i,t}S_{i,t}$) in Panels A, B and C) and the weaker the positive dependence of volatility on the volume (see the coefficients on $v_{i,t}AV_{i,t}B_{i,t}$ ($v_{i,t}BV_{i,t}S_{i,t}$)

in Panels B and C) of the trade. These results not only complement the corresponding findings presented in Table 4.2, but are also stronger than the latter in terms of both magnitude and statistical significance, which is generally in conformance with previous studies (e.g. Ahn et al., 2001, Brogaard et al., 2015, Pham et al., 2017) that show deeper markets support liquidity and mitigate the price impact and return volatility of trades.

In contrast to the opposite-side market depth, we find that larger market depth available on the same side of the LOB as the direction of a trade tends to increase the volatility (see the coefficients on $BV_{i,t}B_{i,t}$ and $AV_{i,t}S_{i,t}$ in Panels A and B) and strengthen the volume-volatility relation (see the coefficients on $v_{i,t}BV_{i,t}B_{i,t}$ and $v_{i,t}AV_{i,t}S_{i,t}$ in Panels B and C) of that trade, even though the proportions of stocks that have significant coefficients are generally remarkably lower. These results are consistent with the findings in the prior literature on order aggressiveness (e.g. Biais et al., 1995, Ranaldo, 2004, Aitken et al., 2007, Duong and Kalev, 2013) that investors tend to submit more (less) aggressive orders when the same-side (opposite-side) market depth increases since the non-execution risk of an incoming limit order is higher (lower). As more aggressive orders typically have a larger impact on prices (Biais et al., 1995, Duong and Kalev, 2013, Brogaard et al., 2018), it follows that larger same-side (opposite-side) market depth increases (decreases) the future trading volatility, as we observe.

In conformance with the results from a combined LOB shown in Table 4.2, the direct effects of the bid and ask depths on return volatility become weaker once one allows for the endogeneity of the volume-volatility relation. Furthermore, the dynamics of the bid and ask depths do play a significant role in explaining future return volatility. While the direct impact of the bid and ask depths on return volatility mostly comes from their lagged information, their current information right before the execution of a trade is the main driver of the volume-volatility relation which constitutes their indirect impact on volatility. Furthermore, the larger the stock, the bigger the cumulative impact of the best bid and ask depths, either direct or indirect, on return volatility.

Table 4.3: Spread, depth and the volume-volatility relation: Bid vs. Ask sides

		Large cap (49 stocks)			**Mid cap** (48 stocks)			**Small cap** (97 stocks)		
		Med	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$
Panel A: Constant contemporaneous volume-volatility relation										
$v_{i,t}$		1.254	0.0%	100.0%	1.238	0.0%	100.0%	1.416	0.0%	100.0%
$Spread_{i,t}$		4.564	16.3%	77.6%	6.546	10.4%	79.2%	3.508	13.4%	73.2%
$BV_{i,t}B_{i,t}$		12.233	26.5%	71.4%	4.921	22.9%	60.4%	3.530	20.6%	55.7%
$BV_{i,t}S_{i,t}$		-9.697	79.6%	6.1%	-8.736	91.7%	6.2%	-7.919	95.9%	0.0%
$AV_{i,t}B_{i,t}$		-6.753	75.5%	8.2%	-6.557	85.4%	10.4%	-5.477	89.7%	0.0%
$AV_{i,t}S_{i,t}$		13.568	6.1%	83.7%	4.846	14.6%	68.8%	3.768	13.4%	59.8%
adj. R^2		0.189	-	-	0.215	-	-	0.212	-	-
Panel B: Endogenous contemporaneous volume-volatility relation										
$v_{i,t}$		-0.357	46.9%	42.9%	-0.760	45.8%	25.0%	0.578	16.5%	40.2%
$v_{i,t}Spread_{i,t}$		18.988	10.2%	75.5%	15.285	6.2%	79.2%	5.354	6.2%	71.1%
$v_{i,t}BV_{i,t}B_{i,t}$		25.892	12.2%	63.3%	5.447	16.7%	50.0%	2.682	9.3%	38.1%
$v_{i,t}BV_{i,t}S_{i,t}$		-83.303	93.9%	0.0%	-36.415	87.5%	0.0%	-23.425	92.8%	0.0%
$v_{i,t}AV_{i,t}B_{i,t}$		-77.159	98.0%	0.0%	-37.500	87.5%	0.0%	-21.382	90.7%	0.0%
$v_{i,t}AV_{i,t}S_{i,t}$		30.906	4.1%	61.2%	4.567	10.4%	41.7%	1.473	12.4%	41.2%
$Spread_{i,t}$		3.339	16.3%	71.4%	4.062	10.4%	77.1%	2.527	15.5%	63.9%
$BV_{i,t}B_{i,t}$		10.111	26.5%	69.4%	2.572	25.0%	56.2%	2.567	21.6%	55.7%
$BV_{i,t}S_{i,t}$		-5.413	63.3%	14.3%	-5.117	70.8%	6.2%	-4.408	86.6%	1.0%
$AV_{i,t}B_{i,t}$		-2.899	57.1%	18.4%	-3.696	66.7%	12.5%	-2.851	67.0%	2.1%
$AV_{i,t}S_{i,t}$		10.542	8.2%	81.6%	3.769	16.7%	64.6%	2.735	11.3%	56.7%
adj. R^2		0.197	-	-	0.226	-	-	0.225	-	-
Panel C: Endogenous dynamic volume-volatility relation										
$v_{i,t}$	Lag 0	-0.257	44.9%	42.9%	-0.748	47.9%	25.0%	0.547	16.5%	40.2%
	$\sum_{1:p}$	-0.066	24.5%	14.3%	0.236	12.5%	10.4%	0.016	6.2%	10.3%
	$\sum_{0:p}$	-0.148	34.7%	26.5%	-0.528	33.3%	20.8%	0.447	8.2%	36.1%
$v_{i,t}Spread_{i,t}$	Lag 0	18.739	10.2%	75.5%	14.545	6.2%	77.1%	5.259	8.2%	71.1%
	$\sum_{1:p}$	9.219	0.0%	59.2%	3.085	2.1%	22.9%	0.986	2.1%	19.6%
	$\sum_{0:p}$	27.570	4.1%	87.8%	17.782	2.1%	77.1%	6.972	5.2%	71.1%
$v_{i,t}BV_{i,t}B_{i,t}$	Lag 0	33.340	12.2%	63.3%	4.931	10.4%	50.0%	2.422	9.3%	39.2%
	$\sum_{1:p}$	-7.249	32.7%	0.0%	-3.391	16.7%	0.0%	-2.095	23.7%	0.0%
	$\sum_{0:p}$	25.947	16.3%	55.1%	3.146	16.7%	31.2%	0.904	10.3%	18.6%
$v_{i,t}BV_{i,t}S_{i,t}$	Lag 0	-79.618	93.9%	0.0%	-35.032	85.4%	0.0%	-23.226	92.8%	0.0%
	$\sum_{1:p}$	-10.397	34.7%	20.4%	-2.641	29.2%	10.4%	-1.571	20.6%	2.1%
	$\sum_{0:p}$	-77.428	77.6%	2.0%	-42.287	83.3%	6.2%	-25.294	91.8%	0.0%
$v_{i,t}AV_{i,t}B_{i,t}$	Lag 0	-77.249	98.0%	0.0%	-35.970	91.7%	0.0%	-21.503	92.8%	0.0%
	$\sum_{1:p}$	-13.508	34.7%	18.4%	-7.367	39.6%	16.7%	-2.007	22.7%	3.1%
	$\sum_{0:p}$	-80.332	81.6%	2.0%	-47.534	89.6%	6.2%	-25.972	87.6%	0.0%
$v_{i,t}AV_{i,t}S_{i,t}$	Lag 0	30.841	4.1%	63.3%	4.478	8.3%	41.7%	1.299	10.3%	40.2%
	$\sum_{1:p}$	-8.706	22.4%	0.0%	-2.598	22.9%	2.1%	-1.830	12.4%	1.0%
	$\sum_{0:p}$	31.542	10.2%	46.9%	0.498	18.8%	31.2%	0.830	15.5%	22.7%
$Spread_{i,t}$	Lag 0	-3.653	55.1%	34.7%	2.411	37.5%	52.1%	-1.181	45.4%	34.0%
	$\sum_{1:p}$	14.497	8.2%	63.3%	2.927	16.7%	37.5%	5.575	14.4%	53.6%
	$\sum_{0:p}$	15.268	0.0%	71.4%	7.144	0.0%	64.6%	4.422	2.1%	66.0%
$BV_{i,t}B_{i,t}$	Lag 0	52.212	18.4%	71.4%	17.953	31.2%	54.2%	20.088	13.4%	55.7%
	$\sum_{1:p}$	-33.574	71.4%	14.3%	-16.805	54.2%	33.3%	-20.849	55.7%	8.2%
	$\sum_{0:p}$	-2.895	16.3%	14.3%	-0.780	0.0%	12.5%	-0.006	11.3%	12.4%
$BV_{i,t}S_{i,t}$	Lag 0	43.679	18.4%	71.4%	10.437	35.4%	50.0%	13.238	15.5%	51.5%
	$\sum_{1:p}$	-44.858	61.2%	4.1%	-21.941	47.9%	18.8%	-23.380	60.8%	5.2%
	$\sum_{0:p}$	-18.327	44.9%	8.2%	-10.195	45.8%	4.2%	-8.925	62.9%	0.0%
$AV_{i,t}B_{i,t}$	Lag 0	50.413	14.3%	77.6%	12.364	29.2%	52.1%	19.656	10.3%	62.9%
	$\sum_{1:p}$	-52.767	79.6%	0.0%	-22.814	50.0%	0.0%	-26.060	66.0%	3.1%
	$\sum_{0:p}$	-22.111	53.1%	6.1%	-12.336	52.1%	6.2%	-8.525	59.8%	0.0%
$AV_{i,t}S_{i,t}$	Lag 0	62.248	14.3%	77.6%	19.345	27.1%	58.3%	20.633	8.2%	62.9%
	$\sum_{1:p}$	-57.520	79.6%	8.2%	-20.757	58.3%	18.8%	-23.948	66.0%	8.2%
	$\sum_{0:p}$	-0.912	20.4%	2.0%	-1.112	14.6%	12.5%	-0.766	12.4%	8.2%
adj. R^2		0.210	-	-	0.236	-	-	0.234	-	-

Continued on next page

Table 4.3 – *continued from previous page*

This table reports the results of the following model, for the constituent stocks of the S&P/ASX200 index during *Jul-Dec 2014*,

$$\sigma_{i,t} = \alpha_0 + \mu_1 Monday_{i,t} + \mu_2 block1_{i,t} + \sum_{j=1}^{12} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{p} [\beta_{0,k} + \delta_k' x_{i,t-k}] v_{i,t-k} + \sum_{k=0}^{p} \gamma_k' x_{i,t-k} + \pi' y_{i,t} + \eta_{i,t},$$

where $\sigma_{i,t}$ is a proxy for return volatility *per unit of time* of the t-th transaction in stock i, which is estimated as the absolute value of the residual $|\widehat{\epsilon}_{i,t}|$ of the following autoregressive model of returns $r_{i,t}$ divided by the duration $T_{i,t}$ of the trade (i.e. $\sigma_{i,t} = |\widehat{\epsilon}_{i,t}|/T_{i,t}$):

$$r_{i,t} = \sum_{k=1}^{5} \psi_{i,k} Day_{k,i,t} + \sum_{k=1}^{5} \phi_{i,k} block_{k,i,t} + \sum_{k=1}^{12} \rho_{i,k} r_{i,t-k} + \varphi' x_{i,t} + \lambda' y_{i,t} + \epsilon_{i,t}.$$

$x_{i,t} = (Spread_{i,t}, BV_{i,t}B_{i,t}, BV_{i,t}S_{i,t}, AV_{i,t}B_{i,t}, AV_{i,t}S_{i,t})'$ is a vector of potential predictors of the volume-volatility relation. $y_{i,t} = (\ln(T_{i,t}), N_{i,t}, ATS_{i,t}, OIB_{i,t}, QTT_{i,t})'$ is a vector of control variables that allow for the effects of the order flow prior to a trade. The lag length p is set to $p = 5$ in Panel C. See Table 4.8 and the notes of Table 4.2 for the definitions of the variables and other notation. The table only reports the coefficient estimates for $v_{i,t}$ and $x_{i,t}$ from the volatility equation.

Table 4.3 indicates that the direction or type of a trade contains useful information about the return volatility of the trade, which is in agreement with previous studies that find strong evidence supportive of the important role played by trading or quoting directions or types in explaining the price formation process (e.g. Hasbrouck, 1991a,b, Dufour and Engle, 2000, Barclay et al., 2003). We also observe some asymmetries in the effects of the bid versus ask depths on return volatility as well as the volume-volatility relation. Incorporating the trade direction information and these asymmetries between the bid and ask sides of the LOB significantly improves the in-sample fit of the volatility regressions, with an average increase of about 3-4 percentage points (or about 20-25%) in the adjusted R^2, in comparison to that in Table 4.2.

Overall, the results presented in Tables 4.2 and 4.3 show that the dynamics of the LOB characteristics at the best quote level, namely the bid-ask spread and the market depth, are informative about the volume-volatility relation and ultimately the return volatility of trades. The positive dependence of volatility on volume is dynamic and positively (negatively) related to the bid-ask spread (market depth). Larger same-side (opposite-side) market depth prior to a trade increases (decreases) the return volatility and the volume-volatility relation of the trade. The effects of market depth on return volatility and the volume-volatility relation are asymmetric between the bid and ask sides of the LOB and tend to increase with a stock's liquidity.

4.4.2 Order book slope and the volume-volatility relation

The previous subsection examines the informativeness of the LOB characteristics at the best level about return volatility and the volume-volatility relation. Prior studies (e.g. Ahn et al., 2001, Kalay et al., 2004, Næs and Skjeltorp, 2006, Duong and Kalev, 2008, Kalay and Wohl, 2009, Pascual and Veredas, 2010) provide empirical evidence that the LOB information that lies beyond the best quotes contains significant predictive power about future returns and volatility. For example, Næs and Skjeltorp (2006) find that the average daily slope of the LOB, which summarizes the LOB information by measuring the average elasticity of the bid and ask order books, is negatively related to daily return volatility, daily trading activity, and the correlation between volatility and trading activity, where daily trading activity is measured by the number of transactions within a day. In this subsection, we investigate the information content of the LOB across different best quote levels by examining how the LOB slopes, computed according to equations (4.5)-(4.7) using the 10 best quotes, explain return volatility and the volume-volatility relation at a transaction level.

The results for an investigation of the power of the LOB slope for explaining the tick-by-tick volume-volatility relation for a combined LOB are presented in Table 4.4. Panel A reports the coefficient estimates of a volatility regression under the assumption that the volume-volatility relation is exogenous and fully contemporaneous. Consistent with the findings discussed in the previous subsection and in prior literature, there is a strong positive contemporaneous dependence of return volatility on trading volume that is statistically significant for almost all stocks. As expected, the return volatility of a trade is negatively related to the slope of the LOB immediately before the trade, with statistical significance obtained for the majority of stocks, which is consistent with the findings of Næs and Skjeltorp (2006) and Duong and Kalev (2008). A larger order book slope implies steeper LOB curves where more shares are supplied closer to the inner bid/ask quotes. Consequently, the larger the LOB slope, the better the LOB is able to absorb a given amount of shares demanded from an incoming trade, and the smaller the price impact and the return volatility of the trade. The negative relation between return volatility and the LOB slope becomes weaker, in magnitude, for larger cap stocks, reflecting the fact that more liquid stocks have a steeper LOB (Næs and Skjeltorp, 2006), and hence for a given change, e.g. one unit increase in the LOB slope, the price of more liquid stocks moves less to accommodate a given trading volume.

Table 4.4: Slope and the volume-volatility relation: Combined limit order book

		Large cap (49 stocks)			**Mid cap** (48 stocks)			**Small cap** (97 stocks)		
		Med	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$
Panel A: Constant contemporaneous volume-volatility relation										
$v_{i,t}$		0.937	0.0%	95.9%	0.874	0.0%	100.0%	1.012	0.0%	100.0%
$Slope_{i,t}$		-0.029	69.4%	24.5%	-0.299	85.4%	10.4%	-0.398	75.3%	17.5%
adj. R^2		0.159	-	-	0.174	-	-	0.179	-	-
Panel B: Endogenous contemporaneous volume-volatility relation										
$v_{i,t}$		3.980	0.0%	98.0%	8.420	2.1%	97.9%	4.941	0.0%	92.8%
$v_{i,t}Slope_{i,t}$		-0.132	77.6%	12.2%	-0.826	85.4%	8.3%	-0.577	82.5%	7.2%
$Slope_{i,t}$		-0.018	61.2%	30.6%	-0.207	79.2%	12.5%	-0.233	67.0%	21.6%
adj. R^2		0.169	-	-	0.182	-	-	0.185	-	-
Panel C: Endogenous dynamic volume-volatility relation										
$v_{i,t}$	Lag 0	3.865	2.0%	95.9%	8.265	2.1%	95.8%	4.931	0.0%	91.8%
	$\sum_{1:p}$	1.395	0.0%	77.6%	1.812	0.0%	47.9%	0.964	3.1%	40.2%
	$\sum_{0:p}$	7.013	0.0%	95.9%	11.046	2.1%	93.8%	6.116	0.0%	94.8%
$v_{i,t}Slope_{i,t}$	Lag 0	-0.128	77.6%	12.2%	-0.786	85.4%	8.3%	-0.574	81.4%	8.2%
	$\sum_{1:p}$	-0.034	65.3%	0.0%	-0.140	41.7%	0.0%	-0.111	39.2%	4.1%
	$\sum_{0:p}$	-0.205	81.6%	10.2%	-1.011	87.5%	4.2%	-0.737	82.5%	3.1%
$Slope_{i,t}$	Lag 0	0.017	30.6%	57.1%	-0.019	41.7%	39.6%	0.188	27.8%	56.7%
	$\sum_{1:p}$	-0.111	85.7%	2.0%	-0.247	70.8%	4.2%	-0.520	72.2%	12.4%
	$\sum_{0:p}$	-0.084	93.9%	0.0%	-0.380	87.5%	0.0%	-0.437	90.7%	1.0%
adj. R^2		0.174	-	-	0.193	-	-	0.194	-	-

This table reports the results of the following model, for the constituent stocks of the S&P/ASX200 index during *Jul-Dec 2014*,

$$\sigma_{i,t} = \alpha_0 + \mu_1 Monday_{i,t} + \mu_2 block1_{i,t} + \sum_{j=1}^{12} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{p} [\beta_{0,k} + \delta_k' x_{i,t-k}] v_{i,t-k} + \sum_{k=0}^{p} \gamma_k' x_{i,t-k} + \pi' y_{i,t} + \eta_{i,t},$$

where $\sigma_{i,t}$ is a proxy for return volatility *per unit of time* of the t-th transaction in stock i, which is estimated as the absolute value of the residual $|\widehat{\epsilon}_{i,t}|$ of the following autoregressive model of returns $r_{i,t}$ divided by the duration $T_{i,t}$ of the trade (i.e. $\sigma_{i,t} = |\widehat{\epsilon}_{i,t}|/T_{i,t}$):

$$r_{i,t} = \sum_{k=1}^{5} \psi_{i,k} Day_{k,i,t} + \sum_{k=1}^{5} \phi_{i,k} block_{k,i,t} + \sum_{k=1}^{12} \rho_{i,k} r_{i,t-k} + \varphi' x_{i,t} + \lambda' y_{i,t} + \epsilon_{i,t}.$$

$x_{i,t} = Slope_{i,t}$ is a potential predictor of the volume-volatility relation. $y_{i,t} = (\ln(T_{i,t}), N_{i,t}, ATS_{i,t}, OIB_{i,t}, QTT_{i,t})'$ is a vector of control variables that allow for the effects of the order flow prior to a trade. The lag length p is set to $p = 5$ in Panel C. See Table 4.8 and the notes of Table 4.2 for the definitions of the variables and other notation. The table only reports the coefficient estimates for $v_{i,t}$ and $x_{i,t}$ from the volatility equation.

We now examine the results for the contemporaneous volume-volatility relation that is allowed to vary with the LOB slope. Panel B of Table 4.4 shows that the positive dependence of return volatility on the trading volume of a trade is negatively associated with the slope of the LOB right before the trade, with statistical significance observed for more than 77% of all stocks. Thus, the positive volume-volatility relation is neither constant nor exogenous as typically assumed in most prior studies but it becomes weaker as the LOB is steeper and more concentrated around the best quotes. This result is consistent with work by Næs and Skjeltorp (2006), who document a negative link between the daily average order book slope and the correlation between daily return volatility and the number of transactions occurring in a day. Our result complements Næs and Skjeltorp's (2006) findings in that it shows that the negative dependence of the volume-volatility relation on the LOB slope prevails even at a tick-by-tick level. Allowing the volume-volatility relation to be endogenously related to the LOB slope significantly weakens the direct effects of the slope on return volatility (see the coefficients on $Slope_{i,t}$ in Panel B in comparison to those in Panel A). It follows from Panel B that the LOB slope right before a trade negatively affects the return volatility of the trade through two channels: a direct channel (captured by the coefficients on $Slope_{i,t}$) and an indirect channel (captured by the coefficients on $v_{i,t}Slope_{i,t}$) that changes the volume-volatility relation. Unlike the results shown in Panel B of Table 4.2, the direct effects of trading volume on return volatility (captured by the coefficients on $v_{i,t}$) do not switch signs but remain strongly positive after one allows the volume-volatility relation to depend on the LOB slope, even though the proportions of significant volume coefficients are slightly smaller.

In support of the theories of Copeland (1976), Jennings et al. (1981), Shalen (1993), and Banerjee and Kremer (2010) and the empirical work of Manganelli (2005), Xu et al. (2006), Nolte (2008), Carlin et al. (2014), and Do et al. (2014), Panel C of Table 4.4 shows that return volatility is positively correlated with both current and lagged trading volumes, implying that this relation is indeed dynamic. In addition, the positive dependence of return volatility on volume also varies with the dynamics of the LOB slope, with larger order book slopes (i.e. steeper LOBs) weakening the volume-volatility relation. Similar to the results in Table 4.2, the negative indirect impact of the LOB slope on future return volatility is primarily contributed by the current slope that prevails right before a trade (see the coefficients on $v_{i,t}Slope_{i,t}$), whereas most of its direct impact on volatility comes from the lagged slope information, as demonstrated by the coefficients on $Slope_{i,t}$. Unlike the bid-ask spread and the depth at the best quotes, both direct and indirect effects of the LOB slope on the volatility of an incoming trade become smaller in magnitude for more liquid stocks (compare mid and small cap stocks with large cap stocks).

Table 4.5 reports the results of an investigation in which we allow for possible asymmetries between the slope of the bid and ask order books in explaining return volatility and the volume-volatility relation. Similar to the results in Table 4.3, it is the opposite side of the LOB that is more predictive of the future return volatility of a trade. In particular, for an incoming purchase (sale) of a given volume, the larger the slope of the ask (bid) order book right before the trade (i.e. the more concentrated the ask (bid) book is around the best quote), the smaller the return volatility (see the coefficients on $Slpa_{i,t}B_{i,t}$ ($Slpb_{i,t}S_{i,t}$) in Panels A and B) and the weaker the volume-volatility relation (see the coefficients on $v_{i,t}Slpa_{i,t}B_{i,t}$ ($v_{i,t}Slpb_{i,t}S_{i,t}$) in Panel B) of the trade. In contrast, the dependence of return volatility on the volume of a trade is positively related to the slope of the LOB that is of the same side as the direction of the trade (see the coefficients on $v_{i,t}Slpb_{i,t}B_{i,t}$ and $v_{i,t}Slpa_{i,t}S_{i,t}$ in Panel B), while the direct impact of the latter on the volatility of the trade becomes ambiguous (see the proportions of significant coefficients on $Slpb_{i,t}B_{i,t}$ and $Slpa_{i,t}S_{i,t}$ in Panels A and B). These results highlight the asymmetric effects of the bid and ask slopes on return volatility and the volume-volatility relation.

Table 4.5: Slope and the volume-volatility relation: Bid vs. Ask sides

		Large cap (49 stocks)			**Mid cap** (48 stocks)			**Small cap** (97 stocks)		
		Med	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$
Panel A: Constant contemporaneous volume-volatility relation										
$v_{i,t}$		1.390	0.0%	100.0%	1.295	0.0%	100.0%	1.417	0.0%	100.0%
$Slpb_{i,t}B_{i,t}$		0.002	36.7%	55.1%	-0.017	39.6%	33.3%	0.019	25.8%	40.2%
$Slpb_{i,t}S_{i,t}$		-0.039	93.9%	2.0%	-0.279	97.9%	2.1%	-0.344	90.7%	1.0%
$Slpa_{i,t}B_{i,t}$		-0.037	89.8%	6.1%	-0.212	89.6%	4.2%	-0.337	82.5%	5.2%
$Slpa_{i,t}S_{i,t}$		0.003	30.6%	55.1%	0.009	25.0%	37.5%	0.073	24.7%	52.6%
adj. R^2		0.235	-	-	0.252	-	-	0.244	-	-
Panel B: Endogenous contemporaneous volume-volatility relation										
$v_{i,t}$		6.916	0.0%	98.0%	11.525	2.1%	97.9%	5.443	0.0%	95.9%
$v_{i,t}Slpb_{i,t}B_{i,t}$		0.101	2.0%	75.5%	0.327	4.2%	66.7%	0.446	0.0%	71.1%
$v_{i,t}Slpb_{i,t}S_{i,t}$		-0.467	91.8%	0.0%	-1.569	93.8%	6.2%	-1.360	96.9%	0.0%
$v_{i,t}Slpa_{i,t}B_{i,t}$		-0.467	95.9%	0.0%	-1.584	93.8%	2.1%	-1.418	92.8%	0.0%
$v_{i,t}Slpa_{i,t}S_{i,t}$		0.157	0.0%	85.7%	0.382	6.2%	72.9%	0.583	1.0%	71.1%
$Slpb_{i,t}B_{i,t}$		0.001	36.7%	51.0%	-0.045	45.8%	25.0%	-0.006	30.9%	29.9%
$Slpb_{i,t}S_{i,t}$		-0.020	89.8%	4.1%	-0.154	83.3%	2.1%	-0.212	79.4%	6.2%
$Slpa_{i,t}B_{i,t}$		-0.027	83.7%	10.2%	-0.113	83.3%	4.2%	-0.191	74.2%	9.3%
$Slpa_{i,t}S_{i,t}$		0.002	36.7%	49.0%	-0.001	27.1%	27.1%	0.024	26.8%	43.3%
adj. R^2		0.270	-	-	0.277	-	-	0.268	-	-
Panel C: Endogenous dynamic volume-volatility relation										
$v_{i,t}$	Lag 0	6.580	0.0%	93.9%	11.348	2.1%	97.9%	5.317	0.0%	95.9%
	$\sum_{1:p}$	1.479	0.0%	77.6%	1.176	0.0%	52.1%	0.799	3.1%	39.2%
	$\sum_{0:p}$	8.276	0.0%	95.9%	12.024	2.1%	95.8%	6.493	0.0%	94.8%
$v_{i,t}Slpb_{i,t}B_{i,t}$	Lag 0	0.102	2.0%	75.5%	0.318	4.2%	66.7%	0.471	0.0%	72.2%
	$\sum_{1:p}$	-0.045	46.9%	2.0%	-0.081	39.6%	2.1%	-0.180	43.3%	1.0%
	$\sum_{0:p}$	0.039	4.1%	46.9%	0.128	6.2%	43.8%	0.284	3.1%	44.3%
$v_{i,t}Slpb_{i,t}S_{i,t}$	Lag 0	-0.468	89.8%	0.0%	-1.576	93.8%	4.2%	-1.354	96.9%	0.0%
	$\sum_{1:p}$	0.006	18.4%	18.4%	0.049	4.2%	18.8%	0.087	2.1%	22.7%
	$\sum_{0:p}$	-0.473	87.8%	0.0%	-1.481	93.8%	6.2%	-1.192	89.7%	0.0%
$v_{i,t}Slpa_{i,t}B_{i,t}$	Lag 0	-0.471	95.9%	0.0%	-1.559	91.7%	2.1%	-1.440	92.8%	0.0%
	$\sum_{1:p}$	0.001	20.4%	14.3%	0.004	8.3%	20.8%	0.052	4.1%	17.5%
	$\sum_{0:p}$	-0.401	89.8%	4.1%	-1.511	93.8%	4.2%	-1.221	87.6%	0.0%
$v_{i,t}Slpa_{i,t}S_{i,t}$	Lag 0	0.163	0.0%	87.8%	0.388	6.2%	75.0%	0.601	1.0%	73.2%
	$\sum_{1:p}$	-0.050	57.1%	0.0%	-0.167	47.9%	0.0%	-0.213	44.3%	0.0%
	$\sum_{0:p}$	0.043	0.0%	46.9%	0.142	10.4%	37.5%	0.327	5.2%	41.2%
$Slpb_{i,t}B_{i,t}$	Lag 0	-0.010	44.9%	44.9%	-0.093	58.3%	29.2%	-0.125	46.4%	30.9%
	$\sum_{1:p}$	-0.020	38.8%	8.2%	0.020	22.9%	27.1%	-0.054	23.7%	25.8%
	$\sum_{0:p}$	-0.039	36.7%	4.1%	-0.036	29.2%	10.4%	-0.021	16.5%	11.3%
$Slpb_{i,t}S_{i,t}$	Lag 0	-0.007	51.0%	40.8%	-0.137	50.0%	31.2%	-0.057	40.2%	35.1%
	$\sum_{1:p}$	-0.009	36.7%	6.1%	-0.020	25.0%	14.6%	-0.084	34.0%	8.2%
	$\sum_{0:p}$	-0.015	28.6%	2.0%	-0.100	37.5%	0.0%	-0.169	43.3%	2.1%
$Slpa_{i,t}B_{i,t}$	Lag 0	-0.009	44.9%	40.8%	-0.037	41.7%	33.3%	0.048	29.9%	44.3%
	$\sum_{1:p}$	-0.041	46.9%	2.0%	-0.045	35.4%	4.2%	-0.196	47.4%	3.1%
	$\sum_{0:p}$	-0.041	40.8%	2.0%	-0.119	39.6%	2.1%	-0.242	49.5%	2.1%
$Slpa_{i,t}S_{i,t}$	Lag 0	0.005	36.7%	49.0%	-0.025	41.7%	27.1%	-0.066	35.1%	40.2%
	$\sum_{1:p}$	-0.034	51.0%	8.2%	-0.044	33.3%	14.6%	-0.063	34.0%	18.6%
	$\sum_{0:p}$	-0.042	51.0%	2.0%	-0.066	22.9%	10.4%	-0.089	22.7%	9.3%
adj. R^2		0.272	-	-	0.288	-	-	0.274	-	-

This table reports the results of the following model, for the constituent stocks of the S&P/ASX200 index during *Jul-Dec 2014*,

$$\sigma_{i,t} = \alpha_0 + \mu_1 Monday_{i,t} + \mu_2 block1_{i,t} + \sum_{j=1}^{12} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{p} [\beta_{0,k} + \delta_k' x_{i,t-k}] v_{i,t-k} + \sum_{k=0}^{p} \gamma_k' x_{i,t-k} + \pi' y_{i,t} + \eta_{i,t},$$

where $\sigma_{i,t}$ is a proxy for return volatility *per unit of time* of the t-th transaction in stock i, which is estimated as the absolute value of the residual $|\widehat{\epsilon}_{i,t}|$ of the following autoregressive model of returns $r_{i,t}$ divided by the duration $T_{i,t}$ of the trade (i.e. $\sigma_{i,t} = |\widehat{\epsilon}_{i,t}|/T_{i,t}$):

$$r_{i,t} = \sum_{k=1}^{5} \psi_{i,k} Day_{k,i,t} + \sum_{k=1}^{5} \phi_{i,k} block_{k,i,t} + \sum_{k=1}^{12} \rho_{i,k} r_{i,t-k} + \varphi' x_{i,t} + \lambda' y_{i,t} + \epsilon_{i,t}.$$

$x_{i,t} = (Slpb_{i,t}B_{i,t}, Slpb_{i,t}S_{i,t}, Slpa_{i,t}B_{i,t}, Slpa_{i,t}S_{i,t})'$ is a vector of potential predictors of the volume-volatility relation. $y_{i,t} = (\ln(T_{i,t}), N_{i,t}, ATS_{i,t}, OIB_{i,t}, QTT_{i,t})'$ is a vector of control variables that allow for the effects of the order flow prior to a trade. The lag length p is set to $p = 5$ in Panel C. See Table 4.8 and the notes of Table 4.2 for the definitions of the variables and other notation. The table only reports the coefficient estimates for $v_{i,t}$ and $x_{i,t}$ from the volatility equation.

Furthermore, there is strong evidence supporting the predictability of the dynamics of the bid and ask slopes about the dynamic volume-volatility relation, with most of the predictive power contributed by the slope information right before the execution of a trade, which is consistent with the results presented in the previous tables. While the negative indirect impact of the bid (ask) slope on the future return volatility of a sell (buy), which is channeled through the dynamic volume-volatility relation, remains economically and statistically significant for the vast majority of stocks (see the coefficients on $v_{i,t}Slpa_{i,t}B_{i,t}$ ($v_{i,t}Slpb_{i,t}S_{i,t}$) in Panel C), its direct effects on volatility, albeit of expected negative signs, are of much less statistical significance (compare the coefficients on $Slpa_{i,t}B_{i,t}$ ($Slpb_{i,t}S_{i,t}$) in Panel C of Table 4.5 with the corresponding results for $Slope_{i,t}$ in Panel C of Table 4.4). Nevertheless, taking into consideration the asymmetries between the bid and ask sides of the LOB as well as the information of the direction of trades brings a considerable enhancement to the in-sample explanatory power of the volatility regressions, with the adjusted R^2 increasing by 7-8 percentage points (or about 40-45%), relative to that for a combined LOB in Table 4.4.

4.4.3 Spread, depth, slope and the volume-volatility relation

Previous subsections 4.4.1 and 4.4.2 demonstrate the informativeness of the dynamics of the LOB characteristics, either at the best quotes (i.e. the bid-ask spread and market depth) or at different quote levels (i.e. the slope of the LOB), about the return volatility and the dynamic volume-volatility relation of trades. In this subsection, we investigate which order book information plays a more important role in explaining the positive dependence of return volatility on trading volume. The results of this analysis are reported in Table 4.6 for a combined LOB, and in Table 4.7 where we allow for the separation of the bid and ask order books.[68]

In conformance with the results discussed in previous subsections, the return volatility of a trade is positively related to not only contemporaneous but also lagged trading

[68]For brevity, we only report in Table 4.7 the estimates of the LOB attributes that are of the opposite side to the direction of a trade (e.g. $Slpa_{i,t}B_{i,t}$) from the volatility equation. The results for the LOB characteristics that are of the same side as the direction of a trade (e.g. $Slpb_{i,t}B_{i,t}$) are of less interest and are often less statistically significant. A complete table of results is available upon request.

volumes while being negatively dependent on the LOB slope prior to the trade, with statistical significance observed for the majority of stocks. In addition, the dynamic volume-volatility relation is not constant but varies inversely with the dynamics of the LOB slope, which is in agreement with Næs and Skjeltorp (2006) and Duong and Kalev (2008). The more concentrated the (opposite-side) order book is around the inner quotes or the larger the (opposite-side) book slope prior to an incoming trade, the more able the market is to absorb the trade. Consequently, there are fewer price revisions, resulting in lower return volatility and a weaker volume-volatility relation. Since larger stocks typically have a steeper LOB (see Table 4.1), it follows that the effects of the LOB slope on return volatility, either direct or indirect, should decrease with stocks' liquidity, which is indeed what we observe (compare mid and small cap stocks with large cap stocks).

Interestingly, after controlling for the LOB slope, both direct and indirect effects of the bid-ask spread and market depth on return volatility and the volume-volatility relation either switch signs or become much less significant, as compared with the corresponding results previously reported in Tables 4.2 and 4.3. This observation can be explained by the fact that the LOB slope, by definition, encompasses the information about the LOB both at and beyond the best quote level. Since the LOB outside the inner quotes is informative about future returns and volatility (Ahn et al., 2001, Kalay et al., 2004, Næs and Skjeltorp, 2006, Kalay and Wohl, 2009, Pascual and Veredas, 2010), the information contained in the LOB slope appears to dominate the bid-ask spread and the market depth in explaining the return volatility and the volume-volatility relation of trades. In fact, this result is in harmony with work by Næs and Skjeltorp (2006), who show that the contemporaneous correlation between daily volatility and the number of trades within a day becomes negatively (positively) related to the bid-ask spread (total depth in the LOB) after the LOB slope is taken into account. It is also consistent with Pascual and Veredas (2010), who find that the ex-post informational volatility of the latent efficient price process is positively (negatively) dependent on the depth available at (beyond) the best quotes, especially when one realizes that the information of the depth beyond the best quotes is incorporated in the LOB slope.

Table 4.6: Spread, depth, slope and the volume-volatility relation: Combined limit order book

		Large cap (49 stocks)			**Mid cap** (48 stocks)			**Small cap** (97 stocks)		
		Med	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$
Panel A: Constant contemporaneous volume-volatility relation										
$v_{i,t}$		0.970	0.0%	98.0%	0.897	0.0%	100.0%	1.047	0.0%	100.0%
$Spread_{i,t}$		-6.335	65.3%	28.6%	-6.745	72.9%	10.4%	-4.682	60.8%	5.2%
$Depth_{i,t}$		5.203	20.4%	67.3%	5.320	14.6%	77.1%	2.025	22.7%	55.7%
$Slope_{i,t}$		-0.096	71.4%	26.5%	-0.572	89.6%	8.3%	-0.582	81.4%	6.2%
adj. R^2		0.159	-	-	0.176	-	-	0.181	-	-
Panel B: Endogenous contemporaneous volume-volatility relation										
$v_{i,t}$		10.725	10.2%	79.6%	22.215	6.2%	85.4%	10.652	0.0%	79.4%
$v_{i,t}Spread_{i,t}$		-32.536	61.2%	14.3%	-24.684	68.8%	6.2%	-7.339	59.8%	3.1%
$v_{i,t}Depth_{i,t}$		7.154	30.6%	40.8%	2.772	18.8%	35.4%	-0.682	35.1%	23.7%
$v_{i,t}Slope_{i,t}$		-0.342	75.5%	16.3%	-1.564	91.7%	6.2%	-1.088	73.2%	5.2%
$Spread_{i,t}$		-6.010	63.3%	28.6%	-5.722	68.8%	10.4%	-3.994	61.9%	7.2%
$Depth_{i,t}$		5.459	10.2%	71.4%	5.466	10.4%	79.2%	2.702	13.4%	58.8%
$Slope_{i,t}$		-0.077	71.4%	26.5%	-0.496	85.4%	8.3%	-0.489	80.4%	6.2%
adj. R^2		0.174	-	-	0.187	-	-	0.191	-	-
Panel C: Endogenous dynamic volume-volatility relation										
$v_{i,t}$	Lag 0	10.027	10.2%	77.6%	21.664	6.2%	85.4%	10.627	0.0%	80.4%
	$\sum_{1:p}$	0.256	28.6%	22.4%	3.037	14.6%	22.9%	1.777	3.1%	17.5%
	$\sum_{0:p}$	12.213	18.4%	67.3%	23.348	6.2%	79.2%	12.633	0.0%	76.3%
$v_{i,t}Spread_{i,t}$	Lag 0	-28.056	61.2%	14.3%	-23.886	64.6%	6.2%	-7.576	59.8%	3.1%
	$\sum_{1:p}$	6.042	18.4%	38.8%	-1.762	22.9%	16.7%	-1.310	9.3%	4.1%
	$\sum_{0:p}$	-21.467	49.0%	26.5%	-28.891	54.2%	8.3%	-9.907	48.5%	1.0%
$v_{i,t}Depth_{i,t}$	Lag 0	4.235	30.6%	40.8%	2.376	20.8%	35.4%	-0.892	34.0%	22.7%
	$\sum_{1:p}$	-8.079	28.6%	4.1%	-1.914	16.7%	6.2%	-0.615	14.4%	1.0%
	$\sum_{0:p}$	-3.300	36.7%	28.6%	-0.689	18.8%	22.9%	-1.016	32.0%	14.4%
$v_{i,t}Slope_{i,t}$	Lag 0	-0.327	73.5%	18.4%	-1.476	91.7%	6.2%	-1.078	73.2%	5.2%
	$\sum_{1:p}$	0.001	26.5%	28.6%	-0.088	25.0%	14.6%	-0.164	19.6%	1.0%
	$\sum_{0:p}$	-0.302	65.3%	22.4%	-1.740	81.2%	8.3%	-1.207	73.2%	2.1%
$Spread_{i,t}$	Lag 0	-5.538	59.2%	10.2%	-2.911	47.9%	12.5%	-0.755	38.1%	19.6%
	$\sum_{1:p}$	5.984	22.4%	40.8%	-9.140	35.4%	16.7%	-7.347	35.1%	8.2%
	$\sum_{0:p}$	3.380	26.5%	36.7%	-13.550	43.8%	12.5%	-6.910	47.4%	6.2%
$Depth_{i,t}$	Lag 0	53.795	12.2%	83.7%	24.443	16.7%	60.4%	21.384	4.1%	67.0%
	$\sum_{1:p}$	-61.702	77.6%	10.2%	-23.948	52.1%	16.7%	-22.482	66.0%	4.1%
	$\sum_{0:p}$	-5.272	30.6%	12.2%	-0.063	10.4%	22.9%	0.209	18.6%	11.3%
$Slope_{i,t}$	Lag 0	0.011	34.7%	51.0%	0.011	35.4%	41.7%	0.308	15.5%	53.6%
	$\sum_{1:p}$	-0.073	44.9%	18.4%	-0.372	58.3%	2.1%	-0.968	75.3%	2.1%
	$\sum_{0:p}$	-0.066	42.9%	24.5%	-0.636	62.5%	10.4%	-0.916	73.2%	0.0%
adj. R^2		0.185	-	-	0.203	-	-	0.204	-	-

This table reports the results of the following model, for the constituent stocks of the S&P/ASX200 index during *Jul-Dec 2014*,

$$\sigma_{i,t} = \alpha_0 + \mu_1 Monday_{i,t} + \mu_2 block1_{i,t} + \sum_{j=1}^{12} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{p} [\beta_{0,k} + \delta_k' x_{i,t-k}] v_{i,t-k} + \sum_{k=0}^{p} \gamma_k' x_{i,t-k} + \pi' y_{i,t} + \eta_{i,t},$$

where $\sigma_{i,t}$ is a proxy for return volatility *per unit of time* of the t-th transaction in stock i, which is estimated as the absolute value of the residual $|\widehat{\epsilon}_{i,t}|$ of the following autoregressive model of returns $r_{i,t}$ divided by the duration $T_{i,t}$ of the trade (i.e. $\sigma_{i,t} = |\widehat{\epsilon}_{i,t}|/T_{i,t}$):

$$r_{i,t} = \sum_{k=1}^{5} \psi_{i,k} Day_{k,i,t} + \sum_{k=1}^{5} \phi_{i,k} block_{k,i,t} + \sum_{k=1}^{12} \rho_{i,k} r_{i,t-k} + \varphi' x_{i,t} + \lambda' y_{i,t} + \epsilon_{i,t}.$$

$x_{i,t} = (Spread_{i,t}, Depth_{i,t}, Slope_{i,t})'$ is a vector of potential predictors of the volume-volatility relation. $y_{i,t} = (\ln(T_{i,t}), N_{i,t}, ATS_{i,t}, OIB_{i,t}, QTT_{i,t})'$ is a vector of control variables that allow for the effects of the order flow prior to a trade. The lag length p is set to $p = 5$ in Panel C. See Table 4.8 and the notes of Table 4.2 for the definitions of the variables and other notation. The table only reports the coefficient estimates for $v_{i,t}$ and $x_{i,t}$ from the volatility equation.

Table 4.7: Spread, depth, slope and the volume-volatility relation: Bid vs. Ask sides

		Large cap (49 stocks)			**Mid cap** (48 stocks)			**Small cap** (97 stocks)		
		Med	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$
Panel A: Constant contemporaneous volume-volatility relation										
$v_{i,t}$		1.381	0.0%	100.0%	1.296	0.0%	100.0%	1.465	0.0%	100.0%
$Spread_{i,t}$		-1.790	49.0%	30.6%	-3.015	52.1%	18.8%	-2.188	44.3%	11.3%
$BV_{i,t}S_{i,t}$		9.272	16.3%	69.4%	5.580	10.4%	72.9%	0.970	20.6%	36.1%
$AV_{i,t}B_{i,t}$		8.400	18.4%	71.4%	6.384	8.3%	66.7%	1.531	14.4%	36.1%
$Slpb_{i,t}S_{i,t}$		-0.065	83.7%	14.3%	-0.383	83.3%	4.2%	-0.433	83.5%	5.2%
$Slpa_{i,t}B_{i,t}$		-0.065	81.6%	14.3%	-0.271	85.4%	8.3%	-0.366	76.3%	1.0%
adj. R^2		0.236	-	-	0.252	-	-	0.246	-	-
Panel B: Endogenous contemporaneous volume-volatility relation										
$v_{i,t}$		29.845	8.2%	87.8%	36.983	6.2%	91.7%	16.660	0.0%	87.6%
$v_{i,t}Spread_{i,t}$		-100.431	83.7%	8.2%	-52.209	83.3%	6.2%	-17.117	73.2%	5.2%
$v_{i,t}BV_{i,t}S_{i,t}$		65.062	16.3%	61.2%	36.890	10.4%	64.6%	4.138	13.4%	41.2%
$v_{i,t}AV_{i,t}B_{i,t}$		80.364	20.4%	63.3%	34.515	8.3%	60.4%	5.152	13.4%	35.1%
$v_{i,t}Slpb_{i,t}S_{i,t}$		-0.820	85.7%	6.1%	-3.160	93.8%	6.2%	-2.671	91.8%	1.0%
$v_{i,t}Slpa_{i,t}B_{i,t}$		-0.909	87.8%	4.1%	-2.933	91.7%	6.2%	-2.575	92.8%	0.0%
$Spread_{i,t}$		-1.719	51.0%	32.7%	-4.408	60.4%	18.8%	-1.985	50.5%	11.3%
$BV_{i,t}S_{i,t}$		11.473	10.2%	83.7%	6.105	8.3%	79.2%	2.166	9.3%	48.5%
$AV_{i,t}B_{i,t}$		11.553	12.2%	79.6%	8.011	4.2%	81.2%	3.102	9.3%	55.7%
$Slpb_{i,t}S_{i,t}$		-0.045	83.7%	8.2%	-0.274	85.4%	2.1%	-0.266	78.4%	4.1%
$Slpa_{i,t}B_{i,t}$		-0.049	81.6%	14.3%	-0.197	83.3%	4.2%	-0.227	70.1%	3.1%
adj. R^2		0.278	-	-	0.297	-	-	0.279	-	-
Panel C: Endogenous dynamic volume-volatility relation										
$v_{i,t}$	Lag 0	30.218	8.2%	87.8%	36.503	6.2%	91.7%	16.639	0.0%	88.7%
	$\sum_{1:p}$	1.260	28.6%	26.5%	1.103	14.6%	25.0%	0.796	4.1%	10.3%
	$\sum_{0:p}$	29.435	12.2%	81.6%	35.541	6.2%	89.6%	17.936	0.0%	86.6%
$v_{i,t}Spread_{i,t}$	Lag 0	-98.626	83.7%	8.2%	-52.148	85.4%	6.2%	-17.010	76.3%	4.1%
	$\sum_{1:p}$	4.377	18.4%	30.6%	0.676	16.7%	16.7%	0.338	7.2%	7.2%
	$\sum_{0:p}$	-96.019	75.5%	16.3%	-49.564	75.0%	6.2%	-16.107	64.9%	2.1%
$v_{i,t}BV_{i,t}S_{i,t}$	Lag 0	64.537	16.3%	61.2%	38.249	10.4%	64.6%	4.512	13.4%	42.3%
	$\sum_{1:p}$	-17.418	30.6%	2.0%	-9.876	18.8%	4.2%	-4.051	29.9%	0.0%
	$\sum_{0:p}$	56.117	18.4%	51.0%	26.766	10.4%	50.0%	1.275	16.5%	23.7%
$v_{i,t}AV_{i,t}B_{i,t}$	Lag 0	78.634	20.4%	63.3%	32.267	8.3%	62.5%	4.986	13.4%	35.1%
	$\sum_{1:p}$	-14.717	20.4%	0.0%	-6.871	20.8%	6.2%	-2.972	14.4%	0.0%
	$\sum_{0:p}$	52.137	20.4%	46.9%	21.558	2.1%	45.8%	1.849	15.5%	22.7%
$v_{i,t}Slpb_{i,t}S_{i,t}$	Lag 0	-0.811	85.7%	6.1%	-3.186	93.8%	6.2%	-2.669	91.8%	1.0%
	$\sum_{1:p}$	0.033	8.2%	42.9%	0.140	4.2%	22.9%	0.180	1.0%	22.7%
	$\sum_{0:p}$	-0.800	81.6%	16.3%	-2.985	87.5%	6.2%	-2.149	81.4%	1.0%
$v_{i,t}Slpa_{i,t}B_{i,t}$	Lag 0	-0.909	87.8%	4.1%	-2.961	91.7%	6.2%	-2.586	92.8%	0.0%
	$\sum_{1:p}$	0.016	8.2%	36.7%	0.056	2.1%	29.2%	0.128	1.0%	15.5%
	$\sum_{0:p}$	-0.839	81.6%	12.2%	-2.652	89.6%	6.2%	-2.273	85.6%	1.0%
$Spread_{i,t}$	Lag 0	-6.091	69.4%	6.1%	-6.988	60.4%	12.5%	-4.086	47.4%	13.4%
	$\sum_{1:p}$	3.844	14.3%	36.7%	0.914	22.9%	18.8%	0.166	12.4%	14.4%
	$\sum_{0:p}$	-3.644	24.5%	32.7%	-10.663	41.7%	14.6%	-4.809	33.0%	10.3%
$BV_{i,t}S_{i,t}$	Lag 0	57.390	10.2%	79.6%	15.468	25.0%	56.2%	14.929	5.2%	54.6%
	$\sum_{1:p}$	-62.621	61.2%	4.1%	-23.179	39.6%	6.2%	-15.343	47.4%	4.1%
	$\sum_{0:p}$	-11.812	24.5%	0.0%	-1.200	10.4%	14.6%	-0.047	15.5%	9.3%
$AV_{i,t}B_{i,t}$	Lag 0	59.366	6.1%	83.7%	24.653	8.3%	60.4%	20.213	4.1%	60.8%
	$\sum_{1:p}$	-53.029	63.3%	0.0%	-17.516	47.9%	6.2%	-14.848	52.6%	2.1%
	$\sum_{0:p}$	-4.502	24.5%	4.1%	1.018	4.2%	10.4%	0.258	12.4%	14.4%
$Slpb_{i,t}S_{i,t}$	Lag 0	-0.022	55.1%	26.5%	-0.119	52.1%	25.0%	-0.043	33.0%	24.7%
	$\sum_{1:p}$	0.021	10.2%	38.8%	-0.008	12.5%	10.4%	-0.126	22.7%	4.1%
	$\sum_{0:p}$	-0.008	20.4%	30.6%	-0.203	20.8%	6.2%	-0.189	27.8%	0.0%
$Slpa_{i,t}B_{i,t}$	Lag 0	-0.026	55.1%	34.7%	-0.063	52.1%	27.1%	0.014	26.8%	25.8%
	$\sum_{1:p}$	-0.003	16.3%	26.5%	-0.042	18.8%	12.5%	-0.162	30.9%	1.0%
	$\sum_{0:p}$	-0.032	32.7%	22.4%	-0.223	31.2%	8.3%	-0.238	35.1%	3.1%
adj. R^2		0.288	-	-	0.314	-	-	0.291	-	-

Continued on next page

Table 4.7 – *continued from previous page*

This table reports the results of the following model, for the constituent stocks of the S&P/ASX200 index during *Jul-Dec 2014*,

$$\sigma_{i,t} = \alpha_0 + \mu_1 Monday_{i,t} + \mu_2 block1_{i,t} + \sum_{j=1}^{12} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{p} [\beta_{0,k} + \delta'_k x_{i,t-k}] v_{i,t-k} + \sum_{k=0}^{p} \gamma'_k x_{i,t-k} + \pi' y_{i,t} + \eta_{i,t},$$

where $\sigma_{i,t}$ is a proxy for return volatility *per unit of time* of the t-th transaction in stock i, which is estimated as the absolute value of the residual $|\widehat{\epsilon}_{i,t}|$ of the following autoregressive model of returns $r_{i,t}$ divided by the duration $T_{i,t}$ of the trade (i.e. $\sigma_{i,t} = |\widehat{\epsilon}_{i,t}|/T_{i,t}$):

$$r_{i,t} = \sum_{k=1}^{5} \psi_{i,k} Day_{k,i,t} + \sum_{k=1}^{5} \phi_{i,k} block_{k,i,t} + \sum_{k=1}^{12} \rho_{i,k} r_{i,t-k} + \varphi' x_{i,t} + \lambda' y_{i,t} + \epsilon_{i,t}.$$

$x_{i,t} = (Spread_{i,t}, BV_{i,t}B_{i,t}, BV_{i,t}S_{i,t}, AV_{i,t}B_{i,t}, AV_{i,t}S_{i,t}, Slpb_{i,t}B_{i,t}, Slpb_{i,t}S_{i,t}, Slpa_{i,t}B_{i,t}, Slpa_{i,t}S_{i,t})'$ is a vector of potential predictors of the volume-volatility relation. $y_{i,t} = (\ln(T_{i,t}), N_{i,t}, ATS_{i,t}, OIB_{i,t}, QTT_{i,t})'$ is a vector of control variables that allow for the effects of the order flow prior to a trade. The lag length p is set to $p = 5$ in Panel C. See Table 4.8 and the notes of Table 4.2 for the definitions of the variables and other notation. The table only reports the coefficient estimates for $v_{i,t}$ and $x_{i,t}$ of the order book that are of the opposite side to the direction of a trade (e.g. $Slpa_{i,t}B_{i,t}$) from the volatility equation.

Overall, the results in subsections 4.4.1-4.4.3 highlight the dynamic nature of the volume-volatility relation which is positive and varies with the dynamics of the LOB. The dependence of return volatility on the trading volume of a trade is positively associated with the bid-ask spread but negatively correlated with the market depth at the best quotes and the slope of the LOB prior to the transaction. Since the LOB slope, by definition, captures the information contained in the bid-ask spread and the market depth at the best quotes, it acts as the dominant explanatory factor of the volume-volatility relation and the return volatility of a trade. The impact of the LOB characteristics on the future return volatility of a trade depends on the liquidity of stocks and is transmitted through two channels: a direct channel that is mainly contributed by the lagged order book information, and an indirect channel that transfers the effects via the volume-volatility relation and is primarily driven by the current order book information that prevails immediately before the trade. There are also asymmetries between the influence of the bid and ask order books on return volatility and the volume-volatility relation, with the opposite-side order book possessing the dominant predictive power about the return volatility of an incoming trade.

4.4.4 Robustness

The above analyses are based on the LOB slopes that are calculated using 10 best bid and ask levels from the LOB. An interesting and natural question is whether the slopes become more or less informative about the return volatility and the volume-volatility relation of trades if they are computed from different sets of the LOB information. To answer this question, we employ different bid and ask levels (5 and 20) from the LOB to calculate the slope measures, then we reexamine our analysis. The results of this exercise are reported in Table 4.9 in the Appendix (i.e. Section 4.7) for a combined LOB, and in Table 4.10 for an order book that is separated into bid and ask sides.[69]

[69]To save space, we only tabulate the results for the case where the volume-volatility relation is allowed to be dynamic and endogenously related to the bid-ask spread, the market depth at the best quotes, and the slope of the LOB right before a trade. In addition, we only report the estimated coefficients for the LOB attributes and their interactions with trading volume $v_{i,t}$ in the volatility equation. In Table 4.10, only the coefficients for the attributes of the opposite side to the trade direction (e.g. $v_{i,t}Slpa_{i,t}B_{i,t}$) are reported. The estimates for other variables are of less interest and are qualitatively similar to the corresponding ones reported in the main text. A complete table of results is available

Overall, the results from Tables 4.9 and 4.10 are qualitatively similar to those reported in Tables 4.6 and 4.7, respectively, with both the dynamic volume-volatility relation and return volatility strongly negatively associated with the dynamics of the slopes of the (opposite-side) order book. The indirect effects of the (opposite-side) order book slope on return volatility, which are transmitted through the volume-volatility relation, tend to decrease with stocks' liquidity (compare mid and small cap stocks with large cap stocks) and mainly stem from the slope information that is available right before a trade. The direct effects of the LOB slope on return volatility are also inversely related to stocks' liquidity; however, they are of less statistical significance than the indirect effects and are mainly explained by lagged slope information (see Table 4.9). These direct effects even play a much smaller statistical role than the corresponding indirect effects when one allows for potential asymmetries between the bid and ask order books (see Table 4.10).

The LOB slope dominates the bid-ask spread and the market depth in explaining the return volatility and the volume-volatility relation of a trade. This result reaffirms the informativeness of the LOB information that is beyond the best inner quotes, as noted in previous studies (e.g. Ahn et al., 2001, Næs and Skjeltorp, 2006, Pascual and Veredas, 2010). Nevertheless, the market depth at the best quotes does possess significant predictive power about volatility, especially for mid and small cap stocks, when 20 best quote levels are used to construct the LOB slope (see the coefficients on $v_{i,t}Depth_{i,t}$ in Panel B of Table 4.9, and the coefficients on $v_{i,t}BV_{i,t}S_{i,t}$ and $v_{i,t}AV_{i,t}B_{i,t}$ in Panel B of Table 4.10)

There is, however, an interesting observation that is worth highlighting. While the impact of the LOB slope on the volume-volatility relation, or equivalently the indirect influence of the slope on return volatility, becomes stronger (in magnitude) for large cap stocks when more order book information is employed to construct the slope measure, the impact is biggest for mid and small cap stocks when the LOB slope is computed using 10 best bid and ask levels. In addition, the proportions of significant coefficients for the mid and small cap stocks are also remarkably lower for the slope

upon request.

measure computed using the 20 best quotes from the book. These results suggest that for almost all stocks, the sixth to tenth best levels of the LOB possess significant predictive power about future return volatility additional to that contained in the first five best quotes. Quotes and depths that are queued beyond the tenth best level (and up to the twentieth best level) are informative about volatility only for highly liquid stocks but not for the less liquid stocks. The reason for this is that for illiquid stocks, quotes outside the 10 best levels are likely stale orders. Consequently, the inclusion of these levels in the calculation of the slope measures reduces the informativeness of the LOB slope for the less liquid stocks, which possibly explains the improvements in the predictability of the depths at the best quotes about volatility and the volume-volatility relation for these stocks as we observe.

The second set of robustness checks the sensitivity of the informativeness of the LOB information to different winsorization cut-off levels. In order to avoid the effect of the outliers, in previous analyses all variables are winsorized, on a stock-by-stock basis, at the 1^{st} and 99^{th} quantiles (i.e. 2% winsorization). We now redo our analyses (with the LOB slope constructed from the 10 best bid and ask levels) adopting two different winsorization cut-off levels, namely the 0.5^{th}-99.5^{th} quantiles (i.e. 1% winsorization) and the 2^{nd}-98^{th} quantiles (i.e. 4% winsorization).[70] The results of this investigation, respectively reported in Tables 4.11 and 4.12 in the Appendix, again qualitatively resemble those reported in Tables 4.6 and 4.7, suggesting that our main finding that the slope of the (opposite-side) LOB is an important determinant of the dynamic volume-volatility relation is robust to different winsorization levels. It is, however, noted that this main finding generally becomes more (less) statistically significant when the 4% (1%) winsorization window is employed.

[70] In addition to the two stocks discussed in footnote 60, another stock GFF (Goodman Fielder Limited - Small cap) is removed after the 0.5^{th}-99.5^{th} winsorization for the reason explained in footnote 65, leaving us with a sample of 197 stocks (49 Large cap, 50 Mid cap, and 98 Small cap). Meanwhile, additional eleven stocks, namely TLS (Telstra Corporation Limited - Large cap), ALZ (Australand Property Group - Mid cap), DUE (DUET Group - Mid cap), DJS (David Jones Limited - Mid cap), ENV (Envestra Limited - Mid cap), AQA (Aquila Resources Limited - Small cap), CMW (Cromwell Property Group - Small cap), GFF (Goodman Fielder Limited - Small cap), HZN (Horizon Oil Limited - Small cap), SIP (Sigma Pharmaceuticals Limited - Small cap), and TEN (Ten Network Holdings Limited - Small cap), are removed after the 2^{nd}-98^{th} winsorization, resulting in a sample of 187 stocks (48 Large cap, 46 Mid cap, and 93 Small cap).

4.5 Why is the order book slope informative?

The previous analyses highlight the significant information content of the LOB slope about the return volatility and the volume-volatility relation of trades. In order to interpret the informativeness of the order book slope given a lack of theoretical guidance, Næs and Skjeltorp (2006) conduct an empirical analysis to identify factors that can explain the slope. Based on an empirical observation that there is a significant negative relation between the average monthly LOB slope and the variation in the analysts' monthly earnings forecasts, these authors suggest that the LOB slope acts as a proxy for disagreements amongst investors. The more traders disagree about the true value of a stock, the wider the range of prices and volumes of the limit or market orders that they will submit, resulting in a less concentrated LOB with a more gentle slope. This conjecture of Næs and Skjeltorp (2006) seems to fit in with a strand of theoretical studies that demonstrate that disagreements amongst investors about asset values are a key factor contributing to the positive correlation between trading volumes and absolute price changes. These disagreements may result from either private information asymmetry (e.g. Grundy and McNichols, 1989, Shalen, 1993) or differences of opinions about public information (e.g. Harris and Raviv, 1993, Kandel and Pearson, 1995, Banerjee and Kremer, 2010). An empirical study of Carlin et al. (2014) also finds that both trading volume and return volatility become larger following an increase in investors' disagreement. Similarly, Wang and Wu (2015) document that the contemporaneous impact of the number of trades on price volatility varies across different corporate bond groups that are classified according to the dispersion of analysts' earnings forecasts, and it is typically larger for bonds that have higher analyst disagreement. Since investor heterogeneity is a driver of the positive dependence of volatility on volume, the informativeness of the LOB slope about the volume-volatility relation can be reasonably explained if the slope is indeed a proxy for the heterogeneity of investors as suggested by Næs and Skjeltorp (2006).

In this chapter, we do not aim to empirically test the above Næs and Skjeltorp's (2006) conjecture, which is connected to the theoretical prediction of Harris and Raviv (1993), Shalen (1993), Banerjee and Kremer (2010), amongst others. Instead, we provide an intuitive graphical illustration that not only directly explains why return volatility and the volume-volatility relation are negatively associated with the slope of the (opposite-side) LOB, but also complements Næs and Skjeltorp's (2006) conjecture.

Consider a market order submitted to the LOB of a hypothetical stock A that immediately results in a trade. Suppose that right before the execution of the market order, the LOB of stock A has the best bid quote of $P_0 - s/2$ and the best ask quote of $P_0 + s/2$, implying that the prevailing mid quote is P_0 and the quoted bid-ask spread is s. To obtain a clearer and simplified picture of how the slope of the LOB that prevails immediately prior to the trade affects the price at which the trade is transacted and the volume-volatility relation of the trade, we assume that (i) the market order that leads to the trade is buyer-initiated so that the ask side of the LOB is relevant for the execution of the order; (ii) the depths queued on the ask order book right before the trade are nicely allocated such that the ask book can be smoothly illustrated by an increasing straight line starting from the best ask;[71] (iii) the last transacted price of stock A is P and it is no greater than the prevailing best ask quote $P_0 + s/2$; (iv) the size of the market order, V_{buy}, is larger than the depth available at the best ask; and (v) the market order is very aggressive such that it walks the LOB and is fully executed.

Figure 4.1 illustrates how the price of stock A adjusts to accommodate the market buy order or the purchase. We consider two scenarios. The first is one for which the ask order book of stock A right before the execution of the purchase has less shares queued close to the best ask quote and hence is relatively flat. The ask order book in this scenario is illustrated with a dashed black line labeled as "Ask order book 1", with Ask Depth_1 shares available at the best ask. The second scenario is one for which the ask order book prior to the trade is more concentrated around the best ask and has a bigger slope, which has the best ask depth of Ask Depth_2 (> Ask Depth_1) and is presented by a solid black line with an "Ask order book 2" label. Note that the "Ask order book 1" ("Ask order book 2") can also be viewed as the state of the LOB for stock A when there is a high (low) degree of disagreement amongst traders whose

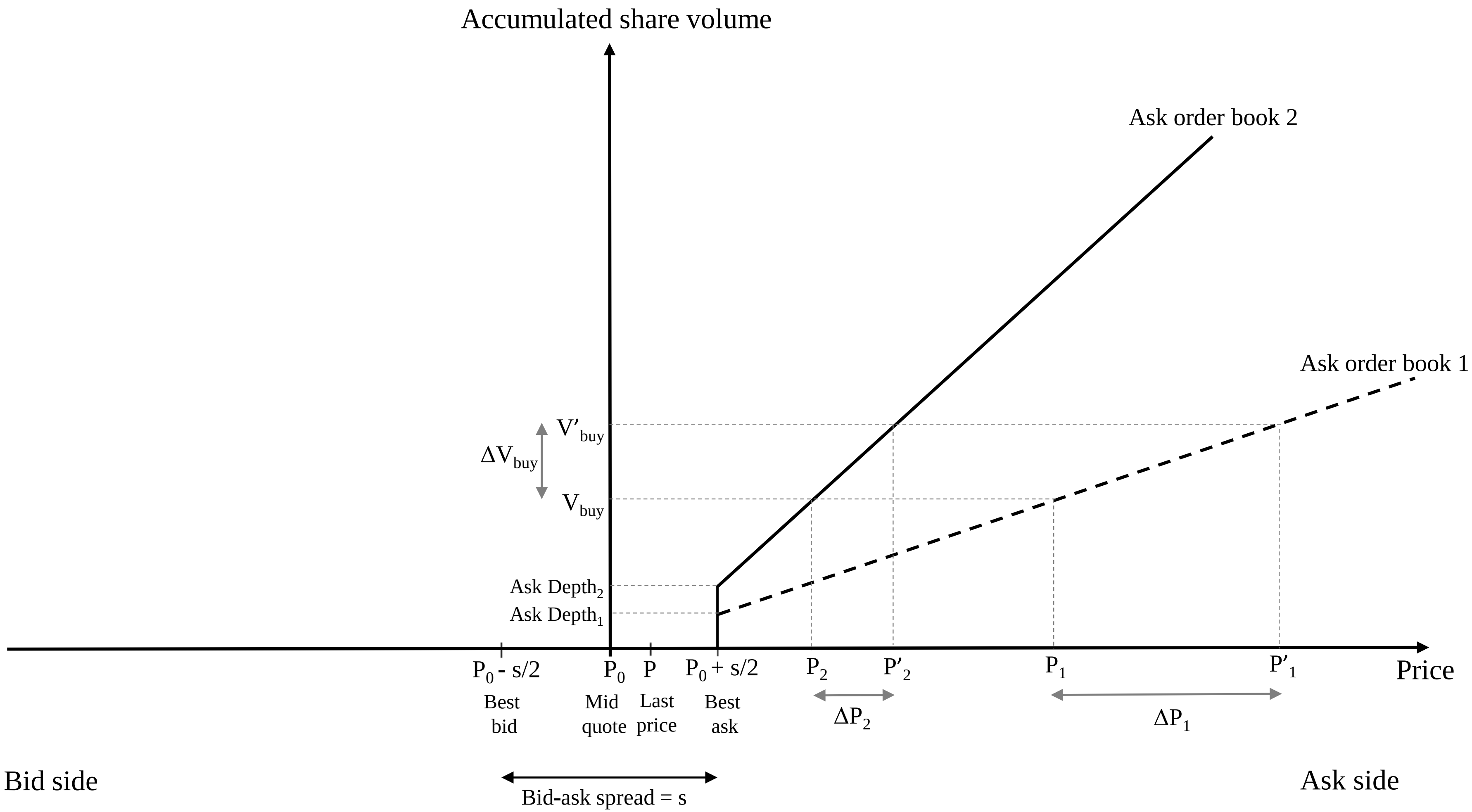
[71] Strictly speaking, the limit order book has a non-decreasing piece-wise linear shape.

orders are placed over a wide (narrow) range of prices. Thus, the two scenarios under consideration here are compatible with Næs and Skjeltorp's (2006) suggestion.

From Figure 4.1, the execution of the buy of size V_{buy} moves the price of stock A to P_1 (P_2) under the first (second) scenario from the previous transaction price P. Clearly, the absolute change in the price of stock A, which is a proxy for volatility that is widely used in the literature, is smaller in the second scenario where the slope of the ask order book that prevails right before the purchase is larger (i.e. $|P_2 - P| < |P_1 - P|$), and this explains the negative correlation between volatility and the LOB slope.

To see how the LOB slope affects the volume-volatility relation, consider a hypothetical increase of ΔV_{buy} in the volume of the purchase from V_{buy} to V'_{buy}. This pushes the price of stock A further to P'_1 (P'_2) under the first (second) scenario, implying an increase in the price of ΔP_1 (ΔP_2), relative to the previous price when the size of the purchase is V_{buy}. The impact of the increase in the buying volume on the stock price, which is essentially a measure of the volume-volatility relation, is $\Delta P_1/\Delta V_{\text{buy}}$ ($\Delta P_2/\Delta V_{\text{buy}}$) in the first (second) scenario. Since $\Delta P_1/\Delta V_{\text{buy}} > \Delta P_2/\Delta V_{\text{buy}}$, it follows that the volume-volatility relation becomes weaker the larger the LOB slope is prior to the trade.

Figure 4.1: Order book slope and the volume-volatility relation



Note: This figure depicts how the order book slope affects the volume-volatility relation, using the ask side of the order book as an illustration.

The main intuition underlying the informativeness of the LOB slope discussed above still holds without the aforementioned simplifying assumptions. Assumption (i) is imposed without a loss of generality so that we only need to focus on the relevant side of the order book. If the market order is a sale, a qualitatively similar graph based on the bid order book can be employed. Assumption (ii) is also trivial and is added to assist the drawing of the graph. It is easy to verify that the above argument from Figure 4.1 remains valid if we use the commonly observed piece-wise linear limit order book instead. Assumption (iii) is also not an unreasonable assumption given that the majority of trades are executed at the inner quotes, since trading volume is often much smaller than the quoted market depth at the best level (see Table 4.1). If the previous transaction before the purchase of size V_{buy} illustrated in Figure 4.1 was a sale, it was certainly executed against the bid order book at a price less than the best ask price. If the last transaction was a purchase, it was very likely transacted either at $P_0+s/2$ (when either (a) the best ask prior to that transaction was also $P_0+s/2$ and the trading volume was less than the depth at $P_0+s/2$; or (b) the best ask prior to the transaction was less $P_0+s/2$ and the trading volume was less than the cumulative ask depths up to $P_0+s/2$), or at one tick lower (when the best ask prior to the transaction was one tick lower than $P_0+s/2$ and the trading volume was exactly equal to the ask depth). However, there is a possibility that the last transacted price might be larger than $P_0+s/2$, which happens if after the last transaction, there were submissions of limit sell orders that pushed the best ask back to $P_0+s/2$. Even in this case, the main idea from Figure 4.1 still holds in general.

Unlike the first three assumptions, assumption (iv) is quite strong and often unrealistic. It is imposed to facilitate the delivery of the main intuition, but it can be relaxed. So far, we have treated the trading volume V_{buy} of the purchase as known and given, but it should be a random variable whose value depends on an investor's liquidity needs and/or information and belief set. If $V_{\text{buy}} \leq \text{Ask Depth}_1 < \text{Ask Depth}_2$, the market buy order will be executed at the best ask $P_0+s/2$ under both scenarios, suggesting that the volatility of the trade will be the same for both situations. However, as $\text{Ask Depth}_2 > \text{Ask Depth}_1$, the probability of V_{buy} being larger than Ask Depth_1 is higher than the probability that V_{buy} is bigger than Ask Depth$_2$, all else being equal, which implies that the main idea from Figure 4.1 remains true in a probabilistic sense and so too on average. Similarly, assumption (v) is rather strong since large aggressive market orders are less often seen in empirical data (e.g. Griffiths et al., 2000, Ranaldo, 2004, Duong and Kalev, 2013). However, if the probability of such large aggressive orders is the same in both scenarios (i.e. for both ask order books 1 and 2), then the main intuition from Figure 4.1 still holds on average without assumption (v).

In summary, we present in Figure 4.1 as a graphical rationale for the negative dependence of the return volatility and the volume-volatility relation of a trade on the prevailing LOB slope right before the trade. This negative dependence is empirically found in this chapter and previous research (see, amongst others, Næs and Skjeltorp, 2006, Duong and Kalev, 2008), and it reaffirms the informativeness of the LOB information about the price formation process.

4.6 Conclusion

This research extends prior literature on the volume-volatility relation by highlighting the significant information content of the LOB about the return volatility and the volume-volatility relation of individual trades. While most existing studies in the literature assume a constant and fully contemporaneous volume-volatility relation, we find strong evidence that the positive dependence of return volatility on the trading volume of a trade is dynamic. In addition, the volume-volatility relation is positively correlated with the bid-ask spread but negatively related to the market depth at the best quotes and the LOB slope prior to the transaction. The dynamics of the LOB characteristics also play a significant role in explaining future return volatility. While their direct impact on volatility is primarily contributed by their lagged information, it is their current information right before a trade that drives the volume-volatility relation, which captures their indirect impact on volatility.

Consistent with the findings in Ahn et al. (2001), Engle and Patton (2004) and Harris and Panchapagesan (2005), there are significant asymmetries between the effects of the bid and ask order books on return volatility and on the volume-volatility relation, with the LOB of the opposite side to the direction of an incoming trade being particularly informative about the return volatility of the trade. The LOB slope plays the dominant role in explaining return volatility and the volume-volatility relation even when we incorporate the information from the bid-ask spread and the market depth at the best quotes. We justify our finding that return volatility and the volume-volatility relation are negatively associated with the LOB slope with a simple intuitive graphical illustration, which is compatible with prior explanations of the informativeness of the order book slope in the literature.

4.7 Appendix

Table 4.8: Definitions of variables

Notation	Description
$r_{i,t}$	Return of the t-th trade in stock i: $r_{i,t} = 100(\ln(q_{i,t+1}) - \ln(q_{i,t}))$, where $q_{i,t}$ is the midpoint of the bid and ask quotes right before the trade
$T_{i,t}$	Time duration (in seconds) between the $(t-1)$-th and t-th trades
$\sigma_{i,t}$	Volatility *per unit of time* of the t-th trade: $\sigma_{i,t} = \lvert\widehat{\epsilon}_{i,t}\rvert / T_{i,t}$, where $\widehat{\epsilon}_{i,t}$ is the residual from the model of returns specified in Equation (4.9)
$v_{i,t}$	Volume *per unit of time* of the t-th trade: $v_{i,t} = V_{i,t}/T_{i,t}$, where $V_{i,t}$ is the number of shares traded (times 1000) divided by the total number of shares outstanding right before the trade, and $T_{i,t}$ is defined above
$x_{i,t}$	A vector of potential predictors of the volume-volatility relation of the t-th trade
$y_{i,t}$	A vector of control variables that allows for the effects of the order flow prior to the t-th trade
$Spread_{i,t}$	Relative spread, defined as quoted spread as a % of the mid-quote right before the t-th trade
$BV_{i,t}$	Total number of shares available at the best *bid* price (times 1000) divided by the total number of shares outstanding right before the t-th trade
$AV_{i,t}$	Total number of shares available at the best *ask* price (times 1000) divided by the total number of shares outstanding right before the t-th trade
$Depth_{i,t}$	Total number of shares available at the best *bid and ask* prices (times 1000) divided by total number of shares outstanding right before the t-th trade: $Depth_{i,t} = BV_{i,t} + AV_{i,t}$
$Slpb_{i,t}$	Slope of the *bid* order book right before the t-th trade, defined in equation (4.6) and calculated using the 10 best bid/ask price levels right before the trade
$Slpa_{i,t}$	Slope of the *ask* order book right before the t-th trade, defined in equation (4.7) and calculated using the 10 best bid/ask price levels right before the trade
$Slope_{i,t}$	Slope of the limit order book right before the t-th trade: $Slope_{i,t} = (Slpb_{i,t} + Slpa_{i,t})/2$
$B_{i,t}$	Buy indicator: equals 1 if the t-th trade is a purchase, 0 otherwise
$S_{i,t}$	Sell indicator: equals 1 if the t-th trade is a sale, 0 otherwise
$N_{i,t}$	Number of transactions during the 5-minute interval right before the t-th trade
$ATS_{i,t}$	Average trade size (times 10^6 and divided by the total number of shares outstanding) during the 5-minute interval right before the t-th trade
$OIB_{i,t}$	Order imbalance (= number of buys - number of sells) during the 5-minute interval right before the t-th trade
$QTT_{i,t}$	Quote to trade ratio (= total number of order submissions, revisions and cancellations divided by number of trades) during the 5-minute interval right before the t-th trade
$Day_{k,i,t}$	Day-of-week dummy variables, $k = 1, \cdots, 5$ for Monday till Friday
$Monday_{i,t}$	Dummy variable for Monday (same as $Day_{1,i,t}$)
$block_{k,i,t}$	Time-of-day dummy variables, $k = 1, \cdots, 6$ for six hourly intervals: 10:10-11:00, 11:00-12:00, 12:00-13:00, 13:00-14:00, 14:00-15:00 and 15:00-16:00
$block1_{i,t}$	Dummy variable for the first trading hour (10:10-11:00) of a day (same as $block_{1,i,t}$)

Table 4.9: LOB and the *endogenous dynamic* volume-volatility relation: Combined LOB

		Large cap (49 stocks)			**Mid cap** (48 stocks)			**Small cap** (97 stocks)		
		Med	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$
Panel A: 5 best bid/ask price levels										
$v_{i,t}Spread_{i,t}$	Lag 0	-28.723	61.2%	14.3%	-24.056	66.7%	6.2%	-8.093	61.9%	3.1%
	$\sum_{1:p}$	6.057	18.4%	38.8%	-2.102	20.8%	16.7%	-1.323	11.3%	3.1%
	$\sum_{0:p}$	-23.129	51.0%	24.5%	-29.233	54.2%	8.3%	-10.510	50.5%	1.0%
$v_{i,t}Depth_{i,t}$	Lag 0	4.584	30.6%	42.9%	2.522	20.8%	35.4%	-0.617	32.0%	23.7%
	$\sum_{1:p}$	-8.132	28.6%	4.1%	-1.808	16.7%	8.3%	-0.641	14.4%	1.0%
	$\sum_{0:p}$	-2.972	36.7%	28.6%	-0.594	18.8%	27.1%	-0.607	32.0%	15.5%
$v_{i,t}Slope5_{i,t}$	Lag 0	-0.164	73.5%	18.4%	-0.747	91.7%	6.2%	-0.553	74.2%	5.2%
	$\sum_{1:p}$	0.000	26.5%	28.6%	-0.057	27.1%	14.6%	-0.102	21.6%	1.0%
	$\sum_{0:p}$	-0.155	65.3%	22.4%	-0.877	81.2%	6.2%	-0.598	73.2%	2.1%
$Spread_{i,t}$	Lag 0	-5.503	59.2%	10.2%	-2.795	47.9%	14.6%	-0.791	39.2%	19.6%
	$\sum_{1:p}$	6.498	22.4%	40.8%	-10.476	35.4%	16.7%	-7.859	38.1%	8.2%
	$\sum_{0:p}$	3.371	26.5%	36.7%	-15.003	43.8%	12.5%	-7.260	51.5%	6.2%
$Depth_{i,t}$	Lag 0	53.812	12.2%	83.7%	24.370	16.7%	60.4%	21.503	4.1%	67.0%
	$\sum_{1:p}$	-61.880	77.6%	10.2%	-23.904	52.1%	16.7%	-22.636	64.9%	4.1%
	$\sum_{0:p}$	-5.628	30.6%	12.2%	0.106	10.4%	22.9%	0.321	18.6%	15.5%
$Slope5_{i,t}$	Lag 0	0.005	34.7%	49.0%	0.004	35.4%	41.7%	0.148	16.5%	51.5%
	$\sum_{1:p}$	-0.035	44.9%	20.4%	-0.185	58.3%	2.1%	-0.490	76.3%	2.1%
	$\sum_{0:p}$	-0.033	42.9%	24.5%	-0.274	60.4%	10.4%	-0.477	75.3%	0.0%
Panel B: 20 best bid/ask price levels										
$v_{i,t}Spread_{i,t}$	Lag 0	-11.023	51.0%	16.3%	-2.512	31.2%	18.8%	-1.513	30.9%	13.4%
	$\sum_{1:p}$	10.137	10.2%	42.9%	0.722	12.5%	16.7%	0.151	6.2%	9.3%
	$\sum_{0:p}$	-4.340	32.7%	28.6%	-2.591	16.7%	20.8%	-1.027	21.6%	10.3%
$v_{i,t}Depth_{i,t}$	Lag 0	-6.680	40.8%	30.6%	-5.688	52.1%	12.5%	-3.294	53.6%	4.1%
	$\sum_{1:p}$	-10.074	38.8%	4.1%	-2.565	33.3%	4.2%	-1.100	21.6%	2.1%
	$\sum_{0:p}$	-17.178	44.9%	18.4%	-6.297	47.9%	8.3%	-4.809	54.6%	2.1%
$v_{i,t}Slope20_{i,t}$	Lag 0	-0.596	73.5%	18.4%	-1.112	81.2%	6.2%	-0.960	63.9%	4.1%
	$\sum_{1:p}$	0.028	14.3%	28.6%	-0.064	14.6%	12.5%	-0.075	16.5%	5.2%
	$\sum_{0:p}$	-0.527	65.3%	22.4%	-1.157	60.4%	10.4%	-1.081	55.7%	2.1%
$Spread_{i,t}$	Lag 0	-5.102	59.2%	8.2%	-1.950	43.8%	16.7%	0.890	26.8%	25.8%
	$\sum_{1:p}$	13.180	12.2%	44.9%	1.271	20.8%	20.8%	-1.556	22.7%	15.5%
	$\sum_{0:p}$	7.863	14.3%	38.8%	-1.634	27.1%	18.8%	-0.389	29.9%	16.5%
$Depth_{i,t}$	Lag 0	53.733	12.2%	83.7%	25.332	20.8%	60.4%	22.235	4.1%	66.0%
	$\sum_{1:p}$	-62.284	77.6%	8.2%	-29.216	56.2%	14.6%	-27.466	69.1%	4.1%
	$\sum_{0:p}$	-8.944	38.8%	4.1%	-2.518	33.3%	12.5%	-2.323	35.1%	3.1%
$Slope20_{i,t}$	Lag 0	0.021	34.7%	51.0%	0.024	31.2%	43.8%	0.747	7.2%	60.8%
	$\sum_{1:p}$	-0.087	32.7%	20.4%	-0.344	50.0%	8.3%	-1.294	66.0%	5.2%
	$\sum_{0:p}$	-0.064	32.7%	24.5%	-0.544	45.8%	10.4%	-0.592	50.5%	1.0%

This table reports the results of the following model, for the constituent stocks of the S&P/ASX200 index during *Jul-Dec 2014*,

$$\sigma_{i,t} = \alpha_0 + \mu_1 Monday_{i,t} + \mu_2 block1_{i,t} + \sum_{j=1}^{12} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{5} [\beta_{0,k} + \delta_k' x_{i,t-k}] v_{i,t-k} + \sum_{k=0}^{5} \gamma_k' x_{i,t-k} + \pi' y_{i,t} + \eta_{i,t},$$

where $\sigma_{i,t}$ is a proxy for return volatility *per unit of time* of the t-th transaction in stock i, which is estimated as the absolute value of the residual $|\widehat{\epsilon}_{i,t}|$ of the following autoregressive model of returns $r_{i,t}$ divided by the duration $T_{i,t}$ of the trade (i.e. $\sigma_{i,t} = |\widehat{\epsilon}_{i,t}|/T_{i,t}$):

$$r_{i,t} = \sum_{k=1}^{5} \psi_{i,k} Day_{k,i,t} + \sum_{k=1}^{5} \phi_{i,k} block_{k,i,t} + \sum_{k=1}^{12} \rho_{i,k} r_{i,t-k} + \varphi' x_{i,t} + \lambda' y_{i,t} + \epsilon_{i,t}.$$

$x_{i,t}$ is a vector of potential predictors of the volume-volatility relation. $y_{i,t} = (\ln(T_{i,t}), N_{i,t}, ATS_{i,t}, OIB_{i,t}, QTT_{i,t})'$ is a vector of control variables that allow for the effects of the order flow prior to a trade. Panel A reports the results when $x_{i,t} = (Spread_{i,t}, Depth_{i,t}, Slope5_{i,t})'$, and Panel B reports the results when $x_{i,t} = (Spread_{i,t}, Depth_{i,t}, Slope20_{i,t})'$, where $Slope5_{i,t}$ ($Slope20_{i,t}$) is the slope of the LOB, calculated using 5 (20) best bid and ask price levels, right before the t-th trade. See Table 4.8 and the notes of Table 4.2 for the definitions of other variables and notation. The table only reports the coefficient estimates for $x_{i,t-k}v_{i,t-k}$ and $x_{i,t-k}$ from the volatility equation.

Table 4.10: LOB and the *endogenous dynamic* volume-volatility relation: Bid vs. Ask sides

		Large cap (49 stocks)			**Mid cap** (48 stocks)			**Small cap** (97 stocks)		
		Med	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$
Panel A: 5 best bid/ask price levels										
$v_{i,t}Spread_{i,t}$	Lag 0	-99.187	83.7%	8.2%	-52.642	85.4%	6.2%	-17.464	76.3%	4.1%
	$\sum_{1:p}$	4.741	18.4%	30.6%	-0.555	16.7%	16.7%	-0.309	8.2%	6.2%
	$\sum_{0:p}$	-93.647	73.5%	16.3%	-53.336	79.2%	6.2%	-16.600	67.0%	2.1%
$v_{i,t}BV_{i,t}S_{i,t}$	Lag 0	65.116	16.3%	61.2%	44.539	8.3%	68.8%	5.124	13.4%	44.3%
	$\sum_{1:p}$	-17.980	30.6%	0.0%	-9.775	16.7%	4.2%	-4.124	29.9%	0.0%
	$\sum_{0:p}$	56.671	18.4%	51.0%	27.645	8.3%	58.3%	1.983	15.5%	27.8%
$v_{i,t}AV_{i,t}B_{i,t}$	Lag 0	80.376	20.4%	63.3%	38.416	6.2%	62.5%	5.807	12.4%	39.2%
	$\sum_{1:p}$	-14.712	20.4%	0.0%	-7.363	20.8%	6.2%	-2.851	14.4%	0.0%
	$\sum_{0:p}$	52.582	20.4%	46.9%	26.468	0.0%	47.9%	2.981	12.4%	26.8%
$v_{i,t}Slpb5_{i,t}S_{i,t}$	Lag 0	-0.407	85.7%	6.1%	-1.737	93.8%	6.2%	-1.392	91.8%	1.0%
	$\sum_{1:p}$	0.016	8.2%	42.9%	0.055	6.2%	20.8%	0.092	1.0%	21.6%
	$\sum_{0:p}$	-0.402	79.6%	16.3%	-1.699	87.5%	6.2%	-1.109	82.5%	1.0%
$v_{i,t}Slpa5_{i,t}B_{i,t}$	Lag 0	-0.456	87.8%	4.1%	-1.532	93.8%	4.2%	-1.306	92.8%	0.0%
	$\sum_{1:p}$	0.009	8.2%	34.7%	0.034	2.1%	29.2%	0.052	2.1%	16.5%
	$\sum_{0:p}$	-0.424	81.6%	12.2%	-1.375	91.7%	6.2%	-1.147	85.6%	1.0%
$Spread_{i,t}$	Lag 0	-6.064	69.4%	6.1%	-7.006	58.3%	12.5%	-4.241	48.5%	14.4%
	$\sum_{1:p}$	3.794	14.3%	36.7%	-1.139	25.0%	18.8%	-0.479	13.4%	14.4%
	$\sum_{0:p}$	-3.643	24.5%	32.7%	-12.190	41.7%	14.6%	-5.701	36.1%	7.2%
$BV_{i,t}S_{i,t}$	Lag 0	57.307	10.2%	79.6%	15.224	25.0%	56.2%	14.002	5.2%	54.6%
	$\sum_{1:p}$	-62.426	61.2%	4.1%	-21.458	39.6%	6.2%	-17.467	46.4%	4.1%
	$\sum_{0:p}$	-11.762	24.5%	0.0%	-0.359	8.3%	14.6%	-0.351	16.5%	10.3%
$AV_{i,t}B_{i,t}$	Lag 0	59.425	6.1%	83.7%	24.629	10.4%	60.4%	20.368	4.1%	60.8%
	$\sum_{1:p}$	-52.693	63.3%	0.0%	-17.531	45.8%	6.2%	-13.444	48.5%	2.1%
	$\sum_{0:p}$	-4.515	24.5%	4.1%	0.758	4.2%	12.5%	0.447	12.4%	14.4%
$Slpb5_{i,t}S_{i,t}$	Lag 0	-0.011	55.1%	26.5%	-0.060	52.1%	25.0%	-0.023	35.1%	24.7%
	$\sum_{1:p}$	0.009	12.2%	38.8%	-0.010	14.6%	10.4%	-0.068	23.7%	3.1%
	$\sum_{0:p}$	-0.004	20.4%	30.6%	-0.120	22.9%	6.2%	-0.098	28.9%	0.0%
$Slpa5_{i,t}B_{i,t}$	Lag 0	-0.013	55.1%	34.7%	-0.032	50.0%	27.1%	0.007	26.8%	25.8%
	$\sum_{1:p}$	-0.002	16.3%	26.5%	-0.015	20.8%	10.4%	-0.087	29.9%	1.0%
	$\sum_{0:p}$	-0.017	32.7%	22.4%	-0.114	31.2%	8.3%	-0.116	35.1%	2.1%
Panel B: 20 best bid/ask price levels										
$v_{i,t}Spread_{i,t}$	Lag 0	-70.844	77.6%	10.2%	-11.350	54.2%	10.4%	-6.832	49.5%	11.3%
	$\sum_{1:p}$	6.504	10.2%	32.7%	1.790	12.5%	14.6%	0.409	7.2%	10.3%
	$\sum_{0:p}$	-54.066	69.4%	18.4%	-10.366	43.8%	10.4%	-4.508	39.2%	7.2%
$v_{i,t}BV_{i,t}S_{i,t}$	Lag 0	39.038	22.4%	51.0%	-1.402	35.4%	29.2%	-7.100	52.6%	6.2%
	$\sum_{1:p}$	-18.042	32.7%	2.0%	-6.790	20.8%	4.2%	-3.085	21.6%	0.0%
	$\sum_{0:p}$	12.826	24.5%	40.8%	-5.186	35.4%	16.7%	-12.292	49.5%	3.1%
$v_{i,t}AV_{i,t}B_{i,t}$	Lag 0	43.981	26.5%	53.1%	0.258	37.5%	29.2%	-4.797	41.2%	10.3%
	$\sum_{1:p}$	-13.644	24.5%	2.0%	-7.017	25.0%	6.2%	-1.576	10.3%	2.1%
	$\sum_{0:p}$	35.467	26.5%	34.7%	0.133	31.2%	22.9%	-5.220	43.3%	7.2%
$v_{i,t}Slpb20_{i,t}S_{i,t}$	Lag 0	-1.588	85.7%	6.1%	-2.259	87.5%	6.2%	-1.871	84.5%	1.0%
	$\sum_{1:p}$	0.066	8.2%	44.9%	0.123	4.2%	18.8%	0.158	3.1%	13.4%
	$\sum_{0:p}$	-1.500	79.6%	16.3%	-2.421	79.2%	6.2%	-1.713	73.2%	1.0%
$v_{i,t}Slpa20_{i,t}B_{i,t}$	Lag 0	-1.580	87.8%	4.1%	-2.529	89.6%	6.2%	-2.210	82.5%	2.1%
	$\sum_{1:p}$	0.044	8.2%	34.7%	0.083	6.2%	16.7%	0.015	4.1%	8.2%
	$\sum_{0:p}$	-1.459	81.6%	12.2%	-2.457	83.3%	6.2%	-2.041	73.2%	2.1%

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Table 4.10 – *continued from previous page*

		Large cap (49 stocks)			**Mid cap** (48 stocks)			**Small cap** (97 stocks)		
		Med	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$
$Spread_{i,t}$	Lag 0	-5.730	65.3%	4.1%	-3.363	47.9%	12.5%	-1.834	40.2%	16.5%
	$\sum_{1:p}$	13.154	8.2%	42.9%	2.147	16.7%	20.8%	1.308	10.3%	19.6%
	$\sum_{0:p}$	2.361	20.4%	36.7%	0.135	27.1%	16.7%	-0.556	19.6%	19.6%
$BV_{i,t}S_{i,t}$	Lag 0	57.306	8.2%	79.6%	12.443	27.1%	56.2%	13.834	5.2%	53.6%
	$\sum_{1:p}$	-62.679	63.3%	4.1%	-24.822	45.8%	10.4%	-18.149	53.6%	2.1%
	$\sum_{0:p}$	-13.791	26.5%	0.0%	-1.664	16.7%	8.3%	-2.397	20.6%	6.2%
$AV_{i,t}B_{i,t}$	Lag 0	59.429	4.1%	83.7%	23.756	10.4%	58.3%	19.773	3.1%	61.9%
	$\sum_{1:p}$	-53.466	69.4%	2.0%	-16.437	43.8%	2.1%	-18.226	53.6%	3.1%
	$\sum_{0:p}$	-6.722	30.6%	6.1%	-0.679	10.4%	16.7%	1.156	13.4%	15.5%
$Slpb20_{i,t}S_{i,t}$	Lag 0	-0.039	53.1%	26.5%	-0.258	52.1%	22.9%	-0.098	32.0%	23.7%
	$\sum_{1:p}$	0.044	10.2%	36.7%	0.010	14.6%	14.6%	-0.115	23.7%	9.3%
	$\sum_{0:p}$	-0.003	20.4%	30.6%	-0.380	31.2%	4.2%	-0.271	35.1%	0.0%
$Slpa20_{i,t}B_{i,t}$	Lag 0	-0.052	53.1%	36.7%	-0.127	52.1%	27.1%	0.011	22.7%	28.9%
	$\sum_{1:p}$	0.030	12.2%	30.6%	-0.031	18.8%	4.2%	-0.462	44.3%	3.1%
	$\sum_{0:p}$	-0.023	28.6%	22.4%	-0.366	39.6%	4.2%	-0.505	51.5%	4.1%

This table reports the results of the following model, for the constituent stocks of the S&P/ASX200 index during *Jul-Dec 2014*,

$$\sigma_{i,t} = \alpha_0 + \mu_1 Monday_{i,t} + \mu_2 block1_{i,t} + \sum_{j=1}^{12} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{5} [\beta_{0,k} + \delta_k' x_{i,t-k}] v_{i,t-k} + \sum_{k=0}^{5} \gamma_k' x_{i,t-k} + \pi' y_{i,t} + \eta_{i,t},$$

where $\sigma_{i,t}$ is a proxy for return volatility *per unit of time* of the t-th transaction in stock i, which is estimated as the absolute value of the residual $|\widehat{\epsilon}_{i,t}|$ of the following autoregressive model of returns $r_{i,t}$ divided by the duration $T_{i,t}$ of the trade (i.e. $\sigma_{i,t} = |\widehat{\epsilon}_{i,t}|/T_{i,t}$):

$$r_{i,t} = \sum_{k=1}^{5} \psi_{i,k} Day_{k,i,t} + \sum_{k=1}^{5} \phi_{i,k} block_{k,i,t} + \sum_{k=1}^{12} \rho_{i,k} r_{i,t-k} + \varphi' x_{i,t} + \lambda' y_{i,t} + \epsilon_{i,t}.$$

$x_{i,t}$ is a vector of potential predictors of the volume-volatility relation. $y_{i,t} = (\ln(T_{i,t}), N_{i,t}, ATS_{i,t}, OIB_{i,t}, QTT_{i,t})'$ is a vector of control variables that allow for the effects of the order flow prior to a trade. Panel A reports the results when $x_{i,t} = (Spread_{i,t}, BV_{i,t}B_{i,t}, BV_{i,t}S_{i,t}, AV_{i,t}B_{i,t}, AV_{i,t}S_{i,t}, Slpb5_{i,t}B_{i,t}, Slpb5_{i,t}S_{i,t}, Slpa5_{i,t}B_{i,t}, Slpa5_{i,t}S_{i,t})'$; Panel B reports the results when $x_{i,t} = (Spread_{i,t}, BV_{i,t}B_{i,t}, BV_{i,t}S_{i,t}, AV_{i,t}B_{i,t}, AV_{i,t}S_{i,t}, Slpb20_{i,t}B_{i,t}, Slpb20_{i,t}S_{i,t}, Slpa20_{i,t}B_{i,t}, Slpa20_{i,t}S_{i,t})'$, where $Slpb5_{i,t}$ ($Slpa5_{i,t}$, $Slpb20_{i,t}$, $Slpa20_{i,t}$) is the slope of the bid (ask, bid, ask) side of the LOB, calculated using 5 (5, 20, 20) best bid (ask, bid, ask) price levels, right before the t-th trade. See Table 4.8 and the notes of Table 4.2 for the definitions of other variables and notation. Panel B of the table only reports the coefficient estimates for $x_{i,t-k}v_{i,t-k}$ and $x_{i,t-k}$ of the order book that are of the opposite side to the direction of a trade (e.g. $v_{i,t-k}Slpa_{i,t-k}B_{i,t-k}$ and $Slpa_{i,t-k}B_{i,t-k}$) from the volatility equation.

Table 4.11: LOB and the *endogenous dynamic* volume-volatility relation: $0.5^{th} - 99.5^{th}$ winsorization

		Large cap (49 stocks)			**Mid cap** (50 stocks)			**Small cap** (98 stocks)		
		Med	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$
Panel A: Combined limit order book										
$v_{i,t}Spread_{i,t}$	Lag 0	-10.180	46.9%	14.3%	-11.612	54.0%	8.0%	-4.296	38.8%	6.1%
	$\sum_{1:p}$	5.977	14.3%	38.8%	-1.483	18.0%	12.0%	-1.167	12.2%	5.1%
	$\sum_{0:p}$	-0.829	34.7%	22.4%	-11.986	42.0%	10.0%	-4.825	36.7%	2.0%
$v_{i,t}Depth_{i,t}$	Lag 0	-2.638	34.7%	28.6%	0.023	24.0%	26.0%	-1.518	36.7%	15.3%
	$\sum_{1:p}$	-5.677	30.6%	4.1%	-0.903	16.0%	8.0%	-0.462	15.3%	0.0%
	$\sum_{0:p}$	-4.775	36.7%	10.2%	-2.678	20.0%	18.0%	-1.592	35.7%	8.2%
$v_{i,t}Slope_{i,t}$	Lag 0	-0.245	67.3%	18.4%	-0.948	84.0%	6.0%	-0.715	69.4%	4.1%
	$\sum_{1:p}$	-0.005	24.5%	28.6%	-0.159	24.0%	8.0%	-0.169	19.4%	1.0%
	$\sum_{0:p}$	-0.267	59.2%	18.4%	-1.185	78.0%	8.0%	-0.883	62.2%	2.0%
$Spread_{i,t}$	Lag 0	-6.439	63.3%	8.2%	-2.256	42.0%	16.0%	-0.362	32.7%	19.4%
	$\sum_{1:p}$	8.843	24.5%	38.8%	-9.753	32.0%	16.0%	-6.381	37.8%	9.2%
	$\sum_{0:p}$	5.251	30.6%	36.7%	-13.720	46.0%	14.0%	-7.332	49.0%	9.2%
$Depth_{i,t}$	Lag 0	57.991	14.3%	83.7%	22.060	20.0%	58.0%	19.503	4.1%	63.3%
	$\sum_{1:p}$	-60.984	81.6%	12.2%	-16.504	54.0%	18.0%	-21.894	65.3%	6.1%
	$\sum_{0:p}$	-5.381	30.6%	10.2%	0.027	8.0%	24.0%	0.079	19.4%	10.2%
$Slope_{i,t}$	Lag 0	0.011	34.7%	53.1%	-0.027	34.0%	42.0%	0.397	15.3%	56.1%
	$\sum_{1:p}$	-0.089	49.0%	4.1%	-0.469	60.0%	2.0%	-1.143	78.6%	3.1%
	$\sum_{0:p}$	-0.062	40.8%	24.5%	-0.733	72.0%	6.0%	-0.938	79.6%	0.0%
Panel B: Bid vs. Ask sides										
$v_{i,t}Spread_{i,t}$	Lag 0	-68.566	81.6%	8.2%	-38.571	82.0%	6.0%	-13.449	70.4%	4.1%
	$\sum_{1:p}$	5.175	10.2%	30.6%	0.936	18.0%	12.0%	0.129	6.1%	5.1%
	$\sum_{0:p}$	-70.439	63.3%	16.3%	-40.211	76.0%	6.0%	-12.769	60.2%	1.0%
$v_{i,t}BV_{i,t}S_{i,t}$	Lag 0	49.395	18.4%	57.1%	26.027	12.0%	62.0%	1.476	19.4%	29.6%
	$\sum_{1:p}$	-14.230	28.6%	0.0%	-4.620	16.0%	4.0%	-3.272	25.5%	0.0%
	$\sum_{0:p}$	33.308	16.3%	40.8%	16.441	12.0%	46.0%	-0.570	20.4%	22.4%
$v_{i,t}AV_{i,t}B_{i,t}$	Lag 0	53.901	18.4%	63.3%	19.920	10.0%	58.0%	3.758	14.3%	30.6%
	$\sum_{1:p}$	-9.068	20.4%	2.0%	-2.448	20.0%	6.0%	-2.613	15.3%	1.0%
	$\sum_{0:p}$	44.352	22.4%	44.9%	11.282	8.0%	44.0%	0.642	19.4%	17.3%
$v_{i,t}Slpb_{i,t}S_{i,t}$	Lag 0	-0.680	85.7%	6.1%	-2.173	90.0%	8.0%	-1.967	83.7%	1.0%
	$\sum_{1:p}$	0.027	6.1%	40.8%	0.042	6.0%	24.0%	0.163	0.0%	22.4%
	$\sum_{0:p}$	-0.639	81.6%	14.3%	-1.976	82.0%	6.0%	-1.624	72.4%	1.0%
$v_{i,t}Slpa_{i,t}B_{i,t}$	Lag 0	-0.840	87.8%	4.1%	-2.460	90.0%	4.0%	-1.768	85.7%	0.0%
	$\sum_{1:p}$	0.018	10.2%	28.6%	0.025	2.0%	22.0%	0.155	1.0%	14.3%
	$\sum_{0:p}$	-0.771	81.6%	12.2%	-2.120	86.0%	6.0%	-1.561	75.5%	2.0%
$Spread_{i,t}$	Lag 0	-6.476	67.3%	2.0%	-9.595	66.0%	8.0%	-5.380	52.0%	10.2%
	$\sum_{1:p}$	1.888	8.2%	40.8%	2.691	14.0%	28.0%	1.769	13.3%	19.4%
	$\sum_{0:p}$	-3.832	26.5%	36.7%	-11.362	42.0%	14.0%	-5.559	38.8%	12.2%
$BV_{i,t}S_{i,t}$	Lag 0	56.916	12.2%	79.6%	11.378	24.0%	52.0%	13.560	7.1%	53.1%
	$\sum_{1:p}$	-63.428	65.3%	6.1%	-15.658	42.0%	12.0%	-14.754	46.9%	5.1%
	$\sum_{0:p}$	-11.321	24.5%	4.1%	3.366	12.0%	14.0%	0.029	15.3%	6.1%
$AV_{i,t}B_{i,t}$	Lag 0	59.774	6.1%	83.7%	22.384	12.0%	60.0%	19.624	4.1%	61.2%
	$\sum_{1:p}$	-50.959	73.5%	0.0%	-13.017	44.0%	8.0%	-17.898	57.1%	3.1%
	$\sum_{0:p}$	-4.421	20.4%	6.1%	4.850	8.0%	12.0%	0.243	14.3%	15.3%
$Slpb_{i,t}S_{i,t}$	Lag 0	-0.016	49.0%	32.7%	-0.056	42.0%	30.0%	0.028	25.5%	36.7%
	$\sum_{1:p}$	0.009	8.2%	32.7%	-0.081	20.0%	6.0%	-0.257	24.5%	2.0%
	$\sum_{0:p}$	-0.015	24.5%	30.6%	-0.292	30.0%	6.0%	-0.264	31.6%	0.0%
$Slpa_{i,t}B_{i,t}$	Lag 0	-0.018	51.0%	36.7%	-0.057	44.0%	24.0%	0.055	16.3%	37.8%
	$\sum_{1:p}$	-0.019	18.4%	18.4%	-0.072	34.0%	4.0%	-0.288	35.7%	2.0%
	$\sum_{0:p}$	-0.032	36.7%	22.4%	-0.250	36.0%	8.0%	-0.288	31.6%	3.1%

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Table 4.11 – *continued from previous page*

This table reports the results of the following model, for the constituent stocks of the S&P/ASX200 index during *Jul-Dec 2014*,

$$\sigma_{i,t} = \alpha_0 + \mu_1 Monday_{i,t} + \mu_2 block1_{i,t} + \sum_{j=1}^{12} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{5} [\beta_{0,k} + \delta_k' x_{i,t-k}] v_{i,t-k} + \sum_{k=0}^{5} \gamma_k' x_{i,t-k} + \pi' y_{i,t} + \eta_{i,t},$$

where $\sigma_{i,t}$ is a proxy for return volatility *per unit of time* of the t-th transaction in stock i, which is estimated as the absolute value of the residual $|\widehat{\epsilon}_{i,t}|$ of the following autoregressive model of returns $r_{i,t}$ divided by the duration $T_{i,t}$ of the trade (i.e. $\sigma_{i,t} = |\widehat{\epsilon}_{i,t}|/T_{i,t}$):

$$r_{i,t} = \sum_{k=1}^{5} \psi_{i,k} Day_{k,i,t} + \sum_{k=1}^{5} \phi_{i,k} block_{k,i,t} + \sum_{k=1}^{12} \rho_{i,k} r_{i,t-k} + \varphi' x_{i,t} + \lambda' y_{i,t} + \epsilon_{i,t}.$$

$x_{i,t}$ is a vector of potential predictors of the volume-volatility relation. $y_{i,t} = (\ln(T_{i,t}), N_{i,t}, ATS_{i,t}, OIB_{i,t}, QTT_{i,t})'$ is a vector of control variables that allow for the effects of the order flow prior to a trade. Panel A reports the results when $x_{i,t} = (Spread_{i,t}, Depth_{i,t}, Slope_{i,t})'$, and Panel B reports the results when $x_{i,t} = (Spread_{i,t}, BV_{i,t}B_{i,t}, BV_{i,t}S_{i,t}, AV_{i,t}B_{i,t}, AV_{i,t}S_{i,t}, Slpb_{i,t}B_{i,t}, Slpb_{i,t}S_{i,t}, Slpa_{i,t}B_{i,t}, Slpa_{i,t}S_{i,t})'$. See Table 4.8 and the notes of Table 4.2 for the definitions of the variables and other notation. The table only reports the coefficient estimates for $x_{i,t-k}v_{i,t-k}$ and $x_{i,t-k}$ of the order book that are of the opposite side to the direction of a trade (e.g. $v_{i,t-k}Slpa_{i,t-k}B_{i,t-k}$ and $Slpa_{i,t-k}B_{i,t-k}$) from the volatility equation.

Table 4.12: LOB and the *endogenous dynamic* volume-volatility relation: $2^{nd} - 98^{th}$ winsorization

		Large cap (48 stocks)			**Mid cap** (46 stocks)			**Small cap** (93 stocks)		
		Med	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$	**Med**	$\%_{-5\%}$	$\%_{+5\%}$
Panel A: Combined limit order book										
$v_{i,t}Spread_{i,t}$	Lag 0	-44.176	68.8%	16.7%	-35.335	87.0%	8.7%	-14.876	71.0%	2.2%
	$\sum_{1:p}$	18.447	10.4%	50.0%	0.696	13.0%	17.4%	-0.230	7.5%	7.5%
	$\sum_{0:p}$	-37.933	52.1%	25.0%	-33.733	67.4%	8.7%	-14.541	57.0%	1.1%
$v_{i,t}Depth_{i,t}$	Lag 0	6.591	29.2%	50.0%	6.578	23.9%	45.7%	-0.006	33.3%	25.8%
	$\sum_{1:p}$	-12.804	47.9%	0.0%	-3.194	28.3%	10.9%	-0.854	11.8%	1.1%
	$\sum_{0:p}$	-2.281	39.6%	29.2%	1.570	23.9%	26.1%	-0.854	31.2%	18.3%
$v_{i,t}Slope_{i,t}$	Lag 0	-0.625	79.2%	12.5%	-2.223	93.5%	6.5%	-1.402	86.0%	2.2%
	$\sum_{1:p}$	0.022	25.0%	47.9%	-0.035	17.4%	17.4%	-0.076	15.1%	6.5%
	$\sum_{0:p}$	-0.441	68.8%	20.8%	-2.492	87.0%	6.5%	-1.825	79.6%	1.1%
$Spread_{i,t}$	Lag 0	-5.461	66.7%	4.2%	-4.639	52.2%	8.7%	-1.428	41.9%	20.4%
	$\sum_{1:p}$	20.453	16.7%	50.0%	-7.943	34.8%	17.4%	-6.250	35.5%	8.6%
	$\sum_{0:p}$	14.913	25.0%	41.7%	-12.604	43.5%	17.4%	-6.849	41.9%	5.4%
$Depth_{i,t}$	Lag 0	53.248	10.4%	83.3%	25.476	21.7%	63.0%	22.611	3.2%	73.1%
	$\sum_{1:p}$	-68.405	79.2%	12.5%	-27.482	58.7%	13.0%	-24.085	68.8%	3.2%
	$\sum_{0:p}$	-12.011	45.8%	6.2%	-1.425	10.9%	21.7%	-0.061	16.1%	10.8%
$Slope_{i,t}$	Lag 0	0.008	35.4%	50.0%	-0.021	34.8%	41.3%	0.235	15.1%	53.8%
	$\sum_{1:p}$	0.009	29.2%	33.3%	-0.251	41.3%	6.5%	-0.886	67.7%	3.2%
	$\sum_{0:p}$	0.013	37.5%	37.5%	-0.523	52.2%	15.2%	-0.726	66.7%	1.1%
Panel B: Bid vs. Ask sides										
$v_{i,t}Spread_{i,t}$	Lag 0	-107.285	83.3%	12.5%	-64.169	91.3%	6.5%	-23.020	81.7%	2.2%
	$\sum_{1:p}$	5.688	12.5%	33.3%	-0.116	15.2%	17.4%	0.262	6.5%	7.5%
	$\sum_{0:p}$	-106.186	81.2%	16.7%	-58.882	84.8%	6.5%	-21.771	71.0%	1.1%
$v_{i,t}BV_{i,t}S_{i,t}$	Lag 0	88.714	18.8%	66.7%	70.362	10.9%	76.1%	12.739	9.7%	49.5%
	$\sum_{1:p}$	-25.104	37.5%	0.0%	-17.585	30.4%	2.2%	-6.438	29.0%	0.0%
	$\sum_{0:p}$	64.403	16.7%	56.2%	50.435	10.9%	60.9%	6.339	14.0%	30.1%
$v_{i,t}AV_{i,t}B_{i,t}$	Lag 0	113.426	20.8%	72.9%	63.300	6.5%	78.3%	14.764	9.7%	47.3%
	$\sum_{1:p}$	-16.346	29.2%	0.0%	-10.552	26.1%	6.5%	-5.335	19.4%	0.0%
	$\sum_{0:p}$	88.396	22.9%	62.5%	48.395	2.2%	63.0%	8.092	8.6%	24.7%
$v_{i,t}Slpb_{i,t}S_{i,t}$	Lag 0	-1.324	85.4%	8.3%	-4.290	93.5%	6.5%	-3.227	93.5%	1.1%
	$\sum_{1:p}$	0.075	4.2%	45.8%	0.292	2.2%	34.8%	0.273	0.0%	26.9%
	$\sum_{0:p}$	-1.175	79.2%	14.6%	-3.899	89.1%	6.5%	-2.806	87.1%	1.1%
$v_{i,t}Slpa_{i,t}B_{i,t}$	Lag 0	-1.397	87.5%	6.2%	-4.014	93.5%	6.5%	-3.435	96.8%	0.0%
	$\sum_{1:p}$	0.037	6.2%	43.8%	0.141	0.0%	37.0%	0.196	1.1%	21.5%
	$\sum_{0:p}$	-1.396	83.3%	14.6%	-3.724	93.5%	6.5%	-3.040	90.3%	1.1%
$Spread_{i,t}$	Lag 0	-3.899	52.1%	16.7%	-5.595	47.8%	13.0%	-2.237	44.1%	12.9%
	$\sum_{1:p}$	5.643	18.8%	39.6%	-0.145	17.4%	21.7%	-1.663	18.3%	17.2%
	$\sum_{0:p}$	-0.891	20.8%	35.4%	-10.563	28.3%	17.4%	-3.724	29.0%	6.5%
$BV_{i,t}S_{i,t}$	Lag 0	59.239	12.5%	77.1%	18.286	21.7%	60.9%	15.339	6.5%	55.9%
	$\sum_{1:p}$	-68.301	62.5%	2.1%	-28.137	43.5%	4.3%	-18.968	51.6%	3.2%
	$\sum_{0:p}$	-19.035	33.3%	0.0%	-6.996	8.7%	6.5%	-1.476	15.1%	8.6%
$AV_{i,t}B_{i,t}$	Lag 0	60.950	2.1%	89.6%	23.818	8.7%	65.2%	21.373	3.2%	65.6%
	$\sum_{1:p}$	-46.679	54.2%	0.0%	-21.144	47.8%	2.2%	-19.823	59.1%	2.2%
	$\sum_{0:p}$	-7.451	20.8%	4.2%	-0.043	6.5%	6.5%	-0.289	10.8%	6.5%
$Slpb_{i,t}S_{i,t}$	Lag 0	-0.043	72.9%	12.5%	-0.150	69.6%	17.4%	-0.157	49.5%	7.5%
	$\sum_{1:p}$	0.066	6.2%	45.8%	0.165	4.3%	13.0%	0.000	8.6%	5.4%
	$\sum_{0:p}$	0.022	14.6%	35.4%	-0.025	13.0%	8.7%	-0.159	22.6%	0.0%
$Slpa_{i,t}B_{i,t}$	Lag 0	-0.039	60.4%	27.1%	-0.208	69.6%	15.2%	-0.077	44.1%	14.0%
	$\sum_{1:p}$	0.026	10.4%	37.5%	0.036	4.3%	28.3%	-0.057	16.1%	7.5%
	$\sum_{0:p}$	-0.019	20.8%	31.2%	-0.047	23.9%	13.0%	-0.239	29.0%	2.2%

Continued on next page

Table 4.12 – *continued from previous page*

This table reports the results of the following model, for the constituent stocks of the S&P/ASX200 index during *Jul-Dec 2014*,

$$\sigma_{i,t} = \alpha_0 + \mu_1 Monday_{i,t} + \mu_2 block1_{i,t} + \sum_{j=1}^{12} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{5} [\beta_{0,k} + \delta_k' x_{i,t-k}] v_{i,t-k} + \sum_{k=0}^{5} \gamma_k' x_{i,t-k} + \pi' y_{i,t} + \eta_{i,t},$$

where $\sigma_{i,t}$ is a proxy for return volatility *per unit of time* of the t-th transaction in stock i, which is estimated as the absolute value of the residual $|\widehat{\epsilon}_{i,t}|$ of the following autoregressive model of returns $r_{i,t}$ divided by the duration $T_{i,t}$ of the trade (i.e. $\sigma_{i,t} = |\widehat{\epsilon}_{i,t}|/T_{i,t}$):

$$r_{i,t} = \sum_{k=1}^{5} \psi_{i,k} Day_{k,i,t} + \sum_{k=1}^{5} \phi_{i,k} block_{k,i,t} + \sum_{k=1}^{12} \rho_{i,k} r_{i,t-k} + \varphi' x_{i,t} + \lambda' y_{i,t} + \epsilon_{i,t}.$$

$x_{i,t}$ is a vector of potential predictors of the volume-volatility relation. $y_{i,t} = (\ln(T_{i,t}), N_{i,t}, ATS_{i,t}, OIB_{i,t}, QTT_{i,t})'$ is a vector of control variables that allow for the effects of the order flow prior to a trade. Panel A reports the results when $x_{i,t} = (Spread_{i,t}, Depth_{i,t}, Slope_{i,t})'$, and Panel B reports the results when $x_{i,t} = (Spread_{i,t}, BV_{i,t}B_{i,t}, BV_{i,t}S_{i,t}, AV_{i,t}B_{i,t}, AV_{i,t}S_{i,t}, Slpb_{i,t}B_{i,t}, Slpb_{i,t}S_{i,t}, Slpa_{i,t}B_{i,t}, Slpa_{i,t}S_{i,t})'$. See Table 4.8 and the notes of Table 4.2 for the definitions of the variables and other notation. Panel B of the table only reports the coefficient estimates for $x_{i,t-k}v_{i,t-k}$ and $x_{i,t-k}$ of the order book that are of the opposite side to the direction of a trade (e.g. $v_{i,t-k}Slpa_{i,t-k}B_{i,t-k}$ and $Slpa_{i,t-k}B_{i,t-k}$) from the volatility equation.

Chapter 5

Conclusions and future research

Research into market microstructure is important not only for investors to create optimal trading strategies that maximize profits and/or minimize trading costs, but also for policy makers to design trading platforms that have higher liquidity and less market friction and manipulation. Market microstructure theory delivers two important predictions about the behavior of security prices in financial markets which are: (1) there is a significant information content from trades that explains the movements in asset prices; and (2) price volatility is positively correlated with the trading activity of market participants. The main objective of this thesis is to develop new models and modeling methods for returns, volatility, trading volume and trade durations using high frequency tick-by-tick data that can be used to test these above predictions in market microstructure. This chapter summarizes the key findings from the three essays presented in this thesis, and it also discusses potential directions for future research.

5.1 Summary

The first essay, presented in Chapter 2, investigates the impact of trades on stock prices (i.e. the first empirical prediction) when the times of trade arrivals are endogenous. This study is motivated by the theoretical finding of Diamond and Verrecchia (1987) and Easley and O'Hara (1992) that the time durations between trades are informative about prices, which is empirically supported by the subsequent work of Dufour and Engle (2000). Unlike Dufour and Engle (2000) and many prior studies in the duration modeling literature that assume that trade durations are independent of (or strictly exogenous to) the dynamics of prices and other trade information such as signs and volumes, we explicitly allow trade durations to depend on these other variables, as suggested by theory. In particular, we model the joint evolution of trade durations, trade characteristics (signs and volumes) and returns with a nonlinear vector autoregression (VAR) that accommodates the feedback effects between these variables. The proposed model is employed to study how trades affect the prices of Australian banking stocks around the releases of monetary policy decisions. The empirical results in Chapter 2 confirm the endogeneity of trade durations, which are found to be positively related to the volatility but negatively affected by the volume of the previous transactions. In agreement with Dufour and Engle (2000), the shorter the time duration between trades, the more serially correlated trades become and the larger the impact they have on prices.

Chapter 2 also highlights the significant impact of interest rate announcements on the trade and price dynamics of major Australian banking stocks. Specifically, trading intensifies significantly and leads to a larger price impact within one minute around interest rate announcements. Conditioning on an average history prior to the releases of the announcements that signal monetary policy decisions, an unanticipated transaction in the banking stocks tend to have a higher (lower) cumulative impact on prices if the trade is submitted to the market earlier (later) than on average. However, this result is only observed when trade durations are endogenously modeled, and there are no differences in the cumulative price response of an unexpected trade to a duration shock if one assumes the strict exogeneity of trade durations.

Finally, based on a generalized forecast error variance decomposition (GFEVD) method developed in Lanne and Nyberg (2016), we find that shocks to trade durations only constitute a modest portion of the forecast error variance of returns of the Australian banking stocks, which is significantly smaller than the proportion explained by other trade attribute shocks. Even though the relative informativeness of trade durations about returns increases markedly either when durations are treated as endogenous or around the monetary policy announcements, trade durations seem to be less important in explaining price dynamics of these banking stocks than other trade attributes.

The second essay, presented in Chapter 3, investigates the second market microstructure empirical prediction about the positive volume-volatility relation by proposing a bivariate stochastic conditional framework to model the interrelationship between trading volume and return volatility. In this chapter, latent conditional expected volume and instantaneous volatility variables are assumed to follow a first order VAR that explicitly incorporates the dynamic feedback effects between these two variables. Our bivariate stochastic model generalizes the widely-used univariate stochastic volatility (SV) and stochastic conditional duration (SCD) models in the literature, and hence it offers higher fitting flexibility than do the multivariate GARCH-type or VAR-type models employed in a few previous studies. The key characteristic of the SV and SCD models is that they relax the conditional deterministic feature assumed by the GARCH- or VAR-type models. We establish analytical expressions for the moments and the autocorrelation functions of the volume and volatility processes in our model, which are all in conformance with those developed previously for the univariate models.

The proposed bivariate stochastic model is employed to study the volume-volatility relation of two Australian stocks. We use quasi maximum likelihood (QML) to estimate the bivariate model, and the validity of this method in our context is demonstrated by simulation. The empirical results show that there is a significant positive dynamic interrelationship between trading volume and return volatility, lending support to market microstructure theory (e.g. Copeland, 1976, Shalen, 1993). However, there is an asymmetry in the dynamic feedback effects between the two variables in that the influence of volume on volatility is much more pronounced. As expected, the incorporation of the joint determination of volume and volatility into our bivariate model significantly enhances the fitting performance of our model, in comparison with its two separate univariate counterparts. Finally, consistent with the findings of Holden and Subrahmanyam (1992) and Manganelli (2005), an initial positive perturbation to either trading volume or return volatility remarkably lifts both quantities to their new long run levels, with the larger and more frequently traded stock converging more rapidly to the full-information equilibrium in both transaction and calendar time.

The third essay, presented in Chapter 4, provides another examination of broad prediction about return volatility put forward by market microstructure theory by focusing on the role of the limit order book (LOB) characteristics in explaining the volume-volatility relation in an LOB market. Employing a high frequency tick-by-tick dataset of Australian stocks in the S&P/ASX200 index, we find that the effects of trading volume on return volatility are positive, dynamic and strongly related to the LOB information. In particular, the larger the market depth at the best quotes and/or the steeper the LOB (i.e. the larger the LOB slope) prior to a transaction, the weaker the volume-volatility relation becomes, and the smaller is the return volatility of the trade. On the other hand, a wider bid-ask spread immediately before a transaction not only strengthens the positive dependence of the return volatility on the trading volume of the trade, but also increases the return volatility of the trade.

There are two channels through which the LOB attributes affect the return volatility of an upcoming transaction: a direct channel that is mainly contributed by lagged LOB information, and an indirect channel that is captured by the volume-volatility relation and is dominantly driven by the most recent LOB information immediately before the trade. Consistent with a few previous studies such as Ahn et al. (2001), Engle and Patton (2004) and Harris and Panchapagesan (2005), the influence of the bid versus ask order books on return volatility and the dependence of volatility on volume is asymmetric. Specifically, the opposite side order book to the sign of an upcoming trade possesses more dominant predictive power about the return volatility and the volume-volatility relation of the trade. We also find that the LOB slope, which summarizes the information of the LOB at all quote levels, dominates both the bid-ask spread and the market depth at the inner quotes in explaining return volatility and the volume-volatility relation. In order to explain why a larger LOB slope reduces return volatility and weakens the volume-volatility relation, we provide an intuitive graphical rationale that is compatible with the justifications related to heterogeneous investors documented in some previous studies such as Næs and Skjeltorp (2006).

5.2 Directions for future research

This thesis presents three chapters in which new models for market microstructure variables such as high frequency returns, volatility, trading volume and trade durations are proposed in order to test the two important predictions implied by market microstructure theory, namely (1) there is a significant information content from trades that explains price movements; and (2) price volatility is positively correlated with the trading activity of market participants. There are several aspects of these papers that could be extended and worth exploring in future work.

First, the nonlinear VAR model of trade durations, trade attributes (signs and volumes) and returns developed in Chapter 2 is employed to investigate (i) the impact of trades on the prices of Australian banking stocks around interest rate announcements when trade arrival times are endogenous; and (ii) the effects of the interest rate announcements on the price formation and trade generation processes of the banking stocks. It will be of interest to extend the analysis in this chapter to study how various types of announcements, either market-wide (macroeconomic) or firm-specific (microeconomic) or both, affect the joint system of trading activities and prices of a wider range of stocks and different asset classes such as bonds and foreign exchanges. Previous studies have shown that news announcements have significant effects on the trade and price behaviors of various assets; however, they primarily examine how news releases affect trade and prices separately, rather than jointly within a system. For example, Andersen et al. (2003, 2007) find that the unexpected component of macro announcements leads to jumps in the conditional mean of exchange rates, whereas it only affects the exchange-rate conditional variance gradually. Similarly, macroeconomic news significantly affects the return and volatility of both equities (e.g. Boyd et al., 2005, Andersen et al., 2007, Berk and Rauch, 2016) and bonds (e.g. Balduzzi et al., 2001, Boyd et al., 2005, Andersen et al., 2007, Nowak et al., 2011). Meanwhile, Nowak and Anderson (2014) document that while macroeconomic news increases the trading frequency of US airline stocks, firm-specific announcements reduce it. A generalization of these studies to account for the joint determination of trade characteristics and prices while examining the impact of news announcements will help enrich our understanding of how information drives trade and price dynamics. It will also be useful to ascertain whether the findings in Chapter 2 for Australian banking stocks generalize in various empirical contexts.

Second, the bivariate stochastic conditional volume-volatility model developed in Chapter 3 is estimated by QML. Although QML estimators are consistent and asymptotically normal under usual regularity conditions and correct model specifications, they are not as efficient as the exact maximum likelihood estimators, and their asymptotic efficiency, relative to the latters', generally deteriorates with the difference between the true non-Gaussian density and its quasi Gaussian approximation (e.g. White, 1982, Hamilton, 1994). A better approximation of the true non-Gaussian distribution as discussed further below will help increase the asymptotic efficiency of the QML estimators. Even though the number of observations in the empirical application in Chapter 3 is sufficiently large to ensure that the estimated results are close to the true values, it is still worthwhile to employ a more effective method to estimate our bivariate stochastic model.

One possible method is the Monte Carlo maximum likelihood (MCML) technique proposed by Durbin and Koopman (1997), which has been applied to various contexts by subsequent studies (e.g. Sandmann and Koopman, 1998, Durbin and Koopman, 2000, Feng et al., 2004, Strickland et al., 2006). This method first approximates the true non-Gaussian density with a Gaussian distribution that has the first two derivatives of the log density equal to those of the true log density, and then corrects for the difference between the exact log likelihood function and the quasi one with a bias correction term computed by simulating from the approximating Gaussian density. However, this method (and other more sophisticated methods that better approximate the exact likelihood such as Kim et al. (1998)) cannot be performed unless the true non-Gaussian density is entirely known, which is generally not the case in our bivariate stochastic volume-volatility model. Nevertheless, we can assume that the true joint non-Gaussian density f_{w} of the measurement innovations $\mathrm{w}_t := (\mathrm{w}_{v,t}, \mathrm{w}_{e^2,t})' = (\log \varepsilon_t, \log \zeta_t^2)'$ in Equation (3.6) can be expressed by a copula of the following form, according to Sklar's (1959) theorem,

$$f_{\mathrm{w}}(\mathrm{w}_t) = f_{\mathrm{w}_v}(\mathrm{w}_{v,t}) f_{\mathrm{w}_{e^2}}(\mathrm{w}_{e^2,t}) c\left(F_{\mathrm{w}_v}(\mathrm{w}_{v,t}), F_{\mathrm{w}_{e^2}}(\mathrm{w}_{e^2,t})\right), \tag{5.1}$$

where f_{w_v} and $f_{\mathrm{w}_{e^2}}$ are respectively the marginal probability densities of $\mathrm{w}_{v,t}$ and $\mathrm{w}_{e^2,t}$ discussed in subsection 3.2.2, F_{w_v} and $F_{\mathrm{w}_{e^2}}$ are the corresponding cumulative density functions (cdf), and $c(\cdot)$ is a bivariate copula density that captures the dependence between $\mathrm{w}_{v,t}$ and $\mathrm{w}_{e^2,t}$. Various types of copula densities with different tail dependence structures can be considered. The use of copulas is increasingly popular in finance and financial econometric literatures (see, amongst others, Patton, 2006, Lee and Long, 2009, Brownlees and Engle, 2017, Oh and Patton, 2017, 2018). The MCML method can be implemented with the assumed copula representation (5.1). However, given the large dataset in our empirical application, it will be computationally challenging and time consuming to implement this method, since the Gaussian approximation and bias correction need to be calculated for each observation. The bivariate structure of our model also adds an extra layer of computational complexity to the implementation of the MCML method, which has primarily been applied to univariate situations. The treatment of the bivariate series of volume and volatility as two sequential univariate time series when running the Kalman filtering (and smoothing), which brings significant computational gains over traditional Kalman filtering (see Section 6.4 of Durbin and Koopman (2012)), may help.

Third, the stochastic conditional methodology in Chapter 3 can be easily extended to incorporate other random variables that have a non-negative support such as trade durations, bid-ask spreads, market depths, or high-low price range. These variables can be modeled with a multiplicative error model (MEM) proposed by Engle (2002) in a similar way to trading volumes in our study. The conditional expectations of these variables, together with the conditional expected volume and instantaneous volatility, will again be assumed to follow a first order VAR model that accommodates the dynamic interrelationships amongst the variables. The joint model is straightforwardly a multivariate version of equation (3.6) in Chapter 3, which can be easily and consistently estimated, assuming correct model specifications, by QML, even though the asymptotic efficiency of the QML estimates, relative to the exact maximum likelihood ones, may not be high because a Gaussian approximation of the true non-Gaussian density becomes less precise as the dimension of the approximation grows. Alternatively, the multivariate stochastic model can be more efficiently estimated by the MCML method discussed above, but at the price of significantly higher computational complexity.

The main analysis in Chapter 4 assumes that the volume-volatility relation (or the impact of trading volume on return volatility) depends linearly on LOB characteristics, denoted by $x_{i,t}$. However, one can relax this assumption and allow such dependence to be nonlinear and nonparametric, as mentioned in footnote 48 of Chapter 4. One way to do this could be based on the following dynamic endogenous volume-volatility equation

$$\sigma_{i,t} = \alpha_0 + \sum_{j=1}^{q} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{p} f_k(x_{i,t-k}) v_{i,t-k} + \sum_{k=0}^{p} \gamma_k' x_{i,t-k} + \eta_{i,t}, \tag{5.2}$$

where $f_k(\cdot)$ is an unknown function to be estimated that measures the marginal effect of trading volume $v_{i,t-k}$ on return volatility $\sigma_{i,t}$, which in turn captures the indirect effect of $x_{i,t-k}$ on $\sigma_{i,t}$. Equation (5.2), however, is too general and computationally challenging to estimate, especially when p is larger than 1 and $x_{i,t-k}$ is high dimensional. We could reduce the computational complexity by assuming that the effect of lagged trading volume $v_{i,t-k}$ on the current volatility is linearly related to $x_{i,t-k}$ for all $k \geq 1$ or 2, and only the current (and perhaps lagged 1) volume has a nonlinear impact on the current volatility that is nonparametrically dependent on $x_{i,t-k}$. Also to avoid the curse of dimensionality, if $x_{i,t-k}$ consists of more than one variable, we could assume that $f_k(x_{i,t-k})$ is additive and separable; that is, $f_k(x_{i,t-k}) = f_{k,1}(x_{i,1,t-k}) + \cdots + f_{k,M}(x_{i,M,t-k})$, where $x_{i,t-k} = (x_{i,1,t-k}, \cdots, x_{i,M,t-k})'$ is an $M \times 1$ vector. The resulting more parsimonious version of equation (5.2) is

$$\sigma_{i,t} = \alpha_0 + \sum_{j=1}^{q} \alpha_j \sigma_{i,t-j} + \sum_{k=0}^{\ell} f_k(x_{i,t-k}) v_{i,t-k} + \sum_{k=\ell}^{p} [\beta_{0,k} + \delta_k' x_{i,t-k}] v_{i,t-k} + \sum_{k=0}^{p} \gamma_k' x_{i,t-k} + \eta_{i,t}, \quad (5.3)$$

where $\ell = 0$ or 1. The semiparametric model (5.3) can be estimated using standard semi- and non-parametric techniques. An alternative method to allow for a nonlinear dependence of the volume-volatility relation on LOB information, which is also mentioned in footnote 48, is to replace $f_k(x_{i,t-k})$ in equation (5.2) with the parametric expression $\beta_{0,k} + \delta' z_{i,t-k}$, where $z_{i,t-k}$ contains $x_{i,t-k}$ and possibly its higher orders, nonlinear transformations, and/or interaction terms that allow for nonlinearities. The resulting model is fully parametric, and it can be consistently estimated by OLS.

Finally, similar to previous studies in the literature, Chapter 4 investigates the effects of the LOB information on the volume-volatility relation using a single-equation approach that precludes the interrelationships amongst the variables of interest. Even though the use of tick-by-tick data and the way in which these variables are computed in our study offer a natural remedy that saves us from the undetermined causality problem often faced by prior studies (see the discussion of this in subsection 4.2.4), it is important to accommodate the joint determination of return volatility, trading volume and LOB characteristics, as highlighted in market microstructure theory. This can be allowed for by employing a system of equations similar to the (nonlinear) VAR model proposed in Chapter 2 to model these variables, as noted in footnote 58. Such a VAR model will provide richer insights into how each of these variables evolves over time, as well as how it behaves if there is a shock to the system, regardless of whether the shock originates from the demand side (e.g. a shock to trading volume) or from the supply side (e.g. a shock to the LOB information such as market depth or the LOB slope). The impulse-response functions obtained from VAR models also offer ways to study market "resiliency", an important concept in finance which is defined as the speed with which prices converge back to their original level after a random, uninformative shock (Kyle, 1985). A subsequent study by Foucault et al. (2005) extends the definition of resiliency to other market dimensions such as bid-ask spreads. Market resiliency measures how quickly the market can recover from or be replenished by liquidity shocks, hence it is an important aspect of market liquidity and efficiency. The extension of the methodology in Chapter 4 to multivariate settings, which can be used to investigate market resiliency, is a worthwhile direction for future research.

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