FOUR ESSAYS ON INTERNATIONAL REAL BUSINESS CYCLE AND ASSET PRICING MODELS

by

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A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

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September 2002

TABLE OF CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

DISCLAIMER

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university and, to the best of my knowledge and belief, the thesis contains no material published or written by another person, except where due reference is made in the text of the thesis.

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Jai Hyung Yoon

ACKNOWLEDGEMENTS

In the writing of my thesis, I have benefited greatly from assistances and supports. I am pleased to acknowledge the assistances and supports as follows:

My supervisor, Dr. Francis In contributed this thesis with his many helpful suggestions and comments on theoretical and technical problems in whole my thesis. I also would like to thank him for supporting and encouraging me whenever I was exhausted while I was writing this thesis.

I wish thank all staffs, the Department of Accounting and Finance, Faculty of Business and Economics, Monash University for financial support, especially Dr. Paul Mather for his excellent administrative support as PhD program coordinator. Sangbae Kim and Doo Yeul Na helped with data collection and computer work.

Some of early work of this thesis was presented in various conferences and seminars as well as journals. I appreciate the helpful comments from business cycle conference participants at University of Rochester and University of Quebec, seminar participants at Melbourne University, Monash University and Deakin University, and the anonymous referees of Journal of Monetary Economics, International Economic Review and Journal of Economic Dynamics and Control.

Finally, this work is dedicated to my parents who always pray for me.

Chapter 1 Introduction

Real business cycle models, which originated in the frontier work of Finn Kydland and Edward Prescott (1982), and John Long and Charles Plosser (1983), have successfully explained some of the key empirical regularities that characterize economic fluctuations. The business cycle theory has been developed in several directions. A major development of business cycle theory is the extension to international real business cycle model initiated by Backus, Kehoe and Kydland (1992). Their international real business cycle model based on large country assumption fairly well explains the international transmission effects of productivity shocks. The international real business cycle theory was further developed to the real business cycle model of small open economy by Mendoza (1991a,1991b). The international real business cycle models developed by Mendoza (1991a,1991b and 1995) focused on two main aspects such as the ability of small open economy to smooth consumption and investment, and the transmission effects from foreign sector.

In another direction, the business cycle research is the study about the driving force of business cycle. Specially, in the real business cycle theory, the productivity shocks that are exogenous shocks to production function have been supported as explaining the main economic fluctuations. In contrast, Greenwood, Hercowitz, and Huffman (1988) and Greenwood, Hercowitz, and Krusell (1997) argue that it is shocks to the marginal efficiency of investment that are important in generating output fluctuations, rather than shocks to the production function. They showed that a positive shock to marginal efficiency of investment stimulates the formation of "new" capital and more intensive utilization and accelerated depreciation of "old" capital. Their model helps to explain the transmission mechanism of shocks. Therefore, their key conclusion of the qualitative and quantitative analysis suggests that shocks to the marginal efficiency of investment can be important elements for business cycles.

In my thesis, I focused on two main aspects. First aspect is an international real business cycle model within a small open economy framework. Second aspect is of driving forces of real business cycle model in relation to investment-specific technology shock.

My thesis is composed of six chapters as follows. Chapter 2 presents a small open economy real business cycle model with investment-specific technology. In chapter 2, we extended the real business cycle model of Greenwood, Hercowitz and Huffman (1988) to a small open economy model. Our model is parameterized, calibrated, incorporating the neoclassical framework with endogenous capacity utilization and foreign financial asset. The endogenous capital utilization variable has a central role to improve our model's ability to transmit the effect of the shock to labour supply, investment and consumption. The foreign financial assets explore the interaction with domestic capital as alternative vehicles to smooth consumption and savings. Our model also is solved and simulated by value function and transition probability iteration method.

In chapter 3, we developed a multisectoral real business cycle model of manufacturing and non-manufacturing industries with the terms of trade shocks. The basic intuition is that a small open economy is greatly affected by foreign sectors through the variables such as imports, the trade balance, the terms of trade and real exchange rates. The main purpose of an international business cycle and multisectoral business cycle model is to explain simultaneously sectoral comovement with respect to the impact of foreign sector. Two main motivations for international business cycle model with two sectors are the report of Whalley (1995) that the share of trade in intermediate goods is larger than the share of trade in final goods and the study of Long and Plosser (1983) that sectoral shocks in a dynamic closed economy model can lead to positive spillover effects in other sectors via changes in demand for intermediate inputs produced by those sectors. We also focused on the indirect effect on the domestic variables of the terms of trade shock through the change of sectoral production and demand. Differently from chapter 2, we solved and simulated the model by using an undetermined coefficients method.

In chapter 4, we build an international real business cycle model with import capital goods and the terms of trade shocks. We emphasized the international transmission of business cycles. With free trade and complete asset markets, the representative consumer can smooth consumption and savings by varying labour effort and the allocation of capital and foreign financial assets. In this chapter, we are concerned of the relationship between investment-specific technology and capital goods import. With free trade, intuitively, investment-specific technology can be achieved by the technology embodied imported capital goods in a small open economy. A recent study by Lee (1995) concludes that foreign capital goods, which are used relatively more than domestic capital goods for the production of capital stock, induce the higher growth rate of income. An important aspect of the above analysis is that changes in business cycles in a small open economy are caused by foreign capital goods that are used for investment and thus production. Accordingly, we focused on the terms of trade shock of capital goods reflecting technological change of import capital goods or the investment-specific technology shocks of foreign sector. Our fundamental question is whether investment-specific technology shock can be substituted with the terms of trade shock of import capital goods.

In chapter 5, we extended our business cycle model to asset pricing model. This chapter aims to integrate existing finance theory on stock and bond (financial asset) prices with macroeconomic model of real business cycles. This chapter unites the study of financial assets and macroeconomics by addressing the issues of asset pricing puzzles. The aim of this chapter is whether a business cycle model with investment-specific technology shock have some success in understanding of the forces determining asset prices. The existence of intertemporal substitution between consumption and investment due to investment-specific technology shock can explain well real financial market. This result can be proved by comparing productivity shock on production function and investment specific-technology shock. This chapter also uses analytical approach for the derivation of closed-form solutions for [illegible] premia for long-term bonds, equity and the short-term riskfree interest rate in a real business cycle model. Finally, chapter 6 concludes

Chapter 2 The Role of Investment-Specific Technology Shock in a Small Open Economy

2.1 Introduction

Real business cycle models, which originated in the frontier work of Finn Kydland and Edward Prescott (1982), and John Long and Charles Plosser (1983), have successfully explained some of the key empirical regularities that characterize economic fluctuations.

Typically, these real business cycle models have focussed on exogenous shocks to the production function. Hence, they support a view that the main economic fluctuations can be explained by productivity shocks. In contrast, Greenwood, Hercowitz, and Huffman (1988) and Greenwood, Hercowitz, and Krusell (1997, 2000) argue that it is shocks to the marginal efficiency of investment that are important in generating output fluctuations, rather than shocks to the production function. By incorporating shocks in a neoclassical framework with endogenous capacity utilization, they show that a positive shock to marginal efficiency of investment stimulates the formation of "new" capital and more intensive utilization and accelerated depreciation of "old" capital. Their model helps to explain the transmission mechanism of shocks. Therefore, their key conclusion from qualitative and quantitative analysis suggests that shocks to the marginal efficiency of investment can be important elements for business cycles.

In this chapter we adopt an international real business cycle model of a small open economy. Our model is parameterized, calibrated, and incorporates the neoclassical framework with endogenous capacity utilization. Our contribution to the existing literature can be outlined in the following way. First, unlike previous neoclassical models, the endogenous capacity utilization variable is central to our model's ability to generate the positive intertemporal co-movement of macroeconomic variables. The utility function we adopt is the stationary cardinal utility function (SCU) formulated initially by Larry Epstein (1983). As in Mendoza (1991a), SCU is introduced to produce a well-defined *stochastic* stationary equilibrium. A contribution of this chapter is that it allows numerical simulation and solves the equilibrium stochastic processes using the stationary cardinal utility. Second, while the analyses by Greenwood, Hercowitz, and Huffman (1988) and Greenwood, Hercowitz, and Krusell (1997, 2000) are based on a closed economy context, our model assumes a small open economy, incorporating the foreign sector. This allows us to explore the interaction of domestic capital and foreign financial assets as alternative vehicles to smoothing the agent's consumption and savings. Third, our analysis is the *first* study which presents both theoretical and quantitative analysis of the small open economy, Australia. This analysis suggests that capacity utilization and the shocks to investment provide a meaningful explanation for Australian economic fluctuations. In other words, we investigate whether investment-specific technology shock can affect economic fluctuations in a small open economy in a similar manner as in a large country such as the USA. Our argument is that investment-specific technological change in a small open economy is affected by both international trade of capital goods, in which

investment-specific technology of the foreign sector is embodied, and domestic technological change.

Mendoza (1991a, 1991b) has addressed the aspect of a small open economy in an international real business cycle model. An important aspect of the model developed by Mendoza (1991a) is that the representative agent's consumption and savings can be smoothed through access to world financial markets. Mendoza's (1991b) model has a flexible intertemporal discount rate which is equal to the world interest rate in the steady state equilibrium.

The quantitative investigation of the importance of investment-specific technological shock in the business cycle fluctuations is addressed in Greenwood, Hercowitz, and Krusell (1997, 2000). Similarly, our model is able to investigate the macroeconomic effects of a direct shock to investment in a neoclassical and small open economy framework. The effects of the investment-specific technological shock are investigated by both qualitative and quantitative analysis. In the qualitative analysis, the effects of the shocks are divided into direct and propagation effects. We examine the relationship between output and other endogenous variables, and the characteristics of the endogenous variables. Further, this chapter focuses on shocks to the marginal efficiency of investment, on capacity utilization and on the smoothness of consumption. The model is solved by dynamic programming, and we simulate the model using value function and transition probability iterations.

This chapter consists of seven sections. Section 2.2 provides a full description of the dynamic stochastic model of a small open economy. Section 2.3 conducts qualitative analysis on the effects of shocks. Section 2.4 sets benchmark parameters, solution techniques, and finds the steady-state conditions for optimal allocation in the dynamic programming problem. Section 2.5 provides the key findings in the empirical analysis. Finally, concluding remarks are presented in Section 2.6.

2.2 An International Real Business Cycle Model

A dynamic stochastic model of a small open economy is developed to investigate shocks to the marginal efficiency of investment. The structure of the model is built in which the economy is represented by large, infinitely-lived consumers with the same preferences, and the same production and technology.

Our model has a few salient features. First, the model adopts Keynes'(1936) view that shocks to the marginal efficiency of investment are important for business fluctuations. This is done by incorporating a neoclassical framework with endogenous capacity utilization. Following Greenwood, Hercowitz, and Huffman (1988), the model allows that an increase in the efficiency of newly produced investment goods stimulates the formation of "new" capital and more intensive utilization and accelerated depreciation of "old" capital. Second, we incorporate foreign financial assets as alternative vehicles to smoothing the agent's consumption and savings. As Mendoza(1991a, 1991b) emphasizes, this reflects an important feature of a small open economy – it can access to world

financial markets without any effect on those markets.

2.2.1 Preferences

Agents who are infinitely-lived with the same identical preferences allocate C_t (private consumption) and L_t (labor supply) intertemporally to maximize stationary cardinal utility.

$$E_0\left[\sum_{t=0}^{\infty}\{U(C_t - G(L_t))\}\exp\left\{\sum_{\tau=0}^{t-1}\upsilon(C_\tau, G(L_\tau))\right\}\right] \tag{2.1}$$

The instantaneous utility function and time-preference functions are as follows:

$$U(C_t - G(L_t)) = \frac{\left[C_t - \frac{L_t^{1+\theta}}{1+\theta}\right]^{1-\gamma} - 1}{1-\gamma} \tag{2.2}$$

$$\upsilon(C_\tau - G(L_\tau)) = \beta \ln\left(1 + C_t - \frac{L_t^{1+\theta}}{1+\theta}\right) \tag{2.3}$$

$$U(\cdot) < 0, \quad U'(\cdot) > 0, \quad U'(0) = \infty, \quad 1+\theta > 1, \quad \gamma > 1$$
$$\upsilon(\cdot) > 0, \quad \upsilon'(\cdot) > 0, \quad \upsilon''(\cdot) < 0, \quad \beta > 0$$
$$U'(\cdot)\exp(\upsilon(\cdot)) \text{ nonincreasing}$$

The parameter β is referred to as the consumption elasticity of the rate of time preference. The parameter θ is the inverse of the intertemporal elasticity of substitution in labor supply. The parameter γ is known as the coefficient of relative risk aversion. The corporation of the stationary cardinal utility into our model implies that our model is a stationary real business model. As Mendoza(1992) stated, this utility function also allows the use of dynamic programming techniques, guarantees the normality of consumption in all periods, and ensures the existence of a unique invariant limiting distribution of the state variables – all without causing a major deviation from the standard time-separable setup. We also assume that the intertemporal discount rate is variable. The distinguishing features of preferences are as follows: first, the intertemporal discount rate equalizes to the world's real interest rate in the steady-state equilibrium. Second, in equilibrium, the marginal product of labor should be equal to the marginal disutility of labor, which indicates that the labor supply decision is separated from the dynamics of consumption.

2.2.2 Production and Technology

Output is given by the following Cobb-Douglas production function using capital and labor. However, it differs from the standard neoclassical specification solely by the inclusion of a variable rate of capacity utilization (h_t) [1]

[1] Greenwood, Hercowitz and Huffman (1988) use a capacity utilization variable for two reasons. First, the capacity utilization decision involves Keynes' notion of 'user cost'. That is, a higher utilization rate causes a faster depreciation rate causes a faster depreciation of capital stock, either because wear and tear increase with use or because less time can be devoted to maintenance. Second, in the business cycle model of Greenwood, Hercowitz, and Huffman (1988), from the first-order condition of the model with investment-specific technology shock, if capacity utilization does not exist in the model, labor expresses as a function of only capital stock, which means labor supply is independent of investment-specific technology shock. Accordingly, by incorporating capacity utilization into the business model, the effect of the shock is transmitted to labor supply *intratemporarily*, and thus labor is expressed as a function of state variables including investment-specific technology shock. In our model, we accept the first reason of Greenwood, Hercowitz, and Huffman (1988). Moreover, without capacity utilization, our model also does not have any transmission mechanism connecting between labor supply and the terms of trade shock *intratemporarily*. Thus capacity utlilization is incorporated into our model.

$$Y_t = (K_t h_t)^{\alpha} L^{1-\alpha} \tag{2.4}$$

where Y_t is the output of the single good in period t, K_t is the capital stock, and L_t is labor input in period t. The parameter α is referred to as the capital share. The variable h_t determines the flow of capital services $(K_t h_t)$ and represents the intensity of the use of capital, that is, the speed of operation or the number of hours per period the capital is used.

2.2.3 Capital Accumulation and Financial Structure

The law of motion for capital stock follows:

$$K_{t+1} = K_t[1-\delta(h_t)] + (1+\varepsilon_t)i_t \tag{2.5}$$

and $\delta(h_t) = \frac{h_t^{\omega}}{\omega} \qquad \omega > 1$

where i_t denotes gross investment and δ is *not* a constant rate of depreciation but a non-negative function of capacity utilization (h_t). ω is an elasticity of depreciation with respect to capacity utilization. The capital accumulation and production capacity in period t+1 depend on both investment and technology shocks (ε_t) affecting the productivity of new capital goods. The technology shocks (ε_t) follow the two-state stochastic process from the stationary Markov distribution function.

The financial structure has links with the trade balance. The financial structure assumes that an agent in the economy is a small participant and has access to a perfectly competitive intertemporal capital market. Holdings of foreign financial assets (A_{t+1}) evolve according to:

$$A_{t+1} = TB_t + (1+r^*)A_t \qquad (2.6)$$

where A_t denotes the holdings of foreign financial assets in period t. TB_t is the balance of trade, and r^* is the world real interest rate, which is determined exogenously. To rule out the possibility of the economy playing a Ponzi game in the intertemporal capital market (these paths allow the representative agent to borrow initially an arbitrarily large amount and then to finance interest payments with further borrowing, thereby never repaying the initially debt), we need the following transversality condition:

$$\lim_{T\to\infty} E_0 \frac{A_T}{(1+r^*)^T} = 0$$

2.2.4 Structure of the Shocks

The stochastic structure of the shocks follows a two-state Markov process. The shocks have the values of two possible points:

$$E = \left\{e^{\xi_1} - 1,\ e^{\xi_2} - 1\right\}$$

The conditional transitional probabilities of moving from one value of the current shock ε to another in the next period's technology shock, ε', are defined by π_{sr}:

$$\text{prob}\left(\varepsilon' = e^{\xi_r} - 1 \mid \varepsilon = e^{\xi_s} - 1\right) = \pi_{sr}$$

where $0 \le \pi_{sr} \le 1$, and $\pi_{s1} + \pi_{s2} = 1$ for s, r = 1, 2

The above stochastic structure is simplified further by imposing the following symmetric conditions: $\pi_{11} = \pi_{22} = \pi$, $\pi_{12} = \pi_{21} = 1 - \pi$, and $\xi_1 = -\xi_2 = \sigma$. Under these symmetric conditions, the asymptotic standard deviation and the first-order autocorrelation coefficient are given by σ and $\rho = 2\pi - 1$, respectively.

2.2.5 Resource Constraint

Finally, the model is completed with the following resource constraint:

$$C_t + i_t + TB_t \le (K_t h_t)^\alpha L_t^{1-\alpha} \qquad (2.7)$$

$$\text{or} \quad C_t + i_t + A_{t+1} - (1+r^*)A_t \le (K_t h_t)^\alpha L_t^{1-\alpha}$$

The aggregate resource constraint of the economy describes that the sum of consumption, investment and the balance of trade cannot exceed gross domestic product.

2.3 The Dynamic Behavior of the Agent

The representative agent's optimization problem is characterized by the optimal decision rules for consumption, capital accumulation, labor supply, foreign asset accumulation, and the capacity utilization rate that maximize equation (2.1), given subject to equations (2.4) – (2.7). Given the state of an economy which descr K_0, A_0, ε_0, ibes the values of K_t, A_t, ε_t, and the stochastic process of the shocks, the representative agent chooses K_{t+1}, h_t, L_t and A_{t+1} in order to solve the following dynamic programming problem.

$$V(K_t, A_t, \varepsilon_t) = \max_{K_{t+1}, h_t, L_t, A_{t+1}} \left[U(C_t, L_t) + B(C_t, L_t) \int V(K_{t+1}, A_{t+1}, \varepsilon_{t+1}) d\Phi(\varepsilon_{t+1} \mid \varepsilon_t)\right] \tag{2.8}$$

subject to

$$C_t = (K_t h_t)^{\alpha} L^{1-\alpha} - (1/1+\varepsilon_t)K_{t+1} + (1/1+\varepsilon_t)(1-\delta(h_t))K_t + (1+r^*)A_t - A_{t+1} \tag{2.9}$$

where $B(\cdot) > 0,\quad B'(\cdot) < 0$ and $B''(\cdot) > 0$

The value function (2.8) has properties that are unique, increasing, concave, and differentiable. Equation (2.9) is obtained by substituting the capital accumulation equation (2.5) into the resource constraint (2.7). Equation (2.9) indicates that gross production is used for consumption, investment, and holding financial assets. Under the

transition equation (2.9), the first-order optimal conditions which maximize the value function with respect to K_{t+1}, h_t, L_t and A_{t+1} are obtained.

$$(K_{t+1}):\ (1/1+\varepsilon_t)\left\{U'(C_t - G(L_t)) + B'(C_t - G(L_t)) \int V(K_{t+1}, A_{t+1}, \varepsilon_{t+1}) d\Phi(\varepsilon_{t+1} \mid \varepsilon_t)\right\}$$

$$= B(C_t - G(L_t)) \int V_1(K_{t+1}, A_{t+1}, \varepsilon_{t+1})\, d\Phi(\varepsilon_{t+1} \mid \varepsilon_t)$$

$$= B(C_t - G(L_t)) \int \left[U'(C_{t+1} - G(L_{t+1})) + B'(C_{t+1} - G(L_{t+1})) \int V(K_{t+2}, A_{t+2}, \varepsilon_{t+2})\, d\Phi(\varepsilon_{t+2} \mid \varepsilon_{t+1})\right]$$

$$\times \left[F_1(K_{t+1}h_{t+1}, L_{t+1})h_{t+1} + (1/1+\varepsilon_{t+1})(1-\delta(h_{t+1}))\right] d\Phi(\varepsilon_{t+1} \mid \varepsilon_t) \tag{2.10}$$

$$(h_t):\ F_1(K_t h_t, L_t) = \delta'(h_t)/(1+\varepsilon_t) \tag{2.11}$$

$$(L_t):\ F_2(K_t h_t, L_t) = G'(L_t) \tag{2.12}$$

$$(A_{t+1}):\ U'(C_t - G(L_t)) + B'(C_t - G(L_t)) \int V(K_{t+1}, A_{t+1}, \varepsilon_{t+1})\, d\Phi(\varepsilon_{t+1} \mid \varepsilon_t)$$

$$= B(C_t - G(L_t)) \int V_2(K_{t+1}, A_{t+1}, \varepsilon_{t+1})\, d\Phi(\varepsilon_{t+1} \mid \varepsilon_t)$$

$$= B(C_t - G(L_t)) \int \left[U'(C_{t+1} - G(L_{t+1})) + B'(C_{t+1} - G(L_{t+1})) \int V(K_{t+2}, A_{t+2}, \varepsilon_{t+2}) d\Phi(\varepsilon_{t+2} \mid \varepsilon_{t+1})\right]$$

$$\times\ (1+r^*)(\varepsilon_{t+1} \mid \varepsilon_t) \tag{2.13}$$

Equation (2.10) shows the intertemporal decision rules between current consumption and future consumption through capital accumulation, that is, the marginal utility of current consumption is equal to the expected marginal utility of future consumption, which results from an abandonment of current consumption to accumulate capital stock. The left-hand side of equation (2.10) represents the loss (with the intertemporal discount rate) in current utility which is realized when an additional current investment is undertaken.

The right-hand side of equation (2.10) indicates the discounted expected future utility (with the change of utility caused by the intertemporal discount rate) obtained from an extra unit of investment today. Equation (2.11) describes the optimal capacity utilization rate condition. It basically sets the marginal benefits of capital services (left-hand side term of equation (2.11)), expressed as the marginal product of capital services, equal to the marginal user cost (right-hand side term of equation (2.11)). The marginal user cost of capital services consists of two components. Specifically, the marginal depreciation rate, $\delta(h_t)$, reflects the marginal cost in terms of increased current depreciation from changing an extra unit of the capacity utilization rate. The term $1/(1+\varepsilon_t)$ is the current replacement cost of old capital in terms of new capital.

Equation (2.12) sets the marginal product of labor equal to the marginal disutility of working. An important characteristic of equation (2.12) is that the labor supply is not dependent on the intertemporal consumption-savings decision, but on the marginal product of labor.

Equation (2.13) indicates the optimal condition of the foreign financial assets. The equation states that the marginal utility of current consumption is equal to the expected marginal utility of future consumption for financial assets, which results from abandoned

current consumption. The right-hand side of equation (2.13) represents the discounted expected future utility obtained from holding an extra unit of current financial assets. The left-hand side of equation (2.13) is made up of two components. The first term is the marginal utility of current consumption. The second term represents the loss in future utility which is caused by an increase in the intertemporal discount rate. Contrary to capital accumulation, the optimal condition of the foreign financial assets is not directly affected by the shock $(1+\varepsilon_t)$. However, a positive shock affects the evolution of foreign financial assets indirectly through the intertemporal substitution effect.

Following Greenwood, Hercowitz, and Huffman (1988), a qualitative analysis of the effects of the investment technology shift, ε_t, governing the marginal efficiency of investment on output, capacity utilization, hours of work, productivity, investment, consumption, and financial assets is conducted. The impact effect of investment shock on the endogenous variables has been analyzed throughout direct or indirect channels.

2.3.1 The Impact Effect of Investment Shock

The analysis of the impact effect of investment shocks are carried out under the assumption that investment shocks, ε_t, are serially uncorrelated and independently distributed over time. This qualitative analysis highlights the main characteristics of the model's propagation mechanism. The model has four control variables and three state variables. Among the four control variables, the capacity utilization variable and the labor supply variable are not intertemporal decision variables. The effects on the variables (capacity utilization rate, labor supply, and productivity) can be calculated from the optimality conditions (2.11) and (2.12).

$$\frac{dh_t}{d\varepsilon_t} = \frac{-\delta'(h_t)[F_{22}(K_t h_t, L_t) - G''(L_t)]}{[(1+\varepsilon_t)^2 \Omega(t)]} > 0 \tag{2.14}$$

$$\frac{dL_t}{d\varepsilon_t} = \frac{F_{12}(K_t h_t, L_t) K_t \delta'(h_t)}{[(1+\varepsilon_t)^2 \Omega(t)]} > 0 \tag{2.15}$$

where $\Omega(t) = -F_{11}(K_t h_t, L_t) K_t G''(L_t) - \delta''(h_t)[F_{22}(K_t h_t, L_t) - G''(L_t)]/(1+\varepsilon_t) > 0$

First, note that $F(\cdot)$ is the concave function while $\delta(\cdot)$ and $G(\cdot)$ are the concave and increasing functions. Accordingly, $F_1, F_2 > 0$, $F_{11}, F_{22} < 0$, $F_{11}F_{22} - F_{12}F_{21} = 0$ and $F_{12} = F_{21}$. The interpretation of equation (2.14) is that a positive investment shock reduces the cost of capacity utilization and hence induces a higher h_t. Similar to the result of Greenwood, Hercowitz, and Huffman (1988), since $F_{12} > 0$, labor's marginal productivity increases, resulting in a higher level of employment. Using (2.14) and (2.15), we can analyze the effect of the investment shock on the marginal product of labor.

$$\frac{dF_2}{d\varepsilon_t} = \frac{F_{12}(K_t h_t, L_t) K_t \delta'(h_t) G''(t)}{[(1+\varepsilon_t)^2 \Omega(t)]} > 0 \tag{2.16}$$

Equation (2.16) indicates that the marginal product of labor, $F_2(K_t h_t, L_t)$, moves upward when a positive unit of investment shock is undertaken. Using (2.9), (2.10) and (2.13), we can analyse the impact effects of the investment shock, ε_t, on the next

period's capital stock, K_{t+1}, the next period's foreign financial asset, A_{t+1}, and the current consumption, C_t.

The resulting expressions are:

$$\frac{dK_{t+1}}{d\varepsilon_t} = \frac{-U'(t) - B'(t)\int V(t+1)d\Phi(t+1)}{(1+\varepsilon_t)^2 B(t)\int U'(t+1)F_{11}h_{t+1}^2 d\Phi(t+1)} > 0 \tag{2.17}$$

$$\frac{dA_{t+1}}{d\varepsilon_t} = \frac{i_t\left[U''(t) + B''(t)\int V(t+1)d\Phi - B'(t)\int V_2(t+1)d\Phi\right]}{(1+\varepsilon_t)[U''(t) + B''(t)\int V(t+1)d\Phi - 2B'(t)\int V_2(t+1)d\Phi + B(t)\int V_{22}(t+1)d\Phi]}$$

$$+ \frac{U'(t) + B'(t)\int V(t+1)d\Phi}{(1+\varepsilon_t)^3 B(t)\int U'(t+1)F_{11}h_{t+1}^2 d\Phi} \gtrless 0 \tag{2.18}$$

$$\frac{dC_t}{d\varepsilon_t} = \frac{F_2(t)F_{21}K_t\delta'(h_t)}{[(1+\varepsilon_t)^2 \Omega(t)]} \tag{2.19}$$

$$+ \frac{i_t B(t)\int V_{22}(t+1)d\Phi - i_t B'(t)\int V_2(t+1)d\Phi}{(1+\varepsilon_t)[U''(t) + B''(t)\int V(t+1)d\Phi - 2B'(t)\int V_2(t+1)d\Phi + B(t)\int V_{22}(t+1)d\Phi]} \gtrless 0$$

The above effects tend to be more complicated, since the intertemporal discount rates are incorporated into the model. In other words, the directions of effects are not straightforward, since the shocks have affected not only the intertemporal discount rates, but also the utility itself. Note that the $V(\cdot)$ function is a twice continuously differentiable concave function in K and A.

As can be seen from equation (2.17), the investment shock (ε_t) has two effects on the future capital stock. The first term provides a positive substitution effect to future capital stock (K_{t+1}). The second term illustrates the positive effect caused by an increase in the intertemporal discount rate. As can be seen from equation (2.18), the substitution effect shows the transfer from foreign financial assets to capital stock. From equation (2.18), the shock on the future foreign financial assets has two effects. The first term of the right-hand side of equation (2.18) shows the income effect, while the second term represents a negative substitution effect. The interesting result is that the increase in the next period's capital stock is not caused by an income effect, but by a substitution effect from the next period's foreign financial assets. In other words, the income effect is absorbed completely by consumption and foreign financial assets. The agent changes the future foreign financial assets and consumption instead of changing the next period's capital stock. Since the income effects are not clear, it is also not clear whether the effects of the shock on the future foreign financial assets are positive or negative. If the income effects covering the effects of a changed intertemporal discount rate are higher than the substitution effect, future financial assets will have a positive effect.

As can be seen from equation (2.19), the technology shock has two effects on consumption. The first term shows the intratemporal substitution effect in that the shock increases the marginal productivity of labor, and raises the opportunity cost of leisure compared with consumption. Hence, the labor supply increases. The second term illustrates that the income effect incorporates the effect of the changed intertemporal

discount rate. The technology shock, given optimal future consumption, makes less investment possible, and so can cause an increment in current consumption. However, a consumption increase induces the intertemporal discount rate to decline, which causes current consumption to reduce again.

Using equations (2.14) and (2.17), the effect of the shock on investment can be analyzed as follows:

$$\frac{di_t}{d\varepsilon_t} = \frac{1}{1+\varepsilon_t}\left[\frac{dK_{t+1}}{d\varepsilon_t} - i_t + K_t\delta'(h_t)\frac{dh_t}{d\varepsilon_t}\right] \gtrless 0 \tag{2.20}$$

The effects of the shock on investment depend on the size of $(dK_{t+1}/d\varepsilon_t)$, i_t and $(dh_t/d\varepsilon_t)$. The first and third terms show positive effects on investment through intertemporal and intratemporal substitution. However, the sign is decided by the size of investment in period t, which represents the income effect.

2.3.2 Dynamic Effects of Investment Shocks

The shock (ε_t) affects the future economic system in two ways. First, a positive shock increases capital accumulation. Second, the shock also changes future financial assets. Once the shock affects future capital stock and foreign financial assets, the changed future capital stock and foreign financial assets have influences on other endogenous variables and on the next period. To analyze the effect of future capital stock and foreign financial assets, it is assumed that the shock (ε_{t+1}) in period t+1 is given.

Using the optimal condition (2.11) and (2.12), and extending to period t+1, we can obtain the following equations:

$$\frac{dh_{t+1}}{dK_{t+1}} = \frac{-F_{11}(t+1)h_{t+1}\left[F_{22}(t+1) - G''(t+1)\right] + F_{12}^2(t+1)h_{t+1}}{\Omega(t+1)} < 0 \qquad (2.21)$$

$$\frac{d(K_{t+1}h_{t+1})}{dK_{t+1}} = \frac{-h_{t+1}\delta''(t+1)\left[F_{22}(t+1) - G''(t+1)\right]}{(1+\varepsilon_{t+1})\Omega(t+1)} > 0 \qquad (2.22)$$

$$\frac{dL_{t+1}}{dK_{t+1}} = \frac{\delta''(t+1)F_{12}(t+1)h_{t+1}}{(1+\varepsilon_{t+1})\Omega(t+1)} > 0 \qquad (2.23)$$

where $\Omega(t+1)$, $F_{12}(t+1)$, $\delta''(t+1)$ and $G''(t+1) > 0$, $F_{11}(t+1) < 0$.

The higher K_{t+1} decreases the marginal productivity of capital service, and hence capacity utilization must reduce to meet the optimal condition. On the other hand, the optimal flow of capital services ($K_{t+1}h_{t+1}$) grows because the decreased effect of the optimal capacity utilization does not completely countervail the increased effect on capital stock (K_{t+1}). As can be seen from equation (2.23), the labor supply increases because the higher capital stock (K_{t+1}) raises the marginal productivity of labor.

In the case of a closed economy model, equations (2.22) and (2.23) prove that the effect of K_{t+1} will persist beyond t+1 under the condition that the higher K_{t+1} increases the next period's capital (K_{t+2}). However, in the case of an open economy model, the propagation channels are more complicated than in a closed economy model.

Under the assumption that A_{t+1} is given, using the optimal condition (2.10) and (2.13), and extending them to period t+1, we obtained the following equations:

$$\frac{dK_{t+2}}{dK_{t+1}} = 0 \qquad (2.24)$$

$$\frac{dA_{t+2}}{dK_{t+1}} = \frac{\left[F_1 h_{t+1} + (1-\delta)/(1+\varepsilon_{t+1})\right]\left[U''(t+1) + B''(t+1)\int V(t+2)d\Phi - B'(t)\int V_2(t+2)d\Phi\right]}{U''(t+1) + B''(t+1)\int V(t+2)d\Phi - 2B'(t+1)\int V_2(t+2)d\Phi + B(t)\int V_{22}(t+2)d\Phi} \gtrless 0 \qquad (2.25)$$

As can be seen from equations (2.24) and (2.25), the increase in capital stock (K_{t+1}) does not affect the next period's capital stock, but rather the next period's foreign financial assets. This explains why an increase of K_{t+1} does not affect the next period's capital stock through the intertemporal channel. This proves that the agent uses foreign financial assets to smooth consumption and investment. Moreover, the complete substitution relationship between capital stock and foreign financial assets provides that the role of foreign financial assets is to absorb the effects of shocks, and so stabilize the economy in the long run.

2.4 Dynamic Programming Problem, Calibration, and Solution Technique

2.4.1 Dynamic Programming Problem and Calibration

The representative household selects paths of consumption and labor supply to maximize the whole life-time utility. To solve the dynamic programming problem, the household's optimal intertemporal decisions are to choose the control variables (K_{t+1}, A_{t+1}, h_t, L_t) in period t, given the state of the economy as described by K_t, A_t and ε_t, where ε_t represents the stochastic shock.

$$V = E_0 \sum_{t=0}^{\infty} \left\{ \frac{\left[C_t - \frac{L_t^{1+\theta}}{1+\theta} \right]^{1-\gamma} - 1}{1-\gamma} \exp\left[-\sum_{\tau=0}^{t-1} \beta \ln\left(1 + C_\tau - \frac{L_\tau^{1+\theta}}{1+\theta} \right) \right] \right\} \quad (2.26)$$

Subjects to:

$$C_t = (K_t h_t)^\alpha L_t^{1-\alpha} - \frac{1}{1+\varepsilon_t} K_{t+1} + \frac{1}{1+\varepsilon_t}(1-\delta(h_t))K_t + (1+r^*)A_t - A_{t+1} \quad (2.27)$$

The first-order conditions for utility maximization are:

$$(K_{t+1}): \frac{1}{1+\varepsilon_t} \times [U'(t) + V_{ct}]$$
$$= \left[\alpha K_{t+1}^{\alpha-1} h_{t+1}^{\alpha} L_{t+1}^{1-\alpha} + \frac{1}{1+\varepsilon_{t+1}} - \frac{1}{1+\varepsilon_{t+1}} \frac{1}{\omega} h_{t+1}^{\omega} \right] \times [B(t)U'(t+1) + B(t)V_{ct+1}] \quad (2.28)$$

$$(L_t): L_t^{\theta} = (1-\alpha)(K_t h_t)^{\alpha} L_t^{-\alpha} \quad (2.29)$$

$$(h_t): \alpha K_t^{\alpha} h_t^{\alpha-1} L_t^{1-\alpha} = \frac{1}{1+\varepsilon_t} \frac{1}{\omega} K_t h_t^{\omega} \quad (2.30)$$

$$(A_{t+1}): [U'(t) + V_{ct}] = (1+r^*) \times [B(t)U'(t+1) + B(t)V_{ct+1}] \quad (2.31)$$

where $B(t) = \left(1 + C_t - \frac{N_t^{1+\theta}}{1+\theta} \right)^{-\beta}$

$$V_{ct} = \beta \left(1 + C_t - \frac{N_t^{1+\theta}}{1+\theta} \right)^{-\beta-1} \{ U(t+1) + B(t+1)U(t+2) + B(t+1)B(t+2)U(t+3) + \cdots \}$$

And, the steady-state equilibrium conditions are:

$$1 = \left(1 + C - \frac{N^{1+\theta}}{1+\theta} \right)^{-\beta} \times \left(\alpha K^{\alpha-1} h^{\alpha} + 1 - \frac{1}{\omega} h^{\omega} \right) \quad (2.28)'$$

$$L^{\theta} = (1-\alpha) K^{\alpha} h^{\alpha} L^{-\alpha} \quad (2.29)'$$

$$\alpha K^{\alpha} h^{\alpha-1} L^{1-\alpha} = \frac{1}{\omega} K h^{\omega} \quad (2.30)'$$

$$1 = \left(1 + C - \frac{N^{1+\theta}}{1+\theta} \right)^{-\beta} \times (1 + r^*) \quad (2.31)'$$

Equation (2.28) shows that marginal utility of current consumption is equal to the marginal utility of future consumption, which results from an abandonment of current consumption. Equation (2.29) shows that the marginal productivity of labor should be equal to the marginal disutility resulting from the abandonment of leisure. Equation

(2.30) indicates that the marginal benefits available from using capital service are equal to the marginal costs, which results from using capital services. Equation (2.31) shows that the marginal utility of current consumption should be equal to the marginal utility of future consumption, which results from the abandonment of consumption in favour of holding foreign financial assets.

2.4.2 Optimal Allocation of Control Variables

To construct the grids and build the value function, optimal allocation equations of control variables and steady state values are necessary. Our model has four control variables ($K_{t+1}, A_{t+1}, h_t, L_t$). Solving the intertemporal dynamic problems, we obtain labor supply (L_t) and capacity utilization (h_t) as the function of state variables (K_t, A_t, ε_t). To obtain other control variables K_{t+1} and A_{t+1} as the function of state variables, we use the value function and transition probability iteration technique. Equations (2.29) – (2.30) are expressed as the function of current capital (K_t) and realized shock (ε_t).

From equation (2.29):

$$L_t = (1-\alpha)^{\frac{1}{\alpha+\theta}}(K_t h_t)^{\frac{1}{\alpha+\theta}} \tag{2.32}$$

Substitute equation (2.32) into equation (2.30), and rearrange equation (2.30):

$$\hat{h}_t = \alpha^{\frac{-(\alpha+\theta)}{\eta}}(1-\alpha)^{\frac{\alpha-1}{\eta}} K_t^{\frac{(1-\alpha)\theta}{\eta}}(1+\varepsilon_t)^{\frac{-(\alpha+\theta)}{\eta}} \tag{2.33}$$

Again, substitute equation (2.33) into equation (2.32), and rearrange equation (2.32):

$$\hat{L}_t = \alpha^{\frac{-\alpha}{\eta}}(1-\alpha)^{\frac{\alpha-\omega}{\eta}} K_t^{\frac{\alpha(1-\omega)}{\eta}}(1+\varepsilon_t)^{\frac{-\alpha}{\eta}} \tag{2.34}$$

where $\eta = \alpha(1+\theta) - \omega(\alpha+\theta)$

2.4.3 Building the Value Function, Parameter Calibration and Steady-State Values

The value function can be obtained by using equation (2.26) subject to equations (2.27), (2.33), and (2.34).

$$V(K_t, A_t, \varepsilon_t^s) = \max \left\{ \frac{\left[C_t - \frac{\hat{L}_t^{1+\theta}}{1+\theta}\right]^{1-\gamma} - 1}{1-\gamma} + \exp\left[-\beta \ln\left(1 + C_t - \frac{\hat{L}_t^{1+\theta}}{1+\theta}\right)\right] \sum_{r=1}^{2} \pi_{sr} V(K_{t+1}, A_{t+1}, \varepsilon_{t+1}^r) \right\} \tag{2.35}$$

Subject to:

$$C_t = (K_t \hat{h}_t)^{\alpha} \hat{L}_t^{1-\alpha} - \frac{1}{1+\varepsilon_t} K_{t+1} + \frac{1}{1+\varepsilon_t}(1-\delta(\hat{h}_t))K_t - A_{t+1} + (1+r^*)A_t \tag{2.36}$$

$$\hat{h}_t = \alpha^{\frac{-(\alpha+\theta)}{\eta}}(1-\alpha)^{\frac{\alpha-1}{\eta}} K_t^{\frac{(1-\alpha)\theta}{\eta}}(1+\varepsilon_t)^{\frac{-(\alpha+\theta)}{\eta}} \tag{2.37}$$

$$\hat{L}_t = \alpha^{\frac{-\alpha}{\eta}}(1-\alpha)^{\frac{\alpha-\omega}{\eta}} K_t^{\frac{\alpha(1-\omega)}{\eta}}(1+\varepsilon_t)^{\frac{-\alpha}{\eta}} \tag{2.38}$$

Once the value function is set, the benchmark parameters must be calibrated. To solve the value function, it is necessary to calibrate the values of α (capital's share in output), β (the consumption elasticity of the rate of time preference), γ (the coefficient of relative risk aversion), θ (the inverse of the intertemporal elasticity of substitution in labor supply), r^* (the world's real interest rate), and ω (the elasticity of depreciation with respect to capacity utilization). In addition, to capture the stochastic structure of the model, ρ (autocorrelation of the shocks) and σ (the standard deviation of the shocks) should be calibrated.

The relevant parameters of preference are principally taken from Mendoza (1991a) and Greenwood, Hercowitz, and Krusell (1997, 2000). Under the restriction that the average ratio of net foreign interest payments to GDP is around 2.0 percent in Australia's case, the consumption elasticity (β) is calculated as 0.77 from the steady-state condition, which equates to the world real interest rate with the discount rate of time preference. The coefficient of relative risk aversion (γ) is set as 2.0. The inverse of the intertemporal elasticity of substitution in labor supply (θ) is set as 0.455 from Mendoza (1991a). The relevant parameters of production are calculated from national income data. An average annual value of 0.31 over the 1965–1999 period is used for capital share (α) of national income. The elasticity of depreciation with respect to utilization (ω) is set as 1.4 under the restriction that the depreciation rate (δ) is 0.1 in a deterministic steady state. The world's real interest rate (r^*) is set as 0.04, which is obtained from Mendoza (1991a). Adjustment costs are set at 1.5 from Greenwood, Hercowitz, and Krusell (2000).

Table 2.1: Benchmark parameter values

Parameter	Definition	Value
β	Consumption elasticity of the rate of time preference	0.77
γ	Coefficient of relative risk aversion	2.0
θ	Inverse of the intertemporal elasticity of substitution in labor supply	0.455
α	Capital share in output	0.31
ω	Elasticity of depreciation with respect to capacity utilization	1.4
r^*	World's real interest rate	0.04
δ	Depreciation rate of capital	0.1
ϕ	Adjustment cost of capital stock	1.5
ρ	Autocorrelation of the stochastic shocks	0.64
σ	Standard deviation of the stochastic shocks	0.035

The stochastic process for investment-specific technology is estimated by using annual data of the ratio of implicit price deflator of non-durable consumption goods to implicit price deflator of producer's durable equipment[2]. Figure 1 shows the reverse of the relative price, indicating both the relatively decreasing equipment price and the abrupt technological change in 1974 that Greenwood and Yorukoglu (1997) emphasized and proved.

[2] Greenwood, Hercowitz and Krusell (2000) and Green and Yorukoglu (1997) use the inverse of the relative price of new equipment (implicit price deflator for non-durable consumption goods and non-housing services divided by Gorden's index of nominal prices for producer durable equipment) to estimate investment-specific technology shock process. They argue that the decrease of the relative price of new equipment reflects the improvement of investment-specific technology. We also estimate the shock process from the relative price (Australian data), which is defined by the logged value of implicit deflator index of equipment investment divided by implicit deflator index of non-durable consumption expenditure.

By using the relative price (q), we estimate the shock process as follows:

$$\ln q_t = \text{constant} + \text{trend} \times \ln \gamma_q + \varepsilon_t$$

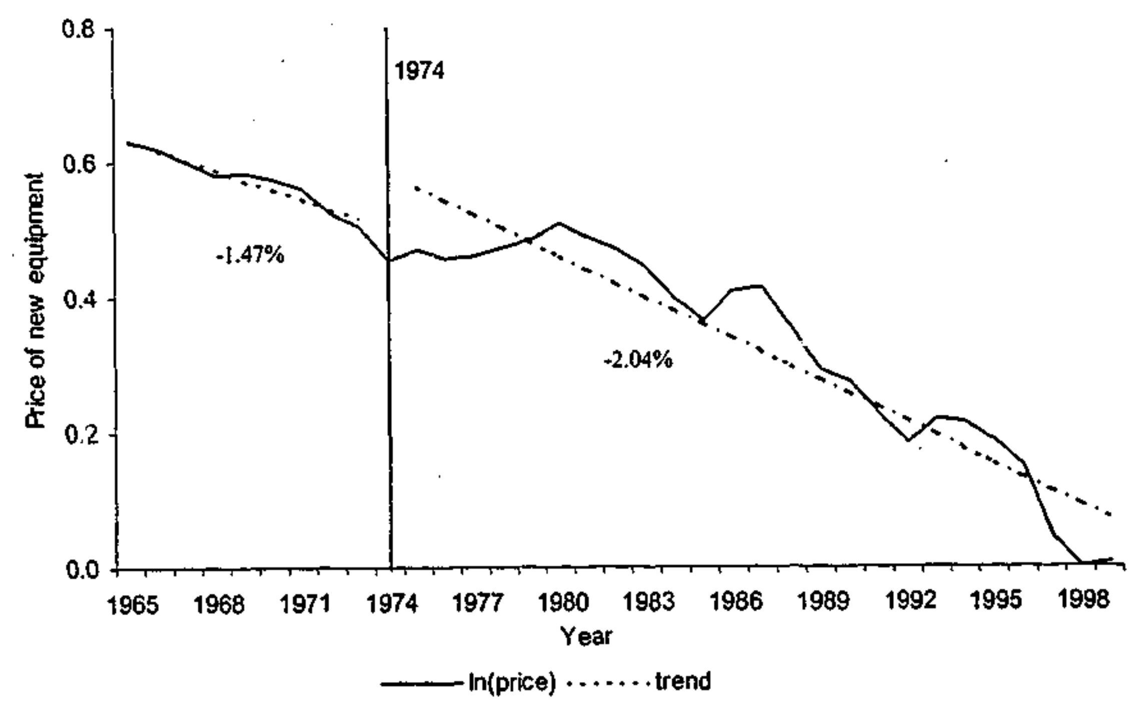
where $\varepsilon_{t+1} = \psi \varepsilon_t + w_t$ with $0 < \psi < 1$ and $\varepsilon_t \sim N(0, \sigma_w)$

Using the annual Australian data from 1965 to1999, the estimated parameters are:

$$\ln \gamma_q = \underset{(17.29)}{0.016}, \quad \psi = \underset{(9.26)}{0.89}, \quad \sigma = 0.030 \quad \text{with} \quad D.W = 1.38,$$

where the numbers in parentheses are *t-statistics*.

Figure 2.1: Price of new equipment[3]



[3]The price of new equipment is computed as the logged value of implicit deflator index of equipment investment divided by implicit deflator index of non-durable consumption expenditure.

Using the parameters of the shock process, two point states and one-step transition probabilities are computed as $\rho = 2\pi - 1$, $\pi_{11} = \pi_{22} = \pi$ and $\xi_1 = -\xi_2 = \sigma$. Accordingly, two–point states of the shocks and one-step transition probabilities of the shocks are expressed as follows:

$$\pi_{sr} = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix} \qquad \text{for all } s, r = 1,2$$

$$\text{by} \quad E = \left\{ e^{\xi_1} - 1, e^{\xi_2} - 1 \right\} = \left\{ e^{0.030} - 1, e^{-0.030} - 1 \right\}$$

2.4.4 Value function and transition probability iterations

We normally use the recursive method to solve the Bellman equation. Because the value function of the model is a Bellman equation, the recursive method is employed to solve the equation. The recursive formulation of the Bellman equation was introduced by Dimitri Bertsekas (1976), and was applied to solve the dynamic macroeconomic models by Thomas Sargent (1980) and Greenwood, Hercowitz, and Huffman (1988).

The value function and transition probability iteration method has the following procedure. First, we define evenly spaced grids for capital (K) and foreign financial assets (A). The grids are set extensively from the deterministic steady-state values of capital (K) and foreign financial assets (A) which are in the center of the grids. Accordingly, the model economy has 2 (the number of the shock grids) × N (the number of capital grids) ×

M (the number of foreign financial asset grids)[4] state space.

Second, we set up equations (2.35), (2.36), (2.37), and (2.38) to use the method of successive approximation to solve the dynamic programming problem. Then we start the value function iteration from initial guess ($V^0 = 0$). Firstly, substitute $V^0(K_i, A_i, \varepsilon_i) = 0$ to the $V^1(K_j, A_j, \varepsilon_j)$ of the right-hand side of equation (2.35). Then, the revised V^0 of the left-hand side of the equation enters into the right-hand side of the equation. The process is iterated until the value function converges. Generally, it takes a long-time value function to converge. Moreover, the purpose of using the value function iteration is to find the representative individual's decision rules ($K' = K'(K, A, \varepsilon)$, $A' = A'(K, A, \varepsilon)$). Therefore, the iterations of the value function are continued only until the decision rules converge. However, because the decision rules sometimes are unstable, the decision rule iteration must be complemented by a suitable convergence of the value function.

Third, once the decision rules have been decided, the one-step transition probabilities are defined. The one-step transition probabilities have a role that moves the values of state variables $(K_i, A_i, \varepsilon_i)$ in period t to the values of state variables $(K_j, A_j, \varepsilon_j)$ in period t+1. The one-step transition probabilities are expressed by a matrix (P) with the possible points of (2×N×M × 2×N×M). The decision rules, which were calculated from the value function iteration, are used to find the transition probability matrix. Given an initial

[4] Capital stocks include 45 elements, foreign financial assets include 22 elements and the shocks include 2 elements. Hence, the state space 2×N×M contains 1980 combinations.

capital stock (K_i), foreign financial assets (A_i) and the shock values (ε_i), next period values for $K' = K'(K_i, A_i, \varepsilon_i) \in K$, $A' = A'(K_i, A_i, \varepsilon_i) \in A$ can be selected by the decision rules. Thus, the prob $[K' = K_j, A' = A_j \mid K = K_i, A = A_i, \varepsilon = \varepsilon_s]$ will be one, and expressed as foilows:

$$\sum_{j=0}^{n} \text{prob}\left[K' = K_j, A' = A_j \mid K = K_i, A = A_i, \varepsilon = \varepsilon_s\right] = 1$$

$$\text{for all } (K, A, \varepsilon) \in K \times A \times E$$

These probabilities are combined with the probabilities of the shocks, thus the transition probabilities are rewritten as follows:

$$\sum_{j=0}^{n} \text{prob}\left[K' = K_j, A' = A_j, \varepsilon = \varepsilon_r \mid K = K_i, A = A_i, \varepsilon = \varepsilon_s\right] = 0.95$$

$$\text{for all } (K, A, \varepsilon) \in K \times A \times E \text{ and } r = s$$

$$\sum_{j=0}^{n} \text{prob}\left[K' = K_j, A' = A_j, \varepsilon = \varepsilon_r \mid K = K_i, A = A_i, \varepsilon = \varepsilon_s\right] = 0.05$$

$$\text{for all } (K, A, \varepsilon) \in K \times A \times E \text{ and } r \neq s$$

Note that the probabilities of the shocks have already been calculated in section 4.

Fourth, using the one-step transition probability matrix, the stationary joint probability function of the capital stock, foreign financial assets, and the shocks are computed by the procedure of transition probability iteration. An initial probability distribution vector is decided, and the vector of $2 \times N \times M$ (p^0) must satisfy the condition that $\sum_{i=0}^{n} p_{ir}^{0} = 1$. Once the point values of the vector are decided, the recursive method is applied by mapping $p^1 = p^0 P$. As a result, p converges to a stationary joint probability distribution (p^*).

Fifth, after the stationary joint probability distribution (p^*) is obtained, we calculate the population moments using the stationary joint probability and next period values of endogenous variables that are decided by the decision rules, which are expressed as follows:

$$E(y) = \sum_{s=1}^{2}\sum_{j=1}^{m}\sum_{i=1}^{n} p_{ijs}^{*}\, y(K_i, A_j, \varepsilon_s)$$

$$Var(y) = \sum_{r=1}^{2}\sum_{j=1}^{m}\sum_{i=1}^{n} p_{ijr}^{*}\,(y(K_i, A_j, \varepsilon_r) - E(y))^2$$

$$Var(y') = \sum_{r=1}^{2}\sum_{j=1}^{m}\sum_{i=1}^{n} \cdot \sum_{s=1}^{2}\sum_{g=1}^{m}\sum_{h=1}^{n} p_{hgs,ijr}\, p_{hgs}^{*}\,(y'(K_i', A_j', \varepsilon_r') - E(y))^2$$

$$E(cy) = \sum_{s=1}^{2}\sum_{j=1}^{m}\sum_{i=1}^{n} p_{ijs}^{*}\,(c(K_i, A_j, \varepsilon_s) - E(c))(y(K_i, A_j, \varepsilon_s) - E(y))$$

$$Var(y'y) = \sum_{r=1}^{2}\sum_{j=1}^{m}\sum_{i=1}^{n} \cdot \sum_{s=1}^{2}\sum_{g=1}^{m}\sum_{h=1}^{n} p_{hgs,ijr}\, p_{hgs}^{*}\,(y'(K_i', A_j', \varepsilon_r') - E(y'))(y(K_h, A_g, \varepsilon_s) - E(y))$$

2.5 Empirical Findings

2.5.1 Simulations and Sensitivity Analysis

This section reports the statistical moments produced by the benchmark model and compares with the actual moments obtained from Australian data. Table 2.2 summarizes the major features of a real economy and the benchmark economies when risk aversion coefficients are 2.0 and 1.6. As shown in Table 2.2, the most salient feature of the standard deviations of the actual data is the well-known fact that investment (5.04) is much more volatile than output (1.51) and consumption is less volatile (0.83). The standard deviation of capital stock shows the lowest value (0.59), while interest payments shows the highest value (14.41).

When the risk aversion coefficient (γ) is 2.0, the benchmark economy has performed fairly well and represents a similar pattern to the real economy in the standard deviation, particularly in the values of investment (5.04 vs. 4.38), labor supply (1.07, 1.04), interest payments (14.41, 11.54) and trade balance (1.20, 0.89). In addition, although consumption and capital stock are overestimated, the benchmark economy replicates the features of the real economy with low values such as consumption (0.83, 1.46) and capital stock (0.59, 1.16). In the case of autocorrelation coefficients, when the risk aversion coefficient (γ) is 2.0, the benchmark economy closely mimics the real economy in consumption (0.37 vs. 0.34), interest payments (0.45 vs. 0.36). The autocorrelation of trade balance in the benchmark economy reflects a low autocorrelation value of the real economy. From the correlations with output in the real economy, it can be seen that investment has the highest correlation with output, but other variables, consumption and interest payments, are fairly close. The feature of highest correlation of labor supply with output is produced by the benchmark model similar to Greenwood, Hercowitz, and Huffman (1988) and Mendoza (1991a).

The major focus of Greenwood, Hercowitz, and Huffman (1988) was capacity utilization; the value of capacity utilization shows a procyclical effect. The correlation coefficient of capacity utilization with output is 0.95. The standard deviation of the capacity utilization is 1.15, lower than the standard deviation of output. This supports the contention that the shock to newly produced capital stock results in the fluctuation of capacity utilization, and that the volatility affects the output in a procyclical way. The correlations of interest payments with output are 0.45 and 0.36 in the real and the benchmark economies when the risk aversion coefficient (γ) is 2.0, respectively. Trade balance shows the correlation values with output of –0.02 and –0.14 in the real and benchmark economies. The coefficient indicates a countercyclical effect of trade balance.

Table 2.2: Statistical moments of Australian data and the benchmark model

Variable	Australian data 1965–1999			The benchmark model ($\gamma = 2.0$)			The benchmark model ($\gamma = 3.0$)		
	σ_x	$\rho_{xt,xt-1}$	$\rho_{xt,GDPt}$	σ_x	$\rho_{xt,xt-1}$	$\rho_{xt,GDPt}$	σ_x	$\rho_{xt,xt-1}$	$\rho_{xt,GDPt}$
Output	1.51 (0.192)	0.34 (0.168)	1.0	1.51	0.34	1.0	1.51	0.34	1.0
Consumption	0.83 (0.092)	0.37 (0.156)	0.44 (0.222)	1.46	0.34	0.98	1.56	0.34	0.92
Investment	5.04 (0.749)	0.52 (0.159)	0.72 (0.071)	4.38	0.18	0.54	4.37	0.18	0.54
Capital stock	0.59 (0.061)	0.76 (0.066)	0.42 (0.119)	1.16	0.34	0.91	1.16	0.35	0.91
Capacity utilization	–	–	–	1.15	0.29	0.95	1.58	0.29	0.95
Labor supply	1.07 (0.115)	0.55 (0.097)	0.39 (0.128)	1.04	0.34	0.99	1.04	0.34	0.99
Interest payments	14.41 (1.876)	0.45 (0.120)	0.45 (0.145)	11.54	0.36	0.36	23.08	0.37	0.19
Trade balance (TB/Y)	1.20 (0.157)	0.29 (0.216)	–0.02 (0.250)	0.89	0.02	–0.14	0.98	0.02	–0.13

Notes: The Australian data are obtained from ABS (Australian Bureau of Statistics), divided by the 15+ population, logged and detrended by a Hodrick-Prescott filter with the smoothing parameter set at 100. Output is real GDP, consumption and investment are total real values based on 1989 – 1990 constant prices. Labor supply is calculated by multiplying the average weekly working hours by non-farm civilian employment. Capital stock is end-year net capital stock, and interest payments are the net of foreign interest paid and received. For each variable, σ_x is the percentage standard deviation, $\rho_{xt,xt-1}$ is the first-order autocorrelation , and $\rho_{xt,GDPt}$ is the contemporaneous correlation with GDP. The values in parenthesis represent the standard deviations estimated by GMM (Generalized Method of Moment) using the Hansen-Heaton-Ogaki gauss program (Ogaki, 1993). The second moments from the benchmark model are computed by value function and transition probability iteration method. As can be seen from Mendoza (1991a, 1991b and 1995), standard deviations of the second moments can not be computed from value function and transition probability iteration method.

When the risk aversion coefficient (γ) increases to 3.0, the standard deviation of consumption increases from 1.46 to 1.56, whereas the correlations with output decreases from 0.98 to 0.92. In general, the risk aversion coefficient (γ) strongly affects interest payments. When the risk aversion coefficient (γ) rises from 2.0 to 3.0, the standard deviation of interest payments increases from 11.54 to 23.08, while correlation with output decreases from 0.36 to 0.19.

Table 2.3: Sensitivity analysis

	Relative volatility			Autocorrelation			Correlation with output		
	C	I	TB	C	I	TB	C	I	TB
Baseline Model	1.46	4.38	0.89	0.34	0.18	0.02	0.98	0.54	-0.14
θ	1.47	4.68	0.95	0.34	0.22	0.05	0.99	0.54	-0.19
β	1.46	4.26	0.88	0.34	0.18	0.02	0.92	0.54	-0.13
α	1.45	4.26	0.88	0.34	0.18	0.01	0.98	0.55	-0.12
δ	1.52	2.84	0.57	0.34	0.08	0.002	0.98	0.63	-0.08
r^*	1.48	3.56	0.71	0.34	0.04	0.01	0.98	0.57	-0.09

Note: Baseline model with each of the following parameters increased by 5%. C, I and TB mean consumption, investment, and trade balance.

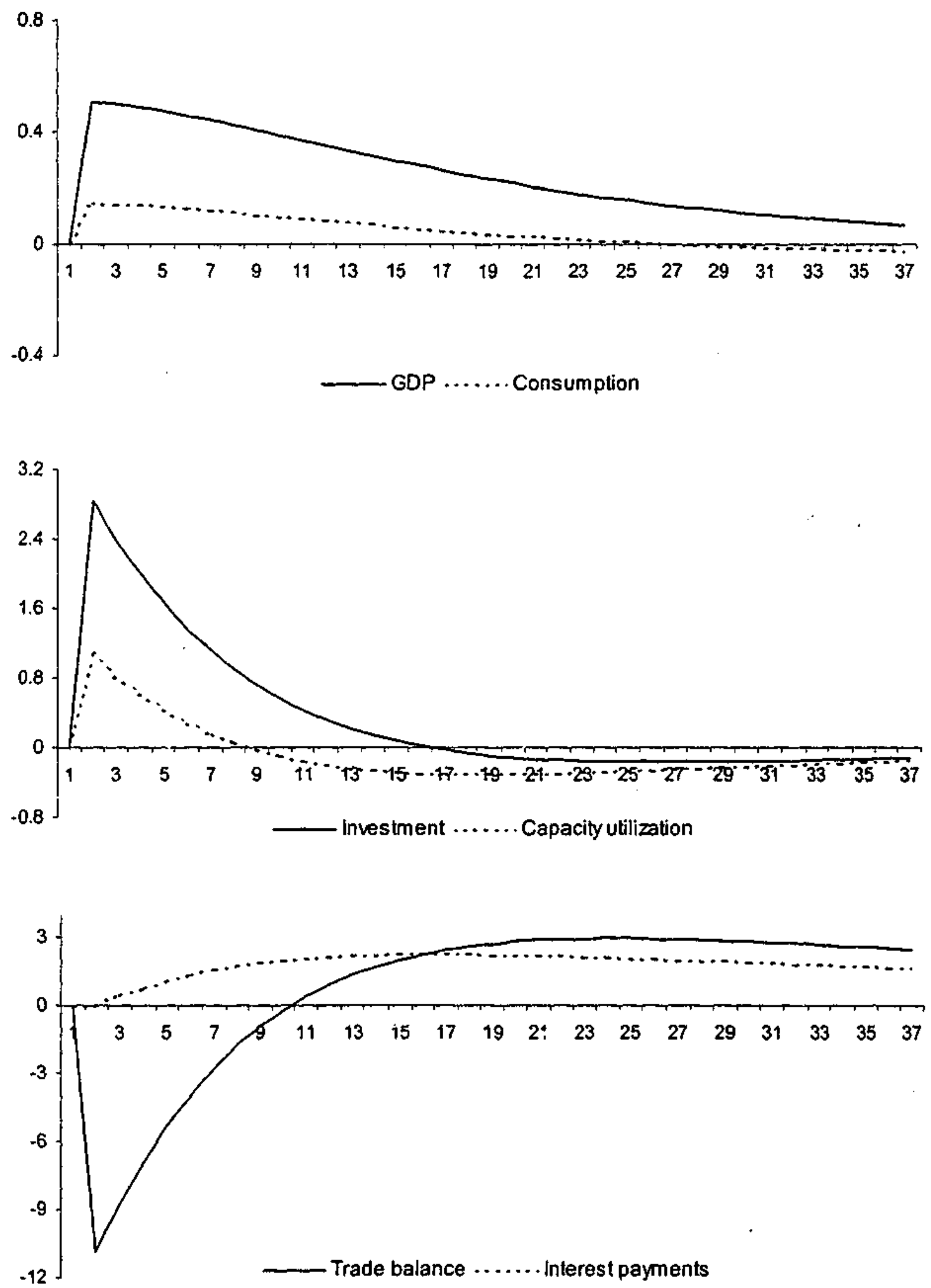
Table 2.3 presents the results of simulation that increase every parameter by 5%. Standard deviations are more sensitive to the change of parameters than autocorrelation and correlation with output. Investment is most sensitive to change of parameters. Investment is relatively more sensitive to the change of depreciation rate and world interest rate in second moments. Trade balance is quite sensitive to the change of depreciation rate and the world interest rate in standard deviations and autocorrelation. The most sensitive parameter is depreciation rate. The depreciation rate greatly affects standard deviations of all variables except for consumption.

2.5.2 Impulse Response Analysis

This section studies the impulse response of macroeconomic aggregates induced by investment-specific technology shock. The impulse response analysis can show the magnitude of the macroeconomic effects from the shock that the qualitative analysis of section 2.3 cannot provide. For impulse response analysis, we utilize the linear policy function derived from the undetermined coefficient method (Christiano, 1998), not value function and transition probability iterations.

Figure 2.2 plots the impulse response of the major macroeconomic variables to 1% positive shock in investment-specific technology shock. As can be seen from Figure 2.2, a positive investment-specific technology shock has a positive effect on GDP, as indicated by the qualitative analysis in section 2.3. The effect of the shock on consumption is ambiguous in the qualitative analysis because consumption has positive income effect and negative substitution effect. From impulse response analysis, the positive income effect overwhelms the negative substitution effect. The impulse response analysis reflects the negative substitution effect of the shock on consumption. Hence, the effect of the shock on consumption is smaller than the effect of the shock on GDP, even though consumption responds positively to the shock. In qualitative analysis, the effect of the shock on investment is ambiguous. In other words, if initial investment is large size, the shock can negatively affect investment. However, in the impulse response analysis, the effect of the shock on investment has a clear positive effect on investment. In the case of investment, a positive substitution effect overwhelms a negative income effect. As can be seen from Figure 2.2, the effect of the shock on investment is more than the effects on GDP, consumption, and capacity utilization. In qualitative analysis, the effect of the shock on foreign financial assets is not clear, because the foreign financial assets also have both positive income effect and negative substitution. For impulse response analysis, the negative substitution effect of the shock on foreign financial assets overwhelms the positive income effect. As a result, from Figure 2.2, the shock has a positive effect on interest payment ($-r^*A$). A positive investment-specific technology shock increases GDP, and thus raises exports. However, the increased GDP also raises imports. Moreover, the increase of investment productivity also increases imports. Consequently, the shock raises imports more than exports, and thus the trade balance decreases.

Figure 2.2: Impulse response analysis: Investment-specific technology shock



2.6 Concluding Remarks

Under the proposition that an economy experiences shocks to the marginal efficiency of investment, and that capital service is dependent on both capital size and capacity utilization, our chapter investigates the central issue as to how economic variables fluctuate in response to shocks to investment.

We first conducted a qualitative analysis on the effects of a shock within a small open economy framework. The findings of the qualitative analysis are that a positive shock affects endogenous variables positively, except for consumption and foreign financial assets. Unlike a closed economy, our qualitative analysis of a small open economy indicates that a capital stock increase resulting from a positive shock is absorbed by the next period's foreign financial assets. This explains how foreign financial assets are used to smooth consumption-savings, thereby stabilizing the economy.

In the quantitative analysis, our benchmark model is parameterized, calibrated, and then simulated. We apply the value functions and transition probability iteration method in order to simulate our benchmark model. In conjunction with the qualitative analysis, using Australian data, we analyze the benchmark model in which the economy has shocks to investment, and output is dependent on both capital size and capacity utilization. From the perspective of the whole structure, the model economy performs well in replicating the key features of the real economy; that is, it demonstrates a similar pattern to the real economy. More importantly, capacity utilization and shocks to

investment provide a meaningful explanation for economic fluctuations. Furthermore, capacity utilization shows a strong procyclical property, and is a significant variable that explains business cycle fluctuations. The qualitative and quantitative results in this chapter are consistent with the results of Greenwood, Hercowitz, and Huffman (1988), suggesting that a variable capacity utilization rate is important for the understanding of business cycle fluctuations. Foreign financial assets provide procyclical effects to output fluctuations, unlike the trade balance. Finally, impulse response analysis provides strong supports for the qualitative analysis, which represents the dynamic effects of the model.

Appendix 2.1: Data Source

The Australian real data are obtained from the ABS (Australian Bureau of Statistics). All series are composed of yearly observations from 1965 to 1999. *Output*: real GDP, 89/90 price. *Consumption*: private final consumption expenditure, 89/90 price. *Investment*: gross capital formation (chain volume measure). *Capital stock*: end-year net capital stock (chain volume measure). *Labor supply*: the number of civilian wage and salary earners × weekly hours worked. *Population*: civilian population aged 15 years and over *Interest payments*: (nominal investment income to overseas – nominal investment income from overseas) ÷ GDP deflator. Trade balance: (export of goods – import of goods)/ GDP

Chapter 3 Terms of Trade, Intermediates Goods and International Real Business Cycles

3.1 Introduction

Since the pioneering work initiated by Backus, Kehoe and Kyland (1992), the study of international real business cycle models has been growing fast in the last several years and has become one of the most active research areas in the business cycle literature.

Research in international real business cycles focuses on the impact of international interdependence on business cycle frequencies. International and open economy perspectives have extended the analysis of behaviour of domestic variables and lead us to study comovement of macroeconomic variables across countries and behaviours of variables, including exports, imports, the trade balance, the terms of trade and real exchange rates.

The main purpose of an international business cycle and multisectoral business cycle model is to explain simultaneously the domestic comovements as well as sectoral comovements. More specifically, it aims to investigate the cross-country correlations of consumption and output, the correlation of imports with output, and the correlation between output and employment, etc.

Backus, Kehoe and Kyland (1992, 1995) ask whether a two-country real business cycle model can account simultaneously for domestic and international aspects of business cycles. Their results indicate that although their models can explain many stylised facts, the quantity anomaly and the price anomaly are quite robust to the choice of parameter values and even to the perturbations of the model structure. In their papers, with free trade and complete asset markets, the representative consumer can smooth consumption by varying labor effort, capital accumulation and the allocation of capital across countries. This tendency leads consumption to be positively correlated across countries.

International real business cycle models are not able to account for the high volatility of exports, imports, the trade balance, the terms of trade, and real exchange rates. To solve this problem, Zimmermann (1999) introduces exogenous exchange rate movements in addition to standard technological shocks, and his model comes much closer to replicating the relatively high volatility observed in the data, but it cannot solve the quantity anomaly.

To mimic the volatile real exchange rate, Mendoza (1991a, 1991b and 1995) introduces exogenous world interest rate shock or/and the terms of trade shock into his small open economy models. However, these shocks cannot be incorporated into two-country international business cycle models in which interest rates and the terms of trade are determined endogenously.

To replicate the salient (international anomalies) features of the data, some researchers introduce other sources of shocks, like oil shocks (Costello and Praschnick (1993) and taste shocks (Stockman and Tesar (1995)). But both of these shocks turn out to have little effect on cross-country correlations and variability of trade-related variables. Devereux, Gregory and Smith (1992) develop a model with a particular type of non-separability between consumption and leisure. They succeed in lowering the cross-country correlation of consumption, by modifying the specification of agents' preference. Canova and Ubide (1997) develop a two-country model with home production. Their results indicate that an international business cycle model with household production can better match cross-country output, consumption and labor correlations.

A few papers build international business cycle models with intermediate inputs (see Costello and Praschnick (1993), Head (1997) and Kouparitsas (1995)). Costello and Praschnick (1993) develop a two-country model which predicts a higher cross correlation of output than Backus, Kehoe and Kyland (1995). Head (1997) builds a two-country model with differentiated intermediate goods and shows that increasing returns to the variety of intermediate goods can lead to a positive international transmission of the business cycle. Kouparitsas (1995) introduces production interdependence via intermediate inputs. In his two-country model, primary products are used as an intermediate input in the production of manufactured goods, and the North imports primary products from the South in exchange for exports of manufactured goods. His results indicate that North-South business cycles emerge because shocks originating from the North are transmitted to the South through trade in goods and assets, and that it can capture the high volatility of the terms of trade and strong positive cross-region comovement.

Recently, Ambler, Cardia and Zimmermann (1998) built a two-country, multi-sector model to examine the role of sectoral shocks and intermediate goods trade in the international propagation of the business cycle. There are two main motivations for this further development of the international business cycle model. First, Whalley (1995) reports that the share of trade in intermediate goods is larger than the share of trade in final goods. For example, trade in intermediate goods represents approximately 60% of total trade. Given its quantitative importance in the data, Ambler, Cardia and Zimmermann (forthcoming) argue that introducing trade in intermediate goods in multi-country models could significantly affect their ability to explain the main features of the intertemporal transmission of the business cycle. Second, according to Long and Plosser (1983), sectoral shocks in a dynamic closed economy model can lead to positive spillover effects in other sectors via changes in demand for intermediate inputs produced by those sectors. In other words, a positive intersectoral transmission of productivity shocks in a closed economy model could lead to a positive international transmission of shocks in a multi-country model (Ambler, Cardia and Zimmermann (forthcoming)).

In this chapter, we build a small open economy, multi-sector model to examine the effects of sectoral shocks and terms of trade shocks in the international transmission mechanism of the business cycle.

Our model has at least *three* salient features. First, in contrast to the model developed by Ambler, Cardia and Zimmermann (forthcoming), which is a two-country international real business cycle model with two sectors that can be solved by the linear quadratic approximation method, our model is a one-country model with two sectors. A two-country model is based on the relation either between large countries or groups of countries, whereas our model is based on a small open economy, thereby treating the shock from overseas as an exogenous variable.

Second, our model has incorporated import goods into it, which can be used for production as input factors. Import goods as input factors imply that the terms of trade shock affects macroeconomic variables through sectoral goods production and marginal productivity of sectoral goods production. For example, terms of trade shock decreases the relative price of import goods, and thus increases imports. The increased imports raise the marginal productivities of capital, labour supply and intermediates, and then increase sectoral productions, and thus GDP, consumption and investment. This procedure shows a different mechanism from the substitution of consumption and investment to imports in a one-sector model.

Third, we incorporate two sectoral productivity shocks in our model. Two sectoral shocks affect macroeconomic variables through the changes of sectoral productivities. The effects of two sectoral productivity shocks are different in magnitude and directions. By incorporating two sectoral productivity shocks, we analyse which sector of manufacturing and non-manufacturing sector impacts more strongly on economic fluctuations.

This chapter is organized as follows: Section 3.2 provides a full description of the dynamic stochastic model of a small open economy. Section 3.3 shows the conditions for optimal allocation of the dynamic programming problem. Section 3.4 provides the key findings and sensitivity analysis, and impulse response analysis. Finally, concluding remarks are presented in Section 3.5.

3.2 An International Real Business Cycle Model with Two Sectors

A dynamic stochastic model of a small open economy is developed to investigate the shock effects of manufacturing and non-manufacturing sectors and terms of trade. The structure of the model is a streamlined version of the model of Ambler, Cardia and Zimmermann (forthcoming) and Mendoza (1991a).

3.2.1 Preferences

Agents of infinite life and with same identical preferences allocate C_t (private consumption) and N_t (labour supply) intertemporarily to maximise the utility.

$$V = E\left[\sum_{t=0}^{\infty} \beta^t U\{C_t, N_t\}\right] \tag{3.1}$$

The instantaneous utility function and time-preference functions are as follows:

$$U(\cdot) = \frac{\left[C_t - \frac{N_t^{1+\theta}}{1+\theta}\right]^{1-\gamma} - 1}{1-\gamma} \tag{3.2}$$

$$U(\cdot) < 0,\ \ U'(\cdot) > 0,\ U'(0) = \infty,\ 1+\theta > 1,\ \gamma > 1$$

The parameter β is the rate of time preference. The parameter θ is the elasticity of substitution between consumption and labour supply. The coefficient γ indicates relative risk aversion. The intertemporal discount rate equalises to the world's real interest rate according to the accumulation of foreign financial assets.

3.2.2 Sectoral Production and Technology

The sectoral productions are given by Cobb-Douglas production function using capital, labour, intermediate goods[5] and import goods.

$$X_{1t} = \varepsilon_{1t} K_{1t}^{\upsilon_1} N_{1t}^{\upsilon_2} MX_{1t}^{\upsilon_3} IM_{1t}^{\upsilon_4} \tag{3.4}$$

$$X_{2t} = \varepsilon_{2t} K_{2t}^{\psi_1} N_{2t}^{\psi_2} MX_{2t}^{\psi_3} IM_{2t}^{\psi_4} \tag{3.5}$$

[5] The incorporation of intermediate goods into our model is to consider the transmission effects of sectoral shocks through intermediate goods. The incorporation causes a circular definition as identified by Ambler, Cardia and Zimmermann (forthcoming), Although it is a circular definition, it does offer an explanation of the transmission effects of sectoral shocks through intermediate goods

where $\sum \upsilon_i = 1$, $\sum \psi_i = 1$ and X_{1t} and X_{2t} are sectoral outputs, respectively. K_{it} and N_{it} are the capital stock and labour supply used in sector i, respectively. MX_i indicates the intermediate goods which are used to produce sectoral good i. υ_i and ψ_i are referred to as the shares of input factors for production in two sectors. Sectoral productions are dependent on the sectoral technology shocks ε_{1t} and ε_{2t} as well as input factors. The sectoral technology shocks, ε_{1t} and ε_{2t}, follow the first-order Markov distribution function.

3.2.3 Intermediate Goods Production

Intermediate goods producers aggregate sectoral good to produce intermediate goods. The intermediate goods are used to produce sectoral goods.

$$MX_t = \left[\omega F_{1t}^{1-\rho} + (1-\omega) F_{2t}^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{3.6}$$

where F_{1t} and F_{2t} are amounts of X_{1t} and X_{2t} to produce intermediate goods MX_t, respectively. ω implies the contribution degree of sector 1 goods in intermediate good production. ρ indicates the substitution elasticity between sectoral goods. The produced intermediate goods are distributed in $MX_t = \sum_{i=1}^{2} MX_{it}$ and used for the production of sectoral goods.

3.2.4 Final Good Production

Final goods producers aggregate sectoral goods to produce final goods, which are used for consumption, investment and exports.

$$Y_t = \left[\pi Q_{1t}^{1-\lambda} + (1-\pi) Q_{2t}^{1-\lambda}\right]^{\frac{1}{1-\lambda}} \tag{3.7}$$

where Q_{it} is the amount of X_{it} used to produce final good Y_t. The final goods aggregator (λ) shows the substitution elasticity between sectoral goods. π represents the share of sector 1 goods in the final goods production. The final goods producer allocates the sectoral goods for profit maximization

3.2.5 Capital Accumulation, Trade Balance and Financial Structure

The law of motion for the capital stock is as follows:

$$K_{it+1} = K_{it}(1-\delta_i) + I_{it} \qquad i = 1, 2 \tag{3.8}$$

where I_{it} denotes sectoral investment, and δ_i is a constant sectoral rate of depreciation. It assumes that capital stock does not move between sectors.

The financial structure has linked with trade balance. Financial structure assumes that an agent in the economy is a small participant and this representative agent has access to world capital markets to borrow and lend foreign financial assets (A_t). Holdings of foreign financial assets (A_{t+1}) evolve according to:

$$A_{t+1} - (1+r^*)A_t = \varepsilon_{3t} p_d EX_t - IM_t \quad (3.9)$$

where EX_t is exports, IM_t is imports, ε_{3t} is terms of trade shock, and r^* is the exogenously determined world's real interest rate. p_d is the relative price of domestic goods to import goods and is assumed as one in the steady state. To rule out the possibility of the economy playing a Ponzi game in the intertemporal capital market (these paths allow the representative agent to borrow initially an arbitrarily large amount and then to finance interest payments with further borrowing, thereby never repaying the initially debt), we need the following transversality condition:

$$\lim_{T\to\infty} E_0 \frac{A_T}{(1+r^*)^T} = 0$$

3.2.6 Resource Constraint

Finally, the model completes with a resource constraint[6] as follows:

[6] To simplify our model, we divide the prices into both domestic good price and import good price.

$$\varepsilon_{3t} p_d C_t + K_{1t+1} - (1-\delta_1)K_{1t} + K_{2t+1} - (1-\delta_2)K_{2t} + IM_t + A_{t+1} - (1+r^*)A_t$$
$$\leq \varepsilon_{3t} p_d Y_t - \frac{\phi_1(K_{1t+1} - K_{1t})}{K_{1t}} - \frac{\phi_2(K_{2t+1} - K_{2t})}{K_{2t}} \quad (3.10)$$

where ϕ_1 and ϕ_2 are parameters to restrict the adjustment costs. The relative price p_d is assumed as one. The aggregate resource constraint[7] of the economy indicates that the sum of consumption, investment, and exports can not be exceeded by final good production.

3.3 Dynamic Programming Problem, Solution Technique and Parameter Calibrations

The social planner selects paths of consumption, labour supply to maximise the whole life-time utility. To solve the dynamic programming problem, the household's optimal intertemporal decisions are to choose the control variables $(C_t, K_{1t+1}, K_{2t+1}, A_{t+1}, N_{1t}, N_{2t}, MX_{1t}, MX_{2t}, IM_{1t}, IM_{2t}, Q_{1t}, Q_{2t}, F_{1t}, F_{2t})$ in period t, given the state of the economy as described by $(K_{1t}, K_{2t}, A_t, \varepsilon_{1t}, \varepsilon_{2t} \text{ and } \varepsilon_{3t})$. To solve the dynamic programming problem, the Lagrangian problem is built as follows:

[7] It should be noted that Y_t is not gross domestic product but $\varepsilon_t Y_t - IM_t$ is gross domestic product. In other words, this implies that import goods are used for not consumption and investment, but intermediate good production.

$$
\begin{aligned}
L = E_t\beta^t \Bigg[& \frac{\left[C_t - \frac{N_t^{1+\theta}}{1+\theta}\right]^{1-\gamma} - 1}{1-\gamma} + \mu_{1t}\big(\varepsilon_{3t}p_d Y_t - \varepsilon_{3t}p_d C_t - K_{1t+1} + (1-\delta_1)K_{1t} \\
& - K_{2t+1} + (1-\delta_2)K_{2t} - IM_t - A_{t+1} + (1+r^*)A_t \\
& - \frac{\phi_1(K_{1t+1} - K_{1t})^2}{K_{1t}} - \frac{\phi_2(K_{2t+1} - K_{2t})^2}{K_{2t}}\Big) + \mu_{2t}(X_{1t} - Q_{1t} - F_{1t}) \\
& + \mu_{3t}(X_{2t} - Q_{2t} - F_{2t}) + \mu_{4t}\left(\left[\omega F_{1t}^{1-\rho} + (1-\omega)F_{2t}^{1-\rho}\right]^{\frac{1}{1-\rho}} - MX_{1t} - MX_{2t}\right)\Bigg]
\end{aligned}
\tag{3.11}
$$

The first-order conditions for utility maximisation are

$$
(C_t):\ \left[C_t - \frac{N_t^{1+\theta}}{1+\theta}\right]^{-\gamma} = \mu_{1t}\varepsilon_{3t}p_d \tag{3.12}
$$

$$
\begin{aligned}
(K_{1t+1}):\ \mu_{1t}\left(1 + \frac{\phi_1(K_{1t+1} - K_{1t})}{K_{1t}}\right) = {} & \beta\mu_{2t+1}\left(\upsilon_1 \frac{X_{1t+1}}{K_{1t+1}}\right) \\
& + \beta\mu_{1t+1}\left((1-\delta_1) + \frac{2\phi_1(K_{1t+2} - K_{1t+1})}{K_{1t+1}} + \frac{\phi_1(K_{1t+2} - K_{1t+1})^2}{K_{1t+1}}\right)
\end{aligned}
\tag{3.13}
$$

$$
\begin{aligned}
(K_{2t+1}):\ \mu_{1t}\left(1 + \frac{\phi_2(K_{2t+1} - K_{2t})}{K_{2t}}\right) = {} & \beta\mu_{3t+1}\left(\psi_1 \frac{X_{2t+1}}{K_{2t+1}}\right) \\
& + \beta\mu_{1t+1}\left((1-\delta_2) + \frac{2\phi_2(K_{2t+2} - K_{2t+1})}{K_{2t+1}} + \frac{\phi_2(K_{2t+2} - K_{2t+1})^2}{K_{2t+1}}\right)
\end{aligned}
\tag{3.14}
$$

$$
(N_{1t}):\ \mu_{1t}\varepsilon_{3t}p_d N_t^{\theta} = \mu_{2t}\left(\upsilon_2 \frac{X_{1t}}{N_{1t}}\right) \tag{3.15}
$$

$$
(N_{2t}):\ \mu_{1t}\varepsilon_{3t}p_d N_t^{\theta} = \mu_{3t}\left(\psi_2 \frac{X_{2t}}{N_{2t}}\right) \tag{3.16}
$$

$$
(MX_{1t}):\ \mu_{2t}\upsilon_3 \frac{X_{1t}}{MX_{1t}} = \mu_{4t} \tag{3.17}
$$

$$
(MX_{2t}):\ \mu_{3t}\psi_3 \frac{X_{2t}}{MX_{2t}} = \mu_{4t} \tag{3.18}
$$

$$
(IM_{1t}):\ \mu_{2t}\upsilon_4 \frac{X_{1t}}{IM_{1t}} = \mu_{1t} \tag{3.19}
$$

$$
(IM_{2t}):\ \mu_{3t}\psi_4 \frac{X_{2t}}{IM_{2t}} = \mu_{1t} \tag{3.20}
$$

$$
(Q_{1t}):\ \mu_{1t}\varepsilon_{3t}p_d\left[\pi Q_{1t}^{1-\lambda} + (1-\pi)Q_{2t}^{1-\lambda}\right]^{\frac{\lambda}{1-\lambda}} \pi Q_{1t}^{-\lambda} = \mu_{2t} \tag{3.21}
$$

$$
(Q_{2t}):\ \mu_{1t}\varepsilon_{3t}p_d\left[\pi Q_{1t}^{1-\lambda} + (1-\pi)Q_{2t}^{1-\lambda}\right]^{\frac{\lambda}{1-\lambda}} (1-\pi) Q_{2t}^{-\lambda} = \mu_{3t} \tag{3.22}
$$

$$
(F_{1t}):\ \mu_{4t}\left[\omega F_{1t}^{1-\rho} + (1-\omega)F_{2t}^{1-\rho}\right]^{\frac{\rho}{1-\rho}} \omega F_{1t}^{-\rho} = \mu_{2t} \tag{3.23}
$$

$$
(F_{2t}):\ \mu_{4t}\left[\omega F_{1t}^{1-\rho} + (1-\omega)F_{2t}^{1-\rho}\right]^{\frac{\rho}{1-\rho}} (1-\omega) F_{2t}^{-\rho} = \mu_{3t} \tag{3.24}
$$

$$
(A_{t+1}):\ \mu_{1t} = \mu_{1t+1} \tag{3.25}
$$

$$
\begin{aligned}
(\mu_{1t}):\ \varepsilon_{3t}p_d\left[\pi_1 Q_{1t}^{1-\lambda} + \pi_2 Q_{2t}^{1-\lambda}\right]^{\frac{1}{1-\lambda}} = {} & \varepsilon_{3t}p_d C_t + K_{1t+1} - (1-\delta_1)K_{1t} \\
& + K_{2t+1} - (1-\delta_2)K_{2t} + IM_t + A_{t+1} - (1+r^*)A_t \\
& + \frac{\phi_1(K_{1t+1} - K_{1t})^2}{K_{1t}} + \frac{\phi_2(K_{2t+1} - K_{2t})^2}{K_{2t}}
\end{aligned}
\tag{3.26}
$$

$$
(\mu_{2t}):\ X_{1t} = Q_{1t} + F_{1t} \tag{3.27}
$$

$$(\mu_{3t}): X_{2t} = Q_{2t} + F_{2t} \tag{3.28}$$

$$(\mu_{4t}): \left[\omega F_{1t}^{1-\rho} + (1-\omega) F_{2t}^{1-\rho}\right]^{\frac{1}{1-\rho}} = MX_{1t} + MX_{2t} \tag{3.29}$$

To solve the dynamic program problem by using the undetermined coefficient method (Christiano, 1998). We should linearise the first order conditions around the steady state values. Then, all control variables and costate variables can be expressed as linear functions of current capital stocks, foreign financial assets and shocks $(K_{1t}, K_{2t}, A_t, \varepsilon_{1t}, \varepsilon_{2t} \text{and } \varepsilon_{3t})$. We can simulate the model by generating the random shocks.

3.3.1 Calibration

We use economic theory extensively as the basis for restricting the general framework for finding numerical values for parameters. The relevant parameters of preference are principally taken from Mendoza (1991a) and Ambler, Cardia and Zimmermann (forthcoming). The world real interest rate is set at 4%, which is obtained from Mendoza (1991a), and thus intertemporal discount rate (β) is calculated as 0.96 (1/(1+r*)). The parameter of the intertemporal elasticity of substitution in labour supply is set at 0.455 from Mendoza (1991a). The coefficient of relative risk aversion (γ) is set to 1.8.

The relevant parameters of sectoral productions are calculated from national income data and input-output tables. By using average annual values of input-output tables, capital

Table 3.1: Benchmark parameter values

Parameter	Definition	Value
β	The rate of time preference	0.96
γ	Coefficient of relative risk aversion	1.8
θ	Inverse of the intertemporal elasticity of substitution in labour supply	0.455
υ_1	Capital shares in manufacturing production	0.11
ψ_1	Capital shares in non-manufacturing production	0.29
υ_2	Labour shares in manufacturing production	0.21
ψ_2	Labour shares in non-manufacturing production	0.33
υ_3	Intermediate goods shares in manufacturing production	0.56
ψ_3	Intermediate goods shares in non-manufacturing production	0.34
υ_4	Import goods shares in manufacturing production	0.12
ψ_4	Import goods shares in non-manufacturing production	0.04
$1/\rho$	Elasticity of substitution in intermediate good production	0.9
ω	Manufactured good shares in intermediate good production	0.43
$1/\lambda$	Elasticity of substitution in final good production	0.9
π	Manufactured good share in final good production	0.26
δ_1	Depreciation rate in capital stock of manufacturing sector	0.12
δ_2	Depreciation rate in capital stock of non-manufacturing sector	0.08
ϕ_1	Adjustment costs in manufacturing sector	1.0
ϕ_2	Adjustment costs in non-manufacturing sector	1.0
r^*	World interest rate	0.04

shares (υ_1 and ψ_1) are set at 0.11 and 0.29, and labour shares (υ_2 and ψ_2) are calculated as 0.21 and 0.33 in manufacturing and non-manufacturing productions. Intermediate goods shares (υ_3 and ψ_3) in manufacturing and non-manufacturing productions are set at 0.56 and 0.34, respectively. Import goods shares (υ_4 and ψ_4) are 0.12 and 0.04, respectively. The elasticities of substitution ($1/\rho$) between sectoral goods in intermediate good production are set as 0.9, and the elasticity of substitution ($1/\lambda$) between sectoral goods in final goods production is also 0.9 obtained from Ambler, Cardia and Zimmermann (forthcoming).

The parameters for intermediate goods production are computed from input-output tables. Sectoral goods shares in intermediate goods production (ω and $(1-\omega)$) are calculated as average annual values of 0.43 and 0.57. Sectoral goods shares also in final goods production (π and $(1-\pi)$) are set as 0.26 and 0.74 from input-output tables. Depreciation rates (δ_1 and δ_2) are set at 0.12 and 0.08[8], respectively. Adjustment cost coefficients (ϕ_1 and ϕ_2) are set at the same value of 1.0.

3.3.2 Shock Process

Parameters of shock process are estimated using Australian sectoral GDP of manufacturing and non-manufacturing sectors, and the terms of trade[9] data, which is defined by the implicit price index of export goods divided by the implicit price index of

[8] These values are obtained from the national account data by using the average ratios of sectoral investments to sectoral capital stocks.

[9] The terms of trade id defined by implicit price index of goods export divided by implicit price index of goods imports.

import goods. It is assumed that the shock process follows the first-order Markov distribution function and is estimated by VAR (vector autoregression). The estimation result is as follows:

$$\begin{bmatrix} \varepsilon_{1t+1} \\ \varepsilon_{2t+1} \\ \varepsilon_{3t+1} \end{bmatrix} = \begin{bmatrix} 0.43 & 0.70 & -0.14 \\ -0.09 & 0.88 & 0.02 \\ 0.48 & 0.53 & 0.51 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} + \begin{bmatrix} w_{1t+1} \\ w_{2t+1} \\ w_{3t+1} \end{bmatrix}$$

The variance-covariance matrix of the innovation is:

$$\begin{bmatrix} 0.00074 & 0.00039 & 0.00064 \\ 0.00039 & 0.00028 & 0.00026 \\ 0.00064 & 0.00026 & 0.00266 \end{bmatrix}$$

and percentage standard deviations are 2.7, 1.7 and 5.2 respectively. The diagonal terms show the variance of the shocks, and the terms below the diagonal terms represent the covariance values.

3.4 Empirical Findings

3.4.1 Business Cycle Properties and Sensitivity Analysis

Table 3.2 represents the major features of the real economy and the benchmark economy. The standard deviations of the real data from Table 3.2 provide that investment, 6.71, is more volatile than output, 5.0, and consumption, 4.46. The standard deviation of trade balance shows the lowest value at 1.0. The highest standard deviation is shown from foreign financial assets at 15.26. In addition, the standard deviation of imports and investment are fairly high at 8.39 and 6.71. The standard deviations of the benchmark economy are similar to real data in consumption (4.46, 3.76), investment (6.71, 6.83) and trade balance (1.00, 1.59). In addition, labour supply shows low values (1.07, 2.74) even though it is overestimated. The standard deviations of foreign sectors such as imports, interest payments and terms of trade correlate well with the real data in size order even though they are underestimated. In the case of first order serial correlation, the benchmark economy fairly well mimics the real data. For instance, the autocorrelations of consumption are similar at 0.51 and 0.39 in the real data and the benchmark economy, respectively. Moreover, the autocorrelations of investment (0.44, 0.45), labour supply (0.55, 0.49), trade balance (0.14, 0.16) and terms of trade (0.43, 0.40) prove that the benchmark economy fairly well replicates the real economy. The correlations of macroeconomic variables with output mimic well the benchmark economy in consumption (0.95, 0.96), investment (0.76, 0.89) and terms of trade (0.76, 0.70). In general, the benchmark economy replicates the distinguished features of real economy. Especially, our model shows a better feature in terms of explaining the persistence of fluctuation than other real business cycle models.

Table 3.2: Statistical moments: Australian data and the benchmark model at constant import prices

	Australian data 1965-1999			The benchmark model		
	σ_x	$\rho_{xt,xt-1}$	$\rho_{xt,GDPt}$	σ_x	$\rho_{xt,xt-1}$	$\rho_{xt,GDPt}$
GDP	5.00 (0.929)	0.45 (0.139)	1.0	5.00 (0.596)	0.45 (0.212)	1.0
Consumption	4.46 (0.912)	0.51 (0.138)	0.95 (0.022)	3.76 (0.455)	0.39 (0.218)	0.96 (0.014)
Investment	6.71 (1.087)	0.44 (0.161)	0.76 (0.076)	6.83 (0.809)	0.45 (0.216)	0.89 (0.039)
Labour supply	1.07 (0.178)	0.55 (0.097)	0.46 (0.198)	2.74 (0.322)	0.49 (0.208)	0.89 (0.034)
Import	8.39 (1.872)	-0.04 (0.103)	0.47 (0.169)	5.70 (0.680)	0.42 (0.216)	0.99 (0.005)
Interest payments	15.26 (2.566)	0.40 (0.138)	0.44 (0.169)	6.41 (0.731)	0.84 (0.151)	0.03 (0.163)
Trade balance (TB/GDP)	1.00 (0.172)	0.14 (0.171)	0.08 (0.133)	1.59 (0.188)	0.16 (0.236)	0.74 (0.075)
Terms of trade (TOT)	5.43 (0.897)	0.43 (0.175)	0.76 (0.147)	3.01 (0.359)	0.40 (0.219)	0.70 (0.085)

Notes: The nominal Australian data are obtained from ABS, divided by the 15+ population, deflated by import prices (see, Mendoza, 1995 and Senhadji, 1998), logged and detrended by a Hodrick-Prescott filter with the smoothing parameter set at 100 (see, Mendoza, 1991b and Senhadji, 1998). σ_x is the percentage standard deviation, $\rho_{xt,xt-1}$ is the first-order autocorrelation , and $\rho_{xt,GDPt}$ is the contemporaneous correlation with GDP. The values in the parenthesis represent the standard deviations estimated by GMM (Generalized Method of Moment) using the Hansen-Heaton-Ogaki gauss program. The model statistics are based on the sample average values of 100 simulations of 36 years. We generated histories of 169 observations and truncated the first 133 observations so that the results do not depend on initial condition of the state variables of the model (Ambler, Cardia and Zimmermann, forthcoming). In the case of the model with the terms of trade shocks, all variables should be deflated by a numeraire. In a small open economy model, the domestic goods prices can not be used as numeraire. All values of Table 2.2 are computed from real Australian data, while all values of Table 3.2 are calculated from the nominal Australian data deflated by the price of import goods. Accordingly, the values of the two tables can not be same. As can be seen from Mendoza (1995), when the nominal data deflated by the price of import goods are used, the volatility of GDP is much higher than real data.

3.4.2 Sensitivity Analysis

Table 3.3 presents the findings of simulation that raise every parameter by 5%. In general, the macroeconomic variables of the benchmark model are stable to the change of parameters. Autocorrelations are relatively more sensitive to the change of parameters than standard deviation and correlations with output.

Investment (from 6.49 to 6.97) is most sensitive to the change of parameter in standard deviation. The intertemporal elasticity of substitution in labour supply affects relatively strongly investment. The adjustment cost of non-manufacturing sector has a strong effect on standard deviation of trade balance. Trade balance is most sensitive to the change of parameters in autocorrelation. Trade balance is also most affected by the change of the adjustment cost of the non-manufacturing sector. Consumption, investment and imports are very stable to the change of parameters in autocorrelation. However, consumption and investment are affected relatively strongly by the change of the intertemporal elasticity of substitution in labour supply, which indicates that labour supply has a role in connecting sectors, thereby allowing spillover effects across sectors. The parameter (ρ), which causes a spillover effect, does not have a significant effect on the major macroeconomic variables.

Table 3.3: Sensitivity Analysis

	Volatility				First-order serial correlation				Correlation with output			
	C	I	M	TB/GDP	C	I	M	TB/GDP	C	I	M	TB/GDP
Baseline Model	3.76	6.83	5.70	1.59	0.39	0.45	0.42	0.16	0.96	0.89	0.99	0.74
γ	3.81	6.97	5.71	1.55	0.39	0.46	0.43	0.14	0.96	0.90	0.99	0.72
θ	3.59	6.51	5.74	1.58	0.45	0.52	0.43	0.17	0.93	0.88	0.93	0.73
R^*	3.75	6.87	5.69	1.57	0.39	0.45	0.42	0.27	0.95	0.89	0.99	0.80
δ_1	3.77	6.78	5.71	1.63	0.39	0.45	0.43	-0.04	0.96	0.89	0.99	0.66
δ_2	3.77	6.71	5.69	1.58	0.39	0.44	0.42	0.19	0.95	0.89	0.99	0.76
ϕ_1	3.75	6.80	5.69	1.55	0.39	0.46	0.43	0.31	0.96	0.89	0.99	0.82
ϕ_2	3.74	6.49	5.68	1.69	0.39	0.45	0.43	-0.15	0.96	0.89	0.99	0.63
ω_1	3.78	6.77	5.66	1.58	0.38	0.45	0.42	0.20	0.95	0.89	0.99	0.77
π_1	3.81	6.80	5.73	1.59	0.38	0.44	0.42	0.19	0.95	0.89	0.98	0.76
ρ	3.76	6.80	5.70	1.59	0.39	0.45	0.42	0.09	0.96	0.89	0.99	0.72
λ	3.71	6.69	5.69	1.58	0.39	0.46	0.42	0.04	0.96	0.90	0.98	0.70

Note: Baseline model with each of the following parameters increased by 5%

3.4.3 Impulse Response Analysis

This section presents the impulse response of economic variables to the shocks of sectoral productions and terms of trade shock. Impulse response analysis is useful to assess the dynamic characterisation of an economic system and to measure the magnitude of the effects of economic variables to the shock.

Figure 3.1 shows the impulse response of each variable to 1% positive shocks of sectoral productivities and the terms of trade. To compare the size of shock effects, Figure 3.1 shows the effect of each variable to three different shocks; namely, the productivity shocks of both manufacturing and non-manufacturing productions, and terms of trade shock.

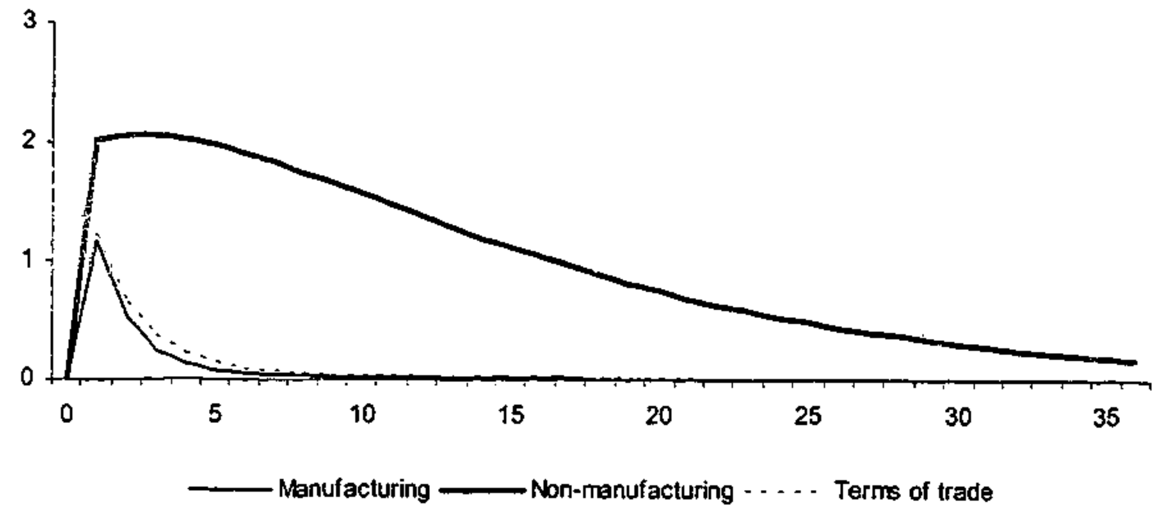
The effect of the productivity shock of the non-manufacturing sector on GDP is stronger than the effects of the productivity shock of manufacturing production and the terms of trade shock[10]. The effects of the productivity shock of manufacturing production and terms of trade shock on GDP are similar in magnitude. For the 1% positive shocks, GDP increases, and then converges to an equilibrium path. The effect of productivity shock of non-manufacturing production on consumption is greater than the effects of the

[10] Among literatures on impulse response function, which are reported using actual Australian data, Dungey and Pagan (2000) show that the terms of trade shocks have a negligible impact on output. Their findings are that the 2.4% standard deviation shock of the terms of trade have positive effects on Australian GDP by maximum 0.5%. While, in the impulse-response function for the model-generated artificial time series, the 1% shock of the terms of trade shock also have positive effects on Australian GDP by maximum 1%. In impulse-response analysis, the effect of the terms of trade on GDP is overestimated compared to real economy.

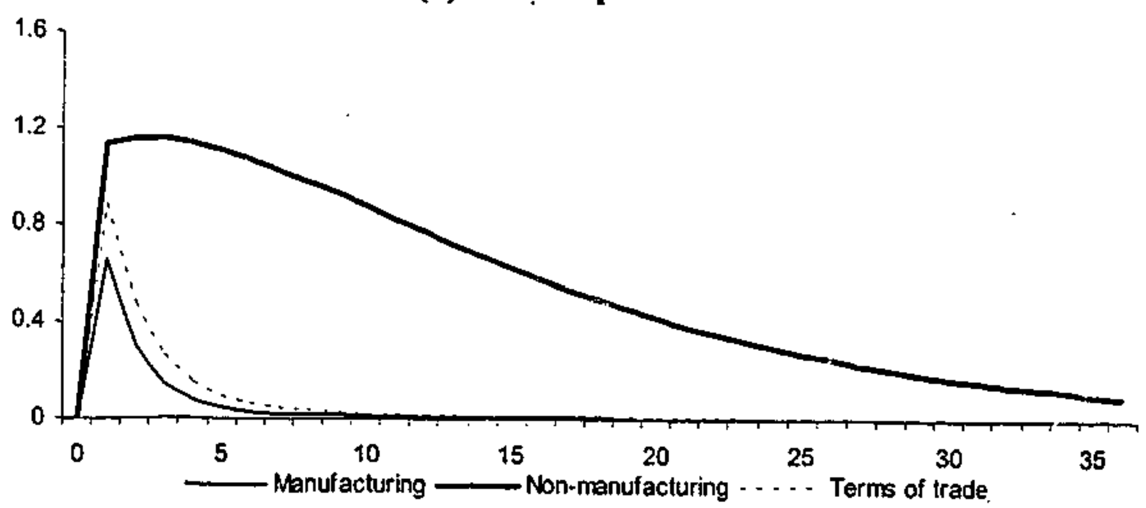
productivity shock of manufacturing production and the terms of trade shock, and is similar to output. Interestingly, terms of trade shock has a stronger effect on consumption than the productivity shock of manufacturing production. The effects of the shocks on investment and imports are in the order of the productivity shock of non-manufacturing production, terms of trade shock and the productivity shock of manufacturing production, and are different from the effect on consumption. A distinguishing feature is that the effect of the productivity shock of non-manufacturing production on investment is relatively higher than on GDP, consumption and imports, in comparison with the effects of the productivity shock of manufacturing production and terms of trade shock. A positive productivity shock in manufacturing production and a positive terms of trade shock increase trade balance. On the other hand, a positive productivity shock in non-manufacturing production decreases trade balance, indicating that the shock increases imports more than exports. A positive productivity shock in manufacturing production has a stronger effect on trade balance than terms of trade shock, dissimilarly to other macroeconomic variables. In other words, the increase of productivity in manufacturing causes exports to rise, whereas the increase of productivity in non-manufacturing causes imports to increase. In general, the productivity shock of non-manufacturing production has stronger effects on macroeconomic variables than the productivity shock of manufacturing production and terms of trade. Moreover, it is an interesting feature that terms of trade shock has a stronger effect than the productivity of manufacturing production on economic variables.

Figure 3.1: Impulse response to sectoral productivity and terms of trade shocks

(a) Output



(b) Consumption



(c) Investment

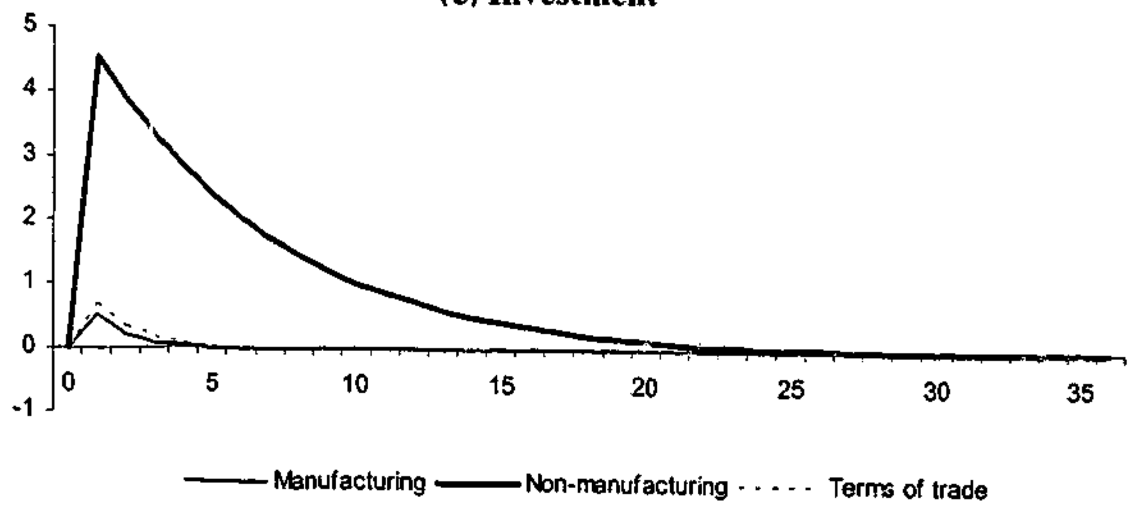
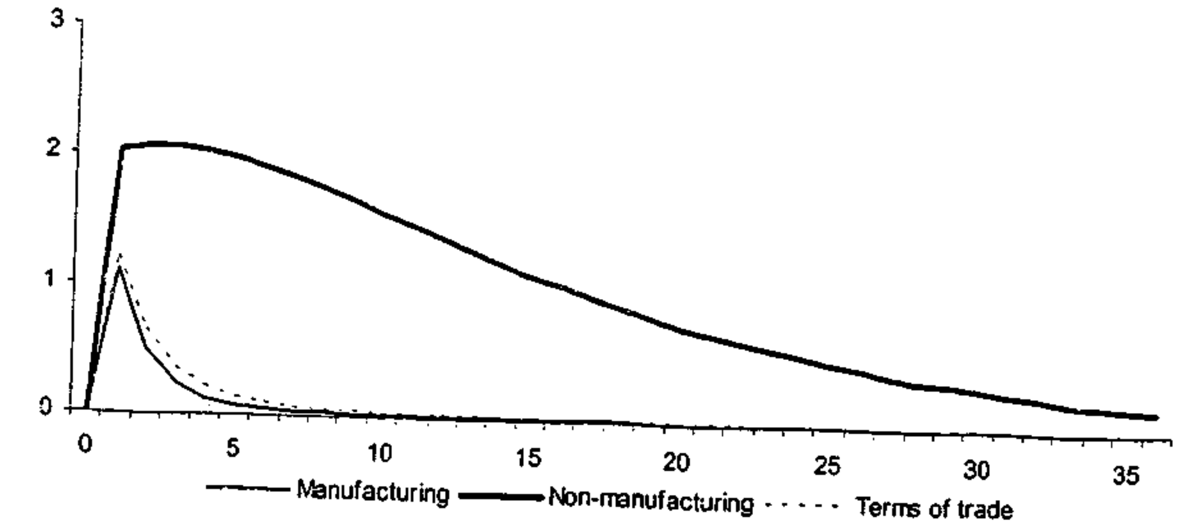
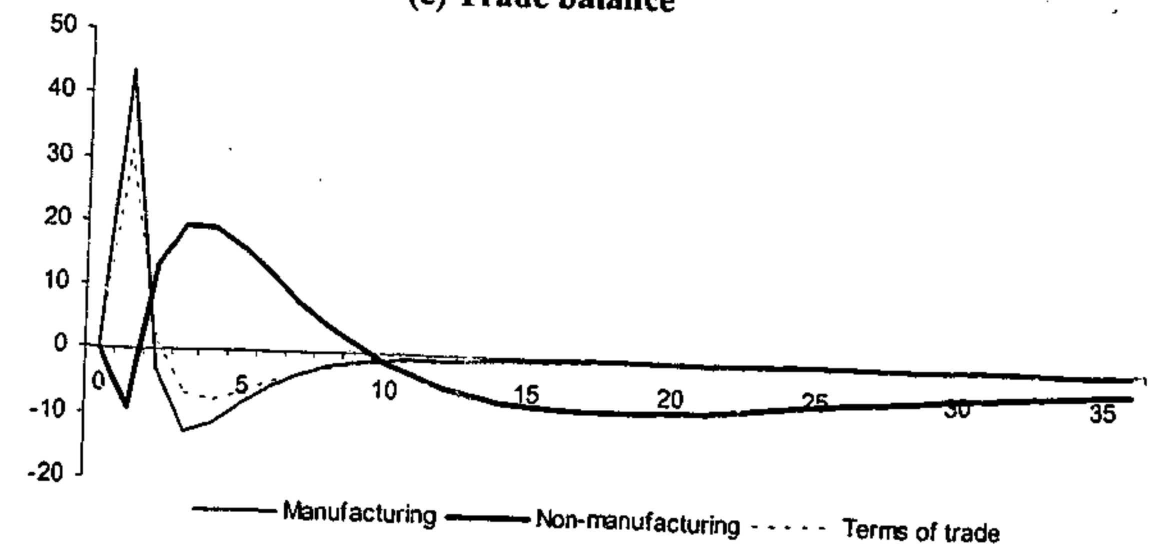


Figure 3.1 (cont'd)

(d) Imports



(e) Trade balance



3.5 Concluding Remarks

This chapter analyses an international real business business cycle model of a small open economy. Under the assumption that the model of the economy produces two goods, and has two sectoral productivity shocks and terms of trade shock, We especially investigate the central issue of how the economic variables fluctuate in response to the shocks.

We conducted the quantitative analysis on the effects of sectoral productivity shocks and the terms of trade shock using the Australian data. The model economy fairly well replicates the real economy in properties of second moments. The finding of sensitivity analysis supports that the model is stable except for trade balance. The autocorrelation of trade balance is relatively sensitive to the change of adjustment cost in non-manufacturing production, which indicates the strong connection between the non-manufacturing sector and the foreign sector. The change of elasticity of substitution in intermediate goods production, which shows spillover effect between sectors, has a small effect on manufacturing. This implies that sectoral transmission effects are very stable. From impulse response analysis, the productivity of non-manufacturing production has a dominant role in the business cycle of Australian economy. The effect is stronger than the productivity shock of manufacturing production and terms of trade shock. Moreover, the strong effect increases imports more than exports, and thus declines trade balance temporally, while the productivity shock of manufacturing production increases exports more than imports.

Appendix 3.1: Data Source

The Australian data are obtained from ABS (Australian Bureau Statistics). All series are composed of yearly observations from 1965 to 1999. All variables except for labour supply are divided by import price. *Output*: nominal GDP. *Consumption:* nominal private final consumption expenditure. *Investment:* nominal gross capital formation. *Labour supply:* the number of non-farm civilian wage and salary earners × index of hours worked. *Population:* civilian population aged 15 years and over. *Interest payments*: (nominal investment income to overseas − nominal investment income from overseas). *Import:* nominal imports of goods. *Trade balance*: (nominal exports of goods −nominal imports of goods)÷nominal GDP. *Terms of trade*: implicit price index of goods export ÷ implicit price index of goods import

Chapter 4 Foreign Capital Goods, the Terms of Trade and International Real Business Cycles

4.1 Introduction

Since the pioneering work initiated by Backus, Kehoe and Kydland (1992), many studies of international real business cycles highlight the transmission of international business cycles (see, Baxter and Crucini (1995) and Kollmann (1996)). The key research often focuses on the impact of international interdependence on business cycle frequencies, based on a two-country model. International interdependence has, in recent years, become an important issue due to free trade, enhanced capital mobility and globalization capital markets. Recently, Mendoza (1991a and 1991b) has introduced and addressed the aspect of a small open economy in the study of international real business cycles. Correia, Neves and Rebelo (1995), Schmitt-Grohé (1998) and Cardia (1991) also contribute to explanation of the business cycles in a small open economy. However, this literature does not focus on the important aspect of the terms of trade as a transmission channel from foreign economies. Mendoza (1995) introduces shocks to terms of trade in a three-sector model which comprises importables, exportables and non-tradables with foreign financial capital markets. Mendoza (1995) accounts for business cycle effects as well as the relationship between the trade balance (TB) and terms of trade (TOT) such as the Harberger-Laursen-Metzler effect. Senhadji (1998) also studies the dynamics of TB and TOT in less-developed countries (LDC). This study focuses on the S-curve, which shows

a S-shaped lead and lag correlation between the trade balance and terms of trade. Schmitt-Grohé (1998) also refers to the roles of the terms of trade in a small open economy.

The literature in small open economy models with terms of trade implicitly suppose that terms of trade reflects foreign productivity shock, which emphasizes income effect rather than substitution effect of technological change. For example, Mendoza (1995), Schmitt-Grohé (1998) and Senhadji (1998) define the terms of trade as the ratio of export price to import price. This definition implies that productivity improvement in foreign production is wholly realized in import goods without any separation between consumption goods and capital goods. In other words, terms of trade should be defined by the prices of capital goods if investment-specific technological change is an important driving force of business fluctuation in large country such as USA.

The small open economy paradigm has a few important features in the study of international real business cycles. First, the representative agent in a small open economy can smooth consumption and savings through access to international financial markets. International financial markets are widely believed to be important for the international transmission of business cycles, since they determine the extent to which individuals can smooth consumption in the presence of country-specific shocks to income. Intuitively, with free trade and complete asset markets, the representative consumer can smooth consumption by varying labour effort, capital accumulation and the allocation of financial capital assets.

Second, the small open economy can not affect global goods and financial markets, but often introduces *exogenous* world interest shock and/or terms of trade shock into the model to analyze its economic fluctuation (Mendoza 1995). Note that these shocks can not be introduced to a two-country model of international business cycles in which interest rates and the terms of trade are determined endogenously.

Third, since the small open economy has been rapidly incorporated through trading goods and integrated into the international financial markets, both foreign capital goods and trade in intermediate goods have an important implication for the small open economy. Often, fluctuations in the foreign trade sector potentially become an important source of real shocks to the small open economy. A recent study by Lee (1995) emphasizes the role of foreign capital goods within an endogenous growth model of a small open economy. Lee (1995) concludes that foreign capital goods, which are used relatively more than domestic capital goods for the production of capital stock, induce the higher growth rate of income. An important aspect of the above analysis is that changes in business cycles in a small open economy are caused by foreign capital goods that are used for investment and thus production. Furthermore, Ambler, Cardia and Zimmermann (forthcoming) emphasize the role of intermediate good trade in the international propagation of the business cycle. This is important given the fact that trade in intermediate goods represents approximately 60% of total trade (Whalley 1995). We argue that introducing foreign capital goods as intermediate goods in a small open economy model could significantly

affect its ability to explain the main features of the intertemporal transmission of business cycles.

In this chapter, we build a small open economy model by incorporating foreign capital goods and the terms of trade into the international transmission mechanism of the business cycle. Our model has at least *three* salient features, distinguished from the model of Mendoza (1991a, 1991b and 1995). First, the analysis in this paper is motivated by the close linkage between foreign capital goods and equipment investment in a small open economy such as Australia. Our intuition is that given that a considerable portion of equipment investment is dependent on foreign capital goods[11], the technology change of equipment investment is far more volatile for foreign capital goods rather than domestic capital goods and is therefore a source of economic fluctuations. Second, this chapter is the first to introduce terms of trade shock of foreign capital goods into a small open economy. Our model is designed to capture the transmission mechanisms of terms of trade shock *via* imports of capital goods. Third, our study is related to work by Greenwood, Hercowitz and Huffman (1988), which emphasizes the importance of investment-specific technology shocks in business cycle fluctuations and endogenous capital utilization in large country such as USA. Our intuition is that a small open economy imports capital goods from large countries in which technology is embodied. While the analysis by Greenwood, Hercowitz and Huffman (1988) and Greenwood, Hercowitz and Krusell (1997) are based on a closed economy, our model assumes a small

[11] According to the recent statistics of ABS (Australian Bureau of Statistics), the portion of foreign capital goods to equipment investment is around 70% on average, during period 1965-99, in Australia.

open economy, incorporating the foreign sector's influences through terms of trade shocks. This chapter consists of six sections. Section 4.2 describes investment-specific technology shock and the terms of trade in our model. Section 4.3 provides a full description of the dynamic stochastic model of a small open economy. Section 4.4 shows the conditions for optimal allocation of the dynamic programming problem. Section 4.5 represents the key findings in the empirical analysis, including the impulse response analysis. Finally, concluding remarks are presented in Section 4.6.

4.2 Investment-Specific Technology and the Terms of Trade

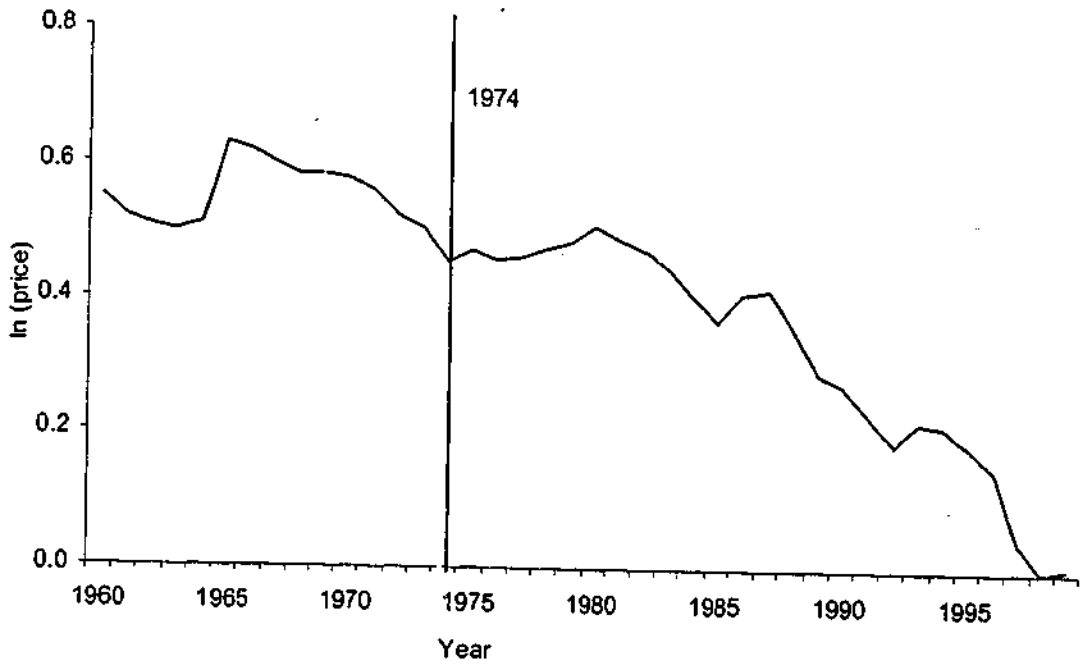
A small open economy is affected by the world economy through several channels, such as exchange rates, the terms of trade and interest rates. As emphasized in previous literature, terms of trade is a significant channel connecting a small open economy with foreign economies. In this chapter, we are interested in whether terms of trade is a channel reflecting the technological change from overseas. Accordingly, we focus on capital goods in which technology is embodied. On the other hand, Greenwood and Orukoglu (1997) emphasize that investment-specific technology progress is an important driving force of business cycles. Especially, they emphasize that the pace of technological change jumped around 1974, and the rapid advance in technology is linked to the further development of information technologies. They proved the rapid technological change around 1974 through the link between the phenomenal rise of IT (information technology) investment and the decline in the price of new equipment.

Accordingly, Greenwood, Hercowitz and Krusell (1997 and 1998) calculated the property of investment-specific technology shock by using price of new equipment, which is defined as the implicit price deflator for non-durable consumption goods and non-housing services divided by Gorden's index of nominal prices for producer durable equipment. Our key research interest is whether investment-specific technology shock can be substituted with the terms of trade shock of foreign capital goods in a small open economy. In other words, in a small open economy, investment is dependent on foreign capital goods in a large portion, and thus the foreign capital goods that reflect overseas technology change are an important factor causing investment-specific technology shock. Accordingly, we define the terms of trade in two ways. First, terms of trade is defined by the relative price of non-durable consumption expenditure to foreign capital goods. This definition emphasizes substitution between consumption goods and capital goods, as in Greenwood and Yorukoglu (1997) and Greenwood, Hercowitz and Krusell (1998), rather than substitution between domestic goods and foreign goods. Second, the terms of trade is defined by the relative price of exports to foreign capital goods. This definition focuses on substitution between domestic goods and foreign goods such as in Mendoza (1995) and Senhadji (1998). Figures 4.1 to 4.3 plot the inverse values of the relative price of new equipment and the terms of trade by using Australian data. Figure 4.1 shows the price of new equipment. The price shows a decreasing trend, though it has short term increase. Moreover, Figure 4.1 shows a turning point of the price around 1974, though it is not an abrupt change like the USA. However, Figures 4.2 and 4.3 show abrupt changes of the terms of trade of foreign capital goods around 1974, as in Greenwood and Orukoglu

(1997) though they do not reflect a decreasing trend. Figures 4.2 and 4.3 show that the terms of trade of foreign

capital goods reflect fairly well foreign technological change. In short, it is interesting to observe that overseas' investment-specific technology shock affects domestic investment-specific technology through the terms of trade, which reflects the technological change in foreign capital goods.

Figure 4.1: Price of new equipment[12]



[12] The price of new equipment is computed as the logged value of implicit deflator index of equipment investment divided by implicit deflator index of non-durable consumption expenditure.

Figure 4.2: Terms of trade[13]

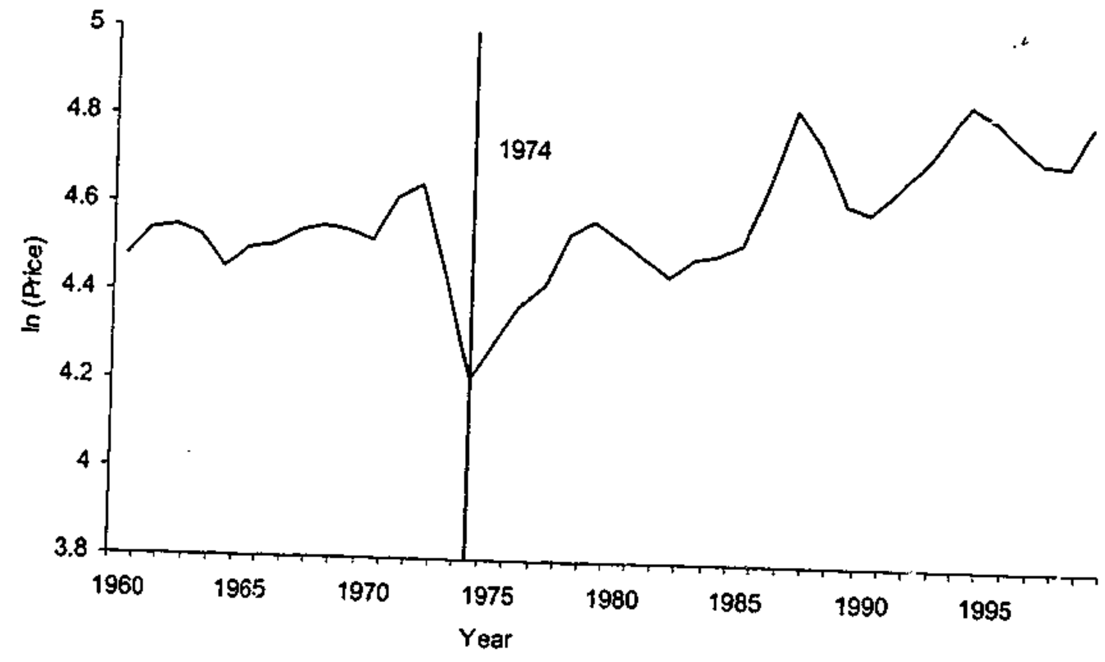
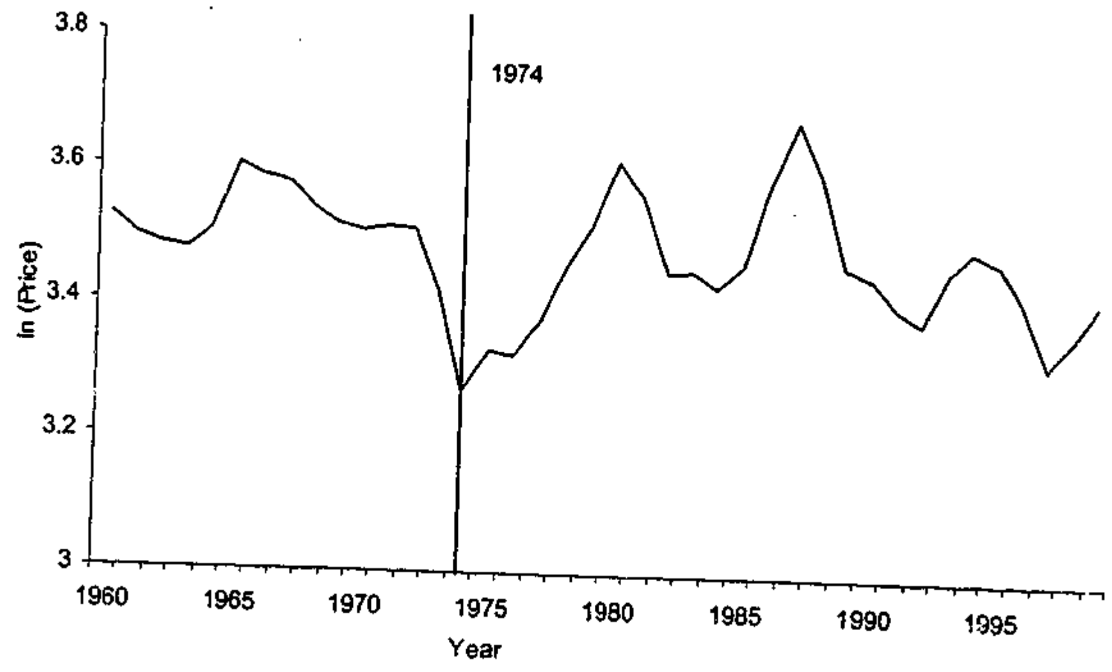


Figure 4.3: Terms of trade[14],



[13] Terms of trade is computed as the logged value of import price of capital goods divided by export price.

[14] Terms of trade is computed as the logged value of import price of capital goods divided by implicit deflator index of non-durable consumption expenditure.

4.3 An International Real Business Cycle Model

A dynamic stochastic model of a small open economy is developed to investigate the effects of productivity and terms of trade shocks. The structure of the model is a further extension from versions of the models of Greenwood, Hercowitz and Krusell (1997) and Mendoza (1995).

4.3.1 Preferences

Agents who are infinite with the same identical preferences, allocate C_t (consumption) and N_t (labour supply) intertemporarily to maximise utility.

$$V = E\left[\sum_{t=0}^{\infty} U\{C_t, N_t\}\exp\left(-\sum_{\tau=0}^{t-1} \upsilon(C_\tau, N_\tau)\right)\right] \qquad (4.1)$$

The instantaneous utility function and time-preference functions are as follows:

$$U(\cdot) = \frac{\left[C_t - \dfrac{N_t^{1+\theta}}{1+\theta}\right]^{\gamma} - 1}{\gamma} \qquad (4.2)$$

$$\upsilon(\cdot) = \beta \ln\left(1 + C_t - \frac{N_t^{1+\theta}}{1+\theta}\right) \qquad (4.3)$$

$U(\cdot) < 0,\ U'(\cdot) > 0,\ U'(0) = \infty,\ 1+\theta > 1,\ \gamma > 1$

$\upsilon(\cdot) > 0 \quad \upsilon'(\cdot) > 0,\ \upsilon''(\cdot) < 0,\ \beta > 0$

$U'(\cdot)\exp(\upsilon(\cdot))$ nonincreasing

The parameter β is referred to as consumption elasticity of the rate of time preference. The parameter θ is the elasticity of substitution between consumption and labour supply. The coefficient γ indicates relative risk aversion. In the steady-state economy, the intertemporal discount rate equates to the world's real interest rate according to the accumulation of foreign financial assets. So the intertemporal discount rate or endogenous discount rate is used to ensure that models of small open economies with time separable preferences have a stationary state with accurate, well-defined dynamics around that steady-state.

4.3.2 Production and Technology

Production technology is given by a Cobb-Douglas production function using structural capital, equipment capital and labour. Capital utilization (h_t) is incorporated into equipment capital.

$$Y_t = Z_t (h_t K_{e,t})^{\alpha_e} K_{s,t}^{\alpha_s} N_t^{1-\alpha_e-\alpha_s} \qquad (4.4)$$

where Y_t is the output of the single good in period t, $K_{e,t}$ and $K_{s,t}$ are the equipment capital stock and structure capital stock, respectively. N_t is labour input in period t. The parameters α_e and α_s represent the shares of capital on equipment and structure. The variable h_t determines the flow of equipment capital services $(h_t K_{e,t})$ and represents the intensity of the use of capital, that is, the speed of operation, or the number of hours per period the capital is used. Z_t is productivity shock, and follows the first-order Markov distribution function.

4.3.3 Capital Accumulation

The laws of motion for the capital stocks are as follows:

$$K_{s,t+1} = i_{s,t} + (1-\delta_s)K_{s,t} \tag{4.5}$$

$$K_{e,t+1} = i_{e,t} + (1-\delta_e(h_t))K_{e,t} \tag{4.6}$$

$$\text{and } \delta_e(h_t) = \frac{h_t^{\omega}}{\omega}$$

where $i_{s,t}$ and $i_{e,t}$ denote the structure[15] and equipment investments, respectively. δ_s is a constant rate of depreciation. δ_e is not a constant rate of depreciation but a non-negative

[15] Structure capital stock is differently from equipment capital stock in that it is composed of capital stock such as buildings and factories. These kinds of capital stock are not greatly used up even though intensive use. Accordingly, it is not the necessary for the depreciation rate of structure capital stock to not be a function of capital utilization.

function of capital utilization (h_t)[16]. ω is an elasticity of depreciation with respect to utilization. Equipment investment goods are produced by a CES production function using domestic capital goods and foreign capital goods.

$$i_{e,t} = \left[\pi m_t^{\rho} + (1-\pi)M_t^{\rho}\right]^{\frac{1}{\rho}} \tag{4.7}$$

where m_t and M_t are domestic capital goods and foreign capital goods, respectively. π implies domestic capital goods share and $1/(1-\rho)$ is elasticity of substitution in the CES aggregator for equipment investment.

Greenwood, Hercowitz and Krusell (1997) and Greenwood and Orukoglu (1997) argue that investment-specific technology is wholly realized in equipment capital stock, such as new machines, by using the decreasing price of new equipment. In our model, it is also supposed that capital stock is composed of structure capital stock and equipment capital stock, and the technological change is embodied into equipment investment. Moreover,

[16] Greenwood, Hercowitz and Huffman (1988) use capital utilization variable in two reasons. First, the capital utilization decision involves Keynes' notion of 'user cost'. That is, a higher utilization rate causes a faster depreciation rate causes a faster depreciation of capital stock, either because wear and tear increase with use or because less time can be devoted to maintenance. Second, in the business cycle model of Greenwood, Hercowitz and Huffman (1988), from the first-order condition of the model with investment-specific technology shock, if capital utilization does not exist in the model, labour expresses as a function of only capital stock, which means labour supply is independent of investment-specific shock. Accordingly, by incorporating capital utilization into the business model, the effect of the shock is transmitted to labour supply intratemporarily, and thus labour is expressed as a function of state variables including investment-specific technology shock. In our model, we accept the first reason of Greenwood, Hercowitz and Huffman (1988). Moreover, without capital utilization, our model also does not have any transmission mechanism connecting between labour supply and the terms of trade shock intratemporally, and thus capital utilization is incorporated into our model.

we further suppose that technological change of foreign equipment capital goods is transmitted to equipment investment through the terms of trade of foreign capital goods.

4.3.4 Trade and Financial Structure

The financial structure is linked to the trade balance. The financial structure assumes that an agent in the economy is a small participant and this representative household has access to world capital markets to borrow and lend foreign financial assets (A_t). Holdings of foreign financial assets (A_{t+1}) evolve according to:

$$A_{t+1} - (1 + r^*)A_t = e_t EX_t - M_t \tag{4.8}$$

where EX_t is exports and M_t is foreign capital goods. e_t is the terms of trade shock, defined as the relative price of exports to foreign capital goods or the relative price of non-durable consumption goods to foreign capital goods. The terms of trade shock of capital goods (e_t) follows the first-order Markov distribution functions, and r^* is the exogenously determined world's real interest rate. To rule out the possibility of the economy playing a Ponzi game in the intertemporal capital market (these paths allow the representative agent to borrow initially an arbitrarily large amount and then to finance interest payments with further borrowing, thereby never repaying the initially debt), we need the following transversality condition:

$$\lim_{T\to\infty} E_0 \frac{A_T}{(1+r^*)^T} = 0$$

4.3.5 Resource Constraint

The model is completed with a resource constraint[17] as follows:

$$Z_t e_t p_d (h_t K_{e,t})^{\alpha_e} K_{s,t}^{\alpha_s} N_t^{1-\alpha_e-\alpha_s} - \frac{\phi_e (K_{e,t+1} - K_{e,t})^2}{K_{e,t}} - \frac{\phi_s (K_{s,t+1} - K_{s,t})^2}{K_{s,t}}$$
$$= e_t p_d C_t + m_t + i_{s,t} + M_t + A_{t+1} - (1 + r^*)A_t \tag{4.9}$$

where ϕ_e and ϕ_s are parameters to restrict the adjustment costs. p_d is the relative price of consumption goods to foreign capital goods, which is assumed as one. Equation (4.9) is a constraint equation[18] with the terms of trade shock computed from the relative price of non-durable consumption goods to foreign capital goods. The aggregate resource

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17 $$Z_t e_t (h_t K_{e,t})^{\alpha_e} K_{s,t}^{\alpha_s} N_t^{1-\alpha_e-\alpha_s} - \frac{\phi_e (e_t K_{e,t+1} - e_t K_{e,t})^2}{e_t K_{e,t}} - \frac{\phi_s (e_t K_{s,t+1} - e_t K_{s,t})^2}{e_t K_{s,t}}$$
$$= e_t C_t + e_t m_t + e_t i_{s,t} + M_t + A_{t+1} - (1 + r^*)A_t \tag{4.9'}$$

Equation (4.9') is constraint equation with the terms of trade shock calculated from the relative price of export to foreign capital goods.

18 From equation (4.9), all variables are divided by the price of foreign capital goods. Equation (4.9) is as follows:

$$Z_t e_t (h_t K_{e,t})^{\alpha_e} K_{s,t}^{\alpha_s} N_t^{1-\alpha_e-\alpha_s} - \frac{\phi_e (K_{e,t+1} - K_{e,t})^2}{K_{e,t}} - \frac{\phi_s (K_{s,t+1} - K_{s,t})^2}{K_{s,t}}$$
$$= e_t C_t + m_t + i_{s,t} + M_t + A_{t+1} - (1 + r^*)A_t$$

We can change the equation (4.9) to another normal equation.

$$Z_t e_t (h_t K_{e,t})^{\alpha_e} K_{s,t}^{\alpha_s} N_t^{1-\alpha_e-\alpha_s} - \frac{\phi_e (K_{e,t+1} - K_{e,t})^2}{K_{e,t}} - \frac{\phi_s (K_{s,t+1} - K_{s,t})^2}{K_{s,t}} = e_t C_t + i_{e,t} + i_{s,t} + e_t EX_t - M_t$$

constraint[19] of the economy describes that the sum of consumption, investment, and the exports can not exceed gross domestic product.

4.4 Dynamic Programming Problem, Solution Techniques and Parameter Calibrations

The social planner selects paths of consumption and labour supply to maximise the whole life-time utility. To solve the dynamic programming problem, the household's optimal intertemporal decisions choose the control variables ($K_{e,t+1}, K_{s,t+1}, h_t, N_t, m_t, M_t, A_{t+1}$) in period t, given the state of the economy as described by $K_{e,t}, K_{s,t}, A_t, Z_t,$ and e_t. To solve the dynamic programming problem of the model with the terms of trade shock

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We can see $i_{e,t} = \left[\pi m_t^{\rho} + (1-\pi)M_t^{\rho}\right]^{\frac{1}{\rho}}$ from equation (4.7).

The constraint equation is expressed by expenditure aspects of GDP, such as consumption and investment, but not production aspects. Moreover, the inputs to produce $i_{e,t}$ are m_t and M_t. Hence, if m_t and M_t have the same prices, $i_{e,t}$ should be substituted with $m_t + M_t$ in the expenditure aspect. Hence, the constraint equation is changed to the following equation:

$$Z_t e_t (h_t K_{e,t})^{\alpha_e} K_{s,t}^{\alpha_s} N_t^{1-\alpha_e-\alpha_s} - \frac{\phi_e (K_{e,t+1} - K_{e,t})^2}{K_{e,t}} - \frac{\phi_s (K_{s,t+1} - K_{s,t})}{K_{s,t}} = e_t C_t + m_t + M_t + i_{s,t} + e_t EX_t - M_t$$

Two imports have to be eliminated, and then the above constraint equation becomes the same as the constraint equation (4.9).

[19] In our model, it is assumed that the model economy imports only foreign capital goods. Because foreign capital goods are used for equipment investment, equipment investment contains foreign capital goods in a production function. Accordingly, output is consumed, invested for structure capital stock, used for equipment investment production and exported. Because equipment investment has a production function, it is not suitable that the constraint equation is expressed as follows:

$$Z_t e_t p_d (h_t K_{e,t})^{\alpha_e} K_{s,t}^{\alpha_s} N_t^{1-\alpha_e-\alpha_s} - \frac{\phi_e (K_{e,t+1} - K_{e,t})^2}{K_{e,t}} - \frac{\phi_s (K_{s,t+1} - K_{s,t})^2}{K_{s,t}}$$
$$= e_t p_d C_t + i_{e,t} + i_{s,t} + A_{t+1} - (1+r^*)A_t$$

computed from the relative price of non-durable consumption goods in terms of foreign capital, the Lagrangian problem is built as follows:

$$\mathcal{L} = \prod_{\tau=0}^{t-1}\left(1 + C_\tau - \frac{N_\tau^{1+\theta}}{1+\theta}\right)^{-\beta} \left\{ \frac{\left(C_t - \frac{N_t^{1+\theta}}{1+\theta}\right)^{\gamma} - 1}{\gamma} \right.$$
$$+ \lambda_{1t}\left((Z_t e_t p_d (h_t K_{e,t})^{\alpha_e} K_{s,t}^{\alpha_s} N_t^{1-\alpha_e-\alpha_s} - \frac{\phi_e (K_{e,t+1} - K_{e,t})^2}{K_{e,t}} - \frac{\phi_s (K_{s,t+1} - K_{s,t})^2}{K_{s,t}} \right. \quad (4.10)$$
$$\left. - e_t p_d C_t - m_t - K_{s,t+1} + (1-\delta_s) K_{s,t} - M_t - A_{t+1} + (1+r^*) A_t \right)$$
$$\left. + \lambda_{2t}\left(- K_{e,t+1} + (1-\delta_e(h_t)) K_{e,t} + \left[\pi m_t^{\rho} + (1-\pi) M_t^{\rho}\right]^{\frac{1}{\rho}} \right) \right\}$$

The first-order conditions for utility maximisation are:

$$(C_t): V_{ct} = \lambda_{1t} e_t p_d \quad (4.11)$$

$$(K_{s,t+1}): \lambda_{1t}\left[\frac{2\phi_s (K_{s,t+1} - K_{s,t})}{K_{s,t}} + 1\right] \quad (4.12)$$

$$= \lambda_{1t+1} B(t) \left[Z_{t+1} e_{t+1} p_d \alpha_s (h_{t+1} K_{e,t+1})^{\alpha_e} K_{s,t+1}^{\alpha_s - 1} N_{t+1}^{1-\alpha_e-\alpha_s} + \frac{2\phi_s (K_{s,t+2} - K_{s,t+1})}{K_{s,t+1}} \right.$$
$$\left. + \frac{\phi_s (K_{s,t+2} - K_{s,t+1})^2}{K_{s,t+1}^2} + 1 - \delta_s \right]$$

$$(K_{e,t+1}): \lambda_{1t}\left[\frac{2\phi_e(K_{e,t+1}-K_{e,t})}{K_{e,t}}\right] + \lambda_{2t}$$

$$= \lambda_{1t+1}B(t)\Big[Z_{t+1}e_{t+1}p_d\alpha_e h_{t+1}(h_{t+1}K_{e,t+1})^{\alpha_e-1}K_{s,t+1}^{\alpha_s}N_{t+1}^{1-\alpha_e-\alpha_s} + \frac{2\phi_e(K_{e,t+2}-K_{e,t+1})}{K_{e,t+1}} \tag{4.13}$$

$$+ \frac{\phi_e(K_{e,t+2}-K_{e,t+1})^2}{K_{e,t+1}^2}\Big] + \lambda_{2t+1}B(t)\big(1-\delta_e(h_{t+1})\big)$$

$$(N_t): N_t^{\theta} = (1-\alpha_e-\alpha_s)Z_t(h_tK_{e,t})^{\alpha_e}K_{s,t}^{\alpha_s}N_t^{-\alpha_e-\alpha_s} \tag{4.14}$$

$$(A_{t+1}): \lambda_{1t} = \lambda_{1t+1}B(t)(1+r^*) \tag{4.15}$$

$$(M_t): \lambda_{1t} = \lambda_{2t}(1-\pi)M_t^{\rho-1}\left[\pi m_t^{\rho}+(1-\pi)M_t^{\rho}\right]^{\frac{1}{\rho}-1} \tag{4.16}$$

$$(m_t): \lambda_{1t} = \lambda_{2t}\pi m_t^{\rho-1}\left[\pi m_t^{\rho}+(1-\pi)M_t^{\rho}\right]^{\frac{1}{\rho}-1} \tag{4.17}$$

$$(h_t): \lambda_{1t}\alpha_e Z_t e_t p_d K_{e,t}(h_tK_{e,t})^{\alpha_e-1}K_{s,t}^{\alpha_s}N_t^{1-\alpha_e-\alpha_s} = \lambda_{2t}h_t^{\omega-1}K_{e,t} \tag{4.18}$$

$$(\lambda_{1t}): Z_te_tp_d(h_tK_{e,t})^{\alpha_e}K_{s,t}^{\alpha_s}N_t^{1-\alpha_e-\alpha_s} - \frac{\phi_e(K_{e,t+1}-K_{e,t})^2}{K_{e,t}} - \frac{\phi_s(K_{s,t+1}-K_{s,t})^2}{K_{s,t}} \tag{4.19}$$

$$- e_tp_dC_t - m_t - K_{s,t+1} + (1-\delta_s)K_{s,t} - M_t - A_{t+1} + (1+r^*)A_t = 0$$

$$(\lambda_{2t}): K_{e,t+1} - (1-\delta_e(h_t))K_{e,t} = \left[\pi m_t^{\rho}+(1-\pi)M_t^{\rho}\right]^{\frac{1}{\rho}} \tag{4.20}$$

where $B(t) = \left(1+C_t-\frac{N_t^{1+\theta}}{1+\theta}\right)^{-\beta}$,

$$V_{ct} = \left(C_t-\frac{N_t^{1+\theta}}{1+\theta}\right)^{-\gamma} - \beta\left(1+C_t-\frac{N_t^{1+\theta}}{1+\theta}\right)^{-\beta-1}\big(U(t+1)+B(t+1)U(t+2)$$

$$+B(t+1)B(t+2)U(t+3)+\cdots\cdots\big)$$

A feature of these first-order conditions is that this model reflects the property of a small open economy business cycle model. The variable intertemporal discount rate[20] is a feature of a small open economy business cycle model. The variable intertemporal discount rate is a concave function as follows, $B(\cdot) > 0$, $B'(\cdot) < 0$ and $B''(\cdot) > 0$. This feature induces consumption and labour to smooth from economic shock. In other words, if consumption increases, the increased consumption decreases the intertemporal discount rate, and in turn the decreased discount rate reduces the utility of current consumption compared with future consumption, and thus decreases current consumption. This interaction causes consumption to smooth, though the effect is small.

4.4.1 Solution Technique and Calibration

To solve the dynamic program problem by using an undetermined coefficient solution method, we need to linearize above first-order conditions around the steady-state values (Christiano 1998). We use economic theory as the basis for restricting the general framework for finding numerical values for parameters.

[20] Mendoza (1991a, 1991b and 1995) adopt a variable intertemporal discount rate, while other literature employs a fixed discount rate. As an anonymous referee pointed out, two results can not be significantly different in the approximation method of our model. However, as can be seen from equation (4.15), if the discount rate is fixed, the Lagrangian multiplier λ is constant. In this case, from equation (4.11), consumption is affected by labour and terms of trade shock. If terms of trade shock does not exist in a business cycle model, consumption is dependent on only labour. As a result, autocorrelation of consumption and correlation of consumption with output are the same as those of labour, except for standard deviation. To avoid this problem in a small open economy model, it is desirable to use variable discount rates, or to incorporate another factor, such as world interest rate shock and terms of trade shock in a small open economy model.

Under the restriction that the average ratio of net foreign interest payments to GDP is around 2%, the consumption elasticity of the rate of time preference (β) is computed as 0.50 from the steady-state condition which equates to the world real interest rate with the discount rate of preference. The coefficient of relative risk aversion (γ) is set to –3.0. The inverse of the intertemporal elasticity of substitution in labour supply (θ) is calculated at 0.64 by using both the restriction that steady state labour supply is 0.33 and the equation derived from the steady-state conditions of equations (4.11) to (4.20).

The relevant parameters of production are calculated from national income data. An average annual value of 0.31 over the 1965–1999 period is used for capital share $(\alpha_s + \alpha_e)$ from the national account. The structure capital share in output (α_s) and equipment capital share in output (α_e) are calculated as 0.18 and 0.13, respectively, by using the ratio of structure investment to GDP from the national account, the steady-state equations (4.10) to (4.20) and the condition that the sum of structure capital and equipment capital shares is 0.31.

The depreciation rate of structure capital (δ_s) and equipment capital (δ_e) are set at 0.084 and 0.139[21]. The elasticity of depreciation with respect to utilization (ω) is set as 1.29 under the restriction that the depreciation rate (δ_e) of equipment capital is 0.139 in a deterministic steady state. Elasticity of substitution in CES aggregator for equipment

[21] These values are obtained by using both the national accounts data and the following capital accumulation formulas

$$\delta_e = 1.0 - \frac{K_{e,t+1} - i_{e,t}}{K_{e,t}} \qquad \delta_s = 1.0 - \frac{K_{s,t+1} - i_{s,t}}{K_{s,t}}$$

Table 4.1: Benchmark parameter values

Parameter	Definition	Values
β	Consumption elasticity of the rate of time preference	0.50
γ	Coefficient of relative risk aversion	–3.0
θ	Inverse of the intertemporal elasticity of substitution in labour supply	0.64
α	Capital share in output	0.31
α_s	Structure capital share in output	0.18
α_e	Equipment capital share in output	0.13
δ_s	Depreciation rate of structure capital	0.084
δ_e	Depreciation rate of equipment capital	0.139
ω	Elasticity of depreciation with respect to utilization	1.29
$1/(1-\rho)$	Elasticity of substitution in CES aggregator for equipment	1.5
$(1-\pi)$	Share of foreign capital goods in equipment investment	0.66
ϕ_s	Adjustment cost coefficient of structure capital	2.3
ϕ_e	Adjustment cost coefficient of equipment capital	2.3
r^*	World's real interest rate	0.04

$(1/(1-\rho))$ is set at 1.5, which is obtained from Ambler, Cardia, and Zimmermann (forthcoming). The share of foreign capital goods in equipment investment $(1-\pi)$ is set as 0.66, which is the ratio of foreign capital goods to equipment investment. The world's real interest rate (r^*) and adjustment cost coefficients are set as 0.04 and 2.3 from Mendoza (1991a) and Greenwood, Hercowitz and Krusell (1998), respectively.

4.4.2 Shock Process

Parameters of the productivity shock process are estimated using Solow residuals of GDP for productivity shocks, and parameters of the terms of trade shock process are estimated using the values that are filtered by linear time trend. It is assumed that the shock process follows the first-order Markov distribution function, and the processes are estimated by VAR (vector autoregression). The estimation result of the terms of trade shock, which is the relative price of non-durable consumption goods to foreign capital goods[22], is as follows:

$$\begin{bmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \end{bmatrix} = \begin{bmatrix} 0.97 & 0.0 \\ 0.0 & 0.72 \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \end{bmatrix} + \begin{bmatrix} w_{1,t+1} \\ w_{2,t+1} \end{bmatrix}$$

[22] The estimation result of the terms of trade shock, which is the relative price of export in terms of foreign capital goods, is as follows:

$$\begin{bmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \end{bmatrix} = \begin{bmatrix} 0.97 & 0.0 \\ 0.0 & 0.71 \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \end{bmatrix} + \begin{bmatrix} w_{1,t+1} \\ w_{2,t+1} \end{bmatrix}$$

The correlation-covariance matrix of the innovation is:

$$\begin{bmatrix} 0.000302 & 0.0 \\ 0.0 & 0.006151 \end{bmatrix}$$

and the percentage standard deviations are 1.74 and 7.84, respectively.

The correlation-covariance matrix of the innovation is:

$$\begin{bmatrix} 0.000292 & 0.0 \\ 0.0 & 0.003729 \end{bmatrix}$$

and the percentage standard deviations are 1.71 and 6.11, respectively.

4.5 Empirical Findings

4.5.1 Business Cycle Properties and Sensitivity Analysis

In this section, the key empirical findings of the statistical moments produced by the benchmark model are reported and compared with the actual moments obtained from Australian data. Table 4.2 summarizes the major statistics of a real economy and the benchmark[23] economy by these categories: the standard deviation, the first-order serial correlations, and the correlation with GDP. The benchmark economy is divided into both non-durable consumption goods price (model A) and export price in terms of foreign capital goods price (model B). All data except for labour and trade balance are deflated by foreign capital good prices. The standard deviations of the actual data from Table 4.2 show that investment (1.09) and structure investment (1.36) are more volatile than output (1.0), and consumption (0.96) is less. However, standard deviation of equipment

[23] The results are based on 100 replications of histories of 100 samples. We generated histories of 169 observations and truncated the first 133 observations so that the results do not depend on initial condition of the state variables of the model (Ambler, Cardia and Zimmermann, forthcoming).

investment (1.0) is similar to output. The variables relating to the foreign sector are more volatile than output. For example, interest payments (1.95), foreign capital goods (1.41) and terms of trade (1.14) defined by export price, while trade balance (0.13) and the terms of trade (0.90) defined by non-durable consumption goods price are less volatile than output. In autocorrelation, domestic variables have higher autocorrelation than foreign variables. For instance, output (0.52), consumption (0.56), investment (0.43), equipment investment (0.40), structure investment (0.60) and labour supply (0.55) are higher than interest payments (0.33), foreign capital goods (0.11) and trade balance (0.14). However, two terms of trade (0.57 and 0.55) are higher than other foreign variables. In correlation with output, domestic variables have higher correlation with output than foreign variables. Specially, consumption (0.98) is highest, investment (0.85), equipment investment (0.71) and structure investment (0.80) are higher than interest payments (0.38), foreign capital goods (0.63) and trade balance (-0.09).

Two benchmark economies show different features. In general, the standard deviations of model A are lower than those of model B. These findings result from the facts that the terms of trade, which are a driving force of the business cycle, affect the standard deviation of the variables. In other words, the higher standard deviation the terms of trade has, the bigger standard deviation the variables have.

Table 4.2: Statistics moments

	Australian data at constant price of foreign capital goods 1965–1999			Benchmark models: Model A			Benchmark models: Model B		
	σ_x	$\rho_{xt,xt-1}$	$\rho_{xt,GDPt}$	σ_x	$\rho_{xt,xt-1}$	$\rho_{xt,GDPt}$	σ_x	$\rho_{xt,xt-1}$	$\rho_{xt,GDPt}$
Output	1.0	0.52 (0.109)	1.0	1.0	0.52 (0.226)	1.0	1.0	0.52 (0.243)	1.0
Consumption	0.96 (0.128)	0.56 (0.108)	0.98 (0.009)	0.53 (0.063)	0.56 (0.217)	0.87 (0.041)	0.50 (0.059)	0.58 (0.233)	0.90 (0.033)
Investment	1.09 (0.145)	0.43 (0.122)	0.85 (0.047)	0.90 (0.106)	0.53 (0.218)	0.86 (0.042)	1.30 (0.153)	0.54 (0.240)	0.99 (0.003)
Equipment investment	1.00 (0.120)	0.40 (0.137)	0.71 (0.076)	1.14 (0.135)	0.52 (0.221)	0.94 (0.019)	1.18 (0.138)	0.57 (0.238)	0.96 (0.012)
Structure investment	1.36 (0.171)	0.60 (0.108)	0.80 (0.041)	0.81 (0.096)	0.53 (0.215)	0.79 (0.063)	1.37 (0.161)	0.52 (0.241)	0.99 (0.003)
Labour supply	0.14 (0.017)	0.55 (0.097)	0.21 (0.164)	0.27 (0.032)	0.59 (0.208)	0.65 (0.096)	0.23 (0.027)	0.63 (0.221)	0.61 (0.104)
Capital utilization	-	-	-	0.73 (0.086)	0.51 (0.227)	0.99 (0.004)	0.56 (0.066)	0.58 (0.235)	0.96 (0.013)
Interest payments	1.95 (0.261)	0.33 (0.152)	0.38 (0.143)	1.96 (0.217)	1.03 (0.129)	0.29 (0.150)	2.06 (0.230)	1.12 (0.128)	0.28 (0.154)
Foreign capital goods	1.41 (0.162)	0.11 (0.190)	0.63 (0.102)	1.24 (0.146)	0.52 (0.221)	0.97 (0.011)	0.91 (0.107)	0.60 (0.263)	0.89 (0.035)
Trade balance (TB/Y)	0.13 (0.022)	0.14 (0.171)	-0.09 (0.184)	0.46 (0.060)	0.51 (0.226)	0.82 (0.063)	0.50 (0.065)	0.52 (0.241)	0.88 (0.044)
Terms of trade (1)	0.90 (0.115)	0.57 (0.103)	0.95 (0.018)	0.78 (0.093)	0.49 (0.227)	0.90 (0.033)	-	-	-
Terms of trade (2)	1.14 (0.167)	0.55 (0.118)	0.91 (0.037)	-	-	-	0.82 (0.097)	0.52 (0.243)	0.93 (0.024)

Note: The nominal Australian data are obtained from ABS, divided by the 15+ population, deflated by the price of foreign capital goods (see, Mendoza, 1995 and Senhadji, 1998), logged and detrended by a Hodrick-Prescott filter with the smoothing parameter set at 100. σ_x is the percentage standard deviation, $\rho_{xt,xt-1}$ is the first-order serial correlation, and $\rho_{xt,GDPt}$ is the contemporaneous correlation with GDP. The values in the parenthesis represent the standard deviations estimated by GMM (Generalized Method of Moment) using the Hansen-Heaton-Ogaki gauss program. Model A represents the benchmark statistics simulated by the terms of trade shock computed from (implicit deflator of non-durable consumption expenditure/foreign capital goods price), Model B provides the benchmark statistics simulated by the terms of trade shock computed from (export price/foreign capital goods price). The first-order serial correlations of benchmark models are adjusted by the ratios of autocorrelation of GDP in real data to that of GDP in benchmark models. All values of Table 4.2 are computed from the nominal Australian data deflated by the price of import capital goods as in Mendoza (1995) and Senhadji (1998).

As can be seen from Table 4.2, the autocorrelation of model B with higher standard deviation of the terms of trade are higher than model A The benchmark models mimic well the real economy in the foreign sector. The standard deviation of interest payments from actual data is 1.95, which is similar to both 1.96 and 2.06 of benchmark models A and B, respectively. The foreign capital goods figure (1.41) from actual data is highly volatile and similar to the benchmark values of 1.24. The standard deviations of the terms of trade (1) and (2) from actual data are 0.90 and 1.14, respectively. These values are slightly higher than those of benchmark models, 0.78 and 0.82, respectively. The variables related to investment are underestimated in benchmark model A and overestimated in benchmark model B. In terms of investment, equipment investment and structure investment, the standard deviation of the actual data are 1.09, 1.00 and 1.36, respectively, while the corresponding values are 0.90, 1.14 and 0.81 for benchmark model A, and 1.30, 1.18 and 1.37 for benchmark model B.

In autocorrelation, benchmark models are overestimated in the foreign sector. Benchmark model A, which emphasizes the substitution between consumption goods and capital goods, performs fairly well and mimics closely the actual data of domestic variables, such as consumption (0.56, 0.56), investment (0.43, 0.53), structure investment (0.60, 0.53) and labour supply (0.55, 0.59). Benchmark model B, which focuses on the substitution between domestic goods and foreign goods, also replicates well the actual data of domestic variables such as consumption (0.56, 0.58), structure investment (0.60, 0.52) and labour supply (0.55, 0.63).

In correlation with output, benchmark models effectively mimic the real economy. Similarly to autocorrelation, benchmark model A relatively effectively replicates domestic variables such investment (0.85, 0.86) and structure investment (0.80, 0.79). Moreover, benchmark model A and B replicate high values of the correlation of domestic variables with output. For instance, the correlation of consumption with output in the real economy and benchmark models A and B show high values of 0.98, 0.87 and 0.90, respectively. In the foreign sector, benchmark model A is similar to benchmark model B. That is, for interest payments, benchmark model A and B show the similar correlation values with output of 0.29 and 0.28 together with foreign capital goods (0.97, 0.89). In the case of terms of trade, both benchmark model A and B effectively replicate the real economy in terms of trade (1) (0.95, 0.90) and terms of trade (2) (0.91, 0.93). Finally, the benchmark models of this chapter incorporate a variable capacity utilization rate introduced by Greenwood, Hercowitz and Huffman (1988). The correlation coefficients of capital utilization with output from the benchmark models show a procyclical effect (0.99, 0.96). The standard deviations of the capital utilization are 0.73 and 0.56, respectively.

Table 4.3 provides the results of simulation that increases every parameter by 5%. In general, the second moments are very stable. Standard deviations are relatively more sensitive to the change of parameters than autocorrelations and correlations with output. Foreign capital goods are most sensitive in volatility to the change of elasticity of substitution between domestic capitals and foreign capital goods, $1/(1-\rho)$. Investment and import capital goods are relatively more sensitive than other variables in standard

Table 4.3: Sensitivity analysis

	Relative volatility				First-order serial correlation				Correlation with output			
	c	i	m	tb	c	i	m	tb	c	i	m	tb
Baseline model	0.53	0.90	1.24	0.46	0.56	0.53	0.52	0.51	0.87	0.86	0.97	0.82
β	0.55	0.93	1.23	0.44	0.57	0.54	0.53	0.50	0.87	0.88	0.97	0.80
γ	0.55	0.90	1.23	0.44	0.56	0.54	0.53	0.50	0.89	0.87	0.97	0.81
θ	0.55	0.92	1.21	0.44	0.57	0.54	0.53	0.50	0.87	0.88	0.97	0.80
α_s	0.54	0.92	1.22	0.46	0.57	0.54	0.53	0.50	0.87	0.87	0.96	0.81
α_e	0.51	0.93	1.21	0.48	0.57	0.54	0.53	0.50	0.85	0.88	0.96	0.83
δ_s	0.53	0.91	1.21	0.47	0.57	0.54	0.52	0.50	0.86	0.88	0.96	0.82
δ_e	0.53	0.90	1.26	0.47	0.57	0.54	0.53	0.50	0.86	0.87	0.97	0.82
$1/(1-\rho)$	0.53	0.93	1.07	0.46	0.57	0.54	0.52	0.50	0.87	0.88	0.94	0.82
π	0.54	0.89	1.24	0.46	0.57	0.54	0.53	0.50	0.87	0.86	0.97	0.81

Note: Baseline model with each of the following parameters increased by 5%

deviation. Autocorrelations are very stable to the change of parameter values. Correlations with output also are stable. However, the change of the elasticity of substitution between domestic capital goods and foreign capital goods, $1/(1-\rho)$, has relatively more influence on correlation of foreign capital goods with output than those of other variables with output.

4.5.2 Impulse Response Analysis

We conduct the impulse response of macroeconomic aggregates induced by terms of trade and productivity shocks. The effects of productivity and terms of trade shocks on the macroeconomic variables can be investigated analytically when the model is not complicated. However, when the model includes various macroeconomic variables and parameters, the direction and magnitude of the macroeconomic effects from the shocks are less straightforward and often produce ambiguous results, depending on the relative size of several parameters. Accordingly, impulse response analysis is a useful and insightful shock evaluation procedure to assess the dynamic characterization of a system. In this study of impulse response analysis, we utilize the linear policy function derived from the undetermined coefficient method.

Figure 4.4 plots the impulse response of the major macroeconomic variables to a positive unit of standard deviation in productivity shock. A positive productivity shock causes an income effect to the economy. An increase in productivity raises GDP higher and then induces consumption and investment to increase. The impulse response functions indicate that positive shocks in productivity are followed by positive responses in the GDP, consumption, foreign capital goods, investment and equipment investment. The responses shown in Figure 4.4 are quite similar in GDP and consumption; however, investment, equipment investment and foreign capital goods differ in pattern and in magnitude, and trade balance shows a negative effect.

It is noticeable that a positive shock in productivity increases the efficiency of investment and has two effects on the economy. First, it augments the demand for domestic capital and foreign capital goods. Second, an increase in the efficiency of investment reduces the cost of capital utilization and thus induces a higher capital utilization. A distinguished feature is that the convergence of GDP and consumption are slow, whereas investment converges quickly to the equilibrium. It appears that a positive productivity shock gives a strong impact on investment and induces capital stock to increase. Later, an increase in capital stock has long-run effects on the major economic variables. Moreover, the initial shock increases investment, and then raises import of capital goods more than the increase of export, and thus decreases trade balance as can be seen from Figure 4.4.

Figure 4.5 shows the response of the major macroeconomic variables to a positive terms of trade shock. A positive terms of trade shock indicates a decrease in foreign capital goods prices, relative to domestic goods prices. This is called the "income effect", resulting from the terms of trade shock. A decrease in the relative price of foreign capital goods increases the budget. The increased budget will increase investment, and thereby increase the use of foreign capital goods and domestic capital goods. Moreover, the income effect increases the capital stock through more investment, and then increases GDP and consumption for the next period. The terms of trade shock has another effect, called the "substitution effect". A decrease in the relative price of foreign capital goods to domestic goods increases the demand for foreign capital goods, but decreases the use of the domestic capital goods in equipment investment. In spite of this substitution effect, the cost of investment declines. The decrease of investment cost then has two effects. The

first effect is the substitution effect across investment, consumption and foreign financial assets. The decrease of investment cost reduces relative consumption and foreign financial assets. The second effect is that a increase of investment raises the capital stock, and then GDP. These effects, together with the income effect, are the components that result in an augmentation in consumption and foreign financial assets. The total effect is determined by the size of both the income effect and substitution effect. Generally, income effect and substitution effect have the same directions, while consumption and foreign financial assets have reverse effects between the income and substitution effects. Nevertheless, the income effect overwhelms the substitution effect, and thus foreign financial assets and consumption increase. Moreover, the augmentation of terms of trade increases exports more than imports and thus has a positive effect on trade balance.

Figure 4.4: Impulse response: productivity shock

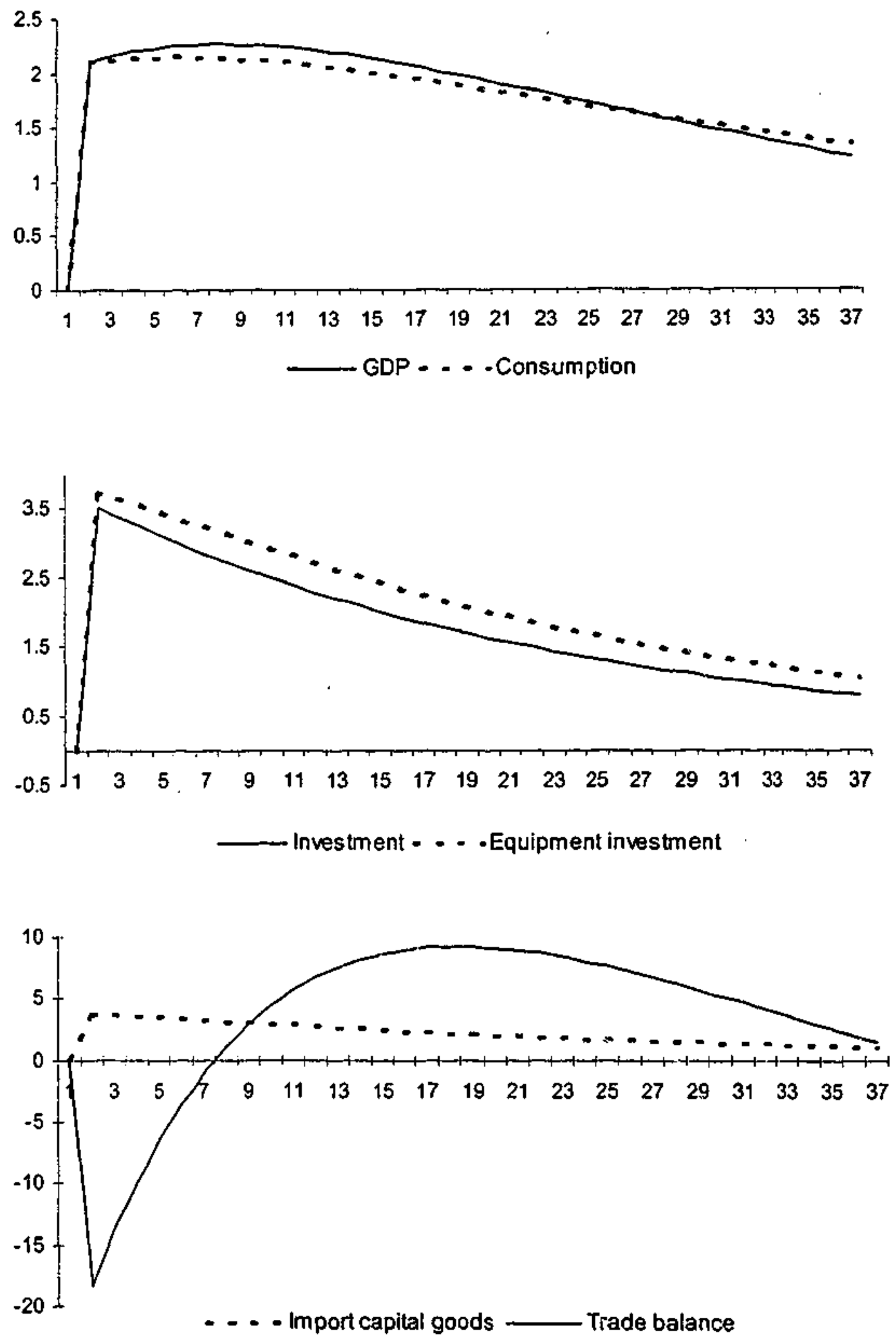
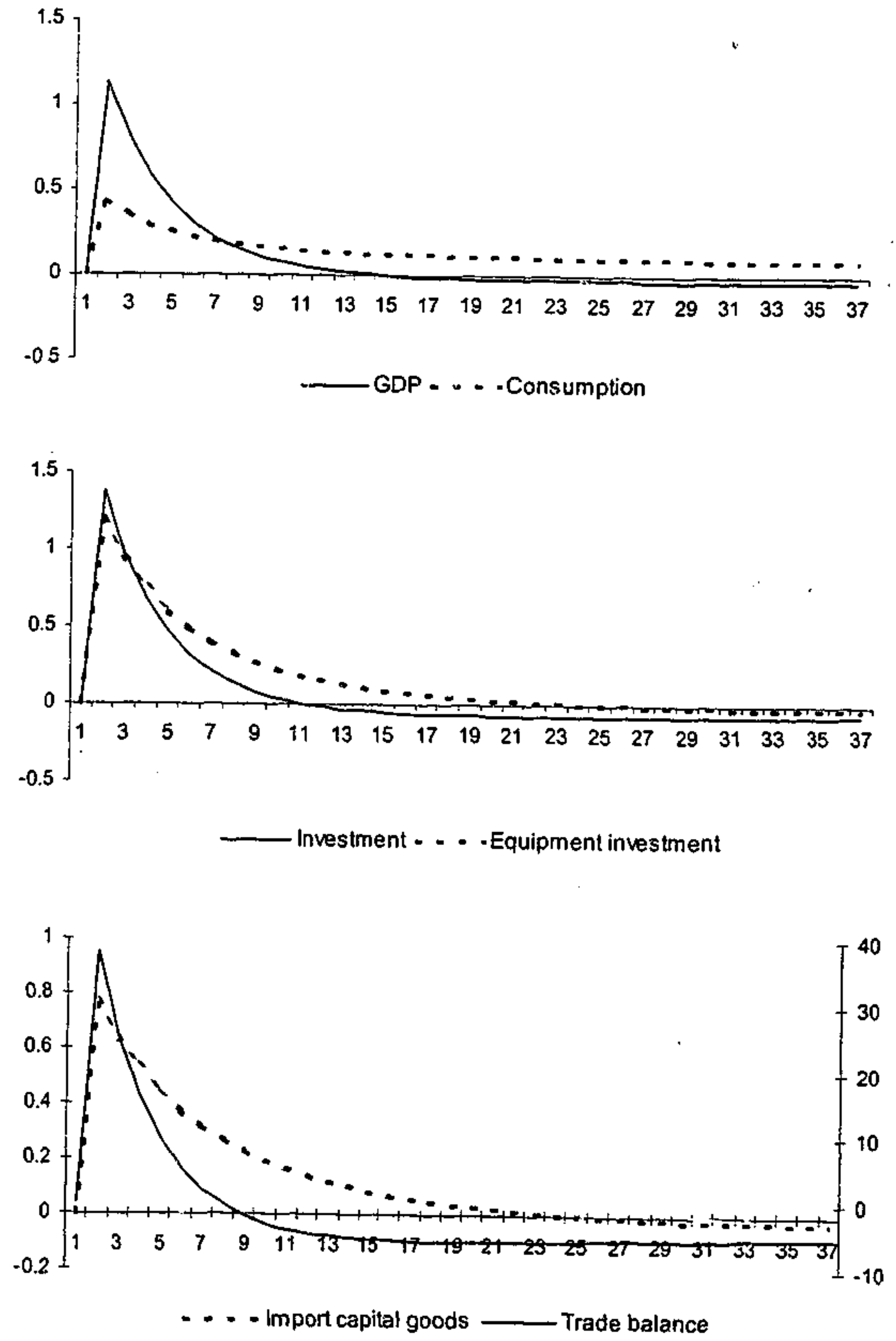


Figure 4.5: Impulse response: the terms of trade shock



4.6 Concluding Remarks

This chapter develops an international real business cycle model of a small open economy. The model has three production factors: structure capital stock, equipment capital stock and labour, and has two shocks (productivity shock and terms of trade shock). Our model is parameterized and calibrated and incorporates the neoclassical framework with endogenous capital utilization and foreign financial assets. We apply Christiano's (1998) undetermined coefficient method in order to simulate our benchmark model. Our model is able to duplicate many of the stylized facts of business cycles in Australia. We investigate two important *a priori* hypotheses. First, we ask whether it is possible to substitute "terms of trade shock", instead of "investment-specific technology shock", and whether terms of trade can transmit overseas investment-specific technology change into the domestic economy and hence cause the business fluctuations in a small open economy. Given the fact that the pattern of foreign capital goods to equipment investment is over 60% in Australia, our quantitative analysis concludes that terms of trade shock provide a meaningful explanation of Australian economic fluctuations. Foreign capital goods increase from productivity shock and terms of trade shock, whereas trade balance decreases as foreign capital goods rise from productivity shock. In addition, capital utilization in a small open economy context plays a key role in explaining economic fluctuation with a strong procyclical property.

Appendix 4.1: Data Sources

The Australian data are obtained from the ABS (Australian Bureau of Statistics). All series are composed of yearly observations from 1965 to 1999. All variables except for labour supply and trade balance are divided by import price of machinery and transport equipment. Full descriptions of the data are as follows: *Output:* nominal GDP. *Consumption:* nominal private final consumption expenditure. *Investment:* nominal gross capital formation. *Equipment investment:* nominal machinery and equipment out of gross capital formation. *Structure investment:* nominal other building and structure of gross capital formation. *Labour supply:* the number of non-farm civilian wage and salary earners × index of hours worked. *Population:* civilian population aged 15 years and over. *Interest payments:* (nominal investment income to overseas – nominal investment income from overseas). *Foreign capital goods:* nominal machinery and transport equipment imports. *Trade balance:* (nominal exports of goods – nominal imports of goods)/ nominal GDP. *Terms of trade (1):* implicit price index of non-durable consumption expenditure ÷ import price index of machinery and transport equipment import. *Terms of trade (2):* export price ÷ import price index of machinery and transport equipment.

Chapter 5 Determination of Asset Prices with an Investment-Specific Technology Model: Implications for the Equity Premium Puzzle

5.1 Introduction

A central task of financial economics is to determine the real risks that drive asset prices and expected returns. The aim of this chapter is to investigate the determination of asset prices and the implications for the equity premium puzzle, based on an investment-specific technology model. Our research objective in the study of asset pricing, within the framework of the dynamic asset pricing model, has at least two important perspectives. First, from the macroeconomic perspective, since asset markets contain valuable information on the intertemporal decision-making processes of investors, the data on financial assets should be used to evaluate the performance of the models. Second, from the finance perspective, standard asset pricing models that explain financial data, while treating either consumption or return as exogenous variables, have their limitations. However, we present a dynamic asset pricing model which allows a simultaneous determination of macroeconomic real variables and asset prices. Furthermore, it provides a more comprehensive economic foundation than the conventional CAPM (Capital Asset Pricing Model). To contribute to a better understanding of the determinants of asset returns and risk premia, this chapter develops a model with investment-specific technology, capacity utilization and capital adjustment costs. Our model has had some

success in revealing the forces determining asset prices. The model also succeeds in replicating financial market data. Previous studies of asset prices using dynamic models in conjunction with asset pricing puzzles are a few. Papers by Abel (1990), Jermann (1994), Boldrin, Christiano and Fisher (1995), Rouwenhorst (1995) and Lettau (2001) have investigated the equity premium in a dynamic model. Abel's (1990) "catching up with the Joneses" specification, power utility and non-expected utility, in particular, require high-risk aversion to account for the asset pricing puzzles. Jermann (1994) develops a model where labor input is fixed and with adjustment costs in capital accumulation. Jermann (1994) suggests that the equity premium is fairly large as long as the adjustment costs are substantial. Boldrin, Christiano and Fisher (1995, 2001) develop a two-sector model with both habit formation preference and limited resource flexibility across sectors. They find that the model is consistent with the observed mean equity premium, mean risk-free and Sharpe ratio on equity. The key feature of Boldrin, Christiano and Fisher (2001) is that by introducing a habit formation preference, a restricted labor assumption, a time-to-plan investment assumption, and an adjustment cost assumption, their model shows the effect of reducing the elasticity of capital supply and reducing the effectiveness of labour by smoothing the response of consumption to shocks.

Rouwenhorst (1995) has explored the asset pricing implications of a general equilibrium business cycle model. However, his model fails to generate a sizeable equity premium with endogenous consumption choice. Studies by Campbell (1994) and Lettau (2001) have recently examined the implications of financial asset prices in their models. Lettau

(2001) studies properties of financial asset prices and addresses the issue of equity premium using Campbell's (1994) approach. Lettau (2001) finds that risk premia are high only for high-risk aversion in the model with high-adjustment costs, and the wedge between the equity premium and the long bond premium continues to be small and often negative, in contrast to real world financial markets. Similar to Campbell (1994) and Lettau (2001), our approach has adopted the method of undetermined coefficients for the model, and as did Lettau (2001), we have derived closed-form solutions for a variety of prices for financial assets.

We develop a model that accounts for the equity premium and average risk-free rate. However, our study differs from previous studies of asset pricing puzzles. However, this study differs from previous studies of asset pricing puzzles in three *novel* ways.

First, we incorporate an investment-specific technology shock into our asset pricing model. Typical economic models support a view that the main economic fluctuations can be explained by productivity shocks. In contrast, Greenwood, Hercowitz and Huffman (1988) and Greenwood, Hercowitz and Krusell (1997, 1998) argue that it is shocks to the marginal efficiency of investment that are important in generating output fluctuations, rather than shocks to the production function. By introducing an investment-specific technology shock into our model, we emphasise the formation of new capital within an asset pricing model. It is important to note how investment-specific technology shock has different effects on a variety of prices for financial assets and the determination of risk pemia, in contrast to the standard productivity shock.

Second, we allow for a capacity utilization variable and variable depreciation rate in the production function to account for the asset pricing puzzles. Technically, capacity utilization has a crucial role for intratemporal propagation of the real shock to other variables under the existence of the investment-specific technology. Moreover, capacity utilization allows that a positive investment-specific technology shock stimulates the formation of "new" capital and a more intensive utilization, and also accelerates depreciation of "old" capital with variable depreciation rates. By incorporating the neoclassical framework with endogenous capacity utilization, we find that a capacity utilization variable has important consequences for real variables as well as the asset pricing variables.

Third, we consider capital adjustment costs in investment to account for the asset pricing puzzles Several papers prove that the introduction of capital adjustment costs improves the prediction ability of asset pricing models. We incorporate capital adjustment costs of the form used in the Greenwood, Hercowitz and Krusell (1997) model into our model. Finally, by incorporating investment-specific technology, capacity utilization and capital adjustment costs in our dynamic asset pricing model, we also derive closed-form solutions for a variety of prices for financial assets in an equilibrium economy. The structure of the article is as follows. Section 5.2 provides some stylized facts from asset markets using US financial data. Section 5.3 develops a dynamic asset pricing model incorporating investment-specific technology, capacity utilization and capital adjustment costs and presents the analytical derivation of closed-form solutions for asset prices.

Section 5.4 addresses the empirical results and Section 5.5 presents our concluding remarks.

5.2 Some Stylized Facts on US Asset Markets

Table 5.1 presents some important and well-known stylized facts about asset markets using US financial data from the first quarter, 1959 to the fourth quarter, 1995. These stylized facts play a key role as a benchmark to evaluate the implications of the asset pricing models. All data are sampled at quarterly frequency and taken from Ibbotson Associates. Returns of assets are reported in real values and percentage per quarter. As shown in Table 5.1, stocks, measured here as Standard & Poor's 500 stock return, have a substantially higher mean return than Treasury bills with one month maturity and are also more volatile. Treasury bills have a low mean return and do not vary much over time. Long Government bonds with 20 years maturity have a slightly higher average return than Treasury bills. However, the volatility of both assets is not much different. Asset premia are asset returns minus returns to the Treasury bills. As expected, the mean equity premium is much higher than the long bond premium. The volatility of equity premiums is also much larger than that of long bond premiums.

Finally, the slope of the capital market line, known as the Sharpe ratio, is 0.12 for the period 1959–95 using Large Company Stocks as a proxy for the market portfolio. It is standard practice to use a broad stock market index as an approximation for the market portfolio.

Table 5.1: Basic statistics of asset real returns

	1959 –74		1975 –95		1959 –95	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
Treasury bills	0.18	0.46	0.45	0.70	0.33	0.68
Long government bond premium	–0.34	3.20	0.89	6.38	0.36	5.27
Equity premium	–0.33	9.75	2.11	7.58	1.05	8.64
Sharpe ratio	0.03		0.28		0.12	

Note: Returns are measured at quarterly frequency. Units are percent per quarter. Risk premia are computed as the difference between the asset return and the Treasury bill rate. The Sharpe ratio is calculated as the mean of equity premium divided by its standard deviation. Source: Ibboston Association

5.3 The Model

This section develops a dynamic asset pricing model for making inferences about the equity premium in the presence of investment-specific technology, capacity utilization and capital adjustment costs. To illustrate the usefulness of our approach, this section starts with the model specification, which incorporates the above features, and is followed by the analysis of steady-state of our model. The model then becomes a system of log-linear differential equations, which can be studied by the method of undetermined coefficients. Finally, we derive closed-form solutions for a variety of prices for financial assets and then investigate the asset pricing implications of our model.

5.3.1 Specification of the Model

In this chapter, our model is a growth model. The stationary cardinal utility of previous chapters is suitable for a stationary model but not for a growth model. In other words, the steady-state growth rate is not computed in the stationary cardinal utility. Accordingly, unlike in previous chapters, we assume that the agent has a time-separable power utility in consumption and labor. A time-separable power utility can catch the steady-state growth rate. We consider a model in which agents derive utility from consumption (C_t) and leisure input $(1-N_t)$:

$$u(t) = \beta^t\left[\frac{C_t^{1-\gamma}-1}{1-\gamma} + \theta\frac{(1-N_t)^{1-\gamma_n}-1}{1-\gamma_n}\right] \quad (5.1)$$

where β is the fixed discount rate, γ and γ_n are coefficients of the risk aversion in consumption and labor, respectively. Note that the special cases $\gamma_n = 0$ and $\gamma_n = 1$ correspond to a model with linear utility in labor and a model with fixed labor, respectively. Output is given by a standard Cobb-Douglas production function using capital and labor. Using the notation Y_t for output, K_t for capital, and N_t for labor input and h_t for capacity utilization, and G for steady-state growth rate, the production function is:

$$Y_t = (h_t K_t)^{\alpha}(G^t N_t)^{1-\alpha} \quad (5.2)$$

Note that it differs from the standard neoclassical specification[24] solely by the inclusion of a variable rate of capacity utilization (h_t). The parameter α is referred to as the capital share. In the spirit of Greenwood, Hercowitz and Huffman (1988), the variable h_t determines the flow of capital services $(h_t K_t)$ and represents the intensity of the use of capital, that is, the speed of operation or the number of hours per period the capital is used. The law of motion for capital stock[25] follows:

$$K_{t+1} = K_t(1-\delta(h_t)) + Z_t i_t \quad (5.3)$$

$$\text{and } \delta(h_t) = \frac{h_t^{\omega}}{\omega} \quad \omega > 1.$$

where i_t denotes gross investment, and δ is not a constant rate of depreciation but a non-negative function of capacity utilization (h_t).

The capital accumulation and production capacity in period t+1 depend on both investment and investment-specific technology shock on (Z_t) affecting the productivity

[24] The standard neoclassical specification defines the production function as follows

$$Y_t = Z_t K_t^{\alpha}(G^t N_t)^{1-\alpha}$$

where Z_t denotes standard productivity shock. To compare the model with investment-specific technology shock with the model with standard productivity shock, we assume that the shocks are stationary and production functions incorporate a steady-state growth factor, G.

[25] In the standard neoclassical specification, the law of motion for capital stock is as follows:

$$K_{t+1} = K_t(1-\delta) + i_t$$

In the standard model, the capacity utilization is not allowed, and the depreciation rate is fixed.

of new capital goods. We define $z_t = \log(Z_t)$ and we assume that investment-specific technology shock follows an AR(1) process with AR-parameter ψ :

$$z_t = \psi z_{t-1} + \varepsilon_t \qquad \sim i.i.d\, N(0,\ \sigma_\varepsilon^2) \tag{5.4}$$

Now, we allow for adjustment cost[26] of investment for output Y_t :

$$Y_t - \frac{\phi(K_{t+1}/Z_t - G K_t/Z_t)^2}{K_t/Z_t} = C_t + i_t \tag{5.5}$$

Using the equations (5.2) and (5.3), we can express C_t as:

$$C_t = (h_t K_t)^\alpha (G^t N_t)^{1-\alpha} - \frac{\phi(K_{t+1}/Z_t - G K_t/Z_t)^2}{K_t/Z_t} - \frac{K_{t+1}}{Z_t} + \frac{K_t}{Z_t}(1 - \frac{1}{\omega} h_t^\omega) \tag{5.6}$$

Using equations (5.1) and (5.6), we derive the first-order conditions for K_{t+1}, N_t and h_t as:

[26] This adjustment costs are zero when the model economy is in the steady-state growth. The adjustment costs of equation (5.5) follow the formation of adjustment costs of Greenwood, Hercowitz and Krusell (1997 and 2000). Moreover, in the case of the model with investment-specific technology shock, from Greenwood, Hercowitz and Huffman (1988), investment is defined by

$$i_t = \frac{K_{t+1}}{\varepsilon_t} - (1-\delta)\frac{K_t}{\varepsilon_t}$$

Our understanding is that capital adjustment costs are used to alleviate high fluctuations of investment. Accordingly, our opinion is that it is desirable that the adjustment cost formation is based on investment.

$$(K_{t+1}):\ C_t^{-\gamma}\left[\frac{1}{Z_t} + \frac{2\phi(K_{t+1}/Z_t - G K_t/Z_t)}{K_t}\right] \tag{6.7}$$

$$= \beta C_{t+1}^{-\gamma}\left[\frac{2\phi(K_{t+2}/Z_{t+1} - G K_{t+1}/Z_{t+1})G}{K_{t+1}} + \frac{\phi(K_{t+2}/Z_{t+1} - G K_{t+1}/Z_{t+1})^2}{K_{t+1}^2/Z_{t+1}} + \alpha h_{t+1}(h_{t+1}K_{t+1})^{\alpha-1}(GN_{t+1})^{1-\alpha} + (1 - \frac{1}{\omega}h_{t+1}^\omega)\frac{1}{Z_{t+1}}\right]$$

$$(N_t):\ \theta(1-N_t)^{-\gamma_n} = C_t^{-\gamma}\left[(1-\alpha)(h_t K_t)^\alpha N_t^{-\alpha}\right] \tag{5.8}$$

$$(h_t):\ \alpha(h_t K_t)^{\alpha-1} K_t N_t^{1-\alpha} = \frac{1}{Z_t} K_t h_t^{\omega-1} \tag{5.9}$$

Using the relationship, $MU_{C,t+1}/MU_{C,t} = \beta R_{t+1}$, we obtain the variable R_{t+1}, the gross rate of return on a one-period investment in capital:

$$R_{t+1} = \left[\frac{2\phi(K_{t+2}/Z_{t+1} - G K_{t+1}/Z_{t+1})G}{K_{t+1}} + \frac{\phi(K_{t+2}/Z_{t+1} - G K_{t+1}/Z_{t+1})^2}{K_{t+1}^2/Z_{t+1}} + \alpha h_{t+1}(h_{t+1}K_{t+1})^{\alpha-1}(GN_{t+1})^{1-\alpha} + (1 - \frac{1}{\omega}h_{t+1}^\omega)\frac{1}{Z_{t+1}}\right] \Big/ \left[\frac{1}{Z_t} + \frac{2\phi(K_{t+1}/Z_t - G K_t/Z_t)}{K_t}\right] \tag{6.10}$$

From equations (5.7) and (5.10), we obtain:

$$C_t^{-\gamma} = \beta E_t C_{t+1}^{-\gamma} R_{t+1} \tag{5.11}$$

These first-order conditions for utility maximization have a number of important features. Equations (5.8) and (5.9) show an intratemporal relationship. Specially, equation (5.9) shows the relationships across labor, capital utilization and current capital stock. Accordingly, by incorporating equation (5.8) into equation (5.9), the effect of the shock is transmitted to labor supply through capacity utilization intratemporarily and consumption intertemporarily.

5.3.2 Steady State Growth

We now describe the steady-state or balanced growth path of our model. At the steady-state growth path, it is supposed that technology, capital, output and consumption all grow at a constant common rate, 0.5% from Lettau (2001). For instance, we express the growth rate G as $G = K_{t+1}/K_t = C_{t+1}/C_t = Y_{t+1}/Y_t$ and express equation (5.11) as $C_{t+1}/C_t = \beta R_{t+1}$. Since the growth rate of return on capital R_{t+1} becomes a constant R in steady-state economy, the condition (5.11) becomes $G = \beta R$ or in logs (denoted by lower-case letters):

$$g = \sigma \log\beta + \sigma r \qquad (5.12)$$

where we define the elasticity of intertemporal substitution in consumption in the same way as Campbell (1994), $\sigma = 1/\gamma$. From equation (5.11), by substituting $C_{t+1} = GC_t$, $C_t^{-\gamma} = \beta E_t C_{t+1}^{-\gamma} R_{t+1}$ and using equation (5.10) with steady-state economy, we obtain:

$$1 = \beta G^{-\gamma}\left[\alpha\frac{Y}{K} + 1 - \frac{1}{\omega}h^{\omega}\right] \qquad (5.13)$$

The steady-state condition of equation (5.8) implies:

$$\frac{\theta}{(1-N)^{\gamma_n}} = \frac{1}{C^{\gamma}}\left[(1-\alpha)(Kh)^{\alpha}N^{-\alpha}\right] \qquad (5.14)$$

Also, the steady-state condition of equation (5.9) shows:

$$\alpha\frac{Y}{K} = h^{\omega} \qquad (5.15)$$

Using the equations (5.13) and (5.15), we obtain the constant steady-state output-capital ratio as:

$$\frac{Y}{K} = \frac{1}{\alpha}h^{\omega} = \frac{\omega r}{\alpha(\omega - 1)} \qquad (5.16)$$

Similarly, we can show the constant steady-state consumption-output ratio as:

$$\frac{C}{Y} = 1 - \frac{\alpha g(\omega - 1) + \alpha r}{\omega r} \qquad (5.17)$$

Note the parameters g, the log technology growth rate; r, the log real return on capital; α, the exponent on labor and capacity utilization in production function, or equivalently labor's share of output; and ω, the elasticity of depreciation with respect to capacity utilization rate.

5.3.3 The Method of Undetermined Coefficients

Campbell (1994) has presented a solution based on log-linear approximation for a stochastic model, using the method of undetermined coefficients (MUC). The key feature of MUC is a feedback rule expressing the endogenous variables (in logs) as a linear function of the logs of the state variables. We utilize Campbell's approach and derive closed-form solutions for the elasticities with respect to current capital and investment-specific technology shock. Hence, we can express the optimal consumption choice and capital choice as a linear function of current capital and investment-specific technology shock as follows:

$$c_t = \eta_{ck} k_t + \eta_{cz} z_t \tag{5.18}$$

$$k_{t+1} = \eta_{kk} k_t + \eta_{kz} z_t \tag{5.19}$$

We use the notation η_{yx} for the partial elasticity of y with respect to x. Note that the consumption elasticities (η_{ck} and η_{cz}) and the capital elasticities (η_{kk} and η_{kz}) are complicated functions of the deep parameters of the model and determine the dynamic behavior of the economy. Using the lag-operator L, we rewrite equations (5.18) and (5.19). Equation (5.19) gives the capital stock as:

$$k_{t+1} = \frac{\eta_{kz}}{1-\eta_{kk}L} z_t \tag{5.20}$$

With investment-specific technology (z_t) following an AR(1) process:

$$z_t = \frac{1}{1-\psi L}\varepsilon_t \tag{5.21}$$

Therefore, these two equations suggest that the capital stock follows an AR(2) process:

$$k_{t+1} = \frac{\eta_{kz}}{(1-\psi L)(1-\eta_{kk} L)}\varepsilon_t \tag{5.22}$$

Similarly, we can show that consumption follows an ARMA(2,1) process:

$$c_t = \frac{\eta_{cz} + (\eta_{ck}\eta_{kz} - \eta_{cz}\eta_{kk})L}{(1-\psi L)(1-\eta_{kk} L)}\varepsilon_t \tag{5.23}$$

Using the quarterly data, we calibrate the benchmark parameters. We compute the elasticities for a variety of parameter values of the model. The relevant parameters are

mainly taken from Lettau (2001). The capital share parameter is chosen as $\alpha = 0.33$, and the growth rate g is set to 0.5% and the steady-state level of the return to capital r is fixed at 1.5%. The coefficient of the consumption elasticity (β) is calculated as 0.99 by using equation (5.12). The elasticity of depreciation with respect to utilization (ω) is set as 1.6 under condition of $\delta = 0.025$.

The stochastic process for investment-specific technology is estimated by using quarterly data of the ratio of implicit price deflator of non-durable consumption goods to implicit price deflator of producer's durable equipment. Greenwood, Hercowitz and Krusell (1997, 1998) first introduced the relative price of equipment to compute the investment-specific technology. However, because they used annual data, we calculate the simplified relative price of equipment for quarterly data following their methodology.

Figure 5.1 shows the reverse of the relative prices, indicating both the relatively decreasing equipment price and the abrupt technological change in 1974.

By using the relative price (q), we estimate the shock process as follows:

$$\ln q_t = \text{constant} + \text{trend} \times \ln\gamma_q + z_t$$

where $z_t = \psi z_t + \varepsilon_t$ with $0 < \psi < 1$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon)$

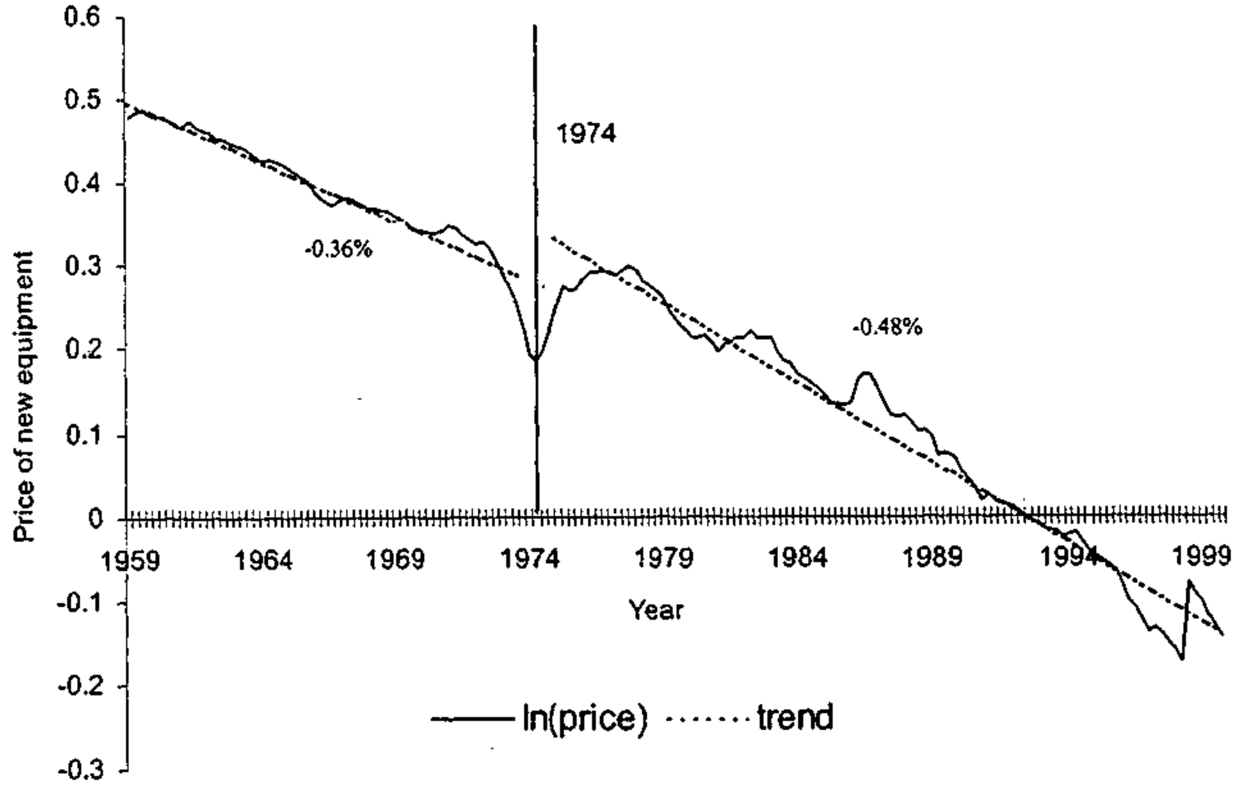
Using the quarterly 1959–99 data, the estimated parameters are:

$$\ln\gamma_q = \underset{(62.61)}{0.004}, \quad \psi = \underset{(37.16)}{0.94}, \quad \sigma = 0.011, \text{ with } D.W = 1.54$$

where the numbers in parentheses are t-statistics.

In the model, the parameters of persistent investment-specific technology shock are set as 0.0, 0.5, 0.95 and 1.0.

Figure 5.1: Price of new equipment[27]



5.3.4 Asset Prices

Despite the fact that dynamic equilibrium models have successfully explained some of the key empirical regularities and comovements of aggregate quantities such as output,

[27] The price of new equipment is computed as the logged value of implicit deflator index of equipment investment divided by implicit deflator index of non-durable consumption expenditure.

consumption, and investment, little attention has been paid to the implications for financial asset prices. Since asset markets and asset prices contain valuable information on intertemporal decisions of agents, data on prices for financial assets as well as real variables should be used to evaluate the performance of models.

The objective of this sub-section is to investigate the asset pricing implications of our model and to derive closed-form solutions[28] for a variety of prices for financial assets in the dynamic asset pricing model. More specifically, we aim to derive explicit solutions for risk premia for equity and long-run bonds as well as the stochastic process of the risk-free short-run interest rate. Let $R_{t,t+1}$ be the gross return on an asset held from period t to t+1 ($R_{t,t+1} = (P_{t+1} + D_{t+1})/P_t$). This definition is straightforward. The price and the dividend of the asset in period t are denoted by P_t and D_t, respectively. Then, we define the risk-free rate, the return of the one-period real bond as R^f. The risk-free interest rate is the inverse of the expected marginal rate of intertemporal substitution in consumption.

$$C_t^{-\gamma} = \beta E_t C_{t+1}^{-\gamma} R_{t+1} \qquad (5.24)$$

Under the supposition that investment-specific technology shock follows log-normal distribution, the model is approximately linear in logs. We express the risk-free rate in logs as:

$$r^f_{t,t+1} = -\log(\beta) + E_t \gamma \Delta c_{t+1} + \frac{\gamma^2}{2} Var_t \Delta c_{t+1} \qquad (5.25)$$

[28] The closed-form solution for the yield-to-maturity on a riskless bond was first derived by Breedon (1986), under log-normality.

The traditional consumption CAPM, or CCAPM, implies that the (log) expected excess return of an asset is determined by the covariance of the asset return with consumption:

$$r^{rp}_{t,t+1} \equiv r^{e}_{t,t+1} - r^{f}_{t,t+1} = \mathrm{Cov}_t(\gamma\Delta c_{t+1}, r_{t,t+1}) \quad (5.26)$$

where $r^{rp}_{t,t+1}$ is log of expected risk premia and $r^{e}_{t,t+1} = \log E_t R_{t,t+1}$ is logarithm of the expected gross return.

Since unexpected consumption growth and asset returns are expressed as a linear function of the investment-specific technology shocks, we now express the log of expected risk premia as:

$$r^{rp}_{t,t+1} = \mathrm{Cov}(\gamma\eta_{cz}\varepsilon_{t+1}, \eta_{rz}\varepsilon_{t+1}) = \gamma\eta_{cz}\eta_{rz}\sigma^2_{\varepsilon} \quad (5.27)$$

Equation (5.27) indicates that the risk premia are determined by the exogenous parameter (σ^2_{ε}), consumption elasticity (η_{cz}) and the elasticity of the asset return with respect to the technology shock (η_{rz}).

Campbell and Shiller (1988) decompose unexpected asset returns in an approximate log-linear framework. The decomposition in our notation is as follows:

$$r_{t,t+1} - E_t r_{t,t+1} \approx (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j} - (E_{t+1} - E_t)\sum_{j=2}^{\infty}\rho^{j-1} r_{t+j-1,t+j} \quad (5.28)$$

Equation (5.28) is derived from a log linearization of the gross return, $(R_{t,t+1} = (P_{t+1} + D_{t+1})/P_t)$. The parameter ρ in equation (5.28), which is a function of the

dividend-price ratio of an asset, can be computed as $\rho = \exp(g - r) \approx 0.99$. Equation (5.28) says that an upward surprise in returns today must correspond to an unexpected dividend on wealth today (the first term in the first sum on the right-hand side of the equation), or to news that future dividends will be higher (the remaining terms in the first sum), or to a downward revision in expected future growth (the second sum on the right-hand side). In other words, equation (5.28) implies that unexpected returns can occur both through revisions in expectations in future dividend growth (with a positive sign) and news about future expected returns (with a negative sign). Therefore, using equation (5.28) we can calculate expected dividends news, expected returns news and the elasticities of asset returns with respect to investment-specific technology shocks. First, we derive the risk-free rate, and then we compute the risk premia for a long-term bond and equity in our dynamic asset pricing model.

5.3.4.1 The Risk-Free Rate

By substituting $\eta^2_{cz}\sigma^2_{\varepsilon}$ term for the conditional variance of consumption growth $(\mathrm{Var}_t\Delta c_{t+1})$ derived from equation (5.18) into equation (5.25), we obtain:

$$r^{f}_{t,t+1} = \gamma E_t\Delta c_{t+1} + \frac{\gamma^2}{2}\eta^2_{cz}\sigma^2_{\varepsilon} - \log\beta \quad (5.29)$$

Given that log consumption follows an ARMA(2,1) process, we can compute the expected consumption growth, and also express the risk-free rate in terms of the parameters as follows:

$$r^f_{t,t+1} = \frac{\gamma}{1-\psi L}\left[\frac{(1-L)}{(1-\eta_{kk}L)}\eta_{ck}\eta_{kz} - (1-\psi)\eta_{cz}\right]\varepsilon_t + \frac{\gamma^2}{2}\eta_{cz}^2\sigma_\varepsilon^2 - \log\beta \qquad (5.30)$$

Equation (5.30) indicates that the risk-free rate follows an ARMA(2,1) process and is determined by expected consumption growth one period in the future. An investment-specific technology shock influences expected consumption growth through direct effect and indirect effect. The direct effect of the shock is measured by the term involving η_{cz}. The indirect effect through capital stock causes the risk-free rate to increase. It is noticeable that the persistent parameter ψ plays an important role in determining the reaction of the risk-free rate. For example, the completely transitory shocks ($\psi = 0$) imply that the risk-free rate is just white noise. The risk-free rate is decreasing after a positive shock because consumption is expected to revert back to its long-term mean very quickly. In contrast, the completely permanent shocks ($\psi = 1$) show that the initial reaction of the risk-free rate after a positive technology shock is unambiguously positive, since η_{cz} and η_{kz} are positive. The risk-free rate will decrease back to its steady-state level at rate η_{kk} after the initial jump upwards.

For the purpose of the computation of risk premia of long bonds and equity, we now rewrite the ARMA (2,1) process of the risk-free rate of equation (5.30) into its MA(∞) representation. The risk-free rate can be expressed as:

$$r^f_{t,t+1} = \frac{1}{\psi - \eta_{kk}}\sum_{s=0}^{\infty}\left[\psi^{s+1} - \eta_{kk}^{s+1} + \theta\left(\psi^s - \eta_{kk}^s\right)\right]w_{t-s} \qquad (5.31)$$

where $\theta = -\frac{(\psi-1)\eta_{kk}\eta_{cz} + \eta_{ck}\eta_{kz}}{(\psi-1)\eta_{cz} + \eta_{ck}\eta_{kz}}$, $w_t = [(\psi-1)\eta_{cz} + \eta_{ck}\eta_{kz}]\varepsilon_t$

The unconditional standard deviation of $r^f_{t,t+1}$ is given by:

$$\sigma\left(r^f_{t,t+1}\right) = \gamma\left[\frac{\{(\psi-1)\eta_{cz} + \eta_{ck}\eta_{kz}\}}{\psi - \eta_{kk}}\right]\left[\frac{(\psi+\theta)^2}{1-\psi^2} - \frac{2(\psi+\theta)(\eta_{kk}+\theta)}{1-\psi\eta_{kk}} + \frac{(\eta_{kk}+\theta)^2}{1-\eta_{kk}^2}\right]^{\frac{1}{2}}\sigma_\varepsilon \qquad (6.32)$$

As in the case of equation (5.30), equation (5.32) can be simplified for the special cases of permanent and transitory shocks. The volatility of the risk-free rate in US data is fairly low, with a standard deviation of 0.68%. In calculating the volatility of the risk-free rate, we utilize equation (5.32).

5.3.4.2 Long-Term Real Bond

A long-term real bond is supposed to have a fixed dividend, and thus it is dependent on the second term of equation (5.28), which shows revision in expectations in future return. Using equation (5.31), the Moving Average (MA) representation of the risk-free rate, we express the unexpected return of the long-term bond as a function of technology shock and the elasticities of the model. Substituting equation (5.31) into equation (5.28), we obtain:

$$r^{lb}_{t,t+1} - E_t r^{lb}_{t,t+1} = -E_{t+1}\sum_{j=0}^{\infty}\rho^j \frac{1}{\psi - \eta_{kk}}\sum_{s=0}^{\infty}\left[\psi^{s+1} - \eta_{kk}^{s+1} + \theta(\psi^s - \eta_{kk}^s)\right]w_{t-s+j} \tag{5.33}$$

$$+E_t\sum_{j=1}^{\infty}\rho^j \frac{1}{\psi - \eta_{kk}}\sum_{s=0}^{\infty}\left[\psi^{s+1} - \eta_{kk}^{s+1} + \theta(\psi^s - \eta_{kk}^s)\right]w_{t-s+j}$$

$$= -\frac{1}{\psi - \eta_{kk}}\left[\frac{\rho(\psi+\theta)}{1-\rho\psi} - \frac{\rho(\eta_{kk}+\theta)}{1-\rho\eta_{kk}}\right]w_{t+1} \tag{5.34}$$

$$= -\frac{\rho(1+\theta\rho)}{(1-\rho\eta_{kk})(1-\rho\psi)}w_{t+1} \tag{5.35}$$

By inserting equation (5.35) into equation (5.27), we obtain:

$$r^{lb}_{t,t+1} = \gamma\eta_{cz}\eta^{lb}_{rz}\sigma^2_\varepsilon \tag{5.36}$$

$$\eta^{lb}_{rz} = \frac{\gamma\rho}{1-\rho\psi}\left[(1-\psi)\eta_{cz} - \frac{1-\rho}{1-\rho\eta_{kk}}\eta_{ck}\eta_{kz}\right] \tag{5.36'}$$

5.3.4.3 Equity

The dividend payment of a firm, defined as D^{eq}_t, is a function of the marginal product of capital net of depreciation.

$$D^{eq}_t = \alpha\frac{Y_t}{K_t} + \frac{1}{Z_t} - \frac{1}{\omega}\alpha\left(\frac{Y_t}{K_t}\right) + \frac{1}{Z_t}\phi\left(\frac{K_{t+1}}{K_t}\right)^2 - \frac{1}{Z_t}\phi G^2 \tag{5.37}$$

In logs-notation, equation (5.37) can be approximated as:

$$\log D^{eq}_t = d^{eq}_t = \frac{r}{1+r}y_t + \frac{2\phi(1+g)^2}{1+r}k_{t+1} - \left(\frac{r+2\phi(1+g)^2}{1+r}\right)k_t - \frac{1}{1+r}z_t \tag{5.38}$$

Log-dividend may be expressed in terms of the state variables:

$$d^{eq}_t = \eta_{dk}k_t + \eta_{dz}z_t \tag{5.39}$$

where $\eta_{dk} = \dfrac{r\eta_{yk} + 2\phi(1+g)^2\eta_{kk} - r - 2\phi(1+g)^2}{1+r}$

$$\eta_{dz} = \frac{r\eta_{yz} + 2\phi(1+g)^2\eta_{kz} - 1}{1+r}$$

The dividend term can now be expressed as an ARMA(2,1) process:

$$d_t = \frac{\eta_{dz} + (\eta_{dk}\eta_{kz} - \eta_{dz}\eta_{kk})L}{(1-\eta_{kk}L)(1-\psi L)}\varepsilon_t \tag{5.40}$$

We use this ARMA(2,1) representation to compute the news to expected discounted dividend growth in equation (5.28). A similar result of the news to expected return yields:

$$(E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j \Delta d_{t+1+j} = \eta^d_{rz}\varepsilon_{t+1} \tag{5.41}$$

where $\eta^d_{rz} = \dfrac{1-\rho}{1-\rho\psi}\left(\eta_{dz} + \dfrac{\rho\eta_{dk}\eta_{kz}}{1-\rho\eta_{kk}}\right)\varepsilon_{t+1}$

Finally, the risk premium for the long-run equity is derived as:

$$r^{eq}_{t,t+1} = Cov(\gamma\eta_{cz}\varepsilon_{t+1}, \eta^{eq}_{rz}\varepsilon_{t+1})$$

$$= \gamma\eta_{cz}(\eta^d_{rz} + \eta^{lb}_{rz})\sigma^2_\varepsilon$$

where $\eta^{eq}_{rz} = \eta^d_{rz} + \eta^{lb}_{rz}$

The long risk premium from equation (5.27) is:

$$r^{steq}_{t,t+1} = \gamma\lambda\eta_{yz}\eta_{cz}\sigma^2_\varepsilon \tag{5.42}$$

So far, we have computed an analytical expression for the stochastic process of the risk-free rate, the risk premia for a real bond with infinite maturity and claims to equity of a firm with infinite lifetime, and a one-period firm in a dynamic asset pricing model.

Finally, the Sharpe ratio in our model is expressed as:

$$SR = \gamma|\eta_{cz}|\sigma_\varepsilon$$

5.4 Empirical Results

Based on the analytical solutions from the previous section, in this section we compute the risk premia for a variety of parameters of the dynamic asset pricing model which incorporates investment-specific technology shocks, capacity utilization and adjustment costs. Then the effect of investment-specific technology shock on risk premia of financial asset is investigated and more importantly, the performance of our model is compared with the models incorporating standard productivity shock with and without adjustment costs. The key feature of our empirical results is that they show that our model performs better than the models with the standard productivity shock and adjustment costs in terms of mimicking the risk premia of firms' equity, long-term real bonds and Sharpe ratio.

5.4.1 The Elasticities in the Three Different Models

In this sub-section, we use the analytical solutions to compute the risk premia for a variety of parameters of our model. Table 5.2.A reports numerical values of the elasticities η_{ck}, η_{cz} and η_{kk}, η_{kz} for the benchmark parameters discussed above and for various ψ when $\gamma = 2.0$ and $\gamma_n = 2.0$. Table 5.2.A also shows the elasticities from three models: model A is with investment-specific technology shock, capacity utilization and

adjustment costs, model B is with productivity shock, and model C is with productivity shock and adjustment costs. To gain more insights into the determination of asset prices, we compare three different models whose relative performance are so far unknown. For example, we show how investment-specific technology shock or technology shocks with and without adjustment costs can effect differently on risk premia of financial assets.

In Table 5.2.A, for the risk-aversion coefficients in consumption and labor, we pick up $\gamma = 2.0$ and $\gamma_n = 2.0$. However, the empirical results using the different γ and γ_n values are reported in the Appendix 5.1. For example, γ and γ_n are set to cover a range of possibilities. The persistence parameter ψ is set equal to 0.0, 0.5, 0.95, and 1.

One distinguished feature of our model is that the effect of an investment-specific technology shock on consumption and next period's capital stock is unlike standard productivity shock. The investment-specific technology shock in our model affects current consumption in *three* channels. First, there are *intratemporal* substitution effects between consumption and leisure, which are caused by the increase of the marginal product of labor and increase in consumption. Second, there are *intertemporal* substitution effects between consumption and investment, which are caused by the rise of marginal product of newly produced capital and decrease in consumption. Third, there are *income* effects, which are caused by productivity increase and raised consumption.

The standard productivity shock has effects on consumption in three channels similar to investment-specific technology shock. However, the difference between investment-specific technology shock and standard productivity shock is that intertemporal substitution effects between consumption and future capital are stronger in the model with the investment-specific technology shock than in the model with productivity shock. Accordingly, a positive investment-specific technology shock can decrease consumption differently from productivity. As can be seen from Table 5.2.A, η_{cz} shows both negative and positive signs.

Another difference between two models is that capacity utilization has a role of *intratemporal* transmission. A positive shock increases capacity utilization rate, and the increased capacity utilization raises labor supply and then next period's capital stock, indicating that consumption decreases. Moreover, by adjusting the use of current capital, capacity utilization makes current capital as if current capital stock is flexible. As can be seen from Table 5.2.A, η_{ck} is lower in model A than in model B and C. On the other hand, η_{kk} is higher than in model A than in model B and model C.

5.4.2 The Implications of the Elasticities

Several aspects of empirical results are worth noting from Table 5.2.A. The coefficients η_{ck} and η_{kk} do not depend on the persistence of technology shocks ψ. On the other hand, η_{cz} increases in ψ. In other words, as the persistence of the shock rises, income effects of

the shock overwhelm substitution effects. Hence, η_{cz} increases with ψ. In another aspect, η_{kz} decreases in ψ. As ψ rises, the substitution effect between consumption and next period's capital is weaker, and thus η_{cz} decreases.

Tables 5.3.A1, 5.4.A1, and 5.5.A1 of the Appendix 5.1 present the values of the elasticities in three models when risk aversion coefficients in consumption are 0.01, 1.0, 2.0, and 100. η_{ck} decreases with γ in three models. As Campbell (1994) defined, risk aversion coefficient γ can be defined as the elasticity of intemporal substitution γ. γ also is an important factor to decide the interest rate from equation (5.24). As γ increases, the interest rate increases, which causes a negative substitution effect on current consumption. Accordingly, η_{ck} decreases in γ. In another aspect, since the decrease of η_{ck} is caused by a negative substitution effect on current consumption, η_{kk}, which causes increase of future consumption, increases in γ. η_{cz} increases with γ for low values of γ, but decreases with γ for high values of γ. The effects of γ on η_{cz} are different according to the sizes of γ and ψ. According to Campbell (1994), a technology shock has an income effect which is stronger when the shock is more persistent at high values of γ, hence η_{cz} increases with ψ.

From Tables 5.3.A1 and 5.3.A1, values of η_{cz} are different in signs and turning point in two models. For example, from Table 5.3.A1, η_{cz} increases in γ except when persistence

ψ is 1, while from Table 5.5.A1, η_{cz} increases in γ for low values of γ, and then decreases in γ for high values of γ. From models A and C, the model with investment-specific technology shock has more dominant substitution effects on consumption than the model with standard productivity shock. η_{kz} decreases with γ in model A, while η_{kz} increases with γ for low values of ψ, but decreases with γ for high values of ψ in model C.

Tables 5.3.B1, 5.4.B1, and 5.5.B1 of Appendix 5.1 illustrate the values of the elasticities of three different models when risk aversion coefficients in labor, γ_n is 0.01, 1.0, 2.0 and 100. η_{ck} and η_{kk} increase with γ_n in all of models A, B, and C even though the increase is not high. γ_n can be defined as the elasticity of intertemporal substitution, $1/\gamma_n$. Increase of γ_n means that the marginal utility of leisure rises, and thus leisure increases and labor supply decreases. In the log-linear approximation of equation (5.8), consumption has a negative relationship with labor supply. Accordingly, as γ_n increases, consumption also increases. η_{cz} increases with γ_n in model A, while η_{cz} decreases with γ_n for permanent persistence (ψ = 1) in model C. As previously explained, when the shock is more persistent, income effect is stronger. Model A shows stronger substitution effect on consumption than models B and C. η_{kz} decreases with γ_n in all of models A, B and C.

Table 5.2.A: Consumption and next period's capital elasticities to current capital and shocks (γ = 2.0, γ_n = 2.0)

ψ	model A		model B		model C	
0.00	0.22	−0.34	0.36	0.06	0.34	0.19
	0.97	0.13	0.95	0.17	0.96	0.14
0.50	0.22	−0.33	0.36	0.09	0.34	0.22
	0.97	0.12	0.95	0.16	0.96	0.13
0.95	0.22	−0.18	0.36	0.36	0.34	0.42
	0.97	0.09	0.95	0.10	0.96	0.09
1.00	0.22	0.04	0.36	0.64	0.34	0.66
	0.97	0.04	0.95	0.05	0.96	0.04

Note: The top two numbers in each group are η_{ck} and η_{cz}, where η_{ck} is the elasticity of consumption with respect to the capital stock and η_{cz} is the elasticity of consumption with respect to investment-specific technology shock. The bottom two numbers in each group are η_{kk}, η_{kz}, where η_{kk} is the elasticity of next period's capital stock with respect to this period's capital stock and η_{kz} is the elasticity of next period's with respect to this period's investment-specific technology shock. Three models are compared: model A with investment-specific technology shock and adjustment costs; model B with productivity shock; model C with productivity shock and adjustment costs.

5.4.3 Risk Premia

Table 5.2.B shows the resulting risk premia for financial assets. Three risk premia from three different models are compared, indicated as model A, model B and model C. To cover a range of possibilities, we vary the persistence of the investment-specific

technology shock from completely transitory ($\psi = 0$), intermediate ($\psi = 0.5, 0.95$) to permanent values ($\psi = 1$). Risk aversion in consumption γ is set at 2.0, the same as risk aversion in labor, γ_n.

First, the Sharpe ratio is dependent on the size of γ (risk-aversion), the absolute value of η_{cz} (the elasticity of consumption with the shock) and σ_ε (standard deviation of the shock). In the productivity shock case, From models B and C, the Sharpe ratio is increasing in ψ because the η_{cz} is increasing in absolute terms. However, from model A, in the case where investment-specific technology shock is incorporated into the asset pricing model, the Sharpe ratio is decreasing in ψ because persistence parameter ψ is positively correlated with η_{cz}, and thus the consumption reaction to shocks η_{cz} is decreasing in absolute terms. Overall, the Sharpe ratio is relatively small, compared to the stylized value of the Sharpe ratio. This implies that risk premia of financial assets will generally be small in the model, since the marginal rate of substitution is not volatile enough. However, an important feature of our empirical simulation is that our model (model A) is performing better than the standard model with productivity shock (model B) and capital adjustment costs (model C). For example, as long as the persistence of the shock has ranged from 0.0 to 0.5, the Sharpe ratio from model A is higher than those of models B and C.

Second, we now compare risk premia of long-term bonds and long-term equity. Both long-term bonds and long-term equities of all three models are decreasing in ψ. As can be

seen from equation (5.36), the risk premium of long-term bonds is affected by consumption elasticity to the shock, η_{cz}, the elasticity of the long-term bond return to the shock, η_{rz}^{lb}, risk aversion of consumption, γ, and standard deviation of shock, σ_ε. Moreover, from equation (5.36′), the elasticity of the long-term bond return to the shock is dependent on η_{cz}. If $\eta_{cz} < 0$, the elasticity is negative, if $\eta_{cz} > 0$, the effect is ambiguous and dependent on other parameters. The risk premium of long-term bonds is decreasing as ψ increases the same as the productivity case.

According to Lettau (2001), the news to future expected returns is the same for bonds and equity since risk premia are constant. Hence the sign of the news to future dividend growth determines whether bonds are riskier than stocks. Productivity shocks have a positive direct effect on dividends. However, investment-specific technology shock has a negative effect on dividends through higher depreciation. In other words, the increase in productivity of newly produced capital intensifies depreciation of old capital. This negative effect causes equity premium to have a higher risk premium than long-term bonds, as in real world financial markets.

The equity premium also is affected by ψ through consumption elasticity to the shock η_{cz}, elasticity of long-term bond premium to the shock η_{rz}^{lb} and elasticity of equity premium to the shock η_{rz}^{d}. Specially, η_{rz}^{d} is dependent on the elasticity of dividend to the shock η_{dz}. Because η_{dz} is negative, the sign of η_{rz}^{d} is ambiguous.

Table 5.2.B: USA data: SR = 0.12, EqPrem = 1.05%, Long Gov't BoPrem = 0.36%, $\sigma(r^f)$ = 0.68%, γ = 2.0, γ_n = 2.0

model	ψ	SR	LT BoPrem	EqPrem	ST EqPrem	σ
A	0.00	0.00753	0.00572%	0.00580%	0.00597%	0.40719%
	0.50	0.00723	0.00563%	0.00578%	0.00578%	0.48554%
	0.95	0.00406	0.00403%	0.00475%	0.00355%	0.19904%
	1.00	0.00079	−0.00191%	−0.00275%	−0.00078%	0.08044%
B	0.00	0.00087	0.00006%	0.00006%	0.00003%	0.00663%
	0.50	0.00129	−0.00009%	−0.00009%	0.00005%	0.02071%
	0.95	0.00500	−0.00251%	−0.00252%	0.00016%	0.09292%
	1.00	0.00898	−0.02063%	−0.02059%	0.00021%	0.07615%
C	0.00	0.00271	0.00069%	0.00069%	0.00062%	0.10244%
	0.50	0.00302	0.00062%	0.00062%	0.00067%	0.10137%
	0.95	0.00586	−0.00151%	−0.00150%	0.00092%	0.04819%
	1.00	0.00922	−0.01853%	−0.01838%	0.00075%	0.06957%

Note: The table shows the conditional asset premia and the conditional Sharpe ratio, LT stands for long-term assets. ST stands for short-term assets. EqPrem is the equity premium and BoPrem is the bond premium. Premia are calculated as excess returns over the riskless Treasury bill. All numbers are in percent per quarter. Three models are compared: model A with investment-specific technology shock and adjustment costs; model B with productivity shock; model C with productivity shock and adjustment costs.

Using the benchmark case with risk aversion coefficients of $\gamma = 2.0$, $\gamma_n = 2.0$ and fairly persistent investment-specific technology shock ($\psi = 0.95$), the risk premium of the long-term equity is higher than that of the long-term bond (0.00475% compared to 0.00403%) from model A. Stocks are riskier than long-term bonds in this version of the model with

investment-specific technology. So, long-term bonds carry a less-riskier premium than equity, which is consistent with real world financial markets. However, in the model B and C cases, in contrast to real financial markets, risk premiums of long-term bonds and long-term equity become negative when the persistence of the stock increases.

Third, it is interesting to compare risk premia of long-term equity and short-term equity. For the benchmark value of $\gamma = 2.0$, long-term equity is decreasing with ψ. In the case of model A, the risk premium of the short-term equity is higher than that of the long-term equity when the persistence of the shock increases from 0.0 to 0.5. When the persistence level of technology shock is large and moves towards 0.95, the risk premium of the short-term equity is not higher than that of the long-term equity. In a comparison of short-term equities for three different models, A, B, and C, the short-term equity is higher in model A than in models B and C except when $\psi = 1.0$.

Overall, the most important implication of our approach is that investment-specific technology shock improves the model to mimic the real economy compared with productivity shock. This improvement comes from the fact that the shock results in strong intertemporal substitution effect between consumption and investment, and thus the effect explains the real economy reasonably well.

5.4.4 Impulse Response Analysis

We consider the impulse response analysis of capital, output, equity, consumption, and the risk-free rate. We consider the benchmark case for $\psi = 0.95$, $\gamma = 2.0$ and $\gamma_n = 2.0$. Figure 5.2(a) presents the complete reactions of capital stock, output and dividends to a unit of investment-specific technology shock. The output and capital reactions are fairly large compared to that of dividends, and capital increases immediately after a positive technology shock while output decreases. An investment-specific technology shock provides the opportunity for increased productivity to work a lot harder to build up capital. Therefore, the impulse response of capital after a positive investment-specific technology shock is to immediately start increasing, hit the maximum positive effects and then bounce back towards its steady-state level. The impulse response of equity to the investment-specific technology shock starts below (negative level), but increases steadily and moves toward its long-run steady-state level. The impulse responses of consumption and risk-free rate to a positive technology shock are shown in Figure 5.2(b). Both of them show a fairly persistent pattern and they reach a steady-state in a longer period of 25 years. Note that consumption is affected by technology shock directly as well as through an increase in the capital stock. As Figure 5.2(b) indicates, the impulse response of consumption is increasing due to the investment-specific technology shock and the accumulation of capital. After about ten years, consumption starts to revert back to its steady-state level. Since the risk-free rate is determined by the expected growth in consumption, the risk-free rate increases and stays below its steady-state level as long as consumption reaction to a technology shock is increasing. The risk-free rate also starts

falling and moves toward to steady-state level once consumption falls. For comparative purposes, we report the empirical results of the values with productivity shock (Figures 5.3(a) and 5.3(b)) and with productivity shock and adjustment (Figures 5.4(a) and 5.4(b)).

5.5 Concluding Remarks

This chapter investigates the determination of financial asset prices in a dynamic asset pricing model. In this chapter, we have introduced investment-specific technology, capacity utilization and capital adjustment costs in a dynamic asset pricing model. Adopting an approximation method based on a log-linear approach for our model, a closed-form solution for the expected risk premia of firm's equity and long-term real bonds is derived. Overall, the model does well in accounting for the business cycle phenomena, as well as other asset pricing properties. This chapter provides a rich picture of the behavior of US financial asset prices and allows a better understanding of the determination of risk premia in models with production. Finally, we are able to show that a model with investment-specific technology, capacity utilization and capital adjustment costs performs better than a model with standard productivity shock and adjustment costs in terms of mimicking the Sharpe ratio and risk premia of firm's equity and long-term real bonds.

Figure 5.2(a): Impulse responses of output, capital, and dividends to shock in investment-specific technology. Parameters used: $\gamma = 2.0, \gamma_n = 2.0$ and $\psi = 0.95$.

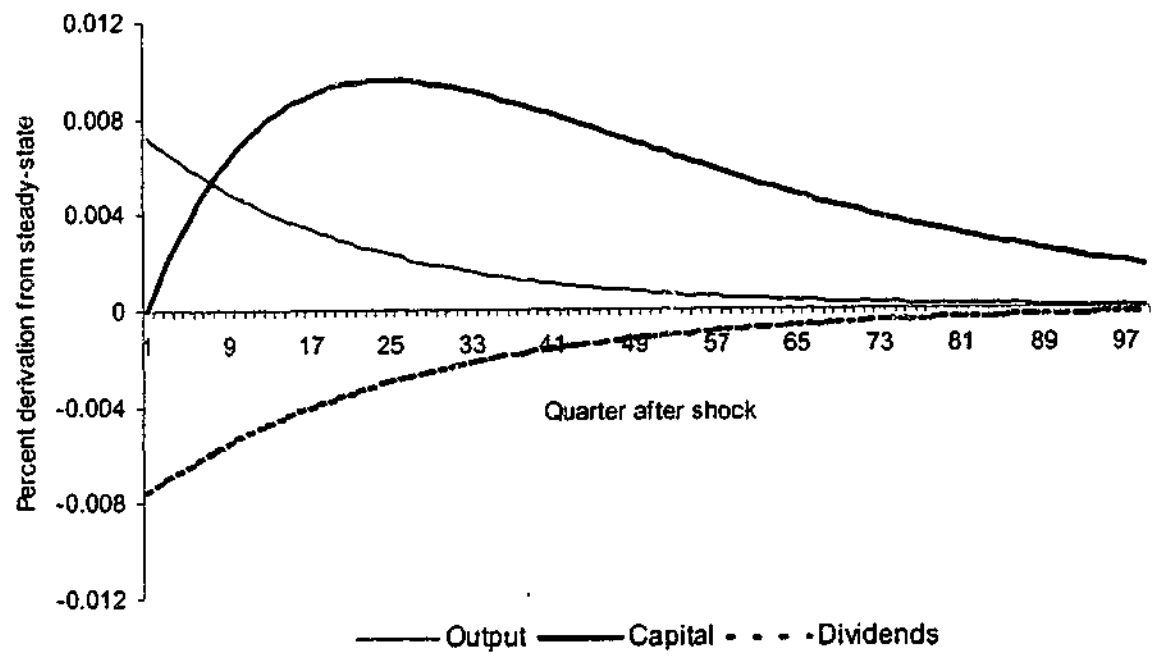


Figure 5.2(b): Impulse responses of consumption and risk-free rate to shock in investment-specific technology. Parameters used: $\gamma = 2.0$, $\gamma_n = 2.0$ and $\psi = 0.95$.

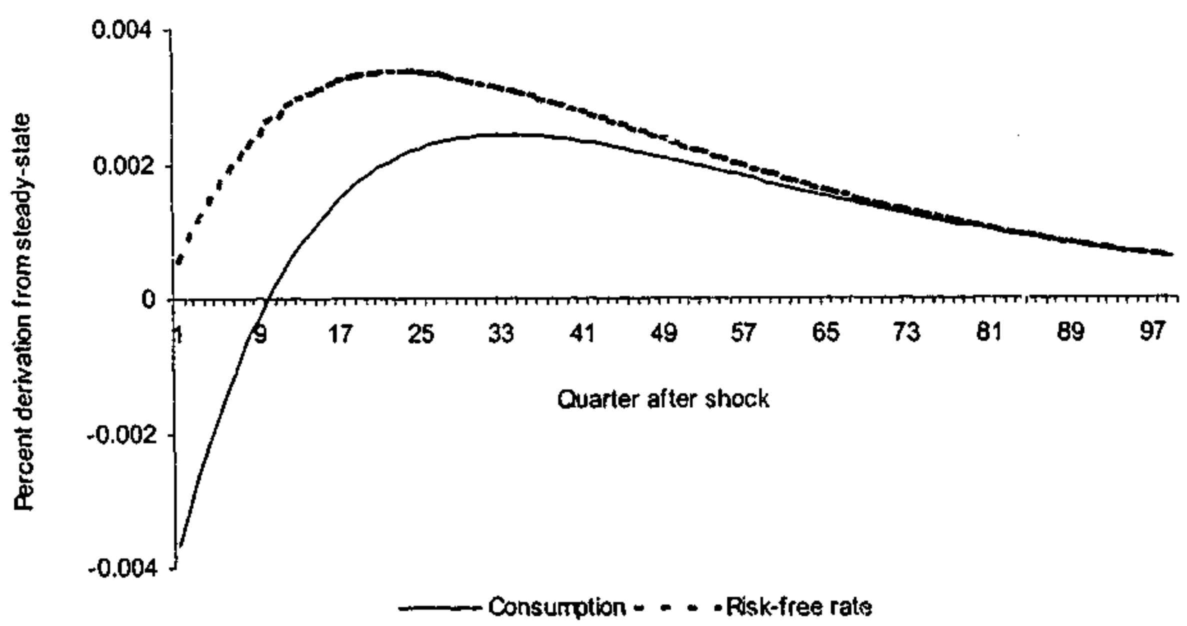


Figure 5.3(a): Impulse responses of output, capital, and dividends to shock in productivity. Parameters used: $\gamma = 2.0$, $\gamma_n = 2.0$ and $\psi = 0.95$.

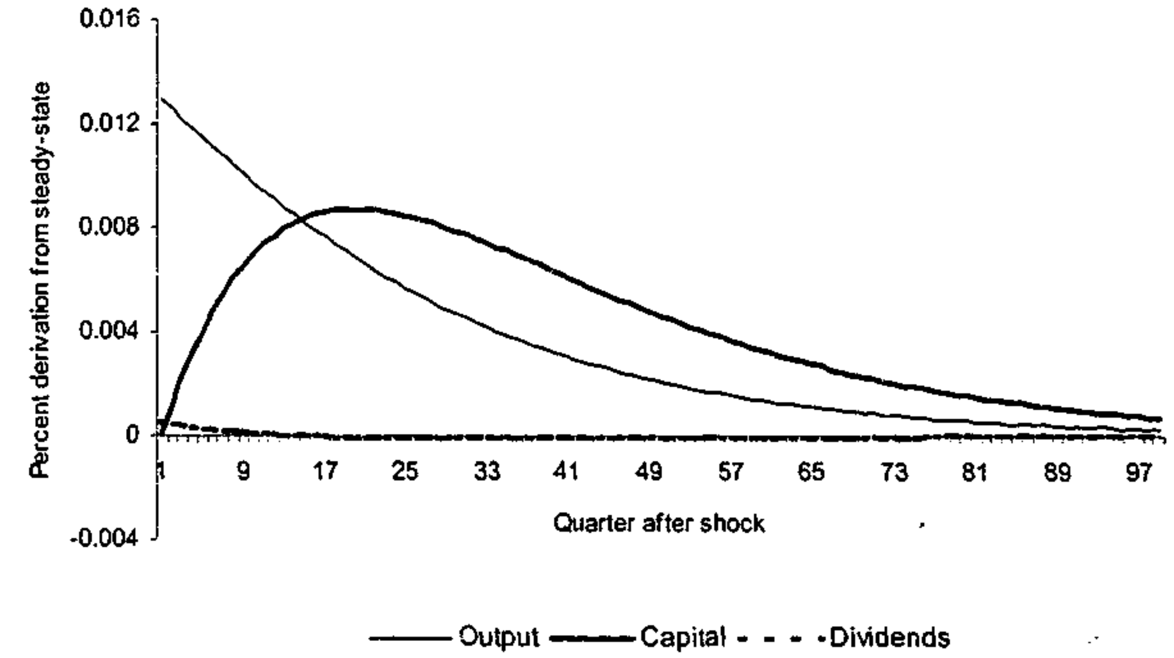


Figure 5.3(b): Impulse responses of consumption and risk-free rate to shock in productivity. Parameters used: $\gamma = 2.0$, $\gamma_n = 2.0$ and $\psi = 0.95$.

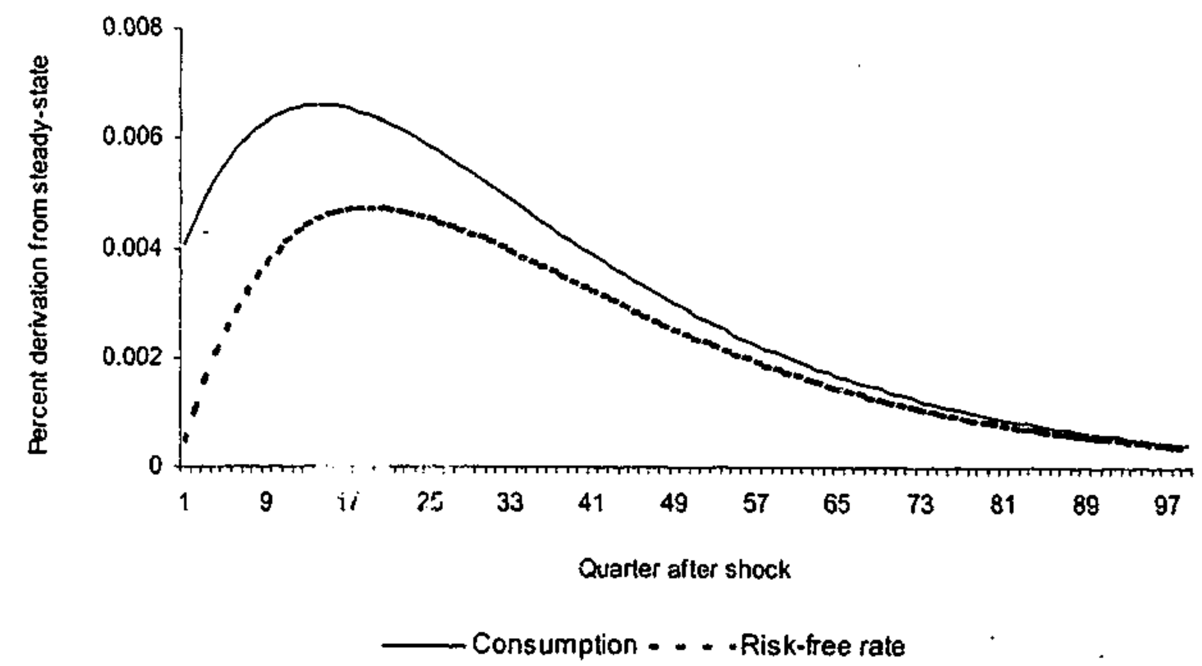


Figure 5.4(a): Impulse responses of output, capital, and dividends to shock in productivity (with adjustment costs). Parameters used: $\gamma = 2.0$, $\gamma_n = 2.0$ and $\psi = 0.95$.

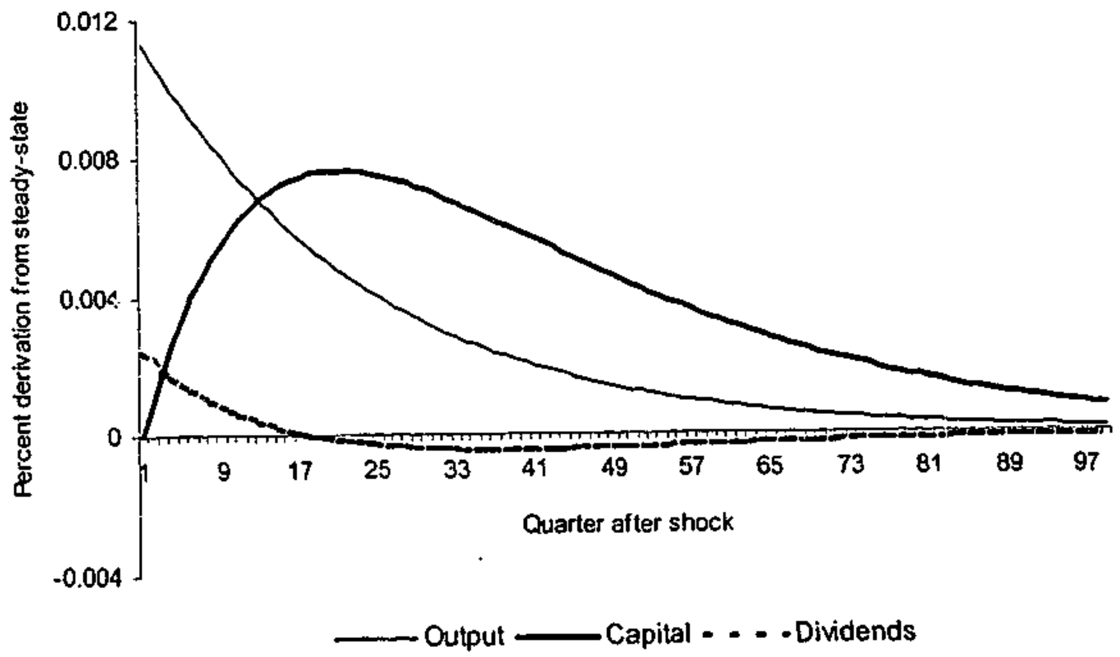
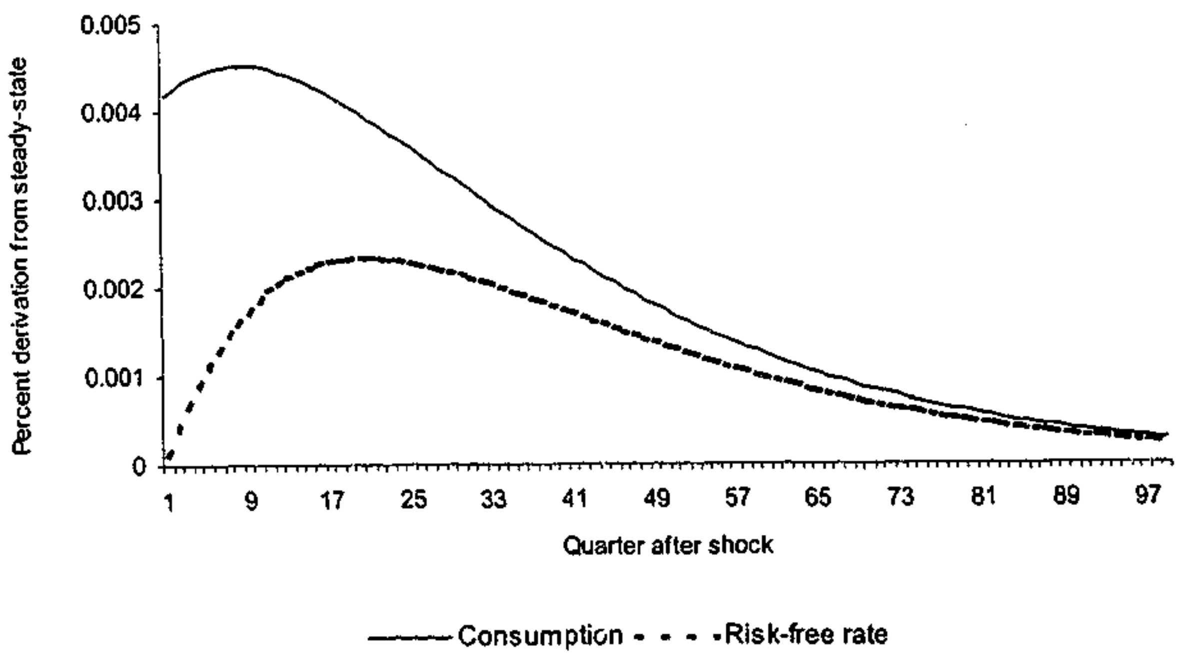


Figure 5.4(b): Impulse response of consumption and risk-free rate to shock in productivity (with adjustment costs). Parameters used: $\gamma = 2.0$, $\gamma_n = 2.0$ and $\psi = 0.95$.



Appendix 5.1: Tables

Table 5.3.A1: Consumption and capital elasticities for the model with investment-specific technology shock and adjustment costs ($\gamma_n = 2.0$)

	γ							
ψ	0.01		1.0		2.0		100	
0.0	0.99	−4.40	0.35	−0.64	0.22	−0.34	0.01	−0.01
	0.93	0.45	0.97	0.15	0.97	0.13	0.98	0.10
0.5	0.99	−4.17	0.35	−0.61	0.22	−0.33	0.01	−0.01
	0.93	0.43	0.97	0.15	0.97	0.12	0.98	0.10
0.95	0.99	−2.46	0.35	−0.37	0.22	−0.18	0.01	−0.003
	0.93	0.27	0.97	0.11	0.97	0.09	0.98	0.07
1.0	0.99	−0.98	0.35	−0.02	0.22	0.04	0.01	0.003
	0.93	0.14	0.97	0.05	0.97	0.04	0.98	0.02

Table 5.3.A2: The model with investment-specific technology shock and adjustment costs. USA data: SR = 0.12, EqPrem = 1.05%, Long Gov't BoPrem = 0.36%, $\sigma(r^f) = 0.68\%$, $\gamma_n = 2.0$

γ	ψ	SR	LT BoPrem	EqPrem	ST EqPrem	σ
0.01	0.00	0.00048	0.00002%	0.00003%	0.00005%	5.32000%
	0.50	0.00046	0.00002%	0.00003%	0.00007%	0.03169%
	0.95	0.00027	0.00002%	0.00007%	0.00013%	0.01302%
	1.00	0.00011	0.00001%	0.00013%	0.00008%	0.00410%
1.00	0.00	0.00703	0.00498%	0.00506%	0.00522%	0.75979%
	0.50	0.00676	0.00491%	0.00505%	0.00508%	0.45300%
	0.95	0.00403	0.00377%	0.00449%	0.00339%	0.18682%
	1.00	0.00020	0.00024%	0.00045%	0.00019%	0.07469%
2.00	0.00	0.00753	0.00572%	0.00580%	0.00597%	0.40719%
	0.50	0.00723	0.00563%	0.00578%	0.00578%	0.48554%
	0.95	0.00406	0.00403%	0.00475%	0.00355%	0.19904%
	1.00	0.00079	−0.00191%	−0.00275%	−0.00078%	0.08044%
100.00	0.00	0.00805	0.00658%	0.00666%	0.00685%	0.00876%
	0.50	0.00767	0.00642%	0.00658%	0.00657%	0.52219%
	0.95	0.00354	0.00366%	0.00429%	0.00325%	0.21007%
	1.00	0.00363	−0.00670%	−0.01061%	−0.00372%	0.08105%

Table 5.3.B1: Consumption and capital elasticities for the model with investment-specific technology shock and adjustment costs ($\gamma = 2.0$)

	γ_n							
ψ	0.01		1.0		2.0		100	
0.0	0.13	−0.14	0.19	−0.30	0.22	−0.34	0.31	−0.41
	0.95	0.31	0.96	0.16	0.97	0.13	0.98	0.07
0.5	0.13	−0.13	0.19	−0.29	0.22	−0.33	0.31	−0.40
	0.95	0.29	0.96	0.16	0.97	0.12	0.98	0.07
0.95	0.13	−0.02	0.19	−0.15	0.22	−0.18	0.31	−0.26
	0.95	0.18	0.96	0.11	0.97	0.09	0.98	0.05
1.0	0.13	0.09	0.19	0.05	0.22	0.04	0.31	0.03
	0.95	0.07	0.96	0.05	0.97	0.04	0.98	0.03

Table 5.3.B2: The model with investment-specific technology shock and adjustment costs. USA data: SR = 0.12, EqPrem = 1.05%, Long Gov't BoPrem = 0.36%, $\sigma(r^f) = 0.68\%$, $\gamma = 2.0$

γ_n	ψ	SR	LT BoPrem	EqPrem	ST EqPrem	σ
0.01	0.00	0.00302	0.00094%	0.00097%	0.00110%	0.19491%
	0.50	0.00275	0.00096%	0.00102%	0.00109%	0.25340%
	0.95	0.00045	0.00022%	0.00030%	0.00030%	0.13557%
	1.00	0.00191	−0.00393%	−0.00598%	−0.00177%	0.05922%
1.00	0.00	0.00662	0.00444%	0.00450%	0.00468%	0.36540%
	0.50	0.00630	0.00437%	0.00450%	0.00453%	0.43999%
	0.95	0.00319	0.00301%	0.00358%	0.00264%	0.19144%
	1.00	0.00104	−0.00240%	−0.00352%	−0.00102%	0.07444%
2.00	0.00	0.00753	0.00572%	0.00580%	0.00597%	0.40719%
	0.50	0.00723	0.00563%	0.00578%	0.00578%	0.48554%
	0.95	0.00406	0.00403%	0.00475%	0.00355%	0.19904%
	1.00	0.00079	−0.00191%	−0.00275%	−0.00078%	0.08044%
100.00	0.00	0.00900	0.00817%	0.00826%	0.00837%	0.47343%
	0.50	0.00874	0.00802%	0.00820%	0.00815%	0.55747%
	0.95	0.00571	0.00601%	0.00703%	0.00548%	0.20737%
	1.00	0.00057	−0.00144%	−0.00206%	−0.00058%	0.09906%

Table 5.4.A1: Consumption and capital elasticities for the model with productivity shock ($\gamma_n = 2.0$)

	γ							
ψ	0.01		1		2.0		100	
0.0	4.05	0.71	0.55	0.10	0.36	0.06	0.01	0.00
	0.66	0.12	0.95	0.17	0.95	0.17	0.96	0.17
0.5	4.05	−1.79	0.55	0.13	0.36	0.09	0.01	0.00
	0.66	0.34	0.95	0.16	0.95	0.16	0.96	0.16
0.95	4.05	−7.76	0.55	0.41	0.36	0.36	0.01	0.01
	0.66	0.88	0.95	0.12	0.95	0.10	0.96	0.09
1.0	4.05	−9.07	0.55	0.67	0.36	0.64	0.01	0.03
	0.66	1.00	0.95	0.08	0.95	0.05	0.96	0.00

Table 5.4.A2: The model with productivity shock. USA data: SR = 0.12, EqPrem = 1.05%, Long Gov't BoPrem = 0.36%, $\sigma(r^f) = 0.68\%$, $\gamma_n = 2.0$

γ	ψ	SR	LT BoPrem	EqPrem	ST EqPrem	σ
0.01	0.00	0.00005	0.00000%	0.00000%	0.00000%	0.22206%
	0.50	0.00013	0.00000%	0.00000%	−0.00001%	0.01703%
	0.95	0.00054	0.00017%	0.00017%	−0.00002%	0.06542%
	1.00	0.00063	0.00125%	0.00125%	−0.00003%	0.03786%
1.00	0.00	0.00067	0.00004%	0.00004%	0.00003%	0.01104%
	0.50	0.00091	−0.00008%	−0.00008%	0.00004%	0.02358%
	0.95	0.00285	−0.00173%	−0.00174%	0.00010%	0.11072%
	1.00	0.00471	−0.01508%	−0.01505%	0.00015%	0.09508%
2.00	0.00	0.00087	0.00006%	0.00006%	0.00003%	0.00663%
	0.50	0.00129	−0.00009%	−0.00009%	0.00005%	0.02071%
	0.95	0.00500	−0.00251%	−0.00252%	0.00016%	0.09292%
	1.00	0.00898	−0.02063%	−0.02059%	0.00021%	0.07615%
100.00	0.00	0.00133	0.00014%	0.00014%	0.00005%	0.00018%
	0.50	0.00217	−0.00012%	−0.00012%	0.00008%	0.01745%
	0.95	0.01030	−0.00316%	−0.00322%	0.00022%	0.05756%
	1.00	0.02060	−0.00169%	−0.00169%	0.00002%	0.00309%

Table 5.4.B1: Consumption and capital elasticities for the model with productivity shock ($\gamma = 2.0$)

	γ_n							
ψ	0.01		1		2.0		100	
0.0	0.29	0.09	0.33	0.07	0.36	0.06	0.43	0.05
	0.93	0.30	0.95	0.20	0.95	0.17	0.97	0.12
0.5	0.29	0.13	0.33	0.10	0.36	0.09	0.43	0.08
	0.93	0.28	0.95	0.19	0.95	0.16	0.97	0.11
0.95	0.29	0.36	0.33	0.35	0.36	0.36	0.43	0.39
	0.93	0.15	0.95	0.12	0.95	0.10	0.97	0.09
1.0	0.29	0.54	0.33	0.60	0.36	0.64	0.43	0.83
	0.93	0.05	0.95	0.05	0.95	0.05	0.97	0.04

Table 5.4.B2: The model with productivity shock. USA data: SR = 0.12, EqPrem = 1.05%, Long Gov't BoPrem = 0.36%, $\sigma(r^f)$ = 0.68%, $\gamma = 2.0$

γ_n	ψ	SR	LT BoPrem	EqPrem	ST EqPrem	σ
0.01	0.00	0.00131	0.00015%	0.00015%	0.00009%	0.01275%
	0.50	0.00178	−0.00023%	−0.00023%	0.00012%	0.03637%
	0.95	0.00508	−0.00337%	−0.00339%	0.00022%	0.12336%
	1.00	0.00750	−0.01684%	−0.01681%	0.00018%	0.05870%
1.00	0.00	0.00097	0.00008%	0.00008%	0.00005%	0.00793%
	0.50	0.00140	−0.00012%	−0.00012%	0.00006%	0.02414%
	0.95	0.00496	−0.00267%	−0.00268%	0.00017%	0.09982%
	1.00	0.00838	−0.01878%	−0.01874%	0.00019%	0.06903%
2.00	0.00	0.00087	0.00006%	0.00006%	0.00003%	0.00663%
	0.50	0.00129	−0.00009%	−0.00009%	0.00005%	0.02071%
	0.95	0.00500	−0.00251%	−0.00252%	0.00016%	0.09292%
	1.00	0.00898	−0.02063%	−0.02059%	0.00021%	0.07615%
100.00	0.00	0.00073	0.00004%	0.00004%	0.00002%	0.00447%
	0.50	0.00114	−0.00006%	−0.00006%	0.00003%	0.01490%
	0.95	0.00540	−0.00240%	−0.00241%	0.00015%	0.08223%
	1.00	0.01170	−0.03133%	−0.03125%	0.00032%	0.10959%

Table 5.5.A1: Consumption and capital elasticities for the model with productivity shock and adjustment costs ($\gamma_n = 2.0$)

	γ							
ψ	0.01		1		2.0		100	
0.0	1.35	1.89	0.51	0.34	0.34	0.19	0.01	0.00
	0.91	0.01	0.95	0.13	0.96	0.14	0.97	0.15
0.5	1.35	1.64	0.51	0.36	0.34	0.22	0.01	0.01
	0.91	0.03	0.95	0.13	0.96	0.13	0.97	0.14
0.95	1.35	0.02	0.51	0.54	0.34	0.42	0.01	0.02
	0.91	0.18	0.95	0.10	0.96	0.09	0.97	0.08
1.0	1.35	−1.04	0.51	0.73	0.34	0.66	0.01	0.03
	0.91	0.27	0.95	0.07	0.96	0.04	0.97	0.00

Table 5.5.A2: The model with productivity shock and adjustment costs
USA data : SR = 0.12, EqPrem = 1.05%, Long Gov't BoPrem = 0.36%, $\sigma(r^f)$ = 0.68%, $\gamma_n = 2.0$

γ	ψ	SR	LT BoPrem	EqPrem	ST EqPrem	σ
0.01	0.00	0.00013	0.00000%	0.00000%	0.00001%	1.32000%
	0.50	0.00011	0.00000%	0.00000%	0.00001%	0.00631%
	0.95	0.00000	0.00000%	0.00000%	0.00000%	0.00334%
	1.00	0.00007	0.00001%	0.00001%	−0.00003%	0.00618%
1.00	0.00	0.00237	0.00054%	0.00054%	0.00051%	0.19062%
	0.50	0.00251	0.00047%	0.00047%	0.00053%	0.09368%
	0.95	0.00376	−0.00114%	−0.00113%	0.00065%	0.05667%
	1.00	0.00511	−0.01339%	−0.01327%	0.00066%	0.08349%
2.00	0.00	0.00271	0.00069%	0.00069%	0.00062%	0.10244%
	0.50	0.00302	0.00062%	0.00062%	0.00067%	0.10137%
	0.95	0.00586	−0.00151%	−0.00150%	0.00092%	0.04819%
	1.00	0.00922	−0.01853%	−0.01838%	0.00075%	0.06957%
100.00	0.00	0.00333	0.00102%	0.00102%	0.00081%	0.00222%
	0.50	0.00404	0.00096%	0.00096%	0.00095%	0.11120%
	0.95	0.01110	−0.00123%	−0.00124%	0.00150%	0.02088%
	1.00	0.02060	−0.00157%	−0.00156%	0.00005%	0.00294%

Table 5.5.B1: Consumption and capital elasticities for the model with productivity shock and adjustment costs ($\gamma = 2.0$)

	γ_n							
ψ	0.01		1		2.0		100	
0.0	0.26	0.27	0.32	0.21	0.34	0.19	0.42	0.16
	0.94	0.20	0.95	0.16	0.96	0.14	0.97	0.11
0.5	0.26	0.29	0.32	0.23	0.34	0.22	0.42	0.18
	0.94	0.19	0.95	0.15	0.96	0.13	0.97	0.10
0.95	0.26	0.43	0.32	0.42	0.34	0.42	0.42	0.44
	0.94	0.12	0.95	0.10	0.96	0.09	0.97	0.08
1.0	0.26	0.55	0.32	0.62	0.34	0.66	0.42	0.85
	0.94	0.05	0.95	0.04	0.96	0.04	0.97	0.04

Table 5.5.B2: The model with productivity shock and adjustment costs
USA data : SR = 0.12, EqPrem = 1.05%, Long Gov't BoPrem = 0.36%, $\sigma(r^f)$=0.68%, γ = 2.0

γ_n	ψ	SR	LT BoPrem	EqPrem	ST EqPrem	σ
0.01	0.00	0.00381	0.00139%	0.00139%	0.00127%	0.15284%
	0.50	0.00405	0.00123%	0.00123%	0.00129%	0.15333%
	0.95	0.00600	−0.00135%	−0.00133%	0.00117%	0.04217%
	1.00	0.00773	−0.01315%	−0.01306%	0.00066%	0.04908%
1.00	0.00	0.00299	0.00085%	0.00085%	0.00077%	0.11525%
	0.50	0.00328	0.00076%	0.00076%	0.00081%	0.11455%
	0.95	0.00585	−0.00145%	−0.00144%	0.00098%	0.04660%
	1.00	0.00862	−0.01630%	−0.01618%	0.00070%	0.06166%
2.00	0.00	0.00271	0.00069%	0.00069%	0.00062%	0.10244%
	0.50	0.00302	0.00062%	0.00062%	0.00067%	0.10137%
	0.95	0.00586	−0.00151%	−0.00150%	0.00092%	0.04819%
	1.00	0.00922	−0.01853%	−0.01838%	0.00075%	0.06957%
100.00	0.00	0.00218	0.00043%	0.00043%	0.00038%	0.07751%
	0.50	0.00253	0.00040%	0.00040%	0.00044%	0.07573%
	0.95	0.00622	−0.00176%	−0.00174%	0.00086%	0.05282%
	1.00	0.01190	−0.02993%	−0.02966%	0.00102%	0.10462%

Chapter 6 Summary and Concluding Remarks

Our research starts with two main topics such as a small open economy and the driving forces of business cycles. A small open economy has properties such as the ability of small open economy to smooth consumption and investment, and the transmission effect from foreign sector. Together with the small open economy properties, we focus on the driving force of business cycle such as investment-specific technology shock and terms of trade shocks. We built international real business cycle model with above two basic ideas and then further developed the models, and also extended to asset pricing model. We found the following conclusions:

In chapter 2, under the proposition that an economy experiences shocks to the marginal efficiency of investment, and that capital service is dependent on both capital size and capacity utilization, we investigates the central issue as to how economic variables fluctuate in response to shocks to investment. We first conducted a qualitative analysis on the effects of a shock within a small open economy framework. The findings of the qualitative analysis are that a positive shock affects endogenous variables positively, except for consumption and foreign financial assets. Unlike a closed economy, our qualitative analysis of a small open economy indicates that a capital stock increase resulting from a positive shock is absorbed by the next period's foreign financial assets. This explains how foreign financial assets are used to smooth consumption-savings, thereby stabilizing the economy. In the quantitative analysis, we apply the value functions and transition probability iteration method in order to simulate our benchmark model. In conjunction with the qualitative analysis, using Australian data, we analyze the benchmark model in which the economy has shocks to investment, and output is dependent on both capital size and capacity utilization. More importantly, capacity utilization and shocks to investment provide a meaningful explanation for economic fluctuations. Furthermore, capacity utilization shows a strong procyclical property, and is a significant variable that explains business cycle fluctuations. Foreign financial assets provide procyclical effects to output fluctuations, unlike the trade balance. Finally, impulse response analysis provides strong supports for the qualitative analysis, which represents the dynamic effects of the model.

In chapter 3, we analyse an international real business business cycle model of a small open economy. Under the assumption that the model of the economy produces two goods, and has two sectoral productivity shocks and terms of trade shock, we especially investigate the central issue of how the economic variables fluctuate in response to the shocks.

We conducted the quantitative analysis on the effects of sectoral productivity shocks and the terms of trade shock using the Australian data. The model economy fairly well replicates the real economy in properties of second moments. The autocorrelation of trade balance is relatively sensitive to the change of adjustment cost in non-manufacturing production, which indicates the strong connection between the non-manufacturing sector and the foreign sector. The change of elasticity of substitution in intermediate goods

production, which shows spillover effect between sectors, has a small effect on manufacturing. This implies that sectoral transmission effects are very stable. From impulse response analysis, the productivity of non-manufacturing production has a dominant role in the business cycle of Australian economy. The effect is stronger than the productivity shock of manufacturing production and terms of trade shock. Moreover, the strong effect increases imports more than exports, and thus declines trade balance temporally, while the productivity shock of manufacturing production increases exports more than imports.

In chapter 4, we develops an international real business cycle model of a small open economy. The model has three production factors: structure capital stock, equipment capital stock and labour, and has two shocks (productivity shock and terms of trade shock). We investigates two important *a priori* hypotheses. First, we ask whether it is possible to substitute "terms of trade shock", instead of "investment-specific technology shock", and whether terms of trade can transmit overseas investment-specific technology change into the domestic economy and hence cause the business fluctuations in a small open economy. Given the fact that the pattern of foreign capital goods to equipment investment is over 60% in Australia, our quantitative analysis concludes that terms of trade shock provide a meaningful explanation of Australian economic fluctuations. Foreign capital goods increase from productivity shock and terms of trade shock, whereas trade balance decreases as foreign capital goods rise from productivity shock. In addition, capital utilization in a small open economy context plays a key role in explaining economic fluctuation with a strong procyclical property.

In chapter 5, we investigate the determination of financial asset prices in a dynamic asset pricing model. In this chapter, we have introduced investment-specific technology, capacity utilization and capital adjustment costs in a dynamic asset pricing model. Adopting an approximation method based on a log-linear approach for our model, a closed-form solution for the expected risk premia of firm's equity and long-term real bonds is derived. Overall, the model does well in accounting for asset pricing properties. This chapter provides a rich picture of the behavior of US financial asset prices and allows a better understanding of the determination of risk premia in models with production. F. ally, we are able to show that a model with investment-specific technology, capacity utilization and capital adjustment costs performs better than a model with standard productivity shock and adjustment costs in terms of mimicking the Sharpe ratio and risk premia of firm's equity and long-term real bonds.

APPENDIX An Undetermined Coefficient Solution Method

Our models are solved by using the undetermined coefficient method proposed by Christiano (1998). The method is explained as follows(see, Christiano 1998):

To solve the dynamic program problem by using an undetermined coefficient solution method, we need to linearize above first order conditions around the steady state values (Christiano 1998). Once all equations are linearized, the undetermined coefficient solution method is used. The undetermined coefficient solution method commences with the proposition that solutions are limited to the following representation:

$$z_t = Az_{t-1} + Bs_t$$

$$z_t = \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix}$$

where, z_{1t} is $n_1 \times 1$ vector of endogenous variables that is determined at time t, and z_{2t} is a $qn_1 \times 1$ vector of q lagged z_{1t}'s. Therefore, z_t is $n_1(q+1) \times 1$ vector including all endogenous and state variables. s_t is an $m \times 1$ vector of exogenous shocks. The undetermined coefficient solution method requires the derivation of the matrix A and B. Christiano (1998) expla.ned the undetermined coefficient method in detail. Hence we attach his explanation as follows:

We limit ourselves to solution which have the following representation:

$$z_t = Az_{t-1} + Bs_t \tag{A.1}$$

where A is the feedback part of the solution and B is the feedforward part., Here, the $n \times n$ matrix A has the following structure:

$$A = \begin{bmatrix} A_1 \\ \cdots \\ A_2 \end{bmatrix},$$

where A_1 is an $n_1 \times n$ matrix of undetermined coefficients and A_2 is the $(n - n_1) \times n$ matrix:

$$A_2 = \begin{bmatrix} I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}, \quad (A.2)$$

Here, $A_1 \equiv A$ when q=0. In (A.2). I and 0 denote the n_1-dimensional identity and zero matrices, respectively. The eigenvalues of A be strictly less than unity in absolute value. The n by m matrix B has the following structure:

$$B = \begin{bmatrix} B_1 \\ \cdots \\ B_2 \end{bmatrix}, \quad (A.3)$$

Here, B_1 is an $n_1 \times m$ matrix composed primarily of as-yet undetermined coefficients, and B_2 is an $(n - n_1) \times m$ of zeros. When q=0, then $B \equiv B_1$. Under the standard assumption about information sets, all elements of B_1 are treated as undetermined coefficients. When information sets differ, a number of elements of B_1 are set to zero.

To solve (A.1) recursively to express z_{t+j}, j = 0,1,.....r-1, as a function of z_{t-1}, s_t and $\varepsilon_{t+k}, k = 1, ..., r-1$. Substituting the resulting expression into the left side of the equality in (A.2), and taking into account $\varepsilon_t \varepsilon_{t+k} = 0,\ k > 0$, we obtain

$$\varepsilon_t\left[\sum_{i=0}^{r} \alpha_i z_{t+r-1-i} + \sum_{i=0}^{r} \beta_i s_{t+r-1-i}\right] = \alpha(A) z_{t-1} + \widetilde{F} s_t \quad (A.4)$$

where $\widetilde{F}$ is defined by

$$\varepsilon_t F s_t = \widetilde{F} s_t, \quad (A.5)$$

and $F = \widetilde{F}$ in the standard case. Here,

$$\widetilde{F} = \sum_{i=0}^{r-1} \left[\beta_i + \overline{Q}_i B_1\right] P^{(r-1-i)} \quad (A.6)$$

and

$$\begin{aligned} &Q_0 \equiv \alpha_0,\ Q_k \equiv Q_{k-1} A + \alpha_k, k = 1,2,......r \\ &\alpha(A) \equiv \alpha_0 A^r + \alpha_1 A^{r-1} + ... + \alpha_r \equiv Q_r \end{aligned} \quad (A.7)$$

In (A.6), $\overline{Q}_i$ is the $n_1 \times n_1$ matrix formed from the first n_1 columns of Q_i. Thus, in addition to the eigenvalue restriction mentioned above, the A that we seek must satisfy:

$$\alpha(A) = 0_{n_1 \times n}. \quad (A.8)$$

Given A, the $n_1 \times m$ matrix B_1 must solve the restriction:

$$\widetilde{F} = 0_{n_1 \times n}. \quad (A.9)$$

A.1 Computing the feedback part

The first-order difference equation form is as follows:

$$aY_{t+1} + bY_t = 0, \quad t \geq 0. \tag{A.10}$$

where Y_t is an $[(r-1)n_1 + n] \times 1$ vector:

$$Y_t = \begin{pmatrix} z_{1,t+r-2} \\ \vdots \\ z_{1,t} \\ z_{t-1} \end{pmatrix} \tag{A.11}$$

Also, the $[(r-1)n_1 + n] \times [(r-1)n_1 + n]$ matrices a and b are defined as follows:

$$a = \begin{bmatrix} \tilde{\alpha}_0 & 0_{n_1 \times [(r-2)n_1+n]} \\ 0_{[(r-2)n_1+n] \times n_1} & I_{(r-2)n_1+n} \end{bmatrix}, \quad b = \begin{bmatrix} \tilde{\alpha}_1 & \tilde{\alpha}_2 & \cdots & \tilde{\alpha}_{r-1} & \alpha_r \\ -I_{n_1} & 0_{n_1} & \cdots & 0_{n_1} & 0_{n_1 \times n} \\ 0_{n_1} & -I_{n_1} & \cdots & 0_{n_1} & 0_{n_1 \times n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{n_1 \times n} & 0_{n_1 \times n} & \cdots & b_1 & b_2 \end{bmatrix} \tag{A.12}$$

where $\tilde{\alpha}_i$ is the left n_1 columns of α_i, I = 1, …., r-1. (When q=0, then $\tilde{\alpha}_i = \alpha_i$ for all I). Also $0_{i \times j}$ and 0_i denote the $i \times j$ and $i \times i$ dimensional matrices of zeros, respectively, and I_i denotes the I dimensional identity matrix. Finally, when q>0, then $n \times n_1$ and $n \times n$ dimensional matrices b_1 and b_2 are defined as follows:

$$b_1 = \begin{bmatrix} -I_{n_1} \\ 0_{qn_1 \times n_1} \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0_{n_1 \times qn_1} & 0_{n_1} \\ -I_{qn_1} & 0_{qn_1 \times n_1} \end{bmatrix},$$

and when q = 0:

$$b_1 = -I_{n_1}, \; b_2 = 0_{n_1}$$

The vector, Y_0 is restricted by the n initial conditions, z_{-1}. The $(r-1)n_1$ elements, $z_{1,0}, \ldots\ldots, z_{1,r-2}$, are free.

A solution (A.10) is a sequence, $\{Y_t : t \geq 0\}$, which satisfies the initial conditions and (A.10) at all dates. A solution is reduced rank if there is an $n_1(r-1) \times nr$ dimensioned matrix D which satisfies a certain rank condition and $DY_t = 0$ for all t ≥0. The rank condition is that the square matrix formed from the first $(r-1)n_1$ columns of D corresponding to the free variables in Y_0 be invertible. A solution is convergent if $Y_t \to 0$ as $t \to \infty$ for all possible initial conditions. The matrix A that we seek corresponds to a reduced rank, convergent solution.

We consider two case, one in which the matrix a is invertible and the other in which it is not.

A.1.1 The invertible "a" case

In the first case, (A.10) implies that all solutions can be expressed as $Y_t = (-a^{-1}b)^t Y_0$, or

$$P^{-1}Y_t = \Lambda^t P^{-1} Y_0, \tag{A.13}$$

where $P\Lambda P^{-1} = -a^{-1}b$ is the eigenvector, eigenvalue decomposition of $-a^{-1}b$. According to (A.13), the set of solutions is of dimension $n_1(r-1)$. This is because each solution, $\{Y_t : t \geq 0\}$, in the set corresponds to a different setting for the $n_1(r-1)$ free parameters in Y_0. Suppose there are exactly $n_1(r-1)$ elements in Λ that exceed unity in absolute value, and let D be composed of the $n_1(r-1)$ rows of P^{-1} associated with the explosive elements of Λ. If D satisfies the rank condition described above, then the set of convergent solutions contains one element, the one corresponding to the Y_0 satisfying $DY_0 = 0$. That Y_0 corresponds to the unique convergent solution of (A.13). This solution is a reduced rank solution because of easily verified fact thatcimplies $\tilde{p}Y_t = 0$ for all $t \geq 0$ when $\tilde{p}$ is one of the row P^{-1}.

If the number of eigenvalues larger than one in absolute value exceeds $n_1(1-r)$, then there is no reduced rank solution satisfying convergence (there are not enough degrees of freedom in the first $n_1(r-1)$ elements of Y_0 to 'zero out' all the explosive eigenvalues in (A.13)). In this case, the solution method developed here does not directly apply. If the number of elements of Λ exceeding unity in absolute value in less than $n_1(r-1)$, then there may be more than one reduced rank, convergent solution.

Write $DY_t = D^1Y_t^1 + D^2z_{t-1}$, where D^1 are the $n_1(r-1)$ first columns of D and D^2 re the remaining n columns and $Y_t = [Y_t^{1\prime}, z_{t-1}']'$ is partitioned conformably.

Then, $DY_t = 0$ implies

$$Y_t^1 = -(D^1)^{-1}D^2z_{t-1}, \qquad (A.14)$$

where $-(D^1)^{-1}D^2$ is an $n_1(r-1) \times n$ matrix. The matrix A_1 that we seek is composed of the bottom n_1 rows of $-(D^1)^{-1}D^2$.

A.1.2 The non-invertible "a" case

Now consider the case when a is not invertible. The procedure we use for handling the case is based on the QZ decomposition, as implemented by Chris Sims (1989). For notational simplicity, let $\omega \equiv (r-1)n_1 + n$, .so that a and b are $\omega \times \omega$ matrices.

The first step is to find the orthonormal matrices Q and Z, and the upper triangular matrices H_0 and H_1 with the properties:

$$QaZ = H_0, \quad QbZ = H_1 \qquad (A.15)$$

The matrix H_0 is structured so that the l zeros on its diagonal are located in the lower right part of H_0. Denote the upper $(\omega - l) \times (\omega - l)$ block of H_0 by G_0. This matrix must be non-singular. Let the corresponding upper left $(\omega - l) \times (\omega - l)$ block in H_1 be denoted G_1. We assume that lower right $l \times l$ block of H_1 is nonsingular. Also, it is useful to partition Z' as follows:

$$Z' = \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}, \qquad (A.16)$$

where L_1 is $(\omega - l) \times \omega$ and L_2 is $l \times \omega$

Inserting $ZZ'(=I)$ before Y_{t+1} and Y_t in (A.10), defining $\gamma_t \equiv Z'Y_t$, and premultiplying (A.10) by Q, (A.10) becomes:

$$H_0\gamma_{t+1} + H_1\gamma_t = 0, \quad t = 0, 1, \ldots\ldots \tag{A.17}$$

Partition γ_t as follows:

$$\gamma_t = \begin{pmatrix} \gamma_t^1 \\ \gamma_t^2 \end{pmatrix}, \tag{A.18}$$

where γ_t^1 is $(\omega - l) \times 1$ and γ_t^2 is $l \times 1$. It is easy to verify that (A.17) implies $\gamma_t^2 = 0,\ t \geq 0$, i.e.,

$$L_2Y_t = 0,\ t = 0,1,\ldots\ldots \tag{A.19}$$

With (A.19) imposed, the last l equations in (A.17) are redundant, so (A.17) can be written

$$G_0\gamma_{t+1}^1 + G_1\gamma_t^1 = 0, \quad t = 0, 1, \ldots\ldots \tag{A.20}$$

The set of solutions to this system can be expressed as $\gamma_t^1 = (-G_0^{-1}G_1)^t\gamma_0^1,\ t \geq 0$, or

$$P^{-1}\gamma_t^1 = \Lambda^t P^{-1}\gamma_0^1, \tag{A.21}$$

where $P\Lambda P^{-1} = -G_0^{-1}G_1$ is the eigenvector, eigenvalue decomposition of $-G_0^{-1}G_1$. The γ_t^1 that solve (A.21) are convergent if, and only if, $\tilde{p}\gamma_0^1 = 0$, where $\tilde{p}$ is composed of the row of P^{-1} corresponding to diagonal terms in Λ that exceed 1 in absolute value. This condition is:

$$\tilde{p}L_1Y_0 = 0 \tag{A.22}$$

Recall that the number of free (i.e., endogenously determined) elements in Y_0 is $n_1(r+1)$. Equation (A.19) for t= 0 represents l restrictions on Y_0, so that to pin Y_0 down uniquely, $n_1(r-1)-l$ more restrictions are needed. Thus, uniqueness requires that there be $n_1(r-1)-l$ explosive eigenvales in Λ, i.e., that $\tilde{p}L_1$ contain $n_1(r-1)-l$ rows. Then, define

$$D = \begin{pmatrix} \tilde{p}L_1 \\ L_2 \end{pmatrix}. \tag{A.23}$$

The matrix A that we seek is then obtained by multipulating D in exactly the same way that was done before.

A.2 Computing the feedforward part

With the A matrix in hand, I now find the B matrix which solves (A.9) conditional on the given matrix, A. The following result is useful:

$$\mathrm{vec}(A_1A_2A_3) = (A_3' \otimes A_1)\mathrm{vec}(A_2) \tag{A.24}$$

where $\otimes$ denotes the Kronecker product and $\mathrm{vec}(\cdot)$ denotes the vectorization operator. Also $\mathrm{vec}(A+B) = \mathrm{vec}(A) + \mathrm{vec}(B)$. Applying these results to (A.6),

We get

$$\mathrm{vec}(F') = \mathrm{vec}\left[\sum_{j=0}^{r-1}(P')^{r-1-j}\beta_j'\right] + \left\{\sum_{j=0}^{r-1}\left[\overline{Q}_j \otimes (P^{r-1-j})'\right]\right\}\mathrm{vec}(B_1'), \tag{A.25}$$

where $P^0 \equiv I$ and denotes the matrix transposition operator. Then $\mathrm{vec}(F') = 0$ implies

$$d + q\delta = 0 \qquad (A.26)$$

where

$$q = \left\{ \sum_{j=0}^{r-1} \left[\overline{Q}_j \otimes (P^{r-1-j})' \right] \right\}, \quad d = \text{vec}\left[\sum_{j=0}^{r-1} (P')^{r-1-j} \beta_j' \right] \text{ and } \delta = \text{vec}(B_1'). \qquad (A.27)$$

Note here that q and d are determined, given A. The solution wee seek is

$$\delta = -q^{-1} d \qquad (A.28)$$

The matrix B_1 can be recovered in a straightforward way from δ.

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