News impacts on financial return distributions: Long memory and regime switching approaches

A thesis submitted for the degree of

Doctor of Philosophy

By

Hung Xuan Do

B.Ec. (Hons), Academy of Finance, Vietnam

Grad.Dip. in Business (Banking and Finance), Monash University, Australia

M.Applied Econometrics, Monash University, Australia

Department of Econometrics and Business Statistics

Monash University

Australia

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Declaration

I hereby declare that this thesis contains no material which has been accepted for the award of any other degree or diploma in any university or equivalent institution, and that to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of this thesis.

Hung Xuan Do

May 2013

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Hung Xuan Do, Melbourne, May 2013

List of publications

The material in Chapter 3 has been published by the *Economics Letters* as a research paper, coauthored with Robert Brooks and Sirimon Treepongkaruna.

The contribution in Chapter 2 was presented in the *24th Australasian Finance and Banking Conference* held in Sydney, Australia in December 2011.

The contribution in Chapter 3 was presented in the *Econometric Society Australasian Meeting* held in Melbourne, Australia in July 2012.

The contribution in Chapter 4 was presented in the *XX Finance Forum* held in Oviedo, Spain in November 2012.

The contribution in Chapter 5 was presented in the *6th CSDA – Computational and Financial Econometrics Conference* held in Oviedo, Spain in December 2012; the *25th Australasian Finance and Banking Conference* held in Sydney, Australia in December 2012 and the *International Conference on the Global Financial Crisis* held in Southampton, United Kingdom in April 2013.

Abstract

Critical roles of return higher moments in financial activities, which have been increasingly documented, suggest that it is worthwhile to analyze the behavior of the financial return distributions under various market conditions. The aim of this thesis is to model the responses of stock and currency return distributions to exogenous shocks under various forms of news which hit the financial markets. Specifically, this thesis is concerned with three scenarios: (i) when each of the return higher moments is shocked; (ii) when the hidden information arrives; and (iii) when the overall sovereign credit ratings change.

Chapter 2 examines the linkages within-between stock and currency (FX) markets via three higher moments: realized volatility, skewness and kurtosis using the Generalized Impulse Response within a Fractionally Integrated Vector Autoregressive (FIVAR) framework. We find evidences of positive linkages within stock and FX markets via all three higher moments in both emerging and developed groups. However, the spread of the FX markets linkages via their 2^{nd} and 4^{th} moment is broader in the developed regions compared with the emerging regions. For the cross-assets linkages, the stock and FX markets in emerging groups are more likely to be negatively linked through the 3^{rd} moment; whereas, those in developed groups are positively transmitted through the 2^{nd} and 4^{th} moment. Finally, in developed markets, the cross-assets linkages are often found to be weaker than the same asset linkages in terms of the magnitude.

Limitations of methodology used in Chapter 2, where the endogenous variables in a FIVAR model need to be fractionally differenced before using the impulse response analysis of a VAR model, lead us to develop a new approach in Chapter 3. We based on the spirit of Peseran and Shin (1998) to derive a generalized impulse response function for the FIVAR model. Chung (2001) has the same purpose but he makes use of the orthogonalized approach proposed by Sims (1980). Our method is different from the methodology shown in Chung (2001) in a sense that it does not require us to orthogonalize the error vector and, therefore, is independent of the ordering of the variables in the system. Consistent with Chung (2001) and the long memory behavior, we show that generalized and orthogonalized impulse responses of FIVAR evolve slowly at the same hyperbolic rates. However, we also note that they are different in a number of aspects. For the purpose of statistical inference in empirical studies, we derive asymptotic theories for both functions. We summarize the results for two scenarios associated with one- and two-step estimation methods, respectively. However, our simulations' results support an application of the two-step estimation procedure in generating the generalized and orthogonalized impulse responses of a FIVAR model.

Chapter 4 utilizes the methodology developed in Chapter 3 to reassess influences of trading volume on stock and FX return distributions while allowing the possibility of interactions among return higher moments. Given the evidence of the higher moments' inter-relationship, the chapter extends the analysis by exploring how trading volume affects the dynamic structure of higher moments' inter-relationship. Our reassessment of volume – volatility interaction supports a complementary property among information theories and further contributes evidence of cross – market relations between volume and volatility. The result for the volume – skewness relationship in conjunction with previous studies leads to a hypothesis that direct impact of volume on the level of negative skewness is less significant for a better diversified portfolio. We further find that the negative interaction between volume and kurtosis can be explained by the differences of opinion hypothesis. Although behavior of the inter-relationship towards significant events and new policies are robust, its strength is mostly decreased by the trading volume. Fundamentally, this finding is consistent with the prominent result found in the volume – GARCH effect literature, which suggests that trading volume is a source of heteroskedasticity in the return volatility.

In Chapter 5, we investigate the effects of credit rating agencies (CRAs)' sovereign credit assessments on stock and currency return distributions by developing a framework that allows a multivariate system of long memory processes to be conditional on specific credit rating regimes. We find heterogeneous effects of sovereign rating actions across regimes, implying the usefulness of our proposed model in accommodating both long memory and regime switching features. Furthermore, we reveal that the total effects (both direct and indirect forces) of sovereign credit assessments on the realized moments can be different to their direct effects. Hence, we develop an impulse response of a transfer function, which can capture these total effects, to investigate which agency has the greatest impact on the EU financial return distributions. We find that the rank orders of CRAs are not unique across rating regimes and even in each realized moment.

Chapter 1

Introduction

1.1 Background and motivation

It has been widely accepted that financial return distributions usually exhibit characteristics of asymmetry and excess kurtosis, which have violated the regular assumption of a normal distribution of financial returns. These stylized facts, thus, exemplify that alongside volatility risk, the asymmetric and fat-tail risks have also played critical roles in many financial activities, such as asset pricing, Value-at-Risk (VaR) calculation and asset allocation. Harvey and Siddique (2000) document that asset returns can be explained by the conditional skewness. Athayde and Flôres (2003) point out the importance of skewness and kurtosis in portfolio optimization. Further, Jurczenko and Maillet (2006) use the four-moment CAPM to demonstrate that a presence of skewness and kurtosis can considerably affect the asset pricing. In addition, Mandelbrot and Hudson (2004) suggest that the estimation of VaR may be flawed if either of the higher moment risks is ignored. Most recently, Brunnermeier and Pedersen (2009) and Conrad et al. (2012) have also emphasized on the importance of the higher moments in financial activities.

An increasing evidence of significant roles of higher moments suggests that it is worthwhile to analyze the behavior of the financial return distributions under various market conditions. However, this type of investigation would require consistent and robust estimates of higher moments of the return distributions. Since the introduction of the Autoregressive Conditional Heteroskedasticity (ARCH) model (Engle, 1982), the volatility clustering behavior of financial return distributions has been successfully described. Thus, the conditional volatility has been extensively measured using the ARCH model and its extensions, for example the family of Generalized ARCH (GARCH) models (first extended by Bollerslev, 1986) and the multivariate GARCH family of models (see Frances and VanDijk, 2000). Inheriting a success of the ARCH model, the other higher moments have been conditionally modeled within parametric frameworks such as the family of time-varying conditional skewness and kurtosis models (see for example, Harvey and Siddique, 1999; Guermat and Harris, 2002; Korkie et al., 2006; Lanne and Saikkonen, 2007; Hashmi and Tay, 2007; and Wilhelmsson, 2009). Utilization of parametric models might be useful for the cases of low frequency data (e.g., monthly, daily data). However, a drawback of the parametric approach has been well recognized that the estimates of conditional higher moments rely heavily on the underlying model assumptions. In addition, this may be more problematic when we come up with a multivariate system due to the large number of parameters that need to be estimated (see Pagan, 1996).

The recent development in methodologies and the increasing availability of high frequency data have provided a better alternative for measuring the higher moments non-parametrically from intraday returns. The use of intraday data compared to daily closing data may lead to a better representation and more robust estimate of the actual price behavior (see for instance, Andersen et al., 2003; Barndorff-Nielsen and Shephard

2004a, 2004b). The realized higher moments, which are the higher moments constructed from intraday returns, can be treated as observable variables and, therefore, are able to be modeled directly within an econometric framework. Further, as estimated non-parametrically, the realized higher moments are free from the distributional and other parametric model assumptions. Therefore, an introduction of the realized measures has facilitated an investigation of the behavior of the financial return distributions.

This thesis exploits the advantages of the realized higher moments to investigate how financial return distributions react to an exogenous shock under various forms of news which hits the financial market. Due to the critical roles of higher moments which have been discussed earlier, a focus on the financial return distributions would help to explain the role of the informational transmission mechanism of the exogenous shock in a variety of financial activities. More specifically, we are mainly interested in the transmission mechanism of the higher moments between financial markets as well as the impacts of hidden information arrival and sovereign credit ratings news on financial return distributions. In other words, our core purposes are to investigate the reactions of financial return distributions under three scenarios: (i) when there is an exogenous shock in each of the higher moments; (ii) when there is an arrival of hidden information to the market; and (iii) when there is a change in the overall sovereign credit quality assessment.

1.2 Research questions

1.2.1 How do financial markets link and cross-link via higher moments?

The recent financial turbulences exemplify the importance of financial market linkages due to an increase in the level of integration among markets. It is likely that one market would be affected by a shock coming to other markets. For example, the recent failures in financial markets around the world were originated from issues in the U.S mortgage markets. Hence, a better understanding of financial markets linkages would be beneficial in forecasting the markets' reaction and managing potential risks in an ever more integrated financial world. In addition, emphasizing on the linkages via higher moments helps to explain the transmission mechanism of volatility, asymmetric and fat-tail risks among financial markets.

Even though empirical evidence of volatility transmission has been extensively witnessed in the literature (e.g., Kearney and Patton, 2000; Speight and Mc Millan, 2001; Cai et al., 2008; and Bubák et al., 2011), it is worthwhile to re-evaluate the issue under different markets' properties (e.g., developed markets vs. emerging markets; stock markets vs. foreign exchange markets). This is due to the introduction of the realized estimates of higher moments and the increasing availability of intraday data as mentioned previously. Besides, the limited number of studies about the transmission of asymmetric and fat-tail risks does not correspond with their importance and motivates to explore these issues in depth.

1.2.2 How does the hidden information arrival affect financial returns distributions?

According to the market microstructure perception, the primary factors that cause movements of assets' price are the arrival of new information and the procedure that incorporates this information into the market (Andersen, 1996). As a proxy of the arrival of hidden information, the trading volume has been widely used to investigate the role of information arrival in determining the financial returns distributions. The relevant information theories, including the mixture of distributions hypothesis (e.g., Clark, 1973; Epps and Epps, 1976; and Tauchen and Pitts, 1983), the sequential arrival of information hypothesis (e.g., Copeland, 1976, 1977) and the differences of opinion hypothesis (e.g., Shalen, 1993; and Harris and Raviv, 1993), suggest a positive and lead – lag relationship between the trading volume and the return volatility. Hong and Stein (2003) employed the differences of opinion hypothesis in conjunction with short – sales constraints to propose that the negative skewness of return will be greater conditional on a high trading volume.

Regarding the volume – volatility relation, empirical studies have consistently confirmed a positive and lead – lag linkage in terms of same – asset markets (within stock or foreign exchange markets) (e.g., Kalev et al., 2004; Bjønnes et al., 2005; Bauwens et al., 2005; and Chan and Fong, 2006). However, the cross – asset markets perspective has not yet received corresponding attention. Besides, a mixture of empirical results is reported for the volume – skewness relationship (see, Chen et al., 2001; Hutson et al., 2008; Hueng and McDonald, 2005; and Charoenrook and Daouk, 2008). Additionally, lack of study on the volume – kurtosis relation in the literature provides further issues which need to be investigated to comprehensively model the impacts of information arrivals on financial return distributions.

1.2.3 How do the sovereign credit quality assessments affect the financial returns distributions?

Sovereign credit ratings are expected to have effects on the behavior of asset prices, especially during the financial turbulences (see for example, Brooks et al., 2001; Ferreira and Gama, 2007; Alsakka and ap Gwilym, 2012). However, the credit rating agencies, providers of specialist information about the credit quality of a sovereign, have often been complained about based upon their slow reaction to the international financial crises as well as their inability to forewarn the market participants (see Mora, 2006; and Gorton, 2008). Hence, it is necessary to evaluate the impact of agency ratings on the stability of financial markets, specifically financial returns distributions. Emphasizing on the financial returns distributions helps to fully understand the role of the informational transmission procedure of sovereign credit ratings in financial decision marking. This is due to the critical roles of return higher moments in financial activities as discussed previously.

The impact of sovereign ratings changes on the first moment of asset returns distributions have been widely studied in the literature (see for example, Brooks et al., 2004; Gande and Parsley, 2005; Ferreira and Gama, 2007; and Hill and Faff, 2010a). Yet, there is a shortage of investigation focusing on the higher moments. A possible reason was the shortcomings of the parametric models used in estimating the conditional higher moments. At present, the utilization of the realized higher moments, constructed non-parametrically from intraday data, should facilitate analyses to fill this gap in the literature. However, it would raise a new challenge in econometric modeling in terms of the rating literature. A set of flexible fractional degrees of integration should be allowed in an econometric framework to accommodate for both short – memory (in cases of realized return and skewness) and long – memory (in cases of realized volatility and kurtosis) behaviors. Simultaneously, the regime switching property of the rating data should also be captured by the framework.

1.3 Outline of the thesis

Chapter 2 provides an assessment on the financial markets linkages via higher moments with a particular focus on stock and currency markets. In this chapter, we utilize the high frequency data to construct the realized volatility, skewness and kurtosis non-parametrically. We investigate their spill-over effect to understand how the financial markets are linked via their higher moments. The fractionally integrated Vector Autoregressive (FIVAR) model is employed to capture any long memory behavior of the realized higher moments. The generalized impulse response function and its bias-corrected bootstrap confidence interval within a VAR (filtered from the FIVAR) model are then obtained for the purpose of statistical inference of the spill-over effect.

In chapter 3, we develop a generalized impulse response (GIR) function for the FIVAR model using the Pesaran and Shin (1998) approach. This function helps to overcome the limitation in terms of methodology used in chapter 2, where the available generalized impulse response function can only be computed in a VAR model. We also reformulate the orthogonalized impulse response (OIR) function developed by Chung (2001) for a comparison purpose. To facilitate statistical inferences in empirical studies, we derive asymptotic theories for both the orthogonalized and generalized functions. We summarize the results for two scenarios associated with one- and two-step estimation methods. Simulation results are also provided in this chapter.

Chapter 4 makes use of methodologies developed in chapter 3 to investigate influences of trading volume on stock and FX return distribution while allowing the possibility of interactions among return higher moments. This chapter also analyses how trading volume affects the dynamic structure of linkages between higher moments of asset returns. These issues are explored in conjunction with the implications of relevant information theories, namely the mixture of distributions hypothesis (MDH), the sequential arrival of information hypothesis (SAIH) and the differences of opinion hypothesis (DOH).

Chapter 5 develops a framework that allows a multivariate system of long memory processes to be conditional on specific regimes. The model is applied to examine the effects of credit rating agencies (CRAs)' sovereign credit re-ratings on European stock and currency return distributions via their first four realized moments. An impulse response of a transfer function is also proposed in this chapter to investigate which agency has the greatest impact on the European stock and currency return distributions.

Chapter 6 concludes the thesis by summarizing the key findings of the main research questions. It also discusses some limitations and provides some directions for the future research.

Chapter 2

Financial Markets Linkages via Higher Moments: A Realized Spill-over Approach

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2.1 Introduction

A profound comprehension about the financial markets linkages has been even more crucial due to an increase in the integration of national markets into international markets. For example, the Subprime Mortgage crisis followed by the recent Global Financial crisis has caused a meltdown in financial markets around the world. More specifically, the contagion originated from the U.S to the rest of the world has almost brought down the global financial market. Accordingly, the clear understanding of financial markets linkages can assist investors, managers and policy makers in forecasting the markets' reaction and managing potential risks if there are adverse shocks coming in. However, whilst to date there has been an extensive empirical research studying on the linkages of financial markets via their volatility, how they interact through their skewness and kurtosis has not been well understood. This chapter aims to contribute directly to this strand of literature by investigating the volatility, skewness and kurtosis transmission while allowing for comparisons between different markets' properties (e.g., developed markets vs. emerging markets, stock markets vs. FX markets).

2.2 Literature review

The information on volatility linkages helps investors, researchers and policy makers in understanding the transmission of volatility risk between financial markets. Meanwhile, the skewness linkages explain how markets are linked through the level of asymmetry of the return distribution. Therefore, the spill-over of downside (upside) risk between financial markets is revealed. Likewise, studies of kurtosis linkages provide better insights into the spread of fat tail risk across financial markets since they provide knowledge about markets' relationship through the occurrence of extreme events. Whilst the importance of volatility risk and downside (upside) risk towards almost all markets' participants and policy makers is well known, the problem of fat tail risk attracts more concerns from hedge funds. This is because of hedge funds' mixed strategies including derivative trading, short selling and illiquidity assets investment which lead to much higher excess kurtosis and fatter tail in returns than a normal distribution[1]. However, since the performance of hedge funds can have a significant impact on the stability of the whole financial system[2], the risk of fat tail also deserves a significant consideration.

The literature has witnessed extensive empirical evidence of volatility transmission across financial markets. Some examples are Hamao et al. (1990), Engle

[1] See Fung and Hsieh (2001) and Amin and Kat (2003) for example.

[2] This fact can be seen clearly from the collapse of Long Term Capital Management L.P. in 1998.

et al. (1990), Kim and Rogers (1995), Alexander (1995), Speight and McMillan (2001), and Wang et al. (2002). These studies normally investigate the spill-over effect by using parametric models, for instance, the GARCH model developed by Bollerslev (1986) and its extension. More specifically, a two-step estimation approach is generally applied whereby the univariate GARCH models are estimated at the first stage, and the volatility spill-over is subsequently investigated by using the fitted conditional variance (in lagged terms) as an independent variable in the GARCH equation for one or more return series. The main disadvantage of this approach is that it may not be fully efficient and the estimated conditional volatility relies heavily on the underlying model assumptions. These drawbacks of the two-step estimation approach have led to a more efficient method, the multivariate GARCH family of models (see Engle and Kroner, 1995; Frances and VanDijk, 2000). Examples of studies using the multivariate GARCH model are Karolyi (1995), Longin and Solnik (1995), Darbar and Deb (1997), Kearney and Patton (2000), and Scheicher (2001). However, the problem of dimensionality may arise in a multivariate GARCH model due to the large number of parameters that need to be estimated (see Pagan, 1996). This suggests that multivariate GARCH is practically applicable only to a small dimensional system.

However, recent development in methodologies for estimating volatility and the increasing availability of high frequency data allow researchers to overcome these problems. These methods, called realized volatility, are fully non-parametric and model free, where volatility is considered to be observable and can be calculated directly from the intraday return[3]. This is in contrast with the parametric models mentioned earlier where volatility is estimated from its past values and treated as an unobserved variable. The new approach, therefore, allows the realized volatility series to be input data for

[3] See for example, Andersen et al. (2003) or Barndorff-Nielsen and Shephard (2004a, 2004b)

standard econometric techniques. The literature shows that most parametric models, which treat volatility as a latent variable, fail to adequately explain a number of observed stylized facts of financial variables (see among others, Bollerslev, 1987; Carnero et al., 2004; and McAleer and Medeiros, 2008). Besides, Wongswan (2006) states that although the utilization of relatively low-frequency data (such as, daily or weekly data) can provide much useful information, both the short-run adjustment effects as well as the effect of fast-processed information may be overlooked. On the other hand, the use of high frequency data helps to improve estimation of volatility and, consequently, the inference about realized volatility's transmission is improved (Bubák et al., 2011). Examples of recent research using realized volatility to investigate the spill-over effect are Cai et al. (2008), Kim and Doucouliagos (2009), McMillan and Speight (2010), and Bubák et al. (2011). Those empirical studies, however, limit their concern to the spill-over in futures markets or across foreign exchange markets. This chapter extends the context by investigating the realized volatility spill-over effect not only within stock and foreign exchange markets but also between them across countries. This scope allows us to understand whether the spill-over effect behaves differently in different types of market. Further, it provides knowledge about the linkages between stock and foreign exchange via realized volatility.

In contrast with the literature on volatility spill-over, there is limited study focusing on the area of skewness and kurtosis linkages in financial markets. Regarding the skewness transmission, whilst Korkie et al. (2006) provides supports of skewness persistence within equity markets, Hashmi and Tay (2007) find little evidence of a skewness spill-over effect from the global and regional factors. In term of kurtosis linkages, most papers have investigated the issue via the interaction of the occurrence of extreme returns between markets. Examples include Longin and Solnik (2001) and

Cumperayot et al. (2006). The common result found is that the occurrence of extreme returns in one market is likely to be positively correlated with that in other markets. These studies analyse the transmission of asymmetric and fat-tail risks in the national country-by-country context by using the parametric and semi-parametric models with low frequency data[4], which rely heavily on the underlying model assumptions as explained earlier. To the best of our knowledge, there has not been any study which makes use of intraday data to analyse the skewness and kurtosis linkages. Further, since Dacorogna et al. (2001) suggests other higher moment measures can be constructed by using intraday return, we extend the idea of realized volatility to estimate realized skewness and realized kurtosis non-parametrically. The realized skewness and realized kurtosis, therefore, are treated as observed variables and they can be used to analyse spill-over effects with standard econometric techniques.

The contribution of this chapter is a thorough investigation of financial markets linkages using the high frequency data in a global context, particularly for within-between stock and FX markets. This analysis involves a broad range of countries in terms of both geography and market development, which allows better comparisons between the linkages due to different markets' properties (e.g., developed markets vs. emerging markets, stock markets vs. FX markets). Further, we investigate the linkages via their three higher moments, which are volatility, skewness and kurtosis, to understand the transmission of volatility risk, downside (upside) risk and the fat tail risk, respectively. In this study, markets' linkages are defined as the spill-over effects which we assess subsequently through the GIR analysis proposed by Pesaran and Shin

[4] For example, Time varying Conditional Skewness model and Extreme Value Theory for the case of skewness and kurtosis spill-over, respectively.

(1998) within a FIVAR model. For a more accurate statistical inference about the existence of a spill-over effect, we construct the 95% bias-corrected bootstrap confidence interval as proposed in Efron and Tibshirani (1993).

The remainder of this chapter is organized as follows. Section 2.3 explains the construction of data for analysis. Section 2.4 describes the estimation of the degree of fractional integration for three realized measures. Section 2.5 and section 2.6 discuss the FIVAR model and the GIR employed, respectively. Section 2.7 provides the empirical results of realized spill-over effects and section 2.8 concludes.

2.3 Data and construction of realized measures

We employ 5-minute interval intraday stock and foreign exchange (FX) market data from the Thomson Reuters database via the Securities Industry Research Centre of Asia-Pacific (SIRCA). The use of 5-minute returns can avoid the problem of measurement error and reduce microstructure biases (see Andersen and Bollerslev, 1998, and Andersen et al., 2001b). In the FX market, the USD is used as the base currency. The sample range is from 01/01/1997 to 20/05/2010. Data on weekends are excluded. Countries are then grouped as suggested by Thomson Reuters. We investigate five main regions, namely Asia Pacific Developed, Asia Pacific Emerging, Asia Emerging, America and Europe Developed. Asia Pacific Developed markets include Australia, New Zealand, Hong Kong, Japan and Korea. Asia Pacific Emerging markets include Indonesia, Malaysia, Philippines, Taiwan and Thailand. Asian Emerging markets include China, India, Pakistan, Taiwan and Thailand. American markets include Brazil, Chile, Peru, Argentina, Canada and the United States. European

Developed markets include Austria, Belgium, France, Germany, Greece, Ireland, Norway, Portugal, Spain, Switzerland, Sweden and the United Kingdom.

In the American market, we also investigate the sub-group, Latin American countries, which represent the American emerging markets in Latin America. Additionally, we divide European developed countries into three sub-groups: North Europe includes Ireland, Sweden, the UK and Norway; South Europe includes Greece, Portugal and Spain; Western Europe includes Austria, Germany, France, Switzerland and Belgium.

Since the stock market is not a non-stop trading market, we consider a trading day as that part of the day when stock markets are open[5]. We therefore define our trading hours as 22:00 GMT to 8:00 GMT for Asia Pacific Developed markets; 1:00 GMT to 9:30 GMT for Asia Pacific Emerging markets; 1:00 GMT to 10:30 GMT for Asian Emerging markets; 7:00 GMT to 17:30 GMT for European markets and 12:00 GMT to 21:00 GMT for American markets[6]. As such, in the Asia Pacific Developed markets, the period from Monday 22:00 GMT to Tuesday 8:00 GMT is considered as our Tuesday sample.

The 5-minute intraday returns are calculated as the change in natural logarithmic of the mid prices. The mid-price, which is the midpoint quote between the Bid and Ask price, is employed to minimize the effect of Bid-Ask bounce (see Roll, 1984). For the

[5] Hansen and Lunde (2005) propose to estimate the realized volatility of a stock market for the whole day to account for the potential latent information during non-trading time. However, since the scope of our study is to analyze the spill-over effects in a wide range of countries, this methodology is not applicable because of different trading and non-trading time in GMT in different stock markets.

[6] Details of GMT Offsets and Stock Markets Trading Times (GMT) for all countries are reported in Table 2.1.

countries in the European Union, we use prices of their own currencies to calculate intraday returns before the day they joined the Union and prices of EUR are used after that day. From the intraday return, we then construct the realized measures for analysis. The daily realized volatility is computed as proposed by Andersen et al. (2003):

$$RV_t = \sum_{i=1}^{D} r_{i,t}^2 \tag{2.1}$$

where $r_{i,t}$ denotes a ith 5-minute return during day t and D denotes the total number of 5-minute return intervals during any trading day.

As suggested in Dacorogna et al (2001), other higher moment measures can also be constructed by using intraday returns. We follow the formula presented in Chen, Hong, and Stein (2001) to compute the realized skewness. The daily realized skewness for any day t is:

$$RS_t = -\frac{D(D-1)^{3/2}(\sum_{i=1}^{D} r_{i,t}^3)}{(D-1)(D-2)(\sum_{i=1}^{D} r_{i,t}^2)^{3/2}} \tag{2.2}$$

This is the negative of the third moment of returns divided by the cubed standard deviation of returns to standardise for differences in variances. The negative sign is included to make sure that an increase in the daily skewness corresponds to an asset return having a more left-skewed distribution (Chen et al., 2001). Therefore, by using this formula we focus on the importance of the downside risk in analysing the spill-over effect.

To compute realized kurtosis, we extend the idea of the realized volatility's methodology. Since the realized volatility is the second moment of realized return, the realized kurtosis, defined as the standardized 4th moment of realized return, can be calculated as:

$$RK_t = \frac{\sum_{i=1}^{D} r_{i,t}^4}{(RV_t)^2} \tag{2.3}$$

Additionally, due to different holidays in different markets included in the model, the Catmull-Rom Spline, a family of the Cubic Spline Interpolation method, is employed to reconstruct missing data due to holidays and days when the number of trades is equal to zero. The Spline Interpolation method has been found to be useful in empirical studies which deal with missing observations in time series data (see Damsleth, 1980; and Pavlov, 2004). The Catmull-Rom Spline can be applied straightforwardly as follows:

$$\begin{aligned} y_t &= (2\lambda^3 - 3\lambda^2 + 1) y_{t-1} + (\lambda^3 - 2\lambda^2 + \lambda)(y_{t+1} - y_{t-2}) \\ &\quad - (2\lambda^3 - 3\lambda^2) y_{t+1} + (\lambda^3 - \lambda^2)(y_{t+2} - y_{t-1}) \end{aligned} \tag{2.4}$$

where y_t is the missing observation at time t that needs to be filled in, λ is the relative position of the missing observation divided by the total number of missing observations in the series. y_{t+1} and y_{t+2} are the next two non-missing observations. y_{t-1} and y_{t-2} are the previous two non-missing observations. In this case, y_t can be the daily series of realized volatility, realized skewness or realized kurtosis which are computed according to the formulas given earlier.

The descriptive statistics of (logged) realized volatility, realized skewness and (logged) realized kurtosis of some selected markets are reported in Table 2.2[7]. Normally, as shown in Andersen et al. (2003), the realized volatility estimates of many markets are approximately normal since their skewness and kurtosis are respectively close to 0 and 3. However, this fact does not hold for all markets examined, since we observe the distribution of realized volatility in some markets are much more leptokurtic (e.g., Brazil stock market or Belgium and France FX market[8]). Regarding the realized skewness measure, despite approximately showing a symmetric shape, its distribution is usually leptokurtic. Interestingly, after transforming to its natural logarithm, the distribution of realized kurtosis is approximately normal in many countries since its skewness and kurtosis are close to 0 and 3, respectively.

The Ljung-Box statistics indicate strong serial correlation in all cases for the realized volatility and most of the cases for the realized kurtosis. In terms of realized skewness, the long-range dependence presents in some cases according to the Ljung-

[7] The realized volatility and kurtosis are transformed to their natural logarithm. The use of realized logarithmic volatility in analysis is supported extensively in the literature (e.g., Andersen and Bollerslev, 1998, Andersen et al., 2001a, and Andersen et al., 2003). In addition, we use realized logarithmic kurtosis to achieve the similar scale of the impulse response analysis between three realized measures in later stage. Therefore, from this stage when we refer to realized volatility and kurtosis, we discuss their natural logarithm.

[8] As we observe, significant extreme values of realized volatility of Brazil stock market mostly appear before the year of 2003. This may be due to the fact that BOVESPA created the New Market in around 2002 to improve market's transparency and, consequently, reduce uncertainties in the capital market. Likewise, in the Belgium and France FX markets, a considerable number of extreme values of realized volatility are observed before the year of 2002, when Belgium and France had not yet switched their currencies to the Euro.

Box test statistics. Hence, the features we observe from realized volatility are consistent with previous studies such as Andersen et al. (2003). However, to our knowledge, the stylized facts of realized skewness and kurtosis have not been pointed out in the literature. Further, it is noteworthy that across all three realized measures, the magnitude of the *Q(20)* statistics of stock markets are smaller than those of FX markets in almost all countries. Additionally, the *Q(20)* statistics of the realized volatility are the largest and those of realized skewness are the smallest overall. These findings reflect the fact that realized measures of the stock market constitute more noisy proxies compared with those of FX markets. Likewise, the realized skewness comprises more noisy proxies relative to the realized kurtosis and realized volatility. In other words, there are more latent dynamic components of realized measures of stock markets hidden in the noises than that of FX markets. Similarly, the noises mask more underlying dynamics of realized skewness than that of realized volatility or kurtosis. Therefore, these findings imply a higher degree of predictability for realized measures of FX markets than that of stock markets. Further, the predictive degree for realized skewness is lower than that for realized volatility and kurtosis. This implication can be explained as a generalisation drawn from Andersen et al. (2004), which evaluates and compares the forecast performance of various volatilites with different degree of serial correlation.

2.4 Estimation of fractionally integrated degree

The evidence of strong serial correlation in realized measures suggests the need to analyse their degrees of fractional integration before estimating any model. A number of recent studies have indicated that the long-range dependence can be efficiently captured by a long-memory, or fractionally-integrated, process (e.g., Ding, Granger and

Engle, 1993; and Andersen and Bollerslev, 1997). The long memory process describes the temporal dependence behaviour in a time series which can be considered as an intermediate between the two classical processes, the short-memory and the unit root processes. In these traditional processes, the degree of integration (d) is equal to 0 and 1 in case of short-memory (known as $I(0)$) and unit root (the so-called $I(1)$), respectively. In case of a long memory process, d, which can receive value of a fractional number, is called as the degree of fractional integration. If $d > 0$, the autocovariances of a time series decay to 0 very slowly that they are not summable. When $d < 0$, although the autocovariances are summable, they still die out more slowly than the exponential rate as shown in the stationary and invertible ARMA processes. In our study, we employ the definition of Brockwell and Davis (1995), and use the term "long memory" whenever the degree of fractional integration $d \neq 0$. Further, the use of fractional degree requires the idea of fractional differencing for an estimation purpose. The fractional difference was defined by Granger and Joyeux (1980) and independently by Hosking (1981). The time series x_t is the d'th fractional difference of time series y_t if it satifies, $x_t = (1-L)^d y_t$, where L is the lag operator. Operationally, the term $(1-L)^{-d}$ can be generated by the following binomial expansion:

$$(1-L)^{-d} = \sum_{i=0}^{\infty} \frac{\Gamma(i+d)}{\Gamma(d)\Gamma(i+1)} L^i = \sum_{i=0}^{\infty} \psi_i^{(d)} L^i$$

where $\Gamma(.)$ is the gamma function; $\psi_0^{(0)} = 1$, and $\psi_i^{(0)} = 0$, for $i \neq 0$.

On the basis of above terminologies, we obtain the degree of fractional integration (d) using the Geweke and Porter-Hudak (1983) (GPH) log-periodogram regression estimator. Table 2.3 and 2.4 report estimates of d as well as the associated t test statistic of their significance for the stock and FX markets in all countries, respectively.

estimator. Table 2.3 and 2.4 report estimates of *d* as well as the associated *t* test statistic of their significance for the stock and FX markets in all countries, respectively.

At the 5% significance level, the estimates of *d* of realized volatility are all significantly greater than 0; whereas, there are 25 out of 27 stock markets and 26 out of 27 FX markets showing significant *d* for the realized kurtosis. In terms of realized skewness, only limited cases show significant estimates of *d* (9 out of 27 stock markets and 13 out of 27 FX markets). Further, we observe that the estimates of *d* of stock markets are normally smaller than those of FX markets across all three realized measures. Likewise, the estimates of *d* for realized volatility are the largest and those of realized skewness are the smallest. Hence, these estimates are consistent with the findings we point out from the *Q(20)* statistic in the previous section. In addition, in each category of realized measures, the estimates of *d* are quite close in terms of stock or FX markets, indicative of a common long-run dependence within each type of market (stock or FX markets).

Figure 2.1 provides a graphical illustration and confirmation of the long-memory results for the three realized measures of a selected stock and FX market. Figure 2.1 graphs the sample autocorrelations of realized volatility, skewness and kurtosis for a lag of 100 days, respectively. For the realized volatility, the evidence of strong serial dependence is shown by the slow hyperbolic autocorrelation decay. On the contrary, the sample autocorrelation of realized skewness decays to zero quickly and then fluctuates around zero during the displacement of 100 days, supportive of the short-memory behaviour. In terms of realized kurtosis, although its sample autocorrelation decays slowly to zero in most cases, the rate of decrease is greater than that of realized volatility. This result indicates that the serial correlation in realized kurtosis is not as strong as in realized volatility. However, the result still confirms the existence of long-memory behaviour of realized kurtosis in most cases.

Figure 2.1 also shows the sample autocorrelations of the realized measures, fractionally differenced by applying the filter $(1-L)^{d_i}$, where d_i is the degree of fractional integration of the i^{th} market. Evidently, this single fractional differencing operator eliminates appropriately the bulk of the serial correlation in the realized volatility and kurtosis series since their sample autocorrelations decay quickly then fluctuate around zero.

2.5 A Fractionally Integrated VAR for modeling the realized measures

The finding of long-range dependence of realized measures, especially realized volatility and kurtosis, pointed out in the previous section suggests that a long-memory model is appropriate to capture those features. Accordingly, Andersen et al. (2003) introduces a simple long-memory K-dimensional VAR for modeling the realized volatility (VAR-RV):

$$A(L)(1-L)^d Y_t = \varepsilon_t \tag{2.5}$$

where $A(L) = I_K - A_1 L - A_2 L^2 - ... - A_p L^p$, A_i is the $K \times K$ matrix of coefficients, p is the order of the lag polynomials in $A(L)$, $Y_t = (y_{1t}, y_{2t}, ..., y_{Kt})'$ is the $K \times 1$ vector of endogenous variables at day t and ε_t is a $K \times 1$ vector of white noise.

Hence, under this form, VAR-RV only allows one common value of d for all endogenous variables in the system. This model, therefore, provides a good description of the dynamics when the estimate of *d* of each endogenous variable is close to each other. In other words, it is not applicable for a high dimensional system with a wide range of values of *d*.

As highlighted in the previous section, in our sample the estimates of *d* in stock markets are significantly different from FX markets. Therefore, with the purpose of providing a broad analysis of spill-over effect within-between stock and FX markets, we apply an extension of the VAR-RV, called the Fractionally Integrated VAR (FIVAR). The FIVAR overcomes the limitation of VAR-RV by allowing more than one value of *d* in the system. The *K*-dimensional FIVAR is expressed in the following form[9]:

$$A(L)D(L)Y_t = \nu + \varepsilon_t \tag{2.6}$$

where *D(L)* is a diagonal $K \times K$ matrix: $D(L) = \text{diag}\left\{(1-L)^{d_1}, (1-L)^{d_2}, ..., (1-L)^{d_K}\right\}$ and *v* is the $K \times 1$ vector of intercepts. The elements of the Y_t vector are the realized measures.

The specification of FIVAR has been discussed previously in Sela and Hurvich (2009). In fact, the model can be considered as a subclass of the vector autoregressive fractionally integrated moving average (VARFIMA(*p*,*d*,*q*)) which has been studied initially by Sowell (1989). Subsequently, the estimations of VARFIMA and FIVAR have received large attention from researchers (e.g., Luceno, 1996; Martin and Wilkins,

[9] For estimation purpose, a further restriction, $|d_j| < \frac{1}{2}$ for all $j = 1,2,\ldots,K$, needs to be satisfied to make the model stationary. This condition can always be obtained by taking an appropriate number of differences. For example, if $\frac{1}{2} < d_j < \frac{3}{2}$ then the first-differenced series has a degree of integration less than $\frac{1}{2}$ in absolute value.

1999; and Chiriac and Voev, 2010). One of the most widely used estimation procedures is the exact time domain maximum likelihood estimation (EMLE) which aims to estimate d, ν and A_i simultaneously. Although the EMLE has its own advantages, such as asymptotic efficiency, it is extremely time-consuming for a high dimensional and higher-order system as well as for large sample sizes. Further, as pointed out in Diebold and Rudebusch (1989), the simultaneous maximum-likelihood estimation of d, ν and A_i may be inconsistent under misspecification of A_i. Accordingly, as the scope of our analysis requires a high dimensional system, we extend the univariate two-step estimation procedure suggested by Geweke and Porter-Hudak (1983) to the multivariate case to estimate the FIVAR model. In the first stage, we obtain a consistent and asymptotically normal estimate of d using the GPH log-periodogram regression estimator. This consistent estimate of d, therefore, does not depend on the lag orders and parameterizations of the A_i in FIVAR. We then transform Y_t to $X_t = (x_{1t}, x_{2t}, ..., x_{Kt})'$ by applying the relationship[10]:

$$x_{it} = \begin{cases} (1-L)^{d_i} y_{it} & \text{if } d_i < \frac{1}{2} \\ (1-L)^{d_i - 1}(1-L) y_{it} & \text{if } \frac{1}{2} < d_i < \frac{3}{2} \end{cases} \tag{2.7}$$

Later, we apply the OLS equation-by-equation to estimate the following unrestricted VAR:

$$A(L) X_t = \nu + \varepsilon_t \tag{2.8}$$

[10] We do not transform realized skewness according to (2.7) since the analysis in section 2.4 reveals its short memory behavior.

So, the model (2.8) is stationary if all the roots of the estimated polynomial $|A(z)| = 0$ are outside the unit circle. The orders of the lag polynomials (*p*) in *A(L)* is chosen based on the AIC criteria. In addition, the correlograms of the residuals are also investigated to make sure each of their elements mimics the white noise process. Since Y_t is the realized measures, X_t can be interpreted as the filtered realized measures after removing the effect of structural changes, crises and other elements which can cause the long-memory behavior.

2.6 Generalized impulse response function for investigating the spill-over effect

To examine the spill-over effect within-between stock markets and FX markets, we conduct the GIR proposed by Pesaran and Shin (1998) within model (2.8) to avoid difficulties in ordering the endogenous variables in a high dimensional system. The GIR can be outlined as follows,

Given the assumption of covariance stationarity, X_t can be rewritten as the infinite moving average representation,

$$X_t = \omega + \sum_{i=1}^{\infty} \Pi_i \, \varepsilon_{t-i} \qquad t = 1, 2, ..., T \tag{2.9}$$

where $\omega = (I_K - \sum_{i=1}^{p} A_i)^{-1} \nu$, and $K \times K$ coefficient matrices Π_i can be computed recursively using the relationship,

$$\Pi_i = \begin{cases} \sum_{j=1}^{i} \Pi_{i-j} A_j & i = 1,2,...p \\ \sum_{j=1}^{p} \Pi_{i-j} A_j & i > p \end{cases} \tag{2.10}$$

where $\Pi_0 = I_K$.

The GIR function of X_t at horizon h is defined as the difference between the conditional expectation of X_{t+h} at time $t+h$ after incorporating the shock's effect at time t and that conditional expectation without the shock's effect, given the information set available at time *t-1*, Ω_{t-1}.

$$GIR(h,\delta,\Omega_{t-1}) = E(X_{t+h} \mid \varepsilon_t = \delta, \Omega_{t-1}) - E(X_{t+h} \mid \Omega_{t-1}) \tag{2.11}$$

where $\delta = (\delta_1, \delta_2, ..., \delta_K)'$ denotes the $K \times 1$ vector of shocks hitting the economy at time t.

Pesaran and Shin (1998) introduce a new approach to calculate the impulse response directly from (2.11) by shocking only one element, say the *j*th element of ε_t, and then extracting the effects of other shocks. This approach, therefore, makes the GIR unique and invariant to the order of variables in the system. The GIR function of X_t at horizon h is now defined as:

$$GIR(h,\delta_j,\Omega_{t-1}) = E(X_{t+h} \mid \varepsilon_{jt} = \delta_j, \Omega_{t-1}) - E(X_{t+h} \mid \Omega_{t-1}) \tag{2.12}$$

Using (2.12) in (2.9), we have

$$GIR(h,\delta_j,\Omega_{t-1}) = \Pi_h E(\varepsilon_t \mid \varepsilon_{jt} = \delta_j) \tag{2.13}$$

With the assumption that ε_t has a multivariate normal distribution, it can be seen that:

$$E(\varepsilon_t \mid \varepsilon_{jt} = \delta_j) = (\sigma_{1j}, \sigma_{2j}, ..., \sigma_{Kj})' \sigma_{jj}^{-1} \delta_j = \sum e_j \sigma_{jj}^{-1} \delta_j \tag{2.14}$$

where e_j is a $K \times 1$ selection vector with unity as its *j*th element and zeros elsewhere.

By setting the unit shock as a one standard deviation shock[11], which is $\delta_j = \sigma_{jj}^{\frac{1}{2}}$, from (2.13) and (2.14) we can obtain the GIR function by

$$\hat{\theta}_j(h) = \sigma_{jj}^{-\frac{1}{2}} \Pi_h \sum e_j \qquad h = 0,1,2,... \tag{2.15}$$

For statistical inference about the existence of the spill-over effect, the bias-corrected bootstrap method presented in Efron and Tibshirani (1993) is employed to construct a 95% confidence interval for the GIR. The bootstrap method has been extensively applied and found to be useful in many econometrics studies (see Berkowitz and Kilian, 2000; and MacKinnon, 2002). The steps to construct a confidence interval can be summarized as follows.

Step 1: Given the filter realized measures X_t, we estimate model (2.8) based on the Least Squares method. The estimator of $A = (\nu, A_1, A_2, ..., A_p)$ and the variance-covariance matrix of the error term, $\sum_\varepsilon$, are $\hat{A}$ and $\hat{\sum}_\varepsilon$ respectively. The residual vector, $\hat{\varepsilon}_t$, can also be obtained. From $\hat{A}$ and $\hat{\sum}_\varepsilon$, we construct the GIR, $\hat{\theta}$, according to the formula (2.15).

Step 2: Draw a residual bootstrap sample, ε_t^B, by random sampling $\hat{\varepsilon}_t$ with replacement. From the first p values of the original realized measures data and ε_t^B we generate the pseudo data through the following recursion:

$$X_t^B = \hat{\nu} + \hat{A}_1 X_{t-1}^B + ... + \hat{A}_p X_{t-p}^B + \varepsilon_t^B \tag{2.16}$$

[11] As such, in this study we refer to the "unit" shock as a shock with size equal to one standard deviation.

Step 3: Re-estimate model (2.8) with the bootstrap realized data, X_t^B, then calculate the bootstrap generalized impulse response $\hat{\theta}^B$. Repeat step 2 and step 3 for 2000 times, which is sufficiently large for the bootstrapping confidence interval (Efron and Tibshirani, 1993). We then obtain a series of bootstrap generalized impulse response, $\{\hat{\theta}_j^B\}_{j=1}^{2000}$.

The 100(1-2α)% bias-corrected bootstrap confidence interval for $\hat{\theta}$ can be constructed as the interval:

$$\text{BC: } [\,\hat{\theta}_{lo}, \hat{\theta}_{up}\,] = [\,\hat{\theta}^B(\alpha_1), \hat{\theta}^B(\alpha_2)\,] \tag{2.17}$$

where $\hat{\theta}^B(q)$ is the q^{th} percentile from the bootstrap distribution of $\{\hat{\theta}_j^B\}_{j=1}^{2000}$; $\alpha_1 = \Phi(2\hat{z}_0 + z^{(\alpha)})$ and $\alpha_2 = \Phi(2\hat{z}_0 + z^{(1-\alpha)})$. $\Phi(.)$ is the standard normal cumulative distribution function and $z^{(\alpha)}$ is the 100α^{th} percentile of a standard normal distribution. Further, $\hat{z}_0$ is called the bias-correction which measures the difference between the median of the bootstrap generalized impulse response, $\hat{\theta}^B$, and the original estimate, $\hat{\theta}$, in the normal unit. The bias-correction can be calculated directly from the proportion of bootstrap replications less than the original estimate $\hat{\theta}$,

$$\hat{z}_0 = \Phi^{-1}\left(\frac{\sum_{j=1}^{B} I(\hat{\theta}_j^B < \hat{\theta})}{B} \right) \tag{2.18}$$

$\Phi^{-1}(.)$ is the inverse function of a standard normal cumulative distribution function. *I(.)* indicates the indicator function and *B* is the number of bootstrap replications, in this chapter it is set to be 2000.

2.7 Realized spill-over effect

The order of the FIVAR model is chosen based on AIC criteria. In addition, we also investigate the correlogram of the residuals to ascertain that each of their elements mimics the white noise process. The estimated FIVAR coefficients are mostly significant, and all the roots of the estimate of the matrix lag polynomial *A(L)* are outside the unit root circle, an indication of covariance stationarity[12]. Therefore, the GIR can be adequately applied to investigate the spill-over effect, which we use as a representation of the markets linkages. In our analysis, the spill-over effect is defined as an effect on one variable in next periods that is caused by an exogenous shock to another variable in the current time. Table 2.5 summarizes the existence and the magnitude of the spill-over effect in Panel A and Panel B, respectively.

2.7.1 Realized Volatility Spill-over

Considerable evidence of a realized volatility spill-over effect within-between stock and foreign exchange (FX) market across countries can be found. Figure 2.2 provides a graphical illustration of the effect in some selected markets. In general, these effects, if they exist, are on a two-way basis and they remain significantly positive for about 3 days then die out quickly. This result is consistent with the behaviour of the short memory as we filtered out the long range dependence of realized volatility in the earlier stage by applying the single fractional differencing operator.

Panel A of Table 2.5 indicates that there is more evidence of a volatility spill-over effect in developed countries than in emerging countries. However, these

[12] Due to unavailability of data, we do not include the China FX market, Pakistan FX market, Belgium stock market and Portugal stock market in this analysis. Further, to conserve space, we do not report the estimated FIVAR coefficients. However, the full set of results is available upon request.

differences mostly come from the FX market linkages (i.e., spill-over effect within FX markets) and the stock-FX market linkages (i.e., spill-over effect between stock and FX markets).

Analyzing the stock market linkages (i.e., spill-over effect within stock markets) via their 2nd moment, which is the realized volatility, we observe a similar pattern between developed and emerging markets. The linkages exist in all cases analyzed in most of regions, except for Asian Emerging and West European Developed group, which show evidence in 44% and 81% of cases, respectively.

Regarding the FX market linkages and the stock-FX market linkages via their 2nd moment, the empirical results consistently show greater evidences of a volatility spill-over effect in developed markets than in emerging markets. Especially, when we exclude the two developed countries, which are the United States and Canada, from the American group to investigate the Latin American group, the rates of spill-over's existence drop from 44% and 54% to 38% and 41% for the FX market linkages and the Stock-FX market linkages, respectively. In addition, no emerging region can offer evidence of the FX and the stock-FX linkages in at least 50% of the cases; whereas all developed regions show evidence in greater than 70% of the cases. Particularly, we note the evidence of the FX market linkages in all cases analysed in all European developed groups. This is consistent with our expectation as many FX markets of European countries have been driven by common dynamics since they joined the European Union.

Panel B of Table 2.5 shows no obvious difference in the magnitude of the volatility spill-over effect is observed for within-between stock and FX markets linkages in emerging market groups. In developed market groups, however, the magnitude of the volatility spill-over effect within FX markets is the largest; followed by the effects within stock markets whose magnitude approximately equals to that of the effects between stock and FX markets.

2.7.2 Realized Skewness Spill-over

Empirical results summarized in Panel A of Table 2.5 provide support for the existence of markets linkages via their 3rd moment in many cases. Figure 2.3 illustrates the GIR analysis in some selected markets. Generally, despite appearing significant initially, the realized skewness spill-over effects die out quickly after 1 to 3 days. This finding is consistent with the short memory behaviour of realized skewness which we discussed earlier. Further, the cross-asset linkages (i.e., spill-over effect between stock and FX market) via their 3rd moment, if it exists, are negative; whereas, these linkages of the same assets (i.e., spill-over effect within stock or FX markets) are positive. Interestingly, it is likely to have more evidence of stock-FX markets linkages via their 3rd moment in emerging market groups than in developed market groups. However, there is no apparent difference between these two types of groups in terms of both stock markets and FX markets linkages.

Regarding the linkages of the stock markets via their 3rd moment, most of the regions show evidence in all cases, except for the Asian Emerging group which provides supports in 88% of cases. A similar pattern can be seen from the empirical results of the FX markets linkages, where existences of skewness spill-over effects are observed in all cases in almost all regions. The exceptions are American (or Latin American) and Asia Pacific Developed groups, which exhibit evidence in around 70% of cases.

An interesting finding comes from the investigation of the linkages between stock and FX markets via their 3rd moment. These relationships are found to be negative in all cases and greater evidence is observed in emerging market regions rather than in developed market regions. The rate of the existence of the relationship in emerging market groups is usually greater than 75%; whereas, it is regularly less than 45% in developed market groups. Since our calculation of skewness focuses on the downside risk, this finding suggests a good strategy for investors to diversify the downside risk is by combining the stock and FX assets in their portfolios, especially in emerging markets. Further, this result is also consistent with the literature which discusses the importance of the downside risk in emerging markets (see Estrada, 2002; and Galagedera and Brooks, 2007).

In terms of the linkages' strength, Panel B of Table 2.5 displays no clear difference between three types of skewness spill-over effects in emerging market groups. However, in developed market groups, we observe a consistent result that the magnitude of the linkages between stock and FX markets is the smallest. Meanwhile, the strength of stock markets and FX markets linkages are approximately equal. This finding suggests that in developed markets, the linkages of the same assets via the downside risk are greater than those of the cross-assets in terms of the magnitude.

2.7.3 Realized Kurtosis Spill-over

The GIR of realized kurtosis with a 95% bias-corrected bootstrap confidence interval in some selected markets is illustrated in Figure 2.4. The existence of the kurtosis spill-over effects is summarized in Panel A of Table 2.5. The empirical results provide support for the existence of realized kurtosis spill-over effects in many cases. These effects, if they exist, are all positive but only last a short period of time, about 2-

3 days. Similar to the realized volatility spill-over effect, this finding confirms the short memory behaviour of the filtered realized kurtosis. Further, the empirical result of a positive kurtosis spill-over effect is consistent with previous studies, which found a positive correlation of extreme returns between markets (see Longin and Solnik, 2001; and Cumperayot et al., 2006).

Generally, we observe more evidence about markets linkages in terms of both the same assets and cross-assets via their 4^{th} moment in developed countries rather than in emerging countries. In addition, both types of group consistently show more evidence of the same asset markets linkages than that of the cross-asset markets linkages.

Regarding the stock markets linkages via their 4^{th} moment, evidence of the linkages can be observed in all cases analysed in all developed market regions. Meanwhile, the emerging market regions provide support in around 65% of the cases, except for the Asia Pacific Emerging group which exhibits significant spill-over effects in all cases. Likewise, in terms of the FX markets linkages, developed market regions also provide strong support for kurtosis spill-over effects since they exist in all cases in 3 out of 4 regions. However, the rates of existence of these linkages are about 40% in the emerging market regions. A similar pattern can be seen from the stock-FX markets linkages as greater evidence is often observed in the developed market groups rather than the emerging market groups.

Additionally, it is noteworthy to point out the fact that there is more evidence of stock markets linkages via their 4^{th} moment than that of FX markets in emerging market groups; whereas, no obvious difference is observed in developed market groups. Further, all regions consistently show greater evidence of the same asset markets linkages than that of the cross-asset markets linkages.

Panel B of Table 2.5 shows mixed results about the magnitude of the developed market linkages. However, they all agree that the magnitude of the stock-FX markets linkages is the smallest. Regarding the emerging markets linkages, no apparent difference in the magnitude of the spill-over effects is observed.

2.7.4 Discussion of results

The strong evidence of the linkages of stock markets via all three higher moments describes their tight relationship in both emerging and developed market groups. The empirical results confirm that the stock markets are positively linked not only through the standard deviation but also through the asymmetric level and the tails of their return distribution. In other words, if there is a shock to a stock market which raises the standard deviation, the level of asymmetry and the occurrence of extreme events of its return distribution, then a broad spill-over effect should be expected to cause an increase in those of other stock markets.

A similar scenario is anticipated for the FX markets linkages. However, the spread of the linkages via the 2^{nd} and 4^{th} moment is narrower in the emerging market regions compared with the developed market regions. The developed market groups provide significantly greater evidence of volatility and kurtosis spill-over effects than the emerging market groups. This result possibly indicates that majority of funds are invested in developed FX markets rather than in emerging FX markets. Further, investors are making FX transactions in order to get in and out of the developed markets more frequently than in the emerging markets. Hence, this result is consistent with our expectation as major currencies are much more liquid than non-major currencies.

In addition, for emerging markets, more evidence of spill-over effects is observed in stock markets rather than in FX markets in terms of all three realized measures. This finding implies that emerging stock markets are more liquid than emerging FX markets counterpart due to benefits of international portfolio diversification.

Regarding the relationship between stock and FX markets, more evidence of the linkages via the 3rd moment are found in the emerging markets rather than in the developed markets; whereas, the developed regions provide greater support for the linkages via the 2nd and 4th moment. Further, in emerging markets, the rates of linkages' existence via the 3rd moment are normally greater than that via the 2nd and 4th moment. Meanwhile, these numbers are regularly the smallest in developed markets. Therefore, in emerging countries, the cross-asset markets linkages are more likely to be transmitted through the asymmetry of the return distribution; whereas, they spread out through the standard deviation and the tail of return distribution in developed countries. These results are consistent with the importance of the downside risk in emerging markets as pointed out in the literature (e.g., Estrada, 2002; and Galagedera and Brooks, 2007). In addition, the negative value of the linkages via the 3rd moment suggests an option for investors to diversify the downside risk by combining the stock and currency assets in their portfolio. However, this strategy might only be applicable for the emerging markets since the developed markets provide greater evidence of linkages via the 2nd and 4th moment rather than via the 3rd moment.

In terms of the strength of the linkages via all three higher moments, while the emerging market groups often show no obvious difference, the developed market groups consistently display that the magnitude of the same asset markets linkages is normally greater than or at least equal to that of the cross-assets markets linkages. This result is consistent with our expectation since there are more common economic factors connecting the same asset markets than the cross-assets markets. Further, along with a lower degree of market transparency, emerging markets often contain much more noisy information than developed markets. Therefore, the difference between impacts of common economic factors on same asset linkages and that on cross-asset linkages is possibly insignificant in emerging markets but significant in developed markets. In fact, this interpretation is consistent with the suggestion in Morck et al. (2000), which states that emerging markets act as less useful processors of economic information than developed markets, particularly in terms of stock markets[13].

Additionally, our significant evidence of spill-over effects via higher moments in terms of both same and cross-assets markets emphasize the necessity to involve the measurement of market linkages via higher moments in many financial activities, especially asset pricing, value-at-risk (VaR) calculation and asset allocation. Jurczenko and Maillet (2006) claim an appearance of skewness and kurtosis risk can significantly affect the asset pricing by introducing the four-moment CAPM model. Likewise, higher moment risks can also have technical implications on VaR models. Thus, the estimation of VaR may be flawed if either of those risks is ignored (see Mandelbrot and Hudson, 2004). Further, a miscalculation in asset pricing or VaR can directly result in an inappropriate asset allocation decision. Therefore, due to the increasing integration

[13] Recently, Büttner et al. (2012) and Hanousek et al. (2009) have documented a close linkage between emerging markets and developed markets which facilitates emerging markets to process information to a large extent. These studies base on the markets of the Czech Republic, Hungary and Poland; which have benefited from the plan of European Union Enlargement 2004-2007. The enlargement plan has led a transfer of massive financial products and assistances from Western European developed markets to European emerging markets during this period. Therefore, their findings may differ from our interpretation and the suggestion of Morck (2000) for the cases of our employed emerging markets.

between countries, an ignorance of markets linkages via higher moments can lead to an underestimation of skewness and kurtosis risk and, consequently, an incorrect financial decision.

2.8 Concluding remarks

This chapter assesses the financial markets linkages with particular focus on stock and FX markets by investigating the spill-over effect not only in the context of volatility but also skewness and kurtosis using high frequency data. The long memory behaviour of realized volatility and kurtosis is well captured by the FIVAR model. For statistical inference, we construct the 95% bias-corrected bootstrap confidence interval for the GIR. The empirical results provide strong support for positive linkages within stock markets via all three higher moment in terms of both emerging and developed markets. Similar properties of the linkages are obtained for the FX markets. However, the spread of the linkages via the 2^{nd} and 4^{th} moment is broader in the developed market regions in comparison with the emerging market regions. In term of cross-assets markets linkages, the stock and FX markets in emerging market groups are more likely to be linked through the 3^{rd} moment; whereas, those in developed market groups are transmitted through the 2^{nd} and 4^{th} moments. Further, the magnitude of the cross-assets markets linkages is often found to be less than that of the same asset markets linkages via all three higher moments. In addition, the fact of negative linkages via the 3^{rd} moment between stock and FX markets suggests that investors can hedge the downside risk by combining both stock and currency assets in their portfolio, especially in emerging markets. Finally, our study highlights the importance of the measurement of financial markets linkages via higher moments in many financial activities, especially asset pricing, VaR estimation and asset allocation.

2.9 APPENDIX

Table 2.1: GMT Offsets and Stock Markets Trading Times (GMT) for all countries examined

Countries	GMT Offset		GMT Trading Hours	
	Standard time	DST	Standard time	DST
Australia	+10	+11	0:00-6:00	23:00-5:00
New Zealand	+12	-	22:00-4:00	-
Hong Kong	+8	-	2:00-8:00	-
Japan	+9	-	0:00-6:00	-
Korea	+9	-	0:00-6:00	-
Indonesia	+7	-	2:30-9:00	-
Malaysia	+8	-	1:00-9:00	-
Philippines	+8	-	1:30-4:00	-
Taiwan	+8	-	1:00-5:30	-
Thailand	+7	-	3:00-9:30	-
China	+8	-	1:30-7:00	-
India	+5.5	-	3:30-10:00	-
Pakistan	+5	-	4:30-10:30	-
Taiwan	+8	-	1:00-5:30	-
Thailand	+7	-	3:00-9:30	-
Austria	+1	+2	8:30-16:30	7:30-15:30
Belgium	+1	+2	8:00-16:30	7:00-15:30
France	+1	+2	8:00-16:30	7:00-15:30
Germany	+1	+2	8:00-16:30	7:00-15:30
Greece	+2	+3	9:00-14:00	8:00-13:00
Ireland	0	+1	8:00-16:30	7:00-15:30
Portugal	0	+1	9:00-17:30	8:00-16:30
Spain	+1	+2	8:00-16:30	7:00-15:30
Switzerland	+1	+2	8:00-16:30	7:00-15:30
The UK	0	+1	8:00-16:30	7:00-15:30
Norway	+1	+2	9:00-15:30	8:00-14:30
The USA	-5	-4	14:30-21:00	13:30-20:00
Brazil	-3	-2	13:00-20:00	12:00-19:00
Chile	-4	-3	13:30-21:00	12:30-20:00
Peru	-5	-	13:30-21:00	-
Argentina	-3	-	14:00-21:00	-

Note: **DST** denotes Daylight Saving Time.

Table 2.2: Descriptive statistics of the realized measures estimates for some countries

	Stock Market				FX markets			
	The UK	Korea	Indonesia	Taiwan	The UK	Korea	Indonesia	Taiwan
Panel A: Descriptive Statistics for the Realized Volatility estimates								
Mean	-9.494	-8.441	-9.073	-8.725	-10.513	-10.973	-9.794	-11.934
Std. Dev.	1.038	1.058	0.972	0.939	0.743	1.843	2.288	1.627
Skewness	0.333	0.223	0.786	0.117	0.362	0.418	0.681	-0.541
Kurtosis	3.375	2.906	4.163	3.103	6.376	4.494	3.346	4.901
Q-stat(20)	29861***	25556***	11262***	19022***	18028***	22842***	44194***	12559***
Panel B: Descriptive Statistics for the Realized Skewness estimates								
Mean	-0.255	-0.250	-0.561	-0.610	-0.021	0.008	0.028	0.216
Std. Dev.	2.384	2.926	2.533	2.591	2.032	2.591	1.254	1.670
Skewness	-0.214	-0.020	0.388	0.142	-0.003	0.007	0.211	0.127
Kurtosis	4.624	3.360	3.544	3.415	5.897	3.346	5.285	5.022
Q-stat (20)	41.3***	64.4***	90.6***	313.0***	21.5	292.6***	153.6***	110.2***
Panel C: Descriptive Statistics for the Realized Kurtosis estimates								
Mean	-2.430	-1.930	-1.783	-1.749	-2.713	-1.677	-2.054	-1.612
Std. Dev.	0.826	0.888	0.697	0.806	0.802	0.960	0.695	0.828
Skewness	0.735	0.413	0.462	0.314	0.931	0.135	0.613	-0.503
Kurtosis	2.612	1.942	2.392	1.956	3.092	1.949	2.974	3.257
Q-stat (20)	779.56***	152.61***	640.46***	262.51***	138.48***	5523.3***	2470.7***	12457***

Notes: Sample period is from 01/01/1997 to 20/05/2010, builds up 3492 observations of daily realized measures. The realized volatility and kurtosis is analysed using their natural logarithm. The Q-stat (20) indicates the Ljung-Box statistics for up to twentieth order serial correlation in the realized measure. *,** and *** denote the serial correlation up to lag 20 is significant at 10%, 5% and 1% significance level, respectively.

Table 2.3: Estimation of fractionally integrated degree for stock markets

	Realized Volatility			Realized Skewness			Realized Kurtosis		
Country	d	t-stat	p-value	d	t-stat	p-value	d	t-stat	p-value
Argentina	0.54	5.75	0.00	0.06	0.69	0.49	0.41	4.37	0.00
Brazil	0.37	3.93	0.00	-0.03	-0.29	0.77	0.32	3.46	0.00
The US	0.58	6.20	0.00	0.10	1.02	0.31	0.27	2.89	0.01
Chile	0.48	5.12	0.00	0.18	1.97	0.05	0.30	3.19	0.00
Peru	0.46	4.93	0.00	0.21	2.28	0.03	0.40	4.29	0.00
China	0.58	6.18	0.00	0.04	0.42	0.67	0.11	1.20	0.24
India	0.47	5.05	0.00	0.10	1.06	0.29	0.23	2.51	0.01
Pakistan	0.43	4.56	0.00	0.24	2.58	0.01	0.39	4.14	0.00
Taiwan	0.72	7.71	0.00	0.35	3.69	0.00	0.26	2.77	0.01
Thailand	0.72	7.71	0.00	0.35	3.69	0.00	0.26	2.77	0.01
Indonesia	0.45	4.80	0.00	0.09	0.98	0.33	0.28	2.99	0.00
Malaysia	0.51	5.43	0.00	0.11	1.14	0.26	0.33	3.47	0.00
Philippines	0.46	4.90	0.00	0.27	2.84	0.01	0.61	6.47	0.00
Australia	0.53	5.67	0.00	0.07	0.73	0.47	0.24	2.53	0.01
Hongkong	0.66	7.05	0.00	0.00	-0.04	0.97	0.21	2.20	0.03
Japan	0.61	6.56	0.00	0.29	3.14	0.00	0.15	1.60	0.12
Korea	0.73	7.79	0.00	-0.05	-0.50	0.62	0.22	2.34	0.02
New Zealand	0.56	5.95	0.00	0.05	0.57	0.57	0.40	4.33	0.00
Ireland	0.57	6.10	0.00	0.23	2.50	0.02	0.54	5.77	0.00
Norway	0.59	6.31	0.00	0.19	1.98	0.05	0.24	2.55	0.01
The UK	0.63	6.69	0.00	0.11	1.19	0.24	0.54	5.81	0.00
Greece	0.47	4.98	0.00	0.30	3.25	0.00	0.56	5.96	0.00
Spain	0.55	5.91	0.00	0.19	2.02	0.05	1.00	10.69	0.00
Austria	0.59	6.30	0.00	0.15	1.56	0.12	0.31	3.35	0.00
France	0.45	4.76	0.00	0.16	1.75	0.09	0.40	4.25	0.00
Germany	0.61	6.48	0.00	0.11	1.22	0.23	0.38	4.11	0.00
Switzerland	0.62	6.59	0.00	0.21	2.21	0.03	0.33	3.48	0.00

Note: *d* denotes for the degree of fractional integration obtained by using GPH (1983) long-periodogram regression estimator. *t-stat* and associated *p-value* are the test statistic and its associated significance value result from testing the null hypothesis $H_0{:}d=0$ against the alternative $H_A{:}\ d \neq 0$.

Table 2.4: Estimation of fractionally integrated degree for FX markets

	Realized Volatility			Realized Skewness			Realized Kurtosis		
Country	d	t-stat	p-value	d	t-stat	p-value	d	t-stat	p-value
Argentina	0.92	9.82	0.00	0.10	1.04	0.30	0.81	8.66	0.00
Brazil	0.68	7.30	0.00	0.36	3.80	0.00	0.71	7.58	0.00
Canada	0.78	8.38	0.00	0.18	1.92	0.06	0.39	4.16	0.00
Chile	0.69	7.37	0.00	0.18	1.92	0.06	0.47	5.01	0.00
Peru	0.67	7.14	0.00	0.09	0.91	0.37	0.21	2.25	0.03
India	0.81	8.69	0.00	0.26	2.75	0.01	0.59	6.35	0.00
Indonesia	0.82	8.73	0.00	0.28	2.97	0.00	0.54	5.79	0.00
Malaysia	0.70	7.45	0.00	0.36	3.82	0.00	0.75	8.02	0.00
Philippines	0.54	5.76	0.00	0.08	0.84	0.40	0.27	2.90	0.01
Taiwan	0.61	6.53	0.00	0.26	2.83	0.01	0.78	8.36	0.00
Thailand	0.82	8.73	0.00	-0.02	-0.26	0.80	0.53	5.65	0.00
Australia	0.67	7.15	0.00	0.07	0.73	0.47	0.29	3.06	0.00
Hongkong	0.68	7.28	0.00	0.11	1.20	0.23	0.50	5.29	0.00
Japan	0.57	6.12	0.00	-0.01	-0.16	0.88	0.20	2.09	0.04
Korea	0.62	6.62	0.00	0.20	2.15	0.04	0.68	7.26	0.00
New Zealand	0.65	6.93	0.00	0.03	0.27	0.79	0.19	2.01	0.05
Ireland	0.77	8.19	0.00	0.18	1.93	0.06	0.49	5.22	0.00
Norway	0.73	7.79	0.00	0.21	2.23	0.03	0.28	3.01	0.00
The UK	0.74	7.87	0.00	0.08	0.90	0.37	0.44	4.65	0.00
Greece	0.76	8.12	0.00	0.17	1.82	0.07	0.31	3.35	0.00
Portugal	0.70	7.48	0.00	0.24	2.53	0.01	0.48	5.12	0.00
Spain	0.69	7.33	0.00	0.24	2.51	0.01	0.40	4.33	0.00
Austria	0.68	7.31	0.00	0.29	3.07	0.00	0.33	3.49	0.00
Belgium	0.73	7.82	0.00	0.25	2.72	0.01	0.26	2.80	0.01
France	0.69	7.32	0.00	0.21	2.28	0.03	0.28	3.00	0.00
Germany	0.68	7.23	0.00	0.27	2.87	0.01	0.25	2.69	0.01
Switzerland	0.69	7.38	0.00	0.03	0.28	0.78	0.28	3.03	0.00

Note: *d* denotes the degree of fractional integration obtained by using GPH (1983) long-periodogram regression estimator. *t-stat* and associated *p-value* are the test statistic and its associated significance value result from testing the null hypothesis $H_0{:}d=0$ against the alternative $H_A{:}\ d \neq 0$.

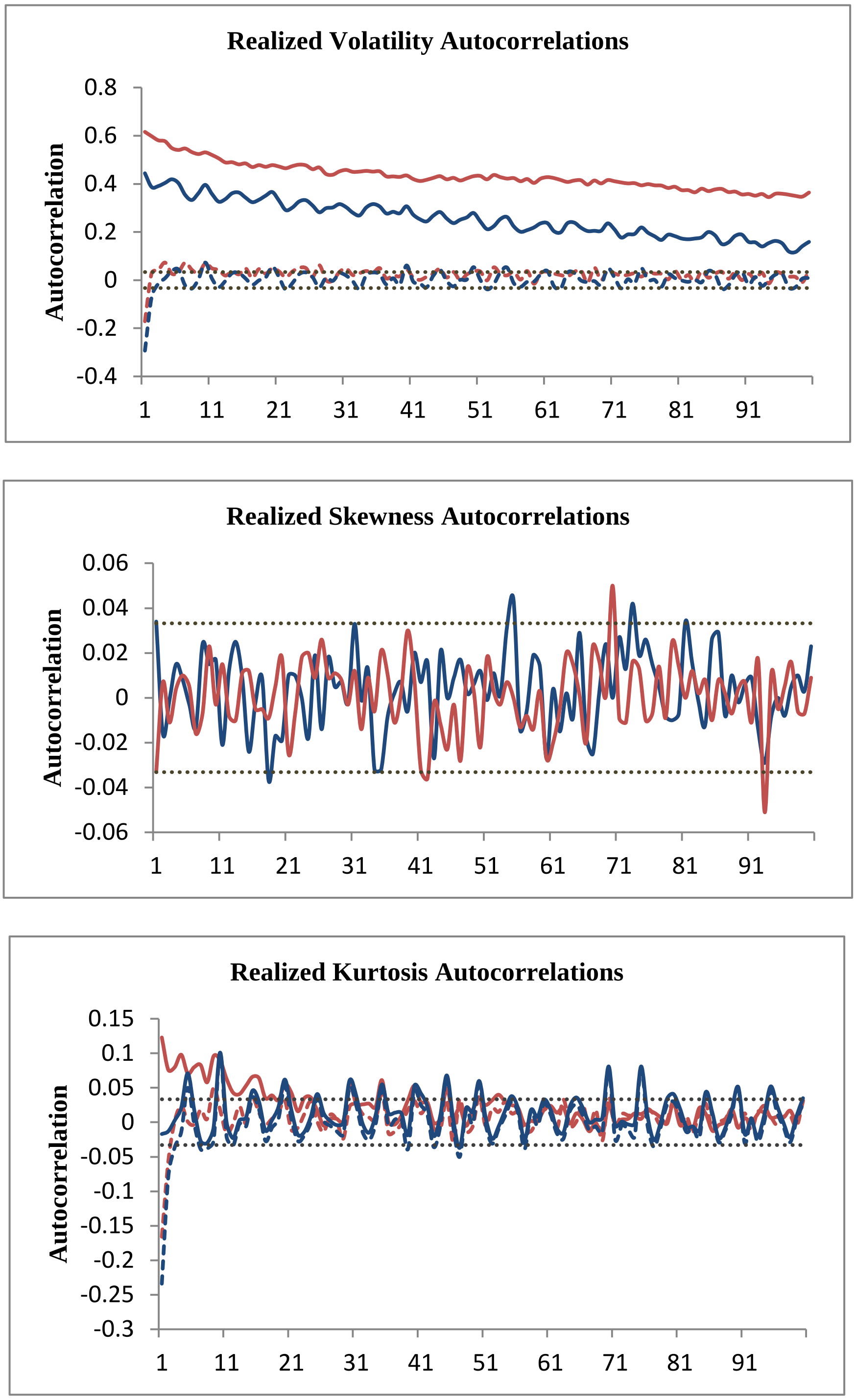
Table 2.5: Existence and Magnitude of the Spill-over Effects

Region	Realized Volatility				Realized Skewness				Realized Kurtosis			
	Panel A: Existence of the Realized Spill-over effects											
	ST	FX	ST-FX	Total	ST	FX	ST-FX	Total	ST	FX	ST-FX	Total
America	25/25	11/25	27/50	63/100	25/25	15/25	40/50	80/100	17/25	9/25	15/50	41/100
	100%	*44%*	*54%*	*63%*	*100%*	*60%*	*80%*	*80%*	*68%*	*36%*	*30%*	*41%*
Latin America	16/16	6/16	13/32	35/64	16/16	10/16	24/32	50/64	10/16	7/16	8/32	25/64
	100%	*38%*	*41%*	*55%*	*100%*	*63%*	*75%*	*78%*	*63%*	*44%*	*25%*	*39%*
Asian Emerging	11/25	3/9	2/30	16/64	22/25	9/9	17/30	48/64	15/25	3/9	4/30	22/64
	44%	*33%*	*7%*	*25%*	*88%*	*100%*	*57%*	*75%*	*60%*	*33%*	*13%*	*34%*
Asian Pacific Emerging	25/25	11/25	11/50	47/100	25/25	25/25	50/50	100/100	25/25	12/25	6/50	43/100
	100%	*44%*	*22%*	*47%*	*100%*	*100%*	*100%*	*100%*	*100%*	*48%*	*12%*	*43%*
Asian Pacific Developed	25/25	19/25	33/50	77/100	25/25	18/25	37/50	80/100	25/25	17/25	16/50	58/100
	100%	*76%*	*66%*	*77%*	*100%*	*72%*	*74%*	*80%*	*100%*	*68%*	*32%*	*58%*
North Europe Developed	9/9	16/16	18/24	43/49	9/9	9/9	8/18	26/36	9/9	9/9	10/18	28/36
	100%	*100%*	*75%*	*88%*	*100%*	*100%*	*44%*	*72%*	*100%*	*100%*	*56%*	*78%*
South Europe Developed	4/4	9/9	10/12	23/25	4/4	9/9	0/12	13/25	4/4	9/9	0/12	13/25
	100%	*100%*	*83%*	*92%*	*100%*	*100%*	*0%*	*52%*	*100%*	*100%*	*0%*	*52%*
West Europe Developed	13/16	25/25	34/40	72/81	16/16	25/25	12/40	53/81	16/16	25/25	33/40	74/81
	81%	*100%*	*85%*	*89%*	*100%*	*100%*	*30%*	*65%*	*100%*	*100%*	*83%*	*91%*

Region	Panel B: Magnitude of the Realized Spill-over Effects		
	Realized Volatility	Realized Skewness	Realized Kurtosis
America	No Obvious Difference	No Obvious Difference	No Obvious Difference
Latin America	No Obvious Difference	No Obvious Difference	No Obvious Difference
Asian Emerging	No Obvious Difference	No Obvious Difference	No Obvious Difference
Asian Pacific Emerging	ST>FX ≈ ST_FX	ST>FX ≈ |ST_FX|	ST>FX ≈ ST_FX
Asian Pacific Developed	ST ≈ FX>ST_FX	ST ≈ FX>|ST_FX|	ST ≈ FX>ST_FX
North Europe Developed	FX>ST ≈ ST_FX	ST ≈ FX>|ST_FX|	FX>ST ≈ ST_FX
South Europe Developed	FX>ST ≈ ST_FX	ST ≈ FX>|ST_FX|	FX>ST ≈ ST_FX
West Europe Developed	FX>ST ≈ ST_FX	ST ≈ FX>|ST_FX|	ST ≈ FX>ST_FX

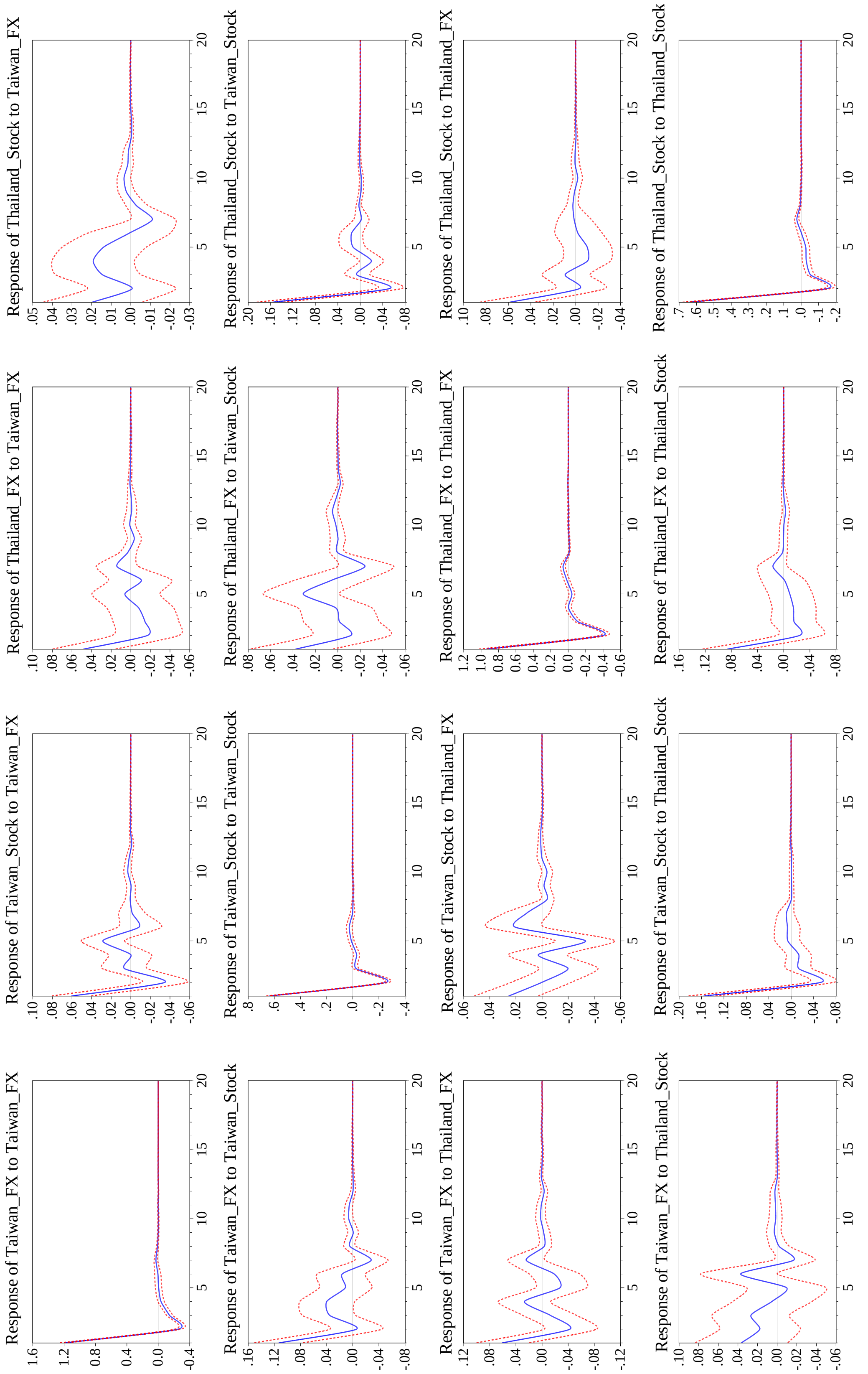
Notes: ST denotes the realized spill-over effects within the stock markets. FX denotes the realized spill-over effects within the FX markets. ST-FX denotes the realized spill-over effects between stock and FX markets. If the value of ST_FX is negative, it's magnitude is noted as |ST_FX|. Panel A reports the number of cases, which show evidences of realized spill-over effect, out of the total cases analysed. The percentage in *italic* indicates the proportion of time we observe the existence of realized spill-over effect. We make conclusion about the significance of spill-over effect using the 95% bias-corrected bootstrap confidence interval bound. We consider the spill-over effects exist when their magnitude is significant greater than 0.05 initially. In Panel B, the symbol ≈ indicates the magnitude of the effects are approximately equivalent.

Figure 2.1: Sample Autocorrelations of Realized Measures



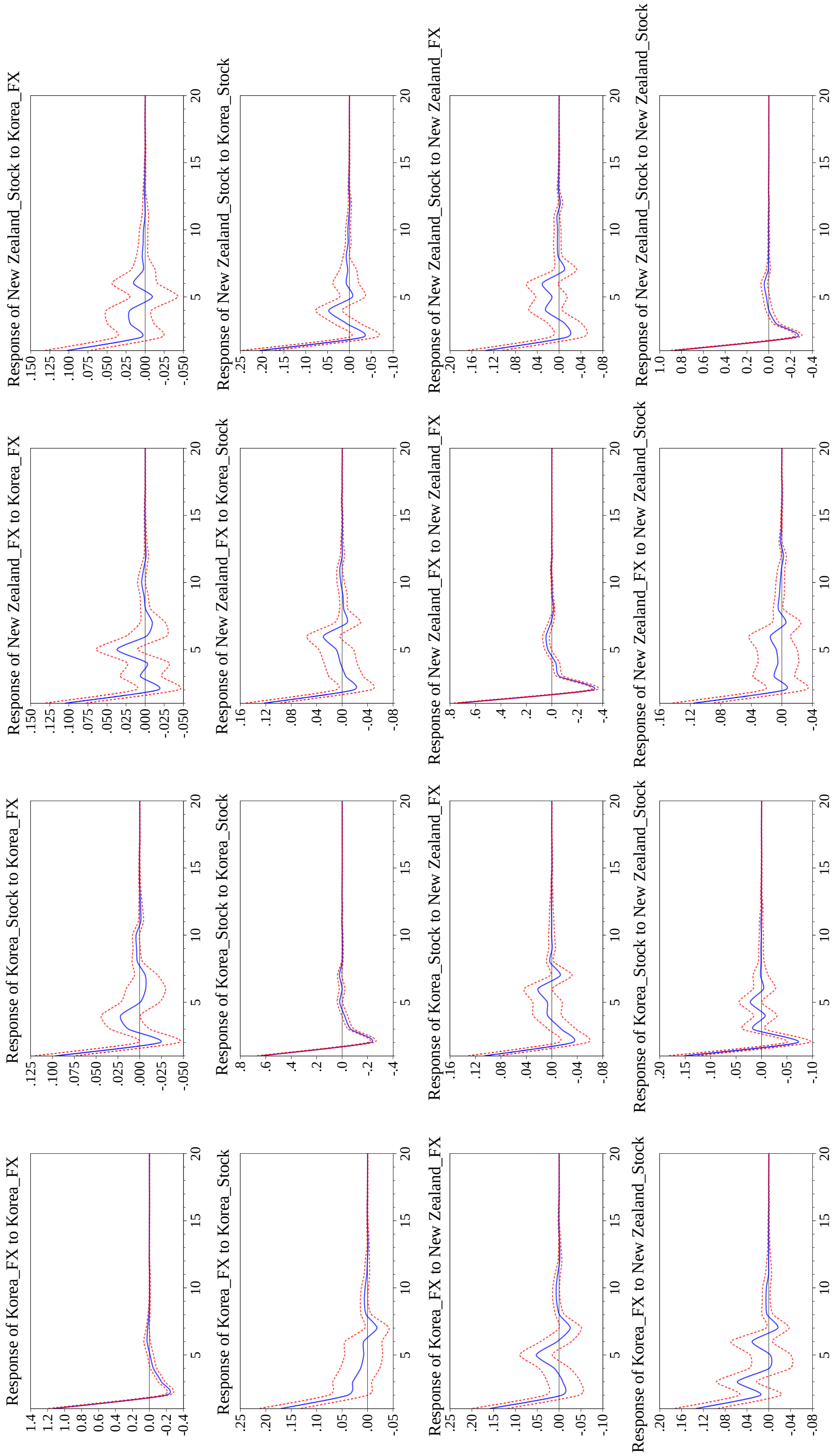
Notes: The figure graphs the sample autocorrelation of daily Australian stock (in red) and FX (in blue) realized measures for a displacement of 100 days. The solid lines give the autocorrelation function of realized measures; whereas the dashed lines refer to the autocorrelation function of realized measures, fractionally differenced by applying the filter $(1-L)^{d_i}$, with d_i is the degree of fractional integration of the i market. The black dotted lines are the confidence bands.

Figure 2.2a: The Realized Volatility GIR of Selected Markets in Asian Pacific Emerging Group



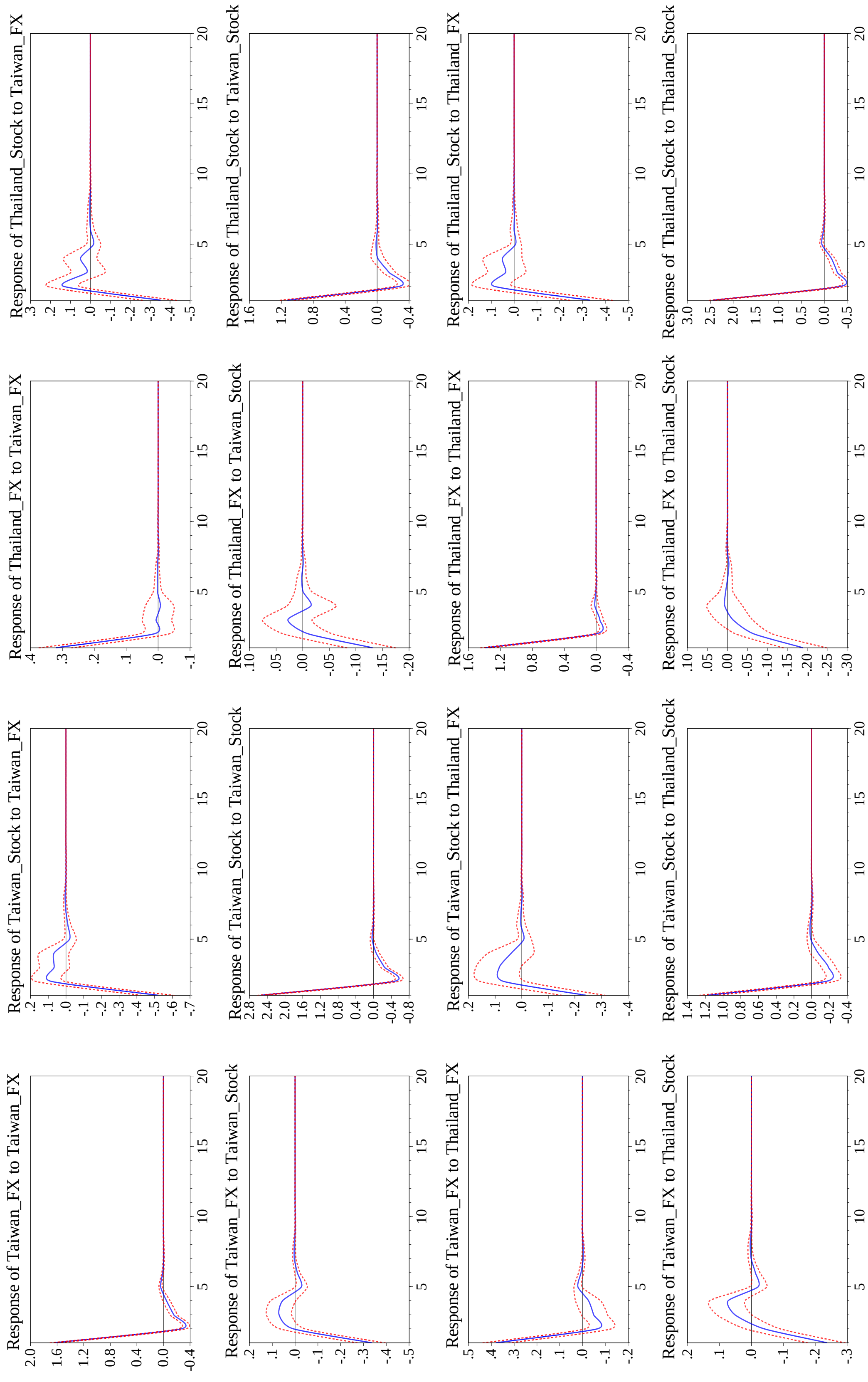
Notes: The figure reports response of a market to one generalized standard deviation shock in another market

Figure 2.2b: The Realized Volatility GIR of selected Markets in Asian Pacific Developed Group



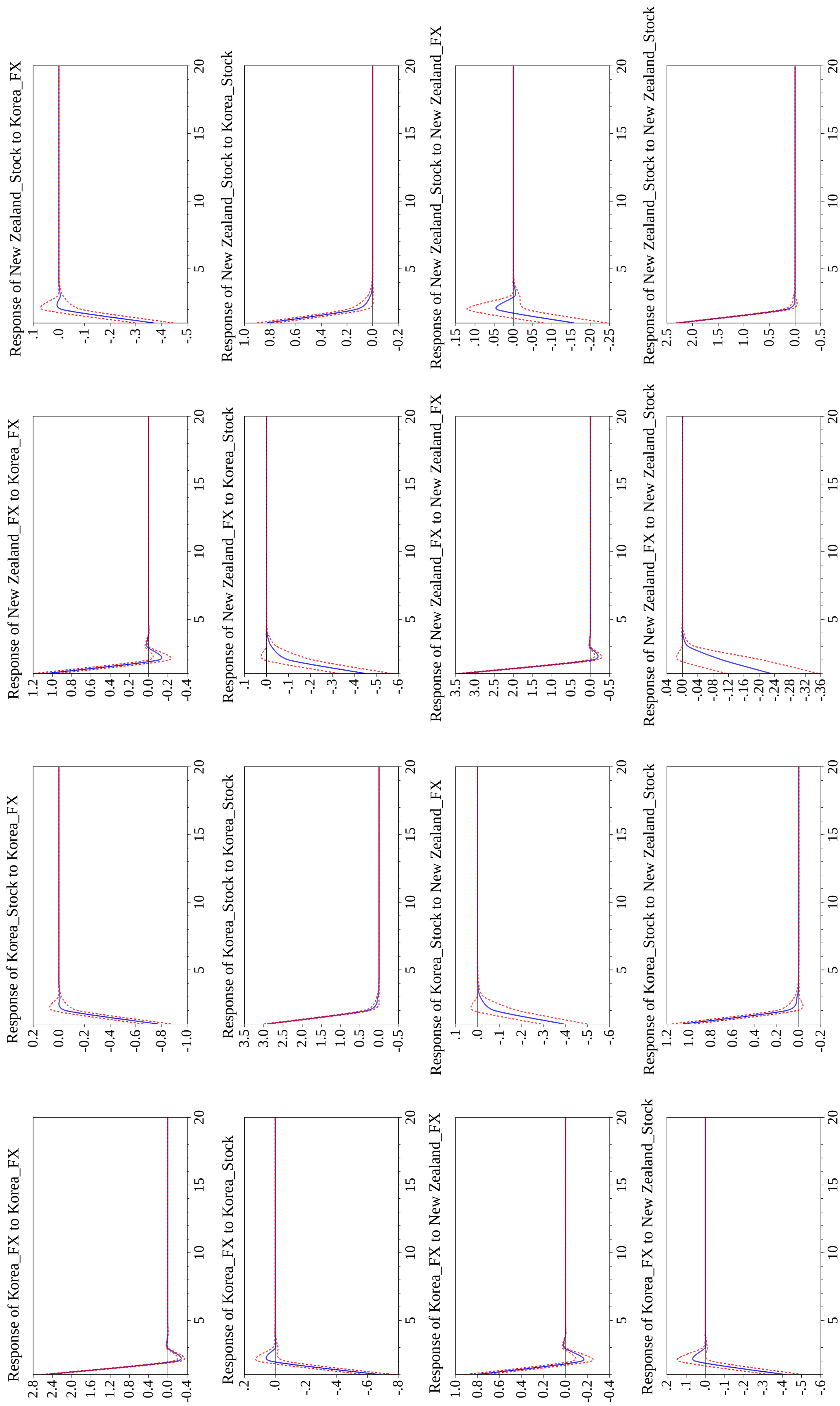
Notes: The figure reports response of a market to one generalized standard deviation shock in another market

Figure 2.3a: The Realized Skewness GIR of selected Markets in Asian Pacific Emerging Group



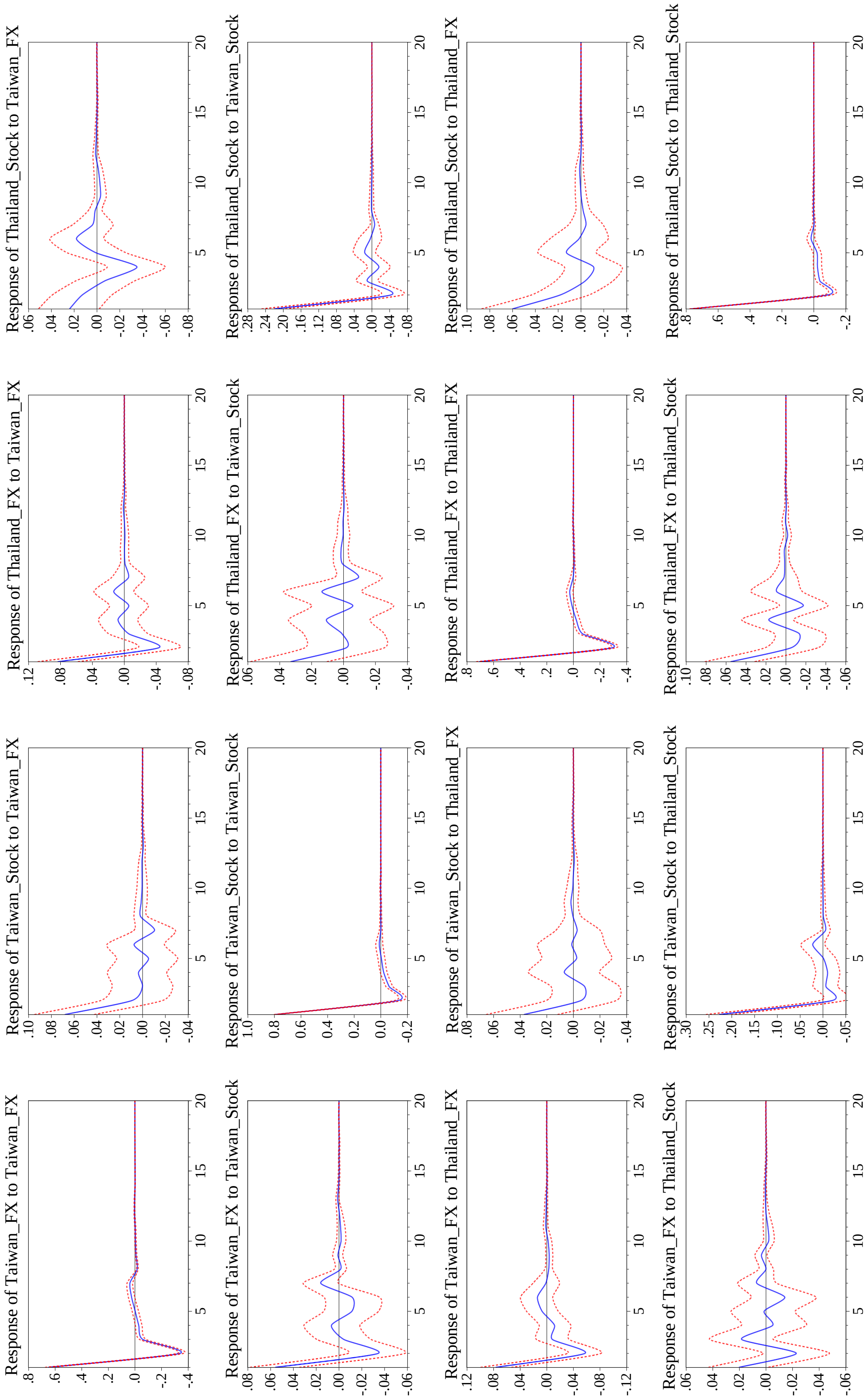
Notes: The figure reports response of a market to one generalized standard deviation shock in another market.

Figure 2.3b: The Realized Skewness GIR of selected Markets in Asian Pacific Developed Group



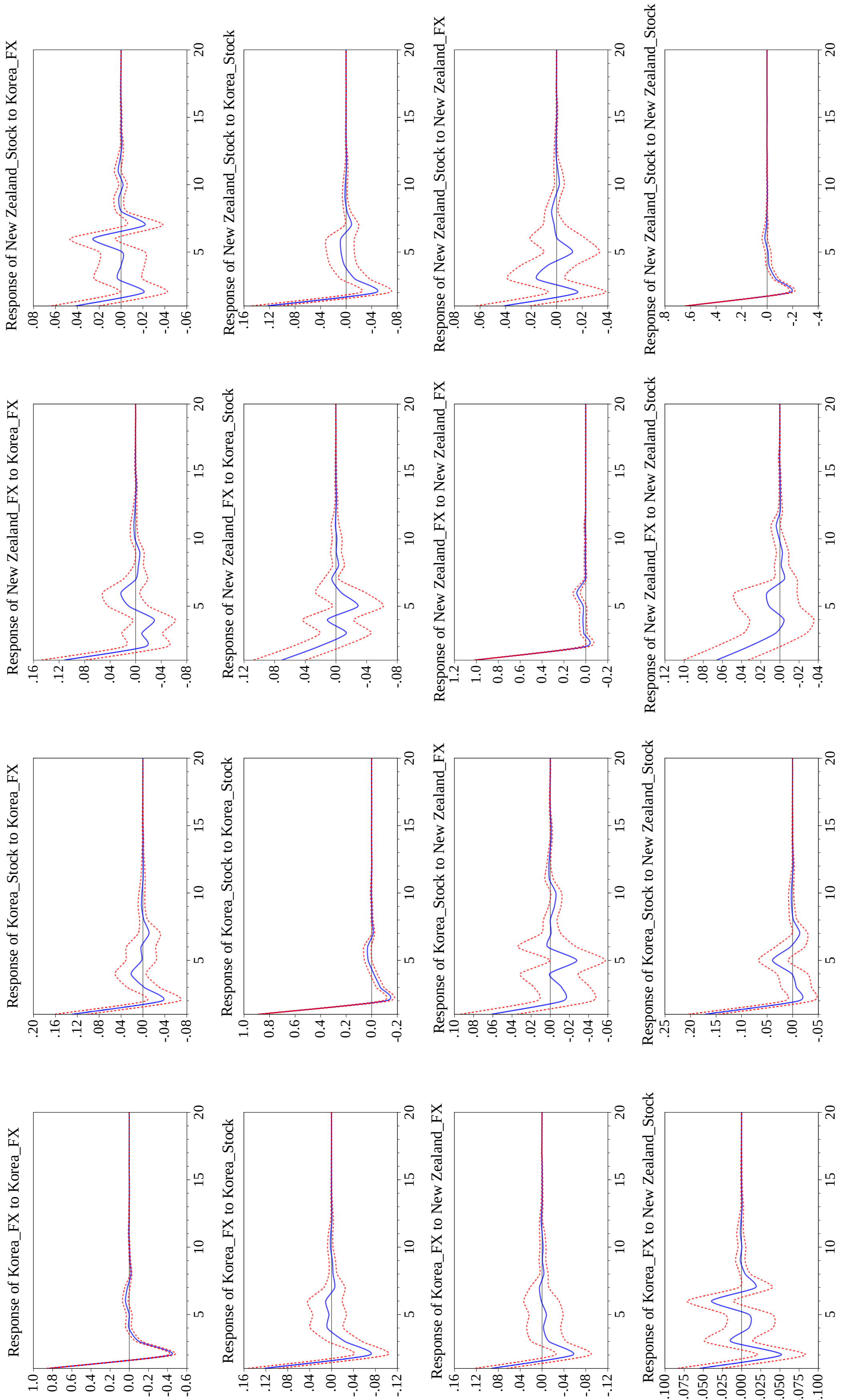
Notes: The figure reports response of a market to one generalized standard deviation shock in another market

Figure 2.4a: The Realized Kurtosis GIR of selected Markets in Asian Pacific Emerging Group



Notes: The figure reports response of a market to one generalized standard deviation shock in another market

Figure 2.4b: The Realized Kurtosis GIR of selected Markets in Asian Pacific Developed Group



Notes: The figure reports response of a market to one generalized standard deviation shock in another market

Monash University

Declaration for Thesis Chapter 3

Declaration by candidate

In the case of Chapter 3, section 3.1-3.5, the nature and extent of my contribution to the work was the following:

Nature of contribution	Extent of contribution (%)
The initiation of idea, formulation of research questions, development of research framework, literature review, derivation of Generalized and Orthogonalized impulse responses, construction of realized volatility, data analysis and interpretation, and writing up the first draft.	75%

The following co-authors contributed to the work (in percentage terms):

Name	Nature of contribution	Extent of contribution (%)
Robert D. Brooks	Revising the draft and acting as a corresponding author with the *Economics Letters*.	15%
Sirimon Treepongkaruna	Revising the draft and contributing to the responses to the reviewers.	10%

Candidate's Signature		Date
	[redacted]	06th May 2013

Declaration by co-authors

The undersigned hereby certify that:

(1) the above declaration correctly reflects the nature and extent of the candidate's contribution to this work, and the nature of the contribution of each of the co-authors.

(2) they meet the criteria for authorship in that they have participated in the conception, execution, or interpretation, of at least that part of the publication in their field of expertise;

(3) they take public responsibility for their part of the publication, except for the responsible author who accepts overall responsibility for the publication;

(4) there are no other authors of the publication according to these criteria;

(5) potential conflicts of interest have been disclosed to (a) granting bodies, (b) the editor or publisher of journals or other publications, and (c) the head of the responsible academic unit; and

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Location(s)	
	Department of Econometrics and Business Statistics, Monash University

	S[redacted]	Date
Robert Brooks	[redacted]	6/5/13
Sirimon Treepongkaruna	[redacted]	6 May 2013

Chapter 3

Generalized Impulse Response Analysis in a Fractionally Integrated VAR Model

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3.1 Introduction

In recent years, studies of fractionally integrated processes have increasingly attracted attention from both theoretical and empirical researchers. A fractional process can effectively provide a suitable description of temporal dependence behaviour in a time series which is shown as an intermediate between two classical processes, short-memory (also known as $I(0)$) and unit root processes (the so-called $I(1)$). Accordingly, the growth of literature on fractional processes can provide more flexible alternatives for modelling the long-memory behaviour in a time series. Empirical studies have found evidence that fractionally integrated processes perform well in describing characteristics of economic and financial data, including volatility of financial asset returns, forward exchange market premia, inflation rates and the interest rate differential (see Henry and Zaffaroni, 2003). In addition, a multivariate framework of fractional processes is able to provide more general tools to investigate the interdependence and feedback relationships between series. One of the most widely used methodologies is the well-known impulse response function.

3.2 Literature review

An analysis based on impulse response functions helps to understand the "persistence effect of shocks" on variables of a system (see Koop et al., 1996). More specifically, it tells us about the dynamic response of a variable to an exogenous shock (impulse) in another variable in a system (Lütkepohl, 2005). Generally, three types of impulse response functions can be employed in a reduced form of a multivariate time series model (for example, a VAR model), namely non-orthogonalized impulse response, orthogonalized impulse response and generalized impulse response function. An utilization of the non-orthogonalized impulse response function requires that the errors (innovations) in the model are contemporaneously uncorrelated, which is practically unusual in a reduced form since economic time series are more likely to be inter-dependent to some extent. Hence, the non-orthogonalized impulse response is normally employed in a structural form, where the variance-covariance matrix of errors is diagonal. As we focus on a reduced form of a multivariate time series model, we solely emphasize on discussions about the orthogonalized and generalized impulse response functions.

The orthogonalized impulse response function (see Sims, 1980) and the generalized impulse response function (see Koop et al., 1996, and Pesaran and Shin, 1998) both accommodate for the fact that innovations in a reduced form of a multivariate time series model are contemporaneously correlated. However, they use different approaches to address the problem of the choice of shocks to a system. The orthogonalized approach decomposes the variance-covariance matrix of errors according to the Cholesky decomposition to orthogonalize the shocks. The process of this decomposition, however, requires an implicit assumption that endogenous variables in a system are in the correct order. In other words, the direction of the instantaneous causalities among endogenous variables needs to be correctly identified before the decomposition. Incorrect orderings of endogenous variables may lead to mistaken conclusions drawn from orthogonalized impulse response analyses (see Lütkepohl, 2005). Nevertheless, this issue may be easily solved in a low dimensional system, where economic theories can be employed to justify the chosen order of endogenous variables. The problem can be more complicated in a case of a high dimensional system, where clear economic guidance on a suitable ordering is not available. In this sense, the generalized approach is developed to overcome the difficulty. Instead of using the Cholesky decomposition to address the choice of shocks, the generalized approach chooses to shock only one element of the error vector, and extract effects of other shocks. Hence, the generalized impulse response is unique and independent of the ordering of the endogenous variables (see Pesaran and Shin, 1998).

In the literature, impulse response functions and their asymptotic distribution are well analysed in a VAR model (e.g., Sims, 1980; Baillie, 1987; Lütkepohl, 1989, 1990; Pesaran and Shin, 1998; and Benkwitz et al., 2000); they, however, have not been widely investigated in a multivariate long memory framework (e.g., fractionally integrated VAR model). More specifically, even though many papers have attempted to develop and apply the estimation and inference of impulse response functions within a univariate long memory model (e.g., autoregressive fractionally integrated moving average (ARFIMA) model) to study economic series[14], only Chung (2001) to date considers the issue within a multivariate long memory model.

Chung (2001) developed an impulse response generating function for the vector fractionally integrated autoregressive moving average (VARFIMA) model. To resolve the problem of the choice of shocks to a system, this methodology follows the suggestion in Sims (1980) by using orthogonalization of shocks, which results from the Cholesky decomposition of the error variance-covariance matrix. As discussed earlier, the underlying requirement of this approach is to determine the most appropriate direction of the contemporaneous relationship between endogenous variables. Alternative re-parameterizations may lead to different results of the impulse response function (see Lütkepohl, 2005, section 2.3.2).

In this chapter, following the Pesaran and Shin (1998) generalized approach; we develop an alternative impulse response function for a FIVAR model. The main advantage of the generalized function is that it is unique and invariant to different orderings of variables in the system. Therefore, the generalized approach provides a good alternative to the orthogonalized approach in the case of an investigation of a high-dimensional system where there is no clear economic guidance on a suitable ordering. Particularly, we show that the generalized and orthogonalized impulse response function for the FIVAR model evolve at the same rate. Therefore, according to the previous work of Chung (2001), the generalized impulse response of the FIVAR model changes at a slow hyperbolic rate. Further, the generalized and orthogonalized impulse response functions for the FIVAR model are equivalent in the case of the diagonal error variance-covariance matrix. When the variance-covariance matrix of

[14] See for example Diebold et al. (1991), Diebold and Rudebusch (1991), Cheung (1993), Baillie et al. (1996), Wright (2000) and Baillie and Kapetanios (2012).

error term is non-diagonal, the two functions generate a coincident response of the system to the first shock but different responses to all other shocks.

To ease the statistical inference of the impulse response functions within a FIVAR framework, we also derive asymptotic theories for both generalized and orthogonalized approaches. Although the bootstrap and Monte Carlo simulation methods can be employed to construct the confidence interval for the impulse response functions, the computational burdens of multivariate fractionally integrated models' estimation procedures make them much less efficient than asymptotic theories. Regarding the orthogonalized impulse response, we introduce a different form for its variance-covariance matrix in comparison with Chung (2001), which we believe can facilitate computational programming. We derive these asymptotic distributions under two different scenarios corresponding to two estimation methods of a FIVAR model. In the first scenario, we assume that degrees of fractional integration are consistently pre-determined before estimating other parameters in a FIVAR model. The results generated in this case, therefore, are applicable for the two-step estimation methods of a FIVAR model. In another scenario, we develop the asymptotic theories when all the parameters of a FIVAR model are estimated simultaneously. Hence, we expect that our results can facilitate the statistical inference and interpretation for the interdependence as well as feedback relationships between endogenous variables in a FIVAR framework.

The remainder of this chapter is organized as follows. In section 3.3, we develop the generalized impulse response function for a FIVAR model and reform the orthogonalized function of Chung (2001) to ease its implementation. We discuss the relationship between the two functions in section 3.4. In section 3.5, we provide an empirical investigation of the realized volatility spill-over effect in Australian stock and currency markets to illustrate our method. Section 3.6 is built up with their asymptotic theories for the purpose of statistical inference. We make some remarks in section 3.7. Section 3.8 presents the simulations' results and we draw conclusions in section 3.9.

3.3 Generalized and orthogonalized impulse response function for FIVAR

3.3.1 The infinite moving average representation of a FIVAR model

Consider a vector of jointly determined dependent variables $Y_t = (Y_{1t}, Y_{2t}, ..., Y_{Kt})'$ that follows a *K*-dimensional FIVAR(*d*, *p*) framework:

$$A(L)D(L)Y_t = \varepsilon_t, \qquad t = 1,2,...,T. \tag{3.1}$$

where L is the lag operator, ε_t is a $K\times 1$ vector of error term. The operator $A(L) = I_K - \sum_{i=1}^{p} A_i L^i$, where A_i is the $K\times K$ matrix of coefficients. The operator $D(L)$ is a diagonal $K\times K$ matrix characterized by the *K*-dimensional vector of degrees of fractional integration $d = (d_1, d_2, ..., d_K)'$ as follows:

$$D(L) = \begin{bmatrix} (1-L)^{d_1} & 0 & \dots & 0 \\ 0 & (1-L)^{d_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & (1-L)^{d_K} \end{bmatrix} \tag{3.2}$$

Operationally, the term $(1-L)^{-d_j}$ can be generated by the following binomial expansion:

$$(1-L)^{-d_j} = \sum_{i=0}^{\infty} \frac{\Gamma(i+d_j)}{\Gamma(d_j)\Gamma(i+1)} L^i = \sum_{i=0}^{\infty} \psi_i^{(d_j)} L^i \tag{3.3}$$

where $\Gamma(.)$ is the gamma function; $\psi_0^{(0)} = 1$, and $\psi_i^{(0)} = 0$, for $i \neq 0$.

As suggested in Sowell (1992), Lütkepohl (2005) and Nielsen (2005), the following standard assumptions have been made:

Assumption 3.1: $E(\varepsilon_t) = 0$, $E(\varepsilon_t \varepsilon_t') = \Sigma_\varepsilon$ *for all t, where* $\Sigma_\varepsilon = \{\sigma_{ij}; i, j = 1,2,..., K\}$ *is an K×K positive definite matrix,* $E(\varepsilon_t \varepsilon_s') = 0$, *for all* $t \neq s$.

Assumption 3.2: *All the roots of* $|A(z)| = \left|I_K - \sum_{i=1}^{p} A_i z^i\right| = 0$ *fall outside the unit circle and* $|d_j| < \dfrac{1}{2}$ *for all* $j = 1, 2,..., K$.

Assumption 3.3: *Let* $Z_t = (Y_{t-1}',..., Y_{t-p}')'$ *be the Kp×1 vector collecting all explanatory variables at time t, the spectral density of* Z_t *exists and satisfies,*

$$f_Z(\lambda) \sim \Lambda^{-1} G \Lambda^{-1} \qquad as \quad \lambda \to 0^+,$$

where $\Lambda = I_p \otimes \Lambda(d)$, *with* $\otimes$ *denotes Kronecker product;* $\Lambda(d) = \mathrm{diag}\{\lambda^{d_1},..., \lambda^{d_K}\}$ *and G is a Kp×Kp real, symmetric and positive definite matrix.*

The Assumption 3.3[15] is to ensure no multicollinearity condition within the components of Z_t (see Nielsen, 2005 and Nielsen and Frederiksen, 2011). Further, under the Assumption 3.2[16], we can represent the model (3.1) under the form of the infinite moving average according to the following two-step process. From (3.1) we have:

$$Y_t = D^{-1}(L) A^{-1}(L) \varepsilon_t \tag{3.4}$$

[15] For the specification and estimation of the matrix G, we refer to Shimotsu (2007).

[16] Our method can be extended to the case of $|d_j| > \frac{1}{2}$ since the condition $|d_j| < \frac{1}{2}$ can be obtained by taking an appropriate number of differences.

Step 1: Set $\Pi(L) = A^{-1}(L)$, where $\Pi(L) = \sum_{i=0}^{\infty} \Pi_i L^i$. The coefficients of the two sequences are matched subsequently. Therefore, the $K \times K$ matrices Π_i can be computed recursively using the relationship,

$$\Pi_i = \begin{cases} \sum_{j=1}^{i} \Pi_{i-j} A_j & i = 1,2,...p \\ \sum_{j=1}^{p} \Pi_{i-j} A_j & i > p \end{cases} \tag{3.5}$$

where $\Pi_0 = I_K$ and I_K is the $K \times K$ identity matrix.

Step 2: Specify $\Phi(L) = D^{-1}(L)\Pi(L)$, where $\Phi(L) = \sum_{i=0}^{\infty} \Phi_i L^i$. Thus, it can be easily seen that ${}^{(j)}\Phi(L) = (1-L)^{-d_j} e_j' \Pi(L),\ j = 1,2,...,K$, where ${}^{(j)}\Phi$ indicates the j^{th} row of the Φ matrix and e_j is a $K \times 1$ vector with unity as its j^{th} element and zeros elsewhere.

Accordingly, from (3.3) we have:

$${}^{(j)}\Phi(L) = \left(\sum_{i=0}^{\infty} \psi_i^{(d_j)} L^i \right) \left(\sum_{i=0}^{\infty} e_j' \Pi_i L^i \right) \tag{3.6}$$

Expanding the multiplication, we have:

$${}^{(j)}\Phi(L) = e_j' \Pi_0 + \left(\sum_{i=0}^{1} \psi_i^{(d_j)} e_j' \Pi_{1-i} \right) L + ... + \left(\sum_{i=0}^{h} \psi_i^{(d_j)} e_j' \Pi_{h-i} \right) L^h + ... \tag{3.7}$$

Matching the coefficient matrices of the lag polynomial $\Phi(L)$ with the expansion of $D^{-1}(L)\Pi(L)$, then

$${}^{(j)}\Phi_h = \begin{cases} \sum_{i=0}^{h} \psi_i^{(d_j)} e_j' \Pi_{h-i} & h = 1, 2, ... \\ e_j' \Pi_0 & h = 0 \end{cases}$$

or

$$\Phi_h = \begin{cases} \sum_{i=0}^{h} \Psi_i^{(d)} \Pi_{h-i} & h = 1, 2, \ldots \\ \Pi_0 & h = 0 \end{cases} \tag{3.8}$$

where $\Psi_i^{(d)}$ is the diagonal $K\times K$ matrix with $\psi_i^{(d_j)}$ as the j^{th} diagonal element.

Hence, the infinite moving average representation of (3.1) can be written as,

$$Y_t = \sum_{i=0}^{\infty} \Phi_i \varepsilon_{t-i} \tag{3.9}$$

3.3.2 Generalized impulse response function for FIVAR

Building on the Pesaran and Shin (1998) approach, we define the generalized impulse response function of FIVAR for *h*-periods ahead as,

$$GIR_{FIVAR}(h, \delta, \Omega_{t-1}) = E(Y_{t+h} \mid \varepsilon_t = \delta, \Omega_{t-1}) - E(Y_{t+h} \mid \Omega_{t-1}) \tag{3.10}$$

where $\delta = (\delta_1, \delta_2, \ldots, \delta_K)'$ denotes the $K\times 1$ vector of shocks hitting the economy at time *t*, and Ω_{t-1} is the information set available at time *t*-1.

As suggested by Pesaran and Shin (1998), we shock only one element, say the j^{th} element of ε_t , then extract the effects of other shocks to make the GIR_{FIVAR} independent of the ordering of endogenous variables in the system. The GIR_{FIVAR} now becomes:

$$GIR_{FIVAR}(h, \delta_j, \Omega_{t-1}) = E(Y_{t+h} \mid \varepsilon_{jt} = \delta_j, \Omega_{t-1}) - E(Y_{t+h} \mid \Omega_{t-1}) \tag{3.11}$$

Using (3.9) in (3.11), we have:

$$GIR_{FIVAR}(h, \delta_j, \Omega_{t-1}) = \Phi_h E(\varepsilon_t \mid \varepsilon_{jt} = \delta_j) \tag{3.12}$$

With the assumption of a multivariate normal distribution of ε_t, it can be seen that[17]:

$$E(\varepsilon_t | \varepsilon_{jt} = \delta_j) = (\sigma_{1j}, \sigma_{2j}, ..., \sigma_{Kj})' \sigma_{jj}^{-1} \delta_j = \Sigma_\varepsilon e_j \sigma_{jj}^{-1} \delta_j \tag{3.13}$$

By setting the unit shock as a one standard deviation shock, which is $\delta_j = \sqrt{\sigma_{jj}}$, from (3.12) and (3.13) we can obtain the scaled GIR_{FIVAR} of the effect of a shock in the j^{th} element at time t on the expected value of Y at time $t+h$ as

$$GIR_{FIVAR}(h, \delta, \Omega_{t-1}) = \Phi_h \Sigma_\varepsilon e_j \sigma_{jj}^{-\frac{1}{2}} \qquad h = 0, 1, 2, ... \tag{3.14}$$

Accordingly, the matrix of response of Y at time $t+h$ to a one generalized standard deviation shock in the system at time t can be fully captured by

$$GIR_{FIVAR}(h, \delta, \Omega_{t-1}) = \Theta_h^g = \Phi_h \Sigma_\varepsilon \Xi = \Phi_h B \qquad h = 0, 1, 2, ... \tag{3.15}$$

where Ξ is a diagonal $K \times K$ matrix characterized by the standard deviation of ε_t,

$$\Xi = \begin{bmatrix} \sigma_{11}^{-\frac{1}{2}} & 0 & \dots & 0 \\ 0 & \sigma_{22}^{-\frac{1}{2}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_{KK}^{-\frac{1}{2}} \end{bmatrix} \tag{3.16}$$

3.3.3 Orthogonalized impulse response function for FIVAR

Chung (2001) develops an impulse response analysis for the FIVAR model by using the orthogonalized approach, which is similar to the suggestion in Sims (1980).

[17]As pointed out in Pesaran and Shin (1998), when the distribution of ε_t is unknown or non-normal, the conditional expectation $E(\varepsilon_t | \varepsilon_{jt} = \delta_j)$ can be obtained by stochastic simulations or by resampling techniques.

This methodology resolves the problem of the choice of δ by using the Cholesky decomposition of of $\Sigma_\varepsilon, \Sigma_\varepsilon = PP'$, where P is a $K\times K$ lower triangular matrix.

Under Assumption 3.2, model (3.1) can be rewritten as an infinite moving average process,

$$Y_t = B(L)u_t \tag{3.17}$$

where $u_t = P^{-1}\varepsilon_t$ and $B(L) = \sum_{i=0}^{\infty} B_i L^i$ with $B_0 = I_K$.

Finally, Chung (2001) shows that any h impulse response in a FIVAR model can be obtained from the h coefficients of "a finite-order power series resulting from truncated power series multiplication and inversion":

$$B_h(L) = \sum_{i=0}^{h} B_i L^i \asymp \Psi_h^{(d)}(L) A^{-1}(L) P \tag{3.18}$$

where $\asymp$ denotes the operation of truncating the series (see Chung, 2001); $\Psi_h^{(d)}(L) = \sum_{i=0}^{h} \Psi_i^{(d)} L^i$.

3.4 The relationship between generalized and orthogonalized impulse response of a FIVAR model

By using the Cholesky decomposition of Σ_ε, the orthogonalized impulse response function of FIVAR (OIR_{FIVAR}) depends on how the endogenous variables are ordered in the system. Generally, many alternative orderings of the variables could be employed to calculate the OIR_{FIVAR}, but there is no clear guidance on which ordering should be used. Hence, this approach would raise difficulties in choosing the suitable parameterization, especially in a high dimensional system. Conversely, as noted in Pesaran and Shin (1998), the GIR_{FIVAR} is unique and invariant to alternative re-parameterizations. Further, the historical relationships observed between different shocks are still fully captured. Apart from this main difference, the two types of impulse responses have a close relationship, which has been pointed out in the following propositions.

Proposition 3.1

The generalized impulse response of FIVAR model evolves at the similar hyperbolic rate with the orthogonalized impulse response of FIVAR.

Proof. Transform $A^{-1}(L)$ to $\Pi(L)$ according to (3.5), then (3.18) becomes,

$$B_h(L) = \sum_{i=0}^{h} B_i L^i \asymp \Psi_h^{(d)}(L)\Pi(L)P \tag{3.19}$$

Let specify $\Delta_h(L) = \sum_{i=0}^{h} \Delta_i L^i \asymp \Psi_h^{(d)}(L)\Pi(L)$, then $B_h(L) = \Delta_h(L)P$. It can be easily seen that,

$$^{(j)}\Delta_h(L) \asymp \left(\sum_{i=0}^{h} \psi_i^{(d_j)} L^i\right)\left(\sum_{i=0}^{\infty} e'_j \Pi_i L^i\right) \tag{3.20}$$

where $^{(j)}\Delta_i(L)$ indicates the j^{th} row of the Δ_i matrix obtained from $\Delta_h(L)$, where *j=1,2,…K.*

Similar to (3.6) and (3.7), we have,

$$^{(j)}\Delta_h(L) = e'_j \Pi_0 + \left(\sum_{i=0}^{1} \psi_i^{(d_j)} e'_j \Pi_{1-i}\right) L + \ldots + \left(\sum_{i=0}^{h} \psi_i^{(d_j)} e'_j \Pi_{h-i}\right) L^h \tag{3.21}$$

Thus,

$$
{}^{(j)}\Delta_h = \begin{cases} \sum_{i=0}^{h} \psi_i^{(d_j)} e_j' \Pi_{h-i} & h = 1, 2, \ldots \\ e_j' \Pi_0 & h = 0 \end{cases}
$$

or

$$
\Delta_h = \begin{cases} \sum_{i=0}^{h} \Psi_i^{(d)} \Pi_{h-i} & h = 1, 2, \ldots \\ \Pi_0 & h = 0 \end{cases} \tag{3.22}
$$

Accordingly, the matrix of response of Y at time $t+h$ to a one orthogonalized standard deviation shock in the system at time t can be fully obtained by

$$
\Theta_h^o = \Delta_h P \tag{3.23}
$$

Since P, Σ_ε and Ξ are constant, the evolving rates of GIR_{FIVAR} and OIR_{FIVAR} are determined by Φ and Δ respectively. From (3.8) and (3.22), we note that $\Phi_h = \Delta_h, \forall h \in \mathrm{N}_0$. Therefore, the GIR_{FIVAR} and OIR_{FIVAR} evolve at the similar hyperbolic rate. ■

Proposition 3.2

In the case where Σ_ε is diagonal then the generalized and orthogonalized impulse responses of FIVAR are equivalent. If Σ_ε is non-diagonal then $\Theta_h^g(j) \neq \Theta_h^o(j)$ for $j = 2, 3, \ldots, K$, and $\Theta_h^g(j) = \Theta_h^o(j)$ for $j = 1$, where $\Theta_h(j)$ denotes the j^{th} column of Θ_h

Proof. In the case where Σ_ε is diagonal: $\Sigma_\varepsilon = \mathrm{diag}\{\sigma_{jj}; j = 1,2,\ldots,K\}$, it can be seen that,

$$\Sigma_{\varepsilon}\Xi = P = \begin{bmatrix} \sqrt{\sigma_{11}} & 0 & \dots & 0 \\ 0 & \sqrt{\sigma_{22}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sqrt{\sigma_{KK}} \end{bmatrix} \tag{3.24}$$

Further, $\Phi_h = \Delta_h, \forall h \in \mathrm{N}_0$; so from (3.15) and (3.23) we obtain $\Theta_h^g = \Theta_h^o$. ■

In the case where Σ_ε is non-diagonal, the proof is similar to Pesaran and Shin (1998). However, we still provide the proof here for completeness. We extract the j^{th} column of Θ_h^g and Θ_h^o as,

$\Theta_h^g(j) = \Phi_h \Sigma_\varepsilon \Xi e_j$ and $\Theta_h^o(j) = \Delta_h P e_j$, for $j = 1, 2, ..., K$,

where $\Phi_h = \Delta_h, \forall h \in \mathrm{N}_0$. We note that,

$\Sigma_\varepsilon \Xi e_j = \sigma_{jj}^{-\frac{1}{2}}(\sigma_{1j}, \sigma_{2j}, ..., \sigma_{Kj})'$, for $j = 1, 2, ..., K$,

$Pe_1 = (p_{11}, p_{21}, ..., p_{K1})', ..., Pe_j = (0, ..., 0, p_{jj}, ..., p_{Kj})', ..., Pe_K = (0, ..., 0, p_{KK})'$

$\therefore \Sigma_\varepsilon \Xi e_j \neq Pe_j$ and $\Theta_h^g(j) \neq \Theta_h^o(j)$ for $j = 2, 3, ..., K$.

For $j = 1$,

$$\Sigma_\varepsilon \Xi e_1 = \sigma_{11}^{-\frac{1}{2}}(\sigma_{11}, \sigma_{21}, ..., \sigma_{K1})' \tag{3.25}$$

Matching the 1st column of the parity $\Sigma_\varepsilon = PP'$, we have,

$$(\sigma_{11}, \sigma_{21}, ..., \sigma_{K1})' = (p_{11}^2, p_{11}p_{21}, ..., p_{11}p_{K1})',\ p_{11} = \sigma_{11}^{\frac{1}{2}} \tag{3.26}$$

$\therefore \Sigma_\varepsilon \Xi e_1 = p_{11}^{-1}(p_{11}^2, p_{11}p_{21}, ..., p_{11}p_{K1})' = (p_{11}, p_{21}, ..., p_{K1})' = Pe_1$

$\therefore \Theta_h^g(1) = \Theta_h^o(1)$. ■

3.5 Illustration

We illustrate our approach by investigating the realized volatility spill-over effect in Australian markets. First, we construct the daily realized volatility for Australian stock and currency markets from 02/01/1997 to 20/5/2010 as proposed by Andersen et al. (2003)[18]:

$$RV_t = \sum_{i=1}^{D} r_{i,t}^2$$

where $r_{i,t}$ denotes a ith 5-minute return during day t and D denotes the total number of 5-minute return intervals during any trading day.

We subsequently model the realized volatility using a bivariate FIVAR framework as specified in (3.1). The estimation is carried out using the two-step estimation procedure. At the first stage, we obtain a consistent estimate of d using the Gweke and Porter-Hudak (1983) (GPH) log-periodogram regression estimator. We then replace d into the FIVAR model and apply OLS equation-by equation to estimate remaining coefficients.

As can be seen from Table 3.1, estimates of d are both significantly greater than 0, indicative of long memory behaviour. Additionally, since both estimates of d are greater than 0.5, in the second step of the estimation we transform the realized volatilities by applying the filter $(1-L)^{d-1}$ to their first difference. The estimation of remaining coefficients in *A(L)*, therefore, is consistent.

The FIVAR's lag length order of 4 is chosen according to the SIC criteria. After consistently estimating coefficient matrices of *A(L)*, we turn to estimate the GIR_{FIVAR}

[18] Our data is extracted from the Securities Industry Research Centre of Asia-Pacific.

and OIR_{FIVAR}. Figure 3.1 shows that both GIR_{FIVAR} and OIR_{FIVAR} confirm the existence of a realized volatility spill-over effect within-between Australian stock and currency market. The spill-over effect remains positive for a long period of time, which is consistent with the long range dependencies behaviour.

Additionally, the behaviour of GIR_{FIVAR} and OIR_{FIVAR} is consistent with propositions pointed out in previous sections. Both GIR_{FIVAR} and OIR_{FIVAR} evolve slowly at a very similar hyperbolic rate. Further, the GIR_{FIVAR} and OIR_{FIVAR} of the system to the shock in realized volatility of Australian stock market are the same; whereas, all other responses are different between the GIR_{FIVAR} and OIR_{FIVAR}.

3.6 Asymptotic theories for the impulse response functions of a FIVAR model

We derive the asymptotic distribution of impulse response functions of FIVAR by using the result from Serfling (1980, p. 122) (also noted in Lütkepohl, 1990, 2005). Let β is an $n \times 1$ vector of parameters and $\hat{\beta}$ is an estimator satisfying,

$$\sqrt{T}(\hat{\beta}-\beta) \xrightarrow{d} N(0, \Sigma_{\hat{\beta}}),$$

where $\xrightarrow{d}$ denotes convergence in distribution, $N(0, \Sigma_{\hat{\beta}})$ denotes the multivariate normal distribution with mean vector 0 and covariance matrix $\Sigma_{\hat{\beta}}$.

Then,

$$\sqrt{T}\left[g(\hat{\beta})-g(\beta)\right] \xrightarrow{d} N\left(0, \frac{\partial g}{\partial \beta'} \Sigma_{\hat{\beta}} \frac{\partial g'}{\partial \beta}\right)$$

where $g(\beta) = (g_1(\beta),\ldots,g_m(\beta))'$ is a continuously differentiable function with values in m-dimensional Euclidean space and $\partial g_i / \partial\beta' = (\partial g_i / \partial\beta_j)$ is non-zero at the true vector β, for $i = 1, 2,\ldots, m$.

In addition, we use the following notation shown in Baillie (1987) and Lütkepohl (1990, 2005) to facilitate our derivation,

$$\underset{(K^2 p\times 1)}{\alpha} := vec(A_1,\ldots,A_p)$$

$$\underset{(Kp\times Kp)}{A} := \begin{bmatrix} A_1 & A_2 & \cdots & A_{p-1} & A_p \\ I_K & 0 & \cdots & 0 & 0 \\ 0 & I_K & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_K & 0 \end{bmatrix}$$

$$\underset{(K(K+1)/2\times 1)}{\sigma} := vech(\Sigma_\varepsilon)$$

$$\underset{(K\times Kp)}{J} := [I_K : 0 : \cdots : 0]$$

where *vec* denotes the column stacking operator and *vech* is the corresponding operator that stacks only the elements on and below the main diagonal of a square matrix.

As usual, we denote $\otimes$ as the Kronecker product; L_m is the $m(m+1)/2\times m^2$ elimination matrix such that, for any $m\times m$ matrix G, $vech(G) = L_m vec(G)$. Further, we define D_m as the $m^2\times m(m+1)/2$ duplication matrix satisfying $D_m vech(G) = vec(G)$ for a symmetric $m\times m$ matrix G; and K_{mn} is the $mn\times mn$ commutation matrix such that, for any $m\times n$ matrix F, $K_{mn} vec(F) = vec(F')$.

In addition, we define a $m^2\times m^2$ matrix S_m, which we call the diagonal-stacking matrix such that, for a $m\times m$ matrix G and a diagonal $m\times m$ matrix H, whose main diagonals are identical, $vec(H) = S_m vec(G)$. The computation of the diagonal-stacking matrix S_m is relatively convenient since it can be calculated as, $S_m = E_m E_m'$, where E_m is a $m^2 \times m$ matrix of 0 and 1 represented as,

$$\underset{(m^2\times m)}{E_m} = \begin{bmatrix} e_1 e_1' \\ e_2 e_2' \\ \vdots \\ e_m e_m' \end{bmatrix},$$

where e_j is the $m\times 1$ vector with 1 in the j^{th} element and 0 elsewhere.

3.6.1 Processes with known order and pre-determined degrees of fractional integration

In this section, we consider the FIVAR(d, p) processes where the lag order p is known and degrees of fractional integration, $d := (d_1, d_2, ..., d_K)'$, are consistently determined before the estimation of remaining parameters in the processes. The result generated, therefore, is applicable to the two-step estimation of a fractionally integrated model, which consistently estimates the differencing parameters in the first step and other parameters in the second step. In the first step, the degrees of fractional integration can be estimated in several ways under an univariate framework, for example, by using log-periodogram regression (see Geweke and Porter-Hudak, 1983), local Whittle estimator (see Künch, 1987), partial autocorrelation function (see Chong, 2000) or exact local Whittle estimator (see Shimotsu et al., 2005, Shimotsu, 2010). In addition, the d vector can also be estimated under a multivariate framework (see Shimotsu, 2007, and Nielsen, 2011). In the second step, the FIVAR(d, p) model can be transformed to a VAR(p) model by applying, for example, the time domain transformation (see Hosking, 1981) or the frequency domain transformation (see Geweke and Porter-Hudak, 1983) and remaining parameters can be subsequently estimated by standard econometric techniques such as multivariate Least Squares or the maximum likelihood estimator. Therefore, under this estimation procedure, the information about the asymptotic distribution of d is not necessarily involved in deriving the asymptotic distribution of the FIVAR impulse response functions. We have the following proposition,

Proposition 3.3

Suppose

$$\sqrt{T}\begin{bmatrix}\hat{\alpha}-\alpha \\ \hat{\sigma}-\sigma\end{bmatrix} \xrightarrow{d} N\left(0, \begin{bmatrix}\Sigma_{\alpha} & 0 \\ 0 & \Sigma_{\sigma}\end{bmatrix}\right)$$

Then,

$$\sqrt{T}vec(\hat{\Pi}_h - \Pi_h) \xrightarrow{d} N(0, G_h \Sigma_{\hat{\alpha}} G_h') \qquad h = 1,2,.... \qquad (3.3.1)$$

where,

$$G_h := \frac{\partial vec(\Pi_h)}{\partial \alpha'} = \sum_{m=0}^{h-1} J(A')^{h-1-m} \otimes \Pi_m ;$$

$$\sqrt{T}vec(\hat{\Phi}_h - \Phi_h) \xrightarrow{d} N(0, V_h \Sigma_{\hat{\alpha}} V_h') \qquad h = 1,2,.... \qquad (3.3.2)$$

where,

$$V_h := \frac{\partial vec(\Phi_h)}{\partial \alpha'} = \sum_{i=0}^{h-1}\left[(I_K \otimes \Psi_i^{(d)})G_{h-i}\right] = \sum_{i=0}^{h-1}\sum_{m=0}^{h-i-1} J(A')^{h-i-1-m} \otimes \Psi_i^{(d)}\Pi_m ;$$

$$\sqrt{T}vec(\hat{\Theta}_h^g - \Theta_h^g) \xrightarrow{d} N(0, H_h \Sigma_{\hat{\alpha}} H_h' + \overline{H}_h \Sigma_{\hat{\sigma}} \overline{H}_h') \qquad h = 1,2,.... \qquad (3.3.3)$$

where,

$$H_h := \frac{\partial vec(\Theta_h^g)}{\partial \alpha'} = (B' \otimes I_K)V_h, \textit{ and}$$

$$\overline{H}_h := \frac{\partial vec(\Theta_h^g)}{\partial \sigma'} = (\Xi^3 \otimes \Phi_h \Sigma_{\varepsilon})S_K D_K + (\Xi \otimes \Phi_h)D_K ;$$

$$\sqrt{T}vec(\hat{\Theta}_h^O - \Theta_h^O) \xrightarrow{d} N(0, C_h \Sigma_{\hat{\alpha}} C_h' + \overline{C}_h \Sigma_{\hat{\sigma}} \overline{C}_h') \qquad h = 1,2,.... \qquad (3.3.4)$$

where,

$$C_h := \frac{\partial vec(\Theta_h^O)}{\partial \alpha'} = (P' \otimes I_K)V_h, \text{ and}$$

$$\overline{C}_h := \frac{\partial vec(\Theta_h^O)}{\partial \sigma'} = (I_K \otimes \Phi_h)\left\{L_K' \left[L_K(I_{K^2} + K_{KK})(P \otimes I_K)L_K'\right]^{-1}\right\};$$

3.6.2 Processes with known order but degrees of fractional integration are not pre-determined

Besides the two-step estimation of a FIVAR(d, p) process, a simultaneous estimation of all FIVAR's parameters has also increasingly attracted researchers' attention. Hosoya (1996) extended the univariate procedure to a multivariate case by proposing a quasi-maximum likelihood estimator in the frequency domain. Ravishanker and Ray (1997) presented a Bayesian inference for Gaussian fractionally integrated VARMA (VARFIMA) process using Markov chain Monte Carlo methods. Martin and Wilkins (1999) used indirect estimation for univariate and vector ARFIMA models to avoid likelihood functions. Sela and Hurvich (2009) employed the preconditioned conjugate gradient algorithm to perform the maximum likelihood estimation for the FIVAR model. They also provide simulations to compare their approach with the most commonly used approximation to the likelihood, the Whittle's method.

In the case when $d := (d_1, d_2, ..., d_K)'$, α and σ of a FIVAR process are estimated simultaneously given the information of lag order of the process, the asymptotic distribution of d needs to be considered in a derivation of the asymptotic distribution of the impulse response functions.

Along with the result from Serfling (1980, p. 122), we use the following proposition to derive the asymptotic distribution of FIVAR's impulse response function.

Proposition 3.4

Given the asymptotic approximation formula of the gamma function (see Abramowitz and Stegun, 1972, section 6.1.39),

$$\Gamma(az+b) \sim \sqrt{2\pi} e^{-az} (az)^{az+b-\frac{1}{2}}, \; a \in \mathrm{N}^*, b \in \mathrm{Z} \text{ and } z \in \mathrm{R} \,,$$

then,

$$\frac{\partial\, \Gamma(z)}{\partial z} \sim \sqrt{2\pi} e^{-z} z^{z-\frac{1}{2}} (\ln z - \frac{1}{2z}) \,,$$

$$\frac{\partial\, \Gamma(z+i)}{\partial z} \sim \sqrt{2\pi} e^{-z} z^{z+i-\frac{1}{2}} (\ln z + \frac{2i-1}{2z}) \;\; \textit{for} \;\; i \in \mathrm{N}_0 \,,$$

$$\overline{\psi}_i^{(z)} = \frac{\partial \psi_i^{(z)}}{\partial z} \sim \begin{cases} \dfrac{z^{i-1}}{(i-1)!} & \text{for } i = 1, 2, \ldots \\ 0 & \text{for } i = 0 \end{cases} \,.$$

Proposition 3.5

Given $V_h, H_h, \overline{H}_h, C_h$ *and* $\overline{C}_h$ *are defined in Proposition 3.3, we have the following results.*

Suppose

$$\sqrt{T} \begin{bmatrix} \hat{d} - d \\ \hat{\alpha} - \alpha \\ \hat{\sigma} - \sigma \end{bmatrix} \xrightarrow{d} N \left(0, \begin{bmatrix} \Sigma_{\hat{d}} & 0 & 0 \\ 0 & \Sigma_{\hat{\alpha}} & 0 \\ 0 & 0 & \Sigma_{\hat{\sigma}} \end{bmatrix} \right)$$

Then,

$$\sqrt{T} vec(\hat{\Phi}_h - \Phi_h) \xrightarrow{d} N(0, V_h \Sigma_{\hat{\alpha}} V_h' + \overline{V}_h \Sigma_{\hat{d}} \overline{V}_h') \qquad h = 1, 2, \ldots \qquad (3.5.1)$$

where,

$$\overline{V}_h := \frac{\partial vec(\Phi_h)}{\partial d'} = \sum_{i=0}^{h} \left[(\Pi_{h-i}' \otimes I_K) \Lambda_i^{(d)} \right]$$

with $\Lambda_i^{(d)}$ *is a* $K^2 \times K$ *matrix where its (j+(j-1)K, j) element is* $\overline{\psi}_i^{(d_j)}$*, j=1, 2, …, K; and 0 elsewhere.* $\overline{\psi}_i^{(d_j)}$ *can be calculated by following Proposition 3.4.*

$$\sqrt{T}vec(\hat{\Theta}_h^g - \Theta_h^g) \xrightarrow{d} N(0, H_h \Sigma_{\hat{\alpha}} H_h' + \overline{H}_h \Sigma_{\hat{\sigma}} \overline{H}_h' + M_h \Sigma_{\hat{d}} M_h') \quad h = 1,2,.... \quad (3.5.2)$$

where,

$$M_h := \frac{\partial vec(\Theta_h^g)}{\partial d'} = (B' \otimes I_K)\overline{V}_h;$$

$$\sqrt{T}vec(\hat{\Theta}_h^O - \Theta_h^O) \xrightarrow{d} N(0, C_h \Sigma_{\hat{\alpha}} C_h' + \overline{C}_h \Sigma_{\hat{\sigma}} \overline{C}_h' + W_h \Sigma_{\hat{d}} W_h') \quad h = 1,2,.... \quad (3.5.3)$$

where,

$$W_h := \frac{\partial vec(\Theta_h^O)}{\partial d'} = (P' \otimes I_K)\overline{V}_h;$$

3.7 Remarks

Remark 1: From Proposition 3.3 and 3.5, approximate variances of the estimated generalized and orthogonalized impulse responses can be simply obtained by dividing the diagonal elements of the asymptotic variance-covariance matrices by the sample size *T*. Therefore, as usual the approximate variances are approaching zero when the sample size increases.

Remark 2: Following comments in Lütkepohl (1990), we note that some matrices of partial derivatives can be zero. For example, in the case when a $K \times 1$ vector $X_t = (X_{1t}, \ldots, X_{Kt})'$ such that, $X_t = D(L)Y_t$ is white noise; if a FIVAR(1) is fitted although the true order *p* is zero, then $G_2 = JA' \otimes I_K + JI_K \otimes \Pi_1 = 0$ because $A = A_1 = 0$ and $\Pi_1 = A_1 = 0$. Therefore, a degenerate asymptotic distribution with a zero variance-covariance matrix is obtained for $\sqrt{T}vec(\hat{\Pi}_2 - \Pi_2)$. This failure occurs when some variables do not respond to the shock in other endogenous variables in the system and, therefore, there are no causal linkages in a particular part of the system. In fact, this problem is similar to that in a VAR system previously discussed in Benkwitz et al. (2000). Further, the potential problems resulting from a degenerate asymptotic distribution are also illustrated in Lütkepohl (2005, section 3.7.1).

Remark 3: If the two-step estimation procedure is applied, after the first step of consistently estimating the degree of fractional integration, the FIVAR(p) process can be transformed to a VAR(p) process, $A(L)X_t = \varepsilon_t$. Subsequently, the transformed VAR process can be estimated as usual by Multivariate Least Squares (LS) or a Maximum Likelihood (ML) estimator. Let us assume that the VAR(p) process X_t is covariance stationary, the ε_t is *Gaussian* white noise and the assumption of the Proposition 3.3 holds, then as shown in Lütkepohl (1990) and Lütkepohl (2005, chapter 3), the variance-covariance matrix of the asymptotic distribution of the parameters are,

$$\Sigma_{\hat{\alpha}} = \Gamma_X^{-1} \otimes \Sigma_\varepsilon ,$$

where,

$$\underset{(Kp\times Kp)}{\Gamma_X} := E\left(\begin{bmatrix} X_t \\ X_{t-1} \\ \vdots \\ X_{t-p+1} \end{bmatrix} \left[X'_t, \ldots, X'_{t-p+1} \right] \right),$$

can be obtained from,

$$vec\Gamma_X = (I_{(Kp)^2} - A\otimes A)^{-1} vec\Sigma_U ,$$

$$\underset{(Kp\times Kp)}{\Sigma_U} = \begin{bmatrix} \Sigma_\varepsilon & 0 \\ 0 & 0 \end{bmatrix},$$

and,

$$\Sigma_{\hat{\sigma}} = 2\, D_K^+ (\Sigma_\varepsilon \otimes \Sigma_\varepsilon) D_K^{+/} ,$$

where $D_K^+ = (D'_K D_K)^{-1} D'_K$ is the Moore-Penrose inverse of the duplication matrix D_K.

Remark 4: In the Proposition 3.4, instead of using the asymptotic approximation of the gamma function, one can apply numerical methods to derive $\overline{\psi}_i^{(z)}$,

$$\overline{\psi}_i^{(z)} = \frac{\partial\Gamma(z+i)}{\partial z}\left(\frac{1}{\Gamma(z)\Gamma(i+1)}\right) - \frac{\partial\Gamma(z)}{\partial z}\left(\frac{\Gamma(z+i)}{\Gamma^2(z)\Gamma(i+1)}\right)$$

where,

$$\begin{aligned}\frac{\partial\Gamma(z)}{\partial z} &= \int_0^\infty \frac{\partial x^{z-1}e^{-x}}{\partial z}\,dx \\ &= \int_0^\infty x^{z-1}e^{-x}\ln x\,dx,\end{aligned}$$

$$\begin{aligned}\frac{\partial\Gamma(z+i)}{\partial z} &= \int_0^\infty \frac{\partial x^{z+i-1}e^{-x}}{\partial z}\,dx \\ &= \int_0^\infty x^{z+i-1}e^{-x}\ln x\,dx\end{aligned}$$

Remark 5: In the case where all vectors of parameters d, α and σ are estimated simultaneously by using the ML estimators, Proposition 3.5 is applied and the variance-covariance matrix of the asymptotic distribution of the parameters can be derived by using the following maximum likelihood theory.

Given the log-likelihood function ln*l* which is a differentiable function of θ, where θ is the vector of parameters, $\theta' := (d', \alpha', \sigma')$, the information matrix for θ is defined as,

$$\Omega(\theta) = -E\left[\frac{\partial^2 \ln l}{\partial\theta\,\partial\theta'}\right],$$

and the asymptotic information matrix for δ, if it exists, is,

$$\Omega_a(\theta) = \lim_{T\to\infty}\Omega(\theta)/T\,,$$

then under general regularity conditions, the ML estimator $\hat{\theta}$ for θ is consistent and,

$$\sqrt{T}(\hat{\theta}-\theta) \xrightarrow{d} N(0, \Omega_a(\theta)^{-1}) .$$

Hence, if the assumption of the Proposition 3.5 holds, we can analytically obtain the following results,

$$\Sigma_{\hat{d}} = \left[\frac{\pi^2}{6} \Sigma_\varepsilon \circ \Sigma_\varepsilon^{-1} - \left(\Sigma_\varepsilon \Psi' \Gamma_X^{-1} \Psi \Sigma_\varepsilon \right) \circ \Sigma_\varepsilon^{-1} \right]^{-1} \tag{3.7.1}$$

$$\Sigma_{\hat{\alpha}} = \Gamma_X^{\ -1} \otimes \Sigma_\varepsilon \tag{3.7.2}$$

$$\Sigma_{\hat{\sigma}} = 2\, D_K^+ \left(\Sigma_\varepsilon \otimes \Sigma_\varepsilon \right) D_K^{+\prime} \tag{3.7.3}$$

where $\Psi = (\Psi_1, \ldots, \Psi_p)'$, $\Psi_i = \sum_{j=i}^{\infty} j^{-1} \Psi_{j-i}^{(d)}$ and $\circ$ denotes the Hadamard product. References for proofs and the missing links are provided in the Appendix part B.

In addition, instead of above analytical expressions, numerical methods can also be used to compute the asymptotic variance-covariance matrix of the ML estimate of θ as the negative inverse of the observed Hessian matrix.

Remark 6: For the purpose of convenience, Y_t in (3.1) is assumed to have zero mean and no polynomial trend or seasonal component. However, we note that all the propositions remain unchanged if a nonzero mean, a polynomial trend or a seasonal component is removed before estimating the FIVAR's parameters.

3.8 Simulations

In this section, we conduct some simulations to examine the finite sample performance of the generalized and orthogonalized impulse response as well as their asymptotic distributions in cases of one- and two-step estimation methods. The sample size was chosen to be T = 100, 200, 500, 1000 and 1500, respectively. We obtain the simulation results based on 1000 replications. The multivariate time series Y_t was simulated to follow the 2-dimensional FIVAR(d, 1) model as below,

$$\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}L\right)\begin{bmatrix}(1-L)^{d_1} & 0\\ 0 & (1-L)^{d_2}\end{bmatrix}\begin{bmatrix}Y_{1t}\\ Y_{2t}\end{bmatrix}=\begin{bmatrix}\varepsilon_{1t}\\ \varepsilon_{2t}\end{bmatrix},\quad \begin{bmatrix}\varepsilon_{1t}\\ \varepsilon_{2t}\end{bmatrix}\sim N\left(\begin{bmatrix}0\\ 0\end{bmatrix},\Sigma_\varepsilon\right)$$

In a FIVAR(d, 1) process, we note that the matrix of coefficients is $A_1=[a_{ij}]$, hence, $\alpha=vec(A_1)$. The parameters are randomly drawn from the following distribution:

$$\begin{bmatrix}d\\ \alpha\\ \sigma\end{bmatrix}\xrightarrow{d}N\left(0,\begin{bmatrix}\Sigma_d & 0 & 0\\ 0 & \Sigma_\alpha & 0\\ 0 & 0 & \Sigma_\sigma\end{bmatrix}\right)$$

To simulate a 2-dimimensional FIVAR(d, 1) model, we choose:

$$\Sigma_d=\begin{bmatrix}0.43 & 0.62\\ 0.62 & 0.91\end{bmatrix},$$

$$\Sigma_\sigma=\begin{bmatrix}0.44 & 0.14 & 0.51\\ 0.14 & 0.96 & -0.61\\ 0.51 & -0.61 & 1.25\end{bmatrix},$$

$$\Sigma_\alpha=\begin{bmatrix}0.23 & 0.36 & -0.04 & 0.09\\ 0.36 & 1.79 & 0.08 & -0.59\\ -0.04 & 0.08 & 0.15 & -0.18\\ 0.09 & -0.59 & -0.18 & 0.55\end{bmatrix}.$$

Further, each element of $d=(d_1,d_2)'$ is restricted to be in the range (–0.5, 0.5) and the variance - covariance matrix of ε_t, Σ_ε, is forced to be positive definite. We then obtain,

$$A_1=\begin{bmatrix}-0.125 & 0.032\\ -0.129 & -0.117\end{bmatrix},\ d=(0.138,0.246)'\text{ and }\Sigma_\varepsilon=\begin{bmatrix}0.735 & 0.065\\ 0.065 & 0.870\end{bmatrix}.$$

We estimate the FIVAR(d, 1) models using the two-step estimation method, the Maximum Likelihood estimation proposed by Sela and Hurvich (2009) and the Whittle likelihood estimation, respectively[19]. In terms of the two-step estimation method, we estimate the vector of degrees of fractional integration, d, in the first step under a multivariate framework proposed by Shimotsu (2007) to capture possible dependencies among the fractional degrees. We then transform the simulated series Y_t to X_t using the relationship, $X_t = D(L)Y_t$. Remaining parameters are subsequently estimated using the Multivariate LS method in a VAR(1) model, $X_t = A_1 X_{t-1} + \varepsilon_t$.

We present the Root Mean-Squared Errors (RSME) of estimates of the parameters in Table 3.2. We denote the 2-step as the estimates obtained from the 2-step estimation method, S-H as the Sela – Hurvich Maximum Likelihood Estimates and Whittle as the Whittle Maximum Likelihood Estimates. As can be seen, the 2-step and the Whittle estimation method do better than the S-H estimation method in estimating the off-diagonal elements of A_1. The RSMEs of off-diagonal estimates of A_1 produced by the 2-step estimation method are even slightly smaller than those obtained from the Whittle estimation method. Regarding the diagonal elements of A_1, the Whittle estimation performs worst in almost cases, whereas, the 2-step and S-H estimation methods are relatively comparable. Similarly, in estimating the elements of Σ_ε, the Whittle estimation does worst among three methods. Estimations of vector d are fairly equivalent among the three estimators. Our results of a comparison between the S-H and the Whittle MLEs are consistent with the outcomes of simulations performed in Sela and Hurvich (2009).

[19] For a description of the Maximum Likelihood estimation proposed by Sela and Hurvich (2009) and the Whittle likelihood estimation, we refer to Sela and Hurvich (2009) section 1.3 and section 4.

In general, the 2-step estimator, therefore, seems to be our best candidate to calculate the generalized and orthogonalized impulse response of a FIVAR model. To determine our ultimate choice of the estimation method, we compare the RSME of the impulse response estimates and their asymptotic standard errors generated by the three estimators. We calculate the overall RMSE as an average of the RMSEs of 10 periods ahead generalized and orthogonalized impulse response estimates. In Table 3.3 and 3.4, we report the RMSEs of the impulse response estimates and their asymptotic standard errors, respectively. These results show that the 2-step estimation method generally performs better than the S-H and the Whittle estimation methods in computing both the generalized and orthogonalized impulse response. Regarding the estimates of asymptotic standard errors of the impulse responses, the 2-step estimation produces comparable estimates with whichever method performs better between the S-H and Whittle.

Overall, we find that the 2-step estimation method produces better estimates of the impulse responses in a FIVAR model; whereas, the estimates of asymptotic standard errors of the impulse responses produced by the 2-step method are as good as either S-H or Whittle method, whichever performs better. In addition, another significant advantage of the 2-step estimation method is that it takes much less time than the maximum likelihood estimation to execute. Accordingly, on a basis of our simulations' results, we would suggest an application of the 2-step estimation method in generating the generalized and orthogonalized impulse response of a FIVAR model.

3.9 Conclusion

In this chapter, the impulse response analysis within a multivariate long memory model has been generalized to be unique and invariant with alternative orderings of variables in the system. This specification of the generalized function is particularly valuable in case of a high-dimensional system where there is no clear economic guidance on a suitable ordering. Further, we even make the implementation of both generalized and orthogonalized impulse response analysis within a FIVAR framework easier by reforming them to more simple representations, which we believe can ease processes of computer programming. For the purpose of statistical inference, we derive the asymptotic theories of both generalized and orthogonalized functions. We summarize results under two situations. First, we assume that the degrees of fractional integration are pre-determined. The second situation allows for the case where the differencing parameters are not pre-determined. However, for both situations, our results are all under the common assumption that the order (p) of FIVAR model is known. Practically, this assumption can be satisfied by using the information criteria such as AIC, HQ or SC to determine a suitable p for a FIVAR model. Hence, the results generated in the first situation are applicable in the case that the two-step estimation procedures are applied, where the degrees of fractional integration are consistently determined before the remaining parameters of FIVAR are estimated in a later stage. Meanwhile, the results reported for the second situation can be applied for the case when all parameters of the FIVAR are estimated simultaneously. In addition, we also summarize the available results of asymptotic theories of the FIVAR's parameters in the literature and provide missing links to make our results readily exploitable for both scenarios. According to our simulations' outcomes, we suggest that the two-step estimation method would be the best choice to generate the impulse responses of a FIVAR model. Hence, our results should facilitate the application of impulse response functions in analysing the interdependence and feedback relationships between fractional processes.

3.10 APPENDIX

Part A – Tables and figures

Table 3.1: Fractionally integrated degree of realized volatility of Australian markets

Name	d	t-stat	P-value
Stock market	0.53	5.67	0.00
Currency market	0.67	7.15	0.00

Note: *d* denotes the degree of fractional integration obtained by using GPH (1983) long-periodogram regression estimator. *t-stat* and associated *p-value* are the test statistic and its associated significance value result from testing the null hypothesis $H_0{:}d=0$ against the alternative $H_A{:}\ d \neq 0$.

Table 3.2: Root Mean-Squared Errors of Estimates of the FIVAR model

Parameters	T=100			T=200			T=500			T=1000			T=1500		
	2-step	S-H	Whittle	2-step	S-H	Whittle	2-step	S-H	Whittle	2-step	S-H	Whittle	2-step	S-H	Whittle
Elements of A_1															
a_{11}	0.205	0.205	0.318	0.135	0.120	0.200	0.089	0.072	0.112	0.068	0.051	0.059	0.055	0.041	0.041
a_{21}	0.111	0.180	0.118	0.078	0.163	0.092	0.049	0.162	0.074	0.034	0.160	0.040	0.029	0.161	0.030
a_{12}	0.097	0.178	0.111	0.066	0.159	0.081	0.041	0.144	0.057	0.029	0.140	0.031	0.023	0.136	0.025
a_{22}	0.189	0.223	0.355	0.130	0.125	0.218	0.087	0.072	0.106	0.067	0.052	0.057	0.055	0.043	0.041
Elements of d															
d_1	0.173	0.174	0.286	0.118	0.095	0.138	0.079	0.055	0.109	0.061	0.038	0.051	0.052	0.031	0.032
d_2	0.166	0.197	0.333	0.117	0.107	0.161	0.078	0.056	0.081	0.061	0.039	0.044	0.051	0.031	0.032
Elements of Σ_ε															
σ_{11}	0.105	0.107	0.622	0.075	0.076	0.622	0.048	0.049	0.624	0.034	0.034	0.618	0.028	0.028	0.617
σ_{12}	0.082	0.085	0.068	0.056	0.060	0.068	0.036	0.041	0.086	0.026	0.031	0.056	0.020	0.025	0.054
σ_{22}	0.130	0.130	0.728	0.090	0.093	0.724	0.057	0.060	0.760	0.039	0.044	0.732	0.033	0.037	0.731

Note: 2-step denotes estimates obtained from 2-step estimation method. S-H denotes the Sela-Hurvich Maximum Likelihood Estimates and Whittle denotes the Whittle Maximum Likelihood estimates. Results are based on 1000 replications.

Table 3.3: Root Mean-Squared Errors of Impulse Response Estimates of the FIVAR model

	T=100			T=200			T=500			T=1000			T=1500		
	2-step	S-H	Whittle	2-step	S-H	Whittle	2-step	S-H	Whittle	2-step	S-H	Whittle	2-step	S-H	Whittle
Panel A: Root Mean-Squared Errors of Estimates of Generalized Impulse Response															
$y_1 \leftarrow y_1$	0.044	0.037	0.041	0.030	0.025	0.059	0.020	0.015	0.033	0.016	0.011	0.018	0.013	0.009	0.017
$y_1 \leftarrow y_2$	0.018	0.028	0.028	0.011	0.022	0.059	0.007	0.019	0.035	0.005	0.018	0.006	0.004	0.017	0.004
$y_2 \leftarrow y_1$	0.023	0.029	0.048	0.015	0.024	0.087	0.009	0.023	0.110	0.006	0.022	0.021	0.005	0.022	0.008
$y_2 \leftarrow y_2$	0.056	0.051	0.074	0.039	0.034	0.097	0.026	0.021	0.107	0.020	0.014	0.045	0.017	0.012	0.043
Panel B: Root Mean-Squared Errors of Estimates of Orthogonalized Impulse Response															
$y_1 \leftarrow y_1$	0.044	0.037	0.041	0.030	0.025	0.059	0.020	0.015	0.033	0.016	0.011	0.018	0.013	0.009	0.017
$y_1 \leftarrow y_2$	0.015	0.027	0.012	0.009	0.022	0.014	0.006	0.019	0.005	0.004	0.018	0.003	0.003	0.018	0.003
$y_2 \leftarrow y_1$	0.023	0.029	0.048	0.015	0.024	0.087	0.009	0.023	0.110	0.006	0.022	0.021	0.005	0.022	0.008
$y_2 \leftarrow y_2$	0.056	0.051	0.065	0.039	0.035	0.058	0.026	0.021	0.046	0.020	0.015	0.045	0.017	0.012	0.044

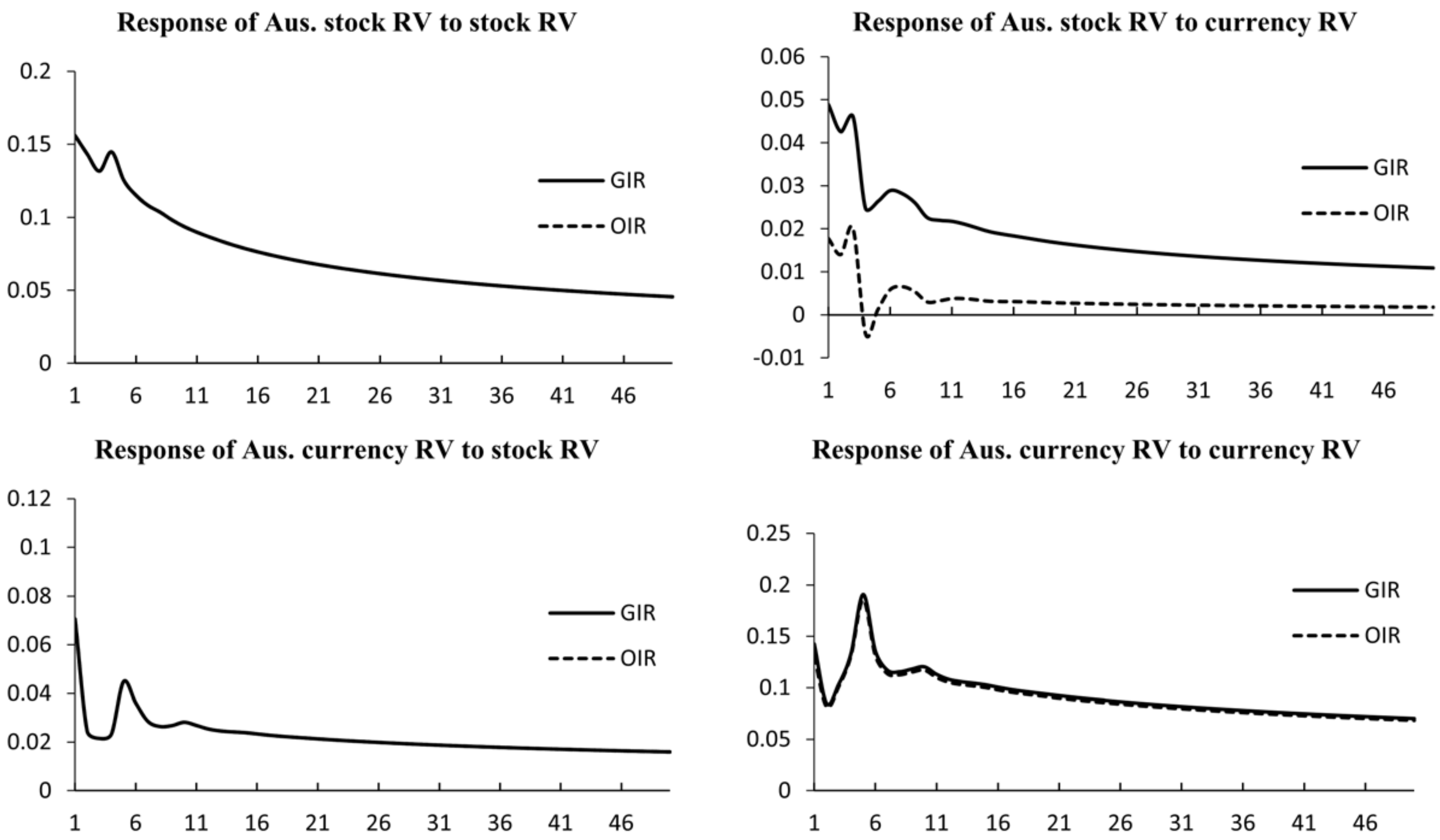
Note: 2-step denotes estimates obtained from 2-step estimation method. S-H denotes the Sela-Hurvich Maximum Likelihood Estimates and Whittle denotes the Whittle Maximum Likelihood estimates. $y_i \leftarrow y_j$ denotes the response of variable y_i to one standard deviation shock to variable y_j. Results are based on 1000 replications.

Table 3.4: Root Mean-Squared Errors of the Asymptotic Standard Errors of the Impulse Response

	T=100			T=200			T=500			T=1000			T=1500		
	2-step	S-H	Whittle	2-step	S-H	Whittle	2-step	S-H	Whittle	2-step	S-H	Whittle	2-step	S-H	Whittle
Panel A: Root Mean-Squared Errors of the Asymptotic Standard Errors of Generalized Impulse Response															
$y_1 \leftarrow y_1$	0.015	0.020	0.018	0.008	0.014	0.015	0.004	0.007	0.005	0.003	0.005	0.001	0.002	0.004	0.001
$y_1 \leftarrow y_2$	0.011	0.013	0.010	0.006	0.008	0.014	0.003	0.004	0.004	0.002	0.003	0.001	0.002	0.002	0.000
$y_2 \leftarrow y_1$	0.009	0.012	0.024	0.005	0.006	0.026	0.002	0.006	0.019	0.001	0.002	0.005	0.001	0.002	0.004
$y_2 \leftarrow y_2$	0.016	0.032	0.026	0.008	0.026	0.029	0.004	0.034	0.019	0.002	0.015	0.008	0.002	0.014	0.006
Panel B: Root Mean-Squared Errors of Asymptotic Standard Errors of Orthogonalized Impulse Response															
$y_1 \leftarrow y_1$	0.010	0.017	0.016	0.006	0.013	0.010	0.003	0.007	0.004	0.002	0.004	0.001	0.002	0.003	0.000
$y_1 \leftarrow y_2$	0.010	0.012	0.007	0.006	0.007	0.028	0.003	0.004	0.012	0.002	0.003	0.001	0.002	0.002	0.000
$y_2 \leftarrow y_1$	0.009	0.012	0.020	0.005	0.006	0.020	0.002	0.006	0.014	0.001	0.002	0.005	0.001	0.002	0.003
$y_2 \leftarrow y_2$	0.009	0.029	0.021	0.005	0.024	0.034	0.002	0.033	0.046	0.001	0.015	0.009	0.001	0.014	0.004

Note: 2-step denotes estimates obtained from 2-step estimation method. S-H denotes the Sela-Hurvich Maximum Likelihood Estimates and Whittle denotes the Whittle Maximum Likelihood estimates. $yi \leftarrow yj$ denotes the response of variable yi to one standard deviation shock to variable yj. Results are based on 1000 replications.

Figure 3.1: The Generalized and Orthogonalized impulse response of realized volatility of Australian stock and currency markets



Notes: The figure reports response of realized volatility (RV) of a market to one standard deviation shock in RV of other market. GIR denotes the Generalized Impulse response; whereas, OIR denotes the Orthogonalized Impulse response.

Part B – Proofs

B1. Proof of proposition 3.3

The result of (3.3.1) is documented previously in Baillie (1987), Lütkepohl (1990) and Lütkepohl (2005). However, our proof is provided for completeness.

Note that: $\Pi_h = JA^h J'$ (see Lütkepohl, 2005, section 2.1.2). So,

$$\frac{\partial vec(\Pi_h)}{\partial \alpha'} = \frac{\partial vec(JA^h J')}{\partial \alpha'} = (J \otimes J)\frac{\partial vec(A^h)}{\partial \alpha'}, \tag{3.29}$$

Since,

$$\frac{\partial vec(A^h)}{\partial \alpha'} = \left[\sum_{m=0}^{h-1}(A')^{h-1-m} \otimes A^m\right]\frac{\partial vec(A)}{\partial \alpha'} \tag{3.30}$$

Then,

$$\frac{\partial vec(\Pi_h)}{\partial \alpha'} = (J \otimes J)\left[\sum_{m=0}^{h-1}(A')^{h-1-m} \otimes A^m\right]\frac{\partial vec(A)}{\partial \alpha'} = \left[\sum_{m=0}^{h-1} J(A')^{h-1-m} \otimes (JA^m)\right]\frac{\partial vec(A)}{\partial \alpha'} \tag{3.31}$$

It can be seen that,

$$\frac{\partial vec(A)}{\partial \alpha'} = I_{Kp} \otimes J'$$

So,

$$\frac{\partial vec(\Pi_h)}{\partial \alpha'} = \sum_{m=0}^{h-1} J(A')^{h-1-m} \otimes (JA^m J') = \sum_{m=0}^{h-1} J(A')^{h-1-m} \otimes \Pi_m . \tag{3.32}$$

For (3.3.2), we have,

$$\frac{\partial vec(\Phi_h)}{\partial \alpha'} = \frac{\partial vec\left(\sum_{i=0}^{h} \Psi_i^{(d)}\Pi_{h-i}\right)}{\partial \alpha'} = \sum_{i=0}^{h}\frac{\partial vec\left(\Psi_i^{(d)}\Pi_{h-i}\right)}{\partial \alpha'} \tag{3.33}$$

It can be seen that,

$$\frac{\partial vec\left(\Psi_h^{(d)}\Pi_0\right)}{\partial\alpha'}=0$$

So,

$$\frac{\partial vec(\Phi_h)}{\partial\alpha'}=\sum_{i=0}^{h-1}\frac{\partial vec\left(\Psi_i^{(d)}\Pi_{h-i}\right)}{\partial\alpha'}=\sum_{i=0}^{h-1}\left[\left(I_K\otimes\Psi_i^{(d)}\right)\frac{\partial vec(\Pi_{h-i})}{\partial\alpha'}\right]=\sum_{i=0}^{h-1}\left[(I_K\otimes\Psi_i^{(d)})G_{h-i}\right]$$

(3.34)

Therefore,

$$\begin{aligned}\frac{\partial vec(\Phi_h)}{\partial\alpha'}&=\sum_{i=0}^{h-1}\left[(I_K\otimes\Psi_i^{(d)})\left(\sum_{m=0}^{h-i-1}J(A')^{h-i-1-m}\otimes\Pi_m\right)\right]\\&=\sum_{i=0}^{h-1}\left(\sum_{m=0}^{h-i-1}J(A')^{h-i-1-m}\otimes\Psi_i^{(d)}\Pi_m\right)\\&=\sum_{i=0}^{h-1}\sum_{m=0}^{h-i-1}J(A')^{h-i-1-m}\otimes\Psi_i^{(d)}\Pi_m.\end{aligned}$$

For 3.3.3, we have,

$$\frac{\partial vec(\Theta_h^g)}{\partial\alpha'}=\frac{\partial vec(\Phi_hB)}{\partial\alpha'}=(B'\otimes I_K)\frac{\partial vec(\Phi_h)}{\partial\alpha'}=(B'\otimes I_K)V_h \tag{3.35}$$

Further,

$$\frac{\partial vec(\Theta_h^g)}{\partial\sigma'}=\frac{\partial vec(\Phi_hB)}{\partial\sigma'}=(I_K\otimes\Phi_h)\frac{\partial vec(B)}{\partial\sigma'} \tag{3.36}$$

We have,

$$\frac{\partial vec(B)}{\partial\sigma'}=\frac{\partial vec(\Sigma_\varepsilon\Xi)}{\partial\sigma'}=(I_K\otimes\Sigma_\varepsilon)\frac{\partial vec(\Xi)}{\partial\sigma'}+(\Xi'\otimes I_K)\frac{\partial vec(\Sigma_\varepsilon)}{\partial\sigma'} \tag{3.37}$$

where,

$$\frac{\partial vec(\Sigma_\varepsilon)}{\partial\sigma'}=\frac{\partial D_K vech(\Sigma_\varepsilon)}{\partial vech(\Sigma_\varepsilon)'}=D_K \tag{3.38}$$

Additionally, it can be easily seen that,

$$vec(\Xi^{-2})=S_K vec(\Sigma_\varepsilon)$$

$$\therefore vec(\Xi\Xi^{-3}) = S_K vec(\Sigma_\varepsilon)$$

Since Ξ is a symmetric and diagonal matrix then,

$$(\Xi^{-3} \otimes I_K) vec(\Xi) = S_K vec(\Sigma_\varepsilon)$$

$$\therefore vec(\Xi) = (\Xi^3 \otimes I_K) S_K D_K vech(\Sigma_\varepsilon) \because vec(\Sigma_\varepsilon) = D_K vech(\Sigma_\varepsilon)$$

$$\therefore \frac{\partial vec(\Xi)}{\partial \sigma'} = \frac{\partial vec(\Xi)}{\partial vech(\Sigma_\varepsilon)'} = (\Xi^3 \otimes I_K) S_K D_K \tag{3.39}$$

Replace (3.37), (3.38) and (3.39) in (3.36), we have,

$$\begin{aligned}\frac{\partial vec(\Theta_h^g)}{\partial \sigma'} &= (I_K \otimes \Phi_h)\left\{(I_K \otimes \Sigma_\varepsilon)\left[(\Xi^3 \otimes I_K) S_K D_K\right] + (\Xi \otimes I_K) D_K\right\} \\ &= (I_K \otimes \Phi_h \Sigma_\varepsilon)\left[(\Xi^3 \otimes I_K) S_K D_K\right] + (\Xi \otimes \Phi_h) D_K \\ &= (\Xi^3 \otimes \Phi_h \Sigma_\varepsilon) S_K D_K + (\Xi \otimes \Phi_h) D_K\end{aligned}$$

For 3.3.4, we have,

$$\frac{\partial vec(\Theta_h^O)}{\partial \alpha'} = \frac{\partial vec(\Phi_h P)}{\partial \alpha'} = (P' \otimes I_K)\frac{\partial vec(\Phi_h)}{\partial \alpha'} = (P' \otimes I_K) V_h \tag{3.40}$$

Further,

$$\frac{\partial vec(\Theta_h^O)}{\partial \sigma'} = \frac{\partial vec(\Phi_h P)}{\partial \sigma'} = (I_K \otimes \Phi_h)\frac{\partial vec(P)}{\partial \sigma'} \tag{3.41}$$

Lütkepohl (1989) and Lütkepohl (1990) show that,

$$\frac{\partial vec(P)}{\partial \sigma'} = L_K'\left[L_K (I_{K^2} + K_{KK})(P \otimes I_K) L_K'\right]^{-1} \tag{3.42}$$

So, replace (3.42) in (3.41) we obtain,

$$\frac{\partial vec(\Theta_h^O)}{\partial \sigma'} = (I_K \otimes \Phi_h)\left\{L_K'\left[L_K (I_{K^2} + K_{KK})(P \otimes I_K) L_K'\right]^{-1}\right\}. \blacksquare$$

B2. Proof of proposition 3.4

Using the asymptotic approximation formula of the gamma function, it can be easily seen that,

$$\Gamma(z) \quad \sim \sqrt{2\pi}e^{-z}z^{z-\frac{1}{2}},$$

$$\Gamma(z+i) \quad \sim \sqrt{2\pi}e^{-z}z^{z+i-\frac{1}{2}} \quad \text{for } i \in \mathrm{N}_0.$$

Applying Leibniz and the generalized power rule for differentiation, we obtain,

$$\frac{\partial\,\Gamma(z)}{\partial z} \quad \sim -\sqrt{2\pi}e^{-z}z^{z-\frac{1}{2}} + \sqrt{2\pi}e^{-z}z^{z-\frac{1}{2}}\left(\frac{z-\dfrac{1}{2}}{z} + \ln z\right)$$

$$\sim \sqrt{2\pi}e^{-z}z^{z-\frac{1}{2}}(\ln z - \frac{1}{2z})$$

Similarly,

$$\frac{\partial\,\Gamma(z+i)}{\partial z} \sim \sqrt{2\pi}e^{-z}z^{z+i-\frac{1}{2}}(\ln z + \frac{2i-1}{2z}),$$

Further,

$$\overline{\psi}_i^{(z)} = \frac{\partial \psi_i^{(z)}}{\partial z} = \frac{\partial \dfrac{\Gamma(z+i)}{\Gamma(z)\Gamma(i+1)}}{\partial z} = \frac{\partial\Gamma(z+i)}{\partial z}\left(\frac{1}{\Gamma(z)\Gamma(i+1)}\right) - \frac{\partial\Gamma(z)}{\partial z}\left(\frac{\Gamma(z+i)}{\Gamma^2(z)\Gamma(i+1)}\right)$$

$$\therefore\ \overline{\psi}_i^{(z)} \sim \frac{z^i}{i!}\left(\ln z + \frac{2i-1}{2z}\right) - \left(\ln z - \frac{1}{2z}\right)\frac{z^i}{i!}$$

$$\therefore\ \overline{\psi}_i^{(z)} \sim \begin{cases} \dfrac{z^{i-1}}{(i-1)!} & \text{for } i = 1, 2, \ldots \\ 0 & \text{for } i = 0 \end{cases} . \blacksquare$$

B3. Proof of proposition 3.5

For 3.5.1 we have,

$$\overline{V}_h := \frac{\partial vec(\Phi_h)}{\partial d'} = \frac{\partial vec\left(\sum_{i=0}^{h} \Psi_i^{(d)} \Pi_{h-i}\right)}{\partial d'} = \sum_{i=0}^{h} \frac{\partial vec\left(\Psi_i^{(d)} \Pi_{h-i}\right)}{\partial d'} = \sum_{i=0}^{h} \left[\left(\Pi'_{h-i} \otimes I_K\right) \left(\frac{\partial vec(\Psi_i^{(d)})}{\partial d'} \right) \right]$$

It then can be seen that,

$$\frac{\partial vec(\Psi_i^{(d)})}{\partial d'} = \Lambda_i^{(d)},$$

$$\therefore \quad \overline{V}_h = \sum_{i=0}^{h} \left[(\Pi'_{h-i} \otimes I_K) \Lambda_i^{(d)} \right];$$

For 3.5.2 we have,

$$M_h := \frac{\partial vec(\Theta_h^g)}{\partial d'} = \frac{\partial vec(\Phi_h B)}{\partial d'} = (B' \otimes I_K) \frac{\partial vec(\Phi_h)}{\partial d'}$$

$$\therefore \quad M_h = (B' \otimes I_K) \overline{V}_h;$$

For 3.5.3 we have,

$$W_h := \frac{\partial vec(\Theta_h^O)}{\partial d'} = \frac{\partial vec(\Phi_h P)}{\partial d'} = (P' \otimes I_K) \frac{\partial vec(\Phi_h)}{\partial d'}$$

$$\therefore \quad W_h = (P' \otimes I_K) \overline{V}_h. \ \blacksquare$$

B4. Proof of results in remark 5 of section 3.7

Proof of (3.7.1) is documented in Nielsen (2004) given a note that, since $X_t = D(L) Y_t$ the infinite moving average of Y_t can be written as, $Y_t = \sum_{i=0}^{\infty} \Psi_i^{(d)} L^i X_t$.

In addition, to prove (3.7.2) and (3.7.3), along with the assumptions that the p pre-sample values for each variable of Y_t, Y_{-p+1}, … ,Y_0, are available and the ε_t is *Gaussian* white noise, we establish some following extra notations,

$$\underset{(K\times T)}{U} := (\varepsilon_1, \ldots, \varepsilon_T)$$

$$\underset{(Kp\times 1)}{X_t^0} := \begin{bmatrix} X_t \\ X_{t-1} \\ \vdots \\ X_{t-p+1} \end{bmatrix}$$

$$\underset{(Kp\times T)}{X} := \left(X_0^0, \ldots, X_{T-1}^0\right)$$

$$\underset{(K\times T)}{\mathrm{x}} := (X_1, \ldots, X_T)$$

$$\underset{(K\times T)}{\mathrm{y}} := (Y_1, \ldots, Y_T)$$

$$x := vec(\mathrm{x})$$

$$y := vec(\mathrm{y})$$

Since $\mathrm{x} = D(L)\mathrm{y}$ then $x = \left[I_K \otimes D(L)\right]y$. Let call $f_x(x)$ and $f_y(y)$ are the probability density of *x* and *y*, respectively. We have,

$$f_y(y) = \left|\frac{\partial x}{\partial y'}\right| f_x(x) = \left|I_K \otimes D(L)\right| f_x(x),$$

Since X_t follows a VAR(p) process, Lütkepohl (2005, section 3.4) shows the probability density of *x* as,

$$f_x(x) = \frac{1}{(2\pi)^{KT/2}}\left|I_T \otimes \Sigma_\varepsilon\right|^{-1/2} \times \exp\left[-\frac{1}{2}\left(x-(X'\otimes I_K)\alpha\right)'(I_T \otimes \Sigma_\varepsilon^{-1})\left(x-(X'\otimes I_K)\alpha\right)\right]$$

So,

$$f_y(y) = \left|I_K \otimes D(L)\right| \frac{1}{(2\pi)^{KT/2}} \left|I_T \otimes \Sigma_\varepsilon\right|^{-1/2}$$
$$\times \exp\left[-\frac{1}{2}\big(x-(X'\otimes I_K)\alpha\big)'(I_T \otimes \Sigma_\varepsilon^{-1})\big(x-(X'\otimes I_K)\alpha\big)\right]$$

Hence, we get the log-likelihood function,

$$\ln l(d,\alpha,\Sigma_\varepsilon) = K\ln\left|D(L)\right| - \frac{KT}{2}\ln 2\pi - \frac{T}{2}\ln\left|\Sigma_\varepsilon\right|$$
$$-\frac{1}{2}\left[x-(X'\otimes I_K)\alpha\right]'(I_T \otimes \Sigma_\varepsilon^{-1})\left[x-(X'\otimes I_K)\alpha\right] \tag{3.43}$$

By using the relationship between the *vec* and the *trace* (*tr*) operator,

$$tr(ABC) = vec(A')'(I \otimes B)vec(C)$$

and note that, $x-(X'\otimes I_K)\alpha = vec(\mathrm{x}-\mathrm{A}X)$, where $\mathrm{A} := (A_1,\ldots,A_p)$; the log-likelihood function can also be represented as,

$$\ln l(d,\alpha,\Sigma_\varepsilon) = K\ln\left|D(L)\right| - \frac{KT}{2}\ln 2\pi - \frac{T}{2}\ln\left|\Sigma_\varepsilon\right| - \frac{1}{2}tr\left[(\mathrm{x}-\mathrm{A}X)'\Sigma_\varepsilon^{-1}(\mathrm{x}-\mathrm{A}X)\right] \tag{3.44}$$

With the representation of (3.43) and (3.44), we can easily obtain the following second order partial derivatives of the log-likelihood[20],

$$\frac{\partial^2 \ln l(d,\alpha,\Sigma_\varepsilon)}{\partial\alpha\,\partial\alpha'} = -(XX'\otimes\Sigma_\varepsilon^{-1}) \tag{3.45}$$

$$\frac{\partial^2 \ln l(d,\alpha,\Sigma_\varepsilon)}{\partial\sigma\,\partial\sigma'} = D_K'\left[\frac{T}{2}(\Sigma_\varepsilon^{-1}\otimes\Sigma_\varepsilon^{-1}) - \frac{1}{2}(\Sigma_\varepsilon^{-1}\otimes\Sigma_\varepsilon^{-1}UU'\Sigma_\varepsilon^{-1})\right.$$
$$\left.-\frac{1}{2}(\Sigma_\varepsilon^{-1}UU'\Sigma_\varepsilon^{-1}\otimes\Sigma_\varepsilon^{-1})\right]D_K \tag{3.46}$$

[20] For details involved in these derivations, we refer to Lütkepohl (2005, section 3.4).

Let the assumption of the Proposition 3.5 holds, the asymptotic variance-covariance matrix of α is,

$$\Sigma_{\hat{\alpha}} = -TE\left(\frac{\partial^2 \ln l(d,\alpha,\Sigma_\varepsilon)}{\partial\alpha\,\partial\alpha'}\right)^{-1} = -TE\left[-\left(XX'\otimes\Sigma_\varepsilon^{-1}\right)\right]^{-1}$$

Noting that $E(XX'/T)=\Gamma_X$, because when $T\to\infty$ the impact of assumed initial p pre-sample values, $Y_{-p+1},\ldots,Y_0$, vanishes. Therefore,

$$\Sigma_{\hat{\alpha}} = \Gamma_X^{-1}\otimes\Sigma_\varepsilon,$$

This completes the proof of (3.7.2).

Similarly, the asymptotic variance-covariance matrix of σ is,

$$\Sigma_{\hat{\sigma}} = -TE\left(\frac{\partial^2 \ln l(d,\alpha,\Sigma_\varepsilon)}{\partial\sigma\,\partial\sigma'}\right)^{-1},$$

Since $E(UU')=T\Sigma_\varepsilon$ then from (3.46) we have,

$$E\left(\frac{\partial^2 \ln l(d,\alpha,\Sigma_\varepsilon)}{\partial\sigma\,\partial\sigma'}\right) = D_K'\left[-\frac{T}{2}\left(\Sigma_\varepsilon^{-1}\otimes\Sigma_\varepsilon^{-1}\right)\right]D_K$$

Hence, the asymptotic variance-covariance matrix of σ can be expressed as,

$$\Sigma_{\hat{\sigma}} = 2\left[D_K'\left(\Sigma_\varepsilon^{-1}\otimes\Sigma_\varepsilon^{-1}\right)D_K\right]^{-1},$$

Finally,

$$\Sigma_{\hat{\sigma}} = 2\,D_K^{+}\left(\Sigma_\varepsilon\otimes\Sigma_\varepsilon\right)D_K^{+/}.$$

This completes the proof of (3.7.3).

Chapter 4

How does trading volume affect financial return distributions?

4.1 Introduction

Influences of trading volume on financial return distributions have been increasingly examined in the finance literature. From the market microstructure point of view, new information arrival and the mechanism that incorporates this information are primary factors causing movements in asset prices (see Andersen, 1996). Since trading volume is widely accepted as a proxy for the arrival of hidden information to the market, knowledge about volume's impact on return distributions helps in understanding the role of information arrival in asset pricing. Fundamentally, these potential interactions between trading volume and return distributions (captured by volatility, skewness and kurtosis) can be explained by three relevant information theories in the literature, including the mixture of distributions hypothesis (MDH), the sequential arrival of information hypothesis (SAIH) and the differences of opinion hypothesis (DOH).

4.2 Literature review

In the literature, a number of information theories have been presented to explain a causal relationship between trading volume and asset prices. The MDH was initially provided by Clark (1973) in an attempt to explain the relationship between trading volume and volatility. Basically, the MDH states that the trading volume and asset prices are jointly driven by common latent information. Clark (1973) argues that trading volume is contemporaneously correlated with the volatility since it can be considered as a proxy for the arrival of events “happen at a random rate over time” (see Mougoué and Aggarwal, 2011). However, Clark (1973) does not directly model this causality. The theory of MDH was then described by different approaches (e.g., Epps and Epps, 1976; Tauchen and Pitts, 1983; and Harris, 1987). While Epps and Epps (1976) model the price change of an individual transaction conditional on the trading volume of that transaction; Tauchen and Pitts (1983) and Harris (1987) formulate the trading volume to be contemporaneously proportional to volatility and vice versa, with their relationship depending on changes of information flow. More recently, Andersen (1996) modifies the MDH by including the liquidity requirements and informational asymmetries among investors, where a stochastic volatility process is employed to model the information flow.

A different approach for justifying the relationship between trading volume and asset prices is the SAIH, which was first introduced by Copeland (1976) and subsequently extended by Jennings et al. (1981) and Smirlock and Starks (1988). The theory of SAIH states that information is circulated to different investors at different times such that the final equilibrium is reached after a sequence of provisional equilibriums. Hence, the SAIH implies a lead-lag relationship between trading volume and volatility, whose strength is determined by levels of dissemination and importance of the information.

Alternatively, the relationship between trading volume and asset prices can be explained by the theory of DOH. The DOH hypothesis may be referred to as the dispersion of beliefs hypothesis (e.g., Chen and Daigler, 2008) or the investor heterogeneity hypothesis (e.g., Hong and Stein, 2003; and Hutson et al., 2008). The DOH was developed by Shalen (1993) for future markets and generalized later by Harris and Raviv (1993). The theory of DOH supposes that different types of investors may interpret the same information differently according to their own signals (e.g., private information and knowledge). Hence, they may have different expectations about the fundamental values of assets, which consequently lead to greater variability in price changes. It is, therefore, expected that the trading volume and volatility are positively related. Hong and Stein (2003) extend the DOH by incorporating short-sales constraints to explain the relationship between trading volume and return asymmetries (skewness). This extension is known as the investor heterogeneity hypothesis, which predicts a positive causality between trading volume and negative skewness of return. In other words, because of the short-sales constraints, high trading volume causes a greater level of negative return skewness, which in turn becomes a source of market crashes.

In summary, the theories of MDH (e.g., Clark, 1973; Epps and Epps, 1976; Tauchen and Pitts, 1983) and DOH (e.g., Shalen, 1993; Harris and Raviv, 1993) suggest a positive contemporaneous linkage between trading volume and volatility; whereas, a lead-lag relationship between them is added by an implication of SAIH (e.g., Copeland, 1976, 1977). Empirically, these theories have been widely tested and accepted in many studies conducted within stock or FX markets (e.g., Kalev et al., 2004; Bjønnes et al., 2005; Bauwens et al., 2005; Chan and Fong, 2006). However, an empirical test of the cross – market relation between volume and volatility has received limited attention so far. Hence, we contribute new findings on the volume – volatility relationship within and between stock and FX markets.

In terms of the volume – skewness relationship, the DOH theory predicts that the negative skewness of return will be greater conditional on a higher trading volume as mentioned earlier (see Hong and Stein, 2003). However, different from the volume – volatility literature, empirical studies on the volume – skewness relationship show mixed results. The theory of Hong and Stein (2003) is supported by Chen et al. (2001) and Hutson et al. (2008) but not supported in Hueng and McDonald (2005) and Charoenrook and Daouk (2008). While a direct volume – skewness relationship is verified with firm – level data, the use of market level data shows little support for the relationship. Even though Hutson et al. (2008) provide empirical evidence on the theory postulated in Hong and Stein (2003) with national stock market data, the direct effect of volume on skewness only exists in 3 out of 11 cases. Therefore, we raise a conjecture that level of portfolio diversification is probably responsible for the difference in results. In this study, we aim to verify our conjecture by reassessing the direct volume – skewness relationship with a particular focus on a regional analysis.

In addition, we further contribute to the literature by testing for the existence of volume – kurtosis interactions and whether it is consistent with the aforementioned information theories. The possibility of a volume – kurtosis relationship is supported from a market microstructure perspective. Since the price movements are mainly caused by new information arrival, an occurrence of extreme returns may be influenced accordingly.

Besides, instead of re-examining volume's impacts on higher moments distinctly, we allow for the possibility of interactions among them in our analysis due to several reasons. A growing integration of national economies with their regions and the rest of the world, collapses of financial institutions and recent financial turbulence consistently suggest that evaluation of a risk needs to be conducted not only in isolation but also by allowing for the possibility that it can interact with and spill-over to amplify other risks. For example, the collapse of Long Term Capital Management L.P. has highlighted the important of hedge fund risks, including fat-tail risk and (possibly) its interaction with other risks (see Fung and Hsieh, 2001; Feix, 2003). Further, a sequence of recent financial crises, including the Sub-prime Mortgage Crisis in 2007 in the U.S, the 2008 Global Financial Crisis and most recently, the European Sovereign Debt Crisis suggest that the assessment of a financial risk is much more complex than just viewing it separately from other risks. A higher degree of integration between economies leads to faster and stronger contagion effects with recent evidence that a downgrade of U.S treasury bonds in late-mid 2011 significantly affected global financial markets. The contagion effects should not only highlight the transmission of a risk across countries but also allow the probability of interaction between risks across markets. Empirically, some preliminary examinations using the correlation approach have revealed prospective interdependence among higher-moment risks (e.g., Cooley et al., 1977; Gupta et al., 2004). In our study, we support this prospect in a more complete context in terms of both static (impulse response analysis) and dynamic (spill-over index) approaches.

Additionally, this evidence of interconnections between higher moments motivates the need to investigate the influences of trading volume on the dynamic structure of cross-moment inter-relationships. The appearance and importance of higher-moment risks have been increasingly recognized in many financial activities, such as asset pricing, value-at-risk calculation and asset allocation (see Athayde and Flôres, 2003; Mandelbrot and Hudson, 2004; Jurczenko and Maillet, 2006 among others). Therefore, such financial activities can benefit from our analysis since the result from our study may help to evaluate volatility risk, downside risk and fat-tail risk under influences of new information arrival more precisely.

We base our study on intraday data to produce a better representation and more robust estimates for higher moments of asset returns. Further, the use of intraday data is also consistent with the aforementioned market microstructure perspective as the market microstructure literature mainly pays attention to intraday patterns rather than inter-day dynamics. The remainder of this chapter is organized as follows. Section 4.3 explains data employed and the construction of variables for analysis. Section 4.4 outlines the econometric framework. Section 4.5 discusses the empirical results of our reassessment of volume's impacts on financial return distribution. Section 4.6 analyses the influence of trading volume on the dynamic structure of the inter-relationships among higher moments and finally, section 4.7 concludes.

4.3 Data

We extract 5-minute intraday mid prices for stock market indexes and FX transactions in 18 countries from the Thomson Reuters Tick History (TRTH) database provided by the Securities Industry Research Centre of Asia-Pacific (SIRCA). The use of 5-minute intervals can overcome the problem of measurement error and reduce microstructural biases (see Andersen and Bollerslev, 1998, and Andersen et al., 2001b). In the FX market, we use the US dollar (USD) as the base currency against which national local currencies are priced in. For stock market indexes, we use the prices denominated in local currencies. The sample extends from January 1, 2002 to February 15, 2010. Data on weekends are excluded. Furthermore, we base our analysis on two sub-sample periods: from January 1, 2002 to Jun 29, 2007 (the 'Stable period') and from July 2, 2007 to February 15, 2010 (the 'Volatile period')[21]. For the purpose of conducting regional analyses, we divide our sample countries into four regional grouping, namely Latin America, Asia Pacific Emerging, Asia Pacific Developed and Western Europe. Furthermore, since the stock market is not a non-stop trading market, we consider a trading day as that part of the day when stock markets are open[22].

We compute the 5-minute intraday returns of each market as the log change in the mid prices. For sample countries in the European Monetary Union (EMU), we use the prices of their own national currencies to calculate intraday returns before they adopted the Euro and prices denominated in Euros thereafter. The intraday returns of regional portfolios are constructed as value-weighted averages of the intraday returns of individual markets in each region where the country weights are based on gross domestic product (GDP)[23]:

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[21] Hence, our Volatile period covers both the Sub-prime mortgage crisis in 2007 and the Global Financial Crisis in 2008. In addition, as can be seen from Figure 4.1, the realized volatilities behave differently across the two periods.

[22] Hansen and Lunde (2005) propose to estimate the realized volatility of a stock market for the whole day to account for the potential latent information during non-trading time. However, since our study focuses on a regional context with different countries, this methodology is not applicable because of the different trading and non-trading times in GMT in different stock markets.

[23] We summarize the details for individual countries in each region, country weights based on GDP and trading hours for all regions in Table 4.1. Besides, we prefer to weight countries by GDP rather than by

$$r_{j,t} = \sum_{i=1}^{q} w_i r_{i,j,t} \tag{4.1}$$

where $r_{j,t}$ denotes the jth 5-minute regional portfolio return during day t, w_i is the weight of market i, $r_{i,j,t}$ denotes the jth 5-minute return of market i during day t and q is the number of markets in the region.

Similar to Chapter 2, section 2.3, we calculate the realized higher moments of regional portfolio return as follow

$$RV_t = \sum_{j=1}^{D} r_{j,t}^2 \tag{4.2}$$

$$RS_t = -\frac{D(D-1)^{3/2}(\sum_{j=1}^{D} r_{j,t}^3)}{(D-1)(D-2)(\sum_{j=1}^{D} r_{j,t}^2)^{3/2}} \tag{4.3}$$

$$RK_t = \frac{\sum_{j=1}^{D} r_{j,t}^4}{(RV_t)^2} \tag{4.4}$$

where D denotes the total number of 5-minute return intervals during any trading day.

Regarding the realized skewness (RS_t), we note that the negative sign is included to make sure an increase in the daily skewness corresponds to an asset return having a more left-skewed distribution (Chen et al., 2001). Therefore, by using this formula we focus on the importance of downside risk in analysing the interdependence with other moments and trading volume. Hence, an utilization of this formula facilitates a comparison between our empirical results and the investor heterogeneity theory of Hong and Stein (2003).

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market capitalization since the GDP figures are likely to be more stable compared to stock market performance with peaks and troughs (see Figure 4.1).

We employ the number of trades as a proxy for trading volume. Furthermore, our choice is also supported by Chan and Fong (2006), who find that the number of trades contains more hidden information than other proxies for volume (e.g., trade size and order imbalance). Theoretically, this is consistent with the hypothesis of stealth trading, which suggests that informed traders may divide a large trade into many smaller transactions to hide their private signals (see for example, Barclay and Warner, 1993, and Chakravarty, 2001). Therefore, we calculate the daily trading volume of a regional portfolio by summing up the total number of trades across all markets within the region over all 5 minute intervals during the day:

$$V_t = \sum_{j=1}^{D} \sum_{i=1}^{m} w_i v_{i,j,t} \tag{4.5}$$

where $v_{i,j,t}$ denotes the jth 5-minute number of trades of market i during day t, w_i is the weight of market i calculated based on its country's GDP.

Additionally, as a result of different holidays in different countries, linear interpolation is employed to reconstruct missing data due to holidays and days of unusually light trading volume. The interpolation method has been found to be useful in empirical studies which deal with missing observations in time series data (see Damsleth, 1980 and Pavlov, 2004).

Table 4.2 and 4.3 provides descriptive statistics for the (logged) realized volatility, realized skewness, (logged) realized kurtosis and (logged) trading volume of

FX and stock markets[24]. As expected, the mean levels of realized volatility and realized kurtosis are consistently higher in the Volatile period than in the Stable period for both stock and FX markets. In addition, the distributions of all the realized measures generally deviate from the normal distribution with some level of asymmetry and excess kurtosis. However, in line with the literature (e.g., Andersen et al., 2003) the distribution of realized volatility is close to normal in many cases. Interestingly, we observe that this fact also applies to realized skewness and realized kurtosis constructed from our international dataset. The Ljung-Box statistics ($Q(20)$) confirm the significance of autocorrelation up to 20 lags in all cases for realized volatility and trading volume. Furthermore, we also observe the existence of a serial correlation problem in most of the cases for realized kurtosis but only in limited cases for realized skewness. In fact, the long-range dependence behaviour of realized volatility and trading volume has been previously documented in the literature (see for example, Andersen et al., 2001a, 2001b, 2003; and Fleming et al., 2011). However, the long memory behaviour of realized kurtosis has not been documented to date.

4.4 Econometric framework

The evidence of long-range dependence in realized measures and trading volume supports the utilization of fractional integration techniques, as fractionally integrated

[24] Realized volatility, kurtosis and trading volume are transformed into their natural logarithm since their non-negativity condition needs to be satisfied when they are modeled. Besides, the use of realized logarithmic volatility in empirical analysis is well supported in the literature (e.g., Andersen and Bollerslev, 1998; Andersen et al., 2001a; and Andersen et al., 2003). In addition, we use realized logarithmic kurtosis and logarithmic trading volume to achieve a similar scale for the subsequent impulse response analyses. Therefore, when we refer to realized volatility, kurtosis and trading volume in our study, they are in their natural logarithmic form.

processes have been found to efficiently capture the long memory behaviour of financial time series (see Ding et al., 1993; and Andersen et al., 1997). Under a fractional process, a series that is an intermediate between a short-memory and an unit root process can be effectively described. Furthermore, in order to investigate the interdependence and feedback relationships in a system including both long- and short-memory series, a multivariate fractional process allowing for multi-memory parameters is useful. Hence, in our study, we consider the specification of a FIVAR model.

4.4.1 Model specification

Suppose a vector of jointly determined dependent variables $Y_t = (Y_{1t}, Y_{2t}, ..., Y_{Kt})'$ that follows a K-dimensional FIVAR framework[25]:

$$A(L)D(L)Y_t = \varepsilon_t, \qquad t = 1,2,...,T. \tag{4.6}$$

where L is the lag operator and ε_t is a $K\times 1$ vector of error term, which is assumed to be white noise and multivariate normally distributed. The variance-covariance matrix of ε_t denoted as $\Sigma_\varepsilon = \{\sigma_{ij}; i, j = 1,2,...,K\}$ is a $K\times K$ positive definite matrix.

The operator $A(L) = I_K - \sum_{i=1}^{p} A_i L^i$, where A_i is the $K\times K$ matrix of coefficients and p is the order of the lag polynomials in $A(L)$. All the roots of $|A(z)| = \left| I_K - \sum_{i=1}^{p} A_i z^i \right| = 0$ are assumed to fall outside the unit circle. The operator $D(L)$

[25] According to Equation (4.6), Y_t is assumed to have no trend and drift. Hence, before modelling the realized measures and the trading volume with FIVAR, they are demeaned and detrended whenever the drift and the trend are statistically significant. Details for the existence of a trend in the realized measures and trading volume series are reported in Table 4.2 and 4.3.

is a diagonal $K \times K$ matrix characterized by the K elements in the degree of fractional integration vector $d = (d_1, d_2, ..., d_K)'$ as follows[26]:

$$D(L) = \begin{bmatrix} (1-L)^{d_1} & 0 & \dots & 0 \\ 0 & (1-L)^{d_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & (1-L)^{d_K} \end{bmatrix}$$

Operationally, we can generate the term $(1-L)^{-d_j}$ with the following binomial expansion:

$$(1-L)^{-d_j} = \sum_{i=0}^{\infty} \frac{\Gamma(i+d_j)}{\Gamma(d_j)\Gamma(i+1)} L^i = \sum_{i=0}^{\infty} \psi_i^{(d_j)} L^i \tag{4.7}$$

where $\Gamma(.)$ is the gamma function; $\psi_0^{(0)} = 1$, and $\psi_i^{(0)} = 0$, for $i \neq 0$.

4.4.2 Model estimation

As discussed in Chapter 3, section 3.6, there are two prominent approaches to estimate a multivariate fractional process. The first approach aims to estimate all the parameters simultaneously (e.g., Hosoya, 1996; Martin and Wilkins, 1999; Nielsen, 2004; Pai and Ravishanker, 2009); whereas, the second approach separates the estimation procedure into two steps, whereby the memory parameters are consistently determined in the first step and the estimation of remaining parameters is subsequently performed with standard econometric techniques. Regarding the second approach, the

[26] For estimation purposes, a further restriction, $|d_j| < \frac{1}{2}$ for all j = 1,2,…,K, needs to be satisfied to make the model stationary. This condition can be obtained by taking an appropriate number of differences. For example, if $\frac{1}{2} < d_j < \frac{3}{2}$ then the first-differenced series has a degree of integration less than $\frac{1}{2}$ in absolute value.

degrees of fractional integration can be estimated under an univariate or a multivariate framework (e.g., Geweke and Porter-Hudak, 1983; Künch, 1987; Chong, 2000; Shimotsu et al., 2005; Shimotsu, 2007; and Nielsen, 2011).

In general, despite its asymptotic efficiency, the simultaneous estimation procedure is time-consuming in cases of high dimensional systems or large sample sizes. Further, simulations' results provied in Chapter 3 suggest an application of the 2-step estimation method in estimating the impulse response within a FIVAR model. Hence, we employ the 2-step estimation approach, in which the memory parameters are estimated under a multivariate framework proposed by Shimotsu (2007) to capture possible dependencies between them. Shimotsu (2007) derives a Gaussian semiparametric estimator of a multivariate fractionally integrated process by using a general form of the spectral density of Y_t.

Let us define the discrete Fourier transform and the periodogram of Y_t evaluated at frequency λ as,

$$w(\lambda) = \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^{T} Y_t e^{\mathrm{i}t\lambda}, \qquad I(\lambda) = w(\lambda) w^*(\lambda),$$

where i is the imaginary unit, $w^*(\lambda)$ denotes the conjugate transpose of $w(\lambda)$.

For the Fourier frequencies $\lambda_a = 2\pi a/T$ with $a = 1,...,m$ where m is the band parameter determined as $m = \mathrm{o}(T)$, Shimotsu (2007) shows the spectral density of Y_t as,

$$f(\lambda_a) \sim \Lambda_a(d) G_s \Lambda_a^*(d),$$

where $\Lambda_a(d) = \mathrm{diag}(\Lambda_{aj}(d))$ with $\Lambda_{aj}(d) = \lambda_a^{-d_j} e^{\mathrm{i}(\pi - \lambda_a)d_j/2}$, $j = 1,\dots,K$; $\Lambda_a^*(d)$ denotes the conjugate transpose of $\Lambda_a(d)$; and,

$$G_s = \frac{1}{m}\sum_{a=1}^{m} \mathrm{Re}\left[\Lambda_a(d)^{-1} I(\lambda_a)\Lambda_a^*(d)^{-1}\right],$$

The objective function can be subsequently obtained as,

$$R(d) = \log\det G_s - 2\sum_{j=1}^{K} d_j \frac{1}{m}\sum_{a=1}^{m}\log\lambda_a \ ,$$

Then, the estimator of the memory parameter vector is defined as,

$$\hat{d} = \arg\min R(d)\,.$$

In order to draw statistical inferences about the significance of the fractional degrees, we employ the asymptotic normal theory built for the estimator. Let d_0 denote the true value of *d* then the asymptotic normal distribution of *d* is defined as,

$$\sqrt{m}(\hat{d} - d_0) \xrightarrow{d} N(0, \Sigma_{ds}^{-1}), \quad \Sigma_{ds} = 2\left[G_s \circ G_s^{-1} + I_K + \frac{\pi^2}{4}(G_s \circ G_s^{-1} - I_K)\right],$$

$$\hat{G}_s \xrightarrow{p} G_s\,,$$

where $\circ$ denotes the Hadamard product.

After consistently estimating *d*, we transform Y_t to $X_t = (x_{1t}, x_{2t}, \dots, x_{Kt})'$ by applying the relationship:

$$x_{it} = \begin{cases} (1-L)^{d_i} y_{it} & \text{if } d_i < \dfrac{1}{2} \\ (1-L)^{d_i - 1}(1-L) y_{it} & \text{if } \dfrac{1}{2} < d_i < \dfrac{3}{2} \end{cases} \tag{4.8}$$

Later, we apply OLS equation-by equation to estimate the following unrestricted Vector Autoregressive (VAR) model:

$$A(L)X_t = \varepsilon_t \tag{4.9}$$

So, model specification (4.9) is stationary if all the roots of the estimated polynomial $|A(z)| = 0$ are outside the unit circle. We determine the orders of the lag polynomials (p) in $A(L)$ based on the lowest AIC. In addition, the correlograms of the residuals are also investigated to ensure each of their elements mimics the white noise process.

4.4.3 Generalized impulse response function and its asymptotic theory

One of the most prevalent tools used to investigate the interdependence between variables in a system is the impulse response function (IRF). The function illustrates how a variable responds to a shock in itself or other variables[27]. Hence, this illustration reveals information regarding feedback relationships between variables under investigation. Generalized IRF (GIRF) and its asymptotic distribution within a FIVAR model developed in Chapter 3 enables us to analyse the inter-relationship among and between short memory and long memory series within a single system.

The GIRF and its asymptotic theories within a FIVAR model are detailed in equation (3.15) and Proposition 3.3. However, we provide a summary of results here to ease reading.

The GIRF for FIVAR at the horizon h can be expressed by,

[27] In impulse response analyses, a shock in a variable is usually referred to as a one standard deviation shock.

$$\Theta_h^g = \Phi_h \Sigma_\varepsilon \Xi = \Phi_h B \qquad h = 0, 1, 2, ... \tag{4.10}$$

where Ξ is a diagonal $K \times K$ matrix characterized by the standard deviation of ε_t,

$$\Xi = \begin{bmatrix} \sigma_{11}^{-\frac{1}{2}} & 0 & \dots & 0 \\ 0 & \sigma_{22}^{-\frac{1}{2}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_{KK}^{-\frac{1}{2}} \end{bmatrix}$$

and,

$$\Phi_h = \begin{cases} \sum_{i=0}^{h} \Psi_i^{(d)} \Pi_{h-i} & h = 1, 2, ... \\ \Pi_0 & h = 0 \end{cases}$$

where $\Psi_i^{(d)}$ is the diagonal $K \times K$ matrix with $\psi_i^{(d_j)}$ as the j^{th} diagonal element, and the $K \times K$ matrices Π_i can be computed recursively using the relationship,

$$\Pi_i = \begin{cases} \sum_{j=1}^{i} \Pi_{i-j} A_j & i = 1,2,...p \\ \sum_{j=1}^{p} \Pi_{i-j} A_j & i > p \end{cases}$$

where $\Pi_0 = I_K$.

Hence, the $\{(i, j), i, j = 1,2,..., K\}$ element in the matrix of impulse responses Θ_h^g is interpreted as the response of the *i*th variable to an innovation in the *j*th variable at horizon *h*.

In addition, for statistical inference on the existence of the relationship, we employ the asymptotic theory of the GIRF as summarized below.

Let $\hat{\Theta}_h^g$ denotes the estimator of the true impulse response matrix Θ_h^g, and,

$$H_h := (B' \otimes I_K)\left(\sum_{i=0}^{h-1}\sum_{j=0}^{h-i-1} J(A')^{h-i-1-j} \otimes \Psi_i^{(d)}\Pi_j\right),$$

$$\overline{H}_h := (\Xi^3 \otimes \Phi_h \Sigma_\varepsilon) S_K D_K + (\Xi \otimes \Phi_h) D_K ,$$

where $\otimes$ denotes the Kronecker product, D_K is the $K^2 \times K(K+1)/2$ duplication matrix, S_K is defined as the $K^2 \times K^2$ diagonal-stacking matrix, $S_K = E_K E_K'$. E_K is a $K^2 \times K$ matrix of 0 and 1 represented as,

$$\underset{(K^2 \times K)}{E_K} = \begin{bmatrix} e_1 e_1' \\ e_2 e_2' \\ \vdots \\ e_K e_K' \end{bmatrix},$$

where e_i is the $K\times1$ vector with 1 in the i^{th} element and 0 elsewhere.

Matrix A and J are represented as,

$$\underset{(Kp \times Kp)}{A} := \begin{bmatrix} A_1 & A_2 & \cdots & A_{p-1} & A_p \\ I_K & 0 & \cdots & 0 & 0 \\ 0 & I_K & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_K & 0 \end{bmatrix},$$

$$\underset{(K \times Kp)}{J} := [I_K : 0 : \cdots : 0],$$

Furthermore, we denote,

$$\Sigma_{\hat{\alpha}} = \Gamma_X^{-1} \otimes \Sigma_\varepsilon ,$$

where Γ_X can be obtained from,

$$vec\Gamma_X = (I_{(Kp)^2} - A \otimes A)^{-1} vec\Sigma_U ,$$

$$\underset{(Kp \times Kp)}{\Sigma_U} = \begin{bmatrix} \Sigma_\varepsilon & 0 \\ 0 & 0 \end{bmatrix},$$

and,

$$\Sigma_{\hat{\sigma}} = 2\, D_K^{+} \left(\Sigma_\varepsilon \otimes \Sigma_\varepsilon\right) D_K^{+/},$$

where $D_K^{+} = \left(D_K' D_K\right)^{-1} D_K'$ is the Moore-Penrose inverse of the duplication matrix D_K.

With these notations, the asymptotic distribution of the generalized impulse responses for a FIVAR model can be written as,

$$\sqrt{T} vec(\hat{\Theta}_h^g - \Theta_h^g) \xrightarrow{d} N(0, H_h \Sigma_{\hat{\alpha}} H_h' + \overline{H}_h \Sigma_{\hat{\sigma}} \overline{H}_h') \qquad h = 1,2,.... \qquad (4.11)$$

4.5 Inter-relationship between trading volume and realized higher-moments

4.5.1 Model estimation outputs

For the purpose of analysing the interdependence between trading volume and realized higher-moments as well as the interaction across stock and FX markets, we estimate all the realized measures and trading volumes for both stock and FX markets in one system. Therefore, we have four FIVAR systems (one for each geographical region) with 8 equations (3 higher moments and trading volume for stock and FX markets). For all systems, we choose the band parameter $m = T^{0.65}$ as suggested by Shimotsu (2007) through the simulation experiments. We report the estimated degree of fractional integration and its associated *z*-statistics as well as the optimum lag lenghts (*p*) in Table 4.4.

The estimated values of memory parameters are generally consistent with information extracted from the *Q*(20) statistics in our preliminary analysis, which indicates that the realized volatility, kurtosis and trading volume series all strongly exhibit long memory behaviour. Furthermore, realized skewness is mostly a short-memory series. Among the long-memory measures, realized kurtosis has the lowest degree of fractional integration. Higher values of memory parameters for realized volatility and trading volume may imply a higher degree of predictability than for realized kurtosis or skewness. This is due to greater persistence in realized volatility and trading volume. In addition, we observe higher degrees of fractional integration for all long memory measures during the Volatile period than in the Stable period for stock markets. Hence, these measures are more serially correlated during the volatile period than in the tranquil period.

The optimal lag lengths identified are reasonably small, which may indicate that the long memory behaviour is well captured for Y_t and the filtered series in X_t are, therefore, free from long range dependence problems. Hence, the VAR specification used for X_t in Equation (4.9) is correctly specified. We confirm this implication by inspecting the sample autocorrelation of X_t (not shown) and see that the autocorrelation dies out quickly and then fluctuates around zero, an indication of short memory processes. In addition, many of the estimated FIVAR coefficients are statistically significant and all inverse roots of the estimate of the lag polynomial matrices in $A(L)$ are inside the unit circle, an indication of covariance stationarity[28]. Therefore, we can employ the GIFR to capture dynamic linkages within the FIVAR system.

We define the generalized impulse response as the spill-over effect (i.e., an exogenous shock in the *i*th variable at the current time, which causes a significant

[28] To conserve space, we do not report the estimated coefficients in the lag polynomial $A(L)$ and their inverse roots. However, the full set of results is available upon request.

change of the *j*th variable in next periods). Hence, we summarize the spill-over effects from realized volatility, skewness, kurtosis and trading volume to other variables in Tables 4.5, 4.6, 4.7 and 4.8, respectively. In our study, we focus on the existence, the sign and the direction of the spill-over in explaining the properties of the interdependence among and between realized measures and trading volume. The existence and the sign of the spill-over effects are inferred from whether the impulse responses are significantly greater or smaller than zero at the 5% significance level. Further, as mentioned earlier, we also conduct sub-sample analyses to analyse the differences between stable and volatile periods in financial markets.

4.5.2 Inter-relationships between trading volume and higher moments

The Volume – volatility inter-relationship

Tables 4.5 and 4.8 generally document a positive relationship between trading volume and realized volatility. In terms of the relationship within the same asset markets, we find a bidirectional spill-over effect between the two measures in all cases. Besides, across asset markets, a bidirectional relationship is also found between trading volume in FX markets and realized volatility in stock markets in nearly all cases[29]. However, we uncover a unidirectional spill-over from trading volume in stock markets to realized volatility in FX markets during the volatile period. In fact, the bidirectional relationship between the two measures within the same asset markets has been shown in the literature (e.g., Mougoué and Aggarwal, 2011). Furthermore, the positive volume-volatility relationship is also widely supported in previous empirical studies conducted within stock and FX markets (see Melvin and Yin, 2000; Bauwens et al.,

[29] The only exception is in the Western European region, which shows a unidirectional spill-over from trading volume in FX markets to stock market realized volatility.

2005; Bjønnes et al., 2005, for examples of FX markets and Kalev et al., 2004; Chan and Fong, 2006, for examples of stock markets). However, no study has addressed the volume-volatility relationship between stock and FX markets which is important for better understanding financial market linkages.

Our findings about the volume-volatility relationship can be explained by the MDH, which predicts that volume and volatility should be positive correlated since they are characterized by the same latent information flows. This explanation is in line with the theory of heterogeneity of beliefs among investors, which shows that new information arrivals in the market may lead to different interpretations between different types of traders. Therefore, traders experience different expectations regarding the fundamental values of assets, which subsequently results in greater variability in price changes (see Shalen, 1993). Furthermore, our results, drawn from an impulse response analysis, imply lead-lag relations between trading volume and realized volatility, which is also consistent with the SAIH. We support the view of Chen and Daigler (2008), who consider the SAIH as being a complementary explanation for the volume-volatility relationship. Information flows may come in sequence to different traders at different times.

The Volume – skewness inter-relationship

Empirical results shown in Tables 4.6 and 4.8 indicates that trading volume has no effect on realized skewness, thus providing a lack of support for the conclusion of Hong and Stein (2003) at the regional level. The only exception that we observe is the case of the Asia Pacific Emerging region during the volatile period, in which the trading volume of stock markets has a positive impact on the realized skewness of FX markets[30]. The investor heterogeneity theory of Hong and Stein (2003), used in explaining the positive impact of volume on negative skewness (i.e., higher trading volume may lead to more negative skewness of returns), is strongly supported in empirical studies using firm-level data (e.g., Chen et al., 2001). However, when the market-level data are employed, the relationship tends to disappear (e.g., Chen et al., 2001; Hueng and McDonald, 2005). More recently, Hutson et al. (2008), in using national stock market indices, provides some empirical evidence on the theory. However, the direct effect of volume on negative skewness only exists in 3 out of 11 cases, implying a weak support for the theory at the national level. Therefore, in conjunction with our findings at the regional level, we hypothesize that the direct influence of trading volume on negative skewness is less significant for a portfolio that is more diversified, conditional on the same market conditions. In order to give some intuition for this hypothesis, we examine the consequences of differences of expectation among investors; say, investor *A* and investor *B* (see Hong and Stein, 2003). Assume that both investors have their own private information, where investor *B* gets more negative signals, so that his expectation about the asset's price is lower than *A*'s. Due to the short-sales constraint, investor *B* will sell all of his assets and sit out of the market. Hence, there is only trade between investor *A* and the arbitrageurs, that leads to the asset price at this time only reflecting the information of investor *A* but not investor *B*. When some of the previously hidden signals of *B* are revealed in the market, the asset price will drop as investor *A* wants to get out of the market at the price matching with what the arbitrageurs learn from when investor *B* gets into the market. Hence, the more pessimistic information of *B* is released, the more an asset's price will

[30] In this specific case, our result is consistent with the theory of Hong and Stein (2003) since we emphasize on the downside risk by utilizing Equation (4.3) to calculate realized skewness.

drop, which leads to higher negative skewness of the return. Therefore, the higher the degree of differences in investors' opinions is, the level of negative skewness will be higher, conditional on high trading volume and vice versa. Intuitively, if a portfolio is better diversified, we should expect a lower degree of differences in investors' valuations of the portfolio's price. Accordingly, the impact of trading volume on the level of negative skewness should be less significant for a better diversified portfolio.

Regarding the opposite direction of the volume-skewness relationship, we find some (but not strong) evidence of the spill-over effect from realized skewness to trading volume during the volatile period. Specifically, realized skewness has a positive impact on trading volume in terms of both within the same and cross-asset markets[31]. One possible explanation is that during the volatile periods, risk-averse investors tend to be more sensitive and panic in response to market downturns, which leads them to evaluate asset prices well below fundamental values. Therefore, they hope to get out of the market before the market gets worse. However, the risk-neutral arbitrageurs are not that pessimistic and are willing to buy at the price at which risk-averse investors want to step out. Therefore, the market experiences an increase in trading volume.

The Volume – kurtosis inter-relationship

Empirical results presented in Tables 4.7 and 4.8 show some evidence of the inter-relationship between trading volume and realized kurtosis. We find a negatively

[31] We find a positive spill-over from realized skewness of FX markets to trading volume of FX markets in cases of Asia Pacific Emerging and Developed regions; and to stock market trading volume in cases of the Asia Pacific Emerging region. Further, similar effects are also observed between realized skewness of stock markets and trading volume of FX markets in cases of Latin American and Asia Pacific Developed regions; and between realized skewness and trading volume within stock markets in the Asia Pacific Developed region.

bidirectional spill-over effect between the two measures within the FX market during both stable and volatile periods in cases of emerging regions. Furthermore, within stock markets, a negatively unidirectional spill-over effect from trading volume to realized kurtosis is observed in Latin American and Western European regions during the volatile period. However, we do not find significant evidence of the cross-asset market relationship between trading volume and realized kurtosis. The negativity of the inter-relationship between the two measures may also be due to the heterogeneity of beliefs among traders, which is used to explain volume-volatility relationships. When a new information flow (e.g., macroeconomic announcements) arrives in the market, different types of traders with their private signals may have different interpretations of the same information. Therefore, dispersion of beliefs among traders appears and leads to different valuations for an asset's price. The more uninformed (noise) traders are present in the market, the higher degree of dispersion of beliefs among traders. Higher dispersion of beliefs, in turn, leads to a lower degree of concentration of price changes around its average value, which is revealed as a decrease in the kurtosis of the return's distribution. Hence, the negative inter-relationship between trading volume and realized kurtosis is, in fact, consistent with the heterogeneity of investors' beliefs in the literature.

4.5.3 Interactions among realized higher moments

Empirical results presented in Table 4.6 consistently show no support for the spill-over effect from realized volatility to realized skewness in all cases. However, regarding the opposite direction of the spill-over effect as shown in Table 4.6, we find some distinctive results between different types of markets as well as for different periods. The spill-over from realized skewness of FX markets to realized volatility (of both stock and FX markets) tends to be negative during the volatile period but insignificant in the stable period. Meanwhile, the realized volatilities of both stock and FX markets respond positively to an innovation in stock market realized skewness. However, the spill-over from stock market realized skewness to FX market realized volatility is only significant during the volatile period; whereas, we observe the unidirectional spill-over from realized skewness to realized volatility in all cases within stock markets.

As can be seen from Tables 4.5 and 4.7, there is strong evidence of a positively bidirectional spill-over effect between realized volatility and realized kurtosis during both tranquil and volatile periods for all regions. However, we only observe this relationship within stock or FX markets but not across asset markets. Hence, the finding indicates that, the volatility risk and fat-tail risk are more likely to interact with each other within the same asset markets. Furthermore, since the interaction is positive, it implies that an innovation in the return's volatility will contribute an increase to the likelihood of extreme events in subsequent periods. Conversely, if there is a shock to the occurrence of extreme events, we should expect a rise in the dispersion of returns.

In contrast with the relationship between realized volatility and realized kurtosis, the results shown in Tables 4.6 and 4.7 do not support the linkages between the 3rd moment (skewness) and the 4th moment (kurtosis). In nearly all cases there is no spill-over effect between realized skewness and realized kurtosis, implying that downside risks and the fat-tail risks are generally not related to each other. However, we observe some exceptions, which are only found in emerging regions during the stable period. We find that there is a positively bidirectional spill-over effect between realized skewness and realized kurtosis within FX markets of the Asia Pacific Emerging region.

Further, a positively unidirectional spill-over from realized skewness of stock markets to realized kurtosis of FX markets exists in the Latin American region.

4.6 Volume impacts on the inter-relationship among higher moments

The evidence of interactions between higher moments motivates the necessity to investigate volume impacts on not only each of the higher moments separately but also the inter-relationship among them. Additionally, based on some recent evidence of time-varying volatility spill-over effects (e.g., Diebold and Yimaz, 2009; Bubák et al., 2011), we are interested in analysing the issue dynamically. Due to both the recent turbulence and evolution of financial markets, a static analysis may only capture the 'average' properties of the inter-relationship for the full-sample but not the behaviour over time. Therefore, it is likely to overlook dynamics of the interactions that are possibly associated with some significant events.

4.6.1 Methodology

The models

In order to model the dynamic influence of trading volume on the inter-relationship among higher moments, we compare the strength of the inter-relationship without and after controlling for the effects of volume. With regards to the case when the volume impact is not controlled for, we employ a FIVAR model as discussed earlier where all realized measures are endogenous variables. To control for the volume impacts, we consider all realized measures and trading volumes as endogenous and exogenous variables in a FIVARX framework, respectively. The specification of a FIVARX model can be represented as follows,

$$A(L)D(L)Y_t = \nabla D_v(L)V_t + \varepsilon_t, \qquad t = 1,2,...,T. \tag{4.12}$$

where ∇ is the $K \times 2$ matrix of coefficients; $D_v(L) = \text{diag}\{(1-L)^{d_{v1}}, (1-L)^{d_{v2}}\}$ and $V_t = (V_{1t}, V_{2t})'$. V_{1t} (V_{2t}) and d_{v1} (d_{v2}) are stock (FX) trading volume and its degree of fractional integration, respectively[32].

Spill-over index

For the purpose of a dynamic analysis, we construct the time-varying spill-over index of the inter-relationship among higher moments as a proxy of its strength. This measure is motivated by some recent studies (e.g., Diebold and Yilmaz, 2009, and Bubák et al., 2011). In these studies, the evolution of volatility spill-over is investigated using the spill-over index, which measures the proportion of the *h* horizontal forecast error of a variable's volatility that can be assigned to innovations in other variables within a VAR framework. Accordingly, this idea, in fact, can be applied to construct not only the volatility spill-over index but also the index for other types of interdependence. However, a drawback of the method proposed in Diebold and Yilmaz (2009) is the requirement to determine the contemporaneous relationship between variables of a system in the first stage. This method, therefore, may face some difficulties in cases of a high dimensional system, where there is no clear economic guidance to order the direction of the contemporaneous relationships between endogenous variables. Hence, we incorporate the method proposed in Diebold and

[32] Since V_t is no longer an endogenous variable in the system, we employ an univariate framework in Shimotsu et al. (2005) to estimate its degree of fractional integration (d_{v1} and d_{v2}). This method can be considered as a special case of a multivariate estimation presented in Shimotsu (2007), which is outlined in section 3. We, therefore, omit a description of the univariate estimation of a fractional degree here.

Yilmaz (2011) in a FIVAR model to investigate these issues. Originally, Diebold and Yilmaz (2011) derived the formula based on the generalized variance decomposition of a VAR process. However, we find that it is in fact straightforward to apply this technique in a FIVAR(X) model when the generalized variance decomposition of a FIVAR(X) model is available.

Similar to the idea of a generalized variance decomposition of a VAR model (see Pesaran and Shin, 1998), we can easily obtain the $\{(i, j), i, j = 1, 2, ..., K\}$ element in the matrix of the *h* step-ahead variance decomposition of a FIVAR(X) process using the generalized approach as follows[33],

$$\theta_{ij}^{g}(h) = \frac{\sigma_{jj}^{-1} \sum_{l=0}^{h-1} \left(e_i' \Phi_l \Sigma_\varepsilon e_j \right)^2}{\sum_{l=0}^{h-1} \left(e_i' \Phi_l \Sigma_\varepsilon \Phi_l' e_i \right)} \tag{4.13}$$

This variance decomposition matrix can be subsequently used to derive the spill-over indices as presented in Diebold and Yilmaz (2011). The total spill-overs index is computed as,

$$S^{g}(h) = \frac{\sum_{\substack{i,j=1 \\ i \neq j}}^{K} \tilde{\theta}_{ij}^{g}(h)}{K} \times 100 \tag{4.14}$$

[33] We omit the derivation of the variance decomposition for FIVAR(X) since it is similar to what has been shown in Pesaran and Pesaran (2009, section 22.6.2) with a note that, for a FIVAR process Y_t has a moving average representation as, $Y_t = \sum_{i=0}^{\infty} \Phi_i \varepsilon_{t-i}$, (see Chapter 3, section 3.3.1).

where $\tilde{\theta}_{ij}^{g}(h)$ is the (i, j) element of the variance decomposition matrix normalized by the row sum,

$$\tilde{\theta}_{ij}^{g}(h) = \frac{\theta_{ij}^{g}(h)}{\sum_{j=1}^{K} \theta_{ij}^{g}(h)}$$

The total spill-overs index evaluates the contribution of all spill-over effects from the innovations across all variables to the total forecast error variance. Therefore, this index can help us to explain the time-varying behaviour of the interdependence among all realized higher-moments. However, since this index cannot separately identify the contribution of spill-overs from shocks in each variable, we also calculate the directional spill-overs index to investigate the contribution of each of the realized higher-moments to the total degree of the inter-relationship among them. The directional spill-overs from variable i to all other variables in the system can be estimated as,

$$S_{-i}^{g}(h) = \frac{\sum_{\substack{j=1 \\ j \neq i}}^{K} \tilde{\theta}_{ji}^{g}(h)}{K} \times 100 \tag{4.15}$$

4.6.2 Empirical results

We construct dynamic spill-over indices for total and directional effects from 1/1/2004 to 15/2/2010 by utilizing the 520-day rolling sample with a 1 step-ahead forecast horizon in a FIVAR(X) model[34].

[34] The choice of window size as 520 (approximately equals to 2 years) is for consistency with the Shimotsu (2007) in estimating the FIVAR model. Given that the choice of window size is somewhat

Time-varying interdependence among realized higher-moments

We first investigate the time-varying behaviour of the interdependence among higher-moment risks by constructing the total spill-over index in a FIVAR system, which includes all realized measures of both stock and FX markets but not the trading volume. The index, a proxy for the degree of the inter-relationship among all realized measures, is graphed in Figure 4.2. As can be seen, the degree of the inter-relationship is clearly changing over time. On average, we observe a higher degree of interdependence in developed regions (ranging from 20-28%) in comparison with emerging regions (varying from 16-24%). Furthermore, we find remarkable movements and radically different properties of the inter-relationship, corresponding to significant economic events[35].

We often find a period with a higher degree of the interaction among higher-moment risks within the region when countries of the region tend to be more integrated. The higher degree of integration between countries may be because of new policies, agreements that enhance the incorporation between national economies and the regional or international economies (e.g., the European Union enlargement plan 2004-2007, Letter of exchange establishing the Japan-ASEAN integration fund in March 2006 and the 2nd ASEAN integration work plan 2009-2015); it can be also due to the highly volatile periods (e.g., the U.S sub-prime mortgage crisis in 2007, Global financial crisis in 2008 and the uncertainty surrounding the onset of the European Sovereign Debt

arbitrary, we also construct the spill-over indices with other window sizes. However, we find that the results are similar to our original choice.

[35] We provide details of the economic events in accordance with periods of high and low degree of the inter-relationship in Figure 4.2.

crisis around the end of 2009). Furthermore, we also find evidence of a sudden increase in the degree of the interaction among higher-moment risks associated with an arrival of pessimistic information in the market (e.g., IMF warnings about the Australian banking system in late 2006).

In the converse situation, a decrease in the degree of the interdependence among higher-moment risks usually starts with events, which lead to a higher degree of an economy's transparency (e.g., database of ASEAN non-tariff measures regularly updated from 2007); or a lower degree of integration between economies (e.g., tight monetary policy of Brazil in Sep 2004, establishment of South America Community of Nations in December 2004 which limits the influence of the U.S on the Latin American region).

To address the contribution of each of higher-moment risks to the total degree of the interdependence among them, we decompose the total spill-over index to the directional spill-over indices, which are plotted in Figures 4.3-4.6. The directional spill-over plot tells us how much (%) of a shock in a realized measure contributes to the forecast error of the whole system. Roughly speaking, we can interpret it as the spill-over from one realized measure to all other measures in the system. As can be seen, the realized volatility and kurtosis of FX markets have contributed the largest spill-over effect to the total degree of the interdependence in developed regions (staying around 5-8%); whereas, the realized skewness of stock and FX markets are the lowest contributors among all (varying from 0.5-3%). Regarding the emerging regions, we find that the realized kurtosis of FX markets tends to have the largest impact on the total spill-over of the system, which varies around 5-7%. However, it is difficult to distinguish between contributions of other realized measures as they are close to each other. Another critical point that we observe is the consistent behaviour of realized skewness of both stock and FX markets in Asia Pacific Developed and Latin American regions. In these regions, the downside risks are likely to have more impact on the total spill-over effect during the high volatility periods. This finding is consistent with our expectation as investors tend to be more sensitive and panic when an extreme loss occurs during the financial turmoil.

Volume impacts

After controlling for the influence of trading volume, the main findings about the dynamic structure of the total spill-over index (e.g., an association with events and new policies) as well as the level of contribution of each realized measure are basically consistent with what has already been discussed in the previous sub-section. Apart from those, Figure 4.2 clearly reports that trading volume has an impact on the strength of the inter-relationship among higher moments.

In particular, trading volume decreases the total spill-over indices of Asia Pacific Developed, Western European Developed and Asia Pacific Emerging region in most times during the analysed period. More specifically, we observe from Figures 4.3-4.5 that this difference is mainly due to a decline in the proportion (%) of spill-over effects from realized kurtosis to other moments. Equivalently, this means that trading volume increases the proportion (%) of spill-over effects from realized kurtosis to itself in future periods[36]. Since realized kurtosis measures the occurrence of extreme returns

[36] The spill-over from one variable is built on two components: (1) spill-over to all other variables in the system and (2) spill-over to itself. The first (1) component is the proportion (%) that a shock in the variable contributes to the forecast error of all others; whereas, the second (2) component is the proportion (%) that a shock in the variable contributes to the forecast error of itself.

(fat-tailedness), such increases may cause more clusters of return volatility, which last for longer periods of time. Therefore, our finding can be fundamentally explained by a prominent result found in the trading volume – GARCH effect literature (initially reported in Lamoureux and Lastrapes, 1990), which indicates trading volume is a source of heteroskedasticity (volatility clustering).

An exceptional case is the Latin American region, where we observe the inter-relationship among higher moments to increase with shocks to trading volume. Even though the behaviour of realized kurtosis under the volume impact is consistent with the above cases, significant elevations in spill-over effects from realized volatilities of both stock and FX market to other higher moments lead to phenomenon major difference between Latin America and other regions.

4.7 Conclusion

This chapter comprehensively examines the effects of trading volume on financial return distributions in a regional context. We assess not only how trading volume affects each higher moments but also how volume impacts on their dynamic inter-relationship. We shed new light on the volume – skewness relationship with a regional portfolio analysis based on high-frequency data. The use of high-frequency data provides us with more robust estimates and treats higher moment return measures as observable variables, which can be appropriately modelled in a FIVAR(X) framework.

Empirical findings in our volume – volatility analysis provide support for current information based theories. Hence, we support Chen and Daigler (2008), who interpret these theories as complementary hypotheses rather than treating them as opponents. Further, we add to the literature on volume – volatility relations by also providing evidence of their positive and lead-lag relationship across stock and FX markets.

Regarding the volume – skewness interaction, the lack of support of Hong and Stein's (2003) conclusion in our regional-level analyses leads us to hypothesize that the direct impact of trading volume on the level of negative skewness is less significant for a better diversified portfolio. This hypothesis, however, may be explained by extending the theory of DOH used in Hong and Stein (2003) with an intuitive expectation that a better diversified portfolio should generate a lower degree of the difference between investors' opinions about its fundamental value. Although this expectation has not been tested in this paper, it suggests an exciting future research direction to extend the theory of Hong and Stein (2003). Additionally, in terms of volume – kurtosis relations, we find evidence of a negatively bidirectional interdependence within the FX markets but unidirectional spill-over from trading volume to kurtosis within stock markets. We suppose that the negativity of the interaction between trading volume and kurtosis may imply an application of the DOH, where higher dispersion of beliefs among traders leads to lower concentration of asset returns around its mean value.

Lastly, we investigate the impact of trading volume on the dynamic linkages between higher moments by using a spill-over index. We find clear evidence that the strength of the linkages between higher moments is affected by trading volume. The level of the inter-relationship in Asia Pacific Developed, Western European Developed and Asia Pacific Emerging region decreases with shocks to trading volume. This is mainly due to a decline in the proportion (%) of spill-over from realized kurtosis to other moments; or equivalently, an increase in the proportion (%) of spill-over from realized kurtosis to itself in next periods. This has policy implications for financial market regulations (like the imposition of short-selling bans) that affect trading volume and in turn, financial return distributions and risks.

4.8 APPENDIX

Table 4.1: Regions, country weights and GMT trading time

Regions	Countries	Average GDP [1]	Weight Value	Trading time (GMT) [2]	
				Standard time	DST
Latin America	Argentina	295	0.14	14:00-21:00	-
	Brazil	1,558	0.73	13:00-20:00	12:00-19:00
	Chile	169	0.08	13:30-21:00	12:30-20:00
	Peru	121	0.06	13:30-21:00	-
Asian Pacific Emerging	Indonesia	511	0.33	2:30-9:00	-
	Malaysia	199	0.13	1:00-9:00	-
	Philippines	163	0.11	1:30-4:00	-
	Taiwan	395	0.26	1:00-5:30	-
	Thailand	262	0.17	3:00-9:30	-
Asian Pacific Developed	Australia	962	0.14	0:00-6:00	23:00 - 5:00
	Hong Kong	209	0.03	2:00-8:00	-
	Japan	4,830	0.68	0:00-6:00	-
	Korea	956	0.13	0:00-6:00	-
	New Zealand	126	0.02	22:00-4:00	-
Western Europe	Austria	373	0.06	8:30-16:30	7:30-15:30
	France	2,571	0.38	8:00-16:30	7:00-15:30
	Germany	3,304	0.49	8:00-16:30	7:00-15:30
	Switzerland	469	0.07	8:00-16:30	7:00-15:30

Note: (1) The average GDP of each country is computed by using its GDP (in billion USD) from 2006 to 2010. We download most of the GDP data from the World Bank, except for Taiwan which we sourced from the Australian Government's – Department of Foreign Affairs and Trade.

(2) We convert the trading times of each stock market to GMT time. In addition, DST denotes the Daylight Saving Time.

Table 4.2: Descriptive Statistics for the Realized measures and trading volume of Foreign Exchange markets

	Stable period					Volatile period				
	Mean	Std. Dev	Skewness	Kurtosis	Q-stat(20)	Mean	Std. Dev	Skewness	Kurtosis	Q-stat(20)
Panel A: Descriptive statistics for the Realized Volatility estimates										
Latin America (‡, ‡‡)	-10.00	1.05	-0.45	6.97	5168.4***	-9.64	1.23	-1.69	13.85	1640.5***
Asian Pacific Emerging (‡, ‡‡)	-11.89	1.69	1.46	5.55	13547***	-10.54	2.27	0.77	2.24	7886.5***
Asian Pacific Developed (‡, ‡‡)	-11.36	0.84	0.29	4.08	674.4***	-10.80	0.93	0.36	3.40	836.5***
Western Europe (‡, ‡‡)	-10.60	0.73	0.04	3.80	2852.8***	-10.27	0.93	-0.06	3.78	3770.3***
Panel B: Descriptive statistics for the Realized Skewness estimates										
Latin America	0.09	1.57	-0.13	7.19	31.2*	0.16	2.15	0.03	4.89	21.69
Asian Pacific Emerging	0.24	1.96	0.54	5.96	36.1**	-0.09	2.25	0.73	6.41	21.59
Asian Pacific Developed	-0.18	4.34	-0.02	2.90	32.1**	-0.13	4.30	-0.08	2.85	17.94
Western Europe	0.27	2.81	0.15	4.07	18.4	0.11	3.07	0.08	3.34	31.1*
Panel C: Descriptive statistics for the Realized Kurtosis estimates										
Latin America (‡, ‡‡)	-2.57	0.73	0.94	3.37	154.7***	-2.46	0.84	0.64	2.56	52.6***
Asian Pacific Emerging (‡)	-2.27	0.76	0.42	2.69	457.64***	-1.97	0.70	0.42	2.67	218.4***
Asian Pacific Developed	-1.98	1.11	0.11	1.58	42.4***	-1.98	1.12	0.04	1.57	107.9***
Western Europe	-2.22	0.90	0.48	2.21	22.7	-2.15	0.96	0.28	1.85	22.00
Panel D: Descriptive statistics for the Trading Volume estimates										
Latin America (‡‡)	6.58	1.00	-2.70	13.64	991.6***	7.69	1.17	-3.33	16.83	215.7***
Asian Pacific Emerging (‡, ‡‡)	4.53	1.12	0.28	1.99	19716***	5.54	0.70	-0.84	5.92	2444.6***
Asian Pacific Developed (‡, ‡‡)	8.54	0.96	-1.02	6.46	18076***	9.65	0.57	-9.14	113.30	182.36***
Western Europe (‡, ‡‡)	9.96	0.72	-4.36	28.37	9627.2***	10.31	0.44	-11.81	196.91	316.6***

Note: Q-stat(20) denotes the Ljung-Box statistics for up to twentieth order serial correlation. *****, ****** and ******* denote the serial correlation up to lag 20 is significant at 10%, 5% and 1% significance level, respectively. Further, **‡** and **‡‡** indicate the linear trend and quadratic trend are significant at 5% significance level, respectively.

Table 4.3: Descriptive Statistics for the Realized measures and trading volume of stock markets

	Stable period					Volatile period				
	Mean	Std. Dev	Skewness	Kurtosis	Q-stat(20)	Mean	Std. Dev	Skewness	Kurtosis	Q-stat(20)
Panel A: Descriptive statistics for the Realized Volatility estimates										
Latin America (‡, ‡‡)	-9.46	0.99	-2.53	14.17	545***	-8.94	1.42	-1.92	10.47	934.8***
Asian Pacific Emerging (‡, ‡‡)	-10.75	0.76	0.16	4.92	2729.6***	-10.09	0.98	0.40	3.97	1678.6***
Asian Pacific Developed (‡‡)	-10.98	1.15	-2.00	18.34	1231.3***	-10.39	1.02	-0.45	4.25	1055.6***
Western Europe (‡, ‡‡)	-9.58	1.12	0.24	3.17	10338***	-8.75	1.04	-0.17	4.31	3465.5***
Panel B: Descriptive statistics for the Realized Skewness estimates										
Latin America	-0.16	1.81	-0.16	5.13	29.8*	-0.21	2.37	-0.05	3.59	21.70
Asian Pacific Emerging	-0.52	2.36	0.34	3.99	39.1***	-0.31	2.83	0.27	2.73	19.80
Asian Pacific Developed	-0.45	3.28	0.05	2.82	22.40	-0.01	3.11	0.07	2.94	26.80
Western Europe (‡, ‡‡)	-0.38	3.13	-0.06	3.45	15.40	-0.24	3.25	-0.16	2.98	13.80
Panel C: Descriptive statistics for the Realized Kurtosis estimates										
Latin America	-2.53	0.61	1.19	4.21	52.6***	-2.30	0.73	0.52	2.34	14.12
Asian Pacific Emerging	-2.25	0.69	0.57	2.70	64.3***	-2.11	0.72	0.22	2.06	45.1***
Asian Pacific Developed	-1.98	0.81	0.30	1.98	51.9***	-1.98	0.72	0.37	2.30	32.2**
Western Europe (‡, ‡‡)	-2.10	0.92	0.37	1.95	449.3***	-2.01	0.92	0.13	1.82	43.2***
Panel D: Descriptive statistics for the Trading Volume estimates										
Latin America (‡, ‡‡)	6.61	0.26	-4.27	100.21	11411***	7.08	0.08	-4.16	28.36	274.9***
Asian Pacific Emerging (‡, ‡‡)	5.95	0.23	-1.60	5.66	23639***	6.54	0.58	0.58	1.43	12834***
Asian Pacific Developed (‡, ‡‡)	5.73	0.27	-1.72	8.31	14666***	5.75	0.16	-0.51	3.44	4303.9***
Western Europe (‡, ‡‡)	8.27	0.81	0.28	4.87	23059***	9.66	0.13	-0.04	3.08	6835.1***

Note: Q-stat(20) denotes the Ljung-Box statistics for up to twentieth order serial correlation; *****, ****** and ******* denote the serial correlation up to lag 20 is significant at 10%, 5% and 1% significance level, respectively. Further, ‡ and ‡‡ indicate the linear trend and quadratic trend are significant at 5% significance level, respectively.

Table 4.4: Multivariate degree of fractional integration and optimal lag order

	Realized Volatility		Realized Skewness		Realized Kurtosis		Volume		Optimal lag	
Periods	Stable	Volatile	Stable	Volatile	Stable	Volatile	Stable	Volatile	Stable	Volatile
Panel A: Multivariate degree of fractional integration in Foreign Exchange markets										
Latin America	0.45***	0.41***	0.01	0.01	0.26***	0.23***	0.38***	0.43***	2	2
	(14.71)	(11.16)	(0.22)	(0.29)	(7.64)	(5.54)	(12.91)	(10.72)		
Asian Pacific Emerging	0.59***	0.67***	0.15***	0.14***	0.32***	0.36***	0.51***	0.48***	2	1
	(17.61)	(14.41)	(3.86)	(3.08)	(7.67)	(7.98)	(14.88)	(10.70)		
Asian Pacific Developed	0.35***	0.40***	-0.01	0.11**	0.28***	0.20***	0.45***	0.50***	3	2
	(11.31)	(11.92)	(-0.30)	(2.21)	(9.08)	(5.68)	(14.09)	(13.81)		
Western Europe	0.41***	0.52***	0.00	0.05	0.14***	0.22***	0.66***	0.54***	6	2
	(13.84)	(15.01)	(-0.02)	(0.94)	(4.32)	(4.67)	(17.73)	(14.49)		
Panel B: Multivariate degree of fractional integration in stock markets										
Latin America	0.36***	0.45***	-0.08**	-0.05	0.14***	0.15***	0.49***	0.50***	2	2
	(13.47)	(10.67)	(-2.05)	(-1.33)	(3.34)	(3.25)	(14.15)	(9.59)		
Asian Pacific Emerging	0.34***	0.54***	0.07*	0.08*	0.18***	0.27***	0.76***	0.91***	2	1
	(11.02)	(13.57)	(1.79)	(1.87)	(5.57)	(5.70)	(18.24)	(19.90)		
Asian Pacific Developed	0.38***	0.51***	0.08*	-0.01	0.19***	0.23***	0.48***	0.68***	3	2
	(10.92)	(14.81)	(1.97)	(-0.29)	(4.76)	(4.82)	(13.12)	(16.91)		
Western Europe	0.59***	0.67***	0.00	0.05	0.30***	0.20***	0.63***	0.75***	6	2
	(17.99)	(17.74)	(0.10)	(1.32)	(7.13)	(4.57)	(18.17)	(16.24)		

Note: *****, ****** and ******* denote that degree of fractional integration is significant at 10%, 5% and 1% significance level, respectively. The *z*-statistics are reported in parentheses. Note that we include both stock and FX markets of a region in one FIVAR system so the optimal lag reported for Panel A and B are the same.

Table 4.5: Spill-over from Realized Volatility to other realized measures and Trading Volume

	Foreign Exchange markets						Stock markets					
	RS		RK		Volume		RS		RK		Volume	
	S	**V**	**S**	**V**	**S**	**V**	**S**	**V**	**S**	**V**	**S**	**V**
Panel A: Spill-over from the Realized Volatility of Foreign Exchange market												
Latin America	●	●	⊕	●	⊕	⊕	●	●	●	●	●	●
Asian Pacific Emerging	●	●	⊕	⊕	⊕	⊕	●	●	●	●	●	●
Asian Pacific Developed	●	●	⊕	⊕	⊕	⊕	●	●	●	⊕	⊕	⊕
West Europe Developed	●	●	⊕	⊕	⊕	⊕	●	●	●	●	●	●
Panel B: Spill-over from the Realized Volatility of Stock market												
Latin America	●	●	ϴ	ϴ	⊕	⊕	●	●	●	●	⊕	⊕
Asian Pacific Emerging	●	●	●	⊕	⊕	⊕	●	●	⊕	⊕	●	⊕
Asian Pacific Developed	●	●	●	●	⊕	⊕	●	●	⊕	⊕	⊕	⊕
West Europe Developed	●	●	●	●	⊕	⊕	●	●	⊕	⊕	⊕	⊕

Notes: **RV**, **RS**, **RK** and **Volume** denote realized volatility, realized skewness, realized kurtosis and trading volume, respectively. **S** and **V** denotes the Stable and Volatile period, respectively. ⊕ denotes the spill-over is positively significant. ϴ denotes the spill-over is negatively significant. ● denotes the spill-over is insignificant. We make conclusion about the significance of spill-over effect using the asymptotic 95% confidence interval of the generalized impulse response in FIVAR derived in Chapter 3, (see Equation (4.11)).

Table 4.6: Spill-over from Realized Skewness to other realized measures and Trading Volume

	Foreign Exchange markets						Stock markets					
	RV		RK		Volume		RV		RK		Volume	
	S	**V**	**S**	**V**	**S**	**V**	**S**	**V**	**S**	**V**	**S**	**V**
Panel A: Spill-over from the Realized Skewness of Foreign Exchange market												
Latin America	ϴ	ϴ	●	●	●	●	ϴ	ϴ	●	●	●	●
Asian Pacific Emerging	●	●	⊕	●	●	⊕	●	ϴ	●	●	●	⊕
Asian Pacific Developed	●	⊕	●	●	●	⊕	●	⊕	●	●	●	●
West Europe Developed	●	ϴ	●	●	●	●	●	ϴ	●	●	●	●
Panel B: Spill-over from the Realized Skewness of Stock market												
Latin America	⊕	⊕	⊕	●	●	⊕	⊕	⊕	●	●	●	●
Asian Pacific Emerging	●	●	●	●	●	ϴ	⊕	⊕	●	●	●	●
Asian Pacific Developed	●	⊕	●	●	⊕	⊕	⊕	⊕	●	●	●	⊕
West Europe Developed	●	⊕	●	●	●	●	⊕	⊕	●	●	●	●

Notes: **RV**, **RS**, **RK** and **Volume** denote realized volatility, realized skewness, realized kurtosis and trading volume, respectively. **S** and **V** denotes the Stable and Volatile period, respectively. ⊕ denotes the spill-over is positively significant. ϴ denotes the spill-over is negatively significant. ● denotes the spill-over is insignificant. We make conclusion about the significance of spill-over effect using the asymptotic 95% confidence interval of the generalized impulse response in FIVAR derived in Chapter 3, (see Equation (4.11)).

Table 4.7: Spill-over from Realized Kurtosis to other realized measures and Trading Volume

	Foreign Exchange markets						Stock markets					
	RV		RS		Volume		RV		RS		Volume	
	S	**V**	**S**	**V**	**S**	**V**	**S**	**V**	**S**	**V**	**S**	**V**
Panel A: Spill-over from the Realized Kurtosis of Foreign Exchange market												
Latin America	●	●	●	●	Ɵ	Ɵ	Ɵ	●	●	●	●	⊕
Asian Pacific Emerging	⊕	⊕	⊕	●	●	Ɵ	●	●	●	●	●	⊕
Asian Pacific Developed	⊕	⊕	●	●	●	●	●	●	●	●	●	●
West Europe Developed	⊕	⊕	●	●	●	●	●	●	●	●	●	●
Panel B: Spill-over from the Realized Kurtosis of Stock market												
Latin America	●	●	●	●	●	●	⊕	●	●	●	●	●
Asian Pacific Emerging	⊕	●	●	●	●	●	⊕	⊕	●	●	●	⊕
Asian Pacific Developed	●	●	●	●	●	●	⊕	⊕	●	●	●	●
West Europe Developed	●	●	●	●	●	●	⊕	⊕	●	●	●	Ɵ

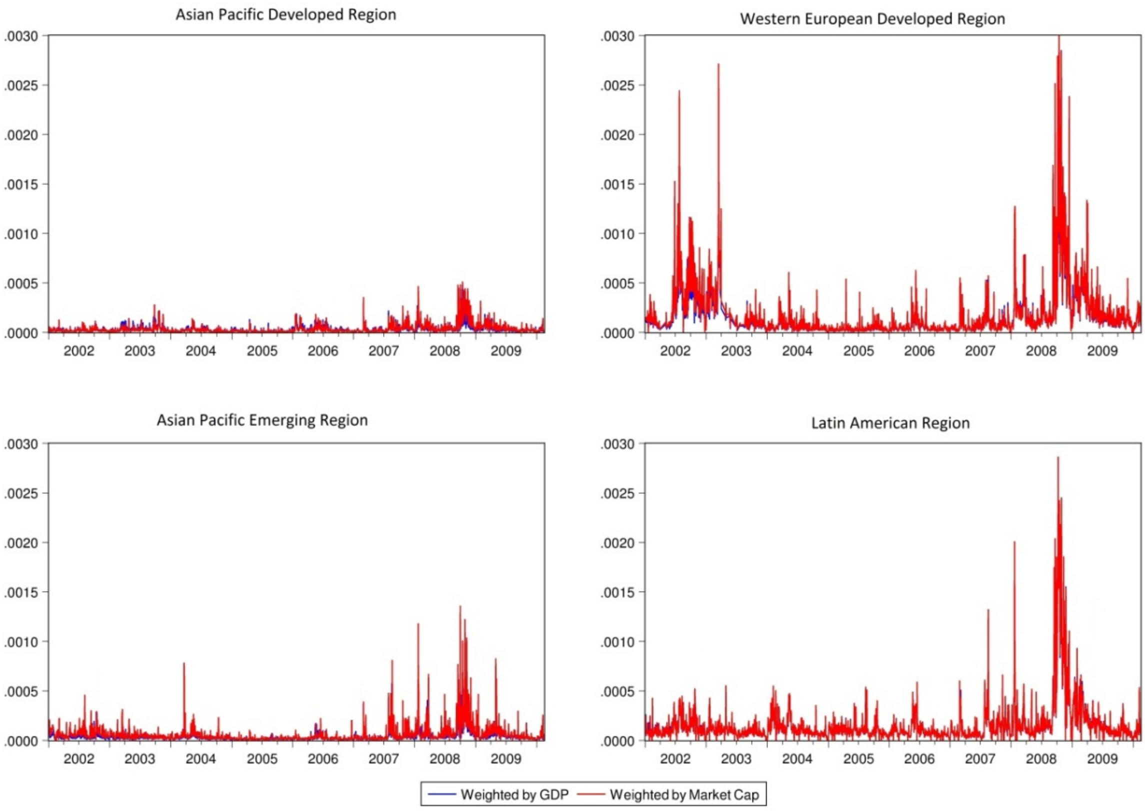
Notes: **RV**, **RS**, **RK** and **Volume** denote realized volatility, realized skewness, realized kurtosis and trading volume, respectively. **S** and **V** denotes the Stable and Volatile period, respectively. ⊕ denotes the spill-over is positively significant. Ɵ denotes the spill-over is negatively significant. ● denotes the spill-over is insignificant. We make conclusion about the significance of spill-over effect using the asymptotic 95% confidence interval of the generalized impulse response in FIVAR derived in Chapter 3, (see Equation (4.11)).

Table 4.8: Spill-over from Trading Volume to realized measures

	Foreign Exchange markets						Stock markets					
	RV		RS		RK		RV		RS		RK	
	S	**V**	**S**	**V**	**S**	**V**	**S**	**V**	**S**	**V**	**S**	**V**
Panel A: Spill-over from the Trading Volume of Foreign Exchange market												
Latin America	⊕	⊕	●	●	Ɵ	Ɵ	⊕	⊕	●	●	●	●
Asian Pacific Emerging	⊕	⊕	●	●	Ɵ	Ɵ	⊕	⊕	●	●	●	●
Asian Pacific Developed	⊕	⊕	●	●	●	●	⊕	⊕	●	●	●	●
West Europe Developed	⊕	⊕	●	●	●	●	●	●	●	●	●	●
Panel B: Spill-over from the Trading Volume of Stock market												
Latin America	●	●	●	●	●	●	⊕	⊕	●	●	●	Ɵ
Asian Pacific Emerging	●	⊕	●	⊕	●	⊕	⊕	⊕	●	●	●	●
Asian Pacific Developed	●	⊕	●	●	●	●	⊕	⊕	●	●	●	●
West Europe Developed	●	⊕	●	●	●	●	⊕	⊕	●	●	●	Ɵ

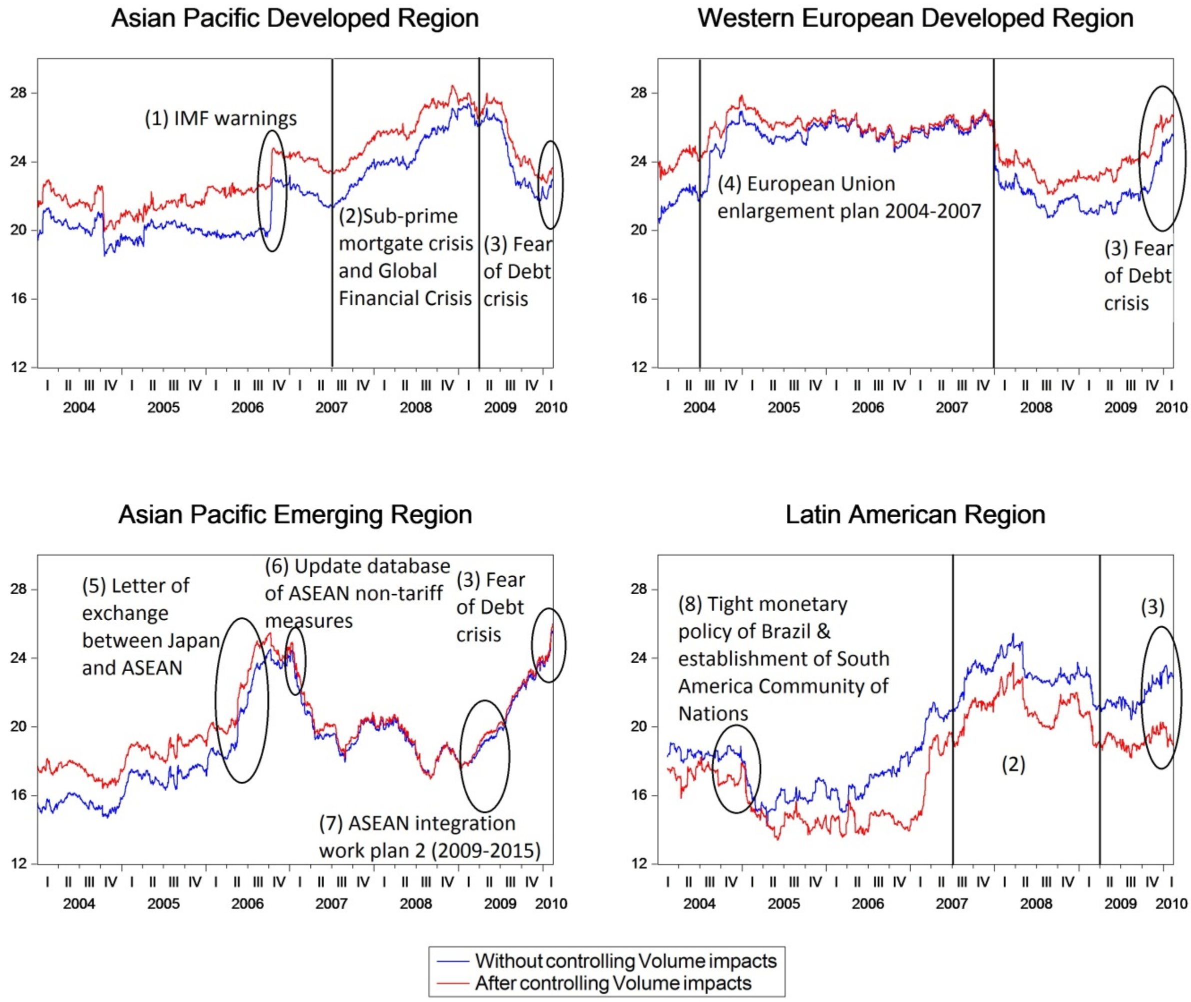
Notes: **RV**, **RS**, **RK** and **Volume** denote realized volatility, realized skewness, realized kurtosis and trading volume, respectively. **S** and **V** denotes the Stable and Volatile period, respectively. ⊕ denotes the spill-over is positively significant. Ɵ denotes the spill-over is negatively significant. ● denotes the spill-over is insignificant. We make conclusion about the significance of spill-over effect using the asymptotic 95% confidence interval of the generalized impulse response in FIVAR derived in Chapter 3, (see Equation (4.11)).

Figure 4.1: Plots of daily realized volatility weighted by GDP and Market Capitalization



Note: We plot the daily realized volatility of stock markets for each region in terms of market weights by GDP and Market Capitalization, respectively. The plot indicates that realized volatility estimates based on GDP weight tends to be smaller than which based on Market Capitalization. This may be due to the GDP figures are likely to be more stable compared to stock markets' performance with peaks and troughs. Further, as can be seen, the realized volatility estimates behave differently from mid-2007 in comparison with previous periods in most of regions.

Figure 4.2: Dynamic inter-relationship among realized higher moments



Notes:

(1) In October 2006, the International Monetary Fund (IMF) started warning Australian Banks about their fragilities, which causes almost immediately worries in Australian markets. Further, as ranked the 3rd and 4th in total investment in Australia at that time, Japan and Hong Kong are also affected because of the bad news, respectively.

(2) The subprime mortgage crisis started in the U.S around mid-2007 then spread globally as the Global Financial Crisis from 2008, which has caused one of the greatest global recessions in financial history.

(3) Fear of a European sovereign debt crisis has risen from late 2009 since many European countries faced a huge problem with budget deficits. Although it is analytically separate from the Global Financial Crisis in 2008, the two crises are linked because many European banks held assets in American banks, which were facing financial troubles.

(4) European enlargement plan from 2004 to 2007 led Western European developed countries to transfer massive financial products (and assistances) to less developed countries during that period.

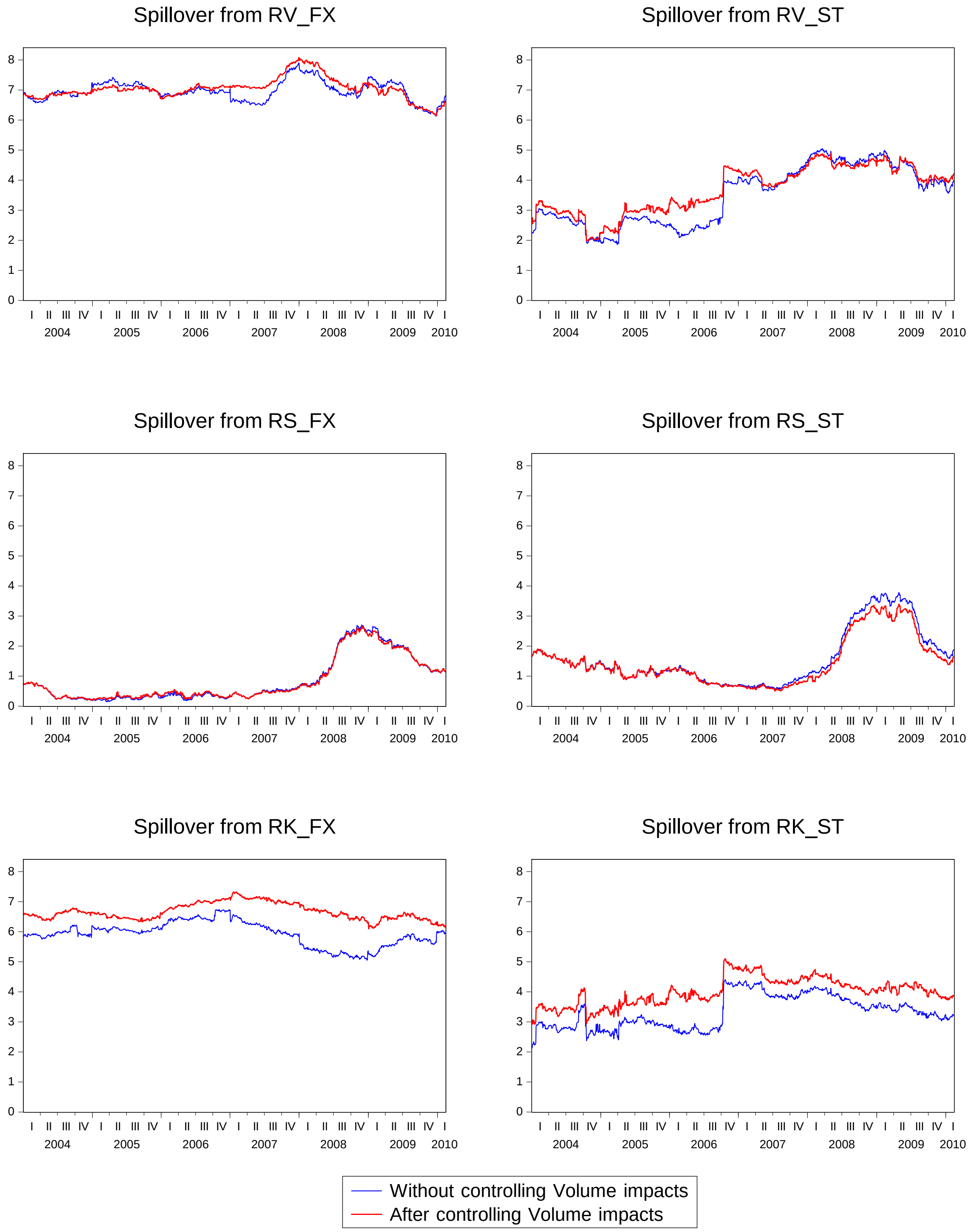
(5) In March 2006, a Letter of Exchange was established between Japan and the Association of Southeast Asian Nations (ASEAN), which stated that Japan would provide a fund of ¥7.5 billion to support ASEAN's integration efforts.

(6) In January 2007, ASEAN started to regularly update the database of its non-tariff measures to enhance transparency.

(7) In early 2009, ASEAN launched the Integration work plan 2 for the period from 2009 to 2015. The plan aims to narrow the development gap and increase the integration between ASEAN's members by allowing the free flow of goods/ services, investment capital and so on.

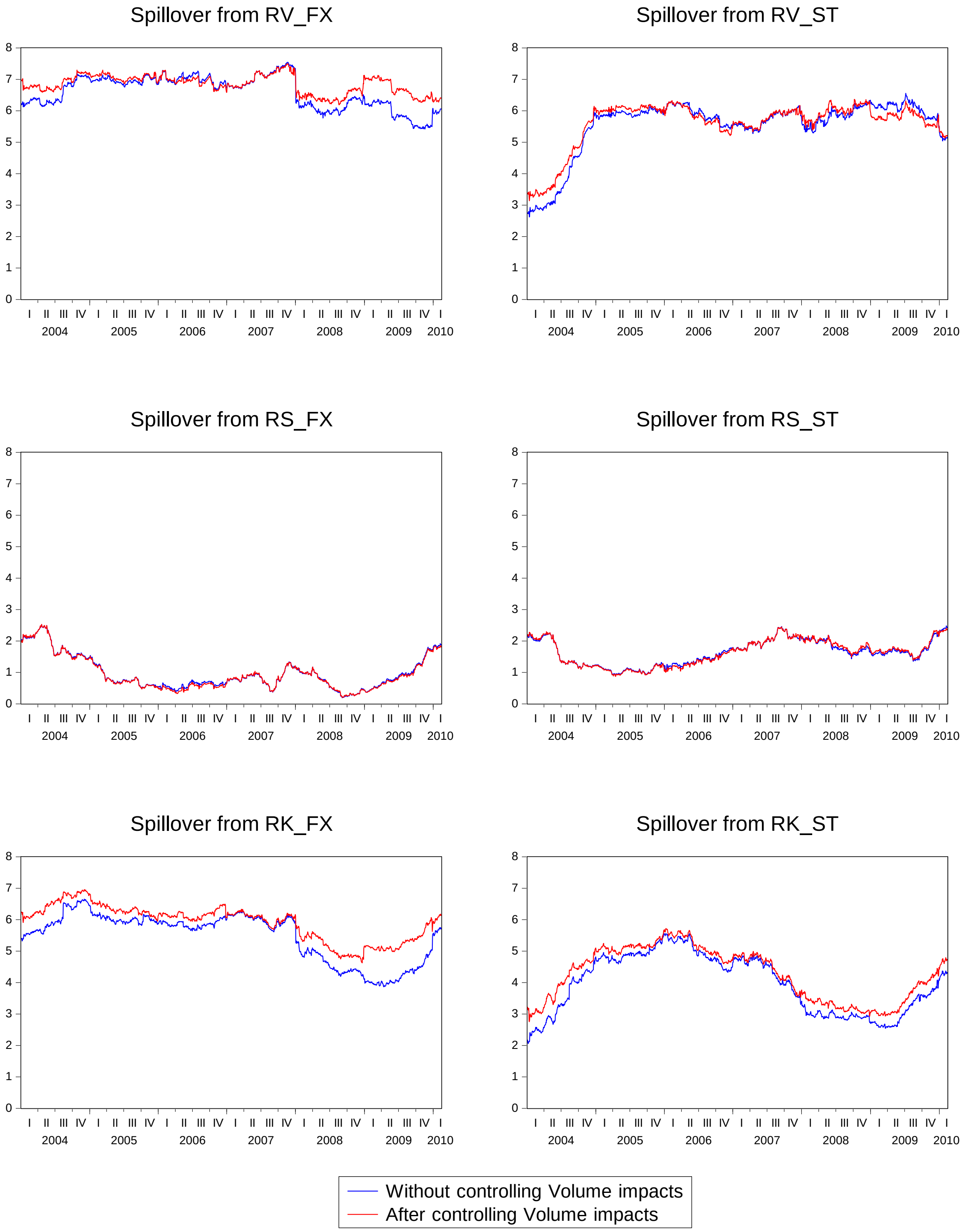
(8) In late 2004, Brazil had tightened its monetary policy (September 2004) and led to reinforce the MERCOSUR by establishing the South America Community of Nations (December 2004). The community acted as a southern hemispheric alternative to NAFTA, which, therefore, limited the influence of the U.S on the Latin American region. Probably, all of these actions would slow down the integration progress of regional members to the U.S and to the international economies.

Figure 4.3: Directional Spill-over effects in the Asia Pacific Developed region



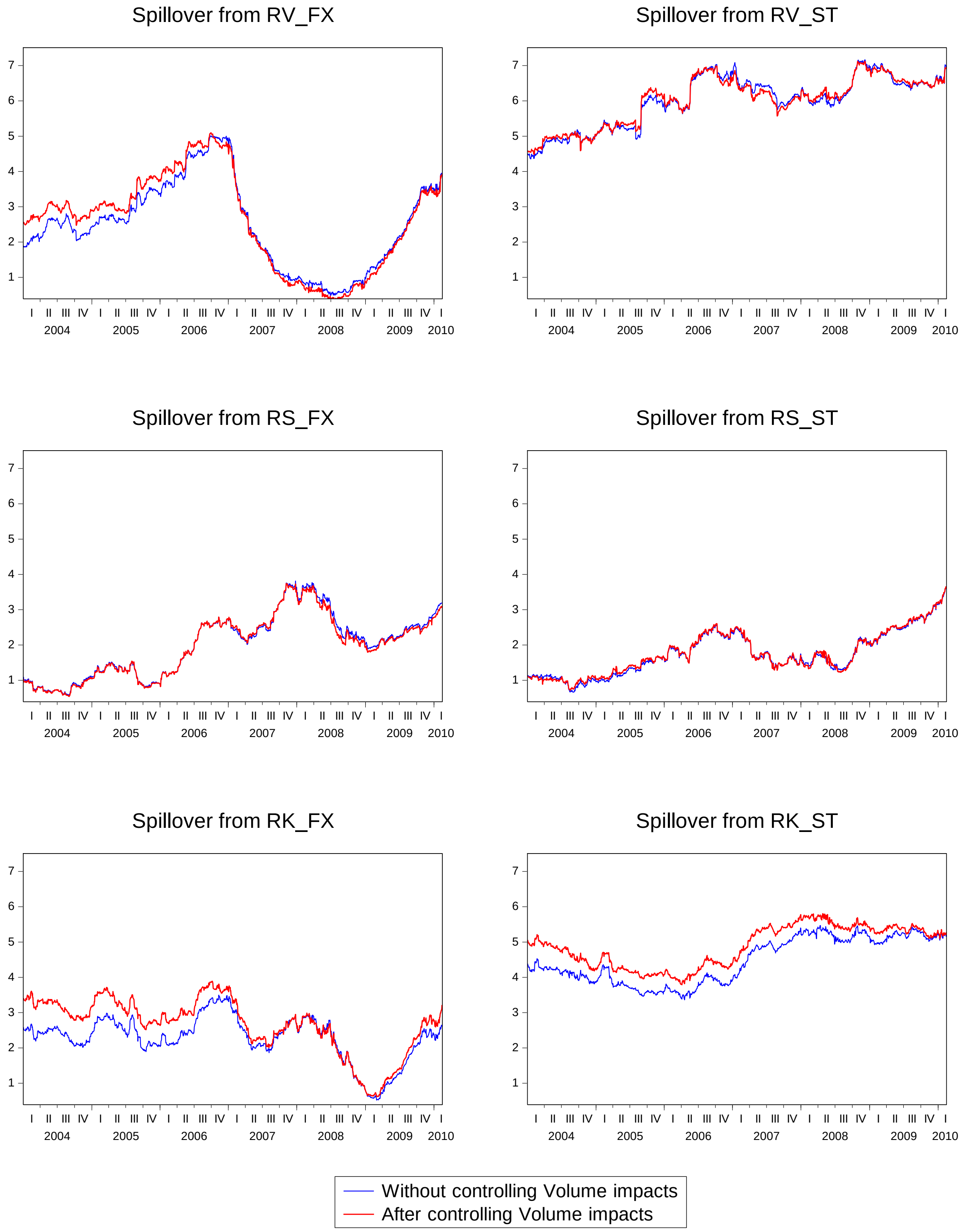
Note: **RV**, **RS** and **RK** denote realized volatility, skewness and kurtosis, respectively. Further, **FX** and **ST** denote the Foreign Exchange and stock markets, respectively.

Figure 4.4: Directional Spill-over effects in the Western European Developed region



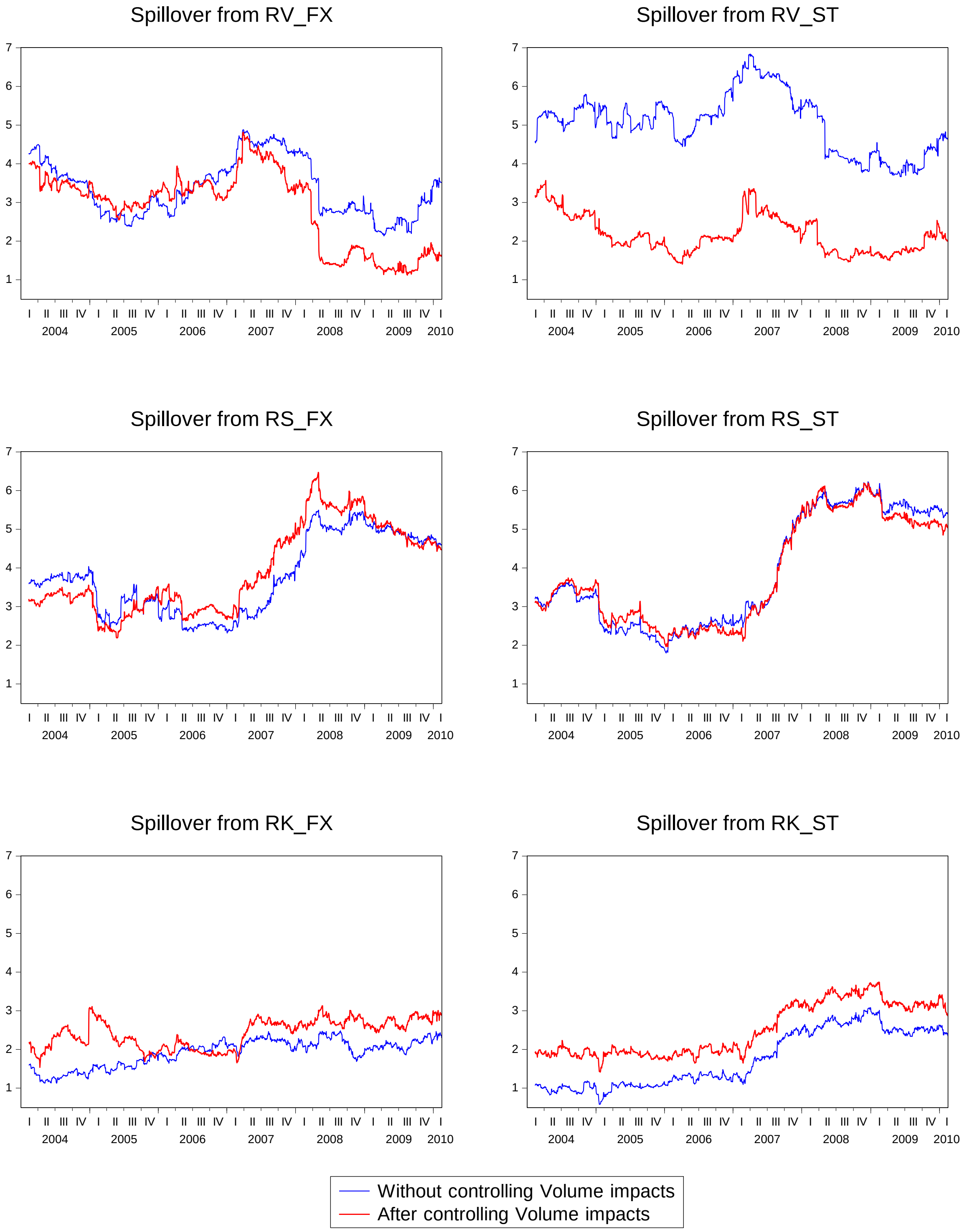
Note: **RV**, **RS**, **RK** and **Volume** denote realized volatility, skewness, kurtosis and trading volume respectively. Further, **FX** and **ST** denote the Foreign Exchange and stock markets, respectively.

Figure 4.5: Directional Spill-over effects in the Asia Pacific Emerging region



Note: **RV**, **RS**, **RK** and **Volume** denote realized volatility, skewness, kurtosis and trading volume respectively. Further, **FX** and **ST** denote the Foreign Exchange and stock markets, respectively.

Figure 4.6: Directional Spill-over effects in the Latin American region



Note: **RV**, **RS**, **RK** and **Volume** denote realized volatility, skewness, kurtosis and trading volume respectively. Further, **FX** and **ST** denote the Foreign Exchange and stock markets, respectively.

Chapter 5

Sovereign credit ratings impacts on financial return distributions: A multivariate regime switching long memory approach

5.1 Introduction

Sovereign credit ratings, which publicly reveal opinions of specialist information intermediaries about the credit quality of a national government, are expected to have influences on the behavior of asset prices, especially during periods of market uncertainty and financial instability. Yet, the credit rating agencies (CRAs), providers of this information, have often been criticized for their slow responses to international financial crises as well as their inability to forewarn market participants of impeding crises (see, e.g., Mora, 2006; Gorton, 2008). It is, therefore, neccessary to assess the informational value of sovereign credit assessments and the impact of agency ratings on the stability of financial markets as represented by moments of asset return distributions. Focusing on financial return distributions enables a much deeper understanding of the role of sovereign ratings information in asset pricing, and hence, can also improve other financial activities such as Value-at-Risk calculation and asset allocation. This is due to the dynamics of higher return moments such as variance, skewness and kurtosis are evidenced to influence asset prices (see among others, Harvey and Siddique, 2000; Athayde and Flôres, 2003; Mandelbrot and Hudson, 2004).

5.2 Literature review

Whilst prior studies have extensively documented the more direct and immediate effects of sovereign credit rating revisions on stock and bond returns (see Reisen and Von Maltzan, 1999; Kaminsky and Schmukler, 1999, 2002; Brooks et. al., 2004; Gande and Parsley, 2005; Pukthuanthong-Le et al., 2007; and Ferreira and Gama, 2007), there is less substantive evidence on the effects on currency markets. To the best of our knowledge, the only exception is Alsakka and ap Gwilym (2012), who investigate the issue in currency markets. As financial crises are invariably related to fluctuations in currency values, this void clearly needs to be addressed to aid our understanding of whether CRAs are capable of playing a stabilising role across different financial markets and under all market conditions.

One of the key findings of the literature on the market impact of rating changes (see for instance, Brooks et. al., 2004) is the asymmetric nature of rating changes, in that rating downgrades have a more significant effect than do upgrades. A natural extension of this finding is to explore whether there are also asymmetries in the rating impacts on currency markets. Also, the findings on stock markets would lead us to predict an asymmetric response to ratings news in currency markets, in that rating downgrades (upgrades) will considerably increase (decrease) currency market volatilities. However, financial impact of rating downgrades on currency volatility is likely to be more significant than that of rating upgrades due to its "bad news" content during financial crises. Thus, this chapter specifically focuses on the effects of sovereign credit assessments on equity and currency markets in the spirit of studies like Brooks et al. (2004), Ferreira and Gama (2007) and Alsakka and ap Gwilym (2012) but in a more completed context by also considering all first four moments of the return distribution.

The existing literature has traditionally examined the effect of sovereign rating changes on the first moment of asset return distributions on a daily data basis (see, e.g., Brooks et al., 2004; Gande and Parsley, 2005; Ferreira and Gama, 2007; Hill and Faff, 2010a; Alsakka and ap Gwilym, 2012); whereas, there is a dearth of attention on the impacts on higher return moments. One possible reason is the limitations of the parametric methods used in estimating the conditional higher moments[37]. In recent times, an increasing availability of high frequency data has facilitated a better alternative for measuring the higher moments non-parametrically from intraday returns as suggested by previous studies (e.g., Dacorogna et al., 2001; Andersen et al., 2003; Barndorff-Nielsen and Shephard, 2004a, 2004b; Amaya et al., 2011; Neuberger, 2012). The use of intraday data compared to daily closing data can give us a better representation and more robust estimate of the actual price behavior (see for instance, Andersen et al., 2003). The realized higher moments, which are the moments constructed from intraday returns, can be treated as observable variables and, therefore, are able to be modeled directly within an econometric framework. As a result, the

[37] Due to limited availability of the high frequency data, the higher moments were often estimated conditionally based on the well-known Generalized Autoregressive Conditional Heteroskedastic (GARCH) models and its variants. The estimates of conditional volatility, skewness and kurtosis, therefore, rely heavily on the underlying model assumptions. In addition, the problem is magnified within a multivariate system due to the large number of parameters that need to be estimated for extracting the outputs of conditional higher moments.

properties of realized higher moments should be taken into account in the empirical modeling process. Our preliminary analyses show that realized returns and skewness exhibit short memory behavior; whereas, realized volatility and kurtosis are more likely to be long memory processes[38]. A long memory process is considered as an intermediate between two classical processes, the short-memory ($I(0)$) and the unit root process ($I(1)$). More precisely, it is defined corresponding to the case of a fractional degree of integration. Therefore, in order to focus on financial return distributions via the first four realized moments, our proposed model allows flexible fractional degrees of integration which can capture both short- and long-memory behavior.

Regarding the sovereign credit ratings, a significant number of studies have modeled sovereign credit rating transitions due to its critical role in modern credit risk management, valuation and international asset allocation (see among others, Bangia et al., 2002; Lando and Skødeberg, 2002; Fuertes and Kalotychou, 2007; Hill et al., 2010b). The estimation of the rating transition probabilities matrix has indicated a regime switching behavior in credit ratings which needs to be accounted for in the modeling of financial market impacts of sovereign credit ratings. In essence, credit ratings, either in levels or first differences (i.e., ratings change), can be categorised into regimes (states), for example, states of ratings level can be defined as each of its letter designations (AAA, AA+,…); whereas, states of ratings changes may include stable (i.e., no change), downgrades or upgrades. Hence, we aim to develop a framework that

[38] Figure 5.3 illustrates the long memory behavior of realized volatility and kurtosis since their autocorrelations die out slowly and their spectral densities are unbounded at the origins; whereas, the realized return and skewness evolve as short memory processes because of their immediate died out autocorrelations and their bounded spectral densities at the origins. This is also consistent with the literature, which documented the stylized fact of realized volatility (e.g., Andersen et al., 2000, 2003), and our findings in Chapter 2 and 4.

not only allows a flexible set of fractional degrees of integration for endogenous variables as mentioned earlier but that also captures the regime switching behaviour of sovereign credit ratings.

We contribute to the existing literature by proposing a new empirical framework that allows a multivariate system of long memory processes to be conditioned on observable regimes, which are defined by characteristics of sovereign credit quality assessments. By accommodating both the long range dependencies of realized higher moments and the regime switching feature of sovereign credit assessments, the properties of these measures can be fully accounted for. An inclusion of both long memory and regime switching properties in one system is challenging. The past studies claim that under certain conditions, non-linear features (such as regime switching) of a time series can be spuriously identified as long memory when measured by the degree of fractional integration (see among others, Granger and Ding, 1996; Bos et al., 1999; and Granger and Hyung, 2004). However, the necessity of combining these features within one framework has been supported in the recent literature, for instance, Diebold and Inoue (2001), Haldrup and Nielsen (2006) and Haldrup et al. (2010). Haldrup and Nielsen (2006) develop an univariate model that allows process to have different degrees of fractional integration in two separate observable regimes. The feature of observable regimes in this model is opposed to the assumption of latent regimes in the traditional Markov switching model proposed by Hamilton (1989, 1990). However, due to some similarities (e.g., the switching behavior), Haldrup and Nielsen (2006) still place their model in the class of a regime switching model. Haldrup et al. (2010) further advances the work of Haldrup and Nielsen (2006) by proposing a bivariate model to analyse the co-movement of two time series, while it still preserves the combination of long memory and regime switching features. To some extent, the work of Haldrup et al.

(2010) can be considered as an extension of the Markov Switching Vector autoregressive framework developed by Krolzig (1997). However, similar to the univariate case discussed earlier, the states in the model of Haldrup et al. (2010) is assumed to be observable, which is different with the latent states identified in Krolzig (1997).

Our model inherits some characteristics of the Haldrup et al. (2010)'s model when it allows a multivariate long memory process to behave differently (i.e., different degrees of fractional integration) across observable states. Yet, the model of Haldrup et al. (2010) mostly focuses on the endogenous variables and employs an estimation procedure that objective functions are optimized over all parameters of the model. We distinguish our approach by allowing for a presence of exogenous variables. This feature is important in the case that we aim to investigate the impact of a variable (e.g., sovereign ratings) which is not determined by the system of endogenous variables (e.g., realized moments). We further advantage our model by proposing a different approach used in the estimation procedure. The proposed technique, which concentrates the likelihood function on fractional degrees of integration, may help to facilitate our model in the case of high dimensionality since the objective function is numerically optimized over a smaller number of parameters in a comparison with existing techniques.

We illustrate our new approach by empirically investigating the impact of sovereign credit assessments on European stock and FX return distributions. We examine the sample period from January 1996 to July 2012, to cover the lead up to the introduction of the Euro and the recent European sovereign debt crisis (hereafter, EDC). All CRAs have been particularly active in downgrading European sovereigns during the debt crisis with on average, nearly 70% of all rating downgrades in our sample taking place since December 2008 (the onset of the EDC) (see Figure 5.1). We aim to contribute comprehensive and new evidence of sovereign rating impacts on European financial markets during the EDC. Moreover, we employ sovereign ratings data from Standard and Poor's, Moody's and Fitch – the three main CRAs in the world - in order to find out which agency has the greatest impact on entire financial return distributions captured by their first four realized moments. Although previous studies have indicated the largest impact is usually from Standard and Poor's (e.g., Reisen and Von Maltzan, 1999; Brooks et al., 2004), recent activities of the CRAs during the EDC may change their rank orders. In line with this view, Alsakka and ap Gwilym (2012) find that over the period from 1994-2010, Fitch's sovereign credit signals induced the most timely currency market responses. Furthermore, previous studies based their analyses on causality tests and event studies, which may only capture the *direct* effects of CRAs' re-rating activities. We argue that the market impact of the CRAs should be measured in terms of their total effects, which include both *direct* and *indirect* forces. In a multivariate framework, where the inter-relationships among realized moments are captured, we define the *indirect* effects of the CRAs on a realized moment as the spillover effect that goes through other realized moments. In this way, we can comprehensively capture the full effects of sovereign credit re-rating activity on entire asset return distributions to reveal which agency truly elicits the greatest market reactions (i.e., has the most influence on the financial return distributions in our context). We believe this is the first study to distinguish between the direct and indirect effects of sovereign credit information in financial markets. This is important given the reliance on rating-contingent financial regulation such as the Basel II and III accord for assessing capital adequacy requirements and for prescribing investment grade only holdings by financial institutions.

The remainder of this chapter is organized as follows. We describe the data construction in section 5.3. Section 5.4 proposes the new econometric model and its estimation procedure. We discuss the findings of our empirical analysis of the EU financial markets in section 5.5. An impulse response of a transfer function is developed to find out the most influential CRA in section 5.6. Finally, we conclude the chapter in section 5.7.

5.3 Data

We capture 5-minute intraday stock and FX market mid prices in some European Union (EU) countries from the Thomson Reuters Tick History (TRTH) database provided by the Securities Industry Research Centre of Asia-Pacific (SIRCA). By using a high frequency of 5-minute intervals, we can minimise the problem of measurement error and reduce microstructure biases[39]. The sample period studied is from January 1996 to July 2012, which covers the period from pre- Asian Financial Crisis until the recent European Sovereign Debt crisis (EDC). In terms of FX markets, we include data from 21 countries: Austria, Belgium, Bulgaria, Cyprus, Czech, Denmark, France, Germany, Greece, Hungary, Ireland, Latvia, Malta, Netherlands, Poland, Portugal, Romania, Slovakia, Spain, Sweden and the United Kingdom. However, the intraday data was only available for 10 stock markets in the European Union (EU), being Austria, France, Germany, Greece, Hungary, Ireland, the Netherlands, Romania, Spain and the United Kingdom.

In addition, we employ historical long-term foreign currency sovereign credit rating and credit outlook and watches from three leading CRAs, Standard and Poor's,

[39] See Andersen and Bollerslev (1998) and Andersen et al. (2001).

Fitch and Moody's. This will enable an assessment on which CRA influences European stock market returns the most via its rating actions. Due to the irregular timing of ratings announcement, we focus our analysis on a monthly basis. We follow the approach of Gande and Parsley (2005) and Ferreira and Gama (2007) among others to transform the sovereign rating and credit outlook and watches into linear scores as presented in Table 5.1. We summarize all rating news released during each month using the comprehensive credit rating (CCR) measure[40]. Figure 5.1 illustrates how active the CRAs are in re-rating EU sovereign obligors. As can be seen, the CRAs have more often upgraded than downgraded EU countries over the entire sample period but not surprisingly most of the downgrade news on EU nations have been released during the most recent sovereign debt crisis (around 70% of all downgrade rating news). Among the three CRAs, Fitch seems to be the least active agency in downgrading the rating level of the EU sovereigns; whereas, the number of upgrades released by Moody's for EU countries is the smallest suggesting that they are the most conservative of the major CRAs. Overall, the absolute number of rating announcements has indicated that Standard and Poor's can be considered as the most active rating agency for countries in the EU (corroborating with prior studies that compare across rating agencies such as Brooks et al., 2004)[41].

To construct a proxy for the opinion of a CRA about the sovereign credit quality of the EU overall, we utilise the sovereign rating drift measure, which is the difference

[40] The CCR is calculated as the sum of linearised sovereign credit ratings and the credit outlook/watches following the approach of Gande and Parsley (2005).

[41] Standard and Poor's released 112 downgrade and 124 upgrade rating news. Meanwhile, Moody's made 109 downgrades and 109 upgrades. Besides, Fitch announced 91 downgrades and 115 upgrades.

between rating upgrades and rating downgrades averaged by the number of sovereigns in our sample. The rating drift across the EU can be calculated as,

$$SRD_t = \frac{\sum_{i=1}^{m} \Delta CCR_{it}}{m} \tag{5.1}$$

where ΔCCR_{it} is the first difference of the CCR measure of country *i*, and m is the number of countries used to construct the rating drift. Since we aim to assess the opinion of a CRA about the whole EU overall, we include historical sovereign ratings data of all 27 EU countries to construct the drift measure. The sovereign rating drift adequately reflects the view of a CRA on the average trend in the credit quality of all sovereign obligors in the EU region on the whole. The plots of the sovereign credit rating drifts for the three major CRAs shown in Figure 5.2 indicate that the rating drifts can be classified into three observable regimes or states over time, which are zero, positive and negative zones. These three zones can be inferred as the regimes of stable, upward and downward trends in sovereign credit quality across the EU as perceived by each of the CRAs. Furthermore, it can be observed that most of the negative rating drifts are in the period of the sovereign debt crisis, consistent with what has been shown in Figure 5.1. We can, therefore, consider the regime of downward sovereign credit quality as primarily the episode of the European sovereign debt crisis (EDC).

To model the stock market and FX return distributions, we construct their higher moments based on intraday returns rather than employing daily close to close prices since the use of intraday data provides us with more consistent and efficient measures (see, e.g., Andersen and Bollerslev, 1998; Barndorff-Nielsen and Shephard, 2001; Andersen et al., 2003 among others).

The daily realized returns constructed from intraday returns are identical to the usual daily returns calculated from daily close to close prices,

$$r_t = \sum_{i=1}^{D} r_{i,t} \tag{5.2}$$

where $r_{i,t}$ denotes the ith 5-minute return during day t and D denotes the total number of 5-minute return intervals during any trading day.

The realized higher moments of returns, namely the realized volatility (RV_t), realized skewness (RS_t) and realized kurtosis (RK_t) are respectively defined as[42],

$$RV_t = \sum_{i=1}^{D} r_{i,t}^2 \tag{5.3}$$

$$RS_t = \frac{\sqrt{D}\sum_{i=1}^{D} r_{i,t}^3}{RV_t^{3/2}} \tag{5.4}$$

$$RK_t = \frac{D\sum_{i=1}^{D} r_{i,t}^4}{RV_t^2} \tag{5.5}$$

To facilitate empirical testing, the monthly realized measures are then simply constructed as averages of corresponding daily realized series.

We graph the sample autocorrelations and spectral densities of realized returns, (logged) realized volatility, realized skewness and (logged) realized kurtosis for a lag of

[42] The properties of realized volatility as defined in Equation (5.3) are well analyzed in the literature (e.g., Andersen and Bollerslev, 1998; Andersen et al., 2003). Meanwhile, the limits of realized skewness and kurtosis under the forms of Equation (5.4) and Equation (5.5) are recently assessed in Amaya et al., (2011).

50 months in Figure 5.3[43]. There is evidence of long memory behaviour in the realized volatility and realized kurtosis series (ie., second and fourth moments)revealed by the slow hyperbolic autocorrelation decay and the most mass at the zero frequency of the spectral densities. Meanwhile, the sample autocorrelations of realized return and realized skewness fluctuates around zero during the displacement of 50 months, exhibiting the property of short memory processes. We can further confirm a high degree of serial correlation in both realized volatility and kurtosis by examining the Ljung-Box statistics in all cases.

5.4 Econometric modelling

The properties and features of the four realized moments of financial returns and the sovereign rating drifts discussed in the previous section, motivate us to develop a flexible multivariate framework that can capture both long memory and regime switching behavior in these series.

Although there have been some studies debating the interchange between the long memory and the non-linear models[44], it is necessary in our case to simultaneously accommodate both long range dependencies and regime switching in order to separate the properties of our variables of interest. The recent literature also supports the importance of including these features within a single framework, for instance, Diebold

[43] We utilize the natural logarithm of realized volatility and kurtosis in our analysis since their applications are extensively supported in the literature (e.g., Andersen and Bollerslev, 1998; Andersen et al., 2003). Further, the use of realized logarithmic volatility and kurtosis help us to avoid the non-negativity conditions in modeling. Therefore, when we refer to the realized volatility and kurtosis measures, they are in natural logarithmic forms.

[44] See for example, Granger and Ding (1996), Bos et al. (1999) and Granger and Hyung (2004).

and Inoue (2001), Haldrup and Nielsen (2006) and Haldrup et al. (2010). In our case, the sovereign rating drifts are clearly distinguished by three separate regimes, which represent the periods of stable, upward and downward trends in sovereign credit quality[45]. In the stable period, sovereign rating drift has no impact on the financial return distribution as it is equal to zero. On the other hand, in the upward and downward periods, the impact of sovereign ratings drift on the financial return distribution as well as the characteristics of the financial return distribution itself can be very different. Accordingly, it would not be a good decision to fix the long memory behavior of the realized moments of asset returns across the three regimes. We should rather allow long memory behavior under the form of fractional integration to vary across these regimes.

Hence, we propose a multivariate long memory model with exogenous variables that are allowed to switch between different regimes. We model the realized moments of asset returns as endogenous variables in the system and we take the view that the sovereign ratings drift is not necessarily explained by the system of those realized moments. The sovereign ratings drift is rather determined by the public information as well as the private information owned and subjectively assessed by the CRAs. Therefore, we treat the sovereign ratings drift as an exogenous variable, which defines the states (regimes) and may help to explain the realized return-based measures. Our model is different to the existing models in the literature (e.g., Haldrup and Nielsen, 2006; Haldrup et al., 2010) in the sense that it allows for the existence of exogenous variables. We further distinguish our model by proposing a different technique used in the estimation procedure. This technique enables our model to be applicable for a

[45] We can also interpret these regimes as the periods in which CRAs release good news and bad news regarding sovereign credit quality across the EU.

higher dimensional system, which is also an advantage over existing models. Instead of numerically optimizing the objective likelihood function with regards to all parameters as in the literature, we further concentrate the objective function with regards to the degrees of fractional integration. Hence, the numerical optimization procedure is much faster and, perhaps, more reliable than previously possible.

5.4.1 Model specification and assumptions

Let the K-dimensional time series, $Y_t = (Y_{1t},...,Y_{Kt})'$, follow a Markov Regime Switching and Fractionally Integrated Vector Autoregressive model with n exogenous variables (MS-FIVARX), $R_t = (R_{1t}, R_{2t},..., R_{nt})'$:

$$A^{(s_t)}(L)D^{(s_t)}(L)Y_t = \nabla^{(s_t)}R_t + \varepsilon_t, \qquad t = 1,2,...,T \tag{5.6}$$

We define $s_t \in \{1,2,...,M\}$ as the observable regime variable which is characterized by the behaviour of one of the exogenous variables R_t and follows an ergodic M-state Markov chain process with an irreducible transition probability matrix,

$$\mathrm{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1M} \\ p_{21} & p_{22} & \cdots & p_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ p_{M1} & p_{11} & \cdots & p_{MM} \end{bmatrix},$$

where, $p_{ij} = \Pr(s_{t+1} = j \mid s_t = i)$ and $\sum_{j=1}^{M} p_{ij} = 1, \forall i, j \in \{1,2,...,M\}$. In other words, p_{ij} is the probability that a regime i is followed by a regime j.

The operator, $A^{(s_t)}(L) = I_K - \sum_{i=1}^{p} A_i^{(s_t)} L^i$, where p is the lag order of the lag polynomial and $A_i^{(s_t)}$ is the K×K matrix of coefficients associated with the endogenous variables. $\nabla^{(s_t)}$ is the $K \times n$ matrix of coefficients associated with the exogenous variables. The operator $D^{(s_t)}(L)$ is a diagonal $K \times K$ matrix characterized by the K-dimensional vector of degrees of fractional integration, $d^{(s_t)} = (d_1(s_t),...,d_K(s_t))'$,

$$D^{(s_t)}(L) = \begin{bmatrix} (1-L)^{d_1(s_t)} & 0 & \cdots & 0 \\ 0 & (1-L)^{d_2(s_t)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (1-L)^{d_K(s_t)} \end{bmatrix}$$

We can employ the binomial expansion to operationally generate the term $(1-L)^{-d_j(s_t)}$ as,

$$(1-L)^{-d_j(s_t)} = \sum_{i=0}^{\infty} \frac{\Gamma(i + d_j(s_t))}{\Gamma(d_j(s_t))\Gamma(i+1)} L^i = \sum_{i=0}^{\infty} \psi_i^{d_j(s_t)} L^i \qquad (5.7)$$

where $\Gamma(.)$ is the gamma function; $\psi_0^{(0)} = 1$, and $\psi_i^{(0)} = 0$, for $i \neq 0$.

As in the representation of the MS-FIVARX, all the coefficient matrices, the degrees of fractional integration as well as the variance – covariance matrix of error terms are assumed to be regime dependent, which means that they are conditional on s_t, for example,

$$\nabla^{(s_t)} = \begin{cases} \nabla^{(1)} & \text{if } s_t = 1 \\ \vdots & \\ \nabla^{(M)} & \text{if } s_t = M \end{cases}$$

Further, since Y_t is assumed to be dependent on regime s_t, the conditional probability density function of Y_t is regime dependent,

$$f(Y_t \mid \Omega_{t-1}, s_t) = \begin{cases} f(Y_t \mid \Omega_{t-1}, \theta_1) & \text{if } s_t = 1 \\ \vdots & \\ f(Y_t \mid \Omega_{t-1}, \theta_M) & \text{if } s_t = M \end{cases},$$

where θ_i is the vector of parameters associated with regime *i*, and Ω_{t-1} is the information set available at time *t-1*.

To ensure the adequacy, stationarity and to avoid the multicollinearity problems, the following additional assumptions have been made for our MS-FIVARX model:

Assumption 5.1: $\varepsilon_t \mid s_t \sim N(0, \Sigma_\varepsilon^{(s_t)})$; $\Sigma_\varepsilon^{(s_t)} = \{\sigma_{ij}^{(s_t)}; i, j = 1,2,...,K\}$ *are (K×K) positive definite matrices,* $E(\varepsilon_r \varepsilon_s' \mid s_t) = 0$, *for all* $r \neq s$.

Assumption 5.2: *All the roots of* $\left|A^{(s_t)}(z)\right| = \left|I_K - \sum_{i=1}^{p} A_i^{(s_t)} z^i\right| = 0$ *fall outside the unit circle and* $d_j^{(s_t)} \in (-0.5, 0.5)$ *for all* $j = 1,2,...,K$.

Assumption 5.3: *Y_t has no deterministic trend.* $Y_{t-1}, Y_{t-2}, ..., Y_{t-p}$ *are not perfectly collinear and each element of* $R_t = (R_{1t}, R_{2t}, ..., R_{nt})'$ *is independent of each other.*

5.4.2 Estimation of transition probabilities

Since the regime variable s_t is assumed to be observable and determined by the behaviour of the exogenous variable R_t, we may exploit R_t to count the number of the observations in each regime as well as the number of transitions among regimes. These figures subsequently can be used to estimate the transition probability matrix P. Therefore, the maximum likelihood estimates (MLEs) of the transition probabilities are simply given as,

$$p_{ij} = \frac{n_{ij}}{\sum_{m=1}^{M} n_{im}}, \quad \forall i, j \in \{1,2,...,M\} \tag{5.8}$$

where n_{ij} is the number of times we observe a regime i that is followed by a regime j.

5.4.3 Estimation of the model's parameters

We obtain the estimates of remaining parameters in the model by using the quasi maximum likelihood via the concentrated log-likelihood function (CLF). For a specific regime, model specification (5.6) follows a Fractionally Integrated Vector Autoregressive framework with exogenous variables (FIVARX). Hence, the CLF of our MS-FIVARX model in a specific regime can borrow the form of the CLF of a FIVARX model.

For simplicity, we ignore the term s_t in constructing the CLF of a MS-FIVARX model in a specific regime since it is in fact under the representation of a FIVARX model. Let us consider,

$$A(L)D(L)Y_t = \nabla R_t + \varepsilon_t, \qquad t = 1,2,..., N \tag{5.9}$$

Further, we assume that the p pre-sample values of each endogenous variable, $Y_{-p+1},...,Y_0$, are available. The following notations are employed to facilitate our derivation,

$$X_t = D(L)Y_t,$$

$$\underset{(K\times N)}{X} = (X_1, X_2,..., X_N),$$

$$\underset{(K\times(Kp+n))}{B} = (A_1, A_2,..., A_p, \nabla),$$

$$\underset{((Kp+n)\times 1)}{Z_t} = \begin{bmatrix} X_t \\ \vdots \\ X_{t-p+1} \\ R_t \end{bmatrix},$$

$$\underset{((Kp+n)\times N)}{Z} = (Z_0, ..., Z_{N-1}),$$

$$\underset{(K\times N)}{U} = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_N),$$

Lemma 5.1:

Let the assumptions 5.1, 5.2, 5.3 hold and the variance-covariance matrix of error terms is written as a function of all parameters as,

$$\Sigma_\varepsilon(d,B) = N^{-1}\sum_{t=1}^{N}(\varepsilon_t \varepsilon_t') = N^{-1}\sum_{t=1}^{N}\left(\left[A(L)D(L)Y_t - \nabla R_t\right]\left[A(L)D(L)Y_t - \nabla R_t\right]'\right)$$

For a given memory parameter d, $\left|\Sigma_\varepsilon(d,B)\right|$ *can be denoted as* $\left|\Sigma_{\varepsilon|d}(d,B)\right|$ *, then the following results hold,*

$\left|\Sigma_{\varepsilon|d}(d,B)\right|$ *is minimized at* $\hat{B} = XZ'(ZZ')^{-1}$*, and,*

$$\left|\Sigma_{\varepsilon|d}(d,B)\right|_{\min} = \left|N^{-1}(X - \hat{B}Z)(X - \hat{B}Z)'\right|.$$

Following Lemma 5.1, we can obtain the CLF with regards to the memory parameter *d* of a FIVARX model as presented in the proposition,

Proposition 5.1:

Let the assumptions 5.1, 5.2, 5.3 hold, the concentrated log-likelihood function with respect to the vector of memory parameters $d = (d_1,...,d_K)'$ *of a FIVARX model is,*

$$l_{FIVARX}^{c}(d) = -\frac{KN}{2}\left[\ln(2\pi)+1\right] - \frac{N}{2}\ln\left|\Sigma_{\varepsilon}(d)\right|$$

where,

$$\Sigma_{\varepsilon}(d) = T^{-1}X(I_N - Z'(ZZ')^{-1}Z)X'$$

and the estimators are obtained by,

$$\hat{d} = \underset{d\in(-0.5,0.5)}{\arg\max}\, l_{FIVARX}^{c}(d)$$

$$\hat{B} = \hat{X}\hat{Z}'(\hat{Z}\hat{Z}')^{-1}$$

According to Proposition 5.1, we can obtain the conditional log-likelihood functions of our MS-FIVARX model, apart from constants, for a specific regime *i* as follows,

$$l^{c}(d^{(s_t=i)}) = -\sum_{t=1}^{T}\frac{I(s_t=i)}{2}\ln\left|\Sigma_{\varepsilon}(d^{(s_t=i)})\right|,$$

where $I(s_t = i)$ is the indicator function returning 1 if $s_t = i$ and 0 otherwise.

The full-sample CLF of a MS-FIVARX model with respect to the vector of memory parameters is given by,

$$l^c(d) = \sum_{i=1}^{M} l^c(d^{(s_t=i)}) \,,$$

Alternatively, we can reform the $l^c(d)$ for the purpose of convenient programing. We collect all the information of the regimes during the sample period in a $M \times 1$ vector ξ_t,

$$\xi_t = \begin{bmatrix} I(s_t = 1) \\ I(s_t = 2) \\ \vdots \\ I(s_t = M) \end{bmatrix},$$

Further, the variance-covariance matrices of error terms concentrated on memory parameters, $\Sigma_\varepsilon(d^{(s_t)})$, for M regimes are collected in the $K \times MK$ matrix Σ,

$$\Sigma = [\Sigma_\varepsilon(d^{(s_t=1)}),\ldots,\Sigma_\varepsilon(d^{(s_t=M)})] \,,$$

Hence, it can be easily seen that,

$$\Sigma_\varepsilon(d^{(s_t)}) = \Sigma(\xi_t \otimes I_K) \,,$$

Accordingly, the regime-specific concentrated log-likelihood function can be represented as,

$$l^c(d^{(s_t=i)}) = -\sum_{t=1}^{T} \frac{I(s_t = i)}{2} \ln\left|\Sigma(\xi_t \otimes I_K)\right| ,$$

The full-sample concentrated log-likelihood function with respect to the memory parameters can be obtained as,

$$l^c(d) = \sum_{i=1}^{M} l^c(d^{(s_t=i)}) = -\left\{ \frac{1}{2} \sum_{t=1}^{T} \ln\left|\Sigma(\xi_t \otimes I_K)\right| \left[I(s_t = 1) + I(s_t = 2) + \ldots + I(s_t = M) \right] \right\},$$

At every specific time $t \in \{1,2,...,T\}$, we always achieve,

$$I(s_t = 1) + I(s_t = 2) + ... + I(s_t = M) = 1,$$

Therefore, we have the ultimate representation of the full-sample CLF of a MS-FIVARX model as,

$$l^c(d) = -\frac{1}{2}\sum_{t=1}^{T} \ln\left|\Sigma(\xi_t \otimes I_K)\right| \tag{5.10}$$

At the first stage, the memory parameters $d^{(s_t)}$ can be obtained by numerically maximizing the $l^c(d)$ with respect to $d^{(s_t)}$,

$$\hat{d}^{(s_t)} = \underset{d \in (-0.5,0.5)}{\arg\max}\ l^c(d),$$

Remaining parameters $\hat{B}$ for each regime are extracted conditional on estimator $\hat{d}^{(s_t)}$ using the results obtained in Proposition 5.1.

5.5 Empirical results

We utilize our proposed model by employing realized return-based measures constructed in section 5.3 to investigate the impact of the sovereign ratings drifts on stock market and FX return distributions within the EU. Since the preliminary analyses performed in section 5.3 affirmed the short memory behaviour of realized returns and skewness, we restrict their memory parameters to be zero. The fractional degrees of integration for realized volatility and kurtosis are allowed to vary across regimes. As discussed in previous sections, we distinguish the relationship between realized return moments and CRA sovereign rating changes into three regimes which are defined by the properties of the sovereign rating drifts. These regimes can be considered as the periods of stable, upward and downward assessments of sovereign credit quality, corresponding to zeros, positive and negative values on sovereign ratings drifts respectively. We focus on the results obtained in the upward and downward regimes. Also, as noted in section 5.3, the time series plots of the sovereign ratings drifts (Figure 5.2) indicate that the period of the EDC is prominent and covers almost the entire downward regime. We, therefore, consider the downward state as a representation of the European sovereign debt crisis.

More importantly, to facilitate the interpretation of the effects of downward sovereign rating drifts on each realized moment, we employ the absolute values of the downward drifts in modelling. Hence, a positive relationship between the drifts and the realized return in the downward regime, for example, can be interpreted as more negative assessments of sovereign credit quality will lead to an increase in the realized return consistent with the basic risk-return trade-off in finance theory.

We choose the optimal lag length *p* for the model so that the innovations mimic the white noise processes and the parsimonious criteria is satisfied. We, therefore, end up with the lag length of order 1 for our models. This result is reasonable as both characteristics of the measures, the long memory and regime switching features, which may require a large number of lag orders have been captured by the specification of the proposed model. The estimated results show that all the roots fall outside the unit circle and the memory parameters are in the range from -0.5 to 0.5, an indication of stationarity[46].

[46] We do not report the full set of our estimation results to conserve space. However, full details are available upon request.

5.5.1 The transition probability matrices

As the regimes are observable, we can easily calculate the estimates of transition probabilities for each regime according to formula (5.8). We present the estimated results of the transition probability matrices in Table 5.2.

The estimates indicate an average level of persistence of the regimes. The probabilities that the sovereign rating drifts stay in one regime are at most 0.5. Among all, the probabilities of staying in the upward regime are the lowest (i.e., 0.25, 0.38 and 0.28 for the Standard and Poor's, Fitch and Moody's respectively). There is a relatively high likelihood of remaining in the stable state (i.e., 0.38, 0.48 and 0.49 for Standard and Poor's, Fitch and Moody's respectively) compared to either upward or downward states, consistent with the view that CRAs provide long-term assessments on sovereign credit quality and the practice of rating through the cycle. These figures in conjunction with the probabilities of residing in the upward regime, however, imply somewhat that the CRAs have not been active in re-assessing sovereign credit quality across the EU prior to the onset of the EDC. In contrast, there are relatively high levels of persistence in the downward regime (i.e., 0.45, 0.50 and 0.39 for Standard and Poor's, Fitch and Moody's respectively) indicating that CRAs seem to have learnt lessons from the Global Financial Crisis and have become more active in downgrading sovereign credit quality throughout the EDC.

5.5.2 Impact of the sovereign credit assessments on financial return distributions

In this section, we analyse the direct impacts of the sovereign ratings drift on each realized moment of the EU stock and FX return distribution by using the Granger

Causality test. Hence, we extract the estimates of the vector $\nabla^{(s_t)}$ and their corresponding *t*-statistics.[47]

Direct impacts on European stock and FX realized returns

We report the effects of sovereign credit quality assessments on stock and FX realized returns across both the upward and downward regime in Table 5.3 and 5.4. As can be seen, the sovereign ratings drifts are likely to have insignificant impacts on stock market realized returns in both upward and downward regimes. This result implies that the overall assessments of CRAs on European sovereign creditworthiness have limited direct contribution to changes in realized stock market returns across the EU. However, if we focus on the direction instead of the significance of the relationship, we find a negative impact of the upward rating drifts on realized stock market returns while downward rating drifts tend to have positive effects. This finding is consistent with the basic risk-return trade off theory in finance since the upward trend in the sovereign credit quality evaluation reveals a tendency of lower credit risk; whereas, the downward trend indicates increasing credit risk.

Interestingly, we find that realized FX returns react significantly to Standard and Poor’s re-ratings in the upward regime but respond more to Moody’s re-ratings in the downward regime (during the EDC). Even though the positive reactions of FX realized returns in the downward regime are consistent with the case of stock realized return, their positive responses to Standard and Poor’s rating drifts in the upward regime are surprising results.

[47] For the purpose of calculating the *t*-statistics, we obtain the asymptotic covariance matrix of the concentrated maximum likelihood estimates as the negative inverse of the observed Hessian matrix.

Direct impacts on European stock market and FX realized volatility

The effects of sovereign credit assessments on stock and FX realized volatility across both upward and downward regimes are shown in Table 5.5 and 5.6. It can be observed that the sovereign rating drifts have limited impacts on both stock and FX realized volatility in the upward regime. However, there is more evidence of their significant effects in the downward regime. This result indicates that the assessments of the CRAs on sovereign credit quality across the EU have greater effects on the uncertainty and/or the dispersion of opinions with respect to the value of European stocks and currencies during the recent EDC.

As expected, we find a consistently negative relationship between the upward rating drifts and realized volatility in both stock and FX markets. Meanwhile, the downward rating drifts have significant and positive effects on realized volatility. The results unambiguously indicate that improvements in CRAs' assessments on sovereign credit quality across the EU reduces stock and FX market uncertainty; whereas continuing negative assessments will increase market uncertainty. This finding is consistent with the empirical results which we obtained in analysing the direct impacts of ratings drift on realized returns from the previous sub-section. The explanation for this consistency can be based on the risk-return trade off theory in finance.

Direct impact on European stock and FX realized skewness

Table 5.7 and 5.8 report the effects of sovereign credit assessments on realized skewness in stock and FX markets across both upward and downward regimes. For the stock market, we find that the case of Standard and Poor's sovereign ratings drift provides strong evidence of the direct effects in the upward regime; whereas, in the downward regime, more evidence of the direct effects is revealed for Fitch's sovereign ratings drift. This result indicates that Standard and Poor's assessments on sovereign creditworthiness within the EU have relatively broader impacts on the asymmetry of stock market return distributions during periods of financial stability. Meanwhile, Fitch has evidently played a more critical role in this regard during the recent EDC. In the FX market, we observe the reverse situation since Fitch's ratings delivers greater direct effects in the upward regime; whereas, Standard and Poor's rating effects are stronger in the downward regime.

Interestingly, in terms of both stock and FX markets, we mostly find a positive relationship between sovereign ratings drift and realized skewness in both upward and downward regimes. Hence, regardless of the upward or downward direction, as long as the ratings drift changes (i.e., more rating news are released), the magnitude of the positive extreme returns in EU stock and FX markets is larger (more right-skewed).

Direct impact on European stock and FX realized kurtosis

The effects of sovereign credit assessments on stock and FX realized kurtosis across both upward and downward regimes are summarized in Table 5.9 and 5.10. We find limited evidence of significant effects in the upward regime but greater evidence of the significant relationship between sovereign ratings drifts and realized kurtosis can be found in the downward regime. Hence, the results show that the assessments of the CRAs on overall sovereign creditworthiness across the EU have greater impacts on the occurrence of extreme returns in stock and FX markets during the EDC.

In addition, we mostly find the negative relationship between the sovereign ratings drift and realized kurtosis in the downward regime; whereas, the upward rating drifts tend to positively affect realized kurtosis. These results indicate that an upsurge in the downward (upward) trend of the CRA's assessments on EU sovereign obligors will significantly lower (increase) the peak of stock and FX return distributions for European countries. This result is consistent with what we have found in the analysis of the direct impacts of sovereign credit assessments on realized volatility. This is because a return distribution with a lower (higher) peak corresponds to a distribution with more (less) return dispersion. Besides, as mentioned in the previous sub-section, we note that an increase in the downward (upward) rating drift will heighten (decrease) stock and FX market volatility across the EU.

5.6 The most dominant credit rating agency

The empirical results discussed so far confirm certain impacts of each CRA's sovereign ratings on financial return distributions via its first four realized moments. It, however, remains questionable which CRA has the largest effect on financial markets. In section 5.5, we assessed the direct impact of CRAs' assessments using Granger Causality tests. Yet, this method is not applicable to address the issue of dominance amongst the CRAs as this should be reflected by their total effects including both direct and indirect forces. Because of the inter-relationship among realized moments, which is also captured in our multivariate system, the indirect effects of the sovereign rating drifts on a realized moment is the spillover effect that goes through other realized moments in the system. In this section, we develop a tool, which we call the impulse response of a transfer function (IRTF), to capture those total effects of the CRAs' assessments. The IRTF describes how endogenous variables react when there is an exogenous shock to the exogenous variables. The function, therefore, is ideal for capturing the total responses of a financial return distribution to a change in the sovereign ratings drift since such a change is usually caused by a shock from outside arriving under the form of public or private information which is assessed by the CRAs.

5.6.1 Impulse response of a transfer function

Under the basic assumptions which have been made in previous sections, we can rewrite model specification (5.6) under an infinite moving average representation (MA(∞)). Similar to what has been derived in Chapter 3, section 3.3, we can easily obtain:

$$Y_t = \Phi_*^{(s_t)}(L)R_t + \Phi^{(s_t)}(L)\varepsilon_t \qquad (5.11)$$

where,

$$\Phi_*^{(s_t)}(L) = \nabla^{(s_t)} + \sum_{h=1}^{\infty} \Phi_h^{(s_t)} \nabla^{(s_t)} L^i \ ,$$

$$\Phi^{(s_t)}(L) = I_K + \sum_{h=1}^{\infty} \Phi_h^{(s_t)} L^i \ ,$$

The $K \times K$ coefficient matrix $\Phi_h^{(s_t)}$ can be calculated using the following relationship,

$$\Phi_h^{(s_t)} = \begin{cases} \sum_{i=0}^{h} \Psi_i^{d(s_t)} \Pi_{h-i}^{(s_t)} & h = 1,2,... \\ \Pi_0^{(s_t)} & h = 0 \end{cases} ,$$

where $\Psi_i^{d(s_t)}$ is the diagonal ($K \times K$) matrix with $\psi_i^{d_j(s_t)}$ (noted in formula (5.7)) as the j^{th} element, and $\Pi_i^{(s_t)}$ is obtained according to the following recursive relationship,

$$\Pi_i^{(s_t)} = \begin{cases} \sum_{j=1}^{i} \Pi_{i-j}^{(s_t)} A_j^{(s_t)} & i = 1,2,..., p \\ \sum_{j=1}^{p} \Pi_{i-j}^{(s_t)} A_j^{(s_t)} & i > p \\ I_K & i = 0 \end{cases}$$

Based on the MA(∞) representation of a MS-FIVARX model, we employ the generalized approach proposed by Koop et al. (1996) to develop our IRTF. The IRTF at a horizon h is, therefore, defined as the difference between the conditional expectation of Y_{t+h}, given the information set available at time $t\text{-}1$ (after incorporating the effect of the shock on exogenous variables) and the conditional expectation without the effect of the shock,

$$IRTF_h = E(Y_{t+h} \mid R_t = \delta, \Omega_{t-1}) - E(Y_{t+h} \mid \Omega_{t-1}) \tag{5.12}$$

where $\delta = (\delta_1, ..., \delta_n)'$ is $(n \times 1)$ vector of exogenous shocks on the exogenous variables R_t.

Derive Y_{t+h} according to representation (5.11) and replace in (5.12). Under an additional assumption that $E(R_t) = 0$, we ultimately obtain the full matrix of impulse responses of a transfer function as,

$$IRTF_h^{(s_t)} = \Phi_h^{(s_t)} \nabla^{(s_t)} \Xi_* \tag{5.13}$$

where Ξ_* is a $(n \times n)$ diagonal matrix characterized by elements of the shocks,

$$\Xi_* = \begin{bmatrix} \delta_1 & 0 & \cdots & 0 \\ 0 & \delta_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \delta_n \end{bmatrix},$$

Accordingly, we can interpret the (i, j) element of $IRTF_h^{(s_t)}$ as the response of the i^{th} endogenous variable at horizon h (i.e., at time $t+h$) to a shock hitting the j^{th} exogenous variable at time t.

It can be easily seen that under equation (5.13), the indirect effects of the exogenous shock in R_t on Y_t are captured in the matrix $\Phi_h^{(s_t)}$; whereas, the direct effects are captured by the matrix $\nabla^{(s_t)}$.

5.6.2 Empirical results on impulse response analyses

We calculate the IRTF based on a one standard deviation shock in the sovereign ratings drift as this is the usual choice in the literature featuring impulse response analyses. We report the average responses of EU stock market and FX realized moments to the shock in the sovereign ratings drift for 20 periods ahead in Figure 5.4 and 5.5, respectively.

As can be seen, Standard and Poor's assessments have the greatest impact on stock market realized returns and skewness for the first 5 periods ahead in the upward regime. This result is consistent with the literature, for example, Reisen and Von Maltzan (1999) and Brooks et al. (2004) also find that the rating actions of Standard and Poor's affect stock market returns more than other CRAs. However, the case of higher moments has not been investigated to date. In our analysis, the empirical results show that the sovereign rating drifts constructed from Fitch ratings have the largest effect on stock market realized volatility in the upward regime; whereas, the magnitude of effects on stock market realized kurtosis is not clearly distinguishable among the major CRAs.

In the most recent sovereign debt crisis represented largely by the downward regime, the rank of the CRAs regarding the magnitude of the effects on realized moments has changed. We find interesting results that Moody's assessments on overall EU sovereign creditworthiness have the greatest impact on almost all stock market realized moments around the first 5 periods. The only exception is the effects on realized volatility, for which Moody's shares the 1st ranking with Fitch ratings since their effects are quite comparable.

In terms of the FX market, we consistently observe that Standard and Poor's and Fitch's ratings drifts have the greatest impact on FX realized higher moments in both upward and downward regimes. Meanwhile, the magnitude of effects on FX realized returns is negligible for all CRAs.

In addition, we note that there is a contradiction in the result between the IRTF (in this section) and the Granger Causality test (in the previous section) in the case of the effects on realized returns in an upward regime. For example, we find a negative relationship between sovereign ratings drift and the stock market realized returns in the upward regime using the Granger Causality test. However, the IRTF confirms this is a positive relationship. The difference in result supports their complementary property. While the Granger Causality only tests the direct causal effect, the IRTF captures both the direct and the indirect effects.

5.7 Conclusions

We have developed a multivariate framework to precisely capture the full effects of CRA sovereign credit assessments on return distributions by allowing endogenous long memory variables to be conditional on observable regime switching in exogenous variables. The model is motivated by the necessity to fully investigate the impacts of sovereign credit quality assessments on financial return distributions as there is a dearth of attention on the impacts of CRA announcements beyond the second moments of asset returns. The consistent and robust estimates of moments of the distribution (i.e., the realized moments) exhibit the long memory behaviour, and the regime switching feature of sovereign ratings has been widely documented. Thus, our proposed model is designed to capture both of these features in order to separately account for the properties of the variables of interest.

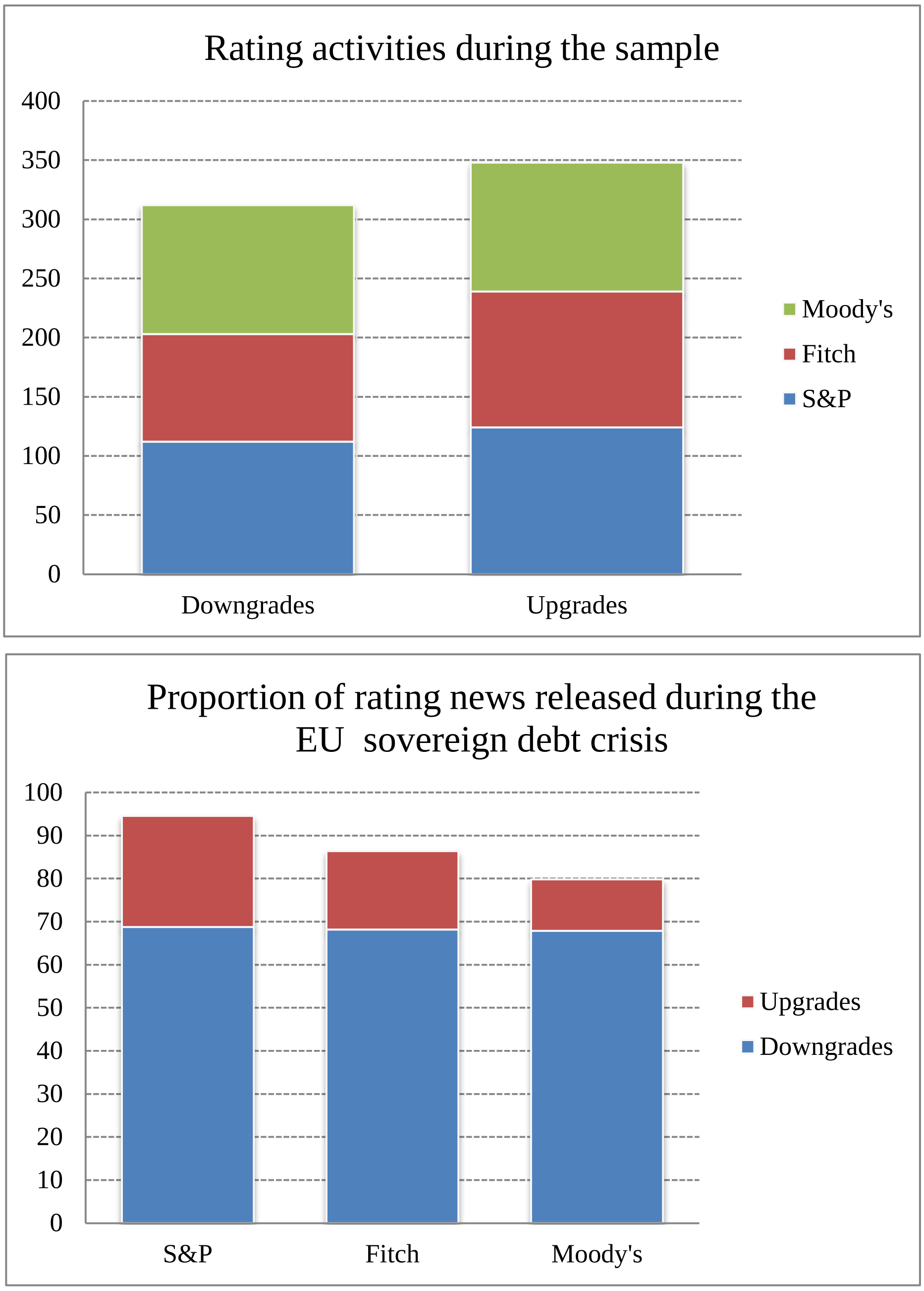
We apply our model to investigate the effects of trends in sovereign credit assessments on stock market and FX return distributions within the EU via their first four realized return moments. The empirical results confirm the heterogeneous effects of rating actions across regimes, which are defined to correspond to the upward and downward trends in sovereign credit assessments by individual CRAs. Hence, these results imply the usefulness of the proposed model since misleading conclusions may be made if the process is not allowed to be conditional on separate states of creditworthiness. More specifically, we mostly find a negative relationship between the overall EU sovereign credit assessments and realized returns in the upward regime, yet the positive relationships are observed in the downward regime. These findings are consistent with the basic risk-return trade off in finance, and are further confirmed by the results of sovereign rating impacts on realized volatility. The evidence mostly shows negative effects of rating drifts on realized volatility in the upward regime but positive effects in the downward regime. Regarding the impacts on realized skewness, as long as the trend (both upward and downward) in overall sovereign credit quality changes, the stock and FX return distributions are more right-skewed. Meanwhile, in terms of realized kurtosis, we find an upsurge in the downward (upward) EU sovereign rating drifts will significantly lower (increase) the peak of the EU stock and FX return distributions. The finding is consistent with empirical results obtained in analysing the impacts on realized volatility.

In this chapter, we also note that the total effects of the sovereign credit assessments on realized moments can be different from their direct effects. This is due to the indirect effects, which are caused by the inter-relationships among the realized return moments. Therefore, we argue that the total effects, rather than the direct one, should be employed to investigate which CRA provides the greatest impact on financial return distributions. We find that the rank orders among the CRAs are not consistent across credit regimes and even in each realized moment. In the periods of financial stability, the assessments of Standard and Poor's have the greatest effect on stock market realized returns and skewness; whereas Fitch's rating actions have the largest impact on stock market realized volatility across the EU. Meanwhile, Moody's rating activities dominate during the recent European sovereign debt crisis. Besides, we consistently find that Standard and Poor's and Fitch share the 1st rank order in having the largest effects on FX realized higher moments. This is possibly due to Fitch being the only major CRA based outside of the US.

5.8 APPENDIX

Part A – Tables and figures

Figure 5.1: Rating activities of the three credit rating agencies

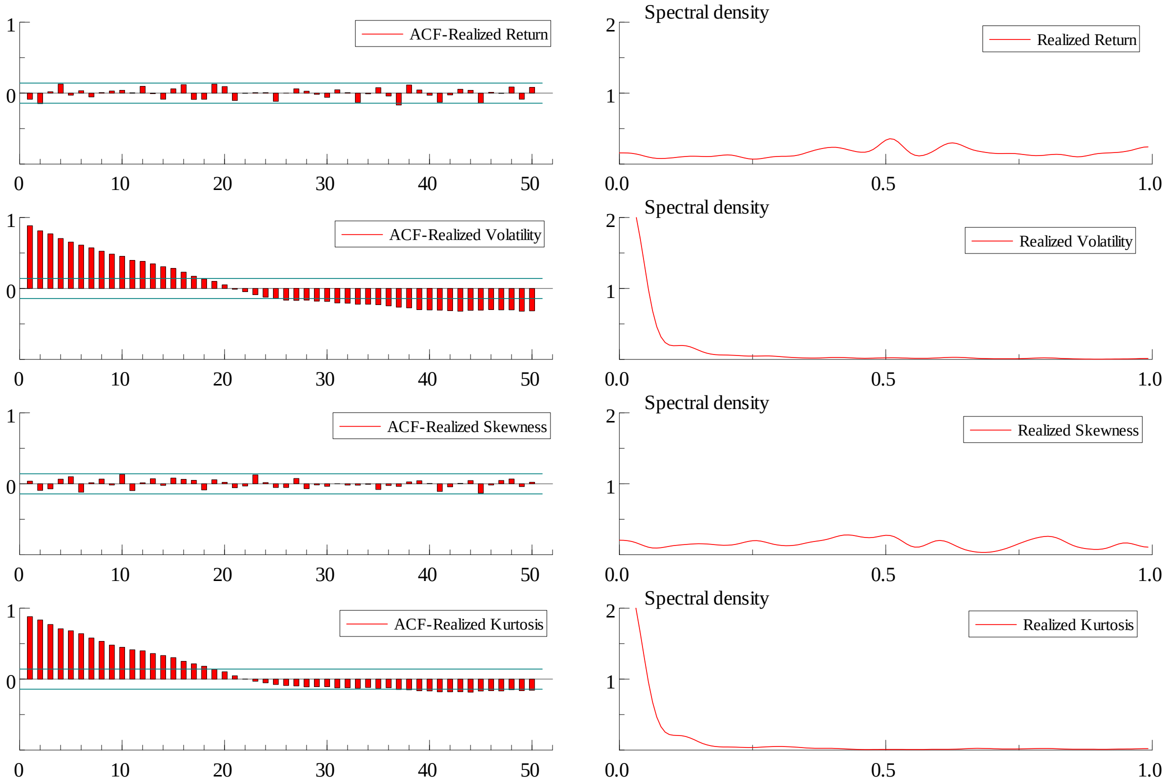


Note: The first chart summarizes the number of rating downgrades and upgrades released by the three credit rating agencies (CRAs), namely Standard and Poor's (S&P), Fitch and Moody's during our full sample period. The second chart reports the proportion of rating events that the CRAs released during the European sovereign debt crisis beginning from October 2008.

Figure 5.2: The European Union sovereign ratings drift

Note: This figure reports the sovereign ratings drifts constructed according to formula (5.1) from historical long-term foreign currency sovereign credit ratings data for all 27 EU countries covered by Standard and Poor's, Fitch and Moody's.

Figure 5.3: Sample autocorrelation functions and spectral densities of the realized moments



Note: This firgure presents sample autocorrelations and spectral densities of a representative stock market realized return, (logged) realized volatility, realized skewness and (logged) realized kurtosis for a lag of 50 months.

Figure 5.4: Average responses of the EU stock realized moments to the sovereign rating drift

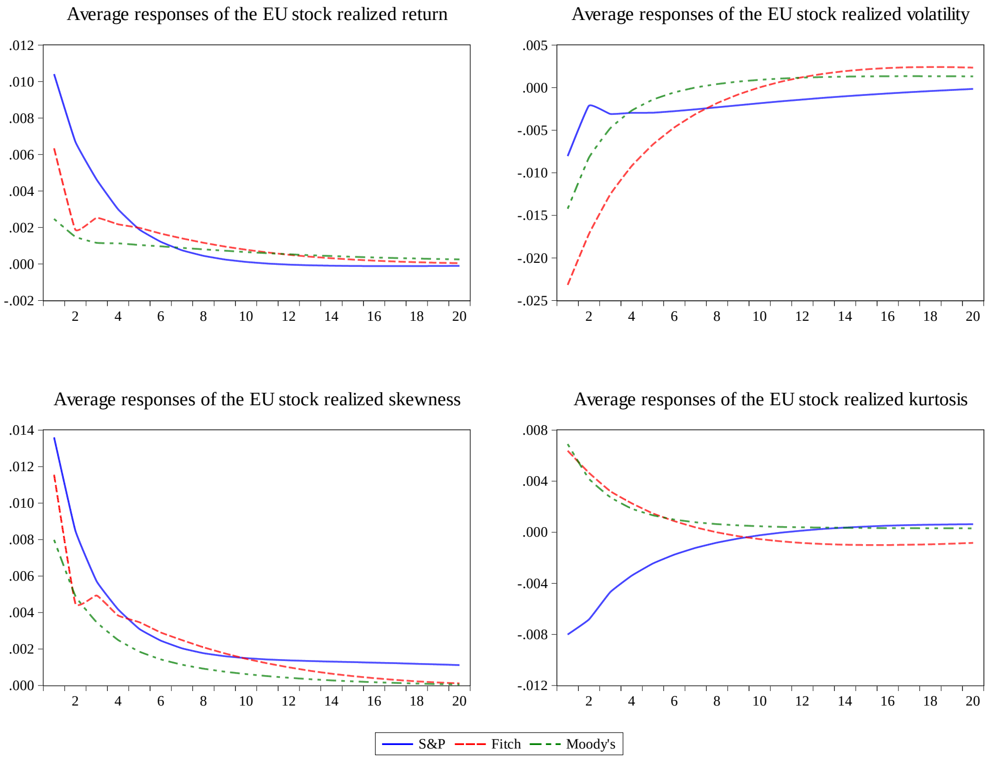


Figure 5.4a: Average responses of the EU stock realized moments to the shock in upward rating drifts

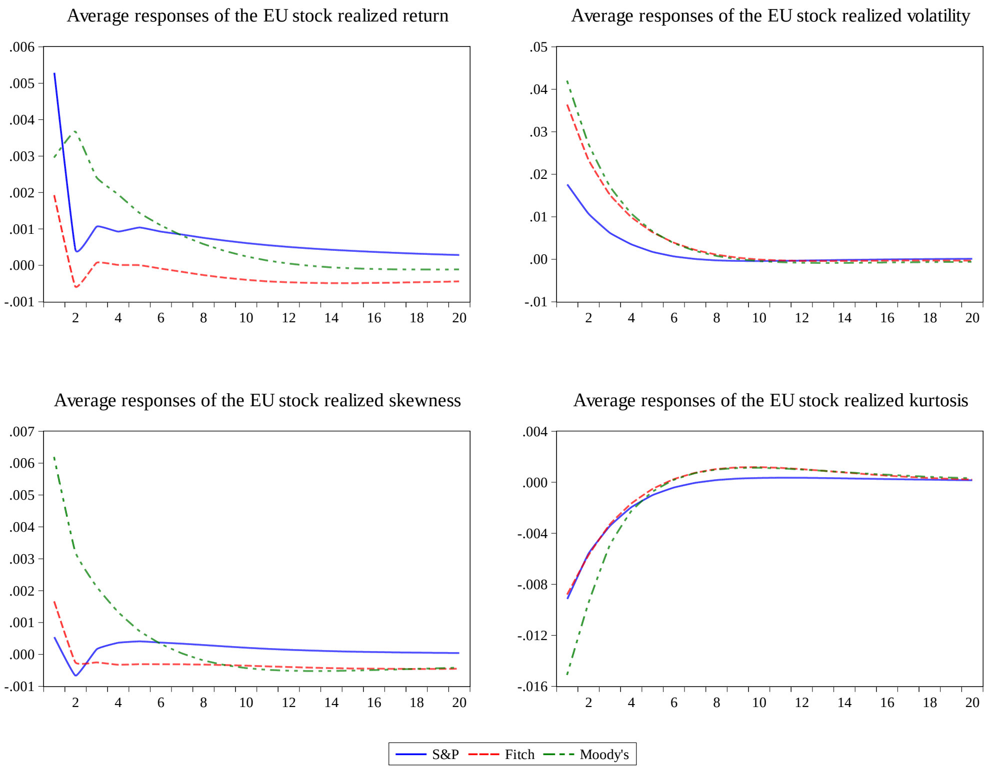


Figure 5.4b: Average responses of the EU stock realized moments to the shock in downward rating drift

Figure 5.5: Average responses of the EU FX realized moments to the sovereign rating drift

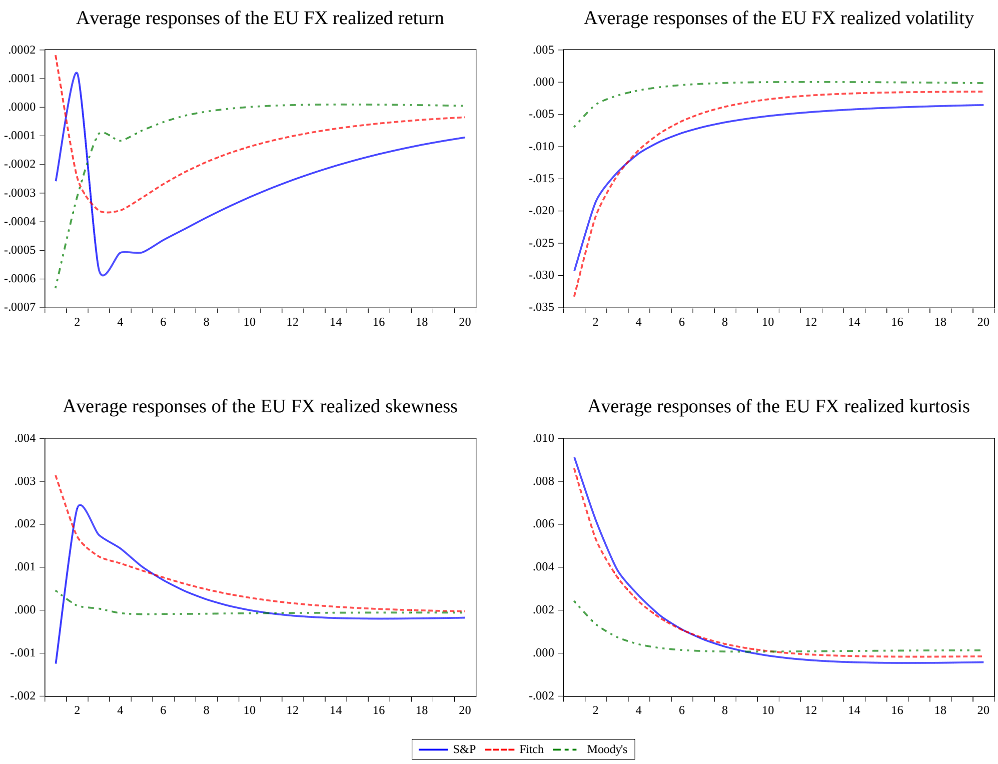


Figure 5.5a: Average responses of the EU FX realized moments to the shock in upward rating drifts

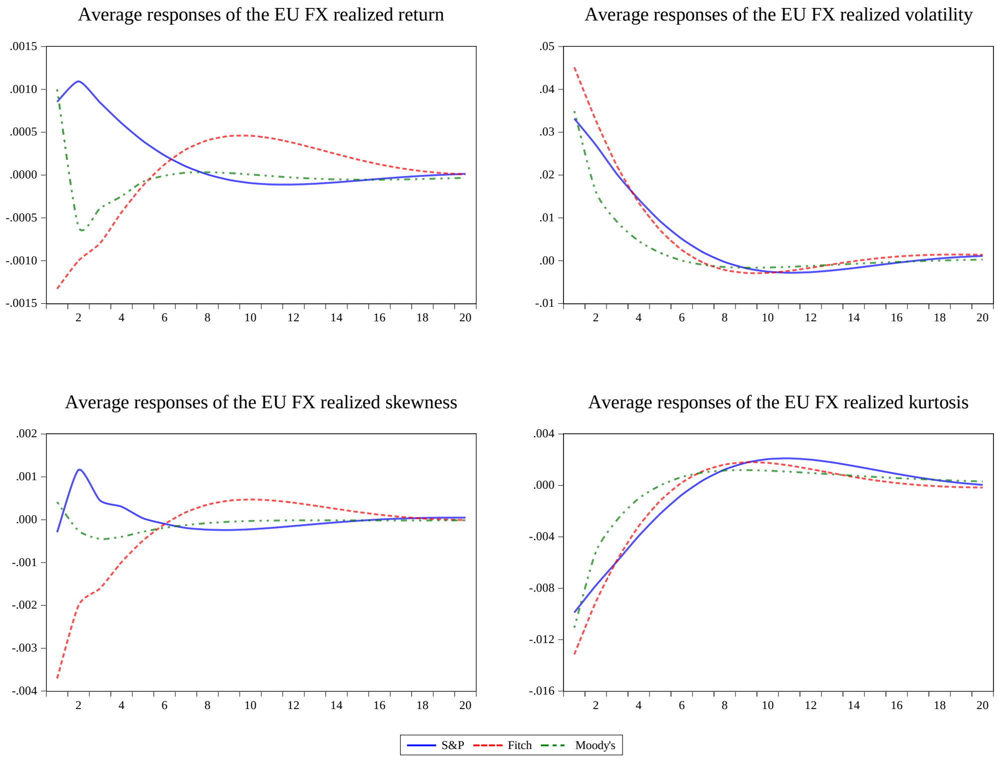


Figure 5.5b: Average responses of the EU FX realized moments to the shock in downward rating drift

Table 5.1: Linear scores transformation of the sovereign credit ratings and credit outlooks

Level of sovereign credit quality	Rating			Linear Scores		Credit outlook		
	S&P	Fitch	Moody's	Rating	Outlook	S&P	Fitch	Moody's
Highest quality	AAA	AAA	Aaa	20	0.5	Positive	Positive	Positive
High quality	AA+	AA+	Aa1	19	0.25	Rating Watch Positive	Rating Watch Positive	Review for Upgrade
	AA	AA	Aa2	18	0	Stable	Stable	Stable
	AA-	AA-	Aa3	17	-0.25	Rating Watch Negative	Rating Watch Negative	Review for Downgrade
Strong payment capacity	A+	A+	A1	16	-0.5	Negative	Negative	Negative
	A	A	A2	15				
	A-	A-	A3	14				
Adequate payment capacity	BBB+	BBB+	Baa1	13				
	BBB	BBB	Baa2	12				
	BBB-	BBB-	Baa3	11				
Likely to fulfil obligations, ongoing uncertainty	BB+	BB+	Ba1	10				
	BB	BB	Ba2	9				
	BB-	BB-	Ba3	8				
High credit risk	B+	B+	B1	7				
	B	B	B2	6				
	B-	B-	B3	5				
Very high credit risk	CCC+	CCC+	Caa1	4				
	CCC	CCC	Caa2	3				
	CCC-	CCC-	Caa3	2				
Near default with possibility of recovery	CC	CC C	Ca	1				
Default	SD	RD D	C	0				

Table 5.2: Transition probability matrices of sovereign rating drifts

Transit	Standard and Poor's			Fitch			Moody's		
from state	Stable	Upward	Downward	Stable	Upward	Downward	Stable	Upward	Downward
Stable	0.38	0.44	0.19	0.48	0.38	0.14	0.49	0.29	0.21
Upward	0.58	0.25	0.17	0.45	0.38	0.17	0.57	0.28	0.15
Downward	0.33	0.22	0.45	0.35	0.15	0.50	0.41	0.20	0.39

Note: This table presents the transition probability matrices of the sovereign rating drifts constructed as in formula (5.1) from sovereign ratings data provided by Standard and Poors, Fitch and Moody's. The drifts are categorised into three observable states, namely the Stable, Upward and Downward assessments of sovereign credit quality corresponding to zeros, positive and negative values of the sovereign ratings drifts. The reported transition probabilities are the probabilities that the states noted in the rows followed by the states noted in the columns of the table.

Table 5.3: Direct impact of sovereign rating drifts on the EU stock realized return

Countries	Upward rating drift			Downward rating drift		
	S&P	Fitch	Moody's	S&P	Fitch	Moody's
Austria	-0.483	-0.445	0.066	-0.009	-0.367	-0.188
	(-0.970)	(-0.747)	(0.354)	(-0.023)	(-0.893)	(-0.679)
France	0.371	0.334	**0.432**	-0.202	0.268	-0.138
	(1.103)	(1.315)	**(3.291)***	(-0.768)	(0.933)	(-0.626)
Germany	-0.309	0.410	0.300	-0.002	0.415	0.343
	(-0.640)	(0.881)	(1.630)	(-0.006)	(1.325)	(1.593)
Greece	**-2.357**	-0.516	0.192	0.571	**1.704**	0.009
	(-2.342)	(-0.651)	(0.765)	(1.292)	**(3.889)***	(0.020)
Ireland	0.135	-0.058	0.186	0.030	0.411	0.233
	(0.290)	(-0.141)	(1.386)	(0.134)	(1.446)	(0.906)
Netherlands	-0.238	-0.045	**0.392**	0.044	0.338	**0.368**
	(-0.619)	(-0.115)	**(2.924)***	(0.173)	(1.397)	**(2.081)**
Spain	**1.946**	**0.973**	0.099	-0.139	0.495	0.063
	(4.994)*	**(2.718)***	(0.500)	(-0.377)	(1.486)	(0.276)
The UK	-0.492	-0.139	-0.047	**-0.526**	0.041	**0.462**
	(-1.193)	(-0.525)	(-0.370)	**(-2.849)***	(0.131)	**(2.871)***
Hungary	-0.704	**-1.020**	0.096	0.343	0.261	-0.042
	(-1.167)	**(-2.494)**	(0.467)	(1.324)	(0.945)	(-0.147)
Romania	-1.084	**1.116**	0.132	0.512	-0.016	-0.099
	(-1.062)	**(1.752)***	(0.356)	(1.298)	(-0.019)	(-0.291)

Note: This table presents the estimates of the first element of the vector $\nabla^{(s_t)}$ and its associated *t*-statistic (in parentheses). These estimates are interpreted as the impact of upward and downward sovereign ratings drifts on the EU stock realized return as computed in formula (5.2). The sovereign rating drifts, which represent the assessments of the CRAs on overall EU sovereign credit quality, are constructed as in formula (5.1) from ratings data provided by Standard and Poor's (S&P), Fitch and Moody's. *, ** and *** denote significance at the 10, 5 and 1% levels, respectively.

Table 5.4: Direct impact of sovereign rating drifts on the EU FX realized return

Countries	Upward rating drift			Downward rating drift		
	S&P	Fitch	Moody's	S&P	Fitch	Moody's
Austria	**0.384**	0.015	0.032	-0.013	-0.038	**0.190**
	(2.967)* **	(0.096)	(0.444)	(-0.139)	(-0.275)	**(1.915)***
Belgium	**0.348**	-0.110	0.101	0.018	0.174	0.086
	(2.619)* **	(-0.683)	(1.376)	(0.220)	(1.441)	(0.931)
Bulgaria	**0.808**	0.249	**0.285**	0.056	0.050	0.118
	(3.482)* **	(1.622)	**(2.605)*** **	(0.564)	(0.365)	(1.271)
Cyprus	0.082	-0.017	-0.032	-0.008	-0.007	**0.165**
	(0.552)	(-0.105)	(-0.393)	(-0.097)	(-0.062)	**(1.846)***
Czech	**0.529**	-0.213	-0.002	0.050	0.141	0.017
	(2.283) **	(-0.888)	(-0.016)	(0.282)	(0.742)	(0.114)
Denmark	0.342	0.024	0.048	-0.043	-0.053	**0.159**
	(1.350)	(0.169)	(0.790)	(-0.423)	(-0.413)	**(1.804)***
France	**0.427**	-0.085	0.070	0.022	0.172	**0.161**
	(3.174)* **	(-0.565)	(1.010)	(0.234)	(1.467)	**(1.702)***
Germany	**0.361**	-0.055	0.016	0.023	0.118	**0.188**
	(2.769)* **	(-0.366)	(0.231)	(0.249)	(0.930)	**(2.021)** **
Greece	**0.385**	-0.034	0.066	-0.031	0.070	**0.210**
	(2.645)* **	(-0.225)	(0.931)	(-0.301)	(0.398)	**(2.429)** **
Hungary	**0.617**	**0.487**	0.061	0.138	0.150	**0.396**
	(2.550) **	**(2.408)** **	(0.438)	(0.664)	(0.615)	**(2.144)** **
Ireland	-0.098	-0.137	-0.019	0.092	0.091	**0.181**
	(-0.494)	(-0.726)	(-0.230)	(1.130)	(0.683)	**(1.857)***
Latvia	**0.657**	-0.030	0.068	0.057	0.066	0.126
	(6.429)* **	(-0.232)	(1.332)	(0.569)	(0.451)	(1.489)
Malta	0.265	0.025	**-0.207**	0.105	0.079	**0.272**
	(1.633)	(0.120)	**(-2.636)*** **	(0.990)	(0.530)	**(2.945)*** **
Netherlands	**0.266**	-0.002	-0.001	0.028	-0.063	0.060
	(2.172) **	(-0.014)	(-0.014)	(0.314)	(-0.467)	(0.560)
Poland	**0.516**	**-0.575**	0.054	0.111	0.137	0.017
	(2.315) **	**(-2.215)** **	(0.434)	(0.491)	(0.500)	(0.082)
Portugal	**0.303**	0.093	**0.138**	0.076	-0.038	**0.200**
	(2.215) **	(0.595)	**(1.978)** **	(0.883)	(-0.247)	**(2.085)** **
Romania	**0.831**	0.070	0.019	0.033	**0.273**	0.121
	(5.035)* **	(0.310)	(0.163)	(0.257)	**(1.767)***	(1.093)
Slovakia	**0.426**	-0.096	0.023	0.056	**0.318**	0.166
	(2.284) **	(-0.569)	(0.314)	(0.511)	**(2.414)** **	(1.368)
Spain	**0.349**	0.089	**0.155**	0.007	0.169	**0.235**
	(2.270) **	(0.550)	**(2.015)** **	(0.077)	(1.393)	**(2.394)** **
Sweden	**0.468**	0.122	0.019	0.018	0.101	0.045
	(2.565)* **	(0.638)	(0.262)	(0.151)	(0.690)	(0.321)
UK	**0.260**	-0.001	-0.012	-0.006	0.008	-0.011
	(2.029) **	(-0.006)	(-0.214)	(-0.066)	(0.076)	(-0.138)

Note: This table presents the estimates of the first element of the vector $\nabla^{(s_t)}$ and its associated *t*-statistic (in parentheses). These estimates are interpreted as the impact of upward and downward sovereign rating drifts on the EU FX realized return as computed in formula (5.2). The sovereign ratings drifts, which represent the assessments of the CRAs on overall EU sovereign credit quality, are constructed as in formula (5.1) from ratings data provided by Standard and Poor's (S&P), Fitch and Moody's. *, ** and *** denote significance at the 10, 5 and 1% levels, respectively.

Table 5.5: Direct impact of sovereign rating drifts on the EU stock realized volatility

Countries	Upward rating drift			Downward rating drift		
	S&P	Fitch	Moody's	S&P	Fitch	Moody's
Austria	-1.120	-1.152	**-0.430**	0.379	**1.234**	**1.071**
	(-1.214)	(-1.188)	**(-1.857)***	(1.611)	**(7.442)*****	**(6.860)*****
France	**0.945**	0.165	-0.255	**0.860**	**0.969**	**0.711**
	(1.740)*	(0.210)	(-0.636)	**(3.191)*****	**(3.224)*****	**(3.479)*****
Germany	0.783	0.378	0.053	**0.737**	0.356	-0.121
	(1.165)	(0.731)	(0.150)	**(2.124)****	(0.986)	(-0.320)
Greece	1.508	-0.912	-0.506	**0.757**	**2.242**	**1.498**
	(1.427)	(-1.072)	(-1.290)	**(2.473)****	**(6.465)*****	**(6.154)*****
Ireland	-0.233	**-1.440**	-0.138	0.134	0.394	**0.592**
	(-0.273)	**(-3.001)*****	(-0.457)	(0.345)	(1.411)	**(2.263)****
Netherlands	**1.525**	0.076	-0.224	0.550	**0.798**	-0.228
	(2.228)**	(0.089)	(-0.988)	(1.602)	**(1.920)***	(-0.926)
Spain	**2.640**	-0.667	-0.262	0.834	**2.054**	0.637
	(1.901)*	(-0.690)	(-0.581)	(1.225)	**(2.609)*****	(1.343)
The UK	-0.158	-0.636	**-0.421**	**0.389**	**0.733**	**0.598**
	(-0.122)	(-0.973)	**(-2.037)****	**(1.692)***	**(1.929)***	**(2.383)****
Hungary	0.842	-0.993	-0.277	0.203	**1.534**	**0.696**
	(0.948)	(-1.382)	(-0.782)	(0.469)	**(4.131)*****	**(1.774)***
Romania	-0.443	**-2.193**	-0.077	-0.414	0.523	0.091
	(-0.531)	**(-2.821)*****	(-0.187)	(-1.549)	(1.178)	(0.211)

Note: This table presents the estimates of the second element of the vector $\nabla^{(s_t)}$ and its associated *t*-statistic (in parentheses). These estimates are interpreted as the impact of upward and downward sovereign ratings drifts on realized stock market volatility as computed in formula (5.3). The sovereign ratings drifts, which represent the assessments of the CRAs on overall EU sovereign credit quality, are constructed as in formula (5.1) from ratings data provided by Standard and Poor's (S&P), Fitch and Moody's. *, ** and *** denote significance at the 10, 5 and 1% levels, respectively.

Table 5.6: Direct impact of sovereign rating drifts on the EU FX realized volatility

Countries	Upward rating drift			Downward rating drift		
	S&P	Fitch	Moody's	S&P	Fitch	Moody's
Austria	-0.799	**-1.140**	-0.082	**0.679**	**1.599**	**0.774**
	(-1.207)	**(-2.406)****	(-0.337)	**(2.895)*****	**(5.294)*****	**(2.949)*****
Belgium	-0.768	**-1.410**	-0.132	**0.871**	**1.430**	**1.364**
	(-1.228)	**(-3.015)*****	(-0.514)	**(3.471)*****	**(4.359)*****	**(4.804)*****
Bulgaria	-0.374	-1.014	-0.044	0.263	0.590	0.314
	(-0.470)	(-1.213)	(-0.120)	(0.608)	(1.317)	(0.708)
Cyprus	-0.532	**-0.828**	-0.126	0.038	**0.764**	0.457
	(-1.129)	**(-1.889)***	(-0.629)	(0.144)	**(2.247)****	(1.302)
Czech	**-1.969**	-0.285	-0.413	**0.711**	**0.719**	**0.476**
	(-2.814)***	(-0.456)	(-1.532)	**(2.728)*****	**(3.596)*****	**(2.270)****
Denmark	-0.630	**-1.143**	0.048	**0.590**	**0.672**	**0.896**
	(-1.501)	**(-2.253)****	(0.179)	**(3.148)*****	**(3.113)*****	**(4.089)*****
France	-0.719	**-1.286**	-0.135	**0.774**	**1.401**	**0.544**
	(-1.113)	**(-2.683)*****	(-0.514)	**(2.924)*****	**(4.467)*****	**(1.849)***
Germany	-0.752	**-1.665**	-0.171	**0.671**	**1.455**	**0.738**
	(-1.213)	**(-3.457)*****	(-0.817)	**(2.842)*****	**(5.970)*****	**(2.961)*****
Greece	-0.498	**-1.295**	0.048	**0.952**	**1.970**	**1.057**
	(-0.755)	**(-2.621)*****	(0.177)	**(3.728)*****	**(6.624)*****	**(3.615)*****
Hungary	**-1.586**	-1.006	-0.026	**0.553**	**1.227**	0.535
	(-2.424)**	(-1.497)	(-0.105)	**(1.854)***	**(4.066)*****	(1.466)
Ireland	-0.883	**-0.971**	0.023	**0.932**	**1.826**	0.461
	(-1.366)	**(-2.017)****	(0.085)	**(4.033)*****	**(6.375)*****	(1.493)
Latvia	**-1.916**	-0.131	-0.060	**0.670**	**0.871**	0.615
	(-2.667)***	(-0.194)	(-0.186)	**(1.845)***	**(2.288)****	(1.421)
Malta	-1.857	**-2.402**	0.178	**0.677**	**0.942**	**0.766**
	(-1.456)	**(-2.708)*****	(0.448)	**(1.723)***	**(2.296)****	**(2.315)****
Netherlands	-0.868	**-1.604**	-0.187	**0.592**	**1.554**	**1.470**
	(-1.401)	**(-3.288)*****	(-0.777)	**(2.305)****	**(5.421)*****	**(5.268)*****
Poland	-0.181	-1.175	0.072	**0.679**	**1.461**	0.598
	(-0.200)	(-1.552)	(0.180)	**(2.035)****	**(3.187)*****	(1.624)
Portugal	-0.807	**-1.107**	-0.145	**0.612**	**2.195**	**0.965**
	(-1.255)	**(-1.954)***	(-0.513)	**(2.056)****	**(7.594)*****	**(3.413)*****
Romania	0.325	0.081	-0.232	0.327	**1.994**	-0.427
	(0.230)	(0.065)	(-0.425)	(0.600)	**(3.318)*****	(-0.804)
Slovakia	**-2.038**	-0.953	-0.079	**0.633**	**1.001**	0.378
	(-3.361)***	(-1.621)	(-0.274)	**(2.056)****	**(4.268)*****	(0.890)
Spain	-0.843	**-1.164**	-0.113	**0.945**	**1.333**	**0.986**
	(-1.223)	**(-2.085)****	(-0.421)	**(3.843)*****	**(3.888)*****	**(3.294)*****
Sweden	-0.188	**-0.746**	-0.165	**0.359**	**0.848**	**0.426**
	(-0.373)	**(-1.864)***	(-0.850)	**(1.961)****	**(3.397)*****	**(2.051)*****
UK	**-1.216**	**-1.785**	-0.159	0.128	**0.574**	0.240
	(-2.415)**	**(-3.541)*****	(-0.686)	(0.546)	**(2.299)*****	(1.149)

Note: This table presents the estimates of the second element of the vector $\nabla^{(s_t)}$ and its associated *t*-statistic (in parentheses). These estimates are interpreted as the impact of upward and downward sovereign rating drifts on the EU FX realized volatility as computed in formula (5.3). The sovereign rating drifts, which represent the assessments of the CRAs on overall EU sovereign credit quality, are constructed as in formula (5.1) from ratings data provided by Standard and Poor's (S&P), Fitch and Moody's. *, ** and *** denote significance at the 10, 5 and 1% levels, respectively.

Table 5.7: Direct impact of sovereign rating drifts on the EU stock realized skewness

Countries	Upward rating drift			Downward rating drift		
	S&P	Fitch	Moody's	S&P	Fitch	Moody's
Austria	**1.089**	-0.620	-0.021	0.027	0.148	-0.213
	(3.453)* **	(-0.784)	(-0.103)	(0.116)	(0.556)	(-0.974)
France	0.557	-0.033	**0.248**	-0.065	0.027	-0.009
	(1.225)	(-0.108)	**(1.970)** **	(-0.319)	(0.134)	(-0.066)
Germany	0.307	-0.321	0.105	0.146	**0.487**	**0.329**
	(1.069)	(-0.974)	(0.669)	(0.510)	**(3.082)*** **	**(2.135)** **
Greece	-0.034	**1.503**	**0.698**	0.051	**0.898**	-0.194
	(-0.035)	**(2.641)*** **	**(3.159)*** **	(0.154)	**(3.308)*** **	(-0.482)
Ireland	**0.834**	**1.019**	-0.007	**0.333**	**0.700**	0.202
	(1.699)*	**(2.693)*** **	(-0.038)	**(1.884)***	**(3.849)*** **	(0.966)
Netherlands	**0.950**	-0.411	**0.307**	0.109	**0.302**	**0.290**
	(2.832)* **	(-0.985)	**(2.089)** **	(0.595)	**(1.715)***	**(2.115)** **
Spain	**3.790**	0.839	0.019	-0.094	**0.456**	**0.585**
	(7.638)* **	(1.353)	(0.087)	(-0.252)	**(1.650)***	**(2.731)*** **
The UK	-0.446	**-1.312**	-0.178	**-0.296**	-0.064	**0.466**
	(-0.724)	**(-2.199)** **	(-0.829)	**(-1.916)***	(-0.304)	**(2.951)*** **
Hungary	**0.622**	**0.719**	0.216	0.223	0.115	-0.124
	(1.645)*	**(2.613)*** **	(1.258)	(1.249)	(0.743)	(-0.573)
Romania	**-1.836**	0.933	0.252	**0.792**	**0.805**	0.292
	(-1.878)*	(1.541)	(0.730)	**(2.271)** **	**(2.567)** **	(0.741)

Note: This table presents the estimates of the third element of the vector $\nabla^{(s_t)}$ and its associated *t*-statistic (in parentheses). These estimates are interpreted as the impact of upward and downward sovereign rating drifts on the EU stock realized skewness as computed in formula (5.4). The sovereign ratings drifts, which represent the assessments of the CRAs on overall EU sovereign credit quality, are constructed as in formula (5.1) from ratings data provided by Standard and Poor's (S&P), Fitch and Moody's. *, ** and *** denote significance at the 10, 5 and 1% levels, respectively.

Table 5.8: Direct impact of sovereign rating drifts on the EU FX realized skewness

Countries	Upward rating drift			Downward rating drift		
	S&P	Fitch	Moody's	S&P	Fitch	Moody's
Austria	**0.481**	**0.392**	**0.286**	0.154	**0.226**	0.028
	(1.915)*	**(1.811)***	**(2.754)*****	(1.585)	**(1.884)***	(0.230)
Belgium	0.371	**0.502**	**0.291**	**0.225**	**0.386**	0.035
	(1.576)	**(2.392)****	**(2.962)*****	**(2.723)*****	**(3.366)*****	(0.371)
Bulgaria	**0.508**	**0.837**	**0.407**	**0.184**	0.162	0.055
	(1.746)*	**(3.729)*****	**(2.824)*****	**(1.670)***	(1.284)	(0.625)
Cyprus	0.050	-0.353	-0.135	**0.167**	**0.228**	0.099
	(0.191)	(-1.201)	(-1.319)	**(1.951)***	**(2.026)****	(1.054)
Czech	-0.060	-0.181	**0.330**	0.132	0.165	**0.188**
	(-0.158)	(-0.598)	**(2.929)*****	(0.984)	(1.270)	**(2.093)****
Denmark	0.184	0.257	0.089	0.057	**0.230**	**0.269**
	(0.107)	(1.310)	(0.979)	(0.479)	**(2.136)****	**(2.833)*****
France	**0.418**	**0.467**	**0.332**	**0.211**	**0.336**	0.050
	(1.685)*	**(2.323)****	**(3.530)*****	**(2.318)****	**(2.857)*****	(0.501)
Germany	0.320	**0.416**	**0.185**	**0.189**	**0.262**	0.065
	(1.350)	**(2.150)****	**(1.797)***	**(2.021)****	**(2.302)****	(0.625)
Greece	0.264	0.262	**0.262**	**0.180**	0.206	0.137
	(1.273)	(1.300)	**(2.633)*****	**(1.817)***	(1.566)	(1.592)
Hungary	-0.551	**0.563**	-0.015	0.051	-0.228	-0.085
	(-1.453)	**(2.267)****	(-0.101)	(0.351)	(-1.034)	(-0.605)
Ireland	-0.335	-0.254	0.012	**0.254**	**0.254**	0.138
	(-1.070)	(-1.074)	(0.112)	**(3.293)*****	**(2.099)****	(1.468)
Latvia	**0.637**	**0.323**	-0.045	0.053	0.108	**0.144**
	(4.068)***	**(2.561)****	(-0.529)	(0.821)	(1.024)	**(2.344)****
Malta	-0.172	-0.147	-0.240	**0.278**	0.180	**0.224**
	(-0.382)	(-0.366)	(-1.450)	**(2.129)****	(0.932)	**(1.753)***
Netherlands	**0.364**	**0.538**	**0.213**	**0.172**	0.155	0.066
	(1.659)*	**(2.516)****	**(2.017)****	**(1.723)***	(1.243)	(0.606)
Poland	-0.373	**-0.699**	-0.174	-0.106	-0.244	-0.205
	(-0.914)	**(-1.995)****	(-1.166)	(-0.600)	(-1.203)	(-1.417)
Portugal	0.317	**0.416**	**0.341**	**0.266**	0.198	0.122
	(1.361)	**(2.034)****	**(3.212)*****	**(3.003)*****	(1.604)	(1.051)
Romania	**1.459**	0.560	0.026	0.224	0.313	0.147
	(2.234)**	(0.781)	(0.112)	(1.499)	(1.432)	(0.503)
Slovakia	**0.650**	**0.532**	**0.312**	0.204	**0.252**	0.131
	(2.434)**	**(2.544)****	**(3.319)*****	(1.491)	**(2.204)****	(0.691)
Spain	0.391	**0.651**	**0.362**	**0.236**	**0.328**	**0.192**
	(1.459)	**(3.123)*****	**(3.487)*****	**(2.766)*****	**(2.901)*****	**(2.051)****
Sweden	0.246	**0.713**	0.110	**0.194**	0.149	-0.114
	(0.984)	**(2.223)****	(1.120)	**(1.735)***	(1.174)	(-1.041)
UK	**0.564**	0.076	0.049	0.130	-0.043	-0.004
	(2.512)**	(0.272)	(0.490)	(1.140)	(-0.328)	(-0.030)

Note: This table presents the estimates of the third element of the vector $\nabla^{(s_t)}$ and its associated *t*-statistic (in parentheses). These estimates are interpreted as the impact of upward and downward sovereign ratings drifts on the EU FX realized skewness as computed in formula (5.4). The sovereign ratings drifts, which represent the assessments of the CRAs on overall EU sovereign credit quality, are constructed as in formula (5.1) from ratings data provided by Standard and Poor's (S&P), Fitch and Moody's. *, ** and *** denote significance at the 10, 5 and 1% levels, respectively.

Table 5.9: Direct impact of sovereign rating drifts on the EU stock realized kurtosis

Countries	Upward rating drift			Downward rating drift		
	S&P	Fitch	Moody's	S&P	Fitch	Moody's
Austria	-0.516	-0.308	0.112	-0.196	**-0.467**	**-0.423**
	(-1.232)	(-0.631)	(1.014)	(-1.308)	**(-3.650)*** **	**(-5.133)*** **
France	-0.386	-0.290	0.297	**-0.673**	**-0.418**	**-0.512**
	(-1.255)	(-0.755)	(1.541)	**(-4.689)*** **	**(-2.368)** **	**(-4.008)*** **
Germany	-0.475	-0.446	-0.061	**-0.455**	-0.180	-0.019
	(-1.380)	(-1.636)	(-0.302)	**(-2.369)** **	(-0.874)	(-0.094)
Greece	**-1.736**	**0.879**	**0.374**	-0.155	**-0.821**	**-1.060**
	(-3.064)* **	**(2.264)** **	**(2.755)*** **	(-0.683)	**(-3.891)*** **	**(-6.451)*** **
Ireland	-0.335	**0.675**	-0.039	-0.191	**-0.346**	**-0.432**
	(-0.883)	**(2.955)*** **	(-0.321)	(-1.151)	**(-2.033)** **	**(-3.247)*** **
Netherlands	**-0.711**	-0.027	**0.171**	**-0.574**	-0.367	0.132
	(-2.063) **	(-0.059)	**(1.672)* **	**(-3.329)*** **	(-1.535)	(1.172)
Spain	-0.201	-0.224	0.069	0.158	**0.807**	**0.577**
	(-0.263)	(-0.312)	(0.307)	(0.412)	**(2.680)*** **	**(2.353)** **
The UK	0.243	0.072	**0.254**	**-0.462**	**-0.593**	**-0.440**
	(0.363)	(0.208)	**(2.804)*** **	**(-3.763)*** **	**(-2.928)*** **	**(-3.150)*** **
Hungary	-0.066	0.269	0.117	-0.126	**-0.650**	**-0.318**
	(-0.171)	(0.806)	(0.783)	(-0.560)	**(-3.215)*** **	**(-1.648)* **
Romania	0.421	**1.102**	-0.090	0.177	-0.122	0.111
	(0.946)	**(2.402)** **	(-0.429)	(0.745)	(-0.214)	(0.384)

Note: This table presents the estimates of the fourth element of the vector $\nabla^{(s_t)}$ and its associated *t*-statistic (in parentheses). These estimates are interpreted as the impact of upward and downward sovereign rating drifts on the EU stock realized kurtosis as computed in formula (5.5). The sovereign rating drifts, which represent the assessments of the CRAs on overall EU sovereign credit quality, are constructed as in formula (5.1) from ratings data provided by Standard and Poor's (S&P), Fitch and Moody's. *, ** and *** denote significance at the 10, 5 and 1% levels, respectively.

Table 5.10: Direct impact of sovereign rating drifts on the EU FX realized kurtosis

Countries	Upward rating drift			Downward rating drift		
	S&P	Fitch	Moody's	S&P	Fitch	Moody's
Austria	0.241	**0.458**	0.084	-0.046	-0.199	-0.183
	(0.913)	**(2.413)****	(0.669)	(-0.451)	(-1.234)	(-1.410)
Belgium	0.134	**0.461**	0.083	**-0.282**	**-0.530**	**-0.339**
	(0.432)	**(1.989)****	(0.545)	**(-2.231)****	**(-2.964)*****	**(-2.471)****
Bulgaria	-0.073	**0.677**	-0.246	0.074	-0.090	-0.192
	(-0.168)	**(1.966)****	(-0.915)	(0.393)	(-0.374)	(-0.837)
Cyprus	0.436	0.002	0.128	**0.231**	0.223	0.070
	(1.551)	(0.008)	(0.958)	**(1.704)***	(1.148)	(0.512)
Czech	**0.830**	0.253	0.168	-0.124	-0.107	0.055
	(2.384)**	(0.776)	(1.469)	(-0.913)	(-0.540)	(0.373)
Denmark	**0.361**	0.201	0.044	-0.087	0.026	-0.316
	(2.540)**	(0.736)	(0.351)	(-0.716)	(0.158)	(-2.733)
France	0.031	0.304	0.089	-0.163	**-0.456**	-0.134
	(0.108)	(1.396)	(0.612)	(-1.471)	**(-2.902)*****	(-1.076)
Germany	0.112	**0.319**	0.106	-0.025	-0.197	**-0.255**
	(0.501)	**(1.702)***	(1.245)	(-0.246)	(-1.640)	**(-2.404)****
Greece	0.375	**0.397**	0.003	**-0.220**	**-0.468**	**-0.421**
	(1.562)	**(1.927)***	(0.023)	**(-2.092)****	**(-3.391)*****	**(-3.661)*****
Hungary	**0.666**	0.296	**0.429**	-0.261	**-0.846**	**-0.379**
	(2.505)**	(1.021)	**(3.732)*****	(-1.429)	**(-4.224)*****	**(-1.983)****
Ireland	0.179	**0.347**	0.045	**-0.321**	**-0.633**	**-0.321**
	(0.653)	**(1.759)***	(0.319)	**(-2.832)*****	**(-4.146)*****	**(-2.111)****
Latvia	-0.185	**-0.396**	-0.015	-0.008	0.086	0.161
	(-0.785)	**(-1.752)***	(-0.136)	(-0.064)	(0.711)	(1.419)
Malta	0.363	**0.922**	0.108	**-0.345**	-0.455	**-0.414**
	(0.765)	**(2.449)****	(0.606)	**(-1.840)***	(-1.427)	**(-2.115)****
Netherlands	0.091	**0.382**	0.094	0.016	-0.175	**-0.320**
	(0.391)	**(1.689)***	(0.889)	(0.155)	(-1.250)	**(-2.697)*****
Poland	0.305	0.569	0.068	-0.232	**-0.654**	-0.061
	(0.621)	(1.529)	(0.332)	(-1.323)	**(-3.577)*****	(-0.317)
Portugal	0.389	0.306	0.081	**-0.308**	**-0.465**	**-0.413**
	(1.224)	(1.066)	(0.506)	**(-2.074)****	**(-2.964)*****	**(-2.560)****
Romania	-0.717	-0.080	-0.116	-0.308	**-0.831**	-0.434
	(-0.996)	(-0.097)	(-0.357)	(-0.825)	**(-2.429)****	(-1.295)
Slovakia	**0.736**	0.355	0.090	-0.127	0.007	-0.205
	(2.909)***	(1.383)	(0.647)	(-0.902)	(0.047)	(-1.340)
Spain	0.291	0.151	0.107	**-0.391**	**-0.613**	**-0.413**
	(0.889)	(0.537)	(0.647)	**(-3.104)*****	**(-3.234)*****	**(-3.040)*****
Sweden	**0.403**	**0.348**	0.113	-0.027	-0.172	**-0.257**
	(1.882)*	**(2.008)****	(1.115)	(-0.241)	(-0.975)	**(-2.407)****
UK	**0.340**	**0.370**	0.084	0.042	-0.145	-0.123
	(1.666)*	**(2.102)****	(0.771)	(0.371)	(-1.127)	(-1.187)

Note: This table presents the estimates of the fourth element of the vector $\nabla^{(s_t)}$ and its associated *t*-statistic (in parentheses). These estimates are interpreted as the impact of upward and downward sovereign rating drifts on the EU FX realized kurtosis as computed in formula (5.5). The sovereign rating drifts, which represent the assessments of the CRAs on overall EU sovereign credit quality, are constructed as in formula (5.1) from ratings data provided by Standard and Poor's (S&P), Fitch and Moody's. *, ** and *** denote significance at the 10, 5 and 1% levels, respectively.

Part B - Proofs

B1. Proof of Lemma 5.1

Under the notations presented in section 5.4.3, the model (5.9) can be written in a compact form as,

$$X = BZ + U \tag{5.14}$$

Given a fixed d, we have $\hat{B} = XZ'(ZZ')^{-1}$ as the Multivariate LS estimator of the model (5.14), then the estimated residuals are, $\hat{U} = X - \hat{B}Z$. We derive the following relationship,

$$X - BZ = X - \hat{B}Z + \hat{B}Z - BZ = \hat{U} + (\hat{B} - B)Z$$

Therefore,

$$(X - BZ)(X - BZ)' = \left(\hat{U} + (\hat{B} - B)Z\right)\left(\hat{U} + (\hat{B} - B)Z\right)' = \hat{U}\hat{U}' + (\hat{B} - B)ZZ'(\hat{B} - B)'$$

It then can be seen that,

$$\left|\Sigma_{\varepsilon|d}(d, B)\right| = \left|N^{-1}(X - BZ)(X - BZ)'\right| = \left|N^{-1}\left(\hat{U}\hat{U}' + (\hat{B} - B)ZZ'(\hat{B} - B)'\right)\right| \geq \left|N^{-1}\hat{U}\hat{U}'\right|$$

Hence, the minimum of $\left|\Sigma_{\varepsilon|d}(d, B)\right|$ is $\left|T^{-1}\hat{U}\hat{U}'\right|$, or equivalently,

$$\left|\Sigma_{\varepsilon|d}(d, B)\right|_{\min} = \left|N^{-1}(X - \hat{B}Z)(X - \hat{B}Z)'\right|, \text{ which is achieved at } \hat{B} = XZ'(ZZ')^{-1}.$$

This completes the proof of Lemma 5.1.

B2. Proof of Proposition 5.1

Under the representation (5.9), the conditional probability density function of Y_t is expressed as,

$$f(Y_t \mid \Omega_{t-1}) = (2\pi)^{-K/2} \mid \Sigma_\varepsilon \mid^{-1/2} \exp\left\{-\frac{1}{2}\left[A(L)D(L)Y_t - \nabla R_t\right]' \Sigma_\varepsilon^{-1}\left[A(L)D(L)Y_t - \nabla R_t\right]\right\}$$

where Σ_ε is the variance-covariance matrix of ε_t in the case of FIVARX model.

The log likelihood function is,

$$l(d,B) = -\frac{KN}{2}\ln(2\pi) - \frac{N}{2}\ln\left|\Sigma_\varepsilon\right| - \frac{1}{2}\sum_{t=1}^{N}\left(\left[A(L)D(L)Y_t - \nabla R_t\right]' \Sigma_\varepsilon^{-1}\left[A(L)D(L)Y_t - \nabla R_t\right]\right)$$

It can be easily to prove that,

$$\frac{1}{2}\sum_{t=1}^{N}\left(\left[A(L)D(L)Y_t - \nabla R_t\right]' \Sigma_\varepsilon^{-1}\left[A(L)D(L)Y_t - \nabla R_t\right]\right) = \frac{N}{2} tr\left[\Sigma_\varepsilon^{-1}\Sigma_\varepsilon(d,B)\right]$$

where the $tr(.)$ indicates the trace operator.

Hence, the log likelihood function can be rewritten as,

$$l(d,B) = -\frac{KN}{2}\ln(2\pi) - \frac{N}{2}\ln\left|\Sigma_\varepsilon\right| - \frac{N}{2} tr\left[\Sigma_\varepsilon^{-1}\Sigma_\varepsilon(d,B)\right]$$

Following Lemma A.6 of Johansen (1995) and the linearity of the trace operator and the strict concavity of a natural logarithm of a matrix determinant noted in Magnus and Neudecker (1988, p.222), the log likelihood function $l(d,B)$ is uniquely maximized by, $\Sigma_\varepsilon = \Sigma_\varepsilon(d,B)$.

We have a concentrated log likelihood function with respect to $\Sigma_\varepsilon(d,B)$ as,

$$l^c(d,B) = -\frac{KN}{2}\ln(2\pi) - \frac{N}{2}\ln\left|\Sigma_\varepsilon(d,B)\right| - \frac{KN}{2}$$

So,

$$l^c(d,B) = -\frac{KN}{2}\left[\ln(2\pi)+1\right] - \frac{N}{2}\ln\left|\Sigma_\varepsilon(d,B)\right| \tag{5.15}$$

According to Lemma 5.1, the variance-covariance matrix of error term concentrated on *d* can be represented under the form as,

$$\begin{aligned}\Sigma_\varepsilon(d) &= T^{-1}(X-\hat{B}Z)(X-\hat{B}Z)' \\ &= T^{-1}X\left(I_N - Z'(ZZ')^{-1}Z\right)X'\end{aligned} \tag{5.16}$$

Replace (5.16) in (5.15), the concentrated log-likelihood function with respect to the vector of memory parameters *d* of a FIVARX model is represented under the form as,

$$l^c_{FIVARX}(d) = -\frac{KN}{2}\left[\ln(2\pi)+1\right] - \frac{N}{2}\ln\left|\Sigma_\varepsilon(d)\right| \tag{5.17}$$

This completes the proof of Proposition 5.1.

Chapter 6

Conclusion

6.1 Introduction

This thesis aims to model the responses of financial return distributions to exogenous shocks under various forms of news which hit the financial markets. More specifically, our studies focus on three scenarios: (i) when there is an exogenous shock in each of the higher moments; (ii) when there is an arrival of hidden information to the market; and (iii) when there is a change in the overall sovereign credit re-ratings. Whilst the issues related to the first moment of return distribution are readily and widely investigated, analyses regarding the higher moments are more challenging as they are unobservable. Traditionally, higher moments of a financial return distribution are conditionally estimated by employing parametric or semi-parametric models. However, this approach may face some drawbacks which have been discussed in previous chapters. Recently, an introduction of realized higher moments, which are the higher moments constructed non-parametrically from intraday returns, has provided an alternative approach to overcome the problems. Hence, we exploit the advantages of these realized measures to serve our investigations of the behavior of the financial return distributions under various market conditions.

As realized measures exhibits a mixture of long- and short-memory behaviors, it is a need to employ a family of multivariate long memory framework, which allow for a set of flexible memory degrees (degrees of fractional integration), for modeling purposes. Similarly, to further capture for the regime switching behavior of the overall sovereign credit assessment in our third analysis, we also propose a multivariate framework that allows for not only a mixture of long- and short-memory but also a Markov regime switching property. Yet, there has not been a suitable tool within the frameworks to aid our investigations, where impulse response analyses need to be conducted but contemporaneous relationships between variables cannot be theoretically pre-determined. Hence, we develop a generalized impulse response function and its asymptotic distribution within a multivariate long memory framework to satisfy our need. This function does not require us to determine the contemporaneous relationships between endogenous variables at the first stage. Still, it provides an unique result regardless of alternative orderings of endogenous variables in the system. Further, the function can adequately captures a mixture of the long- and short-memory properties.

Our studies are important as they can be beneficial for other financial activities such as asset pricing, value-at-risk measurement and asset allocation. This is because the dynamics of higher return moments have been documented to significantly affect asset prices as mentioned in previous chapters. Our research further emphasizes policy implications in light of the increased role of informational transmission mechanism (e.g., higher moment risks transmission, trading volume impacts and sovereign credit ratings impacts) under the Basel II and III banking regulatory framework for assessing capital adequacy requirements and for prescribing investment grade in financial institutions.

The remainder of this concluding chapter is organized as follows. Section 6.2 summarizes key findings in a relation to the research questions noted in Chapter 1. Section 6.3 provides a brief discussion of some issues that require further analysis and some recommendations for future research.

6.2 Key findings

We provide an investigation of the financial markets linkages via higher moments with a particular focus on stock and currency markets in Chapter 2. Chapter 3 develops a new methodology of the impulse response analysis in a multivariate long memory framework to facilitate empirical analyses conducted in Chapter 4, which aims to answer the question "How does trading volume affect the financial return distributions?" in terms of both static and dynamic approach. In Chapter 5, a study on how EU financial return distributions react to overall EU sovereign credit re-ratings changes is conducted by employing our new proposed framework, which allows a multivariate system of long- and short-memory processes to be conditional on observable regimes.

Overall our empirical results show differences in the behavior of developed versus emerging market groups with stronger impacts on realized volatility and realized kurtosis in developed markets and realized skewness in emerging markets. In addition our results show important differences in the behavior of stock and foreign exchange markets, and also the key roles played by trading volume and sovereign credit rating changes in and across markets. A summary of answers for the research questions stated in Chapter 1 is presented as follows.

6.2.1 How do financial markets link and cross-link via higher moments?

Empirical results in Chapter 2 show strong evidence of the positive linkages among stock markets via all three higher moments (i.e., volatility, skewness and kurtosis) in both emerging and developed market groups. Similar results are found regarding the FX markets linkages. However, the developed FX market groups provide considerably greater evidence of realized volatility and kurtosis linkages than the emerging FX markets group.

Regarding the cross-link between financial markets, stock and FX markets in emerging countries groups are more likely to be linked via realized skewness; whereas, their cross-linkages in developed countries groups tend to be established through realized volatility and kurtosis. These results are consistent with the importance of the downside risk in emerging markets, which was documented in the literature (e.g., Estrada, 2002; Galagedera and Brooks, 2007). Notably, the cross-asset market linkages via realized volatility and kurtosis are positive but negative via realized skewness. This empirical result suggests an option for investors to diversify the downside risk by combining both stock and currency assets in their portfolio, especially in emerging markets.

In terms of the strength of the linkages via all three higher moments, whilst the emerging market groups often show no obvious difference, the developed market groups consistently display that the magnitude of the same asset markets linkages is usually greater than or at least equal to that of the cross-asset markets linkages. This is consistent with our expectation as there are more common economic factors that drive the same asset markets than the cross-assets markets. In addition, emerging markets, with its low degree of market transparency, often contain much more noisy information than developed markets. Hence, there may be insignificant difference between impacts of common economic factors on same asset linkages and that on cross-asset linkages in term of emerging market groups.

6.2.2 How does the hidden information arrival affect financial returns distributions?

To answer the question, Chapter 4 employs trading volume as a proxy of hidden information arrival into the market and investigates its impacts on financial return distributions in a regional context. Our empirical findings support the volume – volatility literature, which evidences their positive relationship within stock or FX markets. By the impulse response analyses, we interpret the information based theories (i.e., MDH, SAIH and DOH) as complementary hypotheses and enhance the volume – volatility literature with evidence of their positive and lead-lag relationship not only within but also between stock and FX markets.

We find lack of support for the volume – skewness interactions in regional-level analyses, which leads us to hypothesize that the direct impact of trading volume on the level of negative skewness is less significant for a better diversified portfolio. This hypothesis has not been tested yet, it, however, provide an exciting direction for our future research.

In addition, we find a negative relationship between trading volume and realized kurtosis. We suppose that this result may imply an application of the DOH, where higher dispersion of beliefs among traders leads to lower concentration of asset returns around its mean value.

The evidence of interactions among higher moments leads us to extend the analysis by also investigating an impact of trading volume on such inter-relationships. By using the spill-over index as a proxy for a dynamic structure of the higher moments' inter-relationship, empirical results in this regard provide understandings about how trading volume alters spill-over from one higher moment risk to others. We find that although behaviors of the inter-relationship towards significant events and new policies are robust, its strength is mostly reduced by the trading volume. This is mainly due to a decline in the proportion (%) of spill-over from realized kurtosis to other moments; or equivalently, an increase in the proportion (%) of spill-over from realized kurtosis to itself in next periods. This finding can be fundamentally explained by a prominent result found in the volume – GARCH effect literature (e.g., Lamoureux and Lastrapes, 1990), which documents trading volume is a source of heteroskedasticity problem in the return volatility.

6.2.3 How do the sovereign credit quality assessments affect the financial returns distributions?

The sovereign credit quality assessments are found to have heterogeneous effects on financial return distributions across regimes, which are defined to correspond to the upward and downward trends in sovereign rating drifts by individual CRAs. More specifically, we mostly find a negative relationship between the overall EU sovereign credit assessments and realized returns in the upward regime, yet positive relationships in the downward regime. Even though these relationships tend to be statistically insignificant, their negativity is consistent with the basic risk-return tradeoff theory in finance. This consistency is further confirmed by the empirical results of sovereign ratings impacts on realized volatility, which show negative effects in the upward regime but positive effects in the downward regime. In addition, more evidence of statistically significant effects of sovereign ratings on realized volatility in the downward regime compared to the upward regime indicate an asymmetric response to rating news in both

EU stock and FX markets. In conjunction with the positive direction of effects in the downward regime, ratings may be particularly destabilizing financial markets during chaos periods. Regarding the impacts on realized skewness, the EU stock and FX return distributions will be more skewed to the right as long as the trend of overall EU sovereign credit quality changes, regardless of the direction. Meanwhile, the peak of the EU stock and FX return distributions will be significantly lower (higher) corresponding to an increase in the downward (upward) sovereign rating drifts.

In term of the CRA's reputation, we find that the Standard and Poor's rating actions have the greatest impact on stock market realized return and skewness; whereas, assessments of Fitch have strongest effects on stock market realized volatility across the EU. Meanwhile, Moody's rating activities most influence the EU financial return distributions during the recent European sovereign debt crisis. In the FX markets, Standard and Poor's and Fitch, however, are consistently the most dominant CRAs.

6.3 Future research

6.3.1 Portfolio diversification and volume – skewness relationship

Empirical findings related to the volume – skewness relationship in Chapter 4 lead us to hypothesize that the level of portfolio diversification should be incorporated in the investor heterogeneity hypothesis proposed by Hong and Stein (2003). More specifically, we conjecture that trading volume' impact on the negative skewness is less significant for a better diversified portfolio. Although this hypothesis has not been tested in this thesis, its rationale is briefly explained in Chapter 4, section 4.5.2. Hence, in our future research, it would be interesting to perform empirical tests for the proposed hypothesis under different markets and market' conditions. In case the hypothesis is successfully verified, we would extend the model of Hong and Stein

(2003) to accommodate the level of portfolio diversification in capturing the volume – skewness relationship.

However, how to correctly measure the degree of portfolio diversification would be challenging. One possible approach is to use a proxy of the portfolio diversification such as portfolio residual variance, which is calculated as the difference between the total portfolio variance and the market-related variance, (see for example, Klemkosky and Martin, 1975). Another approach is to construct an index of portfolio diversification (e.g., Woerheide and Persson, 1993; Rudin and Morgan, 2006). Nevertheless, its efficiency and reliability are still questionable. Our plan is first to construct a consistent and reliable proxy for capturing the degree of the portfolio diversification. The two following approaches are then can be employed to verify our hypothesis. Firstly, regression analysis is utilized to capture the interaction between realized skewness and trading volume with and without controlling the degree of portfolio diversification. The empirical results would answer the question whether the level of portfolio diversification plays role in the volume – skewness relationship. Secondly, the dynamic interactions between realized skewness and trading volume can be captured by the spill-over index using the methodology presented in Chapter 4, section 4.6.1. The two indices (i.e., index of portfolio diversification and the spill-over index) are then used to investigate the long- and short-run relationships by performing the (fractional) cointegration and Error Correction Model depending on their degrees of (fractional) integration (see for example, Johansen, 1995; Johansen and Nielsen, 2012).

6.3.2 Estimation in a multivariate long memory model: a hybrid approach

As we have discussed in Chapter 2 and 3, methods for estimating the FIVAR model can be generally classified into two broad classifications: the one- and the two-step estimation approach. Each approach has its own advantages and disadvantages. Although the one-step estimation method, such as the exact maximum likelihood estimation, is theoretically efficient, it is practically limited by the sample size and the dimension of the system. Likewise, even though the two-step estimation method does not accommodate the specification of the model in the first step which aims to estimate the vector of fractional degrees, the optimization procedure converges much faster than the one-step estimation method. Being experienced a difficult choice among the possible estimation methods in Chapter 2; our future plan is to develop an alternative procedure which may overcome the limitations of both methods. In a spirit of Chapter 5, Proposition 5.1, we obtain the conditional log likelihood function of a FIVAR model then concentrate it with respect to the vector of fractional degrees. The concentrated log likelihood function is subsequently maximized to obtain the fractional degrees using the numerical optimization procedure. In the latter stage, estimates of fractional degrees can be used to extract the estimates of remaining parameters. According to this proposed approach, we may preserve the benefit of two-step estimation method (e.g., speed of optimization procedure). Meanwhile, we can still take an advantage of the one-step estimation approach by including the specification of the model in the estimation of the fractional degrees.

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