ABSTRACT

The issue of digital piracy for commercial profit has received a lot of attention in the industrial organisation literature over the last few years. Illegal competition from pirates reduces the revenue of the legal firm and this has been a concern of the digital media and software industry globally. This thesis is motivated by the observation of three important gaps in the literature on commercial piracy. First, pirated products are usually imperfect substitutes of the legal product. This implies that the impact of a change in the legal product’s price on the demand for the pirated product will be different from the impact of a change in the price of the pirated product on the legal firm’s demand. The existing literature on product differentiation, however, does not consider such kind of asymmetric product differentiation. Second, empirical evidences show that firms often need to incur costs to document and report signs of piracy, and then the regulatory authority decides whether or not and how much to invest in monitoring which, in turn, affects the probability that the pirate is being caught and convicted. The implications of costly reporting by the legal firm and ex-post monitoring by the authority have not been investigated. Third, while the importance of piracy in affecting innovation has been widely recognised, there has not been much work that directly models the impact of piracy on firms that engage in competition for innovation. This thesis presents three theoretical essays that attempt to fill these gaps in the literature.

The first essay analyses the effects of the degree of asymmetric product differentiation on the profit ordering of two competing firms in a Stackelberg model to investigate whether a leader earns a higher or a lower profit than a follower. The model captures asymmetric product differentiation by two cross-price effect parameters, and this two-dimensional model generates novel results that are absent in the literature on profit-ordering with symmetric product differentiation. For example, we find that for every level of the cross-price effect of the change in the leader’s price on the follower’s quantity there exists a critical value for the cross-price effect of the change in the follower’s price on the leader’s quantity, such that below this critical level the leader earns a higher profit than the follower, but the reverse is true above this critical value.

The second essay introduces piracy in a market with asymmetric product differentiation. It examines the regulatory authority’s decision on whether to monitor piracy following
the legal firm’s report of the presence of a pirate in the market. In the model, the authority decides whether or not and how much to invest in monitoring in order to maximise a politically motivated objective function. We show that with ex-post monitoring the government will monitor piracy under both price and quantity competition if the legal firm’s political influence is sufficiently high. We also find that, when holding the cross-price effect of the legal firm’s price on the pirate’s output constant, there exists a unique level of the cross-price impact of the pirate’s price on the legal firm’s output, above which monitoring will be higher under quantity competition, and below which monitoring will be higher under price competition. Moreover, we show that when the government can credibly commit to monitor piracy the legal firm’s investment on innovation is higher under quantity competition than under price competition.

The last essay analyses the effect of piracy on innovation when two legal firms engage in R&D competition. When a single firm is deciding how much to invest in R&D which will stochastically determine whether innovation will be successful, piracy unambiguously retards innovation. However, we show that when two legal firms are engaging in a patent race, piracy may enhance innovation. We also show that if the difference between the efficiency parameters of the innovating firms is relatively large then piracy increases aggregate R&D investment and the profit of the less efficient firm.
DECLARATION OF AUTHORSHIP

I hereby declare that this thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other institution, and that to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

ISHITA CHATTERJEE

JANUARY 2010

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TO MY FATHER
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Chapter 1 Introduction
1 Introduction

Digital piracy for commercial profit has been an important issue in the industrial organisation literature over the last few years. That piracy reduces the revenue of the legal firm has been a concern of the digital media and software industry globally. The present thesis looks into three important gaps in the literature on commercial piracy. First, pirated products are usually imperfect substitutes of the legal product. This implies that the impact of a change in the legal product’s price on the demand for the pirated product will be different from the impact of a change in the price of the pirated product on the legal firm’s demand. The existing literature on product differentiation, however, does not consider such kind of asymmetric product differentiation. Second, empirical evidences show that firms often need to incur costs to document and report signs of piracy, and then the regulatory authority decides whether or not and how much to invest in monitoring which, in turn, affects the probability that the pirate is being caught and convicted. The implications of costly reporting by the legal firm and ex-post monitoring by the authority have not been investigated in the literature. Third, while the importance of piracy in affecting innovation has been widely recognised, there has not been much work that directly models the impact of piracy on firms that engage in competition for innovation. This thesis presents three theoretical essays that attempt to fill these gaps in the literature.

The first essay analyses the effects of the degree of asymmetric product differentiation on the profit ordering of two competing firms in a Stackelberg model to investigate whether a leader earns a higher or a lower profit than a follower. Because whether a leader earns a higher or a lower profit than a follower in a duopoly market has important implications for competition strategies and how firms locate their products in the
market, there is a well-established literature that studies this issue of “profit ordering.” This literature features product differentiation but assumes that the cross-price effects are always symmetric. That is, it assumes that the effect of a change in the price of firm 1 on the quantity of firm 2 is the same as the effect of a change in the price of firm 2 on the quantity of firm 1. However, there are many real world examples that feature asymmetric cross price effects. For example, Kang (1989) found that as price of mainframes went up more people switched over to minicomputers, but as price of minicomputers went up, not so many people switched to mainframe, implying that minicomputers are “better” substitutes to mainframes than mainframes are to minicomputers. And, in the case of complements, operating systems are much “better” complements to media softwares as changes in prices of media softwares do not impact the demand for operating systems as price changes of operating systems affect the demand for media softwares.

Building on this observation, we move away from a one-dimensional concept of product differentiation and construct a model with two-dimensional product differentiation and re-examine the question of profit ordering in this new framework. This framework generates some novel non-monotonic results that have not been reported in the literature on profit-ordering with symmetric cross-price effects. The literature on symmetric product differentiation finds that, depending on whether the products are substitutes or complements, one firm always earns higher profit than the other for a given mode of product market competition. However, we find that for every level of the cross-price effect of the follower’s price on the leader’s demand, there exists a critical value of the cross-price effect of the leader’s price on the follower’s demand, such that below this critical level, the leader earns a higher profit than the follower, but the reverse is true above this critical value.
Thus, when the leader’s product is a “better” substitute/complement to the follower’s product, the follower’s price change has only a small effect on the leader’s quantity. The leader is able to capture a large market share even at a higher price whereby the follower earns a lower profit than the leader. Hence, if it wants to capture a larger market share, the follower needs to undertake a large and costly price-cut. On the other hand, when the follower’s product is a “better” substitute/complement to the leader’s product, then a price change in the follower’s product has a large effect on the leader’s quantity. Therefore, a small price cut will be sufficient to generate a large increase in the market share for the follower, enabling the follower to earn a higher profit than the leader.

The second essay introduces piracy in a market with asymmetric product differentiation. In this chapter, we investigate the decision of the government to invest in monitoring commercial piracy when the legal firm has reported the presence of the pirate in the market. The existing literature on monitoring commercial piracy looks at the ex-ante monitoring strategies where the authority, with the objective of maximising ex-ante social welfare, sets and commits to a level of monitoring before the pirate enters the market. However, empirical evidences show that often the authorities take monitoring decisions ex-post after the legal firm has taken the initiative to report piracy. For example\(^1\), Microsoft cooperated with the US Customs Services and the authority of twenty two countries to fight illegal sale of its products. In Argentina, in November 2000, following a report from the anti-piracy body of Microsoft, the Argentinean authority raided and apprehended six illegal sellers who were reported as selling counterfeit Microsoft softwares. In Brazil, Microsoft teamed up with the Specialized

\(^1\) Microsoft Corp (2004)
Police Department in Computer Crimes and initiated a police raid against Mega CD, a company that was selling unauthorised copies of Microsoft products over the web.

This essay examines the government’s decision whether to monitor piracy following the legal firm’s report of the presence of a pirate in the market. In the model, the government decides whether or not and how much, to invest in monitoring in order to maximise a politically motivated objective function. We show that with ex-post monitoring, the government will monitor piracy under both price and quantity competition if the legal firm’s political influence is sufficiently high. We also find that, holding the cross-price effect of the legal firm’s price on the pirate’s output constant, there exists a unique level of the cross-price impact of the pirate’s price on the legal firm’s output, above which monitoring will be higher under quantity competition, and below which monitoring will be higher under price competition. Moreover, when the government can credibly commit to monitor piracy, the legal firm’s investment on innovation is higher under quantity competition than under price competition.

The last essay analyses the effect of piracy on innovation when two legal firms engage in R&D competition. This issue assumes importance because not only does piracy cause huge loss in retail sale but also because piracy has possible detrimental effects on the incentive to innovate. For example, The International Federation of the Phonographic Industry (IFPI) in its Commercial Piracy Report (2005) argues that “the illegal music trade is destroying creativity and innovation”. The literature addressing the impact of piracy on innovation considers only one innovating firm and shows that piracy can have both detrimental and beneficial impact on innovation. This literature on piracy and innovation only looks at the technological uncertainty aspect of innovation where, with some probability, the R&D investment of a firm materialises into a new product.
However, innovation also has a market uncertainty aspect when multiple firms compete simultaneously to develop a new product but the success of a firm in patenting its product is stochastic. We attempt to bridge this gap and bring together these two strands of the literature. When a single firm is deciding how much to invest in R&D which, in turn, determines stochastically whether successful innovation will take place, piracy unambiguously retards innovation. However, we show that when two legal firms are engaging in a patent race, piracy may enhance aggregate innovation. We also show that if the difference between the efficiency parameters of the innovating firms is relatively large then piracy increases aggregate R&D investment and the profit of the less efficient firm.

Overall, this thesis attempts to enhance our understanding on various aspects of asymmetric differentiation between the products of two competing firms. The first essay demonstrates that asymmetric product differentiation affects profit ordering among firms engaging in sequential competition. Interestingly, this study could be extended to analyse how a firm would choose its product differentiation level to endogenously decide its role (leader or follower) in sequential market competition. The second essay introduces a pirate that produces an asymmetric substitute of the legal product and competes with the legal product in the product market. In this set up, we examine how ex-post monitoring decision of the regulatory authority and R&D decisions of the legal firm are affected by the asymmetric product substitutability. In the third essay, we analyse how piracy affects R&D decisions of legal firms when they engage in patent races for their innovations. In the concluding chapter we lay down some directions for future research on asymmetric product differentiation and commercial piracy; such as, how firms should invest on research and development when there is the possibility of a pirate operating in the product market. This study could also be used to empirically test
how this asymmetric degree of product substitutability between the legal firm and the pirate and the political influence of the legal firm on the regulatory authority affect firm-level innovation, commercial piracy and monitoring decisions of the authority.
References


Chapter 2 Profit Ordering in a Sequential Game under Asymmetric Product Differentiation
2.1 Introduction

This paper analyses the impact of asymmetric\(^2\) product differentiation on the profit ordering of firms in a sequential game. Profit ordering, that is whether a leader or a follower captures a higher profit in a sequential competition, has important implications in the formation of competition strategies. There is a substantial literature on profit ordering but they assume symmetric cross-price impacts between the competing products. Symmetric cross-price impacts generates a uni-directional profit ordering; that is, the profit of the leader is always higher than that of the follower or vice versa depending on whether the products are substitutes or complements and on the mode of competition.

However, there are evidences and examples where the cross-price impacts are asymmetric. This captures the idea that the impact of a change in the price of a product on the quantity demanded of a second product differs from the impact of a change in price of the second product on the quantity demanded of the first. Thus, for the substitute example, computers may act as “better” substitute to DVD players due to the several other different functionalities that a computer offers. In such cases, changes in the price of DVD players may have a higher impact on the demand for computers than a change in the price of computers will have on the demand for DVD players. Again, if the consumers are only interested in the media quality, then DVD players would be a “better” substitute to computers and then price changes in the DVD market will have lower impacts on computer demand but price changes in computer market will have greater impacts on DVD sales.

\(^2\) By ‘asymmetric’ we mean asymmetric in magnitude. Similarly, ‘symmetric’ implies symmetric in magnitude.
Similarly, in the case of complements, operating systems are much “better” complements to media softwares as changes in prices of media softwares do not impact the demand for operating systems as price changes of operating systems affect the demand for media softwares. Thus asymmetric cross-price impact could have important impacts on the location of products in the market.

The empirical literature on asymmetric product differentiation is scant. Kang (1989) looks at two substitute computer categories – minicomputers and mainframes. At the time of Kang’s study, microcomputers did not have a significant niche in the US computer market. Thus, his study can be considered to be looking at two substitute products which mutually exhaust the market. There being no other substitute products to minicomputers or mainframes in the market, the cross-price effects can be assumed to be capturing the asymmetric differentiation between the two products in its entirety. Kang uses the annual Industry Market Statistics data on the numbers of mainframes and minicomputers sold in US in the period 1965-1986 for measuring the quantity demanded. Price is measured as an average over each computer category for the same period. In the static model, Kang (1989) finds that the cross-price impact on minicomputers (0.552, significantly different from zero at the 10% level) is more sensitive to the cross-price impact on mainframes (0.06, not significantly different from zero). The magnitude of demand switch from mainframes to minicomputers due to an increase in the price of the former is higher than the switch from minicomputers to mainframes when the price of minicomputers increases.

In view of such real world evidences of cross-price asymmetry, it is relevant to study the impact of the variation in product differentiation on the profit ordering of competing firms. This requires consideration of asymmetric cross-price impact. The literature on
competition strategies and profit ordering, however, considers only symmetric cross-price impact.

The studies on competition strategies of firms with symmetric product differentiation can be divided into sequential and simultaneous games. Because profit ordering is relevant only in the case of sequential games, we will be studying profit ordering under asymmetric product differentiation in a sequential set up. Studies on competition strategies consider proportionate sharing rule (efficient sharing rule)\(^3\) for sequential games (simultaneous games). We discuss the literature on sequential games followed by simultaneous games to relate the findings of our paper to those in the literature on competition strategies and profit ordering between firms.

There is a substantial literature on profit ordering in the context of sequential games and symmetric products. Gal-Or (1985) shows that, when firms compete sequentially on substitute products, the leader’s profit is always higher (lower) than the follower’s profit, if the firms compete in quantities (price). Boyer and Moreaux (1987b) find that the leader always earns a higher profit than the follower if the products have symmetric cross-price impacts on their market demands. Dastidar (2004) shows that, with concave demand and strictly convex cost structures, a homogeneous product duopoly generates equal profits for both firms under price competition and the leader gets a higher profit under quantity competition. Thus, in the context of sequential games and symmetric product differentiation, we see that profit ordering depends on the mode of competition and whether the products are substitutes or complements.

\(^3\) A sharing rule denotes how demand is divided between firms selling a homogeneous product in a market. See Shy (1995, p.108). Proportionate sharing rule implies that the competing firms share the market proportionately depending on the prices. Efficient sharing rule implies that the market will be served by firms in the order of their efficiency ranking which is determined by lower price or lower production costs.
In case of simultaneous games using symmetric cross-price impacts, Singh and Vives (1984), Cheng (1985) and Vives (1985) show that quantity competition, compared to price competition, generates higher profit for both firms in case of substitutes and the reverse is true in case of complements. However, Dastidar (1997) shows that this result is sensitive to the sharing rules assumed. Instead of efficient sharing of the market, if the firms follow an equal sharing rule where the market demand is divided equally between the competing firms (the products being identical) then the Vives (1985) results do not necessarily hold.

The literature on competition strategy is yet to address the issue of how profit ordering behaves when cross-price impacts are asymmetric in magnitude. This paper attempts to fill this gap. Because the impacts of price changes of a product on the demands for the competing product are asymmetric, it implies that one product will be a “better” substitute/complement to the other. Hence, no product will be completely driven out of the market and the market will be shared proportionately. In the current study, we examine, in the context of proportionate market sharing and sequential game, if (i) quantity competition generates higher (lower) profit than price competition for both firms, and if (ii) the profit of the leader is higher (lower) than that of the follower, when the products are asymmetric substitutes (complements).

With asymmetric product differentiation, we find that the results for the symmetric case hold, that is, for substitutes, quantity competition by the leader generates higher profit and, for complements, price competition by the leader generates higher profit for both firms. We also show that it is only the leader’s competition strategy that determines the

---

4 Allen (1992) shows that quantity strategy weakly dominates price strategy for both firms in case of substitutes.
5 Henceforth, asymmetric (symmetric) differentiation would imply that the cross-price effects are unequal (equal) in magnitude.
equilibrium of the game, irrespective of the follower’s competition strategy. We find that for every level of the cross-price effect of the change in the leader’s price on the follower’s quantity, there exists a critical value for the cross-price effect of the change in the follower’s price on the leader’s quantity, such that below this critical level, the leader earns a higher profit than the follower, but the reverse is true above this critical value. This result differs from that in the existing literature that only considers symmetric product differentiation.

This chapter is arranged as follows. In Section 2 we present the model and the basic results. Sections 3 and 4 contain the comparative static analysis and concluding remarks.

2.2 The Model

Let us consider a market for differentiated goods with two firms, firm 1 and firm 2. Let \((q_i, p_i)\) denote firm \(i\)'s quantity and price where \(i \in 1, 2\). Following Singh and Vives (1984) and Boyer and Moreaux (1987b), we consider a differentiated product market characterized by the following demand functions,

\[
\begin{align*}
q_1 &= a_1 - b_1 p_1 + c_1 p_2 \\
q_2 &= a_2 - b_2 p_2 + c_2 p_1
\end{align*}
\]  

(1)

In the above equations, \(b_1\) and \(b_2\) are the own-price impacts and \(c_1\) and \(c_2\) are the cross-price impacts on the quantity demands for the products of firms 1 and 2, respectively. \(a_1\) and \(a_2\) are the intercepts. In the line of Boyer and Moreaux (1987b) we normalise the intercepts and the own-price impacts to 1. However, unlike Singh and Vives (1984) and Boyer and Moreaux (1987b), we allow the cross-price impacts to be
asymmetric, that is, we allow the absolute values of $c_1$ and $c_2$ to differ. We assume that production is costless and $c_1, c_2 < 1$, implying that the own-price impact exceeds the cross-price impact. Goods are substitutes or complements if $c_i > 0$ or $c_i < 0$. Note that the sign of $c_i$ is the same for both firms 1 and 2, that is if the product of firm $i$ is a complement for firm $j$ then the product of firm $j$ cannot be a substitute for firm $i$.

Following Gal-Or (1985) and Boyer and Moreaux (1987b) we consider a proportionate sharing rule where a competitor need not be driven out of the market when the goods are substitutes and the firms are competing in prices. That is, we look at the case where the firms compete sequentially.

Without loss of generality we assume firm 1 to be the first mover and firm 2 to be the second mover. Firm 1 can choose either a price or a quantity and firm 2 can respond by either choosing a price or a quantity. The game that we consider in this paper is represented in Figure 2.1. Let $S_1 = \{p_1, q_1\}$ and $S_2 = \{p_2, q_2\}$ denote firm 1’s and firm 2’s strategy spaces, respectively.

Let $G^i$ denote the subgame beginning with firm 1 choosing a strategy $s_1$, $s_1 \in S_1$. Let $p_1^{i_*}, q_1^{i_*}$, and $\pi_i^*(s_1, s_2)$ denote firm $i$’s equilibria price, quantity, and profit when firm 1 chooses $s_1 \in S_1$ and firm 2 chooses $s_2 \in S_2$. We show that

\[
p_1^{p_1, p_2} = p_1^{p_1, p_1^*}, \quad q_1^{p_1, p_2} = q_1^{p_1^*, p_1^*}, \quad p_2^{p_1, p_2} = p_2^{p_2^*, p_2^*}, \quad q_2^{p_1, p_2} = q_2^{p_2^*, p_2^*} \quad \text{and} \quad p_1^{q_1, p_2} = p_1^{q_1^*, p_2^*},
\]

\(^6\) For income compensated demand curves, symmetric cross-price effects are a necessary condition for the negative semi definiteness of the Hessian matrix. However, since our premise is that cross-price effects can be asymmetric, we are looking at uncompensated demand structures and not focussing on the underlying market utility schedule. We have discussed the issue of utility foundation of demand in greater detail in our concluding chapter.

\(^7\) Our analysis retains the symmetric costs (zero marginal cost) assumption as in Singh and Vives (1984). The difference between our model and that of Singh and Vives (1984) is that the latter assumed symmetric cross-price impact and different own-price impacts in a simultaneous set up; whereas we, like Boyer and Moreaux (1987b), assume away any differences in own-price impacts. However, we focus on cross-price impacts that, unlike Boyer and Moreaux (1987b), cause demand structures to be asymmetric between the firms in a sequential set up.
\[ q_1^{q_1,p_2^*} = q_1^{q_1,q_2^*}, \quad p_2^{q_1,p_2^*} = p_2^{q_1,q_2^*} \quad \text{and} \quad q_2^{q_1,p_2^*} = q_2^{q_1,q_2^*}. \]

This implies that if the leader chose the price \( p \), then, irrespective of the choice of \( p_2 \) or \( q_2 \) by the follower, the equilibrium price and quantity solutions would be the same for both firms. Proposition 1 summarises the results for the games \( G^{p_1} \) and \( G^{q_1} \). The proof is provided in the Appendix.

**Proposition 1:** In equilibrium it is only the leader’s strategic choice between price and quantity that determines the equilibrium of the game.

Proposition 1 implies that the leader’s choice of price or quantity strategy is sufficient to determine the equilibrium of the game. The follower’s choice of price or quantity, for any strategic choice of the leader, results in the same outcome. The intuition is that once the leader has determined its price and quantity in equilibrium, the follower is left to operate as a monopolist in the residual demand curve. Because, along a demand curve, price and quantity have a one-to-one correspondence, determining one will automatically fix the other. Hence, along the residual demand curve, the follower’s
optimal choice of either price or quantity results in the same price-quantity combination in equilibrium.

Let us consider the path, \( p_1, p_2 \) where the leader and the follower both opt for price competition. Firm 2’s reaction function is \( p_2 = \frac{1 + c_2 p_1}{2} \) where \( p_2 \) is increasing or decreasing in \( p_1 \) if the goods are substitutes or complements. Along the path, \( p_1 q_2 \) where the follower chooses a quantity strategy in response to the leader’s choice of price strategy, firm 2’s reaction function is \( q_2 = \frac{1 + c_2 p_1}{2} \). Substituting this in firm 2’s demand curve yields, \( p_2 = \frac{1 + c_2 p_1}{2} \) which is the same as firm 2’s reaction function when it chooses a price in response to firm 1’s choice of \( p_1 \). Consequently, the residual demands for the follower are identical along the paths \( p_1 p_2 \) and \( p_1 q_2 \). The intuition for the case when the leader chooses a quantity strategy is similar.

It follows that, given the leader’s strategy choices, there should be a unique profit combination for the leader and the follower, irrespective of the follower’s strategy choice. Corollary 1 summarises the result for the equilibrium profits.

**Corollary 1:** In the \( G^{p_1} \) game \( \pi_1^{p_1 q_1^*} = \pi_1^{p_1 p_2^*} = \pi_1^{G_{p_1}^*} = \frac{(2 + c_1)^2}{8(2 - c_1 c_2)} \) and

\[
\pi_2^{p_1 q_1^*} = \pi_2^{p_1 p_2^*} = \pi_2^{G_{p_1}^*} = \left(\frac{1}{2} + \frac{c_2(2 + c_1)}{2(4 - 2c_1 c_2)}\right)^2.
\]

Similarly, in the \( G^{q_1} \) subgame

\[
\pi_1^{q_1 q_1^*} = \pi_1^{q_1 p_2^*} = \pi_1^{G_{q_1}^*} = \frac{(c_1 c_2 - c_1 - 2)^2}{8(2 - 3c_1 c_2 - c_1^2 c_2^2)} \quad \text{and}
\]

\[
\pi_2^{q_1 q_1^*} = \pi_2^{q_1 p_2^*} = \pi_2^{G_{q_1}^*} = \frac{((c_1^3 + 3c_2) - 2c_2 - 4)^2}{16(c_1 c_2 - 2)^2 (1 - c_1 c_2)}.
\]
The results stated in Corollary 1 differ from Singh and Vives (1984) who model a simultaneous game using similar linear demand functions and constant (zero) marginal costs. Under the assumption of symmetric cross-price effects and efficient sharing rule, they show that, for each firm, it is optimal to choose the same contract as offered by its rival, otherwise, its rival will undercut in prices or compete more aggressively in quantities, as the case may be, and earn higher profits. That is, 
\[ \pi_i^{q_i, q_j} > \pi_i^{q_i, p_j}, \pi_i^{p_i, p_j} > \pi_i^{p_i, q_j}, \forall i, j = 1, 2; i \neq j. \] 
Our analysis for the sequential game shows that, with asymmetric product differentiation and proportionate sharing rule, the follower is not required to follow the strategic choice of the leader. It does not matter what strategy is chosen by the follower as the leader’s strategic choice alone determines the equilibrium of the game.

We next perform the comparative static analysis of the firms’ profits with respect to the product differentiation parameters in order to examine the leader’s choice of price or quantity strategy. The results are stated in Proposition 2 and are diagrammatically represented in Figure 2.2.

**Proposition 2:** If the products are substitutes then the leader’s optimal choice is the quantity strategy. If the products are complements then the leader’s optimal choice is the price strategy. That is, in equilibrium, 
\[ \pi_i^{Gq_i} > \pi_i^{Gp_i}, \forall c_i > 0 \]
and 
\[ \pi_i^{Gq_i} < \pi_i^{Gp_i}, \forall c_i < 0 \]
In case of substitutes (complements), that is \( c_1 > 0 \) (\( c_1 < 0 \)), we observe that for the leader the quantity strategy (price strategy) dominates the price strategy (quantity strategy). Our finding supports that of Deneckere (1983), Singh and Vives (1984), and Boyer and Moreaux (1987b) who consider symmetric product differentiation. Thus, our result under asymmetric product differentiation conforms to the existing literature regarding the leader’s optimal strategy choice in the symmetric case when the products

\[\pi^*_1 = \begin{cases} \pi_{Gq}^* & c_1 < 0 \\ \pi_{Gp}^* & c_1 > 0 \end{cases}\]

\[c_1 < 0 \quad c_1 = 0 \quad c_1 > 0\]

**Figure 2.2: Optimal strategies of the Leader for Substitutes and Complements**

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8 The same results also hold for the follower. That is if the leader chooses a price (quantity) strategy when the products are complements (substitutes), then the follower would get a higher profit than he would if the leader chose a quantity (price) strategy. The leader’s and follower’s profit curves for quantity and price contracts intersect at \( c_i = 0, i = 1, 2 \) which means that if the goods are completely independent both the price and the quantity strategies yield the same profits for the two firms because each firm is a monopoly.

9 Similar results have been offered by Ono (1978; Ono (1982), Cheng (1985), Vives (1985), Boyer and Moreaux (1987a), Vives (1999), Vives (2005a) and Vives (2005b). Hathaway and Rickard (1979) investigate the duality between Bertrand competition and Cournot competition using the Inverse Function Theorem and show that the price-quantity duality does not hold under certain functional assumptions. Kreps and Scheinkman (1983) assume efficient sharing and review a Cournot-Bertrand strategy mix where the firms play a quantity game in the first place and, once quantity is decided, declare prices following a Bertrand strategy and the authors find that this strategy mix is equivalent to the usual one stage Cournot-Nash quantity strategy. Okuguchi (1987) finds that, under some functional restrictions, the Cournot equilibrium prices are not lower than the Bertrand ones. Hackner (2000) shows that the Singh and Vives (1984) result does not hold if there are more than two firms as then, for a high-quality firm, price strategy dominates a quantity strategy even when the goods are substitutes.
are substitutes or complements. In the next section, we examine whether asymmetric product differentiation between the leader and the follower affects the uni-directional profit ordering as observed in the case of symmetric product differentiation.

2.3 Asymmetric Degree of Product Differentiation

Shy (1995) provides a measure of product differentiation for a two product demand system\(^{10}\) as \(\delta = \frac{c^2}{b^2}\) where \(b\) and \(c\) are the symmetric own-price and cross-price effects, respectively. As \(\delta \to 0\) or as \(c \to 0\), the products are considered to be *highly differentiated*, that is the cross-price impacts are minimal. When \(\delta \to 1\), that is \(c \to b\), the products become *almost homogenous* as the cross-price impact is almost the same as the own-price. However, asymmetric cross-price impacts in our model require that the products will be *highly differentiated* when \(|c_1| - |c_2|\) is high and the products will be *almost homogenous* when \(|c_1| - |c_2|\) \(\to 0\).

The impact of asymmetric product differentiation on the profit ordering of the firms requires the determination of the equilibrium profit positions of both firms for all feasible combinations of \(c_1\) and \(c_2\). If \(c_1 > c_2\) it implies that the demand for firm 1’s product is more affected by a change in the price of firm 2 than the demand for firm 2’s product is affected by a change in firm 1’s price. Thus firm 2’s product is a “better” substitute/complement to firm 1’s product if \(c_1 > c_2\). On the other hand, if \(c_1 < c_2\), then firm 1’s product is a “better” substitute/complement to firm 1’s product. If \(c_1 = c_2 = c\), then the degree of substitutability/complementarity between the two products would be the same or uni-directional. With \(c_1 \neq c_2\) we move from a one-dimensional to a two-

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\(^{10}\) Shy (1995, p136) uses the demand system \(q_i = a - bp_i + cp_j\) with symmetric price effects.
dimensional concept of product differentiation. We map the equilibrium profits, prices, and quantities of the two firms on the \((c_1, c_2)\) plane in Figure 2.3. Lemma 1 defines the graphs in Figure 2.3. We will be comparing the leader and the follower across the equilibrium values of \(X\), where \(X\) denotes the equilibrium prices \((P)\), quantities \((Q)\) and profits \((\pi)\).

**Lemma 1:** There exists a unique \(c_2^x\) that represents the locus of \(c_2\) as functions of \(c_1\) where \(X \in \{P, Q, \pi\}\) such that the equilibrium prices \((P)\), quantities \((Q)\) and profits \((\pi)\) are equal for both the leader and the follower, that is,

\[
\begin{align*}
(i) \quad c_2^p & = \begin{cases} 
 c_2^p(c_1 | G^{p_1}) & \text{for } c_i < 0, \forall i = 1,2 \text{ such that } p_1^{G^{p_1}} = p_2^{G^{p_1}} \\
 c_2^p(c_1 | G^{q_1}) & \text{for } c_i > 0, \forall i = 1,2 \text{ such that } p_1^{G^{q_1}} = p_2^{G^{q_1}} \end{cases} \\
(ii) \quad c_2^p & = \begin{cases} 
 c_2^p(c_1 | G^{p_2}) & \text{for } c_i < 0, \forall i = 1,2 \text{ such that } p_1^{G^{q_2}} = p_2^{G^{q_2}} \\
 c_2^p(c_1 | G^{q_2}) & \text{for } c_i > 0, \forall i = 1,2 \text{ such that } p_1^{G^{q_2}} = p_2^{G^{q_2}} \end{cases} \\
(iii) \quad c_2^p & = \begin{cases} 
 c_2^p(c_1 | G^{q_1}) & \text{for } c_i < 0, \forall i = 1,2 \text{ such that } \pi_1^{G^{q_1}} = \pi_2^{G^{q_1}} \\
 c_2^p(c_1 | G^{q_1}) & \text{for } c_i > 0, \forall i = 1,2 \text{ such that } \pi_1^{G^{q_1}} = \pi_2^{G^{q_1}} \end{cases}
\end{align*}
\]

Proposition 3 summarizes the results for the profit ordering of the firms under the two subgames \(G^{p_i}\) and \(G^{q_i}\) with respect to the product differentiation parameters \(c_i, i = 1,2\).

Figure 2.3 provides a diagrammatic representation of Proposition 3 using Lemma 1. The proof of Proposition 3 is given in the Appendix. In the four quadrant diagram in Figure 2.3, we are only interested in the first and the third quadrants that represent the games \(G^{p_i}\) and \(G^{q_i}\) respectively for varying degrees of \(c_i, i = 1,2\). The first quadrant represents the case where the products are substitutes or \(c_i > 0\) and the third quadrant represents
the case where the products are complements or \( c_i < 0 \). The horizontal axis plots \( c_1 \) and the vertical axis plots \( c_2 \) with \( c_1, c_2 \in [-1,1] \) and \( c_1 c_2 < 1 \).

Figure 2.3: Comparative Static Analysis for Asymmetric Product Differentiation

**Proposition 3:** For every \( c_1 \) there exists a unique critical value of \( c_2 \), say \( c_2^\pi \), such that if \( c_2 \) is below (above) \( c_2^\pi \) then the leader’s (follower’s) profit exceeds that of the follower (leader).

The implications of Proposition 3 are as follows. We begin with substitutes, \( c_i > 0, i = 1,2 \) in the first quadrant. \( c_2^\pi \) is the locus of \( c_1 \) and \( c_2 \) for which \( \pi_1^{G_i^\ast} = \pi_2^{G_i^\ast} \). This is represented by the bold solid curve. When \( c_2 < c_2^\pi \), the leader’s profit exceeds that of the follower. The reverse is true for \( c_2 > c_2^\pi \). Hence, in the case the products are substitutes, there exists a unique critical value of relative product...
differentiation $c_2^* = c_2^*(c_1 \mid Gq_i) \forall c_1 > 0, \forall i = 1,2$ such that $\pi_1^{Gq_i} = \pi_2^{Gq_i}$ beyond which profit ordering changes direction.\(^{11}\)

In the case of substitutes, $c_2^p$ is the locus of $c_2$ for all values of $c_1$ for which $p_1^{Gq_i} = p_2^{Gq_i}$. Thus, for a given $c_1$, if $c_2 > c_2^p$ then the area above the $c_2^p$ locus represents a higher price received by the follower compared to that by the leader. $c_2^o$ is the locus of $c_2$ such that for all $c_1$ the market is equally shared by the leader and the follower; that is, $q_1^{Gq_i} = q_2^{Gq_i}$. In the area below the $c_2^o$ locus, the leader caters to a larger market share than the follower. Note that $c_2^p$ and $c_2^o$ are also unique. Let us slice the graph $c_1 = c_1^s$. At $c_1 = c_1^s$, for $c_2 > c_2^o$, both the price and the quantity of the follower are higher than those of the leader. Thus above the $c_2^o$ locus the follower’s profit exceeds that of the leader. For $c_2 < c_2^p$, the leader’s price and quantity, and, hence, profit exceed those of the follower. Since $c_2^o > c_2^p$, it must be the case that the profit ordering between the leader and the follower switches direction between these two loci. Thus, we find that $c_2^o > c_2^* > c_2^p$.

Thus, for substitutes or $c_1 > 0$, quantity strategy is less profitable to the leader if his products have higher impact on the follower’s demand than the follower’s product have on the leader’s demand, that is $c_2 < c_1$. The asymmetry between the products increase as the gap between $c_2$ and $c_1$ increases and for $c_2$ above $c_2^o$ (which is also above the

\(^{11}\) The expression $c_2^* = c_2^*(c_1 \mid Gq_i)$ such that $\pi_1^{Gq_i} = \pi_2^{Gq_i}$ for $c_1 > 0, \forall i = 1,2$ yields four solutions. We use numerical simulation with a factor of 0.01 to eliminate three of the solutions. That is, we vary $c_2$ within $[0, 1]$ for every level of $c_1$ within $[0, 1]$ with an increment of 0.01 to check for real solutions to $\pi_1^{Gq_i} = \pi_2^{Gq_i}$. This exercise yields only one real solution which fits only one of the roots given by the algebraic expression $c_2^* = c_2^*(c_1 \mid Gq_i)$ such that $\pi_1^{Gq_i} = \pi_2^{Gq_i}$. Again, this solution, though unique, is not helpful for any positive analysis. We thus use numerical approximations for a graphical analysis.
45° line of symmetric product differentiation) the follower ends up with a higher profit than the leader. In the reverse situation the follower ends up capturing a lower share of the market even with a lower price as the leader produces a ‘better’ substitute.

Next we consider complements, represented in the third quadrant of Figure 2.3. Again, \( c_2^{\pi} \) is the locus of \( c_1 \) and \( c_2 \) for which \( \pi_1^{Gh} = \pi_2^{Gh} \). There exists a unique critical value of \( c_2 \) beyond which the leader’s profit is higher than that of the follower. The bold curve shows this reversal in profit between the firms and is represented as

\[
\pi_2^{Gh} > \pi_2^{Gh} .
\]

Further, for complements, if \( |c_2| \leq |c_2^p| \leq |c_2^Q| \), then the leader’s price and quantity is less than that of the follower (\( p_1^{Gh} < p_2^{Gh} \), \( q_1^{Gh} < q_2^{Gh} \)), hence, the leader’s profit is less than that of the follower as well (\( \pi_1^{Gh} < \pi_2^{Gh} \)). Intuitively, for a relatively low absolute level of \( c_2 \) satisfying \( |c_2| \leq |c_2^p| \leq |c_2^Q| \), the follower is able to cater to a larger consumer base despite the high prices since the impact of the leader’s price on the follower’s market demand is very low compared to the impact of the follower’s price on the leader’s market demand. So the follower produces a ‘better’ complement to the leader and, hence, caters to a larger demand and earns higher profit despite high prices. On the other hand, if \( |c_2| \geq |c_2^Q| \geq |c_2^p| \), then the above results are reversed.

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\(^{12}\)The algebraic expression for \( c_2^{\pi} \) is obtained as shown in Part (iii) of Lemma 1. In case of complements, we have a unique solution to \( c_2^{\pi} = c_2^{\pi} (c_1 \mid G^p) \) such that \( \pi_1^{Gh} = \pi_2^{Gh} \) though the algebraic value is not very handy for analytical constructs. Again, this solution, though unique, is not helpful for any positive analysis. We thus use numerical approximations for a graphical analysis.
In the context of complements or $c_i < 0$, the quantity strategy is less profitable to the firms as $c_1$ and $c_2$ move further apart, that is, as the asymmetry between the cross-price impacts on the demand for the products become larger. Note that $c_2^\pi$ lies above the 45° line of symmetry where $c_1 = c_2 = c$. Thus, when the asymmetry between the cross-price impacts is high enough, it does not make sense to compete in capturing market share but makes more sense in increasing the total market size. However, if the impact of the leader’s price on the follower’s demand is less than the impact of the follower’s price on the leader’s demand, or $|c_2| < |c_2^\pi| < |c_1|$, the follower does not have to undercut prices to increase the size of the market. If the reverse is true, that is, $|c_2| > |c_2^\pi| > |c_1|$, then the follower ends up capturing a lower share of the market even with a lower price as the leader produces a ‘better’ complement. In the region $|c_2^p| < |c_2^\pi| < |c_2^0|$ the follower sells at a lower price but captures a much greater share of the market than the leader and at $|c_2| > |c_2^\pi|$ the leader breaks even with the follower and starts earning a higher profit.

The above analysis thus shows that profit ordering switches around the critical level of $c_2$, $c_2^\pi$ which lies between $c_2^p$ and $c_2^0$. This is true for both substitutes and complements. Thus, unlike the conventional literature on sequential games where one firm always earns a higher profit than the other depending on whether the products are symmetric substitutes or complements, the result is not uni-dimensional when the degree of substitutability or complementarity between the two goods are asymmetric. In case of asymmetric product differentiation one firm earns higher profit than its competitor up to a certain level of product differentiation beyond which the competitor starts to earn a higher profit. Thus asymmetric product differentiation results in a two-dimensional profit ordering between firms in a Stackelberg duopoly.
An alternative way of explaining this feature is to explore the impact of complementarity (substitutability) in strategic product networking. If, in absolute terms, the degree of complementarity (or substitutability) is very low (high) for the follower, it implies that a price change by the leader has low (high) effect on the demand for the follower’s product. Then the follower can charge a higher price and also capture a larger market share, thereby harnessing higher profit. This could be because the number of close substitutes available for or the number of features included in the leader’s product is relatively higher (lower) than those for the follower’s product. This can be illustrated with our earlier example of computers and DVD players, for substitutes and operating systems and media softwares, for complements. Typically, computers would be ‘better’ substitutes to DVD players as computers pack more features and operating systems would be ‘better’ complements to media softwares because many media software cater to a very few operating systems.

Let us now consider the case where, in our sequential game structure, the cross-price impacts are symmetric. Along the diagonal in Figure 2.3, we have $c_1 = c_2 = c$. From Proposition 2 we know that the firms will play price competition if the products are complements and the firms will compete in quantities when the products are substitutes.

For $c < 0$ and $c^2 < 1$, we get $\pi_1^{G_1} - \pi_2^{G_1} = \frac{-c^3(4 + 3c)}{16(c^2 - 2)^2} > 0$. Thus, in the price game, for the case of symmetric complements, the leader’s profit exceeds that of the follower.

For $c > 0$ and $c^2 < 1$, we have $\pi_1^{G_2} - \pi_2^{G_2} = \frac{c^3(1 + c)(4 - 3c)}{16(1-c)(c^2 - 2)^2} > 0$. Hence, in the quantity game, for the case of symmetric substitutes, the leader earns a higher profit than the follower. Thus, when the firms compete sequentially, in the case of symmetric cross-price impacts, for both substitutes and complements, the leader’s profit always
exceeds that of the follower. In Figure 2.3, this is shown by the $45^\circ$ line lying below the $c_2^*$ locus for both substitutes and complements.

**Corollary 2:** In case of symmetric product differentiation ($c_1 = c_2 = c$), under sequential game, the leader’s profit exceeds that of the follower, for both price and quantity competitions.

This finding conforms to that of Boyer and Mordeaux (1987b) who only consider the special case of symmetric product differentiation. Thus our analysis of profit ordering with under asymmetric product differentiation also supports the specific case of symmetric product differentiation as has been considered in the literature. Hence, this study provides a generalised investigation of profit ordering under all possible combinations of cross-price impacts and modes of competition between the leader and the follower in the product market.

### 2.4 Conclusions

In this chapter, we model a sequential game where firms choose price and quantity strategies in a generalized environment of asymmetric product differentiation. We show that the leader’s strategic choice uniquely determines the equilibrium of the game. In case of substitutes (complements), quantity (price) is the dominant strategy. Our analysis for the case of asymmetric product differentiation conforms to the special case of symmetric product differentiation that is considered in the existing literature.

Most importantly, we show that the relative degree of complementarity or substitutability determines the profit ordering of the firms. There exists a unique critical level of product differentiation which creates a reversal in the profit ordering between
the leader and the follower in a sequential game. Thus, unlike the existing literature where the leader or the follower always earns a higher profit depending on the degree of product differentiation and the mode of competition, profit ordering ceases to be unidirectional with the introduction of a two-dimensional concept of asymmetric product differentiation.

This concept of asymmetric product differentiation can be translated to the case where a legal firm competes with an illegal firm or a pirate in the product market. The two-dimensional profit ordering would be a useful concept to study the conditions behind the timing of entry into the market by the two firms; that is, whether a firm would legally enter the market with a new product or whether it would want to compete as an illegal follower after the legal product has been introduced. Typically, the impact on the quantity demanded of the legal firm from a change in the price of the pirated product will be different from the impact on the pirate’s demand from a change in the price of the legal product. Thus the cross-price impacts are asymmetric between the legal product and the pirated copy. The firms’ competition strategies in such a case can be modelled under the sequential competition framework and asymmetric cross-price impacts assumption as used in the present chapter. We proceed to do so in the next chapter where we use the set up of the present chapter to examine the conditions on product differentiation and mode of competition under which the legal firm will enter and compete with the pirate in the product market.
References


Appendix

Calculations for these proofs have been done using Mathematica v.5.2, Wolfram Research Inc.

**Proof of Proposition 1 (i):** In the $G^p$ game, firm 1 chooses a price $p_1$ to maximise its profit. In the next stage, the follower can then choose either price or quantity.

**Let us consider the path** $p_1, p_2$: Using the method of backward induction, in stage 2, firm 2 maximises profit $\pi_2$ taking $p_1$ as given.

$\text{Max}_{p_2} \pi_2 = p_2 q_2 = p_2 (1 + c_2 p_1 - p_2)\]

The First Order Condition (FOC): $\frac{\partial \pi_2}{\partial p_2} = 0 \Rightarrow p_2 = \frac{1 + c_2 p_1}{2}$.

The Second Order Condition (SOC): $\frac{\partial^2 \pi_2}{\partial p_2^2} = -2 < 0$.

In stage 1, firm 1 maximises profit $\pi_1$ knowing the price $p_2$ chosen by firm 2.

$\text{Max}_{p_1} \pi_1 = p_1 q_1 = p_1 (1 - p_1 + c_1 p_2) \Rightarrow \text{Max}_{p_1} \pi_1 = p_1 q_1 = p_1 (1 - p_1 + c_1 \frac{1 + c_2 p_1}{2})$.

The First Order Condition (FOC):

$\frac{\partial \pi_1}{\partial p_1} = 0 \Rightarrow p_1^{p_1 p_2} = \frac{2 + c_1}{4 - 2c_1 c_2}$.

Thus, in equilibrium, $p_2^{p_1 p_2} = \frac{1}{2} + \frac{c_2 (2 + c_1)}{2 (4 - 2c_1 c_2)}$ and $q_2^{p_1 p_2} = \frac{1}{2} + \frac{c_2 (2 + c_1)}{2 (4 - 2c_1 c_2)}$. We have $\frac{\partial^2 \pi_1}{\partial p_1^2} = c_1 c_2 - 2 < 0$ and the SOC holds, since $c_1 c_2 < 1$, to ensure positive price.
Let us now consider the path $p_1 q_2$: In stage 2, firm 2 chooses quantity $q_2$ and maximises profit $\pi_2$ taking $p_1$ as given.

$$\text{Max}_{q_2} \pi_2 = p_2 q_2 = q_2 (1 + c_2 p_1 - q_2).$$

The First Order Condition (FOC): \[ \frac{\partial \pi_2}{\partial q_2} = 0 \Rightarrow q_2 = \frac{1 + c_2 p_1}{2} \] and \[ p_2 = \frac{1 + c_2 p_1}{2}. \]

The Second Order Condition (SOC): \[ \frac{\partial^2 \pi_2}{\partial q_2^2} = -2 < 0. \]

In stage 1, firm 1 chooses $p_1$ to maximise $\pi_1$ given the quantity $q_2$ chosen by firm 2.

$$\text{Max}_{p_1} \pi_1 = p_1 q_1 = p_1 (1 - p_1 + c_1 (1 - q_2 + c_2 p_1)) \Rightarrow \text{Max}_{p_1} \pi_1 = p_1 (1 - p_1 + c_1 \frac{1 + c_2 p_1}{2}).$$

The First Order Condition (FOC): \[ \frac{\partial \pi_1}{\partial p_1} = 0 \Rightarrow p_1^{p_1 q_2} = \frac{2 + c_1}{4 - 2c_1 c_2} \] and \[ p_2^{p_1 q_2} = \frac{1 + c_2 (2 + c_1)}{2(4 - 2c_1 c_2)} \Rightarrow q_1^{p_1 q_2} = \frac{1}{2} + \frac{c_1}{4} \] and \[ q_2^{p_1 q_2} = \frac{1}{2} + \frac{c_2 (2 + c_1)}{2(4 - 2c_1 c_2)}. \] Again, we have \[ \frac{\partial^2 \pi_1}{\partial p_1^2} = c_1 c_2 - 2 < 0 \] for a positive price system and thus the SOC for profit maximisation holds.

(ii): In the $G^{th}$ game, firm 1 chooses $q_1$ to maximise profit. In the next stage, the follower can then choose either price or quantity.

Let us consider the path $q_1 p_2$: Using the method of backward induction, in stage 2 firm 2 maximises profit $\pi_2$ taking $q_1$ chosen by firm 1 in stage 1, as given.

$$\text{Max}_{p_2} \pi_2 = p_2 q_2 = p_2 (1 - p_2 + c_2 (1 + c_1 p_2 - q_1)).$$

The First Order Condition (FOC): \[ \frac{\partial \pi_2}{\partial p_2} = 0 \Rightarrow p_2 = \frac{1 + c_2 - c_2 q_1}{2(1 - c_1 c_2)}. \]

The Second Order Condition (SOC) requires \[ \frac{\partial^2 \pi_2}{\partial p_2^2} = 2c_1 c_2 - 2 < 0. \]
In stage 1, firm 1 maximises profit $\pi_1$ knowing the price decision $p_2$ of firm 2.

$$\text{Max}_{q_1} \pi_1 = p_1 q_1 = q_1 \left(1 + \frac{c_1(1 + c_2 - c_2 q_1)}{2(1 - c_1 c_2)} - q_1 \right)$$

The First Order Condition (FOC): $$0 = \frac{\partial \pi}{\partial q_1} \Rightarrow q_1^{q_1,p_2^*} = \frac{1}{2} + \frac{c_1}{4 - 2 c_1 c_2}$$ and

$$p_2^{q_1,p_2^*} = \frac{1}{2} \left(1 + \frac{c_2}{2} - \frac{c_1 c_2}{2(2 - c_1 c_2)} \right)$$.

This implies that in equilibrium we have

$$p_1^{q_1,p_2^*} = \frac{1}{2} + \frac{c_1(1 + c_2)}{4(1 - c_1 c_2)}$$ and $q_2^{q_1,p_2^*} = \frac{1}{2} \left[\frac{c_2}{2} + \frac{4 - 3 c_1 c_2}{4 - 2 c_1 c_2}\right]$.

The Second Order Condition (SOC) has $\frac{\partial^2 \pi}{\partial q_1^2} = -2 - \frac{c_1 c_2}{(1 - c_1 c_2)} < 0$.

Let us consider the path $q_1 q_2$:

In stage 2, firm 2 chooses quantity $q_2$ and maximises profit $\pi_2$ taking the price $q_1$ chosen by firm 1 as given.

$$\text{Max}_{q_2} \pi_2 = p_2 q_2 = q_2 \left(1 - q_2 + c_2 (1 - q_1)\right)$$.

The First Order Condition (FOC): $$0 = \frac{\partial \pi}{\partial q_2} \Rightarrow q_2 = \frac{1 + c_2 - c_2 q_1}{2}$$ and $p_2 = \frac{1 + c_2 p_1}{2}$.

The Second Order Condition (SOC): $$\frac{\partial^2 \pi}{\partial q_2^2} = -2 < 0$$.

In stage 1, firm 1 chooses quantity and maximises profit $\pi_1$ knowing the quantity decision $q_2$ chosen by firm 2.

$$\text{Max}_{q_1} \pi_1 = p_1 q_1 = q_1 \left(1 - q_1 + c_1 - c_1 \left(1 + \frac{1 + c_2 - c_2 q_1}{2}\right)\right)$$. 
The First Order Condition (FOC): \[ \frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow q_1^{q_1*} = \frac{1}{2} + \frac{c_1}{4 - 2c_1c_2} \]

\[ p_2^{q_1*} = \frac{1}{2(1 - c_1c_2)} (1 + \frac{c_2}{2} - \frac{c_1c_2}{2(2 - c_1c_2)}) \]

\[ \Rightarrow p_1^{q_1*} = \frac{1}{2} + \frac{c_1(1 + c_2)}{4(1 - c_1c_2)} \] and \[ q_2^{q_1*} = \frac{1}{2} \left[ \frac{c_2}{2} + \frac{4 - 3c_1c_2}{4 - 2c_1c_2} \right] \].

Thus,

(i) In the \( G^p_1 \) subgame, we have \( p_1^{p_1p_2*} = p_1^{p_1q_1*} = \frac{2 + c_1}{4 - 2c_1c_2}, \quad q_1^{p_1p_2*} = q_1^{p_1q_1*} = \frac{1}{2} + \frac{c_1}{4} \),

\[ p_2^{p_1p_2*} = p_2^{p_1q_1*} = \frac{1}{2} + \frac{c_2(2 + c_1)}{2(4 - 2c_1c_2)} \] and \[ q_2^{p_1p_2*} = q_2^{p_1q_1*} = \frac{1}{2} + \frac{c_2(2 + c_1)}{2(4 - 2c_1c_2)} \].

(ii) Similarly, in the \( G^q_1 \) subgame, we have \( p_1^{q_1p_2*} = p_1^{q_1q_2*} = \frac{1}{2} + \frac{c_1(1 + c_2)}{4(1 - c_1c_2)} \),

\[ q_1^{q_1p_2*} = q_1^{q_1q_2*} = \frac{1}{2} + \frac{c_1}{4 - 2c_1c_2}, \quad p_2^{q_1p_2*} = p_2^{q_1q_2*} = \frac{1}{2} \left( \frac{c_2}{2} - \frac{c_1c_2}{2(2 - c_1c_2)} \right) \]

and \[ q_2^{q_1p_2*} = q_2^{q_1q_2*} = \frac{1}{2} \left( \frac{c_2}{2} + \frac{4 - 3c_1c_2}{4 - 2c_1c_2} \right) \].

Thus, we show that in each of the games \( G^q \) and \( G^p \) there is only one set of optimal price-quantity combination for each of the two firms. \( Q.E.D. \)
Proof of Proposition 2:

For the Leader:

\[ \pi_1^{q_2^{*}} = \pi_1^{p_1^{*}p_2^{*}} = \pi_1^{G_1^{*}} = \frac{(2 + c_1)^2}{8(2 - c_1c_2)} , \]
\[ \pi_1^{q_1^{*}} = \pi_1^{q_2^{*}} = \pi_1^{G_1^{*}} = \pi_1^{q_1^{*}} = \pi_1^{G_1^{*}} = \frac{(c_1c_2 - c_1 - 2)^2}{8(2 - 3c_1c_2 - c_1c_2^2)} . \]
\[ \pi_1^{G_1^{*}} - \pi_1^{G_1^{*}} = \frac{c_1^2c_2(2 + c_1 + c_2)}{8(2 - c_1c_2)(1 - c_1c_2)} \geq 0, \forall c_i \geq 0 : c_1, c_2 < 1. \text{ and} \]
\[ \pi_1^{G_1^{*}} - \pi_1^{G_1^{*}} = \frac{c_1^2c_2(2 + c_1 + c_2)}{8(2 - c_1c_2)(1 - c_1c_2)} \leq 0, \forall c_i \leq 0 : |c_1, c_2| < 1 . \]

For \( c_i < 0 \), the denominator and the term in brackets in the numerator are still positive but the numerator is negative as \( c_2 < 0 \). Hence, for \( c_i > 0 \), \( \pi_1^{G_1^{*}} > \pi_1^{G_1^{*}} \) and for \( c_i < 0 \) we have \( \pi_1^{G_1^{*}} < \pi_1^{G_1^{*}} \).

For the Follower:

\[ \pi_2^{q_2^{*}} = \pi_2^{p_1^{*}p_2^{*}} = \pi_2^{G_2^{*}} = \frac{1}{2} + \frac{c_2(2 + c_1)}{2(4 - 2c_1c_2)} . \]
\[ \pi_2^{q_1^{*}} = \pi_2^{q_2^{*}} = \pi_2^{G_2^{*}} = \frac{(c_1(3 + c_2) - 2c_2 - 4)^2}{16(c_1c_2 - 2)^2(1 - c_1c_2)} \]
and \( \pi_2^{G_2^{*}} - \pi_2^{G_2^{*}} = \frac{c_1^2c_2^2(2 + c_1 + c_2)}{16(2 - c_1c_2)(1 - c_1c_2)} = \frac{c_1^2}{4 - 2c_1c_2} (\pi_1^{G_1^{*}} - \pi_1^{G_1^{*}}) . \]

Let us compare the follower’s profit differential to that of the leader between the two games, that is compare \( \pi_2^{G_1^{*}} - \pi_2^{G_1^{*}} \) to \( \pi_1^{G_1^{*}} - \pi_1^{G_1^{*}} \). Now
\[ \pi_2^{G_1^{*}} - \pi_2^{G_1^{*}} = \frac{c_1^2}{4 - 2c_1c_2} (\pi_1^{G_1^{*}} - \pi_1^{G_1^{*}}) \]
\[ \Rightarrow \frac{\pi_2^{G_1^{*}} - \pi_2^{G_1^{*}}}{\pi_1^{G_1^{*}} - \pi_1^{G_1^{*}}} = \frac{c_1^2}{4 - 2c_1c_2} \]

Since \( c_1c_2 < 1 \), \( \Rightarrow 1 > \frac{c_1^2}{2} > \frac{c_1^2}{4 - 2c_1c_2} > \frac{c_1^2}{4 - 2c_1c_2} > 0 . \)
We thus show that the follower’s gain from switching between price and quantity contracts is less than the gain to the leader from the same switch, that is,
\[
1 > \frac{\pi_i^{G_p^*} - \pi_i^{G_q^*}}{\pi_i^{G_p^*} - \pi_i^{G_q^*}} = \frac{c_2^2}{4 - 2c_1c_2} > 0, \forall c_i.
\]

Since we have \(\frac{c_2^2}{4 - 2c_1c_2} > 0\) for all values of \(c_i, i=1,2\), hence \(\pi_i^{G_p^*} - \pi_i^{G_q^*}\) moves similar to \(\pi_i^{G_q^*} - \pi_i^{G_p^*}\). Hence, for \(c_i > 0\), \(\pi_i^{G_q^*} > \pi_i^{G_p^*}\) and for \(c_i < 0\) we have \(\pi_i^{G_q^*} < \pi_i^{G_p^*}\). This is shown in Figure 2.2. \(Q.E.D.\)

**Proof of Proposition 3:**

**For the Price Game:**

\[
p_1^{G_p^*} = \frac{2+c_1}{4-2c_1c_2} \quad \text{and} \quad p_2^{G_p^*} = \frac{1}{2} + \frac{c_2(2+c_1)}{2(4-2c_1c_2)}
\]

\[
p_1^{G_q^*} - p_2^{G_q^*} = \frac{2c_2 - c_1(2+c_2)}{4c_1c_2 - 8}.
\]

The denominator is always negative.

\[
\therefore p_1^{G_p^*} \geq p_2^{G_p^*} \Rightarrow c_2 \leq \frac{2c_1}{2-c_1} = c_2^p \forall c_i \in [-1,1]
\]

and \(p_1^{G_p^*} \leq p_2^{G_p^*} \Rightarrow c_2 \geq \frac{2c_1}{2-c_1} = c_2^p \forall c_i \in [-1,1]\).

Also, we have \(q_1^{G_q^*} = \frac{1}{2} + \frac{c_1}{4}\) and \(q_2^{G_q^*} = \frac{1}{2} + \frac{c_2(2+c_1)}{2(4-2c_1c_2)}\) such that

\[
q_1^{G_q^*} - q_2^{G_q^*} = \frac{(2+c_1+c_1^2)c_2 - 2c_1}{4c_1c_2 - 8}.
\]

Again, the denominator is always negative and, hence,

\[
q_1^{G_q^*} \geq q_2^{G_q^*} \Rightarrow c_2 \leq \frac{2c_1}{2+c_1+c_1^2} = c_2^q \forall c_i \in [-1,1]
\]
and \( q_1^{G^*_h} \leq q_2^{G^*_h} \Rightarrow c_2 \geq \frac{2c_i}{2 + c_i + c_i^2} = c_2^Q \ \forall c_i \in [-1,1]. \)

Now since the price game will not be played if the goods are substitutes, this part of the analysis is only relevant for \( c_i < 0 \). So for \( c_2 \geq \frac{2c_i}{2 - c_i} \geq \frac{2c_i}{2 + c_i + c_i^2} \ \forall c_i \in [-1,0] \) or for

\[
|c_2| \leq \frac{2c_i}{2 - c_i} \leq \frac{2c_i}{2 + c_i + c_i^2} \ \forall c_i \in [-1,0]
\]

we have \( \pi_1^{G^*_h} \leq \pi_2^{G^*_h} \) and for

\[
c_2 \leq \frac{2c_i}{2 + c_i + c_i^2} \leq \frac{2c_i}{2 - c_i} \ \forall c_i \in [-1,0] \ or \ for \ |c_2| \geq \frac{2c_i}{2 + c_i + c_i^2} \geq \frac{2c_i}{2 - c_i} \ \forall c_i \in [-1,0]
\]

we have \( \pi_1^{G^*_h} \geq \pi_2^{G^*_h} \). Also, when the products are complements, that is, \( c_i \in [-1,0] \) for a critical value of \( c_2 \), \( c_2^\pi \), where \( \frac{2c_i}{2 - c_i} < |c_2^\pi| < \frac{2c_i}{2 + c_i + c_i^2} \) and

\[
c_2^\pi = c_2^\pi(c_i | G^*_h) = \frac{-8 - 4c_i^2 - c_i^3 + \sqrt{64 + 64c_i^3 + 16c_i^2} + 16c_i^3 + 20c_i^4 + 8c_i^5 + c_i^6}{4 - 4c_i + c_i^2}
\]

is a reversal in profit dominance between the leader and the follower. If \( |c_2| \geq |c_2^\pi| \) then \( \pi_1^{G^*_h} \geq \pi_2^{G^*_h} \) and if \( |c_2| \leq |c_2^\pi| \) then \( \pi_1^{G^*_h} \leq \pi_2^{G^*_h} \). This has been shown in the bottom left quadrant of Figure 2.3.

**For the Quantity Game:**

In the quantity game, \( p_1^{G^*_q} = \frac{1}{2} + \frac{c_i(1 + c_2)}{4(1 - c_1c_2)} \) and

\[
p_2^{G^*_q} = \frac{1}{2(1 - c_1c_2)}(1 + \frac{c_i(1 + c_2)}{4 - 2c_i c_2}). \quad p_1^{G^*_q} - p_2^{G^*_q} = \frac{2c_2 - c_1(2 + (1 + c_1)(c_2 - 1)c_1)}{4(c_1c_2 - 2)(1 - c_1c_2)}.
\]
\[ p_1^{G_{q_i}^*} \geq p_2^{G_{q_i}^*} \Rightarrow c_2 \leq \frac{2 + c_1 + c_1^2 - \sqrt{4 + 4c_1 - 3c_1^2 - 6c_1^3 + c_1^4}}{2c_1(1 + c_1)} = c_2^* \forall c_i \in [0,1]. \]

Also

\[ q_1^{G_{q_i}^*} = \frac{1}{2} + \frac{c_1}{4\cdot 2c_1c_2} \quad \text{and} \quad q_2^{G_{q_i}^*} = \frac{1}{2} \left( \frac{c_2}{2} + \frac{4 - 3c_1c_2}{4\cdot 2c_1c_2} \right) \]

\[ q_1^{G_{q_i}^*} - q_2^{G_{q_i}^*} = \frac{2c_2 - c_1(2 + c_2 + c_2^2)}{4c_1c_2 - 8}. \]

\[ \therefore q_1^{G_{q_i}^*} \geq q_2^{G_{q_i}^*} \Rightarrow c_2 \leq \frac{2 - c_1 - \sqrt{4 - 4c_1 - 7c_1^2}}{2c_1} = c_2^Q \forall c_i \in [0,1]. \]

The next step is to find out \( c_2^\pi = c_2^\pi (c_1 \mid G_{q_i}^*) \) such that \( D = \pi_1^{G_{q_i}^*} - \pi_2^{G_{q_i}^*} = 0 \). There are four solutions to this expression. The calculations are not analytically conclusive and comparison is difficult.

So we run a simulation of \( c_1, c_2 \in [0,1] \) with a factor of 0.01 for values of

\[ p_1^{G_{q_i}^*}, q_1^{G_{q_i}^*}, \pi_1^{G_{q_i}^*} \quad \text{and} \quad D = \pi_1^{G_{q_i}^*} - \pi_2^{G_{q_i}^*}. \]

Using these simulation results, we can omit three of the solutions to \( c_2^\pi \mid \pi_1^{G_{q_i}^*} - \pi_2^{G_{q_i}^*} = 0 \), leaving only one unique feasible solution to \( c_2^\pi \mid \pi_1^{G_{q_i}^*} - \pi_2^{G_{q_i}^*} = 0. \) \( Q.E.D. \)

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13 \( p_1^{G_{q_i}^*} - p_2^{G_{q_i}^*} \geq 0 \Rightarrow c_2 \leq \frac{2 + c_1 + c_1^2 + \sqrt{4 + 4c_1 - 3c_1^2 - 6c_1^3 + c_1^4}}{2c_1} \forall c_i \in [0,1]. \)

The solution \( c_2 \leq \frac{2 + c_1 + c_1^2 + \sqrt{4 + 4c_1 - 3c_1^2 - 6c_1^3 + c_1^4}}{2c_1} \) is not feasible in the range \( c_2 \in [0,1]. \)

14 \( c_2 \leq \frac{2 - c_1 + \sqrt{4 - 4c_1 - 7c_1^2}}{2c_1} \) is the other solution; but numerical simulation shows that this solution is not feasible in the range \( c_2 \in [0,1] \).
Chapter 3 Costly Reporting, Ex-Post Monitoring and Commercial Piracy: A Game Theoretic Analysis
3.1 Introduction

In this chapter, we investigate the decision of the regulatory authority to invest in monitoring commercial piracy when the legal firm has reported the presence of the pirate in the market. The existing literature on monitoring commercial piracy looks at ex-ante monitoring strategies where the authority, with the objective of maximising ex-ante social welfare, sets and commits to a level of monitoring before the pirate enters the market. However, empirical evidences show that often the authorities take monitoring decisions ex-post after the legal firm has taken the initiative to report piracy. This study attempts to bridge this gap in the literature by analysing ex-post monitoring decisions when the authority maximises a politically motivated objective function.

We find that the threat of ex-post monitoring can credibly deter piracy. The authority will monitor piracy under both price and quantity competitions if the political influence of the legal firm is sufficiently high. We also show that there exists a unique level of product substitutability between the legal firm and the pirate above which monitoring will be higher under quantity competition and below which monitoring will be higher under price competition. Hence, the more imperfect a substitute the pirated product is to the legal product, the government’s optimal monitoring rate will be higher under price competition than under quantity competition. Alternatively, if the pirated product is a very close substitute to the product of the legal firm, then the government will be monitoring more intensively under quantity competition than under price competition.

Commercial piracy is rampant in the digital industry and is usually observed in sectors such as media and software, where it is easy to replicate a product at a very low or
negligible cost to the pirate. According to an estimate by the International Data Corporation (IDC) and the Business Software Alliance (BSA), global losses to piracy amounted to about $48 billion of which the US - the country most affected by this kind of illegal practice – accounted for over $8 billion during 2007. Weekes (2008) claims that Australia loses over 200 million dollars annually due to piracy in the film and TV show industry despite the large copyright infringement penalty of AUD65000 or five years imprisonment. Thus illegal competition from pirates reduces the revenue of the legal firm and may provide a strong incentive for the legal firms to prevent piracy.

The literature on preventing piracy looks at both technical strategies and monitoring by the authority. Technical strategies imply that the legal firm invests in producing some anti-copying software, such as encryption devices, that would prevent illegal copying of its products. Banerjee (2003) shows that if the legal firm can cost-effectively install an anti-copying device in its product then no monitoring will be the social welfare-maximising equilibrium. However, just as it is easy to copy digital products, it is also easy to decode digital encryption and so technical protection may not be cost-effective. Therefore, the legal producers also put in effort to investigate and report piracy to the regulatory authority. Given the fact that it is costly for the firm to prepare and lodge a

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15 We are not considering end user piracy where copying is done for personal consumption and not for sale such as illegal purchase of the product or free downloading or P2P transfers (in case of software, media files etc) from the internet. There is a relatively large economic literature on end user copying. For a survey, see Peitz and Waelbroeck (2006).
16 See IDC and BSA (2007).
18 A firm may not report piracy if it offers some positive network benefits to the legal firm. Nascimento and Vanhonacker (1988), Conner and Rumelt (1991), and Takeyama (1994) show that in the presence of network externalities, pirates could be helpful to the legal firms as they increase awareness of the legal products. Consumers can sample the less costly pirated products and then decide to buy a legal copy. Conner and Rumelt (1991) show that piracy is a less costly way of advertising the legal product to specific consumers only. Pirates can also increase the market for complementary products which can, as a feedback effect, advance the market for the legal product as well. Another way piracy can be beneficial is through "tacit reciprocity" when the pirate improves on the legal product and the legal producer, in turn, builds on the pirated product (see Kolm 2006, and El Harbi and Grolleau, 2008). In the current study we are abstracting away from such positive and network impacts of piracy.
report, they will do so only if the expected benefit from apprehending the pirate is
greater than the associated cost of investigating and reporting.

On the regulatory front, the authority\textsuperscript{19} is responsible for providing legal protection
through monitoring illegal production. Following a tip-off from an outside source, who
could be an investigator with the legal producer or an employee in the illegal firm or a
concerned individual, the authority conducts raids on the suspect pirate firms. Given the
huge amount of revenue loss due to piracy, the legal producers often find it worthwhile
to investigate copyright infringements in their own domain and then report suspect
activities to the authority.

There are numerous empirical examples of legal firms reporting piracy to the regulatory
authority. Microsoft teamed up with the US Customs Services and the authority of
twenty two countries to fight illegal sale of its products. In Argentina, in November
2000, following a report from the anti-piracy body of Microsoft, the Argentinean
authority raided and apprehended six illegal sellers who were suspected of selling
counterfeit Microsoft software. In Brazil, Microsoft informed the Specialized Police
Department in Computer Crimes and initiated a police raid against Mega CD, a
company that was selling unauthorised copies of Microsoft products over the web. The

\textsuperscript{19} By authority we mean any kind of regulatory authority including the government, the police, and piracy
specific or industry specific regulatory units. Piracy specific or industry specific regulatory units are those
where a firm can report piracy and seek monitoring services to protect their intellectual property/copy or
patent rights, for example the Software & Information Industry Association, the Recording Industry
Association of America and the Motion Pictures Association of America. These organisations work on
behalf of their member firms to counter copyright infringements once such illegal activities are reported
to them. These organisations work together with law enforcement bodies like the authority, the police and
the court. For example, the Indian Music Industry (IMI, see
\url{http://www.indianmi.org/operations.htm}) identifies production and sale of illegal copies of the
digital products of its member firms, provides reports of their investigation to the regulatory authority and
helps the police in conducting raids on the counterfeiters. In the raids conducted by the police following
reports from the IMI, copying devices (computers, CD/DVD writers, blank CDs/DVDs) as well as
counterfeit products (post-production) are confiscated and the counterfeiters arrested. In this paper, we are
including all such organisations under the term ‘authority’.
police apprehended two counterfeiters and confiscated over two thousand CDs with Microsoft software and several unused CDs (pre-production)\textsuperscript{20}.

However, the existing literature on monitoring copyright infringement regarding both end-user and commercial, looks at the case where, before the pirate enters the market, the authority makes its ex-ante social welfare-maximising monitoring decision. Thus, this literature assumes that the authority can commit to the ex-ante choice of the monitoring effort. In this case, the literature finds that the equilibrium would be no monitoring since increased competition from the pirate increases the balance of the consumer surplus vis-à-vis profit of the legal firm as components of social welfare. Thus, an increase in monitoring decreases social welfare due to higher price and under-production of the legal product.

That monitoring reduces social welfare has been shown by Novos and Waldman (1984), Noyelle (1990), Cheng et. al. (1997), Chen and Png (1999) and Harbough and Khemka (2000), in the context of end-user piracy, and by Banerjee (2003, 2006), Banerjee and Mukherjea (2007) and Kiema (2008), in the context of commercial piracy. A general conclusion of the above studies is that a pricing strategy by the legal firm is more effective in preventing piracy than an ex-ante monitoring strategy by the authority whose objective is to maximise social welfare. This is because the high price of legal software products is the dominant reason for piracy and since piracy makes more products available to the consumers at a low price, the loss to the legal firm is

\textsuperscript{20} http://www.cdmediaworld.com/hardware/cdrom/news/0104/ms_piracy.shtml. The fact that pirates are caught before they are able to sell in the market will be used in our study where we will have the legal firm detect the presence of the pirate before the pirate actually operates in the output market.
outweighed by the gain to the consumer surplus in the social welfare function. Hence, monitoring is not a welfare maximising strategy\textsuperscript{21}.

The literature suggests that monitoring is not optimal when the authority is maximising social welfare if this is a simple sum of the profits of the legal firm and the pirate and the consumer surplus. Empirically, we do observe that monitoring by the authority took place. Thus, a natural conjecture is that the objective function of the authority is weighted by the political influence of the legal firm, the pirate and the consumers. Thus, the authority will monitor if it has a politically motivated objective function with a sufficiently high weight for the legal firm’s profit relative to the surplus to the consumers.

The issue of a political lobby influencing monitoring decisions has been addressed by Banerjee (2006) in the context of commercial piracy. He shows that if the authority maximises a simple social welfare function where the profits of the legal firm and the pirate and the consumer surplus are equally weighted, then there will be no monitoring in equilibrium. This is because when the authority maximises such a simple social welfare function, the benefit to the consumers always exceeds the loss to the legal firm when there is piracy. Thus piracy increases the social benefit and it is optimal for the authority to allow the pirate to operate in the market. However, if the legal firm has a higher weight than the pirate or the consumers in the objective function of the authority, then the authority will monitor piracy. In the case of a politically motivated objective

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\textsuperscript{21} Monitoring is even more undesirable when there are some positive network externalities or feedback effects to the legal firm due to the presence of piracy. This issue has been addressed by Church and Gandal (1992), Shurmer (1993), Gandal (1994, 1995), Shy and Thisse (1999), Park and Scotchmer (2005) and Poddar (2005). Piracy can also be treated as a special case of discriminatory pricing where the legal seller considers piracy equivalent to charging zero price to consumers with lower willingness to pay or lower quality preference and charging higher price to consumers with a higher willingness to buy the better quality but more expensive legal product. In this context, for end user piracy, Takeyama (1994) and Slive and Bernhardt (1998) show that allowing piracy is socially optimal. Bae and Choi (2006) show that more originals are sold, and at a lower price, if illegal copies exist in the market and hence optimal equilibrium requires that piracy should be allowed.
function, the trade-off between the surpluses of the consumers and of the legal producers and hence, the monitoring decision of the authority can go either way depending on the relative political influence of the legal firm and the consumers.

Following Banerjee (2006) we also consider a politically motivated objective function of the authority. However, we make a simplifying assumption that the political influence of the pirate is negligible and the political objective function of the authority is the weighted sum of the legal firm’s profit and the consumer surplus. This emphasises the trade-off between the surpluses of only the legal firm and the consumers when the authority monitors piracy vis-à-vis when it does not. Also, Banerjee (2006) considers ex-ante monitoring where the authority sets its anti-piracy strategy before the legal producer or the pirate enter and strategically interact in the market. However, the regulatory authority need not commit to the ex-ante monitoring decision once the pirate enters the market. In this paper, we investigate the case where the threat of monitoring is more credible since the authority takes its monitoring decisions after the legal firm reports the pirate. This is in line with the anecdotal evidences, as presented earlier, where the regulatory authorities act after the piracy has been reported.

Besides examining ex-post monitoring, we also analyse whether the mode of competition between the legal firm and the pirate influences the monitoring decision of the authority. The literature on monitoring commercial piracy only considers price competition. However, there is a substantial literature\textsuperscript{22} that shows that quantity competition is more profitable than price competition if the products are substitutes. In our model, the pirated copy is an imperfect substitute of the original product, and, we

\textsuperscript{22} For example, see Gal-Or (1985) and Boyer and Moreaux (1987a,b) in the context of sequential games; Vives (1985), and Cheng (1985) analyse price and quantity strategies for simultaneous games.
wish to study here if quantity competition in the product market affects the magnitude of the authority’s monitoring decision.

In our study, we assume that the authority gets to know about the presence of a pirate only if the legal firm detects and reports the presence of a pirate in the market. We assume that the probability of detecting the copyright infringer is proportional to the monitoring rate. If there is no regulatory intervention, the pirate and the legal firm compete sequentially in price or quantities with the legal firm as the leader. Our study analyses the effects of the degree of product substitution and the mode of competition between the legal firm and the pirate on i) the entry decision by the pirate, ii) the reporting decision by the legal firm, and iii) the optimal monitoring decision by the authority. We analyse a model where the regulatory authority maximises a politically motivated objective function, given as a weighted sum of the legal firm’s profit and the consumer surplus.

First, we show that for both price and quantity competition, the equilibrium monitoring rate chosen by the authority will depend on i) the degree of product substitutability, ii) the relative weight it puts on the legal firm’s profit vis-à-vis the consumer surplus, iii) the marginal cost of monitoring, and iv) the extent to which piracy changes the legal firm’s profit and the consumer surplus. The magnitudes of these effects depend on whether the firms are engaging in price competition or quantity competition. Second, this paper also finds that there exists a critical degree of product substitutability below which the authority will monitor at a higher intensity under price competition than under quantity competition and vice versa. Finally, we consider the case where the legal product appears in the market only if the legal firm is successful in its R&D investment,
and we show that the legal firm’s investment on innovation is higher under quantity competition than under price competition.

In section 2, we present the model for ex-post monitoring by the regulatory authority when the firms compete in price, and we analyse how the optimal monitoring rate varies with the political influence of the legal firm in the authority’s objective function. In sections 3 and 4, we examine the case for quantity competition and investigate how the results are affected by the mode of competition and by the degree of product substitutability between the legal firm and the pirate. In section 5, we analyse how the decision to invest in R&D by the legal firm is affected by the mode of competition in the product market and the monitoring decision by the authority. In the last section we set out our conclusions.

3.2 Model: Ex-Post Monitoring of Piracy for Price Competition

We assume that in the absence of piracy the market for the legal product is given by \( q = 4 - 2p \), where \( p \) is the price and \( q \) is the quantity demanded. The profit maximising equilibrium for this monopoly case can be described as \( p_m^* = 1 \), \( q_m^* = 2 \), \( \pi_m^* = 2 \) and \( CS_m^* = 1 \), where \( \pi_m \) is the monopoly profit of the legal firm and \( CS \) is the consumer surplus. In this model, we treat piracy as creating a market for differentiated goods where the legal product and the copy are imperfect substitutes. The demands for the products are determined by their own-price and the cross-price impacts. Without loss of generality, let us denote the legal firm as firm 1 and the pirate as firm 2. Let \((q_i, p_i)\) for \( i = 1, 2 \), denote firm \( i \)’s quantity and price. Building on Singh and Vives (1984), Boyer
and Moreaux (1987b) and Chatterjee (2009), we consider a differentiated product market characterized by the following demand functions\(^{23}\):

\[
\begin{align*}
q_1 &= 1 - p_1 + c_1 p_2 \\
q_2 &= 1 - p_2 + c_2 p_1
\end{align*}
\]  

(1)

where we allow for the cross-price impacts, given by \(c_1\) and \(c_2\), to be asymmetric, that is, their absolute values may differ. The own-price effects for both demand functions are normalised to 1.

Note that i) if the products were perfect substitutes or the own-price effects and cross-price effects for both firms were equal, that is \(c_1 = c_2 = 1\), and ii) if the two firms charge the same monopoly price, say, \(p_1 = p_2 = p_m = 1\), then \(q_1 = q_2 = 1 = \frac{q_m^*}{2}\) and \(\pi_1 = \pi_2 = 1 = \frac{\pi_m^*}{2}\). Therefore, the total market \(q = 4 - 2p\) is shared equally by the two firms if their products were perfect substitutes. This confirms that the demand structure is consistent under any form of competition structure. For tractability of analysis, we normalise the demand functions to

\[
\begin{align*}
q_1 &= 1 - p_1 + c p_2 \\
q_2 &= 1 - p_2 + p_1
\end{align*}
\]  

(2)

That is, we assume that the impact of the price of the legal firm on the demand for the pirated product is higher than the impact of the price of the pirated product on the demand for the legal product. Hence, we have \(0 < c_1 = c < 1 = c_2\). The assumption that \(c > 0\) implies that the products are substitutes and \(c < 1\) implies that the own price

---

\(^{23}\) See Chatterjee (2009) for more discussion on these demand functions.
impact exceeds the cross-price impact for the demand of the legal product.\textsuperscript{24} We further assume that production is costless.

Now let us first consider the game in which the regulatory body chooses a level of monitoring, $\alpha$, that maximizes its political objective function subject to the cost of monitoring, $C(\alpha)$. In order that the maximum can exist, we must have $C(0) = 0$, $C'(\alpha) > 0$ and $C''(\alpha) > 0$. Let us assume

\[ C(\alpha) = \frac{k\alpha^2}{2} \] (3)

where $k$ is the institutional parameter representing the efficiency of the authority, with a higher $k$ implying less efficient or more costly regulatory system. We also assume that the probability, $P$, that a pirate gets caught depends positively on the monitoring effort of the authority, that is $P = P(\alpha)$ with $P(0) = 0$, $P'(\alpha) > 0$, $P''(\alpha) < 0$ and $P(\infty) = 1$. For simplicity, we assume $P(\alpha) = \alpha$, i.e. the monitoring rate is also the rate of detecting and apprehending the pirate. Therefore, $P(\alpha \geq 1) = 1$, $1 > P(\alpha) = 0$ and $k$ must be sufficiently large in the cost function to ensure that monitoring cost is not restricted to a fraction. The timing of the game is as follows:

\textbf{Stage 1:} The pirate decides whether to enter the market. If it does not enter, the game ends with the legal firm as a monopolist. If the pirate enters, it has to incur an entry cost of $F$.

\textsuperscript{24}Our analysis retains the symmetric costs (zero marginal cost) assumption as in Singh and Vives (1984) and Boyer and Moreaux (1987b). However, we abstract away from their assumption of same cross-price impact and assume that cross-price impacts are asymmetric between the two demand structures. This demand system also excludes the case of perfect substitutes, that is, when the own-price effects and cross-price effects are same across the two demand functions.
**Stage 2:** If the pirate enters, the legal firm decides to whether to report the pirate to the authority. If it does not report, the game ends. If it reports, it incurs a reporting cost of $R$.

**Stage 3:** If the pirate enters and the legal firm reports, then the authority chooses a monitoring rate $\alpha \in [0,1]$.

**Stage 4:** If piracy cannot be detected or if the legal firm does not report, then the legal firm and pirate compete sequentially in price with the legal firm as the leader. If piracy is detected, the legal firm gets a monopoly in the product market\(^{25}\).

We shall assume that the pirate’s entry cost, $F$, and the legal firm’s reporting cost, $R$, are both sunk costs. The authority monitors after the legal firm reports the presence of piracy. The authority maximises a politically weighted objective function (henceforth, political objective function)\(^{26}\), $W$, which is a weighted sum of profit and consumer surplus (CS) such that $W = \beta \pi_1 + \gamma \pi_2 + (1 - \beta - \gamma)CS$. Here, $\beta$ is the weight of the legal firm, $\gamma$ is the weight of the pirate and $(1 - \beta - \gamma)$ is the weight of the consumers in the political objective function. For simplicity, we assume that the political power of the pirate in influencing the authority is negligible or zero, implying that the authority is not corrupt. That is,

$$W = \beta \pi_1 + (1 - \beta)CS; \quad \gamma = 0. \quad (4)$$

\(^{25}\) We abstract away from the case where the pirate has to pay a fine if caught. The literature on monitoring piracy assumes that the pirate will be penalised if caught. In our model, we are keeping the penalty fixed at zero for simplicity of analysis. A positive penalty will reinforce the findings of this paper.

\(^{26}\) Banerjee (2006) uses a social welfare function which captures lobbying by the legal firm. In his model, the consumer surplus and pirate’s profit are each weighted at one and the legal firm’s profit weighted at more than one. However, our formulation generalises the objective function used by Banerjee (2006) and gives us a sharper trade-off between the legal firm’s profit and consumer surplus with and without piracy. Thus we look at a wider range of political influence of the legal firm vis-à-vis the consumers in the objective function of the authority.
The authority decides upon the monitoring rate $\alpha$ that maximises $W$. If the pirate’s illegal activities are detected with probability $P(\alpha) = \alpha$, then the pirate is apprehended before the production stage and incurs the (sunk) entry cost of $F$. We solve the game by backward induction.

In Stage 4, if the pirate is not detected by the authority, the legal firm and the pirate play a sequential price Stackelberg game with the legal firm as the leader. The equilibrium price, quantity and profit of the legal firm and the pirate, respectively, are given by:

$$
\begin{align*}
    p_1^{*p} &= \frac{2 + c}{4 - 2c}, \\
    q_1^{*p} &= \frac{2 + c}{4}, \\
    \pi_1^{*p} &= \frac{(2 + c)^2}{8(2 - c)} - R, \\
    p_2^{*p} &= \frac{1}{2} + \frac{2 + c}{2(4 - 2c)}, \\
    q_2^{*p} &= \frac{1}{2} + \frac{2 + c}{2(4 - 2c)} \text{ and } \pi_2^{*p} = \frac{(6 - c)^2}{(8 - 4c)^2} - F. \\
\end{align*}
$$

The superscript P indicates price competition in the product market. Thus the total consumer surplus for the price game is given as

$$
CS^{*p} = \frac{52 - 12c - 7c^2 + c^4}{32(2 - c)^2}. 
$$

If the pirate is detected by the authority, then the legal firm gets to be a monopolist in the market and then the equilibrium monopoly outcome will be

$$
\begin{align*}
    p_m^{*} &= 1, \\
    q_m^{*} &= 2, \\
    \pi_m^{*} &= 2 - R \text{ and } CS_m^{*} = 1.
\end{align*}
$$
In Stage 3, following a report from the legal firm, the authority decides on the equilibrium level of monitoring, as illustrated in Figure 3.1. The regulator will choose a monitoring rate $\alpha$ that maximises the expected political objective function (EW) after the legal producer reports.

$$EW = \alpha \left[ \beta (\pi_m^*) + (1 - \beta) CS_m^* - \frac{k\alpha^2}{2} \right] + (1 - \alpha) \left[ \beta (\pi_1^{*p}) + (1 - \beta) CS^{*p} - \frac{k\alpha^2}{2} \right]$$

or,

$$EW = \alpha \left[ \beta (\pi_2^* - \pi^{*p}) + (1 - \beta)(CS_m^* - CS^{*p}) \right] + \beta (\pi_1^{*p}) + (1 - \beta)CS^{*p} - \frac{k\alpha^2}{2}. \quad (8)$$

The expected ‘benefit’ ($EB$) to the authority is

$$EB = \alpha \left[ \beta (\pi_m^*) + (1 - \beta)CS_m^* \right] + (1 - \alpha) \left[ \beta (\pi_1^{*p}) + (1 - \beta)CS^{*p} \right]$$

and the cost is given as

$$C = \frac{k\alpha^2}{2}. \quad \text{The expected marginal benefit is thus given as}$$

Figure 3.1: The Ex-Post Monitoring Game
\[ EMB = \left[ \beta(\pi_m^*) + (1 - \beta)CS_m^* \right] - \left[ \beta(\pi_i^*) + (1 - \beta)CS_i^* \right] \] and the marginal cost of monitoring is given as \[ MC = k\alpha. \] Now \[ MC = k\alpha > 0 \] for \( k, \alpha > 0 \). The authority will never monitor if the marginal benefit from duopoly exceeds that from monopoly. Thus, monitoring will take place only if \( EMB = MC \), or \( \beta(\pi_m^* - \pi_i^*) + (1 - \beta)(CS_m^* - CS_i^*) = k\alpha > 0 \), that is when benefit to the legal firm from the monopoly outweighs the loss in consumer surplus due to reduced competition. Therefore, monitoring will take place only if

\[
\beta > \frac{116c - 39c^2 + c^4 - 76}{148 - 156c + 33c^2 + 4c^3 + c^4} = \beta^p. \tag{9}
\]

If \( \beta < \beta^p \) then the expected marginal benefit from monitoring is negative and, given that monitoring is costly, the authority would choose no monitoring, that is \( \alpha^{*p} = 0 \). If \( \beta > \beta^p \), then the authority will maximise the EW by monitoring at the level where marginal benefit from monitoring (EMB) equals the marginal cost (MC) of monitoring. This optimal level of monitoring is given as

\[
\alpha^{*p} = \frac{\beta(148 - 156c + 33c^2 + 4c^3 + c^4) - 116c + 39c^2 - c^4 + 76}{32k(2 - c)^2}. \tag{10}
\]

**Proposition 1:** (i) There exists a unique critical level of political power, \( \beta^p \), of the legal firm beyond which the authority monitors piracy in equilibrium under price competition in the product market. That is, if \( \beta < \beta^p \) then \( \alpha^{*p} = 0 \); if \( \beta > \beta^p \), then \( \alpha^{*p} > 0 \). (ii) \( \alpha^{*p} \) increases with the level of political power, \( \beta \), and decreases with the cost of monitoring, \( k \).
The proposition is illustrated in Figure 3.2 which shows the expected marginal benefit and the marginal cost of monitoring. It can be seen that if $\beta < \beta^p$ then the EMB is negative, implying that the benefit under duopoly is higher than that under monopoly. Because marginal cost is always positive, the equilibrium outcome should have no monitoring by the authority; however, if $\beta > \beta^p$, then the benefit under monopoly exceeds that from the duopoly, and there exists a unique critical level of monitoring by the authority in equilibrium which is given by $MC = EMB$.

![Figure 3.2: Monitoring Equilibrium under the Price Game](image)

As the political power of the legal firm increases, the weight on the profit of the legal firm increases and thus the weight on the benefit to the consumers go down in the authority’s objective function. Hence with higher $\beta$ the monitoring effort increases. If the inefficiency of the authority increases, that is, with a higher $k$ monitoring becomes more costly, then the equilibrium level of monitoring goes down. In Figure 3.2, the intersection of a higher $MC$ with the $EMB$ will be to the left of $\alpha^{*p}$. 
Figure 3.3 shows how the trade-off between the profit of the legal firm and the consumer surplus with and without piracy changes with the degree of product substitutability, $c$. For a sufficiently high $c$, we have the difference in consumer surplus with piracy vis-à-vis a legal monopoly outweighing the profit difference from monopoly vis-à-vis the illegal competition situation. Hence, when the degree of product substitutability is high, a higher share of the consumers relative to the legal firm in the total surplus would prompt the authority not to monitor piracy. Thus, if $c$ is sufficiently high, the legal firm must have a higher weight in the political objective function of the authority in order to induce monitoring in equilibrium.

In stage 2 of the game, the legal firm will decide to report or not. It will report if the expected profit following the report exceeds that from not reporting. Thus, the legal firm will report if

$$\beta(\pi_m^* - \pi_1^p),$$

$$(1 - \beta)(CS_1^p - CS_m^*).$$
\[ \alpha^{*p}(2-R) + (1-\alpha^{*p}) \left\{ \frac{(2+c)^2}{8(2-c)} - R \right\} > \frac{(2+c)^2}{8(2-c)}. \] (11)

or if \( R < \alpha^{*p} \left\{ 2 - \frac{(2+c)^2}{8(2-c)} \right\} = R^{*p} \) where \( \alpha^{*p} > 0 \). If \( \alpha^{*p} = 0 \), then the legal firm will never report.

**Proposition 2:** (i) Under price competition, if the political influence of the legal firm is sufficiently high to induce monitoring, the legal firm will report piracy only if the cost of reporting is below a critical level. That is, if \( \beta > \beta^{p} \), so that \( \alpha^{*p} > 0 \), then the legal firm will report if \( R < R^{*p} \) and will not report if \( R \geq R^{*p} \). (ii) If \( \beta < \beta^{p} \) so that \( \alpha^{*p} = 0 \) then not reporting is the equilibrium.

In stage 1, the pirate will decide to enter or not. The pirate’s decision depends on the decision of the legal firm to report following the entry of the pirate. If \( \alpha^{*p} > 0 \) and the legal firm has \( R \geq R^{*p} \) or if \( \alpha^{*p} = 0 \) then the legal firm will not report and then the pirate will enter if \( \pi^{*p}_{2} = \frac{(6-c)^2}{(8-4c)^2} - F > 0 \) or if \( \frac{(6-c)^2}{(8-4c)^2} > F \). If \( \alpha^{*p} > 0 \) and the legal firm has \( R < R^{*p} \) then the pirate will enter if his expected duopoly profit following the report exceeds that from not entering. Thus, the pirate will enter if

\[ \alpha^{*p}(-F) + (1-\alpha^{*p}) \left\{ \frac{(6-c)^2}{(8-4c)^2} - F \right\} > 0 \] or if \( (1-\alpha^{*p}) \left( \frac{(6-c)^2}{(8-4c)^2} \right) = F^{*p} > F \). (12)
**Proposition 3:** If $\beta > \beta^p$, $R < R^*$ and $F < F^*$ then the sub-game perfect equilibrium will have the pirate entering, the legal firm reporting and the authority monitoring piracy.

### 3.3 Model: Ex-Post Monitoring of Piracy for Quantity Competition

In this section, we consider the same market structure as in section 2, except that in Stage 4, the legal firm and the pirate play a sequential *quantity* game, instead of a price game, with the legal firm as the leader. The equilibrium price, quantity and profit of the legal firm are given by

\[
p_1^{\text{Q}} = \frac{1}{2(1-c)} \quad q_1^{\text{Q}} = \frac{1}{2-c} \quad \text{and} \quad \pi_1^{\text{Q}} = \frac{1}{2(1-c)(2-c)} - R
\]  

and those for the pirate are given by

\[
p_2^{\text{Q}} = \frac{3 - 2c}{2(2-c)(1-c)} \quad q_2^{\text{Q}} = \frac{3 - 2c}{2(2-c)} \quad \text{and} \quad \pi_2^{\text{Q}} = \frac{(3 - 2c)^2}{4(2-c)^2(1-c)} - F .
\]

Thus the total consumer surplus for the quantity game is given by

\[
CS^{\text{Q}} = \frac{13 - 12c + 4c^2}{8(2-c)^2} .
\]

The superscript *Q* denotes quantity competition in the product market. As in section 2, if the pirate is detected by the authority, then the legal firm gets to be a monopolist in the market and then the equilibrium monopoly outcome will be

\[
p_m^* = 1, \quad q_m^* = 2, \quad \pi_m^* = 2 - R \quad \text{and} \quad CS_m^* = 1 .
\]

In Stage 3, following a report from the legal firm, the authority decides on a monitoring rate $\alpha$ that maximises the expected political objective function (EW) after the legal producer reports.
The expected ‘benefit’ \((EB)\) to the authority is

\[ EB = \alpha \left[ \beta \left( \pi_m^* - \pi_1^{*q} \right) + (1 - \beta)(CS_m^* - CS^{*q}) \right] + (1 - \alpha) \left[ \beta \left( \pi_1^{*q} \right) + (1 - \beta)CS^{*q} - \frac{k\alpha^2}{2} \right]. \]  (17)

The expected marginal benefit is thus given as

\[ EMB = \left[ \beta \left( \pi_m^* - \pi_1^{*q} \right) + (1 - \beta)(CS_m^* - CS^{*q}) \right] - \left[ \beta \left( \pi_1^{*q} \right) + (1 - \beta)CS^{*q} \right] \] and the marginal cost of monitoring is given as \(MC = k\alpha\). Now \(MC = k\alpha > 0\) for \(k, \alpha > 0\). The authority will never monitor if the marginal benefit from duopoly exceeds that from monopoly. Thus, monitoring by the authority will take place only if \(EMB=MC\) or \(\beta \left( \pi_m^* - \pi_1^{*q} \right) + (1 - \beta)(CS_m^* - CS^{*q}) = k\alpha > 0\), when benefit to the legal firm from the monopoly due to reduced competition outweighs the loss in consumer surplus. Therefore, monitoring will take place only if \(\beta \left( \pi_m^* - \pi_1^{*q} \right) + (1 - \beta)(CS_m^* - CS^{*q}) > 0\), that is if the benefit to the legal firm from the monopoly due to reduced competition outweighs the loss in consumer surplus. Hence, monitoring will take place only if

\[ \beta(2 - R) + (1 - \beta) > \beta \left( \frac{1}{2(1-c)(2-c)} - R \right) + (1 - \beta) \frac{13 - 12c + 4c^2}{8(2-c)^2} \]

or \(\beta > \frac{39c - 24c^2 + 4c^3 - 19}{37 - 85c + 56c^2 - 12c^3} = \beta^0\). \quad (18)

If \(\beta > \beta^0\), then the authority will maximise \(EW\) by monitoring at the level

\[ \alpha^{*q} = \frac{\beta(37 - 85c + 56c^2 - 12c^3) - 39c + 24c^2 - 4c^3 + 19}{8k(2-c)^2(1-c)}. \]  (19)
**Proposition 4:** (i) There exists a unique critical level of political power, $\beta^0$, of the legal firm beyond which the authority monitors piracy in equilibrium under quantity competition in the product market. That is, if $\beta < \beta^0$ then $\alpha^* = 0$; if $\beta > \beta^0$, then $\alpha^* > 0$. (ii) $\alpha^*$ increases with the level of political power, $\beta$, and decreases with the cost of monitoring, $k$.

Proposition 4 is illustrated in Figure 3.4 which shows the expected marginal benefit and the marginal cost of monitoring. It can be seen that if $\beta < \beta^0$ then the EMB is negative, implying that the benefit under duopoly is higher than that under monopoly. Marginal cost being always positive, the equilibrium outcome should have no monitoring by the authority. However, if $\beta > \beta^0$, then the benefit under monopoly exceeds that from the duopoly, and there exists a unique critical level of monitoring by the authority in equilibrium such that $MC = EMB$. As the political power of the legal firm increases, the weight on the profit of the legal firm increases and thus the weight on the benefit to the
consumers go down in the authority’s objective function. Hence with higher $\beta$ the monitoring effort increases. If the inefficiency of the authority increases, that is, with a higher $k$ monitoring becomes more costly, then the equilibrium level of monitoring will go down. In Figure 3.4, the intersection of a higher MC with the EMB will be to the left of $\alpha^{*Q}$.

Figure 3.5 shows how the trade-off between the profit of the legal firm and the consumer surplus changes with the degree of product substitutability, $c$. For a sufficiently high $c$, we have the difference in consumer surplus with piracy vis-à-vis a legal monopoly outweighing the profit difference from monopoly vis-à-vis the illegal competition situation. Hence, when the degree of product substitutability is high, a higher share of the consumers relative to the legal firm in the total surplus would prompt the authority not to monitor piracy. Thus, if $c$ is sufficiently high, the legal firm must have a higher weight in the political objective function of the authority in order to induce monitoring in equilibrium.

In stage 2 of the game, the legal firm will decide to report or not. It will report if the expected profit following the report exceeds that from not reporting. Thus, the legal firm will report if

$$\alpha^{*Q} (2 - R) + (1 - \alpha^{*Q}) \left(\frac{1}{2(1-c)(2-c)} - R\right) > \frac{1}{2(1-c)(2-c)}$$

or if $R < \alpha^{*Q} \left(2 - \frac{1}{2(1-c)(2-c)}\right) = R^{*Q}$ where $\alpha^{*Q} > 0$. If $\alpha^{*Q} = 0$, then the legal firm will never report.
**Proposition 5:** (i) Under quantity competition, if the political influence of the legal firm is sufficiently high to induce monitoring, the legal firm will report piracy only if the cost of reporting is below a critical level. That is, if $\beta > \beta^Q$, so that $\alpha^* > 0$, then the legal firm will report if $R < R^Q$ and will not report if $R \geq R^Q$. (ii) If $\beta < \beta^Q$ so that $\alpha^* = 0$ then not reporting is the equilibrium.

In stage 1, the pirate will decide to enter or not. The pirate’s decision depends on the decision of the legal firm to report following the entry of the pirate and the cost of entry. If $\alpha^* > 0$ and the legal firm has $R \geq R^Q$ or if $\alpha^* = 0$ then the legal firm will not report and then the pirate will enter if $\pi^*_2 = \frac{(3-2c)^2}{4(2-c)^2(1-c)} - F > 0$ or if
\[
\frac{(3-2c)^2}{4(2-c)^2(1-c)} > F. \text{ If } \alpha^* > 0 \text{ and the legal firm has } R < R^0 \text{ then the pirate will enter if his expected duopoly profit following the report exceeds that from not entering. Thus, the pirate will enter if}
\[
\alpha^* (-F) + (1-\alpha^*) \left( \frac{(3-2c)^2}{4(2-c)^2(1-c)} - F \right) > 0
\]
or if
\[
(1-\alpha^*) \left( \frac{(3-2c)^2}{4(2-c)^2(1-c)} \right) = F^* > F.
\] (21)

**Proposition 6:** If \( \beta > \beta^0, R < R^0 \) and \( F < F^0 \) then the sub-game perfect equilibrium will have the pirate entering, the legal firm reporting and the authority monitoring piracy.

### 3.4 Comparing the Equilibria under Price and Quantity Competition

This section compares the equilibrium monitoring efforts for the case where the pirate enters and the legal firm reports in equilibrium under both price and quantity competitions. Thus, \( R < R^0, R^p \) and \( F < F^0, F^p \). Whether the monitoring effort under price competition will be higher or lower than the monitoring effort when the firms compete in quantities will depend upon the relative political pressure exerted by the legal firm on the authority.

We find that \( \beta^p > \beta^0 \) if \( c > \bar{c} \). That is, there exists a unique critical degree of substitutability beyond which price competition, compared to quantity competition, requires a higher level of political power of the legal firm to ensure credible monitoring effort by the authority. The closer the pirated copy is as a substitute to the legal product, the less is required of the legal firm to lobby the authority under quantity competition.
This is because when there is a high degree of product substitutability, if the pirate enters the benefit to the consumers is lower under quantity competition and higher under price competition. In this case, for \( c > \tilde{c} \), if \( \beta^p < \beta < \beta^0 \), then the equilibrium monitoring condition will have positive monitoring under quantity competition and no monitoring under price competition. This is shown by the area D in Figure 3.6. If \( \beta > \beta^p \), then, as shown by area B, there will be monitoring in equilibrium for both price and quantity competition, however, monitoring will be higher under quantity competition than under price competition.

![Diagram](image)

**Figure 3.6: Comparing Monitoring between the Price and Quantity Games**

If \( c < \tilde{c} \) then we find that \( \beta^0 > \beta^p \). That is, if the pirated product is not a close substitute of the legal product, then there exists a critical degree of substitutability beyond which the political influence of the legal firm has to be higher under quantity competition than price competition to ensure credible monitoring effort by the authority. If the pirated copy is a significantly inferior substitute to the legal product, the less is
required of the legal firm to lobby the authority because if the pirate enters it increases the total size of the market under price competition. The benefit to the legal firm is higher under price competition and lower under quantity competition when the pirate operates in the market. In this case, for \( c < \tilde{c} \), if \( \beta^0 < \beta < \beta^p \), then the equilibrium monitoring condition will have positive monitoring under price competition and no monitoring under quantity competition. This is shown by the area C in Figure 3.6. If \( \beta > \beta^0 \), then, as shown by area A, there will be monitoring in equilibrium for both price and quantity competition, however, monitoring will be higher under price competition than under quantity competition. The area E illustrates the case when \( \beta < \beta^0 \) and \( \beta < \beta^p \) such that there is no monitoring under both price and quantity competitions.

**Proposition 7:** For a sufficiently high level of political influence of the legal firm such that the authority will choose to monitor piracy under both price and quantity competition, there exists a unique level of product substitutability between the legal firm and the pirate above which monitoring will be higher under quantity competition and below which monitoring will be higher under price competition. That is, if \( \beta > \beta^0 \) and \( \beta > \beta^p \), there exists a unique level of product substitutability \( \tilde{c} \) such that if \( c > \tilde{c} \) then \( \alpha^0 > \alpha^p > 0 \) and if \( c < \tilde{c} \) then \( \alpha^p > \alpha^0 > 0 \).

Thus, there exists a critical degree of substitutability, \( \tilde{c} \), between the demands for the legal product and the pirated product such that if \( c > \tilde{c} \) then higher political pressure is required by the legal firm under price competition compared to quantity competition, to ensure credible monitoring. If \( c < \tilde{c} \) then higher political pressure is required by the
legal firm under quantity competition compared to price competition, to ensure credible monitoring. That is, the more imperfect a substitute the pirated product is to the legal product, the government’s optimal monitoring rate will be higher under price competition than under quantity competition. If the pirated product is a very close substitute to the product of the legal firm, then the government will be monitoring more intensively under quantity competition than under price competition. If the level of political power of the legal firm is high enough to ensure credible monitoring under both price competition and quantity competition, that is if $\beta > \beta^o$ and $\beta > \beta^p$, then monitoring is always optimal in equilibrium; alternatively, if $\beta < \beta^o$ and $\beta < \beta^p$ then not monitoring will be the optimal in equilibrium.

### 3.5 Introducing Innovation by the Legal Firm

Piracy has generally been perceived as having a harmful impact on the digital industry where the products can be copied at a low cost (Marshall, 1999; Straub and Nance, 1990). This issue assumes importance not only because of the high magnitude of the loss in retail sales but also because piracy reduces the incentive to innovate by the legal firms. Novos and Waldman (1984) find that piracy induces higher copyright protection and underproduction in the legal sector thereby reducing social welfare. Park and Ginarte (1996) also support this observation. Qiu (2006) shows that, if copyright protection is weak, then the legal product is only developed for a small number of customers willing to pay a high price and not for general use. In an empirical study, Ding and Liu (2009) find that under weak Intellectual Property Right (IPR) regimes piracy dissuades the legal firms to invest on the development of new technologies.

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27 Piracy may also have beneficial impacts on innovation. Beneficial effects of piracy could be through positive feedback effects on the legal firm’s innovative efforts (Easley et al., 2003; Kolm, 2006; El Harbi and Grolleau, 2008) or through raising awareness of the legal product by providing a cheaper, albeit poorer, substitute (Conner and Rumelt, 1991). We abstract away from such beneficial network effects in the current study.
Jaisingh (2009) shows that innovative investment increases with stronger enforcement of regulatory measures to apprehend copyright infringers.

In this section, we investigate how the mode of competition and the degree of product substitution between the legal firm and the pirate determine the investment in innovation by the legal firm. We consider that the legal firm operates in the product market if the innovation is successful, with the probability of success given by \( \mu(X) \) where \( X \) is the expenditure on Research and Development (R&D) for innovation.

We assume that \( \mu(0) = 0, \mu(\infty) = 1, \mu'(X) > 0 \) and \( \mu''(X) < 0 \). If the new product is successfully developed, then the legal firm faces the threat of a pirate entering with an imperfect copy and competing in the product market. Then the game from the earlier sections, where the authority monitors after a costly report by the legal firm, is played.

We call this the innovation game and its timings can be summarised as follows:

**Stage 0:** The legal firm decides on the extent of R&D investment, \( X \), where \( \mu(X) \in [0,1] \) is the probability of the product being successfully developed. If the R&D investment is unsuccessful, the game stops.

**Stages 1 to 4:** If the product is successfully developed, then the pirate, the legal firm and the authority play the games that we studied earlier for the two cases of price competition and quantity competition.

As before, we solve the innovation game by backward induction. The results for stages 1-4 have already been derived in the earlier sections. Building on these results, we analyse the equilibrium for the initial innovation stage (stage 0) of the game. Let us look at the equilibrium conditions for the sub-game denoted by stages 1-4 for the two
cases\textsuperscript{28} when i) the authority monitors under both price and quantity competitions and ii) the authority never monitors.

Case 1: $\alpha^{*p}, \alpha^{*q} > 0$.

In this case, the authority monitors piracy under both price and quantity competitions. The legal firm will not report for any $R$ such that $R < R^{*p}$ and $R < R^{*q}$, and the pirate will enter for any $F$ such that $F < F^{*q}$ and $F < F^{*p}$. In stage 0, the legal firm maximises its expected profit from investing in R&D. The legal firm can realised its expected profit from stage 4, $E\pi_1$, provided its R&D expenditure, $X$, in stage 0 develops the new product with probability $\mu(X)$. With probability $1 - \mu(X)$ the legal firm cannot develop the product and foregoes the cost of R&D. Thus the legal firm chooses the optimal level of investment $X$ that maximises its expected profit in stage 0 given by:

$$\mu(X)E\pi_1 + (1 - \mu(X))0 - X$$

or $\mu(X)E\pi_1 - X$

or $\mu(X)(\alpha^*(2 - R) + (1 - \alpha^*)(\pi_1 - R)) - X$.  \hspace{1cm} (22)

From the first order conditions of maximising expected profit with respect to $X$, we have

$$\mu'(X^*) = \frac{1}{[\alpha^*(2 - R) + (1 - \alpha^*)(\pi_1 - R)]} = \frac{1}{[\alpha^*(2 - \pi_1) + (\pi_1 - R)]}$$

where $\alpha^* = \alpha^{*p}$ and $\pi_1 = \pi^{*p}_1$ in case of price competition and $\alpha^* = \alpha^{*q}$ and $\pi_1 = \pi^{*q}_1$ for the case of quantity competition. Thus, if the firms were competing in price in the product market, then $\pi_1 = \pi^{*p}_1 = \frac{(2 + c)^2}{8(2 - c)}$ and if they were competing in quantity, then

$$\pi_1 = \pi^{*q}_1 = \frac{1}{2(1 - c)(2 - c)}.$$  Moreover, for $c > 0$, we have $\pi^{*q}_1 > \pi^{*p}_1$. Hence, given

\textsuperscript{28} We are not considering the in-between cases where there is monitoring for one form of competition and no monitoring under the other form of competition.
\[ c = \tilde{c} > 0 \] which implies that \( \alpha^{p^*} = \alpha^{Q^*} > 0 \), we have \( \mu'(X^{p^*}) > \mu'(X^{Q^*}) \). This, in turn, implies that \( X^{Q^*} > X^{p^*} \).

**Proposition 8:** For a sufficiently high level of political influence of the legal firm and a critical degree of product substitutability \( \tilde{c} \) for which the authority will choose to monitor piracy at the same rate under price and quantity competition, the optimal R&D investment by the legal firm will be higher under quantity competition than under price competition for. That is, \( X^{Q^*} > X^{p^*} \) if \( c = \tilde{c} > 0 \) and \( \alpha^{p^*} = \alpha^{Q^*} > 0 \).

**Case 2:** \( \alpha^{p^*}, \alpha^{Q^*} = 0 \).

(i) If \( F > F^{*p^*} \) and \( F > F^{*Q^*} \), then the pirate will never enter under any modes of competition and monitoring will not take place. The legal firm will have a monopoly in the product market and will choose the optimal level of \( X \) that maximises \( \mu(X)2 - X \), implying that \( \mu'(X^{m}) = \frac{1}{2} \).

(ii) If \( F < F^{*p^*} \) and \( F < F^{*Q^*} \) then the pirate will always enter the market irrespective of the mode of competition in the product market. Moreover, if \( R > R^{*p^*} \) and \( R > R^{*Q^*} \), then the legal firm will never report the pirate and hence there will be no monitoring by the authority. The legal firm will thus be acting as a Stackelberg leader in the product market and will choose the optimal level of \( X \) that maximises \( \mu(X)\pi_1 - X \) where \( \pi_1 = \pi^{p^*}_1 \) in case of price competition and \( \pi_1 = \pi^{Q^*}_1 \) for quantity competition. The first order condition of expected profit maximisation with respect to \( X \) gives \( \mu'(X^*) = \frac{1}{\pi_1} \). If the firms were competing in
price, then \( \pi_1 = \pi_1^{*p} = \frac{(2 + c)^2}{8(2 - c)} \) and if they were competing in quantity, then \( \pi_1 = \pi_1^{*q} = \frac{1}{2(1-c)(2-c)} \). For \( c > 0 \) and \( \alpha^{*p} = \alpha^{*q} = 0 \), we have \( \pi_1^{*q} > \pi_1^{*p} \). Thus, \( \mu'(X^{*p}) > \mu'(X^{*p}) \) implying that \( X^{*q} > X^{*p} \).

Note that the monopoly profit of the legal firm is always higher than the profit under a Stackelberg duopoly for both price competition and quantity competition. That is, \( \pi^{*m} = 2 > \pi_1 \) where \( \pi_1 \in [\pi_1^{*p}, \pi_1^{*q}] \). Hence, \( \mu'(X^{*m}) = \frac{1}{2} < \mu'(X^{*}) = \frac{1}{\pi_1} \), implying that \( X^{*m} > X^{*} \) where \( X^{*} \in [X^{*p}, X^{*q}] \). Thus investment in R&D by the legal firm is highest under monopoly and, when it competes with a pirate in the product market, it is higher under quantity competition than under price competition.

### 3.6 Conclusions

In this paper, we investigate the decision of a regulatory authority to invest in monitoring commercial piracy after the legal firm has reported the presence of the pirate in the market. The existing literature on monitoring commercial piracy looks at the ex-ante monitoring strategies where the authority, with the objective of maximising ex-ante social welfare, sets and commits to a level of monitoring before the pirate enters the market. However, empirical evidences show that often the authorities take monitoring decisions ex-post after the legal firm has taken the initiative to report piracy. This study attempts to bridge this gap in the literature by analysing the ex-post monitoring decision of the authority when the legal firm undertakes a costly process of investigating and reporting piracy to the authority.

We examine the case where the objective of the regulatory authority is weighted by the political influence of the legal firm and the consumers. We find that the authority will
monitor piracy under both price and quantity competition if the political influence of the legal firm is sufficiently high. We also show that there exists a unique level of product substitutability between the legal firm and the pirate above which monitoring will be higher under quantity competition and below which monitoring will be higher under price competition. This implies that the extent of political influence the legal firm will exert on the regulatory authority to induce monitoring depends crucially on the mode of competition and on the degree of product substitution.

The mode of competition and the degree of product substitutability also affect R&D decisions of the legal firm in the presence of piracy. We see that piracy unambiguously reduces R&D investment by the legal firm. However, the reduction in R&D is lower under quantity competition than under price competition. In other words, given the degree of product substitutability between the legal firm and the pirate, the legal firm will invest more under quantity competition than under price competition. The magnitude of the legal firm’s R&D investment would be determined by the closeness of its innovated product to the pirated copy, the influence of the legal firm on the political objective of the monitoring authority and the mode of competition between the legal firm and the pirate.

Thus, this study explores how the degree of product substitutability and mode of competition affect the decisions by the legal firm to undertake R&D and to report the presence of piracy in the product market to the authority and also the decision of the authority to monitor piracy, once reported. However, we take the degree of product substitutability as exogenous in this study. An interesting extension of this work would be to endogenise the degree of product substitutability and examine how the pirate and the legal firm locate their products, given the market demand. Thus this study could add
to the literature (for example, Choi and Coughan, 2006; Noh and Moschini, 2006) on quality or brand differentiation and label positioning by considering illegal brands and pirated labels in the product market. It would be interesting to examine how the legal firm and the pirate position their product qualities when they enter the market and how monitoring decision by the authority is affected by such endogenous product locations. A further extension would be to introduce uncertainty on the part of the consumers regarding product quality (see Metrick and Zeckhauser 1996) and examine how the equilibrium is affected if the consumer is unable to distinguish between the legal product and the pirated copy.

Another line of research could be to extend this study to investigate of how the authority’s monitoring decision and the legal firm’s R&D decision are influenced by the relative political influence of the legal firm and the pirate. That is, if we remove the simplifying assumption of $\gamma = 0$ and look at the effect of the ratio of the political powers of the legal firm and the pirate, $\frac{\beta}{\gamma}$, in influencing the political objective and thereby the monitoring decisions of the regulatory authority. Thus, this study could be a step towards empirical and policy studies by bringing together the literature on innovation, product differentiation, commercial piracy and corruption.
References


Chapter 4 On the Impact of Piracy on Innovation in the Presence of Technological and Market Uncertainties
4.1 Introduction

In this chapter we analyse the impact of piracy on innovation, especially in the digital industry, when innovation is associated with both technological and market uncertainties\(^\text{29}\). The literature on piracy and innovation only looks at the technological uncertainty aspect of innovation where the R&D investment of a firm materialises into a new product with some probability. However, innovation also has a market uncertainty aspect when multiple firms compete simultaneously to develop a new product but the success of a firm in patenting its product is stochastic. In other words, a firm might be successful in developing a new product but that does not necessarily mean that it is successful in obtaining a patent for its innovation. We attempt to bridge this gap and bring together these two strands of the literature.

Piracy has generally been perceived as having a damaging influence on software and media industry sectors that have high information and digital content since such products can be copied at a low cost (Marshall, 1999; Straub and Nance, 1990). This issue assumes importance not only because of the high magnitude of loss in retail sale but also of its possible detrimental effects on the incentive to innovate. Business Software Alliance (BSA) believes that “local software industries crippled from competition with high-quality pirated software” and International Federation of the Phonographic Industry (IFPI) in its 2005 Commercial Piracy Reports argues that “the illegal music trade is destroying creativity and innovation”\(^\text{30}\). In an empirical study, Ding and Liu (2009) show that under weak Intellectual Property Rights (IPR) regimes

\(^{29}\) The terms technological uncertainty and market uncertainty have been introduced by Shy (2000).

\(^{30}\) IDC and BSA (2007) claim that around US$48 billion were lost worldwide to piracy. BSA further projects that by 2010 almost US$200 billion worth of software will be pirated globally.
piracy dissuades the legal firms to continue research on the development of new technologies. Park and Ginarte (1996) also support this observation.

In this chapter, we show that, if there is a single innovating firm facing only technological uncertainty, piracy unambiguously retards the incentive to innovate and has adverse effects on profit. However, if there is R&D competition that creates market uncertainty along with technological uncertainty, then piracy may induce the legal firms to put in a higher level of R&D investment than they would in the absence of piracy. This is because, though piracy lowers the competing firms’ reaction functions with respect to the level of R&D investment, the equilibrium R&D investment depends on the relative size of the shifts in the reaction functions due to piracy. We show that if the difference between the efficiency parameters that measures the probability of success of the innovating firms is relatively large then piracy enhances the R&D investment and profit of the less efficient firm.

The literature addressing the impact of piracy on innovation focuses on a single innovating firm and show that piracy can have both detrimental and beneficial impact on innovation. The harmful effect of piracy could be that legal firms would be producing less and spending more on copyright protection. Novos and Waldman (1984) show that increases in copyright protection could increase social welfare loss due to underproduction. Qiu (2006) shows that, if copyright protection is weak, then software is not developed for general use but only for custom demands. Jaisingh (2009) finds that piracy generally harms innovation and it is only under very strict regulatory enforcement policies that the legitimate product quality is improved.

However, piracy can be beneficial in the form of providing insight into emerging market trends and specific consumer requirements, thereby providing direction for R&D
investment to the legal firms. Conner and Rumelt (1991) show that piracy can be a channel of advertising the legal product to consumers who have a high demand for the product. Easley et al. (2003) empirically show that presence of piracy in the music industry induces legal firms to develop internet technologies and electronic modes of distributing music files.\(^{31}\) Duchene and Waelbroeck (2006) show that stronger copyright protection increases the profits of firms practicing the conventional sales and marketing of physical products and lowers the profits of firms who markets its products by allowing consumers to search and sample.

Piracy can also generate a positive feedback effect on innovation with the pirate modifying and improving on the original product thereby inducing further innovation by the legitimate firms (Kolm 2006; El Harbi and Grolleau, 2008).\(^{32}\) In such situations, the legal producers sometimes do not insist on severe punishments against pirates (Barnett, 2005; Raustila and Sprigman, 2006; Barnett et al., 2010).

The above literature on piracy, to the best of our knowledge, has not looked at R&D competition or patent races between innovating firms. The literature on innovation, R&D expenditure and patent races do not touch on the issue of piracy. The literature on firm level R&D mostly concentrates on the firm characteristics that determine the extent and efficiency of R&D investment by a firm. For example, Rosen (1991) finds a positive relation between efficiency and R&D investment of a firm when two firms compete in the product market. This is supported by Cohen (1995) in an empirical

\(^{31}\) When hackers used Valve Software’s Half Life game engine to develop a game called Counter Strike, Valve, a gaming company, took the illegal game software and marketed it themselves, selling over 1.5 million copies (Barnes, 2005). Apple Computer, in a strategic reaction to P2P file sharing technologies, launched the iTunes online music library that was easy to navigate and explore, with free music previews, and allowed flexible download and copying for personal use. See Choi and Perez (2007) for anecdotal evidences on legal firms adopting technologies used by illegal P2P file sharers.

\(^{32}\) This is especially true for design based industries where being pirated is a signal of the high quality of the legal product, and products which ‘are not faked are considered too weak to generate consumer demand and are consequently not produced’ (Whitehall, 2006). Ritson (2007) says that pirated goods are indicative of heralding a brand’s renaissance and a brand dies if no copies appear in the market.
study. However, Patel and Pavitt (1995) show that the relationship between efficiency in production and R&D investment is not very clear-cut. Poyago-Theotoky (1996) show that low-cost firms spend more on cost-reducing R&D and spend less on R&D if the probability of success of such R&D is low.

The literature on patent race and innovation find that there exist positive two-way causal relations between patents and innovations. On one hand, as patenting becomes cheaper it has a direct effect in increasing the incentive to innovate. That is as the price of patenting goes down, demand for R&D investment goes up.\(^33\) On the other hand, Hunt (2006) shows that if the product can be copied without cost then reduction of patent costs may lower R&D activity under certain conditions.\(^34\) Kultti et al. (2006) observes that too much patent race reduces innovation\(^35\). In a study on the computer industry in the U.S., Bessen and Hunt (2003) show that lowering of patent standards raises firms’ propensities to file for more patents.

The closest work in the spirit of our paper is by Shapiro (2006) who models simultaneous and independent inventions, in the context of prior rights, when two firms successfully realize the targeted innovation, but only one firm is able to file for the patent. However, he does not examine the impact of piracy on innovation. In our current chapter we try to link the literature on patent race and innovation to that on innovation and piracy.

This chapter is organised as follows. In Section 2 we present the model with a single innovating firm. In Section 3 we introduce R&D competition between two legal firms.

\(^{33}\) See Maurer and Scotchmer (2002) for a review.
\(^{34}\) This ‘counter-intuitive’ phenomenon can be observed in industries that are highly technology intensive such as, semiconductors, electronics and computers (Hunt 2004, 2006).
\(^{35}\) Cohen et al. (2000) and Gallini (2002) suggest secrecy is a better strategy than patents in protecting innovation and the role of patents lie in encouraging information disclosure rather than R&D investments.
In section 4 we use specific functional forms to illustrate our findings from Sections 2 and 3. Section 5 contains the concluding remarks.

### 4.2 The Basic Model

Let us consider the market for a product, like software, that faces piracy. We first consider the case where there is only one firm investing in R&D technology in order to increase its profit over and above a reservation level, \( \pi \). For simplicity we assume \( \pi = 0 \). Let \( R \) be the R&D investment of the firm, and the probability that the firm is successful in developing the product is \( k\alpha(R) \) such that \( 0 \leq k\alpha(R) \leq 1 \) with the properties \( \alpha'(R) > 0 \) and \( \alpha''(R) < 0 \).\(^{36}\) \( k \) can be viewed as the R&D efficiency parameter.

There is a continuum of consumers indexed by \( \theta, \theta \in [0,1] \). \( \theta \) is assumed to follow a uniform distribution. Each consumer is assumed to purchase only one unit of the product. The utility of a type-\( \theta \) consumer is,

\[
U(\theta) = \begin{cases} 
\theta - p_m & \text{if the consumer buys the product,} \\
0 & \text{if the consumer does not buy the product.}
\end{cases}
\] (1)

\( \theta \) is the consumer’s valuation of the product and \( p_m \) is the price of one unit of the product. Thus, in the model, consumers differ from one another on the basis of their valuation of the product. \( \theta_m \) is the marginal consumer who is indifferent between buying and not buying:

\[
U(\theta_m) = \theta_m - p_m = 0 \Rightarrow \theta_m = p_m.
\] (2)

From equation (2) we get the demand function as,

\[^{36}\] These properties ensure that the second order condition for profit maximization holds.
So the firm’s expected profit is,

$$E\pi = k\alpha(R)(1 - p_m)p_m - R$$  \hspace{1cm} (4)$$

The firm’s realised profit conditional on successful innovation, is the monopoly profit, $r^* = \frac{1}{4}$. So the expected profit is, $E\pi = \frac{k\alpha(R)}{4} - R$. The firm chooses the R&D level that maximises its expected profit. The first order condition for maximization yields $\frac{\partial E\pi}{\partial R} = 0 \Rightarrow \alpha'(R) = \frac{4}{k}$ and $k \geq \frac{4R^*}{\alpha(R^*)}$ ensures non-negative profit. The results for the equilibrium R&D investment and the expected profit denoted as $R^*$ and $E\pi^*$ is summarised in Lemma 1 and the proof is given in the Appendix.

**Lemma 1.** $R^*$ satisfies $\alpha'(R^*) = \frac{4}{k}$ and $E\pi^* = \frac{k\alpha(R^*)}{4} - R^*$. $R^*$ and $E\pi^*$ are increasing in $k$.

This implies that a more efficient firm (with a higher $k$) or one with a higher probability of reaching technological success will invest more in R&D and will have higher expected profit. Let us now introduce piracy in the model. In stage 1 the legal firm chooses a level of R&D investment $R$ and in the second stage engages in price competition with the commercial pirate who sells illegal copies of the legal firm’s product.\(^{37}\) In Chatterjee (2009) it has been shown that the legal firm invests more on R&D under quantity competition than under price competition in Stackelberg games when the legal firm is the leader. In the present model, we look at price competition only to examine, in the context of patent race, how piracy

\[^{37}\text{In Chatterjee (2009) it has been shown that the legal firm invests more on R&D under quantity competition than under price competition in Stackelberg games when the legal firm is the leader. In the present model, we look at price competition only to examine, in the context of patent race, how piracy.}\]
With the availability of the pirated product, a consumer can either purchase the legal firm’s product or the pirated product or nothing. Let $q$ denote the quality of the pirated product which is common knowledge and we assume $q \in (0,1)$. The utility of a type-$\theta$ consumer is as follows.

$$U(\theta) = \begin{cases} 
\theta - p_m, & \text{if the consumer buys the original product}, \\
q\theta - p_c, & \text{if the consumer buys the pirated product}, \\
0 & \text{if the consumer does not buy anything}.
\end{cases} \quad (5)$$

where the subscripts $m$ and $c$ denote the legal firm the pirate, respectively. Here, $q\theta$ is the type-$\theta$ consumer’s effective valuation of the pirated product. $p_m$ and $p_c$ are the prices of the legitimate and the pirated product.

The marginal consumer indifferent between purchasing the legitimate product and the pirated product copy is denoted as $\theta_{cm}$ and it satisfies,

$$\theta_{cm} - p_m = q\theta_{cm} - p_c \Rightarrow \theta_{cm} = \frac{p_m - p_c}{1-q}.$$  

The marginal consumer indifferent between purchasing the pirated product and not buying anything, denoted as $\theta_{c0}$, satisfies,

$$q\theta_{c0} - p_c = 0 \Rightarrow \theta_{c0} = \frac{p_c}{q}. \quad (6)$$

Using the expressions for $\theta_{cm}$ and $\theta_{c0}$ we get the demand functions for the legitimate and the pirated products as follows.

$$D_m(p_m, p_c, q) = \int_{\theta_{cm}}^{1} d\theta = 1 - \theta_{cm} = 1 - \frac{p_m - p_c}{1-q} \quad (6)$$

$$D_c(p_m, p_c, q) = \int_{\theta_{c0}}^{\theta_{cm}} d\theta = \theta_{cm} - \theta_{c0} = \frac{q(p_m - p_c)}{q(1-q)}$$

affects R&D investment by the legal firm. If the firms 1 and 2 competed on quantity, the direction of R&D investment in equilibrium would be similar to the case of price competition but the magnitude would be higher. Hence, price competition captures the maximum reduction to a firm’s expenditure on innovation because of piracy and, therefore, can be considered to be the benchmark case of competition strategy to examine R&D decision by the legal firm when a pirate is present in the product market.

See Banerjee (2003) and Takeyama (1994) for more on the assumption that the legitimate and the pirated products are imperfect substitutes which is captured by the parameter $q$. 

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The expected profit of the legal firm and the pirate are,

\[
E\pi = k\alpha(R)r - R,
\]

\[
E\pi_c = k\alpha(R) \left( \frac{qp_m p_c - p_c^2}{q(1-q)} \right),
\]  

(7)

where \( r = p_m D_m = p_m - \frac{p_m^2 - p_m p_c}{1-q} \) is the realized stage 2 profit of the legal firm conditional on successful innovation in stage 1. The pirate can compete with the legal firm only if the latter is successful in innovation.

We denote this game as the Piracy-Game or the P-Game and solve for the game by backward induction. In stage 2 of the game, the legal firm and the pirate compete in price. The reaction functions are \( p_m = \frac{1-q + p_c}{2} \) and \( p_c = \frac{qp_m}{2} \). Solving the reaction functions yield the equilibrium prices and the legal firm’s realised second stage profit, \( r^* \), as given by,

\[
p_m^* = \frac{2(1-q)}{4-q}, \quad p_c^* = \frac{q(1-q)}{4-q}, \quad \text{and} \quad r^* = \frac{4(1-q)}{(4-q)^2}.
\]  

(8)

Substituting the expressions in (8) in the legal firm’s expected profit function in equation (7) and maximising the expected profit with respect to \( R \) yields the equilibrium R&D investment, denoted as \( R^p \), and the equilibrium expected profit of the legal firm, denoted as \( E\pi^p \). The results are summarized in Lemma 2 and the proof is in the Appendix. \( k \geq \frac{R^p}{\alpha(R^p)r^*} \) ensures non-negative profit.
Lemma 2. The equilibrium R&D investment and the expected profit of the legal firm satisfies, \( \alpha'(R^p) = \frac{1}{kr^{**}} \) and \( E\pi^p = k\alpha(R^p)r^{**} - R^p \). \( R^p \) and \( E\pi^p \) are increasing in \( k \) and decreasing in \( q \).

From Lemma 2 we observe that an improvement in the quality of the pirated product reduces a firm’s incentive to innovate. Let us now compare the R&D levels with and without piracy to understand the impact of piracy on the incentive to innovate by a single innovating firm which is facing technological uncertainty. This leads us to Proposition 1.

**Proposition 1.** Piracy reduces the incentive to innovate when there is a single innovating firm.

**Proof of Proposition 1.** \( \alpha'(R^*) - \alpha'(R^p) = \frac{-8q + q^2}{kr^{**}(4 - q)^2} = \frac{-q(8 + q)}{4kr^{**}(1 - q)} < 0 \). Since \( \alpha'(R) > 0 \) and \( \alpha''(R) < 0 \), it implies that \( R^* - R^p > 0 \) for all \( k > 0 \) because, by assumption, \( q \in (0,1) \). \( Q.E.D. \)

### 4.3 R&D Race and Piracy

Let us now introduce R&D competition between two legal firms, firm 1 and firm 2. In stage 1, firms 1 and 2 compete in R&D investment and the winner of the race receives the patent and is the legal monopolist in stage 2. So in stage 2 the winner of the R&D race faces the demand function as given in equation (3) and the stage 2 realized monopoly profit is, \( r^*_i = \frac{1}{4} \) where \( i = 1,2 \). There is no piracy in the product market.

Hereafter, we will refer to this as the *R&D Race-Game* or the *R-Game*. 
A firm can win the race if it is successful in innovation and the rival firm is
unsuccessful or if both firms are successful then each firm receives the patent with equal
probability. So the probability of firm \( i, i = 1, 2 \), receiving the patent is,

\[
\mu_i(R_i, R_j) = k_i \alpha(R_i)(1 - k_j \alpha(R_j)) + \frac{k_i \alpha(R_i)k_j \alpha(R_j)}{2}, i, j = 1, 2, i \neq j.
\]  

(9)

Hence, the expected profit of firm \( i, i = 1, 2 \), is

\[
E\pi_i = \left( k_i \alpha(R_i)(1 - k_j \alpha(R_j)) + \frac{k_i \alpha(R_i)k_j \alpha(R_j)}{2} \right) r_i^* - R_i, i, j = 1, 2, i \neq j.
\]  

(10)

The first order conditions yield,

\[
\frac{\partial E\pi_i}{\partial R_i} = k_i \alpha'(R_i) \left[ 1 - \frac{k_j \alpha(R_j)}{2} \right] r_i^* - 1 = 0 \Rightarrow k_i \alpha'(R_i)(2 - k_j \alpha(R_j)) = \frac{2}{r_i^*},
\]  

(11)

\[
\frac{\partial E\pi_j}{\partial R_j} = k_j \alpha'(R_j) \left[ 1 - \frac{k_i \alpha(R_i)}{2} \right] r_j^* - 1 = 0 \Rightarrow k_j \alpha'(R_j)(2 - k_i \alpha(R_i)) = \frac{2}{r_j^*}.
\]  

(12)

Let \((R_i^*, R_j^*)\) be the solution to equations (11) and (12).

Let us now consider R&D race in the presence of piracy. We will denote this as the
Race with Piracy Game or the RP-game. In stage 1 of the game, the two firms 1 and 2
compete in R&D investment. The probability that firm \( i \) wins the patent is the same as
in equation (9). The winner of the patent competes in price with the commercial pirate
in stage 2. The utility function and the demand functions facing the winner of the patent
and the pirate are the same as given in equations (5) and (6). So the realised stage 2
profit of the winner of the R&D race is \( r_i^{**} = \frac{4(1 - q)}{(4 - q)^2} \) as given in equation (8). The
expected profit of firm \( i \) is,
\[ E \pi_i = \left( k, \alpha(R_i)(1 - k, \alpha(R_j)) + \frac{k, \alpha(R_i)k, \alpha(R_j)}{2} \right) r_i^{**} - R_i, \quad i, j = 1, 2, \; i \neq j. \] (13)

The first order conditions yield,

\[ \frac{\partial E \pi_i}{\partial R_i} = k, \alpha'(R_i) \left( 1 - \frac{k, \alpha(R_j)}{2} \right) r_i^{**} - 1 = 0 \Rightarrow k, \alpha'(R_i)(2 - k, \alpha(R_j)) = 2 \frac{r_i^{**}}{r_i}, \] (14)

\[ \frac{\partial E \pi_j}{\partial R_j} = k, \alpha'(R_j) \left( 1 - \frac{k, \alpha(R_i)}{2} \right) r_j^{**} - 1 = 0 \Rightarrow k, \alpha'(R_j)(2 - k, \alpha(R_i)) = 2 \frac{r_j^{**}}{r_j}. \] (15)

Let \((R_i^{RP}, R_j^{RP})\) be the solution to equations (14) and (15).

The reaction functions of firm \(i\) in the \(R\)-Game and \(RP\)-Game, as given by equations (11) and (14), can be rewritten, respectively, as

\[ r_i^{*} k, \alpha'(R_i)(2 - k, \alpha(R_j)) = 2 \] (11’)

\[ r_j^{*} k, \alpha'(R_j)(2 - k, \alpha(R_i)) = 2 \] (14’)

Now \(r_i^{**} = r_j^{**} = \frac{4(1 - q)}{(4 - q)^2} < r_i^{*} = r_j^{*} = \frac{1}{4}\). So for a given \(R_j\), \(R_i\) in equation (11’) must be higher than that in equation (14’). This means that piracy downwardly shifts firm \(i\)’s reaction function. Similar analysis holds for firm \(j\)’s reaction functions. This leads us to Proposition 2 which summarizes the comparison between \(R_i^{RP}\) and \(R_i^{R}\), and \(R_j^{RP}\) and \(R_j^{R}\).

**Proposition 2.** (i) If \(R_j^{RP} \geq R_j^{R}\) then \(R_i^{RP} < R_i^{R}\). (ii) If \(R_i^{RP} \geq R_i^{R}\) then \(R_j^{RP} < R_j^{R}\).

(iii) Both \(R_i^{RP} < R_i^{R}\) and \(R_j^{RP} < R_j^{R}\) holds.
Figure 4.1: Shifts in Reaction Curves from the R-game to the RP-game

Proposition 2 can be explained with the help of Figure 4.1. The equilibrium from the RP-game is compared against the equilibrium from the R-game to examine the change in R&D investment by the legal firms caused by the presence of a pirate in the product market. The proof of Proposition 2 follows from the fact that piracy shifts the reaction functions of both the innovating firms downward. Therefore, the position of the intersection point will depend on the relative shifts of the reaction functions. In Figure 4.1, $R^R$ is the equilibrium point of the R-game and the reaction functions are denoted as $BR^R_i$ and $BR^R_j$. The reaction functions in the RP-game lie below $BR^R_i$ and $BR^R_j$. Hence, the intersection of the reaction functions in the RP-game must be below the curve drawn in bold which is the envelope of the top part of firm i’s reaction function in
the $R$-game (labelled as $BR^R_i$) and the bottom part of firm $j$’s reaction function in the $R$-game (labelled as $BR^R_j$).

Let us fix the reaction function of firm $i$ in the $RP$-game at $BR^R_i$ represented by the dotted curve. Now we consider a relatively “small” shift in the reaction function of firm $j$ compared to the shift in the reaction function of firm $i$ when we move from the $R$-game to the $RP$-game. Then the equilibrium is at a point like A where $R^R_j > R^R_i$ and $R^R_i < R^R_j$. This explains part (i) of Proposition 2.

Next we consider the case when, due to piracy, there is a relatively “large” shift in the reaction function of firm $j$, denoted as $BR^R_j$, to intersect the reaction function of firm $i$, given by $BR^R_i$. This is captured by point C where $R^R_j < R^R_i$ and $R^R_i > R^R_j$. This proves part (ii) of Proposition 2. If the shifts are of similar strength then the equilibrium in the $RP$-game will lie in the region labelled B. This explains part (iii) of Proposition 2 where the equilibrium R&D investment in the $RP$-game is less than that in the $R$-game.

Let us consider the symmetric case where $k_i = k_j = k$. In this case the equilibrium R&D investments will be the same for both firms in each of the two games, that is, $R^R_i = R^R_j$ and $R^R_i = R^R_j$. The summary of the comparisons between $R^R_i$ and $R^R_j$, and $R^R_j$ and $R^R_j$ is given in Corollary 1.

**Corollary 1.** If $k_i = k_j = k$ then $R^R_i < R^R_j$ and $R^R_j < R^R_j$. 

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The proof follows from the fact that in the symmetric case the equilibrium R&D investments lie on the 45° line as shown in Figure 4.2. Since piracy shifts down the reaction functions of the two firms equally, it implies that the equilibrium in the \textit{RP}-game lies along the 45° line in the region B where $R_{i}^{RP} < R_{i}^{R}$ and $R_{j}^{RP} < R_{j}^{R}$.

![Figure 4.2: The Symmetric Equilibrium](image)

Proposition 2 and Corollary 1 implies that even if the symmetric condition does not hold still we can get the situation $R_{i}^{RP} < R_{i}^{R}$ and $R_{j}^{RP} < R_{j}^{R}$ if the intersection of the reaction functions in the \textit{RP}-game is in the area labelled B in Figure 4.2. The position of the equilibrium point in the \textit{RP}-game relative to that in the \textit{R}-game depends on the relative strengths of the efficiency parameters $k_{i}$ and $k_{j}$. Let us try to provide an intuition for the results summarized in Proposition 2 and Corollary 1.

A firm’s R&D decision is associated with two costs. One is the loss of the R&D investment if the firm is unsuccessful in developing the new product (technological uncertainty) or if the firm loses the patent race (market uncertainty). In this case the firm
forgoes the profit from the output market. The other is the reduction in profit due to piracy. Hence, provided that the gain exceeds the cost of innovation, the objective of a firm will be to increase the probability of success of the R&D. Market uncertainty being same for both firms and the presence of piracy affecting the profits of the innovating firms similarly, the R&D strategy of each firm would be to increase the probability of technological success in the least cost way.

An innovating firm with higher efficiency can reach the same probability of technological success with a lower level of R&D effort compared to an innovating firm with lower efficiency. Thus, when the firms know that piracy will reduce their profit in stage 2, the more efficient firm will try to lower its cost by reducing R&D expenditure provided that its probability of technological success is sufficiently high. The less efficient firm, in order to increase the probability of its own technological success, responds by increasing its R&D expenditure level (such that its expected profit is positive). This explains parts (i) and (ii) of Proposition 2. However, if the efficiency parameters of the two firms are sufficiently close then it is not profitable for the relatively less efficient firm to increase R&D investment because the cost of doing so exceeds the gain in expected profit.

Thus piracy unambiguously reduces individual and overall R&D investment if the efficiency parameters are sufficiently close. The results for the case where the difference between the efficiency parameters is “large” is summarised in Proposition 3.

**Proposition 3.** If the difference in technological efficiency between firms engaging in patent race is sufficiently large, piracy may increase the total R&D investment by the two firms.
If $R_j^{RP} < R_j^R$ and $R_i^{RP} > R_i^R$, as illustrated by the point C in Figure 4.1, then piracy increases the equilibrium R&D investment of firm $i$. Again, if $R_j^{RP} > R_j^R$ and $R_i^{RP} < R_i^R$, which is represented by the point A in Figure 4.1, then piracy causes firm $j$ to increase its R&D investment in equilibrium. There can be an overall increase in the R&D investment by the two firms if $R_i^{RP} - R_i^R > R_j^R - R_j^{RP}$.

Proposition 3 suggests that piracy may enhance the overall R&D investment under certain conditions where the increase in R&D effort by one firm exceeds the decrease in R&D investment by the other innovating firm, given competition from the pirate in the product market. In the next section, we use a specific functional form of $\alpha(R_i)$ to get further insights into the results summarised in Propositions 2 and 3.

4.4 An Example

In this section we provide an example for the above general analysis. We assume that $k_i \alpha(R_i) = k_i \sqrt{R_i}$. Using this form we get the equilibrium R&D investments and expected profits of firm $i$ for the $R$ and $RP$ games. The results are summarised in Lemmas 3 and 4.

**Lemma 3.** In the $R$-game, $R_i^R = E\pi_i^R = \frac{4k_i^2(16-k_j^2)^2}{(256-(k_i^2k_j^2)^2)}$, $i, j = 1, 2, i \neq j$. $R_i^R$ and $E\pi_i^R$ are increasing in $k_i$ and decreasing in $k_j$.

We get the equilibrium R&D investments for the $R$-game by solving equations (11) and (12) using $k_i \alpha(R_i) = k_i \sqrt{R_i}$. The second order condition of maximization
requires, 
\[ \frac{d^2 E \pi_i}{d R_i^2} = - \frac{k_i (2 - k_j \sqrt{R_j}) r_i^*}{8 R_j^{3/2}} < 0. \] 
This implies that \( 2 - k_j \sqrt{R_j} > 0 \) implying that \( \sqrt{R_j} < \frac{2}{k_j} \). Similarly, \( 2 - k_i \sqrt{R_i} > 0 \) implies that \( \sqrt{R_i} < \frac{2}{k_i} \). Substituting

\[ \sqrt{R_i} = \frac{2k_i (16 - k_j^2)}{256 - (k_i k_j)^2} \] 
in the condition \( \sqrt{R_i} < \frac{2}{k_i} \) yields \( k_i < 4 \).

**Lemma 4.** The equilibrium R&D investments and expected profits of firm \( i \) in the RP-game are, 
\[ R_i^{RP} = E \pi_i^{RP} = \left( \frac{2k_i r_i^{**} (4 - k_j^2 r_j^{**})}{16 - k_i^2 k_j^2 r_i^{**}} \right)^2, \] 
\( i, j = 1, 2, i \neq j \).

For the RP-game we examine the equilibrium by solving equations (14) and (15) using \( k_i \alpha(R_i) = k_i \sqrt{R_i} \). The second order condition of maximization requires

\[ \frac{d^2 E \pi_i}{d R_i^2} = - \frac{k_i (2 - k_j \sqrt{R_j}) r_i^{**}}{8 R_j^{3/2}} < 0. \] 
This implies that \( 2 - k_j \sqrt{R_j} > 0 \Rightarrow \sqrt{R_j} < \frac{2}{k_j} \). Similarly, \( 2 - k_i \sqrt{R_i} > 0 \Rightarrow \sqrt{R_i} < \frac{2}{k_i} \).

Substituting \( \sqrt{R_i} = \frac{2k_i (16 - k_j^2)}{256 - (k_i k_j)^2} \), \( i, j = 1, 2, i \neq j \) in the condition \( \sqrt{R_i} < \frac{2}{k_i} \)
yields \( k_i^2 < \frac{4}{r_i^{**}} < \frac{4}{r_i^*} = 16 \) because \( r_i^{**} = \frac{4(1-q)}{(4-q)^2} < r_i^* = \frac{1}{4} \).

Thus the second order condition of the R-game, given by \( k_i < 4, i, j = 1, 2, i \neq j \), also satisfies the second order conditions in the RP-game and hence this assumption of \( k_i < 4 \) will be retained for the rest of the analysis.
Comparison of $\sqrt{R_i^R} = \frac{2k_i(16-k_j^2)}{256-(k_i k_j)^2}$ and $\sqrt{R_i^{RP}} = \frac{2k_i r_i^{**}(4-k_j^2 r_j^{**})}{(16-k_i^2 k_j^2 r_i^{**2})}$ will allow us to determine the effect of piracy on innovation when there is R&D race. The result is summarized in Proposition 4.

**Proposition 4.** Piracy enhances R&D investment of firm $i$ in the presence of R&D race if

$$r_i^{**}(256-k_i^2 k_j^2) - 4k_j^2 (16-k_i^2)r_i^{**2} - 4(16-k_j^2) > 0.$$ 

**Proof of Proposition 4:**

$$\sqrt{R_i^{RP}} - \sqrt{R_i^R} = \frac{2k_i r_i^{**}(4-k_j^2 r_j^{**})}{(16-k_i^2 k_j^2 r_i^{**2})} = \frac{2k_i(16-k_j^2)}{256-k_j^2 k_j^2} = A_i, \ i, j = 1, 2; i \neq j$$

$$A_i = 2k_i \left( \frac{r_i^{**}(4-k_j^2 r_j^{**})(256-k_j^2 k_j^2) - (16-k_j^2)(16-k_j^2 k_j^2 r_i^{**2})}{(16-k_i^2 k_j^2 r_i^{**2})(256-k_j^2 k_j^2)} \right).$$

The denominator in $A_i$ is always positive because $k_i, k_j < 4$ and $r_j^{**} = \frac{4(1-q)}{(4-q)^2} < \frac{1}{4}$. So the sign of $A_i$ depends on the sign of the numerator. Simplifying the numerator we get

$$B_i = r_i^{**}(256-k_i^2 k_j^2) - 4k_j^2 (16-k_i^2)r_i^{**2} - 4(16-k_j^2)$$

which can be either positive zero or negative. If $B_i > 0$, it implies $\sqrt{R_i^{RP}} - \sqrt{R_i^R} > 0 \Rightarrow R_i^{RP} > R_i^R$. *Q.E.D.*

Let us discuss the implications of Proposition 4 by analysing the impact of the different strengths of $k_i$ and $k_j$ on the incentive to innovate in the presence of piracy when there is R&D race. We will be focusing on the expression $B_i$ for the analysis.
Consider the symmetric case \( k_i = k_j \). So, \( B_i = (16 - k_i^2)(4 - k_i^2 r_i^{**})(4 r_i^{**} - 1) \). Now

\[
(16 - k_i^2) > 0, \quad (4 - k_i^2 r_i^{**}) > 0 \quad \text{and} \quad (4 r_i^{**} - 1) < 0,
\]

since by assumption \( k_i < 4 \) and \( r_i^{**} = \frac{4(1-q)}{(4-q)^2} < \frac{1}{4} \). Therefore, \( B_i = (16 - k_i^2)(4 - k_i^2 r_i^{**})(4 r_i^{**} - 1) < 0 \).

Next let us look at the combinations of \( k_i \) and \( k_j \) that keeps the value of \( B_i \) unchanged.

We will call them the \( B_i \) curves. Total differentiation of \( B_i \equiv r_i^{**} (256 - k_i^2 (16 - k_i^2)(4 r_i^{**})^2 - 4(16 - k_i^2) \) with respect to \( k_i \) and \( k_j \) yields,

\[
2k_i k_j^2 r_i^{**} (4 r_i^{**} - 1) dk_i + 2k_j (1 - 4 r_i^{**})(4 + (16 - k_i^2) r_i^{**}) dk_j = 0.
\]

The first expression on the L.H.S is negative because \( (4 r_i^{**} - 1) < 0 \). The second expression on the L.H.S is positive because \( (1 - 4 r_i^{**}) > 0 \) and \( k_i < 4 \). Therefore, \( \frac{dk_i}{dk_j} > 0 \).

Now

\[
\frac{dB_i}{dk_i} = 2k_i k_j^2 r_i^{**} (4 r_i^{**} - 1) < 0 \quad \text{and} \quad \frac{dB_i}{dk_j} = 2k_i k_j^2 r_i^{**} (4 r_i^{**} + 1) + 8k_j (1 - 16 r_i^{**}) > 0
\]

because \( 4 r_i^{**} < 1 \). This means an increase in \( k_i \) for a given \( k_j \) reduces \( B_i \).

The \( B_i \) curves are represented diagrammatically in Figure 4.3 and the intuition is provided subsequently. Along the \( k_i=k_j \) line \( B_i<0 \). So the \( B_i=0 \) curve which is

\[
k_j = 8 \left( \frac{1 - 4 r_i^{**}}{4 - k_i^2 r_i^{**} - 64 r_i^{**}^2 + 4 k_i^2 r_i^{**}} \right) \text{ lies above the } k_i=k_j \text{ line. Since } \frac{dB_i}{dk_j} > 0, \text{ given } k_i,
\]

as \( k_j \) increases \( B_i \) increases as well. The \( B_i \) curves are upward sloping because

\[
\frac{dk_i}{dk_j} > 0. \ \text{Since } \frac{dB_i}{dk_i} < 0 \quad \text{and} \quad \frac{dB_i}{dk_j} > 0 \text{ it means that higher values of } B_i \text{ are associated with higher } B_i \text{ curves.}
\]

\[39\text{ The concavity of the } B_i \text{ curves as shown in Figure 3.3 is only for illustrative purpose and is not necessary for the analysis.}\]
To understand the intuition behind the result that $\frac{dB_i}{dk_i} < 0$ let us consider the rate of change of $\sqrt{R_i^R}$ and $\sqrt{R_i^R}$ with respect to $k_i$ and $k_j$.

\[
\frac{\partial \sqrt{R_i^R}}{\partial k_i} = \frac{r_i^{**}(4 - k_j^2 r_i^{**})(2r_i^{**}(16 - k_i^2 k_j^2 r_i^{**}) + 4k_j^2 k_i^2 r_i^{**})}{(16 - k_i^2 k_j^2 r_i^{**})^2} > 0
\]

and

\[
\frac{\partial \sqrt{R_i^R}}{\partial k_j} = \frac{16k_j^2 r_i^{**}(4 - k_j^2 r_i^{**})}{(16 - k_i^2 k_j^2 r_i^{**})^2} < 0 \quad \text{since} \quad k_i^2, k_j^2 < 16, \quad r_i^{**} < \frac{1}{4}, \quad \text{thereby}
\]

$k_j^2 r_i^{**} < 4$ and $16 - k_i^2 k_j^2 r_i^{**} > 0$. By similar reasoning,

\[
\frac{\partial \sqrt{R_i^R}}{\partial k_i} = \frac{2(256 + k_i^2 k_j^2)(16 - k_j^2)}{(256 - k_i^2 k_j^2)^2} > 0 \quad \text{and} \quad \frac{\partial \sqrt{R_i^R}}{\partial k_j} = -\frac{64k_i(16 - k_j^2)}{(256 - k_i^2 k_j^2)^2} < 0.
\]

The upward sloping property of the $B_i$-curves is because the rate at which $R_i^R$ increases due to an increase in $k_i$ is less than that of $R_i^R$. Consequently, an increase in $k_i$ reduces the gap between $R_i^R$ and $R_i^R$ thereby lowering the value of $B_i$. The opposite is

\[\text{Figure 4.3: } B_i \text{ curves}\]
the case for \( k_j \). An increase in \( k_j \) reduces both \( R^{RP}_i \) and \( R^{R}_i \) but the rate of reduction of the former is less than that of the latter resulting in an increase in \( B_i \). Thus a decrease in \( k_i \) and an increase in \( k_j \) results in higher values of \( B_i \). This means that if the difference between \( k_i \) and \( k_j \) is “sufficiently high then \( \sqrt{R^{RP}_i} - \sqrt{R^{R}_i} \) is positive.

Table 4.1 provides a numerical example of the comparative static analysis of \( R_i \) with respect to \( k_i \) for different given values of \( q \) (which is the quality of the pirated product) that gives us different values of \( r_i^{**} \). We have fixed \( k_j \) at 3.9 for the entire analysis which satisfies the second order condition \( k_i < 4, i, j = 1, 2, i \neq j \). This allows us to vary \( k_i \) over a large range to capture the effects.
Table 4.1: Comparative Static Analysis of $R_i$ with respect to $k_i$ and $k_j$

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<th>$q$</th>
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<th>$k_i$</th>
<th>$k_j$</th>
<th>$B$</th>
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For any given $q$, we observe that an increase in $k_i$ reduces $B_i$ and increases both $\sqrt{R_{iRP}}$ and $\sqrt{R_{iR}}$ but the increase in $\sqrt{R_{iRP}}$ is less than the increase in $\sqrt{R_{iR}}$. Also, as $q$ decreases from 0.9 to 0.8 to 0.7 we see that the negativity of $B_i$ begins at progressively higher values of $k_i$. That is, for lower qualities of the pirated product the asymmetricity between $k_i$ and $k_j$ required for the enhancement of R&D investment in the presence of piracy decreases.
This example shows that if firm \( i \)’s efficiency parameter is relatively small compared to that of firm \( j \) then \( R_i^{RP} > R_i^R \). However, as the asymmetricity decreases the gap between \( R_i^{RP} \) and \( R_i^R \) decreases and, beyond a critical level, \( R_i^R \) dominates \( R_i^{RP} \).

### 4.5 Conclusions

The literature on piracy and innovation shows that piracy can have both beneficial and adverse impacts on innovative efforts by firms. This paper attempts to link the literature on piracy and innovation to that on innovation and patent race by analysing the impact of piracy on R&D decision of a firm under the assumptions of technological and market uncertainties. In an environment where the success of R&D investment is stochastic, implying presence of technological uncertainty, we show that piracy unambiguously retards innovation under technological uncertainty when there is a single innovating firm. However, with the introduction of R&D competition, which generates market uncertainty on top of technological uncertainty, we show that piracy may enhance innovation.

The last result depends on the relative position of the innovating firms’ reaction functions in the case of R&D race without piracy vis-à-vis that in the R&D race with piracy. We show that it is possible for the technologically less efficient firm’s R&D to increase in the presence of piracy. Using a specific functional form we showed that piracy enhances the R&D investment and profit of a less efficient firm if the difference between the efficiency parameters of the two innovating firms is sufficiently large. If the efficiency parameters of the two firms are sufficiently different, then piracy may also increase the overall R&D investment by the two firms.
References


Appendix

Proof of Lemma 1.

\[ \alpha(R) \frac{dR^*}{dk} = -\frac{4}{k^2} \]. Since \( \alpha(R) < 0 \) hence, \( \frac{dR^*}{dk} > 0 \) and \( \frac{dE\pi^*}{dk} = \frac{\alpha(R^*)}{4} > 0. \)

\[ E\pi^* = \frac{k\alpha(R^*)}{4} - R^* \geq 0 \Rightarrow k \geq \frac{4R^*}{\alpha(R^*)}. \] Q.E.D.

Proof of Lemma 2.

\[ \frac{\partial E\pi}{\partial R} = k\alpha(R) \Rightarrow -1 = 0 \Rightarrow \alpha'(R^p) = \frac{1}{kr^{**}}. \]

\[ \alpha'(R^p) \frac{dR^p}{dk} = -\frac{1}{k^2 r^{**}} \Rightarrow \frac{dR^p}{dk} > 0 \]

because by assumption \( \alpha'(R^p) < 0. \) \( \frac{\partial E\pi^p}{\partial k} = \alpha(R^p) r^{**} > 0. \)

\[ \alpha'(R^p) \frac{dR^p}{dq} = -\frac{1}{kr^{**2}} \Rightarrow \frac{dr^{**}}{dq} > 0. \] because \( \frac{dr^{**}}{dq} = -\frac{4(2 + q)}{(4 - q)^3} < 0. \) Since \( \alpha'(R^p) < 0 \) so

\[ \frac{dR^p}{dq} < 0. \] \( \frac{\partial E\pi^p}{\partial q} = k\alpha(R^p) \frac{dr^{**}}{dq} < 0. \] Q.E.D.

Proof of Lemma 3: Maximizing equation (6) with respect to \( R_i \) yields \( F_i \)'s reaction function which is \( \sqrt{R_i} = \frac{k_i(2 - k_j \sqrt{R_j})}{16} \), \( i, j = 1, 2, i \neq j \). Solving the two gives us \( R_i^R \) which when substituted in equation (6) gives \( E\pi_i^R \). Differentiating \( R_i^R = E\pi_i^R \) with respect to \( k_i \) and \( k_j \) yields \( \frac{dR_i^R}{dk_i} = \frac{dE\pi_i^R}{dk_i} > 0, \frac{dR_j^R}{dk_j} = \frac{dE\pi_i^R}{dk_j} < 0. \) Q.E.D.
Chapter 5 Conclusions
5.1 Concluding Remarks

This thesis builds on the observation that the degree of substitutability or complementarity between two products could be asymmetric. That is to say, a change in the price of one product would affect the demand for the second product differently than would a change in the price of the second product affect the demand for the first. We examine several issues relating to asymmetric product differentiation when two firms compete sequentially in the product market.

In the first essay, we examine how this asymmetry in product differentiation and mode of competition in the product market affect the relative profit positions of the two firms. We find that this profit ordering is not monotonic and each firm can earn a higher profit than the other depending on the relative differentiation between their products. In the second essay, we analyse the case where a legal firm and a commercial pirate compete on asymmetrically differentiated products. We find that the legal firm’s R&D investment decision and the authority’s decision to monitor piracy crucially depend on the degree of product substitutability and the mode of competition between the legal firm and the pirate. The last essay looks at the related question of piracy affecting the decision to innovate by legal firms when the pirated product is an asymmetric substitute of the legal product. We show that, if the difference between the efficiency parameters of the innovating firms is relatively large, piracy may increase the R&D investment of the less efficient firm and even the overall R&D when two legal firms engage in patent races for their innovation.

Thus, this thesis uses the concept of asymmetric product differentiation to address three important issues on commercial piracy. First, pirated products are usually imperfect
substitutes of the legal product implying that the impact of a price change of the pirated product on the demand for the legal product need not be the same as the impact of a price change of the legal product on the pirate’s demand. The existing literature on product differentiation, however, does not consider such kind of asymmetric product differentiation. Second, empirical evidence show that firms often need to incur the costs to document and report signs of piracy, and then the authority decides whether or not, and how much, to invest in monitoring that, in turn, affects the probability that the pirate is being caught and convicted. The implications of costly reporting by the legal firm and ex-post monitoring by the authority have not been investigated. Third, while the importance of piracy in affecting innovation has been widely recognised, there has not been much work that directly models the impact of piracy on firms that engage in competition for innovation. This thesis presents three theoretical essays that attempt to fill these gaps in the literature.

5.2 Directions for Future Research

Several aspects of asymmetric product differentiation that this thesis explores can be extended to apply to various issues on firm competition, product location, R&D investment decisions, policy formulations and corruption.

In the first two core essays, for simplicity of analysis, we have used linear demand functions with asymmetric cross-price parameters. One limitation of using these demand systems is that we have not provided a utility foundation to how such demand structures were generated. An exciting direction for future research would be to find out the underlying utility structure that generates such linear market demand systems with asymmetric cross-price impacts. The literature (see Singh and Vives, 1984; Boyer and
Moreaux, 1987) that uses linear market demands with symmetric cross-price impacts assumes a utility structure of the form

\[ U(q_1, q_2) = \alpha_i q_i + \alpha_j q_j - (\beta_i q_i^2 + 2 \gamma q_i q_j + \beta_j q_j^2) / 2 \]  

(1)

where \( \alpha_i, \beta_i > 0 \) for \( i = 1, 2 \) such that \( \beta_i \beta_j - \gamma^2 > 0 \) and \( \alpha_i \beta_j - \alpha_j \gamma > 0 \) for \( i \neq j, i = 1, 2 \). The representative consumer maximises this utility subject to his budget.

His income-compensated demands for goods 1 and 2 are then given by

\[ q_1 = a_1 - b_1 p_1 + cp_2 \quad \text{and} \quad q_2 = a_2 - b_2 p_2 + cp_1 \]

where \( \delta = \beta_i \beta_j - \gamma^2, \quad c = \frac{\gamma}{\delta}, \quad a_i = \frac{\alpha_i \beta_j - \alpha_j \gamma}{\delta} > 0, \quad \text{and} \quad b_i = \frac{\beta_j}{\delta} \) for \( i \neq j, i = 1, 2 \). In chapters 2 and 3, we assume similar demand structures but, instead of the same cross-price parameter \( c \), we have asymmetric cross price impacts \( c_1 \) and \( c_2 \). The underlying utility structure for such demand systems has not been explored in the literature; however, we do have empirical evidence of demand structures where a price change in one product affects the quantity demanded of the other product differently from how a price change in the latter would affect the quantity demanded of the former. Thus it would be interesting to examine, both theoretically and empirically, the structure of consumer preference that would generate such demand systems.

Another limitation of the present study is that it takes the mode of competition between the competing firms to be exogenously given. In chapters 2 and 3, we find that the mode of competition is important in determining the equilibrium when two firms compete on asymmetric products. Though Singh and Vives (1984), in the context of simultaneous games, show that it is optimal for both firms to choose the same competition strategy – price or quantity – we show (Chapter 2) that in sequential games it is only the leader’s
strategy choice that determines the equilibrium of the game. Again, Tasnádi (2006) shows that in a mixed oligopoly game if the competition strategies could be endogenised then several sub-game perfect Nash equilibria could arise. Therefore, it would be interesting to see, in the context of costly reporting and ex-post monitoring of commercial piracy, if endogenising the mode of competition affects the equilibrium of the game. In other words, given that it is costly for the pirate to enter and for the legal firm to develop new products and to report piracy, different equilibria might appear in the monitoring and innovation games if the firms were allowed to endogenously choose between price and quantity strategies.

A further application of this work would be to endogenise the product differentiation parameter to strategically determine the leader-follower roles of the firms in the market. Because the product differentiation parameter determines which firm would be earning a higher profit, endogenising it could go to determine which firm will enter the market first and also the relative product quality between the leader and the follower. In the current thesis the product differentiation parameters between the two firms have been exogenously given. Hence, the next step would be to examine if firms can strategically choose their relative product locations on the market demand so as to maximise their profit. Thus the concept of asymmetric product differentiation can be used in the context of brand differentiation or label positioning literature (see Choi and Coughlan, 2006; Noh and Moschini, 2006) which investigate how firms producing close substitutes use brand names and labels to demarcate their products from their competitors’.

In the context of commercial piracy, this concept of strategically endogenising the product differentiation parameter would have implications regarding how close a copy of the legal product should the pirate produce, given the mode of competition in the
product market. Alternatively, given the presence of a pirate in the product market, the legal firm could also endogenise the product quality with which it enters the market. This would also influence the R&D decision of the legal firm. The present study abstracts from endogenising the location of the pirated copy relative to the legal product and treats the degree product substitutability as exogenously given.

In our present study we have assumed that the consumers are perfectly aware of the qualities of the legal and pirated products. However, consumers are typically uncertain about the quality of the pirated products before buying and can only examine the contents after their purchase. Moreover, pirated products do not carry any guarantee of quality and hence there is always a certain element of risk in purchasing a pirated product. We could, as a further augmentation, introduce uncertainty on the part of the consumers regarding the product quality (see Metrick and Zeckhauser 1996) and examine how the equilibrium is affected if the consumer is unable to distinguish between the legal product and the pirated copy.

This study could also be extended to empirically investigate how the authority’s monitoring decision and the legal firm’s R&D decision are influenced by the degree of product substitutability and the relative political influence of the legal firm and the pirate. That is, this study could be a step towards policy studies by bringing together the literature on innovation, product differentiation, commercial piracy and corruption.
References


