Development and Validation of Fuzzy Multicriteria Decision Making Models

Yu-Liang Kuo

MBusSys, Monash University, Australia
BBusSys, Monash University, Australia

A Dissertation Submitted to Monash University in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

Faculty of Information Technology
Monash University
Australia

April 2013
Notice 1
Under the Copyright Act 1968, this thesis must be used only under the normal conditions of scholarly fair dealing. In particular no results or conclusions should be extracted from it, nor should it be copied or closely paraphrased in whole or in part without the written consent of the author. Proper written acknowledgement should be made for any assistance obtained from this thesis.
Declaration

I, Yu-Liang Kuo, hereby declare that this dissertation contains no material which has been accepted for the award of any other degree or diploma in any university or equivalent institution, and that, to the best of my knowledge and belief, this dissertation does not contain any material previously published or written by another person except where due reference is made in the text of the dissertation.

Yu-Liang Kuo                Date

12-Apr-2013

Faculty of Information Technology

Monash University
Acknowledgements

Working on the Ph.D. has been a challenging and often overwhelming experience. The completion of my thesis would not have been possible without the help and support of many key individuals. First of all, I am deeply grateful to my parents and siblings for the endless encouragements and enduring belief in my capabilities. Without their continued and unwavering support, my journey would not have been as smooth.

I want to thank Professor Chung-Hsing Yeh. To work with you and be guided by you throughout the years has been my privilege. You have supported me above and beyond an academic capacity and have been patient and encouraging in times of difficulties.

In addition, I would like to thank all my friends and colleagues for their encouragement and motivation. You have all been an invaluable support day in, day out, during all these years. This thesis would not have completed without your help.
Abstract

Fuzzy multicriteria decision making (MCDM) has been widely used in ranking a finite number of decision alternatives characterised by fuzzy assessments with respect to multiple evaluation criteria. The MCDM methods suitable for solving a given decision problem usually differ in their normalisation process and aggregation process for handling the performance ratings of the decision alternatives and the weights of the evaluation criteria. The overall preference of a decision alternative is obtained by aggregating the criteria weights and the performance ratings of the alternatives, on which the ranking is based. Due to their structural differences, these methods often produce inconsistent ranking results for the same fuzzy MCDM problem. To address this issue, this study develops a novel approach for the development and validation of fuzzy MCDM models. The approach incorporates three normalisation methods, three aggregation methods, and a $\alpha$-cut based defuzzification method to develop fuzzy MCDM models. The $\alpha$-cut based defuzzification method allows the decision maker’s attitude on fuzzy assessments to be incorporated into the decision making process. To examine the validity of the fuzzy MCDM models available for a given decision problem, a new validation process is developed based on the fuzzy clustering technique to assist in selecting a valid outcome from the inconsistent ranking results produced by these models. To demonstrate the effectiveness of the fuzzy MCDM model development and validation approach, three practical applications under various decision contexts are conducted.

The first application is about the airport performance evaluation problem. This study selects 12 Asia-Pacific major international airports as the decision alternatives of the evaluation problem and identifies 19 quantitative and qualitative evaluation criteria under the airport operator, passenger, and airline dimensions. Based on three normalisation methods and two aggregation methods, six fuzzy MCDM models are developed which produce
inconsistent ranking results for the evaluation problem. The ranking validity of the six models is examined by the validation process using fuzzy clustering and the most valid model is selected.

The second application is concerned with the scrap metal buyer selection problem. This study considers five recycling companies in southern China as the decision alternatives of the buyer selection problem and identifies four qualitative selection criteria under the economic and environmental dimensions. Based on three normalisation methods and three aggregation methods, seven fuzzy MCDM models are developed which produce inconsistent ranking results for the selection problem. The ranking validity of the seven models is examined by the validation process using fuzzy clustering and the most valid model is selected.

The third application deals with the non-ferrous scrap metal supplier selection problem. This study considers 15 scrap metal suppliers as the decision alternatives of the supplier selection problem and identifies five quantitative and qualitative selection criteria for a non-ferrous scrap metal buyer. Based on three normalisation methods and three aggregation methods, seven fuzzy MCDM models are developed which produce inconsistent ranking results for the selection problem. The ranking validity of the seven models is examined by the validation process using fuzzy clustering and the most valid model is selected.

With the development of the approach and the three empirical applications, this study makes significant methodological and practical contributions. The approach addresses the validity issue of the cardinal rankings generated by different fuzzy MCDM models. In practical applications, the subjective attitude of the decision maker is effectively incorporated into the decision making process. With its simplicity in both concept and computation, the approach has a general applicability for solving general MCDM problems, and is particularly suited to decision situations where the ranking results produced by different fuzzy MCDM models differ significantly.
# Table of Contents

Declaration.................................................................................................................................................. i

Acknowledgements ......................................................................................................................................... ii

Abstract........................................................................................................................................................... iii

Table of Contents ......................................................................................................................................... v

List of Publications ....................................................................................................................................... ix

List of Tables ................................................................................................................................................ x

List of Figures ................................................................................................................................................. xiii

Chapter 1 Introduction .................................................................................................................................. 1

1.1 Research Background .......................................................................................................................... 1

1.2 Research Objectives ............................................................................................................................. 3

1.3 Research Outline .................................................................................................................................. 5

Chapter 2 A Review of Multicriteria Decision Making and Fuzzy Sets in Decision Making .......................................................................................................................................................................................... 9

2.1 Developments in Multicriteria Decision Making .................................................................................. 9

2.2 Simple Additive Weighting (SAW) ....................................................................................................... 12

2.3 Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) ........................................... 13

2.4 Weighted Product (WP) ....................................................................................................................... 16

2.5 Fuzzy Sets in Decision Making .......................................................................................................... 17

2.5.1 Triangular fuzzy numbers .............................................................................................................. 21

2.5.2 Representation of linguistic terms .................................................................................................. 22

2.6 Fuzzy Clustering .................................................................................................................................. 24

2.7 Defuzzification .................................................................................................................................... 26
Chapter 3  Problem Formulation and Methodology Development

3.1 The Fuzzy Multicriteria Decision Making Problem

3.2 Development of Fuzzy MCDM Models

3.2.1 The normalisation process

3.2.2 The aggregation process

3.2.3 The defuzzification process

3.2.4 Development of fuzzy MCDM models

3.3 Ranking Validity of Fuzzy MCDM models

3.4 The Model Development and Validation Approach

3.5 Concluding Remarks

Chapter 4  Application I – Performance Evaluation of Asia-Pacific International Airports

4.1 Introduction

4.2 The Airport Performance Evaluation Problem

4.3 Airports Performance Assessment

4.4 Fuzzy MCDM Models

4.4.1 Model development

4.4.2 Performance ranking results

4.5 Ranking Validity of Fuzzy MCDM Models

4.6 Concluding Remarks
Chapter 5  Application II- Selection of Scrap Metal Buyers ................................. 67
  5.1 Introduction........................................................................................................... 67
  5.2 The Buyer Selection Problem ............................................................................. 68
  5.3 The Buyer Selection Assessment ..................................................................... 69
  5.4 Fuzzy MCDM Models ..................................................................................... 71
    5.4.1 Model development .................................................................................... 71
    5.4.2 Preference ranking results .......................................................................... 74
  5.5 Ranking Validity of Fuzzy MCDM Models ...................................................... 79
  5.6 Concluding Remarks ....................................................................................... 82

Chapter 6  Application III – Selection of Non-Ferrous Scrap Metal Suppliers ............ 83
  6.1 Introduction......................................................................................................... 83
  6.2 Non-Ferrous Scrap Metal Supplier Selection Problem .................................... 85
    6.2.1 The global scrap metal market .................................................................... 85
    6.2.2 The problem formulation ............................................................................ 87
    6.2.3 The supplier selection assessment ............................................................. 90
  6.3 Fuzzy MCDM Models ..................................................................................... 93
    6.3.1 Model development .................................................................................... 93
    6.3.2 Preference ranking results .......................................................................... 95
  6.4 Ranking Validity of Fuzzy MCDM Models ...................................................... 100
  6.5 Concluding Remarks ....................................................................................... 103

Chapter 7  Conclusion .......................................................................................... 105
  7.1 Summary of Research Developments ............................................................. 105
7.2 Research Contributions ........................................................................................................ 108

7.3 Future Research .................................................................................................................. 110

References .................................................................................................................................. 112

Appendix A – The Airport Information for Application I ................................................................. 133
Appendix B – Sensitivity analysis for Application I ..................................................................... 145
Appendix C – Sensitivity analysis for Application II .................................................................... 154
Appendix D – Sensitivity analysis for Application III ................................................................. 165
List of Publications

A. Refereed Journal Papers


B. Refereed Conference Papers


List of Tables

Table 1.1  Decision problem settings of the three applications ......................................5
Table 2.1  Linguistic terms represented by fuzzy numbers..................................................24
Table 3.1  14 fuzzy MCDM models and their corresponding reference code.........................38
Table 4.1  Evaluation criteria under three evaluation dimensions.......................................52
Table 4.2  Linguistic terms for fuzzy importance weighting assessment............................53
Table 4.3  Linguistic terms for fuzzy performance rating assessment..................................53
Table 4.4  Fuzzy weights $w_j$ for evaluation criteria $C_j$..................................................54
Table 4.5  Assessment data $x_{ij}$ for evaluation criteria $C_j$ under the airport operator
dimension.........................................................................................................................54
Table 4.6  Assessment data $x_{ij}$ for evaluation criteria $C_j$ under the passenger dimension....55
Table 4.7  Assessment data $x_{ij}$ for evaluation criteria $C_j$ under the airline dimension......55
Table 4.8  Fuzzy MCDM models and solution procedures..................................................56
Table 4.9  Performance rankings of 12 airports by the six fuzzy MCDM models.................61
Table 4.10 Cluster centers generated by fuzzy clustering...................................................62
Table 4.11 Membership degree and ranking order of 12 airports within the two
clusters..........................................................................................................................63
Table 4.12 Spearman’s correlation coefficients between fuzzy MCDM models and fuzzy
clustering..........................................................................................................................64
Table 5.1  Linguistic terms for fuzzy importance weighting assessment............................69
Table 5.2  Linguistic terms for fuzzy performance rating assessment ($C_1, C_2, C_4$).........70
Table 5.3  Linguistic terms for fuzzy performance rating assessment ($C_3$).......................70
Table 5.4  Assessment results for selection criteria and scrap metal buyers.......................70
Table 5.5  Fuzzy MCDM models and solution procedures..................................................71
Table 5.6  Preference ranking result of the D–N₁–S model (α = 0.5)…………………….75
Table 5.7  Preference ranking result of the D–N₂–S model (α = 0.5)…………………….76
Table 5.8  Preference ranking result of the D–N₃–S model (α = 0.5)…………………….76
Table 5.9  Preference ranking result of the D–N₁–T model (α = 0.5)…………………….77
Table 5.10 Preference ranking result of the D–N₂–T model (α = 0.5)…………………….77
Table 5.11 Preference ranking result of the D–N₃–T model (α = 0.5)…………………….78
Table 5.12 Preference ranking result of the D–W model (α = 0.5)……………………..78
Table 5.13 Preference rankings of seven models (α=0.5, λ = 0.5)……………………..79
Table 5.14 Membership degree and ranking order of five buyers to the two
clusters…………………………………………………………………80
Table 5.15 Spearman’s correlation coefficients between fuzzy MCDM models and fuzzy
clustering…………………………………………………………………80
Table 6.1  Energy and CO₂ saving by using secondary material…………………….85
Table 6.2  Selection criteria for non-ferrous scrap metal supplier selection……………90
Table 6.3  Linguistic terms for fuzzy importance weighting assessment…………………91
Table 6.4  Linguistic terms for fuzzy performance rating assessment (C₃)…………………91
Table 6.5  Linguistic terms for fuzzy performance rating assessment (C₄)…………………91
Table 6.6  Assessment result for weighting the selection criteria………………………92
Table 6.7  Assessment result for the performance ratings of suppliers…………………92
Table 6.8  Fuzzy MCDM models and solution procedures……………………………93
Table 6.9  Preference ranking result of the N₁–S–D model (α = 0.5)…………………..97
Table 6.10 Preference ranking result of the N₂–S–D model (α = 0.5)…………………..97
Table 6.11 Preference ranking result of the N₃–S–D model (α = 0.5)…………………..98
Table 6.12 Preference ranking result of the N₁–T–D model (α = 0.5)…………………..98
Table 6.13 Preference ranking result of the N₂–T–D model (α = 0.5)…………………..99
Table 6.14  Preference ranking result of the $N_3$–T–D model ($\alpha = 0.5$)…………………………99
Table 6.15  Preference ranking result of the W–D model ($\alpha = 0.5$)…………………………100
Table 6.16  Preference rankings of seven models ($\alpha=0.5, \lambda = 0.5$)…………………………101
Table 6.17  Membership degree and ranking order of 15 suppliers to the two clusters……102
Table 6.18  Spearman’s correlation coefficients between fuzzy MCDM models and fuzzy clustering……………………………………………………………………102
List of Figures

Figure 1.1. The research framework.................................................................6
Figure 2.1. A general multicriteria decision making structure.................................10
Figure 2.2. The Euclidean distance for TOPSIS....................................................14
Figure 2.3. Triangular fuzzy number \( \tilde{A} = (a_1, a_2, a_3) \)........................................21
Figure 2.4. An example of two fuzzy triangular numbers........................................22
Figure 2.5. Membership functions of linguistic terms.............................................24
Figure 3.1. The development and validation approach............................................43
Figure 4.1. A hierarchical structure of the airport performance evaluation problem.......50
Figure 4.2. Problem setting and model development.............................................57
Figure 4.3. Overall preference value and ranking under various decision settings using the
              D–N_1–T model.......................................................................................65
Figure 5.1. A hierarchical structure of the buyer selection problem..........................69
Figure 5.2. Problem setting and model development.............................................72
Figure 5.3. Overall preference value and ranking under various decision settings using the
              D–N_1–S model.......................................................................................81
Figure 5.4. Overall preference value and ranking under various decision settings using the
              D–N_3–S model.......................................................................................81
Figure 6.1. World’s scrap aluminium imports by volume...........................................86
Figure 6.2. A hierarchical structure of the supplier selection problem.......................88
Figure 6.3. Problem setting and model development.............................................94
Figure 6.4. Overall preference value and ranking under various decision settings using the
              W–D model..............................................................................................103
Figure 7.1. A comparison of the three applications conducted...............................106
Chapter 1

Introduction

1.1 Research Background

Decision making in the private and public sectors often involve the evaluation and/or selection of available courses of action in an environment characterised by (a) multiple, usually conflicting criteria with non-commensurable units, and (b) both crisp and fuzzy data derived from precise measures of quantitative criteria and imprecise judgements of qualitative criteria signified by human subjectivity. The complexity and generality of the problem have made it one of the most active, international, interdisciplinary fields or research (Dyer et al., 1992; Kasanen, et al., 2000; Zopounidis and Doumpos, 2002).

Multicriteria decision making (MCDM) has been widely used in ranking or selecting one or more alternatives from a finite number of decision alternatives with respect to multiple, usually conflicting criteria or attributes (Yoon and Hwang, 1995). Among its broad range of applications, MCDM has shown advantages in evaluating the performance of the resources and operations of higher education sectors in various decision contexts, with respect to conflicting performance measures or selection criteria (e.g. Blanchard et al., 1989; Davey et al., 1994; Mustafa and Goh, 1996; Saaty and Ramarujam, 1983). In these applications, MCDM provides a systematic means of assisting the decision makers in making more informed decisions about the comparative performance of the resources and operations.
Numerous MCDM models have been proposed for a large variety of selection and evaluation problems (e.g. Hwang and Yoon, 1981; Zeleny, 1982; Colson and de Bruyn, 1989; Dyer et al., 1992; Olson, 1996; Stewart, 1992; Yeh et al., 2000). Bellman and Zadeh (1970) first introduce fuzzy set theory as an effective methodology to deal with the inherent imprecision, vagueness and subjectiveness involved in the human decision making process. Numerous studies have since been conducted on the development of fuzzy MCDM models (Carlsson and Fuller, 1996; Hon et al., 1996; Triantaphyllou and Lin, 1996; Liang, 1999; Hanne, 2001) and their applications to various fuzzy MCDM decision problems (Hwang and Yoon, 1981; Chen and Hwang, 1992; Park, 1997; Yeh et al., 2000; Deng et al., 2000; Yeh and Kuo, 2003a). However, the large number of available fuzzy MCDM models may confuse the decision maker who is new to MCDM methodology, as selecting the right models for solving a particular problem has become another MCDM problem (Zanakis et al., 1998).

Existing studies on the selection of MCDM models have focused on the suitability of the methods when applying to certain MCDM problems, in terms of the characteristics of the method and of the problem (Guitouni and Martel, 1998; Qureshi et al., 1999; Beuthe and Scannella, 2001; Olson, 2001; Zopounidis and Doumpos, 2002). However, the validity of ranking results obtained by suitable fuzzy MCDM models is still an open issue (Yeh, 2002). If the ranking results by different methods are significantly different, the validity issue becomes crucial (Zanakis et al., 1998; Guitouni and Martel, 1998; Qureshi et al., 1999; Beuthe and Scannella, 2001; Yeh, 2002).

Fuzzy MCDM usually involves normalisation and aggregation processes. The normalisation process is required to transform the alternatives’ performance ratings measured by different units to a comparable scale, so that the inter-criteria comparisons can be made.
The overall utility or preference of an alternative is obtained by aggregating the alternative’s performance ratings with the corresponding criteria weights. However, different normalisation and aggregation processes often produce inconsistent ranking results for the same problem (Zanakis et al., 1998; Hanne, 2001). To help select from the ranking results produced by different fuzzy MCDM models, a validation procedure is needed.

Despite a wide variety of fuzzy MCDM models being developed, the development of validation procedures for fuzzy MCDM models remains a challenging research issue (Zanakis et al., 1998). To address this issue, this study proposes a validation process using fuzzy clustering as a basis. The proposed validation process examines the validity of MCDM models in order to assist in selecting a valid outcome for a given evaluation or selection problem. Fuzzy clustering is a technique for grouping alternatives by giving membership degrees to alternatives in each of the cluster instead of separate the alternatives into different clusters (Ruspini, 1969). Thus, all the alternatives can be ranked according to their membership degrees within the cluster. For example, the alternatives in best-performed cluster can be ranked from the highest to the lowest membership degrees, and vice versa to the worst-performed cluster.

1.2 Research Objectives

The objectives of this study are to (a) develop suitable fuzzy MCDM models for solving a given evaluation or selection problem under specific decision contexts, and (b) to establish a context-dependent validation process for validating different decision outcomes produced by different MCDM models. To achieve the research objectives, the following research issues are to be addressed:
(a) How to construct multicriteria decision analysis models for a given MCDM problem?

(b) How to identify appropriate evaluation or selection criteria for a given decision problem?

(c) How to develop suitable fuzzy MCDM models using different normalisation, aggregation, and defuzzification methods for a given decision problem?

(d) How to select the most valid model among suitable fuzzy MCDM models?

To address these research issues, this study develops a new, structured approach for the development and validation of fuzzy MCDM models and applies the approach to three applications under various decision problem settings. These three applications are (a) a performance evaluation problem for Asia-Pacific international airports, (b) a buyer selection problem for scrap metal buyers, and (c) a supplier selection problem for non-ferrous scrap metal suppliers. These three applications are conducted because they represent different decision contexts which can be used to best illustrate how the approach works and demonstrate the merits of the fuzzy MCDM models and of the validation procedure developed. Table 1.1 shows the decision problem settings of the three applications.
Table 1.1
Decision problem settings of the three applications

<table>
<thead>
<tr>
<th>Problem setting</th>
<th>Application I:</th>
<th>Application II:</th>
<th>Application III:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Airport performance evaluation</td>
<td>Buyer selection</td>
<td>Supplier selection</td>
</tr>
<tr>
<td>Decision maker</td>
<td>Unknown</td>
<td>Known</td>
<td>Known</td>
</tr>
<tr>
<td>Alternative numbers</td>
<td>12</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>Criteria numbers</td>
<td>19</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Criteria hierarchy</td>
<td>Three-level</td>
<td>Three-level</td>
<td>Two-level</td>
</tr>
<tr>
<td>Evaluation dimensions</td>
<td>Three</td>
<td>Two</td>
<td>One</td>
</tr>
<tr>
<td>Assessment data</td>
<td>Quantitative and qualitative</td>
<td>Qualitative</td>
<td>Quantitative and qualitative</td>
</tr>
<tr>
<td>Linguistic term sets</td>
<td>Two</td>
<td>Three</td>
<td>Three</td>
</tr>
<tr>
<td>Decision frequency</td>
<td>Annually</td>
<td>Daily, Regularly</td>
<td>Monthly, Periodically</td>
</tr>
<tr>
<td>Subject making assessment</td>
<td>Travel experts</td>
<td>Senior management of the case company</td>
<td>Senior management of the case company</td>
</tr>
</tbody>
</table>

1.3 Research Outline

Figure 1.1 shows the research framework of this study, together with the chapters of this thesis.
Chapter 1 states the theme of this study. The research objectives and associated research issues are identified and discussed. The methodological development of this study for addressing the research issues is outlined.
Chapter 2 reviews the MCDM methodology and its developments, together with fuzzy sets in decision making. Three MCDM methods are discussed: (a) simple additive weighting method (SAW), (b) technique for order preference by similarity to the ideal solution (TOPSIS), and (c) weighted product (WP). The concept of fuzzy set theory, together with the triangular fuzzy numbers and the representation of linguistic terms are discussed. Fuzzy clustering techniques and defuzzification methods are reviewed to establish the theoretical foundation for the methodological development of this study.

Chapter 3 formulates a general fuzzy MCDM problem and develops a number of suitable fuzzy MCDM models for solving the problem. The fuzzy MCDM models differ in their solution procedure involving a normalisation process, an aggregation process, and a defuzzification process. By applying three normalisation methods, three aggregation methods, and one defuzzification method in different combinations or sequences, different decision outcomes are usually generated. To select the most valid model, a validation process is developed based on fuzzy clustering.

Chapter 4 applies the fuzzy MCDM model development and validation approach developed in Chapter 3 to solve an airport performance evaluation problem involving 12 Asia-Pacific international airports. A set of 19 evaluation criteria under three evaluation dimensions in association with the airport operators, passengers, and airlines are identified. Six fuzzy MCDM models are developed for the performance evaluation of the 12 airports and their evaluation outcomes are examined by the validation process.

Chapter 5 applies the fuzzy MCDM model development and validation approach developed in Chapter 3 to solve a buyer selection problem involving five scrap metal buyers.
A set of four selection criteria under the economic and environmental dimensions are identified. Seven fuzzy MCDM models are developed for the preference evaluation of the buyers and their selection outcomes are examined by the validation process.

Chapter 6 applies the fuzzy MCDM model development and validation approach developed in Chapter 3 to solve a supplier selection problem involving 15 scrap metal suppliers. A set of five key selection criteria are identified and seven fuzzy MCDM models are developed for the performance evaluation of the suppliers. The selection outcomes produced by these models are examined by the validation process.

Chapter 7 concludes with a summary of the research work carried out in this study. The methodological and practical contributions achieved by this study are discussed. Limitations of this study and suggestions for future study are discussed.
Chapter 2

A Review of Multicriteria Decision Making and Fuzzy Sets in Decision Making

2.1 Developments in Multicriteria Decision Making

Multicriteria decision making (MCDM) has been widely used in evaluating, selecting or ranking a finite set of decision alternatives characterised by multiple and usually conflicting criteria (Hwang and Yoon, 1981). The MCDM research has been developed to a very large extent over the past few decades. The technical methodologies assist with the decision problems that involve trades offs and conflicts but, while the problems may be solved easily, the outcomes cannot be guaranteed.

Figure 2.1 shows a general structure of the general MCDM problem which consists of a finite number of decision alternatives, multiple non-commensurable and conflicting criteria for each decision problem setting, and different units of measurement among the criteria. There are three steps for numerical analysis of the decision alternatives in using any decision-making method (Triantaphyllou and Mann, 1989). These steps are (a) determining the relevant criteria and decision alternatives, (b) attaching numerical measures to the relative importance to the criteria and the impact of the decision alternatives on these criteria, and (c) processing the numerical values to determine a ranking of each decision alternative.
The cores of the decision making process are evaluation and choice. Selecting among alternatives is made difficult by two factors: uncertainty and constraints upon the information processing capacity (MacCrimmon, 1968). It has become obvious that comparing different methods as to their desirability and the suitability of a decision problem, using a single criterion or a single objective function, can in many cases, not achieve “optimal” solutions to the decision problem. The MCDM research has led to numerous evaluation schemes (for example, in the areas of cost benefit analysis and marketing) and to the formulation of vector-maximum problems in mathematical programming.

Two major areas have evolved to concentrate on decision-making with multiple criteria, which are Multiple Objective Decision-Making (MODM) and Multiple Criteria Decision-Making (MCDM). The main difference between them is that MODM concentrates on continuous decision spaces, primarily on mathematical programming with several objective functions, while MCDM focuses on problems with discrete decision spaces (Wallenius et al., 2008).
In practical decision settings, the human decision making process may contain imprecision, vagueness and subjectiveness inherent in the information. The presence of fuzziness or imprecision in an MCDM problem increases the complexity of the decision situation. The fuzzy set theory, initially proposed by Zadeh (1965), was consequently implemented into the MCDM field by Bellman and Zadeh (1970) to deal with the problems that could not be solved with conventional MCDM techniques. Since then, fuzzy MCDM has been further developed and numerous models have been proposed to solve fuzzy MCDM problems. A review and comparison of many of these fuzzy MCDM models can be found in Chen and Huang (1992), Carlsson and Fuller (1996), Ribeiro (1996), and Triantaphyllou and Lin (1996). The diffusion of the fuzzy set theory into both the MCDM and MODM methods has been reviewed by Kahraman (2008).

Numerous MCDM methods have been developed for solving various types of decision making problems, including widely used compensatory methods such as the simple additive weighting (SAW) method (MacCrimmon, 1968), the technique for order preference by similarity to ideal solution (TOPSIS) (Hwang and Yoon, 1981), and the weighted product (WP) method (Yoon, 1989). These MCDM methods are based on the multiattribute utility theory or multiattribute value theory (MAVT) (Dyer and Sarin, 1979; Keeney and Raiffa, 1993), which is probably the most widely used theory in solving MCDM problems. With simplicity in both concept and computation, MAVT-based MCDM methods are intuitively appealing to the decision makers in practical applications. These methods are particularly suited to decision problems where a cardinal preference or ranking of the decision alternatives is required. In addition, these methods are the most appropriate quantitative tools for group decision support systems (Bose et al., 1997; Matsatsinis and Samaras, 2001).
2.2 Simple Additive Weighting (SAW)

The Simple Additive Weighting (SAW) is probably the best-known and widely used MCDM method (Hwang and Yoon, 1980). Due to its simplicity and effectiveness, the SAW method has often been applied in solving selection problems (Kabassi and Virvou, 2004; Sener et al., 2006; Afshari et al., 2010; Savitha and Chandrasekar, 2011; Xu and Yeh, 2012). The basic logic of the SAW method is to obtain a weighted sum of the performance ratings of each alternative over all criteria. Since two items with different measurement units cannot be added, a normalisation procedure is required to permit addition among criteria values. The decision maker often assigns weights to the corresponding criteria to reflect the relative importance of the criteria. The total value for each alternative is then computed by multiplying its comparable rating with respect to each criterion by the corresponding criterion weight and then summing these products over all the criteria. The procedure of SAW can be summarised as follows:

Step 1: Construct the normalised decision matrix. This step is to convert the various criterion dimensions into non-dimensional criteria. The normalised rating \( r_{ij} \) of alternative \( A_i \) (\( i = 1, 2, \ldots, m \)) with respect to the criterion \( C_j \) (\( j = 1, 2, \ldots, n \)) is calculated by:

\[
   r_{ij} = \begin{cases} 
   \frac{x_{ij}}{\sqrt{\sum_{m=1}^{m} x_{ij}^2}}, & \text{if } j \text{ is a benefit criterion} \\
   1 - \frac{x_{ij}}{\sqrt{\sum_{m=1}^{m} x_{ij}^2}}, & \text{if } j \text{ is a benefit criterion}
   \end{cases} \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n. \quad (2.1)
\]

Step 2: Evaluate the total value \( V_i \) of each alternative \( A_i \) by using the value function as:

\[
   V_i = \sum_{j=1}^{n} r_{ij} w_j \quad i = 1, 2, \ldots, m \quad (2.2)
\]
where

\[ V_i = \text{the total value of alternative } A_i. \]

\[ r_{ij} \ (0 < r_{ij} < 1) = \text{the normalised performance rating of alternative } A_i \text{ on criterion } C_j. \]

\[ w_j = \text{the assigned weight of criterion } C_j. \]

In the SAW method, the criteria are preferentially independent. The contribution of each individual criterion to the total value is independent of other criteria values (Yoon and Hwang 1995), and the alternative with the highest total value is the preferred alternative.

### 2.3 Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

The Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS) is first introduced by Hwang and Yoon (1981) for solving MCDM problems. The basic principle of TOPSIS is the chosen alternative should have the shortest distance from the positive ideal solution and the longest from the negative ideal solution. Figure 2.2 shows the concept of the Euclidean distance for TOPSIS. TOPSIS considers the distances to both the positive ideal solution and negative ideal solution simultaneously by defining the relative closeness to the ideal solutions. TOPSIS then evaluates the relative closeness of the alternatives derived by comparing the relative distances. An assumption of TOPSIS is that each criterion has a tendency toward monotonically increasing or decreasing utility (Triantaphyllou and Lin, 1995).
The procedure of TOPSIS can be summarised as follows:

Step 1: Construct the normalised decision matrix. This step converts the various criterion dimensions into non-dimensional criteria, which allows comparisons across criteria. An element, $r_{ij}$, of the normalised decision matrix $R$, is calculated as

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{m=1}^{m} x_{ij}^2}} , \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \quad (2.3)$$

Step 2: Construct the weighted normalised decision matrix. A set of weights $W = (w_1, w_2, \ldots, w_n)$, specified by the decision maker, is used, in conjunction with the previous normalised decision matrix, to determine the weighted normalised matrix $V$ defined as:

$$V = (r_{ij}w_j) , \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \quad (2.4)$$

Step 3: Determine the positive ideal ($A^*$) and negative ideal ($A^-$) solutions defined as:
\( A^* = \left\{ \left( \max v_{ij} \mid j \in J \right), \left( \min v_{ij} \mid j \in J' \right) \text{ for } i = 1, 2, \ldots, m \right\} = \{ v_1^*, v_2^*, v_3^*, \ldots, v_n^* \}, \) (2.5)

\( A^- = \left\{ \left( \min v_{ij} \mid j \in J \right), \left( \max v_{ij} \mid j \in J' \right) \text{ for } i = 1, 2, \ldots, m \right\} = \{ v_1^-, v_2^-, v_3^-, \ldots, v_n^- \}, \) (2.6)

where

\( J = \{ j = 1, 2, \ldots, n \mid j \text{ associated with the benefit criteria} \} \)

\( J' = \{ j = 1, 2, \ldots, n \mid j \text{ associated with the cost criteria} \} \)

For benefit criteria (\( J \)), the decision maker desires a maximum value among the alternatives, for cost criteria (\( J' \)), the decision maker desires a minimum value among the alternatives.

Step 4: Calculate the separation measure. The concept of the \( n \)-dimensional Euclidean distance is used to measure the separation distances of each alternative to the positive ideal solution and the negative ideal solution, respectively, as:

\[
S_i^* = \sqrt{\sum_{j=1}^{n} \left( v_{ij} - v_j^* \right)^2}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n. \] (2.7)

where \( S_i^* \) is the separation (the Euclidean distance) of alternative \( A_i \) from the positive ideal solution, and

\[
S_i^- = \sqrt{\sum_{j=1}^{n} \left( v_{ij} - v_j^- \right)^2}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n. \] (2.8)

where \( S_i^- \) is the separation (the Euclidean distance) of alternative \( A_i \) from the negative ideal solution.
Step 5: Calculate the relative closeness to the ideal solution. The relative closeness of alternative $A_i$, with respect to the positive ideal solution $A^*$, is defined as:

$$C_i^* = \frac{S_i^-}{S_i^* + S_i^-}, \quad 0 \leq C_i^* \leq 1, \quad i = 1, 2, \ldots, m. \quad (2.9)$$

Step 6: Rank the preference order. This step decides the most satisfactory alternative according to a preference rank order of $C_i^*$, the shortest distance to the positive ideal solution. This alternative is guaranteed to have the longest distance to the negative ideal solution.

The concept of TOPSIS has been widely used in various MCDM models for solving practical decision problems, such as in service performance evaluation (e.g. Feng and Wang, 2000; Kuo et al., 2007; Lee and Lin, 2011; Tseng, 2011; Buyukozkan and Cifci, 2012), supplier performance evaluation (e.g. Awasthi et al., 2011a; 2011b; 2011c), and supplier selection (e.g. Dalalah et al., 2011). Behzadian et al. (2012) provide a state-of-the-art literature survey on TOPSIS applications and methodologies, where 266 papers published in 103 scholarly journals since 2000 have been categorised into nine areas. TOPSIS has received much interest from researchers and practitioners mainly due to (a) its simplicity and comprehensibility in concept, (b) its computational efficiency, and (c) its ability to measure the relative performance of the decision alternatives in a simple mathematical form.

### 2.4 Weighted Product (WP)

The Weighted Product (WP) method was introduced by Bridgeman (1922). The WP method is similar to the SAW method, while the main difference is that the different
measurement units of criteria do not have to be normalised into a dimensionless scale. Each alternative is compared with the others by multiplying a number of ratios, one for each criterion. Each ratio is raised to the power equivalent to the relative weight of the corresponding criterion (a positive power for benefit criteria and a negative power for cost criteria) (Yoon and Hwang, 1995).

The procedure of WP can be summarised as follows:

Step 1: The total value of alternative $A_i$ is given by

$$V_i = \prod_{j=1}^{n} (r_{ij})^{w_j}$$

(2.10)

Step 2: To compare the alternatives $A_k$ and $A_l$, the following product is obtained by

$$R\left(\frac{A_k}{A_l}\right) = \prod_{j=1}^{n} \left(\frac{r_{kj}}{r_{lj}}\right)^{w_j}$$

(2.11)

In the maximisation case, if $R\left(\frac{A_k}{A_l}\right)$ is greater than one, then alternative $A_k$ is more desirable than alternative $A_l$. The best alternative is the one that is better than or at least equal to the other alternatives (Triantaphyllou and Lin, 1996).

2.5 Fuzzy Sets in Decision Making

Fuzzy set theory, first introduced by Zadeh (1965), has been applied to many disciplines in a variety of ways including operations research, management science, control engineering, artificial intelligence, expert systems, decision theory, robotics and pattern recognition (Chen and Hwang, 1991; Bandermer and Gottwald, 1995; Klir and Yuan, 1995). Fuzzy sets are generalised sets in which each element is in the set to some degree, as opposed to classic or crisp sets in which each element is either completely in or completely out (Kosko, 1994).
In crisp set theory, anything involved either in the set or not, can be represented by a binary membership function $\mu_A$ (characteristic function of $x$ in $A$). The membership function maps elements of $X$ to elements of the set $\{0, 1\}$, expressed by $\mu_A: X \rightarrow \{0,1\}$, where $\{0,1\}$ is the set values 0 and 1, define as:

$$\mu_A(x) = \begin{cases} 
1; & \text{if } x \in A \\
0; & \text{if } x \notin A 
\end{cases} \quad (2.12)$$

A crisp set only deals with the two-value logic 1 or 0. If $x$ is an element of $A$, then the membership function of $x$ is 1. Otherwise, the membership function of $x$ is 0. The crisp set, which assigns a value of either 1 or 0 to each individual in a universal set, results in discrimination between members and non-members of the crisp set under consideration.

In fuzzy set theory, a fuzzy set is described by a membership function $\mu_A(x)$ of $A$. The membership function maps the element $x$ of a universal set $X$ into real numbers in $[0, 1]$, expressed by $\mu_A: X \rightarrow [0,1]$. A fuzzy set is a class of objects with a continuum of membership grades. The membership function is assigned to each element with a grade of membership associated with that fuzzy set. Therefore, fuzzy elucidation of a data structure is a very natural and intuitively reasonable way to formulate and solve real world problems.

In fuzzy MCDM problems, the membership function may be represented by (Abd El-Wahed, 2008):

$$\mu_A(Z^A(x)) = \begin{cases} 
1, & Z^A(x) \leq L_A \\
\frac{u_k - Z^A(x)}{u_k - l_k}, & L_k < Z^A(x) < U_A \\
0, & Z^A(x) \leq U_k
\end{cases} \quad (2.13)$$
where

$$U_A = \max_{x \in X} Z^A(x) = \text{is the upper bound of the membership function.}$$

$$L_A = \min_{x \in X} Z^A(x) = \text{is the lower bound of the membership function.}$$

For example, for the temperature over 30 degrees Celsius, we consider this to be hot weather; while for the temperature less than 15 degrees Celsius, we consider this to be cold weather. However, does this mean that a temperature of 29 degrees Celsius would not be considered hot? The bivalent logic does not have the flexibility to handle such a case. In fuzzy terms, the use of a fuzzy set can effectively interpret the degree of the membership function. Based on Equation (2.13), one reasonable model of the fuzzy set “hot weather temperature” would be:

$$\mu(x) = \begin{cases} 
1, & x \leq 30 \\
\frac{x - 15}{15}, & 15 < x < 30 \\
0, & x \leq 15 
\end{cases} \quad (2.14)$$

When a membership function $\mu(x)$ is 1, the weather temperature over 30 degrees Celsius is absolutely in the set. While, when the $\mu(x)$ is 0, 30 degrees Celsius is absolutely not in the set. The weather temperature between 15 to 30 degrees Celsius would be in the set with different degrees.

Bellman and Zadeh (1970) subsequently introduce fuzzy set theory into multicriteria analysis as a means of effectively dealing with the inherent imprecision, vagueness and subjectiveness of the human decision making process. Since then, the applications and relevant approaches coping with various decision-making problems in fuzzy environments have grown (e.g. Hon et al., 1996; Park, 1997; Yeh et al., 1999a; Liang, 1999; Yeh et al., 2000; Chen, 2001; Al-Najjar and Alsyouf, 2003; Hsieh et al., 2004; Yeh and Deng, 2004;
Chiou et al., 2005; Ding and Liang, 2005; Chou, 2006; 2007; Chang et al., 2007; Yang et al., 2008; Chu and Lin, 2009; Wu et al., 2009; Alipour et al., 2010; Chen et al., 2011; Dheena and Mohanraj, 2011; Chou and Cheng, 2012). Particularly, the fuzzy TOPSIS model, which is believed as a unique and useful tool, has been applied to wide-ranging applications (e.g. Chen, 2000; Mahdavi et al., 2008; Gumus, 2009; Kannan et al., 2009; Dagdeviren et al., 2009; Amiri, 2010; Kelemenis and Askounis, 2010; Sun, 2010; Torfi et al., 2010; Yusuf and Yurdakul, 2010; Chamodrakas et al., 2011; Kelemenis et al., 2011; Kaya and Kahraman, 2011; Yang et al., 2011; Rouhani et al., 2012; Yeh and Xu, 2012).

Basically, fuzzy MCDM is used to evaluate, rank, or select a set of alternatives with respect to multiple conflicting criteria involving fuzzy assessments. A large part of the published work concentrates on finding an optimal alternative among the available alternatives with respect to predetermined criteria. For example, Park (1997) uses the fuzzy linguistic approach to analyse Asian airports’ competitiveness. Chu and Tsao (1999) propose a fuzzy MCDM model with a three-level hierarchy structure to solve a car selection problem. Yeh et al. (2000) develop a fuzzy MCDM model for the performance evaluation of a bus company. Tsaur et al. (2002) use fuzzy MCDM models for evaluating the airline service quality. Torlak et al. (2011) use a fuzzy TOPSIS method for analysing the performance of the Turkish domestic airline industry. Yang et al. (2008) propose an integrated fuzzy MCDM method to address a vendor selection problem by considering the relationships between criteria. Deng and Chan (2011) develop a new fuzzy MCDM method to deal with the supplier selection problem.
2.5.1 Triangular fuzzy numbers

Among the various shapes of fuzzy number, triangular fuzzy number is the most popular one mainly due to their simplicity in both concept and computation. The merits of using triangular fuzzy numbers in fuzzy modeling have been well justified by Pedrycz (1994). As shown in Figure 2.3, a triangular fuzzy number is represented with three points as \( \tilde{A} = (a_1, a_2, a_3) \), where \( a_1 \leq a_2 \leq a_3 \). A triangular fuzzy number is a convex fuzzy set (Zadeh, 1965) with its membership function defined as

\[
\mu_A(x) = \begin{cases} 
(x - a_1)/(a_2 - a_1), & a_1 \leq x \leq a_2, \\
(a_3 - x)/(a_3 - a_2), & a_2 \leq x \leq a_3, \\
0, & \text{otherwise.} 
\end{cases}
\]  

(2.15)

![Figure 2.3. Triangular fuzzy number \( \tilde{A} = (a_1, a_2, a_3) \)](image)

The basic operation on fuzzy triangular numbers, which were developed by Van Laarhoven and Pedrycz (1983), are defined as follows:

Let \( \tilde{M} = (a_1, a_2, a_3) \) and \( \tilde{N} = (b_1, b_2, b_3) \) be two positive triangular fuzzy numbers as shown in Figure 2.4.
The basic fuzzy arithmetic operations on these fuzzy numbers are defined as:

(a) Addition: \( \tilde{M} + \tilde{N} = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \)

(b) Subtraction: \( \tilde{M} - \tilde{N} = (a_1 - b_3, a_2 - b_2, a_3 - b_1) \)

(c) Multiplication: \( \tilde{M} \times \tilde{N} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3) \)

(d) Division: \( \tilde{M} / \tilde{N} = (a_1 / b_3, a_2 / b_2, a_3 / b_1) \)

2.5.2 Representation of linguistic terms

Linguistic terms have been found intuitively easy to use in expressing the subjectiveness and imprecision of the decision maker’s assessments (Zadeh, 1975a; 1975b; Zimmermann, 1996; Yeh et al., 1999b; Herrera and Herrera-Viedma, 2000). A linguistic variable is the one whose values are not numbers (as in the case of a deterministic variable), but rather, linguistic terms. The contents of these terms are defined by fuzzy sets over a base variable. For example, the expressions to describe the temperature will be “cold”, “warm”, “hot” and “very hot”.

Figure 2.4. An example of two fuzzy triangular numbers
Linguistic variables can be interpreted by fuzzy numbers. A linguistic variable encapsulates the properties of approximate or imprecise concepts in a systematic and computationally useful way. It reduces the apparent complexity of describing a system by matching a semantic tag to the underlying concept. A linguistic variable always represents a fuzzy space, which is another way of saying that, when we evaluate a linguistic variable, we come up with a fuzzy set (Klir and Yuan, 1995).

The fuzzy linguistic approach, which deals with qualitative aspects that are represented in qualitative terms by means of linguistic variables, provides an important tool for solving decision problems in different areas (Zadeh, 1975a). In each fuzzy linguistic approach, appropriate linguistic descriptors for the term set and their semantics have to be chosen (Herrera et al., 2008). The semantics of linguistic terms is given by fuzzy numbers defined in the $[0, 1]$ interval, which are usually described by membership functions.

Table 2.1 shows an example of five linguistic terms characterized by triangular fuzzy numbers, which are used for the fuzzy assessment. A 1-9 ratio scale is used as it has been proven to be an effective measurement scale for reflecting the qualitative information of a decision problem and for enabling the unknown weights to be approximated (Yeh and Chang, 2009). Triangular fuzzy numbers are used to present membership function because they are simpler than trapezoidal fuzzy numbers. Figure 2.5 shows the membership functions of these linguistic terms defined in Table 2.1 (Klir and Yuan, 1995; Yeh et al., 2000).
Table 2.1
Linguistic terms represented by fuzzy numbers

<table>
<thead>
<tr>
<th>Linguistic terms (Variable)</th>
<th>Extremely Poor (EP)</th>
<th>Poor (P)</th>
<th>Fair (F)</th>
<th>Good (G)</th>
<th>Excellent (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membership Function (Linguistic value)</td>
<td>(1, 1, 3)</td>
<td>(1, 3, 5)</td>
<td>(3, 5, 7)</td>
<td>(5, 7, 9)</td>
<td>(7, 9, 9)</td>
</tr>
</tbody>
</table>

Figure 2.5. Membership functions of linguistic terms

2.6 Fuzzy Clustering

Clustering is an unsupervised classification of patterns (Jain et al., 1999). The clustering technique is used to organise data (quantitative, qualitative, or a mixture of both) into groups (clusters) based on similarities among the individual data items. Based on whether the clusters (subsets) of the data set are fuzzy or crisp, clustering methods can be classified into crisp clustering and fuzzy clustering.
The classic or crisp cluster analysis is based on classical set theory, which separates a
data set into constituent groups. However, this often leaves the problem of objects not being
able to be unequivocally assigned to any particular cluster. The fuzzy clustering analysis
allows the objects to belong to several clusters by giving the dominant a degree of
membership to each cluster (Bezdek, 1981; Höppner et al., 1999).

Most analytical fuzzy clustering algorithms are based on the optimisation of the basic \( c \)-
means objective function. The fuzzy \( c \)-means algorithm is based on the isodata method of
Ball and Hall (1967), which allows one piece of object to belong to two or more clusters.
That is, the number of clusters, \( c \), needs to be given at the beginning of the process, where \( c \)
is greater than, or equal to two, and less than or equal to the number of the objects, \( k \). This
method is developed by Dunn (1973) and improved by Bezdek (1981).

The algorithm for the clustering process of the fuzzy \( c \)-mean algorithm is given as follows:

Step 1: Let \( X = \{x_1, x_2, \ldots, x_k\} \) be a set of given alternatives which are regarded as the objects
to be clustered. Initialise the membership values \( \mu_{ik} \) of the alternatives \( x_k \) \((k = 1, 2, \ldots, n)\) to each of the \( c \) clusters \( v_i \) \((i = 1, 2, \ldots, c)\) (randomly) as:

\[
\sum_{i=1}^{c} \mu_{ik} = 1, \; \forall k = 1, \ldots, n \quad \text{and} \quad \mu_{ik} \in [0,1] \quad \forall i = 1, 2, \ldots, c
\]
\[\forall k = 1, 2, \ldots, n. \tag{2.16}\]

Step 2: Calculate the cluster centres \( v_i \) using these membership values \( \mu_{ik} \) as:

\[
v_i = \frac{\sum_{k=1}^{n} (\mu_{ik})^m x_k}{\sum_{k=1}^{n} (\mu_{ik})^m}, \forall i = 1, 2, \ldots, c; \; m > 1 \tag{2.17}\]
Step 3: Calculate the new membership values $\mu_{ik}^{new}$ using the cluster centres $v_i$:

$$\mu_{ik}^{new} = 1/\sum_{i=1}^{c} \left( \frac{\|v_i - x_k\|}{\|v_j - x_k\|} \right)^{2/(m-1)}, \quad \forall i = 1, 2, \ldots, c \quad \forall k = 1, 2, \ldots, n. \tag{2.18}$$

where the Euclidean distance is used to measure the vector distance between $v_i$ (or $v_j$) and $x_k$.

Step 4: Compare $\mu$ and $\mu^{new}$. If the distance between these two successive membership matrices is smaller than a small positive number $\varepsilon$ (e.g. 0.01) as the stipulated convergence threshold (i.e. if $\|\mu^{new} - \mu\| < \varepsilon$), then stop; otherwise let $\mu = \mu^{new}$ and go to Step 2.

As a result of fuzzy clustering, every alternative is assigned to the $c$ clusters with different membership values.

2.7 Defuzzification

Many of fuzzy systems incorporate a defuzzification as the last step that maps a fuzzy set (the output of fuzzy systems) into a crisp value. The defuzzification methods are normally used to (a) defuzzify the single fuzzy output set from a fuzzy system, or (b) combine with other processes in a fuzzy system to obtain a more efficient computation scheme. Van Leekwijck and Kerre (1999) review 18 different defuzzification methods and classify them into four categories, which are (a) maxima methods and derivatives, (b) distribution methods and derivatives, (c) area methods, and (d) miscellaneous methods. This section briefly
reviews some commonly used defuzzification methods, including Mean of Maximal (MOM), Center-of-Area (COA), and alpha-cut and lambda function.

The MOM defuzzification method computes the average of the fuzzy outputs that have the highest degrees. It is used when maxima of the membership functions are not unique. Let \( \mu_m(x) \) be the point at which the membership function is maximum, \( m \) is the number of times the output distribution reaches the maximum level, the crisp value \( \mu^*(x) \) can be obtained by
\[
\mu^*(x) = \frac{\sum_{m=1}^{M} \mu_m(x)}{M}.
\] (2.19)

The COA is the most commonly used defuzzification technique. The idea of COA is to give a numerical value \( x_0 = x_{COA}(A) \) for a fuzzy set \( A \), which divides the area under the membership function in two (approximately) equal parts. To utilize the COA method to find out the best non-fuzzy performance value (BNP) is a simple and practical method as there is no need to bring in the preferences of any evaluators (Tzeng and Teng, 1993; Tsaur et al., 1997; Opricovic and Tzeng, 2003; Chiou et al., 2005; Yu et al., 2005; Chen et al., 2011; Hu and Liao, 2011; Doumpos and Grigoroudis, 2013). Given a triangular fuzzy number \( \tilde{A} = (a_1, a_2, a_3) \), the BNP (crisp) value \( BNP(\tilde{A}) \) by the COA method can be obtained by
\[
BNP(\tilde{A}) = \frac{((a_2 - a_1) + (a_3 - a_1))}{3} + a_1 \quad \text{or} \quad BNP(\tilde{A}) = \frac{a_1 + a_2 + a_3}{3}.
\] (2.20)

The concept of alpha-cut (\( \alpha \)-cut) is often used to extrapolate fuzzy functions from crisp ones (Chang, 1981; Zhao and Govind, 1991). \( \alpha \)-cuts are slices through a fuzzy set producing non-fuzzy (crisp) sets (Buckley, 2004). The \( \alpha \)-cut of a fuzzy set is the (crisp) set of all elements that have a membership value greater than or equal to \( \alpha \). For a fuzzy set \( A \) in \( X \) and any real number \( \alpha \in [0, 1] \), the \( \alpha \)-cut set of \( A \) is represented by
\[
A_\alpha = \{ x \in X : \mu_A(x) \geq \alpha \}.
\] (2.21)
For example, let $A$ be a fuzzy set whose membership function is given by Equation (2.15). To obtain the $\alpha$-cut of $A$, $\alpha$ is first set to both left and right membership functions of $A$ (Dutta et al., 2011), that is, $\alpha = (x - a_1)/(a_2 - a_1)$ and $\alpha = (a_3 - x)/(a_3 - a_2)$. Then, $x$ can be expressed in terms of $\alpha$ by $x = (a_2 - a_1)\alpha + a_1$ and $x = a_3 - (a_3 - a_2)\alpha$, which gives the $\alpha$-cut of $A$ as

$$A_\alpha = [A_\alpha^L, A_\alpha^R] = [(a_2 - a_1)\alpha + a_1, a_3 - (a_3 - a_2)\alpha]$$  \hspace{1cm} (2.22)

Through $\alpha$-cut analysis on triangular fuzzy numbers, it will obtain two values $A_\alpha^L$ (minimum range) and $A_\alpha^R$ (maximum range), which need to be converted into a crisp value. The lambda ($\lambda$) (the concept of an optimism index) is then introduced to obtain the crisp output (Cheng and Mon, 1994; Jie et al., 2006; Ayağ and Özdemir, 2012) as

$$A_\lambda^\alpha = \lambda \times A_\alpha^R + [(1 - \lambda) \times A_\alpha^L], \hspace{1cm} 0 \leq \lambda \leq 1.$$  \hspace{1cm} (2.23)

### 2.8 Concluding Remarks

This chapter has reviewed the background concepts and methods used in this study. The review of the widely used MCDM methods, fuzzy set theory in multicriteria analysis, the fuzzy $c$-mean clustering method, and defuzzification methods provides theoretical foundations for developing and validating the MCDM evaluation and selection models to be presented in the following chapters. Chapter 3 will present a procedure for developing various fuzzy MCDM models and a validation process for selecting among inconsistent ranking results produced by these models.
Chapter 3

Problem Formulation and Methodology Development

3.1 The Fuzzy Multicriteria Decision Making Problem

Fuzzy multicriteria decision making (MCDM) can be used for evaluating or selecting a finite number of decision alternatives, in which the cardinal preference or ranking of all the alternatives is required. A typical fuzzy MCDM problem usually involves a set of \( m \) decision alternatives \( A_i (i = 1, 2, \ldots, m) \), which are to be evaluated based on \( n \) criteria (performance measures) \( C_j (j = 1, 2, \ldots, n) \). Assessments are to be given to determine (a) the weight vector \( W = (w_1, w_2, \ldots, w_j, \ldots, w_n) \), which represents the relative importance \( w_j \) of criteria \( C_j \) for the problem, and (b) the decision matrix \( X = \{x_{ij}, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \} \), which represents the performance ratings \( x_{ij} \) of alternatives \( A_i (i = 1, 2, \ldots, m) \) with respect to criteria \( C_j (j = 1, 2, \ldots, n) \). Given the weight vector \( W \) and decision matrix \( X \), the objective of the problem is to rank all the alternatives by giving each of them an overall preference value with respect to all criteria.

In practical applications, the weight vector \( W \) and decision matrix \( X \) can contain both crisp (quantitative) and fuzzy (qualitative) data. For quantitative measurements such as passenger volume or financial data, crisp values are used. For qualitative measurements such as customer-perceived service quality, subjective assessments represented by fuzzy data are often used. This is because fuzzy modeling has proven to be an effective way for formulating decision problems where the information available is subjective and imprecise (Bellman and Zadeh, 1970).
To facilitate the making of subjective assessments on criteria weights and performance ratings using fuzzy data, a set of linguistic terms characterised by triangular fuzzy numbers are used in this study for expressing the subjectiveness and vagueness of the decision maker’s assessments. Triangular fuzzy numbers are used to represent the approximate value range of a linguistic term, denoted as \((a_1, a_2, a_3)\), where \(a_1 \leq a_2 \leq a_3\). \(a_2\) is the most possible assessment value, and \(a_1\) and \(a_3\) are the lower and upper bounds respectively for reflecting the fuzziness of the assessment.

With the use of linguistic terms in assessing the criteria weights and performance ratings of alternatives, the fuzzy weight vector \(W\) and the fuzzy decision matrix \(X\) are thus constructed. A performance matrix is then obtained by multiplying the fuzzy weight vector \(W\) by the fuzzy decision matrix \(X\), as given in Equation (3.1). With the use of triangular fuzzy numbers, the multiplication operation is based on interval arithmetic (Kaufmann and Gupta, 1991).

\[
Z = \begin{bmatrix}
    w_1 x_{11} & w_2 x_{12} & \ldots & w_n x_{1n} \\
    w_1 x_{21} & w_2 x_{22} & \ldots & w_n x_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    w_1 x_{m1} & w_2 x_{m2} & \ldots & w_n x_{mn}
\end{bmatrix}
\]  \hspace{1cm} (3.1)

### 3.2 Development of Fuzzy MCDM Models

The solution procedure for the fuzzy MCDM problem presented in the previous section typically involves three key processes: normalisation, aggregation, and defuzzification. In general applications, quantitative performance ratings of the alternatives are often assessed by different measurement units. A normalisation process is often required to make the
comparison across performance ratings under different units in a decision matrix compatible. The normalisation process transforms the performance ratings of different units to a comparable scale, so that the inter-criteria comparisons can be made. The MCDM aggregation process is used to synthesise the fuzzy weight vector and the fuzzy decision matrix in order to obtain an overall fuzzy preference value for each alternative. The defuzzification process is used to obtain a crisp preference value for each alternative, on which the ranking of all the alternatives can be based. The following sections present commonly used methods in each of these three key processes.

3.2.1 The normalisation process

Three widely used normalisation methods described below can be used for the normalisation process.

(a) Vector normalisation ($N_1$)

This method divides the performance ratings of each criterion in the decision matrix by its norm. The normalised performance ratings ($r_{ij}$) of $x_{ij}$ in the decision matrix are calculated as

For benefit criteria (the larger $x_j$, the greater the preference),

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \ i = 1, 2, ..., m; \ j = 1, 2, ..., n. \quad (3.2)$$

For cost criteria (the smaller $x_j$, the greater the preference),

$$r_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \ i = 1, 2, ..., m; \ j = 1, 2, ..., n. \quad (3.3)$$
The vector normalisation method implies that all criteria have the same unit length of vector. The main advantage of this method is that every criterion is measured in dimensionless units, thus making it easier for inter-criteria comparisons. The main disadvantage is that it does not lead to a measurement scale of equal length because the minimum and maximum values of the scales are not equal to each criterion. Due to a non-linear scale transformation, a straightforward comparison is hard to make. This procedure is often employed in ELECTRE and TOPSIS (Hwang and Yoon, 1981).

(b) Linear scale transformation between 0 to 1 ($N_2$)

This method uses the following formulas to normalise the decision matrix ($x_{ij}$) for benefit (the larger $x_j$, the greater the preference) criteria and cost criteria (the smaller $x_j$, the greater the preference) respectively:

$$r_{ij} = \frac{x_{ij} - x_{j}^{\text{min}}}{x_{j}^{\text{max}} - x_{j}^{\text{min}}}, \ i = 1, 2, ..., m; \ j = 1, 2, ..., n.$$  (3.4)

$$r_{ij} = \frac{x_{j}^{\text{max}} - x_{ij}}{x_{j}^{\text{max}} - x_{j}^{\text{min}}}, \ i = 1, 2, ..., m; \ j = 1, 2, ..., n.$$  (3.5)

where $x_{j}^{\text{max}}$ and $x_{j}^{\text{min}}$ are the maximum and minimum values of the $j^{\text{th}}$ criterion respectively. The advantage of this method is that the scale of measurement ranges precisely from 0 to 1. The worst normalised performance rating of a criterion is 0, while the best-normalised performance rating is 1. A possible drawback of this process is that the scale transformation does not lead to a proportional change in performance ratings (Hwang and Yoon, 1981; Yoon and Hwang, 1995).
(c) Linear scale transformation \((N_3)\)

This method divides the performance ratings of a criterion by its maximum value. The normalised value of \(x_{ij}\) for benefit criteria is given as

\[
 r_{ij} = \frac{x_{ij}}{x_j^{\text{max}}}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n. \tag{3.6}
\]

where \(x_j^{\text{max}}\) is the maximum value of the \(j^{\text{th}}\) criterion. In the case of cost criteria, \(r_{ij}\) is computed as

\[
 r_{ij} = 1 - \frac{x_{ij}}{x_j^{\text{max}}}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n. \tag{3.7}
\]

The value of the normalised \(r_{ij}\) ranges from 0 to 1, and the criterion is more favorable as \(r_{ij}\) approaches 1. The significance of the scale transformation is that all performance ratings are transformed in a linear (proportional) way, so that the relative order of magnitude of the performance ratings remains equal.

### 3.2.2 The aggregation process

Three widely used MAVT-based MCDM methods described below can be used for the aggregation process.

(a) The simple additive weighting (SAW) method

The basic logic of the SAW method is to obtain a weighted sum of the performance ratings of each alternative over all criteria. The SAW method normally requires normalising the fuzzy decision matrix \((X)\) to allow a comparable scale for all ratings in \(X\). The overall fuzzy preference value \((V_i)\) of each alternative is obtained by
$$V_i = \sum_{j=1}^{n} r_{ij}w_j \quad i = 1, 2, \ldots, m$$  \hspace{1cm} (3.8)$$

where

$$r_{ij} = \text{the normalised performance rating of alternative } A_i \text{ on criterion } C_j.$$  

$$w_j = \text{the assigned weight of criterion } C_j.$$  

(b) The technique for order preference by similarity to ideal solution (TOPSIS)

The basic concept of the TOPSIS method is that the most preferred alternative should not only have the shortest distance from the positive ideal solution, but also have the longest distance from the negative ideal solution. With the TOPSIS method, the positive ideal solution and the negative ideal solution can be determined as

$$r^+ = \left( \max_{i}(r_{i1}), \max_{i}(r_{i2}), \ldots, \max_{i}(r_{im}) \right) = (r_1^+ , r_2^+ , \ldots, r_m^+)$$  \hspace{1cm} (3.9)$$

$$r^- = \left( \max_{i}(r_{i1}), \max_{i}(r_{i2}), \ldots, \max_{i}(r_{im}) \right) = (r_1^-, r_2^-, \ldots, r_m^-)$$  \hspace{1cm} (3.10)$$

The Hamming distance between each alternative $$A_i$$ and the positive ideal solution $$r^+$$, and between $$A_i$$ and the negative ideal solution $$r^-$$, can be calculated respectively as

$$d_i^+ = \sum_{j=1}^{m} w_j (r_{ij}^+ - r_{ij}); \quad d_i^- = \sum_{j=1}^{m} w_j (r_{ij} - r_{ij}^-)$$  \hspace{1cm} (3.11)$$

where $$d_i^+$$ is the Hamming distance of alternative $$A_i$$ from the positive ideal solution, and $$d_i^-$$ is the Hamming distance of the alternative $$A_i$$ from the negative ideal solution.
The overall performance value for each alternative across all criteria can then be determined by

\[ V_i = \frac{d_i^-}{d_i^+ + d_i^-}, \quad i = 1, 2, ..., m. \]  \hfill (3.12)

(c) The weighted product (WP) method

The WP method uses multiplication for connecting criteria ratings, each of which is raised to the power of the corresponding criteria weight. This multiplication process has the same effect as the normalisation process for handling different measurement units. The fuzzy preference value of each alternative is given by

\[ V'_i = \prod_{j=1}^{n} (r_{ij})^{w_j} \]  \hfill (3.13)

where \( \sum_{j=1}^{n} w_j = 1 \). \( w_j \) is a positive power for benefit criteria and a negative power for cost criteria. In this study, for easy comparison with the preference values generated by the other two methods, the overall fuzzy preference value \( (V_i) \) of each alternative is given by

\[ V_i = \frac{\prod_{j=1}^{n} (r_{ij})^{w_j}}{\prod_{j=1}^{n} (r_j^*)^{w_j}} \]  \hfill (3.14)

where

\[ r_j^* = \max_i r_{ij} \]

\[ 0 \leq V_i \leq 1. \]
3.2.3 The defuzzification process

In this study, the $\alpha$-cut and $\lambda$ function is applied in the defuzzification process as (a) the value of $\alpha$ can represent the decision maker’s degree of confidence in the fuzzy assessments with respect to the criteria weights and performance ratings of the alternatives, and (b) the use of $\lambda$ allows the decision outcome to reflect the decision maker’s attitude towards risk, which may be optimistic, pessimistic or somewhere in between.

By using the concept of $\alpha$-cut on the fuzzy numbers in the performance matrix given in Equation (3.1), an interval performance matrix is derived as given in Equation (3.15), where $0 \leq \alpha \leq 1$. For a given $\alpha$, $z_{ijl}^\alpha$ and $z_{ijr}^\alpha$ ($i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$) are the average of the lower bounds and upper bounds of the crisp intervals respectively, resulted from all $\alpha$-cuts using the alpha values equal to or greater than the specified value of $\alpha$.

$$Z_{\alpha} = \begin{bmatrix}
[z_{11l}^\alpha, z_{11r}^\alpha] & [z_{12l}^\alpha, z_{12r}^\alpha] & \cdots & [z_{1nl}^\alpha, z_{1nr}^\alpha] \\
[z_{21l}^\alpha, z_{21r}^\alpha] & [z_{22l}^\alpha, z_{22r}^\alpha] & \cdots & [z_{2nl}^\alpha, z_{2nr}^\alpha] \\
\vdots & \vdots & \ddots & \vdots \\
[z_{ml1}^\alpha, z_{m1r}^\alpha] & [z_{m2l}^\alpha, z_{m2r}^\alpha] & \cdots & [z_{mnl}^\alpha, z_{mnr}^\alpha]
\end{bmatrix} \tag{3.15}
$$

A larger $\alpha$ value indicates that the decision maker is more confident in choosing a crisp value interval to represent the corresponding fuzzy number, as the interval is smaller and has a higher possibility. This implies that a confident decision maker would not consider less possible values embedded in a fuzzy number.

To reflect the decision maker’s relative preference between $z_{ijl}^\alpha$ and $z_{ijr}^\alpha$ in Equation (3.15), a preference index $\lambda$ in the range of 0 and 1 is incorporated into the solution procedure. As a result, a crisp performance matrix is obtained as
where $z_{ij}^{\lambda'} = \lambda z_{ij}^\alpha + (1 - \lambda) z_{ij}^{\alpha'}$, $0 \leq \lambda \leq 1$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$. In actual decision settings, $\lambda = 1$, $\lambda = 0.5$, or $\lambda = 0$ can be used to indicate that the decision maker has an optimistic, moderate, or pessimistic view, respectively, on fuzzy assessment results. An optimistic decision maker is apt to prefer higher values of the crisp value interval derived from fuzzy assessments, while a pessimistic decision maker tends to favour lower values. With the defuzzification process, a crisp preference value for each alternative can be obtained, on which the ranking of all the alternatives can be based.

### 3.2.4 Development of fuzzy MCDM models

Combining the three methods for normalisation with three aggregation methods (SAW, TOPSIS, and WP) and one defuzzification method ($\alpha$-cut and $\lambda$ function) will result in 7 fuzzy MCDM models. It is noted that normalisation is not required if the aggregation process uses WP. If the defuzzification process is conducted first to obtain a crisp performance matrix given in Equation (3.16), followed by the normalisation and aggregation processes, another set of 7 fuzzy MCDM models can be generated. Table 3.1 shows these 14 fuzzy MCDM models, each is associated with a code for easy reference.
Table 3.1
14 fuzzy MCDM models and their corresponding reference code

<table>
<thead>
<tr>
<th>Process 1</th>
<th>Process 2</th>
<th>Process 3</th>
<th>Model Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalisation</td>
<td>Aggregation</td>
<td>Defuzzification</td>
<td>N1–S–D</td>
</tr>
<tr>
<td>N1</td>
<td>SAW</td>
<td>D (α-cut and λ)</td>
<td>N1–T–D</td>
</tr>
<tr>
<td>N1</td>
<td>TOPSIS</td>
<td>D (α-cut and λ)</td>
<td>N1–T–D</td>
</tr>
<tr>
<td>N2</td>
<td>SAW</td>
<td>D (α-cut and λ)</td>
<td>N2–S–D</td>
</tr>
<tr>
<td>N2</td>
<td>TOPSIS</td>
<td>D (α-cut and λ)</td>
<td>N2–T–D</td>
</tr>
<tr>
<td>N3</td>
<td>SAW</td>
<td>D (α-cut and λ)</td>
<td>N3–S–D</td>
</tr>
<tr>
<td>N3</td>
<td>TOPSIS</td>
<td>D (α-cut and λ)</td>
<td>N3–T–D</td>
</tr>
<tr>
<td>WP</td>
<td></td>
<td>D (α-cut and λ)</td>
<td>W–D</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Defuzzification</th>
<th>Normalisation</th>
<th>Aggregation</th>
</tr>
</thead>
<tbody>
<tr>
<td>D (α-cut and λ)</td>
<td>N1</td>
<td>SAW</td>
</tr>
<tr>
<td>D (α-cut and λ)</td>
<td>N1</td>
<td>TOPSIS</td>
</tr>
<tr>
<td>D (α-cut and λ)</td>
<td>N2</td>
<td>SAW</td>
</tr>
<tr>
<td>D (α-cut and λ)</td>
<td>N2</td>
<td>TOPSIS</td>
</tr>
<tr>
<td>D (α-cut and λ)</td>
<td>N3</td>
<td>SAW</td>
</tr>
<tr>
<td>D (α-cut and λ)</td>
<td>N3</td>
<td>TOPSIS</td>
</tr>
<tr>
<td>D (α-cut and λ)</td>
<td>WP</td>
<td></td>
</tr>
</tbody>
</table>

These models can be used to solve the general fuzzy MCDM problem that requires cardinal preference or ranking of all the alternatives. Due to the structural differences among these fuzzy MCDM models, these models often produce inconsistent ranking outcomes for a given weight vector and decision matrix. To address this inconsistency issue, a validation process using fuzzy clustering is developed for selecting more valid ranking outcome among inconsistent results produced by different fuzzy MCDM models for a given problem.
3.3 Ranking Validity of Fuzzy MCDM models

The validation process is based on a fuzzy clustering algorithm known as fuzzy $c$-means (Bezdek, 1981). Examples and applications of the fuzzy $c$-means algorithm have been widely illustrated and reported in the literature (e.g. Bellman et al., 1966; Yen and Langari, 1999; Zimmermann, 2001; Zopounidis and Doumpos, 2002). Clustering is a technique for grouping objects (alternatives) based on multiple features (criteria). Fuzzy clustering algorithms such as fuzzy $c$-means provide an adequate means for representing real data structures, in particular when clusters are overlapping and objects (alternatives) are having multiple and non-dichotomous cluster memberships (Bellman et al., 1966; Chau and Yeh, 2000).

With fuzzy clustering, it is admissible for an alternative to belong to more than one cluster with different degrees of membership. An alternative is said to be most typically belong to a cluster in which it has the highest membership degree as compared to its relatively lower partial memberships of other clusters. Alternatives within a cluster indicate a strong relationship among them, thus implying their closeness in a ranking relationship. As the ranking relationship indicated by fuzzy clustering truly reflects the structure of the problem data set, it provides an objective reference for validating ranking results of fuzzy MCDM models. This suggests that the ranking of alternatives generated by fuzzy MCDM models should be consistent with the ranking relationship of the alternatives indicated by clusters generated by fuzzy clustering. It is this notion on which the development of the validation procedure is based (Yeh and Kuo, 2003b).

Applying fuzzy $c$-means to a clustering problem requires a prior determination of the number of clusters. In an unsupervised setting where knowledge about the number of ideal clusters is unavailable, this can be determined using various cluster validity measures
(Windham, 1981). As the purpose is to validate the ranking of alternatives within a continuum between two extremes, the best and the worst, the setting of the validation process can thus be intuitively formulated as a fuzzy clustering problem with two clusters, namely the best-performed cluster and the worst-performed cluster.

In the fuzzy $c$-means algorithm for validating the ranking results of fuzzy MCDM models, the alternatives being evaluated in a fuzzy MCDM problem are regarded as the objects to be clustered. Each alternative $A_i$ ($i = 1, 2, ..., m$) is represented by a vector of $n$ features as $[f_{i1}, f_{i2}, ..., f_{in}]$, where the $n$ features correspond to the $n$ criteria $C_j$ ($j = 1, 2, ..., n$) of the problem, and each feature value $f_{ij}$ of alternative $A_i$ is the corresponding weighted performance rating of alternative $A_i$ on the $j^{th}$ criterion. Given the weighted performance matrix $F = f_{ij}$ ($i = 1, 2, ..., m; j = 1, 2, ..., n$) obtained by a fuzzy MCDM model, the fuzzy $c$-means algorithm for validating fuzzy MCDM models works as follows:

Step 1: Initialise the membership values $\mu_{ki}$ of the $m$ alternatives $A_i$ ($i = 1, 2, ..., m$) to each of the two clusters $v_k$ ($k = 1, 2$), such that

$$\sum_{k=1}^{2} \mu_{ki} = 1, \mu_{ki} \in [0,1], i = 1, 2, ..., m; k = 1, 2. \quad (3.17)$$

Step 2: Calculate the cluster centers $v_k$ (each is represented by a vector) for the best-performed and worst-performed clusters respectively, using these membership values $\mu_{ki}$, as

$$v_k = \sum_{i=1}^{m} (\mu_{ki})^2 \cdot A_i \left/ \sum_{i=1}^{m} (\mu_{ki})^2 \right., k = 1, 2. \quad (3.18)$$
where \( A_i = [f_{i1}, f_{i2}, \ldots, f_{in}] \).

Step 3: Calculate the new membership values \( \mu_{ki}^{\text{new}} \) using the two cluster centers \( v_k \) as

\[
\mu_{ki}^{\text{new}} = \frac{1}{\sum_{k=1}^{2} \left( \frac{v_k - A_i}{v_k - A_i} \right)^2}, \quad k = 1, 2; i = 1, 2, \ldots, m. \tag{3.19}
\]

where \( A_i = [f_{i1}, f_{i2}, \ldots, f_{in}] \). The Euclidean distance is used to calculate the vector distance between \( v_k \) (or \( v_y \)) and \( A_i \) in Equation (3.19).

Step 4: Compare \( \mu \) and \( \mu^{\text{new}} \). If the distance between these two successive membership matrices is smaller than a small positive number \( \varepsilon \) (e.g., 0.01) as the stipulated convergence threshold (i.e., if \( \| \mu^{\text{new}} - \mu \| < \varepsilon \)), then stop; otherwise let \( \mu = \mu^{\text{new}} \) and go to Step 2.

As a result of fuzzy clustering, every alternative is assigned to the two clusters with different membership values. An ordering of these alternatives based on their resulting membership values thus reveals a ranking that signifies their closeness/similarity towards the best-performed alternative and the worst-performed alternative respectively in each corresponding fuzzy MCDM model. Since clusters are groupings of similar alternatives, the comparison and ranking of membership values have a practical meaning (Peneva and Popchev, 1998; Chau and Yeh, 2000). As such, it serves as an objective validation tool to help select among inconsistent ranking results produced by different fuzzy MCDM models.

To validate the ranking order of a fuzzy MCDM model based on that of fuzzy clustering, the Spearman’s rank correlation analysis is carried out. The Spearman's rank order correlation
is the nonparametric version of the Pearson product-moment correlation. Spearman’s correlation coefficient, signified by \( \rho \), measures the strength of association between two ranked variables, where monotonic relationship is observed. The formula to calculate Spearman’s rank correlation coefficient is as follows

\[
\rho = 1 - \frac{6 \sum d_i^2}{n(n^2-1)}
\]  

(3.20)

where

\( d_i \) = the difference in paired ranks.

\( n \) = the number of variables.

### 3.4 The Model Development and Validation Approach

Figure 3.1 shows the framework of the fuzzy MCDM model development and validation approach. The solution procedure of the approach is summarised as follows:

Step 1: Obtain the fuzzy weight vector \( W \) for decision criteria \( C_j \) and the fuzzy decision matrix \( X \) for decision alternatives \( A_i \) based on the fuzzy assessment results by the decision maker for a given decision problem.

Step 2: Apply different normalisation, aggregation, and defuzzification methods to develop various fuzzy MCDM models for solving the formulated decision problem.

Step 3: Use fuzzy clustering based validation process to examine the ranking validity of the fuzzy MCDM models and compare the consistency degree between the ranking outcomes obtained by each MCDM model and by the fuzzy clustering using Spearman’s rank correlation coefficient.
Step 4: Select the fuzzy MCDM model that produces the highest correlation coefficient at Step 3.

3.5 Concluding Remarks

This chapter has formulated the general MCDM problem that requires cardinal ranking of the decision alternatives. This chapter has also presented a structured procedure for
developing fuzzy MCDM models to solve the problem. These models use different normalisation, aggregation, and defuzzification methods, thus often producing different ranking results of the decision alternatives for a given problem. To validate these ranking results for selecting the most valid model, a validation process based on fuzzy clustering has been developed. Chapters 4 to 6 will apply this fuzzy MCDM model development and validation approach to three practical applications, which are airport performance evaluation (Application I), buyer selection (Application II), and supplier selection (Application III).
Chapter 4

Application I – Performance Evaluation of Asia-Pacific International Airports

4.1 Introduction

The operation efficiency and service quality of international airports are critical in modern day context where businesses are international and people are more mobile. To help an airport identify functional areas for improvement, it is advisable to evaluate its performance relative to other comparable airports with respect to a number of manageable evaluation criteria. As the evaluation is based on a comparative process, the evaluation result can be used as a service benchmarking and management tool for airports. The most comprehensive study of comparing service performance of international airports is probably the International Air Transport Association (IATA) Global Airport Monitor. Despite a wide coverage of performance indicators for airport services, no attempt was made by IATA to develop an integrated airport performance value.

Quite a few studies on airport performance evaluation have been reported in the literature with focus on specific functional areas of airport operations, such as productivity (Gillen and Waters II, 1997; Hooper and Hensher, 1997; Gillen and Lall, 1997; Oum et al., 2003), capacity and delays (Brunetta et al., 1999), efficiency (Sarkis, 2000; Pels et al., 2003; Yu, 2010), airline services (Adler and Berechman, 2001), competitiveness (Park, 1997; Park, 2003), financial performance (Vasigh and Hamzaee, 1998), service quality (Yeh and Kuo, 2003a; Fernandes and Pacheco, 2010; Chou, 2011). Although there is no universal and exact
definition for airport performance, the overall performance of airports can be measured from the viewpoints of passengers, airlines, and the airport operator (Lemer, 1992). The most widely used airport performance evaluation methods are perhaps the data envelopment analysis (DEA) and total factor productivity (TFP) (Gillen and Lall, 1997; Humphreys and Francis, 2000; Sarkis, 2000; Pels et al., 2003; Chi-Lok and Zhang, 2009; Yu, 2010). Although these methods are well suited for measuring the efficiency of airport operations based on the resources as input measures and the performance as output measure, they do not intend to address the effectiveness issue with respect to the overall performance of airports from the viewpoints of the airport operator, passengers, and airlines as a whole.

The evaluation of the overall performance of airports is a complex decision-making process, as it usually involves (a) multiple conflicting criteria with non-commensurable units, and (b) both crisp and fuzzy data derived from precise measures of quantitative criteria and imprecise judgements of qualitative criteria resulting from human subjectivity. Multicriteria decision making (MCDM) has proven to be an effective approach for ranking a finite number of alternatives characterized by multiple conflicting criteria (Hwang and Yoon, 1981; Olson, 1996). The most widely used theory in solving MCDM problems is multiattribute utility theory (Keeney and Raiffa, 1993) or multiattribute value theory (MAVT) (Dyer and Sarin, 1979), with which a cardinal preference or ranking of the decision alternatives is generated. Fuzzy set theory has proven to be a powerful modeling tool for coping with the subjectiveness and imprecision of human judgments (Bellman and Zadeh, 1970; Zimmermann, 1996). In the applications of MAVT-based MCDM for evaluation and selection problems involving subjective judgments of the decision maker, fuzzy MCDM has demonstrated its applicability in ranking decision alternatives such as transportation systems.
In this chapter, the airport performance evaluation is to be formulated as a fuzzy MCDM problem that requires cardinal ranking of the airports evaluated. A number of evaluation criteria are to be identified for the performance evaluation of the airports. The methodological development presented in Chapter 3 will then be used to obtain a valid airport performance ranking.

4.2 The Airport Performance Evaluation Problem

Air travel demand in the Asia-Pacific region has had an average annual growth rate of 10.1% during the past decade, the highest in the world. A high rate of economic growth in Asia has spurred the rapid expansion of commercial aviation industries serving the Asia-Pacific region. With 16 of the world’s 25 busiest air routes, Asia’s major airports are already near capacity. Asia’s explosive economic growth has been accompanied by rapid expansion and the transformation of the region’s aviation industry. Air transportation is expected to play a larger role in this region more than anywhere else in the world. First, the high population and income growth rates in many Asian countries are expected to produce an astounding increase in the demand for air transportation services. The region already accounts for more than 50 per cent of the world’s population. Second, vast distances separate many Asia-Pacific countries. About 60 per cent of the air routes in the region are between cities that are at least 2,000 kilometres apart, and a number of countries in the region are islands or archipelagos with few alternatives to air passenger travel (Croix and Wolff, 1995).
This study uses 12 Asia-Pacific airports as the decision alternatives of the airport performance evaluation problem for illustrating the effectiveness of the fuzzy MCDM model development and validation approach presented in Chapter 3. These 12 airports include most of the major international airports in the region which process more than half of the region’s international traffic. These airports are $(A_1)$ Don Muang International Airport - Bangkok (BKK), $(A_2)$ Chek Lap Kok International Airport - Hong Kong (HKG), $(A_3)$ Kansai International Airport - Osaka (KIX), $(A_4)$ Tullamarine International Airport - Melbourne (MEL), $(A_5)$ Ninoy Aquino International Airport - Manila (MNL), $(A_6)$ Narita International Airport - Tokyo (NRT), $(A_7)$ Capital International Airport - Beijing (PEK), $(A_8)$ Incheon International Airport - Seoul (SEL), $(A_9)$ Hongqiao International Airport - Shanghai (SHA), $(A_{10})$ Changi International Airport - Singapore (SIN), $(A_{11})$ Kingsford Smith International Airport - Sydney (SYD), and $(A_{12})$ Chiang Kai-Shek International Airport - Taipei (TPE).

The use of performance measures or evaluation criteria in the airport performance evaluation problem is crucially important because of its specific economic characteristics. In the business competitive environment, optimal performance can be equated with profitability. However, the conditions under which airports operate are far from competitive. Regulatory, geographical, economic, social and political constraints all hinder direct competition between airports (Doganis and Graham, 1987). Various evaluation criteria can be used to evaluate the airport performance from the perspectives of different stakeholders of the airport operation. For example, the airport operators (service providers) would focus on the measures concerned with the operational efficiency of the airport. Passengers are more interested in the measures in relation to service quality and safety. Airlines are more concerned about the facilities provided by the airport. As such, to reflect the operating characteristics of international airports, the criteria for performance evaluation are constructed from the
perspectives of the airport operators, passengers, and airlines. These three dimensions have been commonly used in the performance evaluation of airports, which are normally applied individually for a specific evaluation problem, such as airport operational efficiency (Sarkis, 2000), airport productivity (Gillen and Lall, 1997; Hooper and Hensher, 1997), airport competitiveness (Park, 1997), airline services (Adler and Berechman, 2001), and airport passenger service performance (Rhoades, 2000; Yeh and Kuo, 2003a). In this study, we integrate these three dimensions to obtain an overall relative evaluation value for each of the 12 airports using fuzzy MCDM.

Figure 4.1 shows a hierarchical structure of the airport performance evaluation problem, including the three evaluation dimensions (Level 1) and their associated evaluation criteria $C_j (j = 1, 2, ..., n)$ (Level 2), and 12 airports (decision alternatives) $A_i (i = 1, 2, ..., m)$ to be evaluated (Level 3). Subjective assessments are to be made by the decision maker to determine (a) the weight vector $W = (w_1, w_2, ..., w_j, ..., w_n)$, which represents the relative importance of $n$ evaluation criteria $C_j (j = 1, 2, ..., n)$ for the problem, and (b) the decision matrix $X = \{x_{ij}, i = 1, 2, ..., m; j = 1, 2, ..., n\}$, which represents the performance ratings of airport $A_i$ with respect to evaluation criteria $C_j$. Given the weight vector $W$ and decision matrix $X$, the objective of the problem is to rank all the airports by giving each of them an overall performance value with respect to all evaluation criteria.
The airport operator dimension is mainly concerned with airport operational management and the facilities provided for the airport users. The airport operator is assessed quantitatively based on the airport operational data, which are revenue \( (C_1) \) – total operating revenue, annual growth rate \( (C_2) \) – annual growth rate of total passenger volume, airport terminal size \( (C_3) \) – total terminal surface area of airport, navigation aids \( (C_4) \) – category of the navigation aid system, ground access facilities \( (C_5) \) – types of ground transportation to/from city, distance to CBD \( (C_6) \) – distance from the airport to the central business district, car park \( (C_7) \) – number of parking car bays, noise pollution \( (C_8) \) – number of aircraft movement per day, and passenger volume \( (C_9) \) – number of passengers (millions).

The passenger dimension relates to the services provided by the airport and the quality level as perceived by the passengers. Under this dimension, the evaluation criteria are comfort \( (C_{10}) \) – congestion level and cleanliness in the terminal building, processing time \( (C_{11}) \) – total time required during check-in, immigration inspection, and customs, convenience \( (C_{12}) \) – availability and accessibility of airport facilities within the terminal.
building, courtesy of staff ($C_{13}$) – helpfulness and friendliness of airport staff, information
visibility ($C_{14}$) – information display for flights, airport facilities, and signposting, and
security ($C_{15}$) – safety measures and security facilities. These criteria are to be measured
based on passengers’ perception, which are to be assessed subjectively by a survey using a set
of linguistic terms characterised by fuzzy numbers.

The airline dimension is mainly concerned with the facilities provided by airports such as
gates, operating hours, aircraft movement and runways. Under this dimension, the evaluation
criteria are aircraft movement ($C_{16}$) – total number of aircraft movement, gates ($C_{17}$) –
number of gates, operating hours ($C_{18}$) – total daily operating hours of the airport, and
runway ($C_{19}$) – number of runways.

Table 4.1 summarises the identified evaluation criteria. These criteria involve both
quantitative and qualitative assessments, for which numerical data and fuzzy numbers are to
be used respectively. These criteria are independent of each other, thus suitable for use in an
MAVT-based MCDM model.
Table 4.1
Evaluation criteria under three evaluation dimensions

<table>
<thead>
<tr>
<th>Evaluation dimension</th>
<th>Evaluation criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airport Operator</td>
<td>$C_1$ Revenue/Cost</td>
</tr>
<tr>
<td></td>
<td>$C_2$ Annual growth rate</td>
</tr>
<tr>
<td></td>
<td>$C_3$ Airport terminal size</td>
</tr>
<tr>
<td></td>
<td>$C_4$ Navigation aids</td>
</tr>
<tr>
<td></td>
<td>$C_5$ Ground access facilities</td>
</tr>
<tr>
<td></td>
<td>$C_6$ Distance to CBD</td>
</tr>
<tr>
<td></td>
<td>$C_7$ Car park</td>
</tr>
<tr>
<td></td>
<td>$C_8$ Noise pollution</td>
</tr>
<tr>
<td></td>
<td>$C_9$ Passenger volume (Mil)</td>
</tr>
<tr>
<td>Passenger</td>
<td>$C_{10}$ Comfort</td>
</tr>
<tr>
<td></td>
<td>$C_{11}$ Processing time</td>
</tr>
<tr>
<td></td>
<td>$C_{12}$ Convenience</td>
</tr>
<tr>
<td></td>
<td>$C_{13}$ Courtesy of staff</td>
</tr>
<tr>
<td></td>
<td>$C_{14}$ Information visibility</td>
</tr>
<tr>
<td></td>
<td>$C_{15}$ Security</td>
</tr>
<tr>
<td>Airline</td>
<td>$C_{16}$ Aircraft movement</td>
</tr>
<tr>
<td></td>
<td>$C_{17}$ Gates</td>
</tr>
<tr>
<td></td>
<td>$C_{18}$ Operating hours</td>
</tr>
<tr>
<td></td>
<td>$C_{19}$ Runway</td>
</tr>
</tbody>
</table>

4.3 Airports Performance Assessment

The relative importance $w_j$ of the 19 evaluation criteria $C_j$ are to be assessed via survey. The performance ratings $x_{ij}$ of the 12 airports $A_i$ with respect to the evaluation criteria under the airport operator and airline dimensions are obtained based on relevant websites or airport publications. For the passenger dimension, a survey questionnaire is designed to ask the international travel experts (tour guides) to assess the performance rating of the 12 airports with respect to the corresponding evaluation criteria, using a set of linguistic terms.
To facilitate the experts in airport authorities, airlines, and research institutions to make subjective assessments on the criteria weights and performance ratings using fuzzy data, two linguistic variables, importance and performance, are used respectively. A set of linguistic terms are used for each linguistic variable to represent the value range of the variable. As discussed in Chapter 2, triangular fuzzy numbers are used to represent the approximate value range of the linguistic term, denoted as \((a_1, a_2, a_3)\), where \(a_1 \leq a_2 \leq a_3\).

Table 4.2 shows the set of linguistic terms using 1-9 ratio scale, together with their corresponding membership functions, for the linguistic variable “importance”, which is used to assess the relative importance of the evaluation criteria. To assess the performance rating of the airports with respect to each evaluation criterion, another set of linguistic terms for the linguistic variable “performance” is given in Table 4.3. Equal weights are given for the three evaluation dimensions. The assessment data are shown in Tables 4.4 to 4.7.

**Table 4.2**
Linguistic terms for fuzzy importance weighting assessment

<table>
<thead>
<tr>
<th>Linguistic Term</th>
<th>Not Important</th>
<th>Somewhat Important</th>
<th>Important</th>
<th>Very Important</th>
<th>Extremely Important</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membership Function</td>
<td>(NI)</td>
<td>(SI)</td>
<td>(I)</td>
<td>(VI)</td>
<td>(EI)</td>
</tr>
<tr>
<td></td>
<td>(1, 1, 3)</td>
<td>(1, 3, 5)</td>
<td>(3, 5, 7)</td>
<td>(5, 7, 9)</td>
<td>(7, 9, 9)</td>
</tr>
</tbody>
</table>

**Table 4.3**
Linguistic terms for fuzzy performance rating assessment

<table>
<thead>
<tr>
<th>Linguistic Term</th>
<th>Very Poor (VP)</th>
<th>Poor (P)</th>
<th>Fair (F)</th>
<th>Good (G)</th>
<th>Very Good (VG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membership Function</td>
<td>(1, 1, 3)</td>
<td>(1, 3, 5)</td>
<td>(3, 5, 7)</td>
<td>(5, 7, 9)</td>
<td>(7, 9, 9)</td>
</tr>
</tbody>
</table>
Table 4.4
Fuzzy weights $w_j$ for evaluation criteria $C_j$

<table>
<thead>
<tr>
<th>Evaluation dimension</th>
<th>Evaluation criteria weight $w_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_1$ $w_2$ $w_3$ $w_4$ $w_5$ $w_6$ $w_7$ $w_8$ $w_9$</td>
</tr>
<tr>
<td>Airport operator</td>
<td>VI VI I I I I I SI</td>
</tr>
<tr>
<td></td>
<td>$w_{10}$ $w_{11}$ $w_{12}$ $w_{13}$ $w_{14}$ $w_{15}$</td>
</tr>
<tr>
<td>Passenger</td>
<td>VI VI VI EI I VI</td>
</tr>
<tr>
<td></td>
<td>$w_{16}$ $w_{17}$ $w_{18}$ $w_{19}$</td>
</tr>
<tr>
<td>Airline</td>
<td>I VI VI SI</td>
</tr>
</tbody>
</table>

Table 4.5
Assessment data $x_{ij}$ for evaluation criteria $C_j$ under the airport operator dimension

<table>
<thead>
<tr>
<th>Airport</th>
<th>$C_1$</th>
<th>$C_3$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_8$</th>
<th>$C_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1 BKK</td>
<td>2.23</td>
<td>9</td>
<td>621</td>
<td>2</td>
<td>2</td>
<td>25</td>
<td>3,500</td>
<td>28</td>
<td>18</td>
</tr>
<tr>
<td>A_2 HKG</td>
<td>2.15</td>
<td>7.40</td>
<td>1,248</td>
<td>2</td>
<td>3</td>
<td>28</td>
<td>3,000</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>A_3 KIX</td>
<td>1.24</td>
<td>14.2</td>
<td>511</td>
<td>2</td>
<td>3</td>
<td>50</td>
<td>6,133</td>
<td>24</td>
<td>10</td>
</tr>
<tr>
<td>A_4 MEL</td>
<td>3.07</td>
<td>6.3</td>
<td>2,369</td>
<td>2</td>
<td>1</td>
<td>22</td>
<td>6,000</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>A_5 MNL</td>
<td>1.63</td>
<td>12.9</td>
<td>631</td>
<td>1</td>
<td>1</td>
<td>10</td>
<td>654</td>
<td>23</td>
<td>7</td>
</tr>
<tr>
<td>A_6 NRT</td>
<td>1.01</td>
<td>4</td>
<td>710</td>
<td>2</td>
<td>2</td>
<td>66</td>
<td>9,144</td>
<td>28</td>
<td>24</td>
</tr>
<tr>
<td>A_7 PEK</td>
<td>3.5</td>
<td>7.1</td>
<td>960</td>
<td>2</td>
<td>1</td>
<td>30</td>
<td>700</td>
<td>29</td>
<td>16</td>
</tr>
<tr>
<td>A_8 SEL</td>
<td>1.51</td>
<td>15.52</td>
<td>732</td>
<td>2</td>
<td>2</td>
<td>17</td>
<td>4,964</td>
<td>43</td>
<td>14</td>
</tr>
<tr>
<td>A_9 SHA</td>
<td>3.5</td>
<td>19</td>
<td>447</td>
<td>1</td>
<td>1</td>
<td>12</td>
<td>400</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>A_10 SIN</td>
<td>1.63</td>
<td>7.9</td>
<td>1,663</td>
<td>2</td>
<td>1</td>
<td>20</td>
<td>3,600</td>
<td>37</td>
<td>27</td>
</tr>
<tr>
<td>A_11 SYD</td>
<td>1.74</td>
<td>5</td>
<td>881</td>
<td>2</td>
<td>1</td>
<td>9</td>
<td>11,500</td>
<td>28</td>
<td>20</td>
</tr>
<tr>
<td>A_12 TPE</td>
<td>5.42</td>
<td>6.6</td>
<td>1,200</td>
<td>2</td>
<td>1</td>
<td>31</td>
<td>2,303</td>
<td>22</td>
<td>15</td>
</tr>
</tbody>
</table>
Table 4.6
Assessment data \( x_{ij} \) for evaluation criteria \( C_j \) under the passenger dimension

<table>
<thead>
<tr>
<th>Airport</th>
<th>( C_{10} )</th>
<th>( C_{11} )</th>
<th>( C_{12} )</th>
<th>( C_{13} )</th>
<th>( C_{14} )</th>
<th>( C_{15} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁ BKK</td>
<td>G</td>
<td>G</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>A₂ HKG</td>
<td>P</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>A₃ KIX</td>
<td>F</td>
<td>G</td>
<td>G</td>
<td>VG</td>
<td>VG</td>
<td>G</td>
</tr>
<tr>
<td>A₄ MEL</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
</tr>
<tr>
<td>A₅ MNL</td>
<td>F</td>
<td>F</td>
<td>G</td>
<td>P</td>
<td>P</td>
<td>F</td>
</tr>
<tr>
<td>A₆ NRT</td>
<td>F</td>
<td>G</td>
<td>G</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
</tr>
<tr>
<td>A₇ PEK</td>
<td>G</td>
<td>F</td>
<td>G</td>
<td>P</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>A₈ SEL</td>
<td>F</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>A₉ SHA</td>
<td>G</td>
<td>F</td>
<td>G</td>
<td>F</td>
<td>F</td>
<td>G</td>
</tr>
<tr>
<td>A₁₀ SIN</td>
<td>G</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
</tr>
<tr>
<td>A₁₁ SYD</td>
<td>F</td>
<td>G</td>
<td>G</td>
<td>VG</td>
<td>VG</td>
<td>G</td>
</tr>
<tr>
<td>A₁₂ TPE</td>
<td>G</td>
<td>G</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>G</td>
</tr>
</tbody>
</table>

Table 4.7
Assessment data \( x_{ij} \) for evaluation criteria \( C_j \) under the airline dimension

<table>
<thead>
<tr>
<th>Airport</th>
<th>( C_{16} )</th>
<th>( C_{17} )</th>
<th>( C_{18} )</th>
<th>( C_{19} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁ BKK</td>
<td>178,458</td>
<td>33</td>
<td>24</td>
<td>2</td>
</tr>
<tr>
<td>A₂ HKG</td>
<td>171,191</td>
<td>38</td>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>A₃ KIX</td>
<td>118,984</td>
<td>33</td>
<td>24</td>
<td>3</td>
</tr>
<tr>
<td>A₄ MEL</td>
<td>155,236</td>
<td>20</td>
<td>24</td>
<td>2</td>
</tr>
<tr>
<td>A₅ MNL</td>
<td>169,871</td>
<td>14</td>
<td>24</td>
<td>2</td>
</tr>
<tr>
<td>A₆ NRT</td>
<td>127,618</td>
<td>49</td>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>A₇ PEK</td>
<td>161,019</td>
<td>20</td>
<td>24</td>
<td>2</td>
</tr>
<tr>
<td>A₈ SEL</td>
<td>211,852</td>
<td>20</td>
<td>17</td>
<td>2</td>
</tr>
<tr>
<td>A₉ SHA</td>
<td>128,095</td>
<td>13</td>
<td>24</td>
<td>1</td>
</tr>
<tr>
<td>A₁₀ SIN</td>
<td>177,372</td>
<td>69</td>
<td>24</td>
<td>2</td>
</tr>
<tr>
<td>A₁₁ SYD</td>
<td>279,152</td>
<td>24</td>
<td>24</td>
<td>3</td>
</tr>
<tr>
<td>A₁₂ TPE</td>
<td>109,777</td>
<td>22</td>
<td>24</td>
<td>3</td>
</tr>
</tbody>
</table>
With the use of these linguistic terms in assessing the criteria weights and performance ratings of the airports, a fuzzy weight vector $W = (w_1, w_2, \ldots, w_j)$ and a fuzzy decision matrix $X = (x_{ij}, x_{12}, \ldots, x_{ij})$ can be constructed.

### 4.4 Fuzzy MCDM Models

#### 4.4.1 Model development

To solve the airport performance evaluation problem, fuzzy MCDM models can be developed based on one defuzzification method ($\alpha$-cut and $\lambda$), three normalisation methods (Vector Normalisation $N_1$, Linear Scale Transformation between 0 to 1 $N_2$, and Linear Scale Transformation $N_3$) and two MCDM aggregation methods (SAW and TOPSIS). This model development produces six fuzzy MCDM models: $D$–$N_1$–$S$, $D$–$N_1$–$T$, $D$–$N_2$–$S$, $D$–$N_2$–$T$, $D$–$N_3$–$S$, and $D$–$N_3$–$T$ as shown in Table 4.8. The airport performance evaluation problem setting and the fuzzy MCDM model development process are shown in Figure 4.2.

<table>
<thead>
<tr>
<th>Model</th>
<th>Process 1</th>
<th>Equation</th>
<th>Process 2</th>
<th>Equation</th>
<th>Process 3</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>D–$N_1$–$S$</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>$N_1$</td>
<td>(3.2)-(3.3)</td>
<td>SAW</td>
<td>(4.1)</td>
</tr>
<tr>
<td>D–$N_1$–$T$</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>$N_1$</td>
<td>(3.2)-(3.3)</td>
<td>TOPSIS</td>
<td>(4.2)-(4.5)</td>
</tr>
<tr>
<td>D–$N_2$–$S$</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>$N_2$</td>
<td>(3.4)-(3.5)</td>
<td>SAW</td>
<td>(4.1)</td>
</tr>
<tr>
<td>D–$N_2$–$T$</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>$N_2$</td>
<td>(3.4)-(3.5)</td>
<td>TOPSIS</td>
<td>(4.2)-(4.5)</td>
</tr>
<tr>
<td>D–$N_3$–$S$</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>$N_3$</td>
<td>(3.6)-(3.7)</td>
<td>SAW</td>
<td>(4.1)</td>
</tr>
<tr>
<td>D–$N_3$–$T$</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>$N_3$</td>
<td>(3.6)-(3.7)</td>
<td>TOPSIS</td>
<td>(4.2)-(4.5)</td>
</tr>
</tbody>
</table>
The solution procedure of the fuzzy MCDM models can be summarised as follows:

**Step 1:** Generate a fuzzy performance matrix \( Z = \{w_jx_{ij}, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n\} \) by multiplying the fuzzy weight vector \( W \) by the fuzzy decision matrix \( X \), as given in Equation (3.1). With the use of triangular fuzzy numbers, the multiplication operation is based on interval arithmetic.

57
Step 2: Use the defuzzification method ($\alpha$-cut and $\lambda$), as given in Equations (3.15)-(3.16) on
the fuzzy numbers in the performance matrix, a crisp performance matrix $Z^i_{\alpha} = \{z^i_{j,\alpha}, i = 1, 2, ..., m; j = 1, 2, ..., n\}$ can be derived.

Step 3: Apply the three normalisation methods $N_1$ (Equations (3.2)-(3.3)), $N_2$ (Equations (3.4)-(3.5)), and $N_3$ (Equations (3.6)-(3.7)), respectively, to obtain the normalised performance matrix.

Step 4: Apply the two aggregation methods SAW (Equation (4.1)) and TOPSIS (Equations (4.2)-(4.5)), respectively, to obtain the overall performance value of the airports.

For a given decision maker’s degree of confidence level $\alpha$ in fuzzy assessments and a given index of optimism $\lambda$, the SAW method used to calculate the overall performance value of the airports can be represented as

$$V_i = \sum_{j=1}^{n} z^i_{j,\alpha}, i = 1, 2, ..., m; j = 1, 2, ..., n. \quad (4.1)$$

Similarly, the TOPSIS method can be represented as:

$$A^+_{\alpha} = (z^+_{1,\alpha}, z^+_{2,\alpha}, ..., z^+_{n,\alpha})$$
$$A^-_{\alpha} = (z^-_{1,\alpha}, z^-_{2,\alpha}, ..., z^-_{n,\alpha}) \quad (4.2)$$

where

$$z^+_{j,\alpha} = \max(z^\lambda_{1,\alpha}, z^\lambda_{2,\alpha}, ..., z^\lambda_{m,\alpha})$$
$$z^-_{j,\alpha} = \min(z^\lambda_{1,\alpha}, z^\lambda_{2,\alpha}, ..., z^\lambda_{m,\alpha}), j = 1, 2, ..., n. \quad (4.3)$$
The positive ideal solution $A_d^+$ and the negative ideal solution $A_d^-$ can be determined from the crisp performance matrix by selecting the maximum value and the minimum value respectively across all airports with respect to each criterion. They represent the performance vectors of the best possible airport and the performance vectors of the worst possible airport on $n$ criteria, respectively.

The overall performance of airport $A_i$ on criteria $C_j$ ($j=1, 2, ..., n$) can be expressed as a performance vector of $n$ elements. As such, the vector matching technique is used to measure how close a particular airport is to the best possible airport and the worst possible airport in terms of its performance, given as

$$S_{ia}^+ = \frac{A_{ia}^+ A_d^+}{\max(A_{ia}^+ A_d^+, A_{ia}^+ A_d^-)}$$

$$S_{ia}^- = \frac{A_{ia}^- A_d^-}{\max(A_{ia}^- A_d^+, A_{ia}^- A_d^-)} , i = 1, 2, ..., m.$$  

where $A_{ia}^+$ is the $i$th row of the crisp performance matrix, representing the corresponding performance vector of airport $A_i$ with respect to criteria $C_j$. $S_{ia}^+$ and $S_{ia}^-$ represent the degree of similarity between airport $A_i$ and the positive ideal solution and the negative ideal solution respectively.

For a given decision maker’s degree of confidence level ($\alpha$) in fuzzy assessments and a given index of optimism ($\lambda$), an overall performance value for each airport can be determined by

$$V_i = \frac{S_{ia}^+}{S_{ia}^+ + S_{ia}^-} , i = 1, 2, ..., m.$$  

(4.5)
### 4.4.2 Performance ranking results

The airline dimension is used as an example to illustrate Steps 1 and 2. With the fuzzy weight vector $W$ and the fuzzy decision matrix $X$ obtained from the fuzzy assessments as given in Tables 4.4 – 4.7, the fuzzy performance matrix for the 12 airports with respect to the four criteria ($C_{16}$, $C_{17}$, $C_{18}$, and $C_{19}$) under this dimension is obtained as

$$
Z = \begin{bmatrix}
(511579.6, 844701.2, 1201617.2) & (147.4, 213.4, 275) & (132.8, 180.8, 216) & (2.8, 6.8, 10.8) \\
(490747.5, 810304.1, 1152686.1) & (169.7, 245.7, 316.7) & (94.1, 128.1, 153) & (2.8, 6.8, 10.8) \\
(341087.5, 563190.9, 801158.9) & (147.4, 213.4, 275) & (132.8, 180.8, 216) & (4.2, 10.2, 16.2) \\
(445009.9, 734783.7, 1045255.7) & (89.3, 129.3, 166.7) & (132.8, 180.8, 216) & (2.8, 6.8, 10.8) \\
(486963.5, 804056.1, 1143798.1) & (62.5, 90.5, 116.7) & (132.8, 180.8, 216) & (2.8, 6.8, 10.8) \\
(365838.3, 604058.5, 859294.6) & (218.9, 316.9, 408.3) & (94.1, 128.1, 153) & (4.2, 10.2, 16.2) \\
(461587.8, 762156.5, 1084194.6) & (89.3, 129.3, 166.7) & (132.8, 180.8, 216) & (2.8, 6.8, 10.8) \\
(607303.3, 1002756.7, 1426456.7) & (89.3, 129.3, 166.7) & (94.1, 128.1, 153) & (2.8, 6.8, 10.8) \\
(367205.7, 606316.3, 862506.3) & (58.1, 84.1, 108.3) & (132.8, 180.8, 216) & (1.4, 3.4, 5.4) \\
(508466.4, 839560.8, 1194304.8) & (308.2, 446.2, 575) & (132.8, 180.8, 216) & (2.8, 6.8, 10.8) \\
(800235.7, 1321319.5, 1879623.5) & (107.2, 155.2, 200) & (132.8, 180.8, 216) & (4.2, 10.2, 16.2) \\
(314694.1, 519611.1, 739165.1) & (98.3, 142.3, 183.3) & (132.8, 180.8, 216) & (4.2, 10.2, 16.2)
\end{bmatrix}
$$

To obtain a crisp performance matrix, $\alpha = 0$ and $\lambda = 0.5$ are applied to Equations (3.15)-(3.16). This setting reflects that the decision maker has no particular preference for the fuzzy assessment results. $\alpha = 0$ implies that the mean value of a fuzzy number (Yager, 1981; Dubois and Prade, 1987); that is, the average of value intervals of all $\alpha$-cuts on the fuzzy number is used. $\lambda = 0.5$ indicates that the decision maker has a moderate attitude towards the fuzzy assessments; that is, the decision maker weights all the values resulted from fuzzy assessments equally. This defuzzification process thus produces a crisp performance matrix as
By applying three normalisation methods ($N_1$, $N_2$, and $N_3$) and two aggregation methods (SAW, TOPSIS), respectively, at Steps 3 and 4, six performance rankings of 12 airports are generated as shown in Table 4.9.

### Table 4.9

Performance rankings of 12 airports by the six fuzzy MCDM models

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td></td>
<td>4.986 (9)</td>
<td>8.419 (8)</td>
<td>11.902 (9)</td>
<td>0.433 (9)</td>
<td>0.720 (8)</td>
<td>0.470 (9)</td>
</tr>
<tr>
<td>A₂</td>
<td></td>
<td>5.454 (5)</td>
<td>7.911 (5)</td>
<td>12.712 (5)</td>
<td>0.487 (6)</td>
<td>0.793 (6)</td>
<td>0.514 (6)</td>
</tr>
<tr>
<td>A₃</td>
<td></td>
<td>5.774 (3)</td>
<td>11.496 (3)</td>
<td>13.143 (4)</td>
<td>0.505 (4)</td>
<td>0.852 (4)</td>
<td>0.543 (3)</td>
</tr>
<tr>
<td>A₄</td>
<td></td>
<td>5.255 (6)</td>
<td>11.450 (4)</td>
<td>13.250 (3)</td>
<td>0.502 (5)</td>
<td>0.949 (1)</td>
<td>0.525 (5)</td>
</tr>
<tr>
<td>A₅</td>
<td></td>
<td>3.879 (12)</td>
<td>5.580 (12)</td>
<td>10.249 (12)</td>
<td>0.371 (12)</td>
<td>0.667 (11)</td>
<td>0.389 (12)</td>
</tr>
<tr>
<td>A₆</td>
<td></td>
<td>5.812 (1)</td>
<td>8.898 (7)</td>
<td>12.200 (7)</td>
<td>0.511 (3)</td>
<td>0.720 (9)</td>
<td>0.526 (4)</td>
</tr>
<tr>
<td>A₇</td>
<td></td>
<td>4.640 (10)</td>
<td>7.058 (10)</td>
<td>10.994 (10)</td>
<td>0.389 (11)</td>
<td>0.629 (12)</td>
<td>0.440 (10)</td>
</tr>
<tr>
<td>A₈</td>
<td></td>
<td>5.187 (7)</td>
<td>8.345 (9)</td>
<td>11.937 (8)</td>
<td>0.465 (8)</td>
<td>0.682 (10)</td>
<td>0.493 (8)</td>
</tr>
<tr>
<td>A₉</td>
<td></td>
<td>4.276 (11)</td>
<td>6.985 (11)</td>
<td>10.961 (11)</td>
<td>0.408 (10)</td>
<td>0.769 (7)</td>
<td>0.436 (11)</td>
</tr>
<tr>
<td>A₁₀</td>
<td></td>
<td>5.800 (2)</td>
<td>11.508 (2)</td>
<td>13.468 (2)</td>
<td>0.544 (2)</td>
<td>0.855 (3)</td>
<td>0.554 (2)</td>
</tr>
<tr>
<td>A₁₁</td>
<td></td>
<td>5.658 (4)</td>
<td>12.781 (1)</td>
<td>14.114 (1)</td>
<td>0.565 (1)</td>
<td>0.936 (2)</td>
<td>0.571 (1)</td>
</tr>
<tr>
<td>A₁₂</td>
<td></td>
<td>5.141 (8)</td>
<td>9.256 (6)</td>
<td>12.351 (6)</td>
<td>0.473 (7)</td>
<td>0.823 (5)</td>
<td>0.503 (7)</td>
</tr>
</tbody>
</table>
With the fuzzy MCDM models used, the confidence level (represented by the $\alpha$ value) and the risk attitude (represented by the $\lambda$ value) may affect the evaluation result. A sensitivity analysis for the performance ranking results of the 12 airports is shown in Appendix B.

### 4.5 Ranking Validity of Fuzzy MCDM Models

In Table 4.9, the ranking results produced by the six fuzzy MCDM models are not consistent. The fuzzy clustering based validation method as given in Equations (3.17)-(3.19) is then applied to select a more valid ranking result among the six inconsistent results generated by the six fuzzy MCDM models. By assigning 12 airports to the best-performed and worst-performed clusters respectively based on the performance matrix obtained by each fuzzy MCDM model, a ranking of 12 airports based on their membership degrees is obtained.

Table 4.10
Cluster centers generated by fuzzy clustering

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_8$</th>
<th>$C_9$</th>
<th>$C_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best-Performed</td>
<td>0.293</td>
<td>0.357</td>
<td>0.416</td>
<td>0.951</td>
<td>0.390</td>
<td>0.348</td>
<td>0.517</td>
<td>0.431</td>
<td>0.604</td>
<td>0.406</td>
</tr>
<tr>
<td>Worst-Performed</td>
<td>0.382</td>
<td>0.425</td>
<td>0.204</td>
<td>0.591</td>
<td>0.156</td>
<td>0.315</td>
<td>0.175</td>
<td>0.448</td>
<td>0.299</td>
<td>0.506</td>
</tr>
<tr>
<td>$C_{11}$</td>
<td>$C_{12}$</td>
<td>$C_{13}$</td>
<td>$C_{14}$</td>
<td>$C_{15}$</td>
<td>$C_{16}$</td>
<td>$C_{17}$</td>
<td>$C_{18}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Best-Performed</td>
<td>0.701</td>
<td>0.646</td>
<td>0.753</td>
<td>0.724</td>
<td>0.847</td>
<td>0.433</td>
<td>0.477</td>
<td>0.666</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worst-Performed</td>
<td>0.273</td>
<td>0.498</td>
<td>0.276</td>
<td>0.272</td>
<td>0.341</td>
<td>0.273</td>
<td>0.168</td>
<td>0.789</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4.10 shows the cluster centre of the best-performed cluster and worst-performed cluster. The membership degree and ranking order of the 12 airports within the best-performed cluster and worst-performed cluster are listed in Table 4.11.

Table 4.11
Membership degree and ranking order of 12 airports within the two clusters

<table>
<thead>
<tr>
<th>Airport</th>
<th>Best-Performed Cluster</th>
<th>Worst-Performed Cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.331 (9)</td>
<td>0.669 (9)</td>
</tr>
<tr>
<td>A2</td>
<td>0.721 (2)</td>
<td>0.279 (2)</td>
</tr>
<tr>
<td>A3</td>
<td>0.682 (5)</td>
<td>0.318 (5)</td>
</tr>
<tr>
<td>A4</td>
<td>0.616 (6)</td>
<td>0.384 (6)</td>
</tr>
<tr>
<td>A5</td>
<td>0.196 (11)</td>
<td>0.804 (11)</td>
</tr>
<tr>
<td>A6</td>
<td>0.694 (4)</td>
<td>0.306 (4)</td>
</tr>
<tr>
<td>A7</td>
<td>0.186 (12)</td>
<td>0.814 (12)</td>
</tr>
<tr>
<td>A8</td>
<td>0.509 (7)</td>
<td>0.491 (7)</td>
</tr>
<tr>
<td>A9</td>
<td>0.240 (10)</td>
<td>0.760 (10)</td>
</tr>
<tr>
<td>A10</td>
<td>0.743 (1)</td>
<td>0.257 (1)</td>
</tr>
<tr>
<td>A11</td>
<td>0.713 (3)</td>
<td>0.287 (3)</td>
</tr>
<tr>
<td>A12</td>
<td>0.370 (8)</td>
<td>0.630 (8)</td>
</tr>
</tbody>
</table>

The ranking order of 12 airports within the best-performed cluster is the same as that within the worst-performed cluster. This suggests that only one ranking order of fuzzy clustering will be used to validate the fuzzy MCDM models in this application. In applications where the two ranking orders indicated by two clusters of fuzzy clustering are different, the validation results of fuzzy MCDM models based on the two ranking orders individually have to be averaged.
To validate the ranking order of the six fuzzy MCDM models based on that of fuzzy clustering, the Spearman’s rank correlation analysis is carried out as given in Equation (3.20). Table 4.12 shows the validation result. The ranking result of the D–N₁–T model has the highest correlation coefficients with the fuzzy clustering result. This suggests that the ranking result produced by the D–N₁–T model is the most valid one for this application.

Table 4.12
Spearman’s correlation coefficients between fuzzy MCDM models and fuzzy clustering

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.895</td>
<td>0.839</td>
<td>0.846</td>
<td>0.902</td>
<td>0.650</td>
<td>0.881</td>
</tr>
</tbody>
</table>

According to the ranking result produced by the D–N₁–T model shown in Table 4.9, the Kingsford Smith International Airport - Sydney (SYD) (A₁₁) has the best overall performance in terms of the airport operator, passenger, and airline dimensions, while the Ninoy Aquino International Airport - Manila (MNL) (A₃) has the worst overall performance.

Figure 4.3 shows some representative evaluation results under various settings of α and λ using the D–N₁–T model. It is noted that the ranking results of the D–N₁–T model remain consistent with different values of α and λ. This implies that the decision maker’s attitude towards the handling of the uncertainty associated with the fuzzy assessments in this case study does not have influence on the evaluation result in terms of relative ranking. This would give the decision maker a reliable assurance of the performance rankings of the 12 airports evaluated.
Figure 4.3. Overall preference value and ranking under various decision settings using the D–N1–T model

4.6 Concluding Remarks

As presented in this Chapter, the airport performance evaluation problem requires considering multiple evaluation criteria structured in a two-level hierarchy involving the airport operator, passenger, and airline dimensions. The evaluation requires using both quantitative and qualitative assessment data. In this study, the airport performance evaluation problem has thus been formulated as a fuzzy MCDM problem that requires cardinal ranking of all airports. Six fuzzy MCDM models, which differ in normalisation and aggregation processes, have been developed for solving the evaluation problem.
Different fuzzy MCDM models may result in different ranking results for a specific decision problem. As evidenced in this study, the six fuzzy MCDM models have resulted in six different performance rankings of the 12 Asia-Pacific international airports. The validation process based on fuzzy clustering has thus been used to validate inconsistent ranking results and the most valid ranking result has been selected for this application. The evaluation result validated and selected would provide the airports with indicative information about their relative strengths and weaknesses in terms of the evaluation criteria associated with the airport operator, the passenger, and the airline.
Chapter 5

Application II- Selection of Scrap Metal Buyers

5.1 Introduction

Over the past decade, U.S. steel industry has an average of 66.5% scrap metal recycling (Stundza, 2000). Scrap metal has become an important secondary source to the industry metal supply. It is important for the recycling company to select their scrap metal buyer carefully as they need to send out offers to the buyers on a daily basis. Some scrap metal buyers are metal broker agencies which trade the scrap metal. And some are scrap metal recovery companies, which process the metal reclamation. During the scrap metal recycling process, toxic chemical may be released by improper recycling methods (Slade, 1980; Chen et al., 2000). Therefore, both economic and environmental issues should be considered for selecting appropriate scrap metal buyers. Due to the core characteristic of the problem involving multiple and conflicting criteria, MCDM has been found to be effective in handling this buyer selection problem.

In this chapter, the selection of scrap metal buyers is to be formulated as a fuzzy MCDM problem that requires cardinal ranking of the potential buyers. A number of selection criteria are to be identified for the buyer selection problem. The methodological development presented in Chapter 3 will be used to obtain the most valid preference ranking of the buyers.
5.2 The Buyer Selection Problem

The buyer selection problem involves a set of \( m \) buyers (as decision alternatives) \( A_i \) \((i = 1, 2, ..., m)\), which are to be evaluated based on a set of \( n \) selection criteria \( C_j \) \((j = 1, 2, ..., n)\). Subjective assessments are to be made by the decision maker to determine (a) the weight vector \( W = (w_1, w_2, ..., w_j, ..., w_n) \), which represents the relative importance of \( n \) selection criteria \( C_j \) \((j = 1, 2, ..., n)\) for the problem, and (b) the decision matrix \( X = \{x_{ij}, i = 1, 2, ..., m; j = 1, 2, ..., n\} \), which represents the performance ratings of buyer \( A_i \) with respect to selection criteria \( C_j \). Given the weight vector \( W \) and decision matrix \( X \), the objective of the problem is to rank all the buyers by giving each of them an overall preference value with respect to all selection criteria.

In this application, five anonymous recycling companies in southern China are selected for evaluation. The scrap metal buyers are to be ranked based on economic and environmental criteria. These criteria are: \((C_1)\) Bidding prices, which is the price offered to the scrap metal; \((C_2)\) Financial credibility, which is related to the financial background of the potential buyer; \((C_3)\) Processing facility, which is concerned with the company’s reclamation facility, need to be environmental friendly; \((C_4)\) Possible long term relationship, which is preferred if the buyer is willing to build up a long term relationship as scrap metal trading is an on-going business. Figure 5.1 shows a hierarchical structure of the scrap metal buyer selection problem, including economic and environmental dimensions (Level 1) and their associated selection criteria (Level 2), and five scrap metal buyers (decision alternatives) (Level 3).
5.3 The Buyer Selection Assessment

The relative importance $w_j$ of the four selection criteria $C_j$ and the performance ratings $x_{ij}$ of the five scrap metal buyers $A_i$ with respect to the four evaluation criteria are to be assessed by three sets of linguistic terms, respectively. A 1-9 ratio scale is used and the approximate value range of the linguistic terms is represented by triangular fuzzy numbers. Table 5.1 shows the linguistic terms for the linguistic variable “importance”, used to assess the relative importance of the selection criteria. Tables 5.2 and 5.3 show the linguistic terms for the linguistic variable “performance”, used to assess the performance rating of scrap metal buyers.

Table 5.1
Linguistic terms for fuzzy importance weighting assessment

<table>
<thead>
<tr>
<th>Linguistic Term</th>
<th>Not Important</th>
<th>Somewhat Important</th>
<th>Important</th>
<th>Very Important</th>
<th>Extremely Important</th>
</tr>
</thead>
<tbody>
<tr>
<td>(NI)</td>
<td>(SI)</td>
<td>(I)</td>
<td>(VI)</td>
<td>(EI)</td>
<td></td>
</tr>
<tr>
<td>Membership</td>
<td>(1, 1, 3)</td>
<td>(1, 3, 5)</td>
<td>(3, 5, 7)</td>
<td>(5, 7, 9)</td>
<td>(7, 9, 9)</td>
</tr>
</tbody>
</table>
Table 5.2
Linguistic terms for fuzzy performance rating assessment ($C_1$, $C_2$, $C_4$)

<table>
<thead>
<tr>
<th>Linguistic Term</th>
<th>Very Low (VL)</th>
<th>Low (L)</th>
<th>Medium (M)</th>
<th>High (H)</th>
<th>Very High (VH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membership Function</td>
<td>(1, 1, 3)</td>
<td>(1, 3, 5)</td>
<td>(3, 5, 7)</td>
<td>(5, 7, 9)</td>
<td>(7, 9, 9)</td>
</tr>
</tbody>
</table>

Table 5.3
Linguistic terms for fuzzy performance rating assessment ($C_3$)

<table>
<thead>
<tr>
<th>Linguistic Term</th>
<th>Very Poor (VP)</th>
<th>Poor (P)</th>
<th>Fair (F)</th>
<th>Good (G)</th>
<th>Very Good (VG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membership Function</td>
<td>(1, 1, 3)</td>
<td>(1, 3, 5)</td>
<td>(3, 5, 7)</td>
<td>(5, 7, 9)</td>
<td>(7, 9, 9)</td>
</tr>
</tbody>
</table>

Table 5.4 shows the fuzzy weights assessed for the four selection criteria and the performance ratings for the five potential buyers with respect to each selection criterion. With the use of these linguistic terms in assessing criteria weights and performance ratings of scrap metal buyers, a fuzzy weight vector $W = (w_1, w_2, \ldots, w_j)$ and a fuzzy decision matrix $X = (x_{11}, x_{12}, \ldots, x_{ij})$ can be constructed.

Table 5.4
Assessment results for selection criteria and scrap metal buyers

<table>
<thead>
<tr>
<th>$C_j$</th>
<th>Selection criteria</th>
<th>$w_j$</th>
<th>Weights</th>
<th>Buyer $A_i$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>Bidding prices</td>
<td>$w_1$</td>
<td>EI</td>
<td>VH</td>
<td>H</td>
<td>VH</td>
<td>M</td>
<td>VH</td>
<td></td>
</tr>
<tr>
<td>$C_2$</td>
<td>Financial credibility</td>
<td>$w_2$</td>
<td>VI</td>
<td>H</td>
<td>VH</td>
<td>VH</td>
<td>H</td>
<td>VH</td>
<td></td>
</tr>
<tr>
<td>$C_3$</td>
<td>Processing facility</td>
<td>$w_3$</td>
<td>SI</td>
<td>G</td>
<td>VG</td>
<td>F</td>
<td>G</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>$C_4$</td>
<td>Possible long term relationship</td>
<td>$w_4$</td>
<td>I</td>
<td>VH</td>
<td>M</td>
<td>VL</td>
<td>L</td>
<td>H</td>
<td></td>
</tr>
</tbody>
</table>
5.4 Fuzzy MCDM Models

5.4.1 Model development

To solve the buyer selection problem formulated above, fuzzy MCDM models can be developed based on one defuzzification method ($\alpha$-cut and $\lambda$), three normalisation methods (Vector Normalisation $N_1$, Linear Scale Transformation between 0 to 1 $N_2$, and Linear Scale Transformation $N_3$) and three MCDM aggregation methods (SAW, TOPSIS, and WP). This model development produces seven fuzzy MCDM models: D–N$_1$–S, D–N$_1$–T, D–N$_2$–S, D–N$_2$–T, D–N$_3$–S, D–N$_3$–T, and D–W as shown in Table 5.5.

Table 5.5
Fuzzy MCDM models and solution procedures

<table>
<thead>
<tr>
<th>Model</th>
<th>Process 1</th>
<th>Equation</th>
<th>Process 2</th>
<th>Equation</th>
<th>Process 3</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Defuzzification</td>
<td>Process 2</td>
<td>Normalisation</td>
<td>Aggregation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D–N$_1$–S</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>$N_1$</td>
<td>(3.2)-(3.3)</td>
<td>SAW</td>
<td>(4.1)</td>
</tr>
<tr>
<td>D–N$_1$–T</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>$N_1$</td>
<td>(3.2)-(3.3)</td>
<td>TOPSIS</td>
<td>(4.2)-(4.5)</td>
</tr>
<tr>
<td>D–N$_2$–S</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>$N_2$</td>
<td>(3.4)-(3.5)</td>
<td>SAW</td>
<td>(4.1)</td>
</tr>
<tr>
<td>D–N$_2$–T</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>$N_2$</td>
<td>(3.4)-(3.5)</td>
<td>TOPSIS</td>
<td>(4.2)-(4.5)</td>
</tr>
<tr>
<td>D–N$_3$–S</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>$N_3$</td>
<td>(3.6)-(3.7)</td>
<td>SAW</td>
<td>(4.1)</td>
</tr>
<tr>
<td>D–N$_3$–T</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>$N_3$</td>
<td>(3.6)-(3.7)</td>
<td>TOPSIS</td>
<td>(4.2)-(4.5)</td>
</tr>
<tr>
<td>D–W</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td>-</td>
<td>-</td>
<td>WP</td>
<td>(5.1)-(5.2)</td>
</tr>
</tbody>
</table>

Figure 5.2 shows the problem setting of the buyer selection and the fuzzy MCDM model development process.
The solution procedure of fuzzy MCDM models can be summarised as follows:

Step 1: Generate a fuzzy performance matrix \( Z = \{w_i x_{ij}, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n\} \) by multiplying the fuzzy weight vector \( W \) by the fuzzy decision matrix \( X \), as given in Equation (3.1). With the use of triangular fuzzy numbers, the multiplication operation is based on interval arithmetic.
Step 2: Use the defuzzification method (α-cut and λ), as given in Equations (3.15)-(3.16) on the fuzzy numbers in the performance matrix, a crisp performance matrix \( Z_{\alpha}^{\lambda'} = \{z_{ij\alpha}, i = 1,2,...,m; j = 1,2,...,n\} \) can be derived.

Step 3: Apply the three normalisation methods \( N_1 \) (Equations (3.2)-(3.3)), \( N_2 \) (Equations (3.4)-(3.5)), and \( N_3 \) (Equations (3.6)-(3.7)), respectively, to obtain the normalised performance matrix.

Step 4: Apply the two aggregation methods SAW (Equation (4.1)) and TOPSIS (Equations (4.2)-(4.5)) for the normalised performance matrix, respectively, to obtain the overall preference value of the buyers.

Step 5: Apply the aggregation method WP (Equations (5.1)-(5.2) on the crisp performance matrix \( Z_{\alpha}^{\lambda'} \) to obtain the overall preference value of the buyers. With three \( \lambda \) values, 21 ranking results are to be generated.

For a given decision maker’s degree of confidence level \( \alpha \) in fuzzy assessments and a given index of optimism \( \lambda \), the WP method used to calculate the overall preference value of the buyers can be represented as

\[
S_{\alpha i}^{\lambda} = \prod_{j=1}^{n} z_{ij\alpha}^{\lambda'} \quad i = 1,2,...,m; \quad j = 1,2,...,n. \tag{5.1}
\]

For easy comparison with the preference values generated by the other methods, the relative preference value \( (V) \) of each buyer is given by
\[ V_{ai}^{\lambda} = \prod_{j=1}^{n} z_{ija}^{\lambda'}, \quad i = 1, 2, ..., m; \quad j = 1, 2, ..., n. \]  

where \( z_{ija}^{\lambda'} \) is the most favorable value for \( j \)th criteria.

### 5.4.2 Preference ranking results

With the fuzzy weight vector \( W \) and the fuzzy decision matrix \( X \) obtained from the fuzzy assessments as given in Table 5.4, the fuzzy performance matrix for the five buyers with respect to the four criteria (\( C_1, C_2, C_3, \) and \( C_4 \)) at Step 1 is obtained as

\[
Z = \begin{bmatrix}
(49, 81, 81) & (25, 49, 81) & (5, 21, 45) & (21, 45, 63) \\
(35, 63, 81) & (35, 63, 81) & (7, 27, 45) & (9, 25, 49) \\
(49, 81, 81) & (35, 63, 81) & (3, 15, 35) & (3, 5, 21) \\
(21, 45, 63) & (25, 49, 81) & (5, 21, 45) & (3, 15, 35) \\
(49, 81, 81) & (35, 63, 81) & (1, 9, 25) & (15, 35, 63)
\end{bmatrix}
\]

To obtain a crisp performance matrix, \( \alpha = 0 \) and \( \lambda = 0.5 \) are applied to Equations (3.15)-(3.16) at Step 2. This setting reflects that the decision maker has moderate attitude for the fuzzy assessment results. The defuzzification process thus produces a crisp performance matrix as

\[
Z_{ai}^{\lambda'} = \begin{bmatrix}
73 & 51 & 23 & 43.5 \\
60.5 & 60.5 & 26.5 & 27 \\
73 & 60.5 & 17 & 8.5 \\
43.5 & 51 & 23 & 17 \\
73 & 60.5 & 11 & 37
\end{bmatrix}
\]
By applying three normalisation methods ($N_1$, $N_2$, and $N_3$) and three aggregation methods (SAW, TOPSIS, and WP), respectively, at Steps 3 and 4, the seven fuzzy MCDM models generate seven sets of preference rankings for the five scrap metal buyers.

With the fuzzy MCDM models used, the confidence level (represented by the $\alpha$ value) and the risk attitude (represented by the $\lambda$ value) may affect the evaluation result. The value of $\alpha$ represents the decision maker’s degree of confidence in the fuzzy assessments, where $0 \leq \alpha \leq 1$. The value of $\lambda$ reflects the decision maker’s attitude towards risk, such as optimistic ($\lambda = 1$), moderate ($\lambda = 0.5$), or pessimistic ($\lambda = 0$). With different sets of $\alpha$ and $\lambda$ value incorporated at Step 2, different preference rankings would be generated. A sensitivity analysis for the preference ranking results of the five scrap metal buyers is shown in Appendix C. An example is given in Tables 5.6 to 5.12 to show the evaluation results produced by the seven fuzzy MCDM models under three typical settings of $\lambda$ ($\lambda = 0$, 0.5, and 1) with a moderate confidence level (i.e. $\alpha = 0.5$).

### Table 5.6
Preference ranking result of the D–$N_1$–S model ($\alpha = 0.5$)

<table>
<thead>
<tr>
<th>Buyer</th>
<th>$\lambda = 0$ Overall preference index</th>
<th>Ranking</th>
<th>$\lambda = 0.5$ Overall preference index</th>
<th>Ranking</th>
<th>$\lambda = 1$ Overall preference index</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.886</td>
<td>1</td>
<td>0.932</td>
<td>1</td>
<td>0.894</td>
<td>1</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.798</td>
<td>2</td>
<td>0.848</td>
<td>2</td>
<td>0.817</td>
<td>2</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.605</td>
<td>4</td>
<td>0.666</td>
<td>4</td>
<td>0.612</td>
<td>4</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.556</td>
<td>5</td>
<td>0.658</td>
<td>5</td>
<td>0.594</td>
<td>5</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.737</td>
<td>3</td>
<td>0.804</td>
<td>3</td>
<td>0.755</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 5.7
Preference ranking result of the D–N₂–S model (α = 0.5)

<table>
<thead>
<tr>
<th>Buyer</th>
<th>λ = 0</th>
<th>Overall preference index</th>
<th>Ranking</th>
<th>λ = 0.5</th>
<th>Overall preference index</th>
<th>Ranking</th>
<th>λ = 1</th>
<th>Overall preference index</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>0.333</td>
<td>3</td>
<td></td>
<td>0.306</td>
<td>3</td>
<td></td>
<td>0.289</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>A₂</td>
<td>0.263</td>
<td>5</td>
<td></td>
<td>0.224</td>
<td>5</td>
<td></td>
<td>0.187</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>A₃</td>
<td>0.417</td>
<td>2</td>
<td></td>
<td>0.403</td>
<td>2</td>
<td></td>
<td>0.395</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>A₄</td>
<td>0.790</td>
<td>1</td>
<td></td>
<td>0.746</td>
<td>1</td>
<td></td>
<td>0.716</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>A₅</td>
<td>0.319</td>
<td>4</td>
<td></td>
<td>0.296</td>
<td>4</td>
<td></td>
<td>0.280</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.8
Preference ranking result of the D–N₃–S model (α = 0.5)

<table>
<thead>
<tr>
<th>Buyer</th>
<th>λ = 0</th>
<th>Overall preference index</th>
<th>Ranking</th>
<th>λ = 0.5</th>
<th>Overall preference index</th>
<th>Ranking</th>
<th>λ = 1</th>
<th>Overall preference index</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>0.880</td>
<td>1</td>
<td></td>
<td>0.928</td>
<td>1</td>
<td></td>
<td>0.955</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>A₂</td>
<td>0.817</td>
<td>2</td>
<td></td>
<td>0.862</td>
<td>2</td>
<td></td>
<td>0.894</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>A₃</td>
<td>0.663</td>
<td>4</td>
<td></td>
<td>0.709</td>
<td>4</td>
<td></td>
<td>0.734</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>A₄</td>
<td>0.575</td>
<td>5</td>
<td></td>
<td>0.674</td>
<td>5</td>
<td></td>
<td>0.737</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>A₅</td>
<td>0.763</td>
<td>3</td>
<td></td>
<td>0.816</td>
<td>3</td>
<td></td>
<td>0.845</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.9
Preference ranking result of the D–N$_1$–T model ($\alpha = 0.5$)

<table>
<thead>
<tr>
<th>Buyer</th>
<th>$\lambda = 0$</th>
<th>$\lambda = 0.5$</th>
<th>$\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Overall preference value</td>
<td>Ranking</td>
<td>Overall preference value</td>
</tr>
<tr>
<td>$A_1$</td>
<td>0.708</td>
<td>1</td>
<td>0.666</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.624</td>
<td>3</td>
<td>0.596</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.469</td>
<td>5</td>
<td>0.468</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.450</td>
<td>4</td>
<td>0.473</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.600</td>
<td>2</td>
<td>0.590</td>
</tr>
</tbody>
</table>

Table 5.10
Preference ranking result of the D–N$_2$–T model ($\alpha = 0.5$)

<table>
<thead>
<tr>
<th>Buyer</th>
<th>$\lambda = 0$</th>
<th>$\lambda = 0.5$</th>
<th>$\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Overall preference value</td>
<td>Ranking</td>
<td>Overall preference value</td>
</tr>
<tr>
<td>$A_1$</td>
<td>0.389</td>
<td>3</td>
<td>0.403</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.282</td>
<td>5</td>
<td>0.322</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.444</td>
<td>2</td>
<td>0.453</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.666</td>
<td>1</td>
<td>0.708</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.384</td>
<td>4</td>
<td>0.395</td>
</tr>
</tbody>
</table>
### Table 5.11
Preference ranking result of the D–N₃–T model (α = 0.5)

<table>
<thead>
<tr>
<th>Buyer</th>
<th>λ = 0 Overall preference value</th>
<th>Ranking</th>
<th>λ = 0.5 Overall preference value</th>
<th>Ranking</th>
<th>λ = 1 Overall preference value</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>0.642</td>
<td>1</td>
<td>0.536</td>
<td>1</td>
<td>0.476</td>
<td>1</td>
</tr>
<tr>
<td>A₂</td>
<td>0.533</td>
<td>4</td>
<td>0.438</td>
<td>4</td>
<td>0.382</td>
<td>4</td>
</tr>
<tr>
<td>A₃</td>
<td>0.469</td>
<td>5</td>
<td>0.415</td>
<td>5</td>
<td>0.384</td>
<td>5</td>
</tr>
<tr>
<td>A₄</td>
<td>0.642</td>
<td>2</td>
<td>0.533</td>
<td>2</td>
<td>0.467</td>
<td>2</td>
</tr>
<tr>
<td>A₅</td>
<td>0.642</td>
<td>2</td>
<td>0.533</td>
<td>2</td>
<td>0.467</td>
<td>2</td>
</tr>
</tbody>
</table>

### Table 5.12
Preference ranking result of the D–W model (α = 0.5)

<table>
<thead>
<tr>
<th>Buyer</th>
<th>λ = 0 Overall preference value</th>
<th>Ranking</th>
<th>λ = 0.5 Overall preference value</th>
<th>Ranking</th>
<th>λ = 1 Overall preference value</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>0.081</td>
<td>1</td>
<td>0.967</td>
<td>1</td>
<td>0.984</td>
<td>1</td>
</tr>
<tr>
<td>A₂</td>
<td>0.006</td>
<td>2</td>
<td>0.068</td>
<td>2</td>
<td>0.094</td>
<td>2</td>
</tr>
<tr>
<td>A₃</td>
<td>0.000</td>
<td>5</td>
<td>0.000</td>
<td>5</td>
<td>0.000</td>
<td>5</td>
</tr>
<tr>
<td>A₄</td>
<td>0.000</td>
<td>4</td>
<td>0.001</td>
<td>4</td>
<td>0.001</td>
<td>4</td>
</tr>
<tr>
<td>A₅</td>
<td>0.011</td>
<td>3</td>
<td>0.052</td>
<td>3</td>
<td>0.052</td>
<td>3</td>
</tr>
</tbody>
</table>

It is noted that the ranking results generated by the seven fuzzy MCDM models are significantly different. In this case, the validity issue becomes crucial (Beuthe and Scannella, 2001; Guitouni and Martel, 1998; Yeh and Willis, 2001; Zanakis et al., 1998).
5.5 Ranking Validity of Fuzzy MCDM Models

As shown in Table 5.13, the ranking results using $\lambda = 0.5$ and $\alpha = 0.5$ are used as an example for demonstrating the validation process. The fuzzy clustering based validation method as given in Equations (3.17) to (3.19) is applied to select the most valid ranking result among these inconsistent results generated by the seven fuzzy MCDM models.

Table 5.13
Preference rankings of seven models ($\alpha=0.5$, $\lambda = 0.5$)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A2</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>A3</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>A4</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>A5</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

By using the validation process, the five scrap metal buyers can be assigned to the best-possible and worst-possible clusters respectively. Table 5.14 shows the membership degree and ranking order of the five scrap metal buyers within the two clusters. The ranking order of the five suppliers within the best-possible cluster is the same as that within the worst-possible cluster. This suggests that only one ranking order of fuzzy clustering will be used to validate the fuzzy MCDM models in this application.

To validate the ranking order of the seven fuzzy MCDM models based on that of fuzzy clustering, the Spearman’s rank correlation analysis is carried out as given in Equation (3.20). Table 5.15 shows the validation result.
Table 5.14
Membership degree and ranking order of five buyers to the two clusters

<table>
<thead>
<tr>
<th>Buyer</th>
<th>Membership degree in best-possible cluster</th>
<th>Membership degree in worst-possible cluster</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>0.946</td>
<td>0.053</td>
<td>1</td>
</tr>
<tr>
<td>A₂</td>
<td>0.882</td>
<td>0.117</td>
<td>2</td>
</tr>
<tr>
<td>A₃</td>
<td>0.166</td>
<td>0.833</td>
<td>4</td>
</tr>
<tr>
<td>A₄</td>
<td>0.087</td>
<td>0.912</td>
<td>5</td>
</tr>
<tr>
<td>A₅</td>
<td>0.816</td>
<td>0.183</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 5.15
Spearman’s correlation coefficients between fuzzy MCDM models and fuzzy clustering

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ</td>
<td>1</td>
<td>0.7</td>
<td>1</td>
<td>0.8</td>
<td>0.7</td>
<td>0.25</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The ranking result of both the D–N₁–S and D–N₃–S models has the highest correlation coefficients with the fuzzy clustering result. This suggests that the ranking result of these two models is the most valid one for this application. According to the ranking result by these two models shown in Table 5.13, buyer A₁ has the highest preference index and will be selected.

Figures 5.3 and 5.4 show some representative evaluation results under various settings of α and λ of the D–N₁–S and D–N₃–S models.
Figure 5.3. Overall preference value and ranking under various decision settings using the D–N$_1$–S model

Figure 5.4. Overall preference value and ranking under various decision settings using the D–N$_3$–S model
It is noted that the ranking results of the two models are more consistent when the risk attitude of the decision maker is pessimistic or moderate (i.e. $\lambda = 0$ or 0.5). The ranking result changes slightly only when the risk attitude is optimistic (i.e. $\lambda = 1$). This implies that the decision maker’s attitude towards the handling of the uncertainty associated with the fuzzy assessments in this case study does not have a significant influence on the evaluation result in terms of relative ranking, with an exception of the ranking change between the buyers $A_3$ and $A_4$ when $\lambda = 1$. This would give the decision maker a reasonable assurance of the preference rankings of the five buyers evaluated.

5.6 Concluding Remarks

In this buyer selection application, the selection of scrap metal buyers based on the criteria under the economic and environmental dimensions has been formulated as a fuzzy MCDM problem that requires cardinal ranking of all available buyers. Seven fuzzy MCDM models, which differ in normalisation and aggregation processes, have been developed for solving this buyer selection problem. With different ranking results generated, the fuzzy clustering process has conducted for validating these inconsistent ranking results. The D–N$_1$–S and D–N$_3$–S models have been selected as their ranking result is the most valid one for this application.
Chapter 6

Application III – Selection of Non-Ferrous Scrap Metal Suppliers

6.1 Introduction

It is a common perception that recycling is an environmentally friendly activity. For over half a decade, people worldwide have recycled paper, plastic and metals. The collection and processing of post-consumer recyclables have grown rapidly over the past four decades (Zhang and Forssberg, 1997). As one major recycling stream, non-ferrous scrap metal recycling is not simply a collection or processing of wastes, because it encompasses the reintroduction of the recyclables into the cycle of industrial production. The goal of reducing landfills is only achieved if there is demand for the secondary materials by producing industrials. In this case, recycling will not occur until someone buys or gets paid to take back the sorted materials, reprocess the collected scrap metal into metal ingot, and resells those materials back to the market.

The scrap metal trade (at the national or international level) is based on the use and reuse of waste materials (Grace et al., 1978). Scrap dealers/suppliers search for sources of affordable raw materials from certain wastes generated by producing industries (including the steel, copper, aluminium, rubber, and paper industries). Mills seek out recyclable materials by salvaging scrap generated in their own production (home scrap) and by acquiring scrap from outside sources such as independent scrap suppliers (purchased scrap). Scrap suppliers obviously constitute an important part of recycling, acquiring post-consumer and industrial
scrap (in materials ranging from rags, waste paper, bottles and cans to old automobiles to scrapped railcars, ships and old mill equipment), processing the materials, and selling them to mills for reuse. In this case, the mills (scrap metal buyers) will, whether consciously or subconsciously, consider a number of aspects (criteria) when deciding which supplier(s) to select to conduct business with.

A reliable and stable supplier is a crucial entity and a key success factor in supply chain management. The supplier selection process consequently becomes an integral component and a core issue in supply chain management decisions (Shahroodi et al., 2012). Selecting the most suitable supplier often involves multiple selection criteria (Benyoucef et al., 2003; Ng, 2008; Tahriri et al., 2008; Carter et al., 2010; Ho et al., 2010; Kontis and Vrysagotis, 2011; Tektas and Aytekin, 2011; Yadav, 2011). The supplier selection problem can thus be formulated as a multicriteria decision making (MCDM) problem due to its characteristics including (a) multiple selection criteria, and (b) both quantitative and qualitative data (Ghodsypour and O’Brien, 1998). Most studies on the buyer-seller relationship focus on supplier selection and evaluation (Pearson and Ellram, 1995; Swift and Gruben, 2000; Park and Krishnan, 2001). The main purpose of the supplier selection is to find the most suitable supplier from a number of available suppliers who will best meet the requirements of the selection criteria identified by the decision maker.

In this chapter, the selection of non-ferrous scrap metal suppliers is to be formulated as a fuzzy MCDM problem that requires cardinal ranking of the potential suppliers. The methodological development presented in Chapter 3 will be used to obtain a valid preference ranking of the suppliers. An international non-ferrous metal trading company is used to
demonstrate the applicability of the fuzzy MCDM model development and validation approach in solving the non-ferrous scrap metal supplier selection problem.

6.2 Non-Ferrous Scrap Metal Supplier Selection Problem

6.2.1 The global scrap metal market

A significant and key advantage for using scrap metal (secondary metal) is that it will greatly increase the level of energy savings and reduce the emission of CO2, as shown in Table 6.1.

Table 6.1
Energy and CO2 saving by using secondary material

<table>
<thead>
<tr>
<th>Material</th>
<th>Energy saving</th>
<th>CO2 Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>&gt; 95%</td>
<td>Aluminium</td>
</tr>
<tr>
<td>Copper</td>
<td>&gt; 85%</td>
<td>Copper</td>
</tr>
<tr>
<td>Steel</td>
<td>&gt;74%</td>
<td>Steel</td>
</tr>
<tr>
<td>Zinc</td>
<td>&gt; 60%</td>
<td>Nickel</td>
</tr>
<tr>
<td>Lead</td>
<td>&gt; 65%</td>
<td>Zinc</td>
</tr>
</tbody>
</table>

*Source: BIR Study on the Environmental Benefits of Recycling, 2009

The recycle and reuse of end-of-life electronics has progressively become a major challenge globally. In the US alone, it is reported that 75% of all used electric and electronic equipment is stored, 15% is landfilled, 7% is resold and 3% is recovered. In 1991, a study by Carnegie-Mellon University estimated that if the current rate at which the US discards scrap computers (10 million per year) continues, around 150 million old personal computers (PCs) and workstations will have been sent to landfills by the year 2005 (Riggle, 1993; McAdams,
However, this represents only a small portion of the electronic equipment that will require disposal in the US. Based on the result of a 1992 survey for the consumption of electric and electronic equipment in Western Europe, it shows that approximately 7 million tons of electric and electronic equipment items were consumed and the total waste approached 4 million tons, which accounted for 2-3% of the entire European waste stream. An annual increase of 3% is expected over the next decade for the amount of consumption by weight (Zhang and Forssberg, 1999). Figure 6.1 shows the percentages of scrap aluminium imported by 11 countries.

![Figure 6.1. World’s scrap aluminium imports by volume](image)

(2010 Data Source: Global Trade Information Services Inc.)

It is noted that China is the largest consumer of aluminum in the world mainly because China’s policies on environment and raw material procurement favor the use of scrap aluminum for energy efficiency. For example, the objectives of Chinese government’s 12th Five-Year Plan are (a) to increase infrastructure and targeted industrial development, (b) to
rapidly expand the automotive industry and clean energy vehicles, and (c) to develop the aerospace infrastructure. The scrap aluminum is a cost effective and energy efficient raw material source to achieve these objectives as it (a) requires only 5% of the energy used to make new aluminum, (b) reduces alloying costs, and (c) is infinitely recyclable. Given the growing needs and comparatively high price of the non-ferrous scrap metal in today's market, buyers need to pay close attention to the supplier selection in order to obtain high quality materials and services.

6.2.2 The problem formulation

In this application, a Chinese non-ferrous scrap metal buyer in Nanhai, Guangdong province, China, is selected to illustrate the selection problem. This company has a monthly purchasing budget of 1,000mt of Zorbas (equivalent of around USD 2 million), and they need to assess their suppliers monthly when they receive an offer from the suppliers. Therefore, it is necessary for the senior management of the company to perform monthly selection assessments of the available suppliers in order to rank them according to their performance and allocate the budget.

The supplier selection problem involves a set of m suppliers (as decision alternatives) $A_i$ ($i = 1, 2, \ldots, m$), which are to be evaluated based on a set of n selection criteria $C_j$ ($j = 1, 2, \ldots, n$). Subjective assessments are to be made by the decision maker of the company to determine (a) the weight vector $W = (w_1, w_2, \ldots, w_j, \ldots, w_n)$, which represents the relative importance of n selection criteria $C_j$ ($j = 1, 2, \ldots, n$) for the problem, and (b) the decision matrix $X = \{x_{ij}, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n\}$, which represents the performance ratings of supplier $A_i$ with respect to selection criteria $C_j$. Given the weight vector $W$ and decision matrix $X$, the objective of the problem is to rank all the suppliers by giving each of them an
overall preference value with respect to all selection criteria. Figure 6.2 shows a hierarchical structure of the scrap metal supplier selection problem.

![Supplier selection](image)

Figure 6.2. A hierarchical structure of the supplier selection problem

A comprehensive investigation is conducted to identify a set of supplier selection criteria. The purchase price of a specific scrap item, naturally, is the most prolific factor that plays a crucial role in the selection decision (Edwards and Pearce, 1978). The volatility of the commodity market often makes the purchase price an important selection criterion.

However, if the buyer expects to receive good quality materials with every shipment, they may agree to pay a premium price for the scrap metals. As lodging material claims and finding unwanted scraps of other materials can be time-consuming and problematic, scrap metal of a high quality of is always preferred by the buyer. The quality of non-ferrous scrap metal is based on the recovery rate of the non-ferrous metal. Contaminations such as dirt, wood, oil and moisture will lower the overall recovery rate of the metal content.

The scrap metal buyer/processor will, in most cases, be recycling the materials they purchase. For the purposes of this study, only companies recycling and consuming the metals
and not being traders are being examined. Subsequently, the metals are processed in order to create a new product. Despite the fact that many suppliers may be used at one time, on time shipments are of high importance. Therefore, an immediate (or close to immediate) shipment time is preferred. The scrap metal buyer also prefers to do business with suppliers who can ensure that the materials are going to be routed to the destination as quickly as possible. This is referred to as lead time. Given these two factors, if a scrap metal supplier has the reputation for frequent late shipments or long lead times, they may not be selected. Late shipments and long lead times often result in production delays, which could in turn lead to significant problems with customers, who purchase the finished product at the end of the process. In this case, the shipping time is an important selection criterion that needed to be taken into consideration when purchasing the material from a supplier.

Another criterion that must be considered is the volume of the material that the supplier may have. A certain scrap metal buyer may need a monthly supply of 300mt of a certain grade of aluminium or copper scrap in order to meet their production quotas. The buyer can achieve this amount by dealing with a number of suppliers, but if the buyer needs to purchase a number of different scrap items to produce a finished product (for example, four different grades of aluminium scrap), the buyer may intend to pay a premium to do business with a certain supplier because they can buy many items from the same place. It is all a matter of their priorities. In this case, a stable and consistent supplier with adequate quantity of material is preferred. In addition, if any issues do arise, it is imperative that the selected supplier is able to deal with problems expediently. The buyer will, at times, have to deal with late shipments, material quality issues and other headaches. It is important for the buyer to feel that the supplier(s) will assist her through this process and provide compensation for any losses in profits that may incur.
As shown in Table 6.2, five key selection criteria are identified in this application, including Yield ($C_1$), Price ($C_2$), Quality ($C_3$), Volume ($C_4$), and Shipping time ($C_5$).

Table 6.2
Selection criteria for non-ferrous scrap metal supplier selection

<table>
<thead>
<tr>
<th>Selection criteria $C_j$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield ($C_1$)</td>
<td>Twelve month average selling price of the Zorbas from each yard in use.</td>
</tr>
<tr>
<td>Price ($C_2$)</td>
<td>The purchase price of a specific scrap metal material.</td>
</tr>
<tr>
<td>Quality ($C_3$)</td>
<td>The cleanness of the scrap metal material.</td>
</tr>
<tr>
<td>Volume ($C_4$)</td>
<td>The volume of scrap metal available from each yard.</td>
</tr>
<tr>
<td>Shipping time ($C_5$)</td>
<td>The length of the travel time from origin yard to destination yard.</td>
</tr>
</tbody>
</table>

6.2.3 The supplier selection assessment

For the quantitative criteria such as yield, price and shipping time, the corresponding selection assessments are determined based on the historical data of the company and the data from the shipping line for the shipping time. For the qualitative criteria such as quality and volume, the data was gathered from the senior management of the company. A highly structured questionnaire was designed and handed out to the senior trading manager to get his subjective assessments. The relative importance $w_j$ of the five selection criteria $C_j$ and the performance ratings $x_{ij}$ of the 15 scrap metal suppliers $A_i$ with respect to the selection criteria are to be assessed by three sets of linguistic terms, respectively. A 1-9 ratio scale is used and the approximate value range of the linguistic terms is represented by triangular fuzzy numbers. Table 6.3 shows the linguistic terms for the linguistic variable “importance”, used to assess the relative importance of the selection criteria. Tables 6.4 and 6.5 show the
linguistic terms for the linguistic variable “performance”, used to assess the performance rating of non-ferrous scrap metal suppliers.

Table 6.3
Linguistic terms for fuzzy importance weighting assessment

<table>
<thead>
<tr>
<th>Linguistic Term</th>
<th>Not Important</th>
<th>Somewhat Important</th>
<th>Important</th>
<th>Very Important</th>
<th>Extremely Important</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(NI)</td>
<td>(SI)</td>
<td>(I)</td>
<td>(VI)</td>
<td>(EI)</td>
</tr>
<tr>
<td>Membership Function</td>
<td>(1, 1, 3)</td>
<td>(1, 3, 5)</td>
<td>(3, 5, 7)</td>
<td>(5, 7, 9)</td>
<td>(7, 9, 9)</td>
</tr>
</tbody>
</table>

Table 6.4
Linguistic terms for fuzzy performance rating assessment ($C_3$)

<table>
<thead>
<tr>
<th>Linguistic Term</th>
<th>Very Bad (VB)</th>
<th>Bad (B)</th>
<th>Average (A)</th>
<th>Good (G)</th>
<th>Very Good (VG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membership Function</td>
<td>(1, 1, 3)</td>
<td>(1, 3, 5)</td>
<td>(3, 5, 7)</td>
<td>(5, 7, 9)</td>
<td>(7, 9, 9)</td>
</tr>
</tbody>
</table>

Table 6.5
Linguistic terms for fuzzy performance rating assessment ($C_4$)

<table>
<thead>
<tr>
<th>Linguistic Term</th>
<th>Very Low (VL)</th>
<th>Low (L)</th>
<th>Medium (M)</th>
<th>High (H)</th>
<th>Very High (VH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membership Function</td>
<td>(1, 1, 3)</td>
<td>(1, 3, 5)</td>
<td>(3, 5, 7)</td>
<td>(5, 7, 9)</td>
<td>(7, 9, 9)</td>
</tr>
</tbody>
</table>

Tables 6.6 and 6.7 show the fuzzy weights assessed for the five selection criteria and the performance ratings for the 15 potential suppliers with respect to each selection criterion.
respectively. With the use of these linguistic terms in assessing the criteria weights and performance ratings of the suppliers, a fuzzy weight vector $W = (w_1, w_2, \ldots, w_j)$ and a fuzzy decision matrix $X = (x_{ij})$ can be constructed.

Table 6.6
Assessment result for weighting the selection criteria

<table>
<thead>
<tr>
<th>Selection criteria</th>
<th>$C_1$</th>
<th>$C_3$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criteria weight</td>
<td>$w_1$</td>
<td>$w_2$</td>
<td>$w_3$</td>
<td>$w_4$</td>
<td>$w_5$</td>
</tr>
<tr>
<td>EI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.7
Assessment result for the performance ratings of suppliers

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Yield ($C_1$)</th>
<th>Price ($C_2$)</th>
<th>Quality ($C_3$)</th>
<th>Volume ($C_4$)</th>
<th>Shipping Time ($C_5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.8975</td>
<td>1,866</td>
<td>VG</td>
<td>L</td>
<td>40</td>
</tr>
<tr>
<td>$A_2$</td>
<td>2.1588</td>
<td>1,876</td>
<td>VG</td>
<td>L</td>
<td>20</td>
</tr>
<tr>
<td>$A_3$</td>
<td>1.8913</td>
<td>1,893</td>
<td>A</td>
<td>L</td>
<td>30</td>
</tr>
<tr>
<td>$A_4$</td>
<td>1.5525</td>
<td>1,826</td>
<td>B</td>
<td>L</td>
<td>25</td>
</tr>
<tr>
<td>$A_5$</td>
<td>1.9454</td>
<td>1,913</td>
<td>VG</td>
<td>L</td>
<td>20</td>
</tr>
<tr>
<td>$A_6$</td>
<td>0.9772</td>
<td>1,833</td>
<td>G</td>
<td>VL</td>
<td>40</td>
</tr>
<tr>
<td>$A_7$</td>
<td>0.3079</td>
<td>1,863</td>
<td>G</td>
<td>VL</td>
<td>45</td>
</tr>
<tr>
<td>$A_8$</td>
<td>0.0288</td>
<td>2,103</td>
<td>G</td>
<td>A</td>
<td>30</td>
</tr>
<tr>
<td>$A_9$</td>
<td>0.7483</td>
<td>2,100</td>
<td>B</td>
<td>A</td>
<td>30</td>
</tr>
<tr>
<td>$A_{10}$</td>
<td>0.9179</td>
<td>1,870</td>
<td>A</td>
<td>H</td>
<td>40</td>
</tr>
<tr>
<td>$A_{11}$</td>
<td>0.0189</td>
<td>1,820</td>
<td>A</td>
<td>H</td>
<td>50</td>
</tr>
<tr>
<td>$A_{12}$</td>
<td>0.0047</td>
<td>1,815</td>
<td>B</td>
<td>A</td>
<td>55</td>
</tr>
<tr>
<td>$A_{13}$</td>
<td>0.6625</td>
<td>1,748</td>
<td>A</td>
<td>H</td>
<td>65</td>
</tr>
<tr>
<td>$A_{14}$</td>
<td>0.2988</td>
<td>1,815</td>
<td>A</td>
<td>VH</td>
<td>70</td>
</tr>
<tr>
<td>$A_{15}$</td>
<td>0.6880</td>
<td>1,848</td>
<td>VB</td>
<td>VH</td>
<td>70</td>
</tr>
</tbody>
</table>
6.3 Fuzzy MCDM Models

6.3.1 Model development

To solve the selection problem formulated above, fuzzy MCDM models can be developed based on three normalisation methods (Vector Normalisation $N_1$, Linear Scale Transformation between 0 to 1 $N_2$, and Linear Scale Transformation $N_3$), three MCDM aggregation methods (SAW, TOPSIS, and WP), and one defuzzification method ($\alpha$-cut and $\lambda$). This model development produces seven fuzzy MCDM models: $N_1$–S–D, $N_2$–S–D, $N_3$–S–D, $N_1$–T–D, $N_2$–T–D, $N_3$–T–D, and W–D as shown in Table 6.8.

Table 6.8
Fuzzy MCDM models and solution procedures

<table>
<thead>
<tr>
<th>Model</th>
<th>Normalisation</th>
<th>Process 1</th>
<th>Equation</th>
<th>Process 2</th>
<th>Equation</th>
<th>Process 3</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_1$–S–D</td>
<td>$N_1$</td>
<td>(3.2)-(3.3)</td>
<td>SAW</td>
<td>(3.8)</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td></td>
</tr>
<tr>
<td>$N_2$–S–D</td>
<td>$N_1$</td>
<td>(3.2)-(3.3)</td>
<td>SAW</td>
<td>(3.9)-(3.12)</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td></td>
</tr>
<tr>
<td>$N_3$–S–D</td>
<td>$N_2$</td>
<td>(3.4)-(3.5)</td>
<td>SAW</td>
<td>(3.8)</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td></td>
</tr>
<tr>
<td>$N_1$–T–D</td>
<td>$N_2$</td>
<td>(3.4)-(3.5)</td>
<td>TOPSIS</td>
<td>(3.9)-(3.12)</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td></td>
</tr>
<tr>
<td>$N_2$–T–D</td>
<td>$N_3$</td>
<td>(3.6)-(3.7)</td>
<td>TOPSIS</td>
<td>(3.8)</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td></td>
</tr>
<tr>
<td>$N_3$–T–D</td>
<td>$N_3$</td>
<td>(3.6)-(3.7)</td>
<td>TOPSIS</td>
<td>(3.9)-(3.12)</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td></td>
</tr>
<tr>
<td>W–D</td>
<td>-</td>
<td>-</td>
<td>WP</td>
<td>(3.13)-(3.14)</td>
<td>$\alpha$-cut and $\lambda$</td>
<td>(3.15)-(3.16)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.3 shows the problem setting of the supplier selection and the fuzzy MCDM model development process.
The solution procedure of fuzzy MCDM models can be summarised as follows:

Step 1: Generate a fuzzy performance matrix $Z = \{w_jx_i, i = 1, 2, ..., m; j = 1, 2, ..., n\}$ by multiplying the fuzzy weight vector $W$ by the fuzzy decision matrix $X$, as given in Equation (3.1). With the use of triangular fuzzy numbers, the multiplication operation is based on interval arithmetic.
Step 2: Apply the three normalisation methods $N_1$ (Equations (3.2)-(3.3)), $N_2$ (Equations (3.4)-(3.5)), and $N_3$ (Equations (3.6)-(3.7)), respectively, to obtain the normalised fuzzy performance matrix.

Step 3: Apply the two aggregation methods SAW (Equation (3.8)) and TOPSIS (Equations (3.9)-(3.12)) for the normalised fuzzy performance matrix, respectively, to obtain the overall fuzzy preference value of the suppliers.

Step 4: Apply the aggregation method WP (Equations (3.13)-(3.14) on the fuzzy performance matrix $Z$ to obtain the overall fuzzy preference value of the suppliers.

Step 5: Use the defuzzification method ($\alpha$-cut and $\lambda$), as given in Equations (3.15)-(3.16) on the overall fuzzy performance value of the suppliers, a crisp overall preference value can be derived. With three $\lambda$ values, 21 ranking results are to be generated.

6.3.2 Preference ranking results

With the fuzzy weight vector $W$ and the fuzzy decision matrix $X$ obtained from the fuzzy assessments as given in Tables 6.6 and 6.7, the fuzzy performance matrix for the five suppliers with respect to the five criteria ($C_1$, $C_2$, $C_3$, $C_4$ and $C_5$) at Step 1 is obtained as
Apply three normalisation methods \((N_1, N_2, \text{and } N_3)\) and three aggregation methods (SAW, TOPSIS, and WP), respectively, at Steps 3 and 4 for obtaining the overall fuzzy preference value of the suppliers. Then defuzzify the fuzzy outcomes into crisp preference values at Step 5 by using \(\alpha\)-cut and \(\lambda\) value.

The global commodity market is very volatile after the global financial crisis in 2008, in representing the market condition \(\lambda\) value is incorporated, where \(\lambda = 1\) represents the bullish market, \(\lambda = 0.5\) represents the moderate market, and \(\lambda = 0\) represents the bearish market. For representing the decision maker’s confidence level, \(\alpha\) value is applied, where \(\alpha = 1\) means the decision maker is in aggressive mood, \(\alpha = 0.5\) means the decision maker is in moderate mood, and \(\alpha = 0\) means the decision maker is in pessimistic mood.

Tables 6.9 to 6.15 show the preference rankings of the 15 suppliers obtained by the seven fuzzy MCDM models in the bullish market, the moderate market, and the bearish market, respectively.
Table 6.9
Preference ranking result of the N$_1$–S–D model ($\alpha = 0.5$)

<table>
<thead>
<tr>
<th>Supplier</th>
<th>$\lambda = 0$</th>
<th></th>
<th>$\lambda = 0.5$</th>
<th></th>
<th>$\lambda = 1$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Overall</td>
<td>Ranking</td>
<td>Overall</td>
<td>Ranking</td>
<td>Overall</td>
<td>Ranking</td>
</tr>
<tr>
<td>$A_1$</td>
<td>1.314</td>
<td>5</td>
<td>1.319</td>
<td>5</td>
<td>1.316</td>
<td>5</td>
</tr>
<tr>
<td>$A_2$</td>
<td>1.654</td>
<td>1</td>
<td>1.659</td>
<td>1</td>
<td>1.658</td>
<td>1</td>
</tr>
<tr>
<td>$A_3$</td>
<td>1.375</td>
<td>3</td>
<td>1.434</td>
<td>3</td>
<td>1.464</td>
<td>3</td>
</tr>
<tr>
<td>$A_4$</td>
<td>1.201</td>
<td>7</td>
<td>1.274</td>
<td>6</td>
<td>1.312</td>
<td>6</td>
</tr>
<tr>
<td>$A_5$</td>
<td>1.610</td>
<td>2</td>
<td>1.615</td>
<td>2</td>
<td>1.613</td>
<td>2</td>
</tr>
<tr>
<td>$A_6$</td>
<td>1.155</td>
<td>10</td>
<td>1.191</td>
<td>10</td>
<td>1.208</td>
<td>10</td>
</tr>
<tr>
<td>$A_7$</td>
<td>0.992</td>
<td>14</td>
<td>1.028</td>
<td>14</td>
<td>1.046</td>
<td>14</td>
</tr>
<tr>
<td>$A_8$</td>
<td>1.191</td>
<td>8</td>
<td>1.216</td>
<td>8</td>
<td>1.227</td>
<td>9</td>
</tr>
<tr>
<td>$A_9$</td>
<td>1.161</td>
<td>9</td>
<td>1.213</td>
<td>9</td>
<td>1.240</td>
<td>8</td>
</tr>
<tr>
<td>$A_{10}$</td>
<td>1.364</td>
<td>4</td>
<td>1.381</td>
<td>4</td>
<td>1.389</td>
<td>4</td>
</tr>
<tr>
<td>$A_{11}$</td>
<td>1.125</td>
<td>12</td>
<td>1.142</td>
<td>11</td>
<td>1.150</td>
<td>11</td>
</tr>
<tr>
<td>$A_{12}$</td>
<td>0.891</td>
<td>15</td>
<td>0.942</td>
<td>15</td>
<td>0.969</td>
<td>15</td>
</tr>
<tr>
<td>$A_{13}$</td>
<td>1.229</td>
<td>6</td>
<td>1.245</td>
<td>7</td>
<td>1.253</td>
<td>7</td>
</tr>
<tr>
<td>$A_{14}$</td>
<td>1.143</td>
<td>11</td>
<td>1.124</td>
<td>12</td>
<td>1.115</td>
<td>12</td>
</tr>
<tr>
<td>$A_{15}$</td>
<td>1.082</td>
<td>13</td>
<td>1.064</td>
<td>13</td>
<td>1.056</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 6.10
Preference ranking result of the N$_2$–S–D model ($\alpha = 0.5$)

<table>
<thead>
<tr>
<th>Supplier</th>
<th>$\lambda = 0$</th>
<th></th>
<th>$\lambda = 0.5$</th>
<th></th>
<th>$\lambda = 1$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Overall</td>
<td>Ranking</td>
<td>Overall</td>
<td>Ranking</td>
<td>Overall</td>
<td>Ranking</td>
</tr>
<tr>
<td>$A_1$</td>
<td>0.523</td>
<td>8</td>
<td>0.511</td>
<td>8</td>
<td>0.504</td>
<td>8</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.480</td>
<td>10</td>
<td>0.468</td>
<td>10</td>
<td>0.461</td>
<td>10</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.567</td>
<td>5</td>
<td>0.539</td>
<td>5</td>
<td>0.520</td>
<td>6</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.712</td>
<td>1</td>
<td>0.686</td>
<td>1</td>
<td>0.668</td>
<td>1</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.479</td>
<td>12</td>
<td>0.467</td>
<td>11</td>
<td>0.460</td>
<td>11</td>
</tr>
<tr>
<td>$A_6$</td>
<td>0.638</td>
<td>4</td>
<td>0.621</td>
<td>4</td>
<td>0.607</td>
<td>4</td>
</tr>
<tr>
<td>$A_7$</td>
<td>0.663</td>
<td>3</td>
<td>0.646</td>
<td>2</td>
<td>0.633</td>
<td>2</td>
</tr>
<tr>
<td>$A_8$</td>
<td>0.490</td>
<td>9</td>
<td>0.467</td>
<td>12</td>
<td>0.450</td>
<td>14</td>
</tr>
<tr>
<td>$A_9$</td>
<td>0.536</td>
<td>7</td>
<td>0.516</td>
<td>7</td>
<td>0.501</td>
<td>9</td>
</tr>
<tr>
<td>$A_{10}$</td>
<td>0.478</td>
<td>13</td>
<td>0.462</td>
<td>13</td>
<td>0.450</td>
<td>13</td>
</tr>
<tr>
<td>$A_{11}$</td>
<td>0.550</td>
<td>6</td>
<td>0.534</td>
<td>6</td>
<td>0.522</td>
<td>5</td>
</tr>
<tr>
<td>$A_{12}$</td>
<td>0.666</td>
<td>2</td>
<td>0.645</td>
<td>3</td>
<td>0.631</td>
<td>3</td>
</tr>
<tr>
<td>$A_{13}$</td>
<td>0.471</td>
<td>14</td>
<td>0.455</td>
<td>15</td>
<td>0.443</td>
<td>15</td>
</tr>
<tr>
<td>$A_{14}$</td>
<td>0.447</td>
<td>15</td>
<td>0.455</td>
<td>14</td>
<td>0.457</td>
<td>12</td>
</tr>
<tr>
<td>$A_{15}$</td>
<td>0.480</td>
<td>11</td>
<td>0.505</td>
<td>9</td>
<td>0.519</td>
<td>7</td>
</tr>
</tbody>
</table>
### Table 6.11
Preference ranking result of the N₃-S-D model (α = 0.5)

<table>
<thead>
<tr>
<th>Supplier</th>
<th>λ = 0 Overall preference value</th>
<th>Ranking</th>
<th>λ = 0.5 Overall preference value</th>
<th>Ranking</th>
<th>λ = 1 Overall preference value</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>0.656</td>
<td>5</td>
<td>0.656</td>
<td>5</td>
<td>0.688</td>
<td>6</td>
</tr>
<tr>
<td>A₂</td>
<td>0.841</td>
<td>1</td>
<td>0.841</td>
<td>1</td>
<td>0.873</td>
<td>1</td>
</tr>
<tr>
<td>A₃</td>
<td>0.683</td>
<td>4</td>
<td>0.683</td>
<td>4</td>
<td>0.751</td>
<td>3</td>
</tr>
<tr>
<td>A₄</td>
<td>0.606</td>
<td>8</td>
<td>0.606</td>
<td>8</td>
<td>0.679</td>
<td>7</td>
</tr>
<tr>
<td>A₅</td>
<td>0.824</td>
<td>2</td>
<td>0.824</td>
<td>2</td>
<td>0.856</td>
<td>2</td>
</tr>
<tr>
<td>A₆</td>
<td>0.566</td>
<td>11</td>
<td>0.566</td>
<td>11</td>
<td>0.620</td>
<td>9</td>
</tr>
<tr>
<td>A₇</td>
<td>0.488</td>
<td>14</td>
<td>0.488</td>
<td>14</td>
<td>0.542</td>
<td>12</td>
</tr>
<tr>
<td>A₈</td>
<td>0.641</td>
<td>6</td>
<td>0.641</td>
<td>6</td>
<td>0.688</td>
<td>5</td>
</tr>
<tr>
<td>A₉</td>
<td>0.607</td>
<td>7</td>
<td>0.607</td>
<td>7</td>
<td>0.664</td>
<td>8</td>
</tr>
<tr>
<td>A₁₀</td>
<td>0.690</td>
<td>3</td>
<td>0.690</td>
<td>3</td>
<td>0.726</td>
<td>4</td>
</tr>
<tr>
<td>A₁¹</td>
<td>0.566</td>
<td>10</td>
<td>0.566</td>
<td>10</td>
<td>0.603</td>
<td>11</td>
</tr>
<tr>
<td>A₁²</td>
<td>0.430</td>
<td>15</td>
<td>0.430</td>
<td>15</td>
<td>0.486</td>
<td>14</td>
</tr>
<tr>
<td>A₁₃</td>
<td>0.577</td>
<td>9</td>
<td>0.577</td>
<td>9</td>
<td>0.613</td>
<td>10</td>
</tr>
<tr>
<td>A₁₄</td>
<td>0.533</td>
<td>12</td>
<td>0.533</td>
<td>12</td>
<td>0.539</td>
<td>13</td>
</tr>
<tr>
<td>A₁₅</td>
<td>0.492</td>
<td>13</td>
<td>0.492</td>
<td>13</td>
<td>0.486</td>
<td>15</td>
</tr>
</tbody>
</table>

### Table 6.12
Preference ranking result of the N₁-T-D model (α = 0.5)

<table>
<thead>
<tr>
<th>Supplier</th>
<th>λ = 0 Overall preference value</th>
<th>Ranking</th>
<th>λ = 0.5 Overall preference value</th>
<th>Ranking</th>
<th>λ = 1 Overall preference value</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>0.546</td>
<td>5</td>
<td>0.521</td>
<td>6</td>
<td>0.507</td>
<td>7</td>
</tr>
<tr>
<td>A₂</td>
<td>0.760</td>
<td>1</td>
<td>0.742</td>
<td>1</td>
<td>0.733</td>
<td>1</td>
</tr>
<tr>
<td>A₃</td>
<td>0.616</td>
<td>3</td>
<td>0.628</td>
<td>3</td>
<td>0.634</td>
<td>3</td>
</tr>
<tr>
<td>A₄</td>
<td>0.507</td>
<td>7</td>
<td>0.532</td>
<td>5</td>
<td>0.544</td>
<td>4</td>
</tr>
<tr>
<td>A₅</td>
<td>0.733</td>
<td>2</td>
<td>0.713</td>
<td>2</td>
<td>0.703</td>
<td>2</td>
</tr>
<tr>
<td>A₆</td>
<td>0.457</td>
<td>10</td>
<td>0.462</td>
<td>8</td>
<td>0.464</td>
<td>8</td>
</tr>
<tr>
<td>A₇</td>
<td>0.356</td>
<td>14</td>
<td>0.363</td>
<td>14</td>
<td>0.365</td>
<td>14</td>
</tr>
<tr>
<td>A₈</td>
<td>0.445</td>
<td>12</td>
<td>0.443</td>
<td>11</td>
<td>0.440</td>
<td>11</td>
</tr>
<tr>
<td>A₉</td>
<td>0.417</td>
<td>13</td>
<td>0.431</td>
<td>13</td>
<td>0.440</td>
<td>10</td>
</tr>
<tr>
<td>A₁₀</td>
<td>0.552</td>
<td>4</td>
<td>0.545</td>
<td>4</td>
<td>0.542</td>
<td>5</td>
</tr>
<tr>
<td>A₁¹</td>
<td>0.446</td>
<td>11</td>
<td>0.440</td>
<td>12</td>
<td>0.436</td>
<td>13</td>
</tr>
<tr>
<td>A₁²</td>
<td>0.305</td>
<td>15</td>
<td>0.327</td>
<td>15</td>
<td>0.338</td>
<td>15</td>
</tr>
<tr>
<td>A₁₃</td>
<td>0.516</td>
<td>6</td>
<td>0.513</td>
<td>7</td>
<td>0.511</td>
<td>6</td>
</tr>
<tr>
<td>A₁₄</td>
<td>0.485</td>
<td>8</td>
<td>0.460</td>
<td>9</td>
<td>0.449</td>
<td>9</td>
</tr>
<tr>
<td>A₁₅</td>
<td>0.470</td>
<td>9</td>
<td>0.448</td>
<td>10</td>
<td>0.438</td>
<td>12</td>
</tr>
</tbody>
</table>
Table 6.13
Preference ranking result of the N₂–T–D model (α = 0.5)

<table>
<thead>
<tr>
<th>Supplier</th>
<th>λ = 0 Overall preference value</th>
<th>Ranking</th>
<th>λ = 0.5 Overall preference value</th>
<th>Ranking</th>
<th>λ = 1 Overall preference value</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>0.650</td>
<td>8</td>
<td>0.662</td>
<td>8</td>
<td>0.669</td>
<td>8</td>
</tr>
<tr>
<td>A₂</td>
<td>1.000</td>
<td>1</td>
<td>1.000</td>
<td>1</td>
<td>1.000</td>
<td>1</td>
</tr>
<tr>
<td>A₃</td>
<td>0.751</td>
<td>6</td>
<td>0.778</td>
<td>6</td>
<td>0.797</td>
<td>5</td>
</tr>
<tr>
<td>A₄</td>
<td>0.770</td>
<td>5</td>
<td>0.786</td>
<td>5</td>
<td>0.796</td>
<td>6</td>
</tr>
<tr>
<td>A₅</td>
<td>1.000</td>
<td>1</td>
<td>1.000</td>
<td>1</td>
<td>1.000</td>
<td>1</td>
</tr>
<tr>
<td>A₆</td>
<td>0.469</td>
<td>12</td>
<td>0.501</td>
<td>11</td>
<td>0.536</td>
<td>11</td>
</tr>
<tr>
<td>A₇</td>
<td>0.372</td>
<td>14</td>
<td>0.394</td>
<td>14</td>
<td>0.432</td>
<td>13</td>
</tr>
<tr>
<td>A₈</td>
<td>0.851</td>
<td>3</td>
<td>0.867</td>
<td>3</td>
<td>0.878</td>
<td>3</td>
</tr>
<tr>
<td>A₉</td>
<td>0.804</td>
<td>4</td>
<td>0.818</td>
<td>4</td>
<td>0.827</td>
<td>4</td>
</tr>
<tr>
<td>A₁₀</td>
<td>0.695</td>
<td>7</td>
<td>0.712</td>
<td>7</td>
<td>0.725</td>
<td>7</td>
</tr>
<tr>
<td>A₁₁</td>
<td>0.523</td>
<td>9</td>
<td>0.547</td>
<td>9</td>
<td>0.565</td>
<td>9</td>
</tr>
<tr>
<td>A₁₂</td>
<td>0.257</td>
<td>15</td>
<td>0.277</td>
<td>15</td>
<td>0.302</td>
<td>15</td>
</tr>
<tr>
<td>A₁₃</td>
<td>0.505</td>
<td>10</td>
<td>0.524</td>
<td>10</td>
<td>0.540</td>
<td>10</td>
</tr>
<tr>
<td>A₁₄</td>
<td>0.498</td>
<td>11</td>
<td>0.474</td>
<td>12</td>
<td>0.466</td>
<td>12</td>
</tr>
<tr>
<td>A₁₅</td>
<td>0.466</td>
<td>13</td>
<td>0.418</td>
<td>13</td>
<td>0.391</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 6.14
Preference ranking result of the N₃–T–D model (α = 0.5)

<table>
<thead>
<tr>
<th>Supplier</th>
<th>λ = 0 Overall preference value</th>
<th>Ranking</th>
<th>λ = 0.5 Overall preference value</th>
<th>Ranking</th>
<th>λ = 1 Overall preference value</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>0.529</td>
<td>6</td>
<td>0.527</td>
<td>6</td>
<td>0.525</td>
<td>6</td>
</tr>
<tr>
<td>A₂</td>
<td>0.670</td>
<td>1</td>
<td>0.662</td>
<td>1</td>
<td>0.657</td>
<td>1</td>
</tr>
<tr>
<td>A₃</td>
<td>0.529</td>
<td>5</td>
<td>0.549</td>
<td>4</td>
<td>0.561</td>
<td>3</td>
</tr>
<tr>
<td>A₄</td>
<td>0.456</td>
<td>11</td>
<td>0.467</td>
<td>10</td>
<td>0.479</td>
<td>10</td>
</tr>
<tr>
<td>A₅</td>
<td>0.650</td>
<td>2</td>
<td>0.643</td>
<td>2</td>
<td>0.637</td>
<td>2</td>
</tr>
<tr>
<td>A₆</td>
<td>0.440</td>
<td>12</td>
<td>0.461</td>
<td>12</td>
<td>0.473</td>
<td>11</td>
</tr>
<tr>
<td>A₇</td>
<td>0.387</td>
<td>14</td>
<td>0.411</td>
<td>14</td>
<td>0.425</td>
<td>14</td>
</tr>
<tr>
<td>A₈</td>
<td>0.489</td>
<td>10</td>
<td>0.504</td>
<td>7</td>
<td>0.512</td>
<td>7</td>
</tr>
<tr>
<td>A₉</td>
<td>0.436</td>
<td>13</td>
<td>0.455</td>
<td>13</td>
<td>0.466</td>
<td>12</td>
</tr>
<tr>
<td>A₁₀</td>
<td>0.550</td>
<td>3</td>
<td>0.550</td>
<td>3</td>
<td>0.552</td>
<td>4</td>
</tr>
<tr>
<td>A₁₁</td>
<td>0.491</td>
<td>9</td>
<td>0.494</td>
<td>9</td>
<td>0.497</td>
<td>8</td>
</tr>
<tr>
<td>A₁₂</td>
<td>0.365</td>
<td>15</td>
<td>0.387</td>
<td>15</td>
<td>0.400</td>
<td>15</td>
</tr>
<tr>
<td>A₁₃</td>
<td>0.530</td>
<td>4</td>
<td>0.532</td>
<td>5</td>
<td>0.535</td>
<td>5</td>
</tr>
<tr>
<td>A₁₄</td>
<td>0.517</td>
<td>7</td>
<td>0.499</td>
<td>8</td>
<td>0.493</td>
<td>9</td>
</tr>
<tr>
<td>A₁₅</td>
<td>0.492</td>
<td>8</td>
<td>0.466</td>
<td>11</td>
<td>0.455</td>
<td>13</td>
</tr>
</tbody>
</table>
Table 6.15
Preference ranking result of the W–D model (α = 0.5)

<table>
<thead>
<tr>
<th>Supplier</th>
<th>λ = 0 Overall preference value</th>
<th>λ = 0.5 Overall preference value</th>
<th>λ = 1 Overall preference value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ranking</td>
<td>Ranking</td>
<td>Ranking</td>
</tr>
<tr>
<td>A1</td>
<td>0.260</td>
<td>0.238</td>
<td>0.209</td>
</tr>
<tr>
<td>A2</td>
<td>0.594</td>
<td>0.594</td>
<td>0.598</td>
</tr>
<tr>
<td>A3</td>
<td>0.331</td>
<td>0.346</td>
<td>0.343</td>
</tr>
<tr>
<td>A4</td>
<td>0.186</td>
<td>0.207</td>
<td>0.216</td>
</tr>
<tr>
<td>A5</td>
<td>0.553</td>
<td>0.552</td>
<td>0.554</td>
</tr>
<tr>
<td>A6</td>
<td>0.169</td>
<td>0.155</td>
<td>0.137</td>
</tr>
<tr>
<td>A7</td>
<td>0.067</td>
<td>0.058</td>
<td>0.047</td>
</tr>
<tr>
<td>A8</td>
<td>0.022</td>
<td>0.019</td>
<td>0.014</td>
</tr>
<tr>
<td>A9</td>
<td>0.147</td>
<td>0.155</td>
<td>0.150</td>
</tr>
<tr>
<td>A10</td>
<td>0.273</td>
<td>0.265</td>
<td>0.242</td>
</tr>
<tr>
<td>A11</td>
<td>0.013</td>
<td>0.010</td>
<td>0.007</td>
</tr>
<tr>
<td>A12</td>
<td>0.002</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>A13</td>
<td>0.186</td>
<td>0.170</td>
<td>0.142</td>
</tr>
<tr>
<td>A14</td>
<td>0.100</td>
<td>0.083</td>
<td>0.063</td>
</tr>
<tr>
<td>A15</td>
<td>0.082</td>
<td>0.070</td>
<td>0.056</td>
</tr>
</tbody>
</table>

To examine whether the confidence level (the α value) and the market condition (the λ value) may affect the evaluation result, a sensitivity analysis for the preference ranking results of the 15 scrap metal suppliers by changing the values of α and λ is shown in Appendix D.

### 6.4 Ranking Validity of Fuzzy MCDM Models

It is clearly that the preference ranking results produced by the seven fuzzy MCDM models are not consistent. As shown in Table 6.16, the ranking results using λ = 0.5 and α = 0.5 are used as an example for demonstrating the validation process. The fuzzy clustering based validation method given in Equations (3.17)-(3.19) is then applied to select the most valid ranking result among the inconsistent results.
Two clusters are used to represent the best-preferred and worst-preferred clusters. Table 6.17 shows the membership degree and ranking orders of the 15 suppliers to the two clusters. The ranking order of the 15 suppliers within the best-preferred cluster is the same as that within the worst-preferred cluster. This suggests that only one ranking order of fuzzy clustering will be used to validate the fuzzy MCDM models in this application.

Table 6.16
Preference rankings of seven models (α = 0.5, λ = 0.5)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>5</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>A₂</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A₃</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>A₄</td>
<td>6</td>
<td>1</td>
<td>8</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>A₅</td>
<td>2</td>
<td>11</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>A₆</td>
<td>10</td>
<td>4</td>
<td>11</td>
<td>8</td>
<td>11</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>A₇</td>
<td>14</td>
<td>2</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>A₈</td>
<td>8</td>
<td>12</td>
<td>6</td>
<td>11</td>
<td>3</td>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>A₉</td>
<td>9</td>
<td>7</td>
<td>7</td>
<td>13</td>
<td>4</td>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>A₁₀</td>
<td>4</td>
<td>13</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>A₁₁</td>
<td>11</td>
<td>6</td>
<td>10</td>
<td>12</td>
<td>9</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td>A₁₂</td>
<td>15</td>
<td>3</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>A₁₃</td>
<td>7</td>
<td>15</td>
<td>9</td>
<td>7</td>
<td>10</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>A₁₄</td>
<td>12</td>
<td>14</td>
<td>12</td>
<td>9</td>
<td>12</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>A₁₅</td>
<td>13</td>
<td>9</td>
<td>13</td>
<td>10</td>
<td>13</td>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

To validate the ranking order of the seven fuzzy MCDM models based on that of fuzzy clustering, the Spearman’s rank correlation analysis is carried out as given in Equation (3.20). Table 6.18 shows the validation result. The ranking result of the W–D model has the highest
correlation coefficients with the fuzzy clustering result. This suggests that the ranking result of the W–D model is the most valid one for this application. According to the ranking result by the W–D model shown in Table 6.16, supplier $A_2$ has the highest preference value and will be selected.

Table 6.17
Membership degree and ranking order of 15 suppliers to the two clusters

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Membership degree in best-preferred cluster</th>
<th>Ranking</th>
<th>Membership degree in worst-preferred cluster</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.851</td>
<td>5</td>
<td>0.149</td>
<td>5</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.875</td>
<td>3</td>
<td>0.125</td>
<td>3</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.898</td>
<td>1</td>
<td>0.102</td>
<td>1</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.750</td>
<td>6</td>
<td>0.250</td>
<td>6</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.898</td>
<td>2</td>
<td>0.102</td>
<td>2</td>
</tr>
<tr>
<td>$A_6$</td>
<td>0.866</td>
<td>4</td>
<td>0.134</td>
<td>4</td>
</tr>
<tr>
<td>$A_7$</td>
<td>0.696</td>
<td>7</td>
<td>0.304</td>
<td>7</td>
</tr>
<tr>
<td>$A_8$</td>
<td>0.556</td>
<td>8</td>
<td>0.444</td>
<td>8</td>
</tr>
<tr>
<td>$A_9$</td>
<td>0.527</td>
<td>9</td>
<td>0.473</td>
<td>9</td>
</tr>
<tr>
<td>$A_{10}$</td>
<td>0.234</td>
<td>10</td>
<td>0.766</td>
<td>10</td>
</tr>
<tr>
<td>$A_{11}$</td>
<td>0.073</td>
<td>15</td>
<td>0.927</td>
<td>15</td>
</tr>
<tr>
<td>$A_{12}$</td>
<td>0.010</td>
<td>13</td>
<td>0.900</td>
<td>13</td>
</tr>
<tr>
<td>$A_{13}$</td>
<td>0.104</td>
<td>12</td>
<td>0.896</td>
<td>12</td>
</tr>
<tr>
<td>$A_{14}$</td>
<td>0.074</td>
<td>14</td>
<td>0.926</td>
<td>14</td>
</tr>
<tr>
<td>$A_{15}$</td>
<td>0.131</td>
<td>11</td>
<td>0.869</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 6.18
Spearman’s correlation coefficients between fuzzy MCDM models and fuzzy clustering

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.675</td>
<td>0.236</td>
<td>0.611</td>
<td>0.643</td>
<td>0.555</td>
<td>0.404</td>
<td>0.704</td>
</tr>
</tbody>
</table>

102
Figure 6.4 shows some representative evaluation results under various settings of $\alpha$ and $\lambda$ using the W–D model. It is noted that the ranking result is more consistent when the global commodity market moderate and bullish (i.e. $\lambda = 0.5$ or 1). The ranking results change slightly only for suppliers $A_4$, $A_9$ and $A_{13}$, and would not affect the selection decision. This would give the decision maker of the company a reasonable assurance of the preference rankings of the 15 suppliers evaluated.

- **Degree of confidence level ($\alpha$)**

Figure 6.4. Overall preference value and ranking under various decision settings using the W–D model

### 6.5 Concluding Remarks

Selecting non-ferrous scrap metal suppliers involves a supplier assessment process based on multiple selection criteria. MCDM has shown advantages in ranking the performance of a set of decision alternatives with respect to multiple criteria in various decision contexts. In
this chapter, the non-ferrous scrap metal supplier selection problem has been formulated as a fuzzy MCDM problem. Given the identified selection criteria weights and the performance ratings assessed for 15 available suppliers, seven fuzzy MCDM models (N₁–S–D, N₂–S–D, N₃–S–D, N₁–T–D, N₂–T–D, N₃–T–D, and W–D) have been applied for generating different performance value rankings. To deal with the ranking inconsistency problem, the fuzzy clustering based validation process has been used and the W–D model has been selected as the most valid one due to its highest ranking correlation with the ranking result generated by fuzzy clustering.
Chapter 7

Conclusion

7.1 Summary of Research Developments

As presented in Chapter 3, this study has developed a new fuzzy multicriteria decision making (MCDM) model development and validation approach for solving the general evaluation and selection problem under various decision settings. This approach has proposed a structured procedure for developing fuzzy MCDM models by applying three normalisation methods, three aggregation methods, and a $\alpha$-cut based defuzzification method. As evidenced in the three empirical applications presented, different fuzzy MCDM models often result in different ranking results for a specific decision problem. In this study, a new validation process based on fuzzy clustering has been developed for validating inconsistent ranking results for a given problem data set.

Three empirical applications have been conducted in Chapters 4 to 6 to illustrate how this model development and validation approach can be used to develop fuzzy MCDM models and help select the most valid ranking result for a given problem. Figure 7.1 shows a comparison of the problem settings and model developments of the three applications. In each application, context-specific performance measures and decision settings have been identified, and the effectiveness of the approach has been highlighted.
Figure 7.1. A comparison of the three applications conducted

In Chapter 4, the fuzzy MCDM model development and validation approach has been used to address the performance evaluation problem of 12 Asia-Pacific international airports. This application has the following developments:

(a) Nineteen quantitative and qualitative evaluation criteria for airport performance evaluation are identified under the airport operator, passenger, and airline dimensions.

(b) Assessments are conducted using two sets of linguistic terms with fuzzy numbers for measuring the importance of each evaluation criterion and the performance of each airport with respect to the evaluation criteria.

(c) Six fuzzy MCDM models are developed based on three normalisation methods ($N_1$, $N_2$, $N_3$), two aggregation methods (SAW, TOPSIS), and one defuzzification method.
(α-cut and λ), using the solution procedure of defuzzification, normalisation and aggregation, for solving the airport performance evaluation problem.

(d) The fuzzy clustering based validation process is applied for validating inconsistent ranking results produced by the six fuzzy MCDM models, and the D-N1-T model is selected as the most valid model.

In Chapter 5, the fuzzy MCDM model development and validation approach has been used to address the buyer selection problem of five scrap metal buyers. This application has the following developments:

(a) Four qualitative selection criteria for buyer selection are identified under the economic and environmental dimensions.

(b) Assessments are conducted using three sets of linguistic terms with triangular fuzzy numbers for measuring the importance of each selection criterion and the performance of each potential buyer with respect to the selection criteria.

(c) Seven fuzzy MCDM models are developed based on three normalisation methods (N1, N2, N3), three aggregation methods (SAW, TOPSIS, WP), and one defuzzification method (α-cut and λ), using the solution procedure of defuzzification, normalisation and aggregation, for solving the buyer selection problem.

(d) The fuzzy clustering based validation process is applied for validating inconsistent ranking results produced by the seven fuzzy MCDM models, and the D-N1-S and D-N3-S models, producing the same result, are selected as the most valid models.

In Chapter 6, the fuzzy MCDM model development and validation approach has been used to address the supplier selection problem of 15 scrap metal suppliers. This application has the following developments:
(a) Five quantitative and qualitative selection criteria for supplier selection are identified.

(b) Assessments are conducted using three sets of linguistic terms with triangular fuzzy numbers for measuring the importance of each selection criterion and the performance of each potential supplier with respect to the selection criteria.

(c) Seven fuzzy MCDM models are developed based on one defuzzification method (\(\alpha\)-cut and \(\lambda\)), three normalisation methods (\(N_1\), \(N_2\), and \(N_3\)), and three aggregation methods (SAW, TOPSIS, and WP), using the solution procedure of normalisation, aggregation, and defuzzification, for solving the supplier selection problem.

(d) The fuzzy clustering based validation process is applied for validating inconsistent ranking results produced by the seven fuzzy MCDM models, and the W-D model is selected as the most valid model.

### 7.2 Research Contributions

This study makes new, significant methodological and practical contributions to MCDM research and applications, outlined below:

(a) A structured approach for the development and validation of fuzzy MCDM models.

The new fuzzy MCDM model development and validation approach addresses the challenging issue of validating the inconsistent ranking results obtained by various fuzzy MCDM models for a given decision problem. The fuzzy MCDM model development process can be used to develop fuzzy MCDM models by incorporating any number of available normalisation, aggregation, and defuzzification methods which are acceptable to the decision maker, not necessarily limited to the methods used in this study. With its simplicity in both concept and computation, the fuzzy MCDM validation process can be applied to the general fuzzy MCDM cardinal ranking problem. In solving a given fuzzy MCDM problem with many
models available and acceptable to the decision maker (not necessarily limited to the models
developed in this study), the validation process can be applied to all available models for
identifying the most valid ranking result. It is particularly suited to decision situations where the
ranking results produced by different fuzzy MCDM models differ significantly.

(b) A novel method for incorporating the decision maker’s attitude on fuzzy assessments
into the evaluation or selection decisions.

The airport performance evaluation, the buyer selection, and the supplier selection
problems involve the decision maker’s subjective assessments which are vague and imprecise
in nature. The use of linguistic terms in fuzzy assessments is an intuitive, yet effective
approach to the evaluation and selection process.

In the defuzzification process, the \( \alpha \) and \( \lambda \) values are used for incorporating the decision
maker’s attitude into the evaluation or selection decisions. In Chapter 4, as the decision
maker has no particular preference for the fuzzy assessments, the value of \( \alpha \) and \( \lambda \) are set to
an average value 0 and 0.5, respectively. In Chapter 5, the value of \( \alpha \) is used for representing
the decision maker’s degree of confidence in the assessments, such as high (\( \alpha = 1 \)), moderate
(\( \alpha = 0.5 \)), and low (\( \alpha = 0 \)). The value of \( \lambda \) is used for representing the decision maker’s
attitude towards risk, such as optimistic (\( \lambda = 1 \)), moderate (\( \lambda = 0.5 \)), and pessimistic (\( \lambda = 0 \)).
In Chapter 6, the value of \( \alpha \) is used for representing the decision maker’s confidence level
towards the assessments, such as aggressive (\( \alpha = 1 \)), moderate (\( \alpha = 0.5 \)), and pessimistic (\( \alpha = 0 \)).
The \( \lambda \) value is used for representing the market conditions, such as bullish market (\( \lambda = 1 \)),
moderate market (\( \lambda = 0.5 \)), and bearish market (\( \lambda = 0 \)). The use of the \( \alpha \) and \( \lambda \) values
effectively incorporates the subjective attitude of the decision maker into the evaluation or
selection decisions.
7.3 Future Research

The fuzzy MCDM model development and validation approach developed in this study has addressed the research issues in developing various MCDM models and validating the corresponding ranking results for a given decision problem. With its general applicability, this approach can be future explored and expanded as follows:

(a) The approach developed only incorporates one defuzzification method (the $\alpha$-cut and $\lambda$ function), three commonly used normalisation, and three widely used aggregation methods for developing the fuzzy MCDM models. There are other defuzzification methods that may be considered if applicable, such as the right value, the degree of optimality, the fuzzy integral, the area centre, and the degree of dominance. Similarly, other suitable normalisation and aggregation methods may also be considered for solving the given decision problems. In this future study, the approach can be further expanded by involving other applicable methods into the normalisation, aggregation, or defuzzification processes. It is to be noted that this future research work is still based on the logic and notion of the approach developed in this study, but further demonstrates the methodological applicability of the approach.

(b) Due to the limited time and data available, Application I focuses only on the major international airports in the Asia-Pacific region. Applications II and III consider only the non-ferrous scrap metal companies. In the future study, the empirical application of the airport performance evaluation can be conducted for global major international airports. The empirical application of the buyer and supplier selection can be conducted for both non-ferrous and ferrous buyers and suppliers, as most of the metal recycling companies deal with both non-ferrous and ferrous metals.
(c) In the future study for the scrap metal buyer and supplier selection problem, a hybrid model incorporating neural network (NN) models with MCDM methods may be developed. By using NN models on the ten years period of London Metal Exchange (LME) historical data on LME aluminium and LME copper, a pricing trend forecasting model can be constructed for estimating if the market is in a bullish market or bearish market.

(d) Since the decision maker in industrial settings may not be familiar with fuzzy MCDM theories or mathematics involved, it would be advisable to develop a fuzzy MCDM decision support system that can cope with multicriteria decision problems involving both crisp and fuzzy data. The decision support system can help the decision maker define the problem and its data set with empirically tested membership functions. The system can also provide adequate connectives that can be chosen by the decision maker depending on the decision context to facilitate the problem formulation and decision scenario analysis.
References


117


Appendix A – The Airport Information for Application I

Twelve Asia-Pacific major international airports are selected for the airport performance evaluation problem in this study. A brief background of each airport, including the general information, geographic location, capacity, and special features are outlined below.

A1: Bangkok Suvarnabhumi Airport
(Bangkok, Thailand)

Suvarnabhumi Airport (BKK) is located about 25 km east of downtown Bangkok. It commenced operations in 2006 and it has the world’s tallest free-standing control tower. It accommodated almost 48 million passengers in 2011 from 96 airlines and operates on a 24-hour basis. BKK is the 6th busiest airport in Asia in 2011.

The two parallel runways, 01R/19L and 01L/19R are 4,260 meters and 3,810 meters long respectively. There are two parallel taxiways and a total of 120 parking bays (51 with contact gates and 69 remote gates) and 8 parking bays (5 contact gates and 3 remote gates). BKK is capable of accommodating Airbus A380 aircrafts and handle 76 flight operations per hour.

In the international passenger terminal, the ground floor and the second floor are allocated as the departure area. The floor houses restaurants and the airport spectator area.
A2: Hong Kong (Chek Lap Kok) International Airport

(Hong Kong)

The Hong Kong International Airport (HKG) was officially opened on July 6, 1998 and operated by Airport Authority Hong Kong. The access to Chek Lap Kok is via a high speed rail system and a six lane highway that joins North Lantau with Kowloon and Hong Kong Island. The state-of-art airport is designed for efficiency and passenger convenience, including:

- Two runways
- Capacity to accommodate up to 87 million passengers (35 million upon opening) and nine million tones of cargo annually
- A 1.3 km long passenger terminal and 2.5 km of moving walkways
- 288 passenger check-in counters
- Baggage handling system capacity of 20,000 luggage item an hour
- 48 bridge-served gates for wide-bodied aircraft and 27 fixed gates
- Aircraft handling capacity of 40 aircraft per hour
- 30,000 square meters of terminal space for shops, food and beverage outlets, banks and currency changers
- 3,000 vehicles parking space
- Airport Express railway reaches Hong Kong’s Central district in 23 minutes
- A Modular construction of passenger terminals that allows for modification and expansion to meet future capacity needs
A3: Kansai International Airport

(Osaka, Japan)

Kansai International Airport (KIX) is the first offshore airport. It was purposely built to relieve overcrowding at Osaka International Airport. Kansai Airport is located on an artificial island in the middle of Osaka Bay, 38 km southwest of the Osaka Train Station. The Phase I development completed in September 1994, approximately 511 hectares of land was reclaimed from the Osaka bay to serve as the airport platform. A 3,500 meters long runway and a variety of related airport support facilities were built in this phase. Another 4,000 meters runway was opened in 2007.

The airport was designed to allow for expedite connections between international and domestic by housing within the same terminal building. This new design concept on passenger terminal permits passenger to transfer to any major airports in Japan within two hours. In 2011, KIX had 107,791 aircraft movements, 13,857,000 passenger movements and 712,116 tonnes of cargo.

The Kansai International Airport Company, a joint venture in financing and staffing among national and local governments and private sectors, is responsible for managing the airport. The airport aims to make the Kansai region Japan’s leading international commercial centre.
A4: Melbourne Tullamarine International Airport

(Melbourne, Australia)

The Melbourne Airport (MEL) opened in 1970, replacing Essendon Airport as the major international airport and a year later as the domestic airport for Victoria. The 2,369 hectares site is located 23 km northwest of the city of Melbourne. Melbourne Airport is currently used by 27 international airlines and four major domestic airlines (Qantas Airlines, Virgin Blue Airlines, Jetstar Airways and Tiger Airways) and a number of commuter services. The airport operates 24 hours a day, with parallel 23 meters wide taxiways.

In 2012, there were a total of 29,297,387 passengers and 206,798 aircraft movements, which include the international, domestic and general aviation aircraft movements. There are four terminals at the Melbourne Airport with 56 gates (40 domestic and 16 international). MEL has two intersecting runways: one 3,657 meters north-south and one 2,286 meters east-west.

Due to increasing traffic, several runway expansions are planned, including extending the north-south runway to 4,500 meters, and extending the east–west runway to 3,500 meters. Two new 3,000 meters runways are also planned, with an expected cost of $500 million, will be opened by 2020.
A5: Manila Ninoy Aquino International Airport

(Manila, Philippines)

NAIA has two runways of crosswind configuration. The first, 06/24, handles all international and most domestic operations, while the second, 13/31, serves mostly the general aviation and domestic departures for domestic taxing purposes.

The present main runway 06/24, originally constructed in 1953, was strengthened and extended in 1995 to a total length of 3,737 meters. NAIA is the gateway to the country and responsible for about 96% of international flights departure and arrival. It has an International Passenger Terminal (IPT) with a capacity of 4.5 million passengers but presently serves about 7.2 million passengers per year.

NAIA Terminal 2, completed in 1998, can accommodate 9 million domestic passengers per annum, although it was initially intended to accommodate only 2.5 million international passengers.

NAIA Terminal 3 officially opened to selected domestic flights from July 2008. This is the biggest terminal in NAIA complex and is built on a 63.5 hectare lot. The terminal is capable of servicing 33,000 passengers daily at peak or 6,000 passengers per hour.
**A6: Tokyo Narita International Airport**

(Tokyo, Japan)

The Tokyo Narita International Airport (NRT) opened on May 20, 1978 and is located about 57 km east from the Tokyo Train Station. A total of 1,065 hectares were reserved for the development of the airport of which, approximately 550 hectares were developed during Phase I, including a 4,000 meter runway, parallel taxiway, passenger terminal building and apron, cargo terminal, navigational aids, and aircraft maintenance facilities. In 2011, the airport processed around 28 million passengers and almost 1.9 metric tonnes of cargo. It is the second-busiest passenger airport in Japan, busiest air freight hub in Japan, and ninth-busiest air freight hub in the world. 54 carriers, including Japanese and foreign airlines use the airport. In 2011, aircraft take-offs reached 235,000. Because the airport is located in a noise-sensitive area, substantial noise abatement policies are enforced including curfew, relocation assistance, land purchase, and soundproofing work.

The Narita International Airport Corporation operates the airport. In Phase II, the airport has built up a 280,000 square meter main terminal building, a 60,000 square meter satellite building, 49 aircraft boarding gates, and an over one million square meter apron. To support new terminal operations, a 3,200-meter runway has been constructed when land acquisition issues were settled. As a key airport for northeast Asia, it is a gateway for Japan, contributing greatly to the economic growth of the nation and the region.
A7: Beijing Capital International Airport

(Beijing, China)

The Beijing International Airport (PEK) is owned and operated by the Beijing Capital International Airport Company Limited, a state-owned company. It began operations in 1958 and is located 32 km northeast of Beijing’s city centre. PEK has three parallel runways, of 3,900, 4,200 and 4,500 meters, respectively. In 2012, nearly 82 million passengers were handled by the airport. The aircraft movement in 2012 was 517,584.

By 2008, PEK has expanded to include 3 terminals and has overtaken Tokyo Haneda to be the busiest airport in Asia based on scheduled seat capacity. Terminal 1, with 72,000 m² of space, opened on January 1, 1980, replaced the smaller existing terminal which was in operation since 1958. It serves the domestic routes of Hainan Airlines and its subsidiaries. Terminal 2 was opened on November 1, 1999, with a floor area of 396,000 m². It serves China Southern Airlines, China Eastern Airlines, Sky Team members and other domestic and international flights. Terminal 3 became fully operational in March 2008. It serves Air China, Star Alliance and Oneworld members, and some other domestic and international airlines.
A8: Seoul Incheon International Airport

(Seoul, South Korea)

Incheon International Airport (ICN) commenced operations in 2001. It is run by the Incheon International Airport Corporation and owned by the government of South Korea. ICN is located 48 km west of Incheon’s city centre, on an artificially created piece of land. The access road to the airport is the Incheon International Airport expressway - a freeway with eight lanes. The airport includes three parallel runways, two of which are 3,750 meters and 4,000 meters in length.

The passenger terminal, as the central terminal of the airport, comprises 44 gates, among which 38 are reserved for international traffic. All gates can accommodate the new Airbus 380. There are 50 customs inspection ports, 2 biological quarantine counters, 6 stationary and 14 portable passenger quarantine counters, 120 arrival passport inspection counters, 8 arrival security ports, 28 departure security ports, 252 check in counters, and 120 departure passport inspection counters. The cargo terminal complex has six cargo terminals. It was designed to be able to process 1.7 million tons of cargo per year. The airport operates 24 hours a day, 7 days a week.
A9: Shanghai Hongqiao International Airport

(Shanghai, China)

Shanghai International Airport (SHA) began its operations in April 1964. The distance between the airport and the city centre is 13 km. It has two runways both 3,400 meters long. SHA has two terminals and a total capacity of 40 million passengers. Terminal 2 is four times the size of the original Terminal 1 and now houses 90 percent of all airlines at the airport. With the new runway, Shanghai became the first city in China to have five runways for civilian use (Shanghai-Pudong and Shanghai-Hongqiao combined). In 2012 it handled a total number of 33,851,200 passengers.

The airport is hosting 22 airlines and serving 82 scheduled passenger destinations. SHA is the 4th busiest airport in mainland China and the 41st busiest in the world. The airport was also mainland China's 5th busiest airport in terms of cargo traffic and the 7th busiest by traffic movements.
A10: Singapore Changi International Airport

(Singapore)

Singapore Changi International Airport (SIN) is located at the eastern tip of Singapore, which is 20 km from the city centre and occupies 1,663 hectares of land, although half of it is reclaimed from the sea. Terminals 1 to 3 were opened in July 1981, November 1990, and January 2008, respectively.

The Changi Airport Group operates the airport and it is owned by the Government of Singapore. The airport’s three parallel runways and other airport facilities operate 24 hours a day. The terminals have a total annual handling capacity of 66 million. In 2012, the airport handled 51 million passengers and served more than 100 airlines operating 6,100 weekly flights. Changi is ranked the seventh busiest airport by international passenger traffic in the world and the second busiest in Asia in 2011. The airport is one of the busiest cargo airports in the world, handling 1.9 million tonnes of cargo in 2011. Changi Airport has a total of 144 parking bays, among which 92 are aerobridges, 10 are contact and 42 remote.

On 1 March, 2012, Changi Airport Group announced plans for expansion. Terminal 4 will have a handling capacity of 16 million passengers and will open in 2017.
A11: Sydney Kingsford Smith International Airport

(Sydney, Australia)

The Sydney (Kingsford Smith) Airport (SYD) was officially opened in 1920, although there were aircrafts that landed at the present site of Sydney Airport as early as 1911. The 640 hectares site is located 8 km from Sydney city centre. The airport is the busiest airport in Australia, handling nearly 37 million passengers, 289,006 aircraft movements and 471,000 metric tonnes of cargo in 2012. It is believed that one half of the nation’s international passengers and one third of the nation’s domestic passengers pass through Sydney. Currently, 47 airlines and a number of commuter services are using this airport. Sydney Airport has three terminals and three runways, which are 2,530 meters, 2,438 meters and 3,962 meters respectively.
A12: Taiwan Taoyuan International Airport
(Taipei, Taiwan)

Taiwan Taoyuan International Airport (TPE), operated by the Taoyuan International Airport Corporation, opened on February 26, 1979. It occupies 1,223 hectares in the Taoyuan country’s northern shore, which is about 40 km from Taipei city, a 40 minutes drive from the CBD.

There are two runways in the airport - the northern runway is 3,660 meters in length and 60 meters in width and the southern runway is 3,350 meters in length and 60 meters in width. The passenger terminal building has four levels above ground and one basement level with the total floor area of 169,500 square meters. In 2011, total passenger movement was 24,947,551 and the aircraft movement was 163,199.

Terminal 1 was originally designed to handle 8 million passengers per year (including arrivals, departures and transits). However, the air traffic grows significantly due to its geographical advantage and the fast development of Taiwan industrials. To reduce congestion, a new terminal was opened in 2000. The total capacity of the two terminals together is around 34 million passengers per year.
Appendix B – Sensitivity analysis for Application I

B.1 Fuzzy MCDM model D–N$_1$–S ($\lambda = 0$)

B.2 Fuzzy MCDM model D–N$_1$–S ($\lambda = 0.5$)
B.3 Fuzzy MCDM model D–N1–S (λ = 1)

![Graph D-N1-S (λ = 1)](image)

B.4 Fuzzy MCDM model D–N2–S (λ = 0)

![Graph D-N2-S (λ = 0)](image)
B.5 Fuzzy MCDM model D–N$_2$–S ($\lambda = 0.5$)

\[ \text{D-N2-S (}$\lambda = 0.5$\text{)} \]

B.6 Fuzzy MCDM model D–N$_2$–S ($\lambda = 1$)

\[ \text{D-N2-S (}$\lambda = 1$\text{)} \]
B.7 Fuzzy MCDM model D–N$_3$–S ($\lambda = 0$)

B.8 Fuzzy MCDM model D–N$_3$–S ($\lambda = 0.5$)
B.9 Fuzzy MCDM model $D-N_3-S$ ($\lambda = 1$)

B.10 Fuzzy MCDM model $D-N_1-T$ ($\lambda = 0$)
B.11 Fuzzy MCDM model D–N₁–T (λ = 0.5)

![Graph showing overall performance values for different α values with λ = 0.5.](image)

B.12 Fuzzy MCDM model D–N₁–T (λ = 1)

![Graph showing overall performance values for different α values with λ = 1.](image)
B.13 Fuzzy MCDM model D–N₂–T (λ = 0)

D-N2-T (λ = 0)

B.14 Fuzzy MCDM model D–N₂–T (λ = 0.5)

D-N2-T (λ = 0.5)
B.15 Fuzzy MCDM model D–N₂–T (\(\lambda = 1\))

\[
D-N2-T (\lambda = 1)
\]

B.16 Fuzzy MCDM model D–N₃–T (\(\lambda = 0\))

\[
D-N3-T (\lambda = 0)
\]
B.17 Fuzzy MCDM model D–N₃–T (λ = 0.5)

D-N3-T (λ = 0.5)

B.18 Fuzzy MCDM model D–N₃–T (λ = 1)

D-N3-T (λ = 1)
Appendix C – Sensitivity analysis for Application II

C.1 Fuzzy MCDM model D–N₁–S (λ = 0)

C.2 Fuzzy MCDM model D–N₁–S (λ = 0.5)
C.3 Fuzzy MCDM model D–N₁–S (λ = 1)

![Graph D-N1-S (λ = 1)]

C.4 Fuzzy MCDM model D–N₂–S (λ = 0)

![Graph D-N2-S (λ = 0)]

155
C.5 Fuzzy MCDM model D–N₂–S (λ = 0.5)

D-N2-S (λ = 0.5)

C.6 Fuzzy MCDM model D–N₂–S (λ = 1)

D-N2-S (λ = 1)
C.7 Fuzzy MCDM model $D-N_3-S \ (\lambda = 0)$

![Graph for $D-N_3-S \ (\lambda = 0)$](image)

C.8 Fuzzy MCDM model $D-N_3-S \ (\lambda = 0.5)$

![Graph for $D-N_3-S \ (\lambda = 0.5)$](image)
C.9 Fuzzy MCDM model D–N₃–S (λ = 1)

\[ \text{D-N3-S (λ = 1)} \]

\[ \begin{array}{cccccccccccc}
\alpha = 0 & \alpha = 0.1 & \alpha = 0.2 & \alpha = 0.3 & \alpha = 0.4 & \alpha = 0.5 & \alpha = 0.6 & \alpha = 0.7 & \alpha = 0.8 & \alpha = 0.9 & \alpha = 1
\end{array} \]

C.10 Fuzzy MCDM model D–N₁–T (λ = 0)

\[ \text{D-N1-T (λ = 0)} \]

\[ \begin{array}{cccccccccccc}
\alpha = 0 & \alpha = 0.1 & \alpha = 0.2 & \alpha = 0.3 & \alpha = 0.4 & \alpha = 0.5 & \alpha = 0.6 & \alpha = 0.7 & \alpha = 0.8 & \alpha = 0.9 & \alpha = 1
\end{array} \]
C.11 Fuzzy MCDM model D–N₁–T (λ = 0.5)

C.12 Fuzzy MCDM model D–N₁–T (λ = 1)
C.13 Fuzzy MCDM model D–N<sub>2</sub>–T (λ = 0)

C.14 Fuzzy MCDM model D–N<sub>2</sub>–T (λ = 0.5)
C.15 Fuzzy MCDM model $D-N_2-T$ ($\lambda = 1$)

C.16 Fuzzy MCDM model $D-N_3-T$ ($\lambda = 0$)
C.17 Fuzzy MCDM model D–N₃–T (λ = 0.5)

\[ \text{Overall preference value} \]

\[ \alpha = 0 \quad \alpha = 0.1 \quad \alpha = 0.2 \quad \alpha = 0.3 \quad \alpha = 0.4 \quad \alpha = 0.5 \quad \alpha = 0.6 \quad \alpha = 0.7 \quad \alpha = 0.8 \quad \alpha = 0.9 \quad \alpha = 1 \]

C.18 Fuzzy MCDM model D–N₃–T (λ = 1)

\[ \text{Overall preference value} \]

\[ \alpha = 0 \quad \alpha = 0.1 \quad \alpha = 0.2 \quad \alpha = 0.3 \quad \alpha = 0.4 \quad \alpha = 0.5 \quad \alpha = 0.6 \quad \alpha = 0.7 \quad \alpha = 0.8 \quad \alpha = 0.9 \quad \alpha = 1 \]
C.19 Fuzzy MCDM model $D$–$W$ ($\lambda = 0$)

![Graph of Fuzzy MCDM model $D$–$W$ ($\lambda = 0$) showing overall preference values for different $\alpha$ values.]

C.20 Fuzzy MCDM model $D$–$W$ ($\lambda = 0.5$)

![Graph of Fuzzy MCDM model $D$–$W$ ($\lambda = 0.5$) showing overall preference values for different $\alpha$ values.]

163
C.21 Fuzzy MCDM model D–W ($\lambda = 1$)
Appendix D – Sensitivity analysis for Application III

D.1 Fuzzy MCDM model $N_1$–$S$–$D$ ($\lambda = 0$)

D.2 Fuzzy MCDM model $N_1$–$S$–$D$ ($\lambda = 0.5$)
D.3 Fuzzy MCDM model N₁–S–D (λ = 1)

![Graph of N₁-S-D (λ = 1)]

D.4 Fuzzy MCDM model N₂–S–D (λ = 0)

![Graph of N₂-S-D (λ = 0)]
D.5 Fuzzy MCDM model $N_2$–$S$–$D$ ($\lambda = 0.5$)

D.6 Fuzzy MCDM model $N_2$–$S$–$D$ ($\lambda = 1$)
D.7 Fuzzy MCDM model $N_3$–$S$–$D$ ($\lambda = 0$)

D.8 Fuzzy MCDM model $N_3$–$S$–$D$ ($\lambda = 0.5$)
D.9 Fuzzy MCDM model $N_3$–$S$–$D$ ($\lambda = 1$)

![Graph of N3-S-D (\lambda = 1)]

D.10 Fuzzy MCDM model $N_1$–$T$–$D$ ($\lambda = 0$)

![Graph of N1-T-D (\lambda = 0)]
D.11 Fuzzy MCDM model $N_1$–$T$–$D$ ($\lambda = 0.5$)

![Graph showing overall preference values for different $\alpha$ values in $N_1$–$T$–$D$ ($\lambda = 0.5$)]

D.12 Fuzzy MCDM model $N_1$–$T$–$D$ ($\lambda = 1$)

![Graph showing overall preference values for different $\alpha$ values in $N_1$–$T$–$D$ ($\lambda = 1$)]
D.13 Fuzzy MCDM model $N_2$–$T$–$D$ ($\lambda = 0$)

D.14 Fuzzy MCDM model $N_2$–$T$–$D$ ($\lambda = 0.5$)
D.15 Fuzzy MCDM model N₂–T–D (λ = 1)

D.16 Fuzzy MCDM model N₃–T–D (λ = 0)
D.17 Fuzzy MCDM model $N_3$–T–D ($\lambda = 0.5$)

![Graph showing overall preference values for different $\alpha$ values]

D.18 Fuzzy MCDM model $N_3$–T–D ($\lambda = 1$)

![Graph showing overall preference values for different $\alpha$ values]
D.19 Fuzzy MCDM model W–D ($\lambda = 0$)

D.20 Fuzzy MCDM model W–D ($\lambda = 0.5$)
D.21 Fuzzy MCDM model $W$–$D$ ($\lambda = 1$)