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**MONASH UNIVERSITY**  
**THESIS ACCEPTED IN SATISFACTION OF THE**  
**REQUIREMENTS FOR THE DEGREE OF**  
**DOCTOR OF PHILOSOPHY**

ON..... 12 April 2002 .....

.....  
Dev Sec. Research Graduate School Committee

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## AMENDMENTS

The following amendments are under the requirements of the Examiner's Report:

1. In line 10 on page 65, "the optimal utility function of" should be "the optimal utility of".
2. In his Examiner's Report, Dr. Shuntian Yao states: "

There is, however, a small mathematical error in his model concerning the measure of the population of individuals and the utility function of the Monarch. In fact with a continuum of individuals,  $M$  is just a cardinal number of the set of population. As a result the factor  $M/2$  in equation (3.3.3b) and the factor  $M$  in (3.3.3h) have vague meanings. Unless the preference of the Monarch is completely different from an ordinary individual, otherwise the author has to explain what is the utility of the Monarch when she consumes an infinitely large amount of each of the goods.

Answer: The reason for putting the factor  $\frac{M}{2}$  in equation (3.3.3b) and the factor  $M$  in (3.3.3h) is because for simplicity it is assumed in this model that there is only One Monarch, i.e. a single person, in this large economy with population size of  $M$ . The preference of the Monarch is the same as the general populace. However, when the Monarch imposes taxation, she faces a trade-off in terms of what to do with the revenue. The trade-off is between using the revenue for her own personal consumption and using the revenue to finance protection of property rights, which in turn will increase future tax revenue through more effective enforcement of criminal laws. The Monarch imposes taxation on all of her subjects, and the number of good  $x$  or  $y$  specialists is  $\frac{M}{2}$  in equilibrium respectively. Thus, in (3.3.3b) the total taxation revenue collected from her subjects should be the tax collected from each good  $x$  or  $y$  specialist times their total number respectively, i.e.  $\frac{M}{2}$ . Based on the same reason, equation (3.3.3h) also needs to include the population size  $M$  in its expression which is the optimal utility of the Monarch's decision problem under previous assumptions.

**A General Equilibrium Analysis of the Division  
of Labour: Violation and Enforcement of  
Property Rights, Impersonal Networking  
Decisions and Bundling Sale**

***A THESIS SUBMITTED TO  
MONASH UNIVERSITY  
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY***

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## Abstract of Thesis

This thesis applies inframarginal analysis to study the relationship among the division of labor, theft behavior and bundling sale within the new classical microeconomic framework. It will address three issues: the effects of stealing and self-protection from being stolen on the network size of division of labor (Chapter 2); the effects of the property rights system that penalizes stealing on the network size of division of labor and on economic development (Chapter 3); the effects of bundling sale on the network size of division of labor (Chapter 4).

Chapter 1 provides a review of the literature of inframarginal analysis of the division of labor. A brief outline of Chapters 2 to 4 is also included in this introductory chapter.

Chapter 2 develops a general equilibrium model with endogenous specialization and endogenous theft behavior to investigate effects of theft on the equilibrium network size of division of labor, on aggregate productivity, and on per capita real income. If an individual can steal from her neighbors or her trade partners, then an increase in transportation efficiency or a decrease in stealing efficiency will increase the level of division of labor where each individual's resources allocated to theft may be either lower or higher than in autarky. This shows the conventional wisdom that "wealth reduces the desire for stealing and poverty stimulates theft" is not always consistent with a sophisticated general equilibrium analysis of interdependence between per capita real income and equilibrium levels of division of labor and stealing activity. An increase in transportation efficiency and/or a decrease in stealing efficiency will raise the equilibrium level of division of labor, thereby enlarging the extent of the market, and increase aggregate productivity and per capita real income.

Chapter 3 shows how an improvement in institutional efficiency from the third party's protection for property rights can promote the development of division of labor, specialization of workers and the enhancement of aggregate productivity and per capita real income. Contrary to the assumptions underlying ideas of technology and investment fundamentalism, this chapter stresses the importance of institutional factors, especially the enforcement of property rights that captures the relationship between

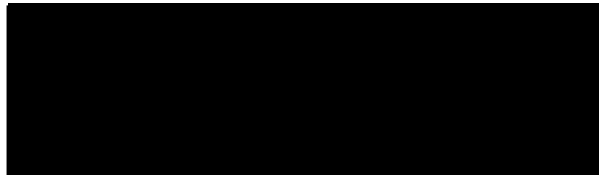


structural transformation that occurs in an economy and the level of division of labor. This model is compared with the model of Hobbes' jungle in Chapter 2 to investigate the effects of the Monarch's power in enforcing property rights on network size of division of labor and productivity. By comparing self-protection and the third party's protection, this chapter shows that the government can endogenously emerge from taxation that is used to finance the judicial system and enforcement of laws that penalize theft. This chapter has examined the trade off between positive network effects of the laws and their enforcement, and negative effects of taxation on the network size of division of labor. It is shown that the improvement in institutional efficiency expands the demand for transactions, which requires third party's protection. The market mechanism for goods as well as for the third party's protection, determines the selection of the protection system of property rights. In this model, aggregate productivity is determined by the network size of division of labor (extent of the market). The network size of division of labor is determined by the enforcement of property rights, which is dependent on tax revenue and in turn dependent on per capita real income, aggregate productivity, and network size of division of labor.

Chapter 4 develops a general equilibrium model of impersonal networking decisions and bundling sale. It departs from the other models of bundling and tying by allowing substitution between goods, flexible quantities of goods, resale of any goods, competitive market, and ex ante identical utility function for all individuals. Hence, interactions and feedback loops between quantities, prices, network effects of division of labor, transaction costs, self-interested decisions, income, and productivity can be investigated. Inframarginal analysis (total cost-benefit analysis across corner solutions in addition to marginal analysis of each corner solution) of the model shows that the function of bundling sale in a competitive market is to get intangible information goods involved in the division of labor and commercialized production, meanwhile avoiding direct pricing of such goods, thereby promoting division of labor and aggregate productivity. According to this theory of bundling, bundling in a competitive market is Pareto efficient and it plays a very important role to utilize positive network effects of division of labor on aggregate productivity. Antitrust prosecution should pay more attention to the intention to block free entry rather than bundling itself.

## Statement of Originality

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university, and to the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except when due reference is made in the text of the thesis.



**LI Ke**

January, 2001

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## CHAPTER 1. Introduction

### 1.1 Specialization and Division of Labour

Houthakker (1956, p.182) expressed the belief that "Most economists have probably regarded the division of labour, in Schumpeter's words, as an 'external common place,' yet there is hardly any part of economics that would not be advanced by a further analysis of specialization." This implies that the analysis of specialization and division of labour is not merely one of many fields of economics, but rather is at the core of classical mainstream economics. The focus of classical economics was on the implications of specialization and division of labour for economic growth and welfare<sup>1</sup>. Before Adam Smith, the role of the market and population size in permitting specialization and the advantages of the division of labour, like improving the skill of individual workers, saving the time and effort involved in having to switch from one operation to another, and facilitating the invention of machinery, were spelt out by Josiah Tucker (1755, 1774), Denis Diderot (1713), Henry Maxwell (1721), and Anne-Robert-Jacques Turgot (1766).

Adam Smith (1776) explicitly explored the central role of specialization and division of labour in economic analysis by systematically investigating their implications for economic growth and prosperity. In the beginning of his book written in 1776, *An*

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<sup>1</sup> Plato (380BC, book 2, pp. 102-6) considered welfare implication of division of labour and specialization and the connection between the division of labour, the market, and money. Xenophon also examined the connection between cities and the division of labour (see Gordon, 1975, p. 41). William Petty (1671, pp. 260-61) noted that specialization contributes to skillful clothmaking and pointed out that Dutch could convey goods cheaply because they specialized each ship for a specific function. In another place, Petty gave a more striking example of the division of labour in the manufacture of a watch. He indicated (1683, pp. 471-2) that cities can promote the division of labour by reducing transaction costs. Joseph Harris (1757) and Josiah Tucker (1755, 1774) referred directly to the productivity implications of the division of labour, the possibility for the subdivision of labour, and the intimate relationship between a greater variety of goods, production roundaboutness, and a higher level of division of labour [See Groenewegen (1987), Meek and Skinner (1973), and Rashid (1986) for more details of this classical literature].

*Inquiry into the Nature and Causes of the Wealth of Nations*, he stated that "The greatest improvement in the productive powers, and the greater part of skill, dexterity and judgement with which it is anywhere directed, or applied, seem to have been the effect of the division of labour" (1776, book I, chapter 1). He proposed the conjecture that the extent of the market is determined by transportation efficiency (1776, pp.31-32) and the proposition that the division of labour is limited by the extent of the market (1776, chapter 3 of book I). According to Smith's notion of specialization and comparative advantage, now referred as "endogenous comparative advantage", economies of specialization and division of labour may exist even if all individuals are ex ante identical and the differences in productivity among individuals are the result rather than cause of the division of labour (1776, p.28).<sup>2</sup>

During the 19<sup>th</sup> century and the first half of the 20<sup>th</sup> century, the division of labour remained to play an important role in the economic analysis, particularly in the works of David Ricardo (1817), John Rae (1834), Charles B. obage (1832), Karl Marx (1867), Amasa Walker (1874), Alfred Marshall (1890), and Allyn Young (1928). Marshall (1890) attempted to formalize classical economic thinking within a mathematical framework. Marshall's *Principles* (1890) consists of two parts. One part (chapters 8-12) is full of classical insights on the economic implications of specialization and division of labour without mathematical formalization. The second part successfully formalized the

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<sup>2</sup> Adam Smith explained the difference in productivity between the agricultural sector and industrial sector as determined by the relative difference in the benefits of specialization compared to the seasonal adjustment cost caused by specialization between the two sectors. This theory explains economic structure by the different balance points in trading off economies of division of labour against coordination cost of the division of labour instead of by tastes, income, or exogenous technical conditions. An extension of the theory implies that a decline in income share of the agricultural sector occurs not because of a change in tastes, in income, or in exogenous technical conditions, but because the agricultural sector has a higher coordination cost of the division of labour compared to the benefits derived from the division of labour, and it can improve productivity only by importing an increasingly larger number of industrial goods. These goods are produced by a high level of division of labour in the manufacturing sector where transaction costs are more likely to be outweighed by economies of division of labour. This theory of Smith is formalized by Shi and Yang (1995), and some empirical implications of the Smith conjecture is investigated by Kaldor (1967).

relatively unimportant part of classical economic thinking on the problem of resource allocation. This part is based on an unrealistic dichotomy between pure consumers and pure producers (firms), which is essential for avoiding corner solutions<sup>3</sup>. Here, the problem of resource allocation is to find the efficient relative quantities of different goods and the efficient relative quantities of factors allocated to produce different goods with a given degree of scarcity (or a given transformation function) and a given pattern and level of division of labour. The problem of organization by contrast is to find the efficient level and pattern of division of labour in order to expand the production possibility frontier (or to reduce scarcity) against transaction costs for a given relative quantities of different goods consumed and produced. However, Marshall's formalization of the resource allocation problem established the mainstream of economics in the following years. As Buchanan (1994, p. 6) observed, "with one part of his mind always in classical teachings, Marshall recognized that this genuinely marvellous neoclassical construction requires that the Smithian proposition on labour specialization be abandoned". Marshall's neoclassical framework is characterized by the dichotomy between pure consumers and firms, the replacement of the concept of economies of specialization with the concept of economies scale, and marginal analysis of demand and supply.<sup>4</sup> The debate on external vs. internal economies of scale and on other issues within the framework clarified some confusion.

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<sup>3</sup>As Yang and Y-K. Ng (1993) show, the absence of formalization of classical economics on problems of economic organization in Marshall's work was because the formalization must involve corner solutions, but the technique for handling corner solutions and related inframarginal analysis was not available until the 1950s, which can answer Stigler's (1976, pp. 1209-1210) talk about "Adam Smith's failure". Here, a corner solution to an optimisation problem is a solution that involves upper and/or lower bound values of some decision variables. Inframarginal analysis is defined as total benefit-cost analysis across corner solutions in addition to marginal analysis of each corner solution. Buchanan and Stubblebine (1962) propose this concept. The application of inframarginal analysis to a decision problem can be found from Rosen (1983). The application of inframarginal analysis to general equilibrium models can be found from Yang (1991), Yang and Wills (1990), Yang and Borland (1991), Yang and Shi (1992), and Yang and Ng (1993). Koopman (1957) suggests that the concept of production function should be replaced with the notion of activity analysis when inframarginal analysis is conducted.

<sup>4</sup>The modern Arrow-Debreu model of general equilibrium, which is featured with the first two properties of Marshall's framework, has generalized and consolidated Marshall's framework. Arrow and Debreu use the concept of non-convex production set to generalize the concept of economies of scale.

As an unexpected consequence of Marshall's success in formalizing problems of resource allocation, the core of classical economics concerning specialization and division of labour has been forgotten. Samuelson's (1948) work followed Marshall's marginal analysis of demand and supply and macroeconomics that incorporates Keynesian economics. Unfortunately, little attention was given to problems of specialization and division of labour in his books (Samuelson, 1947, 1955). No formal models of Smith's endogenous comparative advantage were developed to endogenize individuals' levels of specialization in the textbooks although they covered formal models of Ricardo's exogenous comparative advantage.

Allyn Young's work (1928) is the unique exception to Schumpeter's claim (1954, p.187).<sup>5</sup> As Buchanan (1994, p.7) indicated, "Allyn Young sensed that the focus of economists' attention was shifting too readily and too rapidly toward clarification of analysis within neoclassical structure and away from classical emphasis". Young's paper (1928) is regarded by Rosen (1983, p.44) as "the zenith of the analysis of the connection between specialization and economic development". Young emphasized the concepts of specialization, roundaboutness, and division of labour, and criticized the concept of economies of scale or increasing returns to scale which had been already very popular in economics teaching and research due to the success of Marshall's book (1890) of neoclassical economics.<sup>6</sup> As Young argued, the replacement of Smith's concept of economies of division of labour with Marshall's concept of economies of scale obscures the distinction, thereby misleading the discipline of economics.

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<sup>5</sup> Schumpeter (1954, p.187) claimed that Smith's first doctrine that economic development mainly springs from the progressive division of labour and increasing specialization, "has not received the attention it deserves: nobody, either before or after Smith, ever thought of putting such a burden upon division of labour".

<sup>6</sup> It is interesting to note that the classical economists did not use the concept of economies of scale or increasing returns to scale. The concepts that they used are specialization, division of labour, and related benefits and costs. A careful reading of Alfred Marshall (1890) and Allyn Young (1928) indicates that the subtle distinction between the concepts of economies of specialization and economies of scale was crucial for the subsequent development of economics. According to Groenewegen (1987), the works of Senior (1836, pp. 74-5, 181-2), Mill (1848, p. 13), Fawcett (1863), and Nicholson (1893) started the process that replaces the concept of economies of division of labor with the concept of economies of scale.



Young's concept of "social increasing returns" is very similar to Buchanan's (1994) concept of "generalized increasing returns" and to Rosen's (1978) concept of "superadditivity". Young stated several times that the increasing returns with which he was concerned are not caused by the scale of a firm or an industry. According to him, they are generated by specialization and the division of labour rather than by economies of scale.<sup>7</sup> He used three concepts to describe the division of labour. The first one is the individuals' specialization. An individual's level of specialization increases as his scope of activities is narrowed down. The second one is the length of a roundabout production chain, or so-called roundaboutness. The third one is the number of intermediate goods in each link of the chain. Certainly, the three concepts are related to, but distinct from the concept of economies of scale. Indeed, Young's concept of social increasing returns based on specialization and division of labour is equivalent to the modern concept of a positive network effect of division of labour.<sup>8</sup> Young's conjecture (1928, p.534, p.539) consists of the following three statements. "The securing of increasing returns depends on the progressive division of labour"; "Not only the division of labour depends upon the extent of the market, but the extent of the market also depends upon the division of labour";<sup>9</sup> "Demand and supply are two sides of the division of labour".<sup>10</sup> Young's conjecture represents the view that takes economies of division of labour as a network effect.

Young suggested that the extent of the market is determined not only by population size, but also by purchasing power, which is determined by productivity, which is in turn dependent on the extent of division of labour. He then went on to argue that the circle that the division of labour depends upon the extent of the division of labour

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<sup>7</sup> Young (1928, p. 533) even argued that the use of the notion of large-scale-production misses the phenomenon of economies of division of labor.

<sup>8</sup> Young (1928, p. 539) spelled this out as follows. "The mechanism of increasing returns is not to be discerned adequately by observing the effects of variations in the size of an individual firm or of a particular industry, for the progressive division of labor and specialization of industries is an essential part of the process by which increasing returns are realized. What is required is that industrial operations be seen as an interrelated whole."

<sup>9</sup> Roumasset and Smith (1981) provide evidence for the proposition that individuals' levels of specialization determine the extent of the market.

<sup>10</sup> This relates to Say's law. Yang and Ng (1993, chap. 18) show that a new classical dynamic equilibrium model may generate efficient business cycles and unemployment even if Say's law holds.

implies that a dynamic mechanism generates progressively increasing division of labour and the extent of the market. On the other hand, this circle implies that the size of the market network and the degree of division of labour are simultaneously determined. Young's conjecture explores a typical feature of network effects of the division of labour and related market.<sup>11</sup> He implicitly, therefore, set up a research agenda to use economic models to explain how the size of the market network based on specialization and division of labour is determined in a decentralized system. Another more explicit target set by Young is to formalize the concept of economies of division of labour that include economies of individuals' specialization, economies of roundaboutness, and economies of the variety of producer goods. On the basis of the formalization, a dynamic equilibrium model may be able to simultaneously explain the three aspects of the division of labour.

Young argued (1928, p.531) that "the view of the nature of the processes of industrial progress which is implied in the distinction between internal and external economies is necessarily a partial view. Certain aspects of those processes are illuminated, while, for that very reason, certain other aspects, important in relation to other problems, are obscured." Hence, it seemed to Young that the concept of external economies of scale is a misrepresentation of the classical concept of economies of specialization and division of labour. Since Young and Marshall, the research of specialization has developed along two lines. One is associated with Marshall's concept of (external or internal) economies of scale and with his marginal analysis, and the other follows Young's concept of economies of specialization and division of labour.<sup>12</sup>

Houthakker (1956, p. 182) further developed Smith's proposition that the extent of the market is determined by transportation conditions to suggest that a tradeoff between economies of specialization and transaction costs can be used to explain the level of division of labour. If the transaction cost coefficient for one unit of goods is very large,

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<sup>11</sup>The concept of network effect here is consistent with Katz and Shapiro's (1985, 1986) definition, which implies that performance of a network depends on the number of participants and that a participant's decision depends on his expectation of other participants' decisions.

<sup>12</sup>Economies of scale and economies of division of labor may coexist. But the latter is much more important than the former, since the latter enhances the capacity of society in acquiring information and knowledge by exploiting interpersonal complementarity, while the former is a pure technical concept that may have nothing to do with endogenous technical progress generated by the knowledge acquisition process of society.

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then the economies of specialization are outweighed by the transaction costs caused by the division of labour, and the equilibrium level of division of labour will be very low. In this case, the extent of the market is small and market demand and supply are both zero. As the transaction cost coefficient falls, the efficient level of division of labour and productivity will increase, so that the extent of the market and demand and supply in the market place will increase.<sup>13</sup> This is a promising direction toward the formalization of Young's concepts of demand and supply. Houthakker drew the distinction between economies of specialization and economies of scale when he discusses the necessity of a new analytical approach. "We have increasing returns to the extent that if several activities are replaced by a single one, there is less need for (internal) coordination and switching time and more scope for acquiring experience. The output of the single activity may thus be raised above the combined outputs of the several activities." Houthakker complained that the evolution of specialization and division of labour in an economic system seems more significant and important than the evolution of species, but research of the former in economics is far behind studies of the latter in biology. This complaint still has important implications for current economic research. However, Houthakker argued (1956, p.182) "such an analysis (of specialization) involves the use of methods that are rather unlike those by which the classical questions of economics are discussed".

Another important paper that followed Young's research direction is Stigler (1951). Like Houthakker, Stigler used a graph to emphasize the distinctive feature of specialization that a firm's productivity increases as it narrows down its range of production activities. He demonstrated that a firm's cost function will be endogenously and discontinuously changed by its decision on the level of specialization. The discontinuous change in the cost function that is caused by a change in a firm's level of specialization is similar to the inframarginal analysis developed by Rosen (1983) and Yang (1991). However, Stigler still followed Marshall's approach of separating the analysis of demand from the analysis of decision making regarding the level of specialization. He emphasized internal increasing returns to specialization against Marshall's concept of external economies of scale. When discussing the problem of

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<sup>13</sup>Rosen (1983, p. 48) also states that market constraints on specialization must arise from transactions costs that limit the size of a person's market.

vertical integration, he combined the concept of economies of specialization with the concept of economies of scale, departing from Young's research direction.

Since Stigler and Houthakker, the literature on specialization has developed along three directions. The first one has developed formal models based on Ricardo's concept of exogenous comparative advantage and on Marshall's framework with a dichotomy between pure consumers and firms and marginal analysis of demand and supply, which are focusing on the division of labour between countries rather than on the endogenization of individuals' levels of specialization. This turned out to be a field called the theory of international trade. Dixit and Norman's textbook (1980) is representative of this research direction. The second one has developed formal models using the concept of economies of scale and Marshall's framework to endogenize one aspect of Young's concept of division of labour: the number of consumption and producer goods. Representative works along this direction are Dixit, Stiglitz (1977), Ethier (1982), Krugman (1979), Judd (1985), Romer (1986), and Grossman, Helpman (1989, 1990).

The third direction has endogenized individuals' levels of specialization and the level of division of labour for society as a whole, following Smith, Young, and Houthakker's ideas. Some of the formal models applied inframarginal analysis to endogenize the level of division of labour and explain other economic phenomena by the level and pattern of division of labour. In the last two decades, the primary effort to bring formal economic research back to the original ideas of Smith, Young, and Houthakker, might be attributed to Rosen (1978) and Becker (1981). Rosen (1978) extended the Ricardo model to the case with many goods and many individuals. Applying linear programming rather than marginal analysis to handle the problem of corner solutions, he explored the implications of corner solutions for endogenization of individuals' levels and patterns of specialization instead of getting around corner solutions. He showed that economies of division of labour that are endogenously determined by individuals' decisions on their levels and patterns of specialization look like external economies of scale, but may exist in the absence of economies of scale. Becker (1981) developed a model to endogenize individuals' decisions on specialization within a family. This model is solved using inframarginal analysis of many corner and interior solutions. The positive interactions between labour and human capital allocated to produce certain goods

generate a pattern of complete specialization for each member of the family except one who will not completely specialize when an integer condition for the numbers of different specialists is not satisfied. Although this model is not explicitly specified as a dynamic decision model, and human capital plays a role similar to the one with difference in endowment between agents in neoclassical models, it focuses on the endogenization of individuals' patterns of specialization and emphasizes the role of endogenous comparative advantage. This might be taken to be a starting point for formalizing Smith and Babbage's idea that the division of labour can be used to avoid duplicate fixed learning and training costs.<sup>14</sup>

Yang (1988, 1990) developed a model to endogenize individuals' levels of specialization and the level of division of labour in society by abandoning the dichotomy between pure consumers and firms. In this model, each individual is a consumer-producer who prefers diverse consumption and specialized production because of economies of specialization. A tension between specialized production and diverse consumption for each consumer-producer generates a tradeoff between economies of specialization and transaction costs. A central planner may trade off economies of specialization against transaction cost to achieve the efficient level of division of labour by equalizing marginal benefit of the division of labour and marginal transaction cost. Since this is a planning model, marginal analysis can be used for decision making, and corner solutions that may emerge from a decentralized market are avoided. In terms of mathematics, the Becker and Murphy's model (1992) is very similar to this model if the coordination cost in their model is interpreted as the transaction costs in the Yang's model (1988, 1990). These models showed that the efficient level of division of labour is determined not only by the population size which is usually considered as the extent of the market, but also by the efficient balance between the economies of division of labour and the coordination or transaction costs.

Since the middle of the 1980s, the endogenization of individuals' levels of specialization has been developing along two distinctive directions. One is to endogenize individuals' levels of specialization on the basis of the neoclassical dichotomy between pure consumers and pure producers or to endogenize individuals' levels of specialization

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<sup>14</sup>Also, Schultz (1993) has explored the intrinsic connection between human capital and specialization.

A combination of several configurations<sup>15</sup> that is compatible with the market clearing conditions for traded goods is referred to by Yang (1990) as a "market structure" or a structure for short. For each structure, a market clearing condition can be established for each traded good by specifying the numbers of individuals selling different goods and by equalizing total market demand and supply. Also, utility equalization conditions can be established by competition for a higher income between specialties (configurations). Hence, for each structure, there may exist a set of relative prices of traded goods and a set of numbers of individuals selling different goods that satisfy the market clearing conditions. Yang refers to the set of relative prices and the set of numbers of individuals selling different goods in each structure as a corner equilibrium. Each corner equilibrium is associated with a certain network of the market. Different corner equilibria are associated with different numbers of traded goods for society, different degrees of interdependence between different specialists, and different productivity levels. A Walrasian regime is assumed because the number of ex ante identical individuals is large and economies of specialization are individual specific. Within this framework, a general equilibrium is defined as a fixed point that satisfies the following conditions. First, each individual uses inframarginal analysis to maximize his utility with respect to configurations and quantities of each good produced, consumed, and traded for a given set of relative prices of traded goods and a given set of the numbers of individuals selling different goods. Second, the set of relative prices of traded goods and the set of numbers of individuals selling different goods clear the markets for traded goods and equalize utility for all individuals selling different goods. There are two steps in solving for the general equilibrium. First, a corner equilibria is solved for each structure. Second, the general equilibrium is identified as the corner equilibria that generates the highest utility level since it satisfies the two conditions for the definition of general equilibrium. The other corner equilibria are not general equilibrium since they do not satisfy the first condition. A rigorous proof of the proposition that individuals have an incentive to deviate from these inefficient corner equilibria can be found in Yang (1990) and Yang and Ng (1993, Chapter 6).

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<sup>15</sup> A profile of zero and non-zero variables that is compatible with the Wen theorem (1997b) is referred to as a "configuration"

There are two types of comparative statics of the general equilibrium. The first type of comparative statics implies that the general equilibrium, demand and supply functions, and indirect utility function will discontinuously shift between corner equilibria as transaction costs and production function parameters have reached some critical values. The discontinuous jump of the supply function is consistent with Stigler's conjecture (1951) that a change in the level of division of labour will discontinuously shift the cost function and it can be interpreted as an endogenous technical progress. Another type of comparative statics of the general equilibrium implies that the equilibrium relative prices, quantities of goods, and number of individuals selling different goods will change continuously in response to continuous changes of the parameters within the ranges demonstrated by the first type of comparative statics. The second type of comparative statics is analogous to neoclassical comparative statics of equilibrium based on marginal analysis. It generates the implications for resource allocation for a given level and pattern of division of labour. But there is no neoclassical counterpart of the first type of comparative statics based on inframarginal analysis. From this framework, it is easy to tell that the efficient extent of the market and efficient level of specialization, productivity, scarcity, and per capita real income are different aspects of the level of division of labour. The efficient level of division of labour is determined by the trade-off between economies of division of labour and transaction costs.

The first type of comparative statics substantially enhances the power of general equilibrium models in explaining changes in patterns of market network. The Yang's model (1988, 1990) showed that the invisible hand can efficiently exploit network effects of division of labour and transaction costs. Which network of the market and related division of labour is efficient depends on the transaction efficiency coefficient. If the transaction efficiency is low, the positive network effect of the division of labour is outweighed by the transaction costs. Autarky or a low level of division of labour, which is associated with a small size of the network of the market, is efficient and will be chosen by the invisible hand. On the other hand, if the transaction efficiency is improved, the efficient and equilibrium level of division of labour and related efficient size of market network will increase. Hence, whether or not the positive network effects can be



utilized, will fully depend on where is the efficient tradeoff between the positive network effects and the transaction costs.

Under this new classical economic framework, a great deal of research has already been undertaken, such as a trade theory (Yang and Shi 1992), a theory of the firm (Yang and Ng 1995), urban economics (Yang and Rice 1994), growth models (Yang and Borland 1991), a theory of contract and property rights (Yang and Wills 1990, Lio 1996), a theory of capital (Yang, 1999), a theory of money (Cheng, 1998), a theory of business cycles and unemployment (Yang and Ng 1993), a theory of insurance and uncertainty (Lio, 1996), a theory of development (Shi and Yang 1995), a theory of hierarchy (Shi and Yang 1998, Yang 1994), etc.. Sun, Yang, and Yao (1999) have proven the existence theorem and the first welfare theorem for a general class of the Yang's model (1988, 1990) with a continuum of individuals and without an explicit specification of functional forms. Zhou, Sun, and Yang (1999) extended the results to a general class of the Yang's model allowing ex ante different individuals. Also, they have proven that the set of equilibrium allocations is equivalent to the set of core allocations. These models have simultaneously formalized many of the original ideas of Smith, Young, and Houthakker. They have showed that demand and supply are two sides of the division of labour, and the extent of the market (absolute level of aggregate demand) can be endogenized as one aspect of the level of division of labour. The most important function of the market is to choose the efficient size of the market network based on the division of labour. The concept of endogenous comparative advantage is formalized. The absolute level of aggregate demand of each person, which is one aspect of the extent of the market, is determined by each person's level of specialization.

However, two gaps need to be filled in the literature of newclassical economics. First, stealing is not endogenized and the effects of stealing on network size of division of labour are not examined. Second, the effects of bundling sale on network size of division of labour are not investigated.

### **1.2 Brief Outline of Chapters 2 to 4**

This thesis applies inframarginal analysis to study the relationship among the division of labour, theft behaviour and bundling sale within the new classical

jungle in Chapter 2 to investigate the effects of the Monarch's power in enforcing property rights on network size of division of labor and productivity. By comparing self-protection and the third party's protection, this chapter shows that the government can endogenously emerge from taxation that is used to finance the judicial system and enforcement of laws that penalize theft. This chapter has examined the trade off between positive network effects of the laws and their enforcement, and negative effects of taxation on the network size of division of labor. It is shown that the improvement in institutional efficiency expands the demand for transactions, which requires third party's protection. The market mechanism for goods as well as for the third party's protection, determines the selection of the protection system of property rights. In this model, aggregate productivity is determined by the network size of division of labor (extent of the market). The network size of division of labor is determined by the enforcement of property rights, which is dependent on tax revenue and in turn dependent on per capita real income, aggregate productivity, and network size of division of labor. Hence, the notion of general equilibrium is a powerful vehicle to investigate circular causation and related economics of state.

Chapter 4 develops a general equilibrium model of impersonal networking decisions and bundling sale. It departs from the other models of bundling and tying by allowing substitution between goods, flexible quantities of goods, resale of any goods, competitive market, and ex ante identical utility function for all individuals. Hence, interactions and feedback loops between quantities, prices, network effects of division of labour, transaction costs, self-interested decisions, income, and productivity can be investigated. Inframarginal analysis (total cost-benefit analysis across corner solutions in addition to marginal analysis of each corner solution) of the model shows that the function of bundling sale in a competitive market is to get intangible information goods involved in the division of labour and commercialised production, meanwhile avoiding direct pricing of such goods, thereby promoting division of labour and aggregate productivity. According to this theory of bundling, bundling in a competitive market is Pareto efficient and it plays a very important role to utilize positive network effects of division of labour on aggregate productivity. Antitrust prosecution should pay more attention to the intention to block free entry rather than bundling itself.

## CHAPTER 2. Division of Labor, Specialization, and Theft Behavior: A General Equilibrium Analysis

### 2.1 Introduction

The traditional theory of competitive markets relies on an implicit assumption that a large number of anonymous traders engaged in mutually beneficial transactions are under a perfect legal umbrella that protects and enforces property rights without cost. However, such circumstances are rare. Even in a bilateral exchange of goods with commonly known quality, it is still possible that one party does not execute the contractual promise contingent on the other party's performance. Moreover, one party may attempt to steal the other party's goods besides buying. Coase (1960) and Demsetz (1964, 1967) raise the issues of social cost. As Demsetz points out, based on the social transaction cost, the systems enforcing the exchangeable property rights provide an attractive basis for economizing on the costs of allocating resources in society. The emergence of the property rights system can reduce the resources that individuals devote to conflicts over distribution.

According to Hobbes (1651) and Rosseau (1762), human society has a primordial "state of nature" where property rights do not exist. Individuals under such circumstances may devote some of their endowments to steal goods from other parties, as well as to self-protect their own goods. According to Hobbes, the state of nature has the characteristics of the law of the jungle, that is,

...to a time of Warre, where every man is Enemy to every man, ... wherein men live without other security, than what their own strength, and their own invention shall furnish them withall. ...that nothing can be Unjust. The notions of Right and Wrong, Justice and Injustice have there no place. Where there is no common power, there is no law. ... but onely that to be every mans, that he can get; and for so long, as he can keep it. ... The Right of Nature, is the Liberty each man hath, to use his own power, as he will himself, for the preservation of his own Nature; that is to say, of his own Life; and consequently, of doing anything, which in his own Judgement, and Reason... (Leviathan, pp.64-66)

Moreover, Hobbes described the mechanism of "Laws of Nature" which is to establish property rights and mandate punishment for those who violate the rights of others. However, establishing social order involves trade-offs between positive network

effects of establishment of public law and its enforcement, and negative effects of reducing the resources devoted to productive activities, and on the network size of division of labor. The decision whether or not to establish social order is dependent on comparing the costs of enforcement, the cost of the attempt to transfer benefit from others and the cost of self-protection. Just as the statement made by Rousseau,

...What a man loses as a result of the social contract is his natural liberty and his unqualified right to lay hands on all that tempts him, provided only that he can encompass its possession. What he gains is civil liberty and the ownership of what belongs to him... (The Social Contract, Chapter VIII)

Because each individual is a potential criminal as well as a potential victim, the social contract therefore should have the following components: (i) rules to establish property rights, (ii) a mechanism for enforcing the sanctions for violations of property rights, (iii) rules which specify the taxation system made by each individual to support this property rights system, such as a "criminal justice system" (Skogh and Stuart, 1982). However, Hobbes and Rosseau did not tell us how the Commonwealth superpower will endogenously emerge from economic development, or in their words, how the Man or the Assembly of Men emerges from the society.

In addition to the above research, there have been several more studies done in recent decades. Becker (1968) sets up an economic approach for crime and punishment. Alchian (1983) emphasizes that implementing the property rights system needs the might of the state or the government. Buchanan (1975) analyses the bases for a society where the people want to be free but who recognize the inherent limits that social interdependence places on them. Further he points out "Men want freedom from constraints, while at the same time they recognize the necessity of order" (*The Limits of Liberty*, preface). Barzel (1997) applies contract theory to analyze property rights and the evolution of the state. Guth and Kliemt (1995) establish and apply an indirect evolutionary approach to show that institutions of enforceable adjudication in themselves may enable higher levels of contract compliance than would occur in their absence even though adjudicators are no better behaved than ordinary traders.

Umbeck (1981) states, even before the "War where every man is enemy of every man" (Hobbes, 1651, p.64), human society already sets up contractual arrangement and the superpower agents which effectively protect and enforce the implementation of the

contracts. Moreover, the markets emerge as the intermediate of the whole society also during this progress. Although Hobbes presented to us some aspects of the principal nature of the human society, they rarely happened or were only special cases at some periods in human history (Demsetz, 1964). However, the above studies do not endogenize stealing and network size of division of labor and the emergence and evolution of property rights in their models.

The purpose of this chapter is twofold. First, using inframarginal analysis, this chapter develops a general equilibrium model with endogenous stealing and endogenous network size of division of labor. It can examine the effect of stealing on the network size of division of labor and productivity; Second, this model shall formalize the notion of Hobbes' Jungle using a mixed Nash - Walrasian equilibrium model, and take a small step toward the formalization of the economics of state and constitutional economics.

The rest of this chapter is arranged as follow: Section 2 presents a basic model with two final goods. It will consider the cases with and without stealing and self-protection. This model doesn't concern legal protection from the third party, like the Government or the Monarch. Moreover, this model is under a mixed regime of Nash and Walras; Section 3 will solve for equilibrium and its inframarginal comparative statics; Section 4 concludes this chapter.

## **2.2 A General Equilibrium Model with Division of Labor, Increasing Return to Specialization, and the Attempts to Steal the Possessions of Other Parties**

Consider a large economy with  $M$  ex ante identical consumer-producers, and two final goods  $x$  and  $y$ , where the set of individuals is assumed a continuum. Assume these goods can be self-produced or purchased from the market. The self-provided consumption of good  $x$  is denoted as  $x^s$ , the amount sold in the market is  $x^d$ , and the amount purchased from the market is  $x^m$ . The transaction efficiency coefficient is  $k$  for each unit of good purchased from the market, and  $kx^d$  is the quantity of good  $x$  received from the purchase for consumption, where  $k \in (0,1)$ . Let  $x^i$  and  $y^i$  denote an individual's amounts of labor allocated to steal other parties' two goods, respectively. Furthermore,  $t$

represents the stealing efficiency coefficient and it is assumed that  $t \in (0,1)$ .  $t \cdot x'$  is the consumption of good  $x$  from stealing. Strictly speaking, parameter  $t$  represents relative efficiency of an individual's stealing activities compared to others' self-protection activities. Other variables for good  $y$  have the similar meaning. Moreover, because each individual cannot expect to only steal the other parties' goods without having their goods stolen, let  $x^T$  and  $y^T$  denote the amounts of the two goods stolen from her by all others.

Moreover, in this model we make the following assumption.

### Assumption 2.1

**Each individual may choose stealing from her neighbors in autarky or stealing from her trading partners in a structure with the division of labor.**

It is assumed that all individuals are ex ante identical, and they are different only after they choose to specialize in the production of different final goods ( $x$  or/and  $y$ ). Finally, we assume that the whole economy is symmetrical, although all the later results can be applied to a non-symmetrical economy except that the calculation will be more strenuous.

Each consumer-producer has identical, non-satiated, continuous, and rational preference represented by the following utility function:

$$(2.1.1) \quad u = f(x^c, y^c),$$

where  $x^c \equiv (x + k \cdot x^d + t \cdot x')$  and  $y^c \equiv (y + k \cdot y^d + t \cdot y')$  are the amounts of the two goods that are consumed.  $f(\cdot)$  is continuously increasing and quasi-concave.<sup>1</sup> For simplicity, it is assumed  $f(\cdot) = (x^c)^\theta \cdot (y^c)^{1-\theta}$  and  $\theta = 1/2$ .

The individual's production functions with fixed learning costs, are specified as,

$$(2.1.2) \quad \begin{aligned} x^p &= x + x^s + x^T = \max\{l_x - \alpha, 0\} \text{ and } \alpha \in (0,1), \\ y^p &= y + y^s + y^T = \max\{l_y - \alpha, 0\} \text{ and } \alpha \in (0,1). \end{aligned}$$

<sup>1</sup> The specification of such iceberg transaction cost is a common practice in the equilibrium models with the trade-off between increasing returns and transaction costs (see Krugman, 1995). This specification avoids notoriously formidable index sets of destinations and origins of trade flows.

Here,  $x^p$  and  $y^p$  are the total output levels of the individual in producing goods  $x$  and  $y$ , respectively.  $x^s$  and  $y^s$  are respectively the amounts of the two goods sold in the market.  $x^T$  and  $y^T$  are respectively the amounts of the two goods that are stolen from the individual by others. Term  $l_x$  represents the amount of time used in the production of good  $x$  and the individual's level of specialization in producing good  $x$ ; the parameter  $\alpha$  is a fixed learning cost and is related to the degree of economies of specialization. Term  $l_y$  and the parameter  $\alpha$  of good  $y$  have the similar meaning.

(2.1.2) implies that the received self-provided consumption of the two goods respectively are,

$$(2.1.3) \quad \begin{aligned} x &= \max\{x^p - x^s - x^T, 0\}, \\ y &= \max\{y^p - y^s - y^T, 0\}. \end{aligned}$$

This person takes  $x^T$  and  $y^T$  as given when she chooses her own stealing activity level. In this sense, the stealing game among all these individuals which is part of this model is a Nash game, despite the Walrasian regime where individuals choose the quantities of production, trade, and consumption for given market prices. The above functions indicate that if an individual does not produce good  $x$  and/or good  $y$ , then the other individuals can't steal them from her.

The individual's stealing functions are as follows.

$$(2.1.4) \quad \begin{aligned} x^t &= l_x^\beta, \text{ and} \\ y^t &= l_y^\beta, \end{aligned}$$

where  $x^t$  and  $y^t$  are the total amounts of good  $x$  and  $y$  which the individual steals from the other parties. It is assumed that  $\beta \in (0,1)$ , which means stealing activities exhibit decreasing returns to scale.

The endowment constraint for labor is:

$$(2.1.5) \quad l_x + l_y + l_x + l_y = 1.$$

This system of production functions and endowment constraint implies the existence of economies of specialization in production since labor productivity increases with an individual's level of specialization. Meanwhile, the stealing functions display

decreasing returns. Intuitively, from endowment constraint (2.1.5), we can also derive the following constraint,

$$(2.1.6) \quad 0 \leq l_x, l_y, l_x^s, l_y^s \leq 1,$$

The budget constraint for her is,

$$(2.1.7) \quad p_x x^s + p_y y^s = p_x x^d + p_y y^d.$$

Finally, all the variables, parameters and coefficients are non-negative.

There are  $2^{10} = 1024$  combinations of zero and non-zero values of  $x, x^s, x^d, x^t, y, y^s, y^d, y^t$ . In Yang and Ng (1993), Yang and Shi (1992), and Wen (1997), Lemma 2.1 has been established in a model similar to the one in this chapter.

### Lemma 2.1

**According to the Kuhn-Tucker condition, for an individual's optimum decision, each person sells at most one good, and does not buy the same good, or buy and self-provide the same good.**

Then, we define a configuration as a combination of zero and non-zero variables, which is compatible with Lemma 2.1 and Assumption 2.1; a market structure, or a structure for short, is a division of individuals in the economy among the configurations that are compatible with the market clearing condition. Here, Lemma 2.1 and Assumption 2.1 imply that a specialist producer of good  $x$  cannot steal from another  $x$  specialist since  $x$  specialists trade only with  $y$  specialists.

Let us examine all structures that might occur in equilibrium.

#### 2.2.1 Autarky without stealing (A)

Individual autarky without stealing is a structure where every individual chooses the configuration A, which implies  $x^s = x^d = y^s = y^d = x^t = x^r = y^t = y^r = 0$  for all the consumer-producers. All the people in this structure self-provide the two final goods by themselves, and no market transactions happen among them. The decision problem for an individual in configuration A can be specified as follows,



$$(2.2.1) \quad \text{Max: } u_A = x^{\frac{1}{2}} \cdot y^{\frac{1}{2}},$$

subject to the following constraints:

$$(2.2.2) \quad x^p = x = \max\{l_x - \alpha, 0\} \text{ and } \alpha \in (0,1),$$

$$y^p = y = \max\{l_y - \alpha, 0\} \text{ and } \alpha \in (0,1),$$

$$l_x + l_y = 1,$$

$$0 \leq l_x, l_y \leq 1.$$

The optimal resource allocation in this situation of autarky is,

$$(2.2.3) \quad l_x = l_y = \frac{1}{2},$$

$$u_A = \left(\frac{1}{2} - \alpha\right).$$

Here, the individual's maximum utility  $u_A$  equals the maximum per capita output level of the two final goods. As every individual is only endowed with one unit of labor,  $u_A$  is the per capita real income as well as the maximum average labor productivity of the final goods in this structure.

### 2.2.2 Autarky with Stealing ( $A^S$ )

Autarky with stealing is a structure where every individual chooses the configuration autarky with stealing  $A^S$ , which implies  $x^r = x^d = y^r = y^d = 0$  for all the consumer-producers. All the people in this structure self-provide the two final goods and steal them from their neighbors as well. There are no market transactions happen among them. Meanwhile, for the purpose of simplicity without losing generality, we assume  $\beta = \frac{1}{2}$  from now on. The decision problem of an individual who chooses the configuration autarky with stealing  $A^S$  becomes,

$$(2.3.1) \quad \text{Max: } u_{(AS)} = (x + t \cdot x')^{\frac{1}{2}} \cdot (y + t \cdot y')^{\frac{1}{2}},$$

subject to the following constraints:

$$(2.3.2) \quad x = \max\{l_x - \alpha - x^r, 0\} \text{ and } \alpha \in (0,1),$$

$$y = \max\{y - \alpha - y^T, 0\} \text{ and } \alpha \in (0,1),$$

$$x' = l_x^{\frac{1}{2}},$$

$$y' = l_y^{\frac{1}{2}},$$

$$l_x + l_y + l_x + l_y = 1,$$

$$0 \leq l_x, l_y, l_x, l_y \leq 1.$$

Taking symmetry into account, which generates  $x^T = x'$  and  $y^T = y'$ , the optimal resource allocation in autarky with stealing is,

$$(2.3.3) \quad l_x = l_y = \left(\frac{t}{2}\right)^2,$$

$$u_{(AS)} = \left(\frac{1}{2} - \alpha - \frac{t \cdot (2-t)}{4}\right).$$

Here, the individual's maximum utility  $u_{AS}$  equals the maximum per capita output level of the two final goods. As every individual is endowed with one unit of labor,  $u_{AS}$  is the real per capita income as well as the maximum average labor productivity of utility in this structure.

Moreover, since  $u_{(AS)} > 0$ , a positive utility in  $A^S$  requires the following inequality.

$$(2.3.4) \quad f(\alpha, k, t) \equiv (1-t)^2 + 2 \cdot (1-2\alpha) > 0,$$

which requires the following conditions to be met,

$$(2.3.5) \quad \alpha < \frac{1}{4}, \text{ or}$$

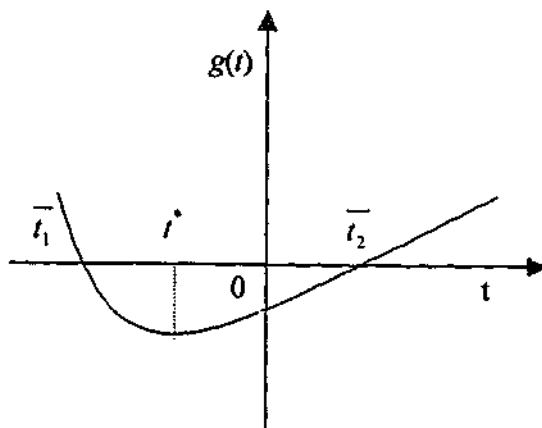
$$\frac{1}{2} > \alpha > \frac{1}{4} \text{ and } t < t_1, \text{ or}$$

$$\frac{1}{2} > \alpha > \frac{1}{4} \text{ and } t > t_2,$$

where  $t_1 = 1 - \sqrt{4\alpha - 1}$ , and  $t_2 = 1 + \sqrt{4\alpha - 1}$ . It can be shown that  $x > 0$  if and only if  $g(t) \equiv t^2 + 2t - 2(1-2\alpha) < 0$ , which holds if  $t \in (0, \sqrt{1+2(1-2\alpha)} - 1)$ . Here,  $g(t)$  is a convex U-shape curve with the minimum point on the left side of the vertical axis since

$\partial g(t)/\partial t > 0$  at  $t=0$  and  $\partial^2 g(t)/\partial t^2 > 0$  within this region of parameter values. As shown in Figure 2.1,  $\bar{t}_1$  and  $\bar{t}_2$  are cutting points of the curve to the horizontal axis, given by  $g(t) = 0$  and  $\bar{t}_1 < \bar{t}_2$ . It can be seen that  $\bar{t}_1 < 0$  and  $\bar{t}_2 > 0$ , and  $\bar{t}_1 = -\sqrt{1+2(1-2\alpha)} - 1$  and  $\bar{t}_2 = \sqrt{1+2(1-2\alpha)} - 1$ .

Figure 2.1 The Graph of  $g(t)$



Since  $\bar{t}_2 > t_2$  cannot hold when  $\alpha > \frac{1}{4}$ , this implies  $t > t_2$  is irrelevant for  $u_{(AS)} > 0$  when

$$\frac{1}{2} > \alpha > \frac{1}{4}.$$

### 2.2.3 Complete Division of Labour without Stealing ( $D^N$ )

The structure involving complete division of labor without stealing is denoted as  $D^N$ . It consists of configurations  $(x/y)$  and  $(y/x)$ . All the individuals in these configurations specialize in producing one final good, which means self-providing and selling only one final good, meanwhile buying the other final goods from the market. The symbols in the parenthesis denote the nature of the configuration. Taking  $(x/y)$  as an example, the first symbol  $x$  means that the individual self-provides and sells good  $x$ ; the second symbol  $y$  after the slash indicates that the individual buys good  $y$  from the

Based on Lemma 2.1, there are two configurations  $(xx^T / y^d y')$ , and  $(yy^T / x^d x')$  involved in the complete division of labor with stealing. The configurations in the structure are the same as in structure  $D^N$  except that the amounts of stealing activities are positive. The symbols in the parenthesis, taking  $(xx^T / y^d y')$  as an example are as follows:  $x$  means that the individual self-provides and sells good  $x$ ;  $x^T$  represents that some of good  $x$  are stolen by others;  $y^d$  after the slash, indicates that the individual buys good  $y$  from the market;  $y'$  represents the amount that the individual steals of good  $y$ . This configuration can be defined as  $x^d = x' = y = y' = y^T = 0$ , and  $x, x^s, x^T, y^d, y' > 0$ .

Considering the configuration  $(xx^T / y^d y')$  in the structure  $D^S$ , an individual has the following decision problem,

$$(2.5.1) \quad \text{Max: } u_{(xx^T / y^d y')} = x^{\frac{1}{2}} \cdot (k \cdot y^d + l \cdot y')^{\frac{1}{2}},$$

subject to the following constraints:

$$(2.5.2) \quad x^p = x + x^s + x^T = \max\{l_x - \alpha, 0\} \text{ and } \alpha \in (0,1),$$

$$y' = l_y^{\frac{1}{2}},$$

$$x = \max\{l_x - \alpha - x^s - x^T, 0\} \text{ and } \alpha \in (0,1),$$

$$l_x + l_y = 1,$$

$$0 \leq l_x, l_y \leq 1,$$

$$p_x x^s = p_y y^d.$$

Therefore, the optimal solution for the individual choosing configuration  $(xx^T / y^d y')$  is,

$$(2.5.3) \quad l_y = \frac{l^2}{4k^2},$$

$$x^s = \frac{(1 - \alpha - x^T)}{2} - \frac{3l^2}{8k^2}.$$

Taking symmetry into account, which generates  $x^T = y'$ , we have,

$$\alpha > \frac{3}{4} \text{ and } t < t_3, \text{ or}$$

$$\alpha > \frac{3}{4} \text{ and } t > t_4,$$

where  $t_3 = k \cdot (1 - \sqrt{4\alpha - 3})$  and  $t_4 = k \cdot (1 + \sqrt{4\alpha - 3})$ . Moreover, it can be shown that  $x^s > 0$  if and only if  $g(t) \equiv 3t^2 + 2kt - 4k^2(1 - \alpha) < 0$ , which holds if  $t \in (0, \frac{k\sqrt{1+12k^2(1-\alpha)} - k}{3})$ . Here,  $g(t)$  is a convex U-shape curve with the minimum point on the left side of the vertical axis since  $\partial g(t)/\partial t > 0$  at  $t=0$  and  $\partial^2 g(t)/\partial t^2 > 0$  within this region of parameter values. However,  $t_4 > \frac{k\sqrt{1+12k^2(1-\alpha)} - k}{3}$  when  $\alpha > \frac{3}{4}$ , which implies that  $t > t_4$  is irrelevant for  $u_{(DS)} > 0$  when  $\alpha > \frac{3}{4}$ .

Per capita real incomes of these structures are summarized in Table 2.1.

**Table 2.1. Per Capita Real Income in Different Economic Structures**

Structures	Per capita real income
$A$	$u_A = \left(\frac{1}{2} - \alpha\right)$
$A^S$	$u_{(AS)} = \left(\frac{1}{2} - \alpha - \frac{t \cdot (2-t)}{4}\right)$
$D^N$	$u_{(DN)} = \frac{k^{\frac{1}{2}} \cdot (1-\alpha)}{2}$
$D^S$	$u_{(DS)} = \frac{[4k^2 \cdot (1-\alpha) - t \cdot (2k-t)]}{8k^{\frac{3}{2}}}$

### 2.3 General Equilibrium and Its Inframarginal Comparative Statics

As far as stealing activities are concerned, the regime is similar to a Nash game, which involves conjectured variations of quantities stolen by other players. Hence, it can be assumed that each individual chooses her stealing level for a given amount of her goods that is stolen by others. It can be claimed that this is a mixed regime of Nash and Walras. As far as quantities traded and produced and prices are concerned, it is a Walrasian regime; while as far as stealing activity levels are concerned, it is a Nash regime with a continuum of players. Each player makes her decision according to the price vector announced by the referee and her conjecture on the quantities of her production that is stolen by others (see Appendix 2.1, *Mathematical Description of a mixed Nash - Walrasian Equilibrium Model with Endogenous Stealing and Endogenous Specialization*).

A general equilibrium is a consequence of the interactions between prices and behaviors that simultaneously determines both prices and the quantities of goods and factors. As shown in Yang and Ng (1993), since each individual chooses only the optimum one from the multiple optimal solutions, the general equilibrium is one of multiple corner equilibria. In the new classical framework, a general equilibrium not only counts all interactions between prices and quantities, between the markets for different goods and factors, and between individuals' self-interested behaviors, but also is the mechanism that simultaneously determines the network size of division of labor, demand and supply as two sides of the network, productivity, and per capita real income. A general equilibrium can be defined as a set of relative prices of traded goods, a set of numbers of individuals choosing different configurations that constitute a structure, and individuals' quantities of goods produced, traded, consumed and the stealing level, which satisfies the following conditions: (1) each individual's decision of the configurations and the quantities of goods produced, traded, consumed and stealing level, maximizes her utility for a given set of relative prices of traded goods, a given set of the numbers of individuals choosing different configurations, and the amount of goods stolen; (2) the set of relative prices of traded goods and the numbers of individuals in different configurations clear the markets for all traded goods and equalize all individuals' utilities; (3) the stealing plan of every individual is realized.

According to the Yao Theorem (Yang, 2001, Chapter 6), the general equilibrium in this model is the corner equilibrium that yields the highest per capita real income and in which nobody has an incentive to unilaterally deviate from it. Other corner equilibria are not general equilibrium. Therefore, we will compare per capita real incomes among corner equilibria in all structures and check if individuals have incentives to deviate from it. In order to do it, we will partition the parameter space  $(\alpha, k, t)$  into several sub-spaces within each of which a particular corner equilibrium generates the highest per capita real income and therefore is the general equilibrium. This is referred to as total cost-benefit analysis, which is the second step in the inframarginal analysis.

Comparing the per capita real incomes in structure  $D^N$  and  $D^S$ , it is easy to show that,

$$(2.6.1) \quad u_{DN} > u_{DS},$$

for  $t \leq 2k$ , which must hold for a feasible corner equilibrium in  $D^S$ .

However, we can show that an individual has an incentive to deviate from the corner equilibrium in  $D^N$ . If we consider the  $x$  specialist's utility function in structure  $D^S$ , which can be derived as follows,

$$(2.6.2) \quad u_{(x^T, y^d, y^f)} = (l_x - \alpha - x^s - x^T)^{\frac{1}{2}} \cdot [kx^s + t \cdot (1 - l_x)^{\frac{1}{2}}]^{\frac{1}{2}}.$$

From this utility function, its first order conditions are,

$$(2.6.3)$$

$$\begin{aligned} \left. \frac{\partial u_{(x^T, y^d, y^f)}}{\partial l_x} \right|_{l_x=1} &= \frac{1}{2 \cdot u_{(x^T, y^d, y^f)}} \cdot \left\{ [kx^s + t \cdot (1 - l_x)^{\frac{1}{2}}] - (l_x - \alpha - x^s - x^T) \cdot \frac{t}{2 \cdot (1 - l_x)^{\frac{1}{2}}} \right\} \Bigg|_{l_x=1} \\ &= \frac{1}{2 \cdot u_{(x^T, y^d, y^f)}} \cdot (kx^s - \infty) < 0, \end{aligned}$$

$$(2.6.4)$$

$$\left. \frac{\partial u_{(x^T, y^d, y^f)}}{\partial l_y} \right|_{l_y=0} = \frac{1}{2 \cdot u_{(x^T, y^d, y^f)}} \cdot \left\{ -(kx^s + t \cdot l_y^{\frac{1}{2}}) + (1 - l_y - \alpha - x^s - x^T) \cdot \frac{t}{2 \cdot l_y^{\frac{1}{2}}} \right\} \Bigg|_{l_y=0}$$

$$= \frac{1}{2 \cdot u_{(x^T, y^d, y^s)}} \cdot (-kx^T + \infty) > 0 .$$

According to the Kuhn - Tucker condition, the above results imply that the optimum value of  $l_x$  is not 1, and the optimum value of  $l_y$  is not 0, which means when an individual specializes in good  $x$ , she will still steal good  $y$  from others. Since  $l_y = 0$  in structure  $D^N$ , this result implies that an individual has an incentive to deviate from the corner equilibrium in structure  $D^N$ . Applying the same procedure, it also can be derived that,

(2.6.5)

$$\begin{aligned} \left. \frac{\partial u_{(x^T, y^d, y^s)}}{\partial \alpha} \right|_{l_y=1} &= \frac{1}{2 \cdot u_{(x^T, y^d, y^s)}} \cdot \left\{ -(kx^T + t \cdot l_y^{\frac{1}{2}}) + (1 - l_y - \alpha - x^T - x^T) \cdot \frac{t}{2 \cdot l_y^{\frac{1}{2}}} \right\} \Bigg|_{l_y=1} \\ &= \frac{1}{2 \cdot u_{(x^T, y^d, y^s)}} \cdot \left[ -(kx^T + t) - (\alpha + x^T + x^T) \cdot \frac{t}{2} \right] < 0 . \end{aligned}$$

This result implies that it is not optimal to specialize in stealing.

In structure  $D^S$ , the optimum  $l_x \neq 1$  and the optimum  $l_y \neq 0$ . In other words, structure  $D^N$  never occurs in equilibrium due to the "prisoners' dilemma" in a Nash game of choosing a stealing level, i.e., people still have incentives to steal the other parties' possessions even if this stealing makes everybody worse off.

Meanwhile, from (2.6.3) and Table 2.1, it follows that the individual will not devote all her labor to stealing activities. In addition, a comparison between  $u_{DN}$  and  $u_{DS}$  in Table 2.1 shows that the corner equilibrium in structure  $D^N$  is Pareto superior to the corner equilibrium in structure  $D^S$  where stealing occurs.

Comparing the per capita real incomes in structure  $A$  and  $A^S$ , if we follow the same method as the above analysis, it is easy to prove that  $\left. \frac{\partial u_{AS}}{\partial l_x} \right|_{l_x=0} > 0$  and

$$\left. \frac{\partial u_{AS}}{\partial l_x} \right|_{l_x=1} < 0, \text{ together with } \left. \frac{\partial u_{AS}}{\partial l_y} \right|_{l_y=0} > 0 \text{ and } \left. \frac{\partial u_{AS}}{\partial l_y} \right|_{l_y=1} < 0. \text{ These results indicate that}$$

structure  $A$  never occurs in equilibrium due to the "prisoners' dilemma" in a Nash game of



choosing the stealing level. Hence, people in autarky also have the temptation to steal their neighbors' possession even if stealing makes everybody worse off.

If we define a departure of equilibrium from the Pareto optimum as endogenous transaction costs, then our preceding analysis can be summarized in the following proposition.

**Proposition 2.1**

The corner equilibrium in structure  $D^N$  is Pareto superior to the one in structure  $D^S$ , and the corner equilibrium in structure  $A$  is Pareto superior to the one in structure  $A^S$ .  $D^N$  and  $A$  never occur in equilibrium due to prisoners' dilemma, which implies that each individual has temptation to steal despite stealing makes everybody worse off.

Next, we compare per capita real incomes of structure  $A^S$  and  $D^S$  that will occur in equilibrium. It can be shown that  $u_{DS} > u_{AS}$ , if and only if the following inequality holds,

$$(2.6.6) \quad g(\alpha, k, t) = \gamma \cdot t^2 + \theta \cdot t + \rho > 0$$

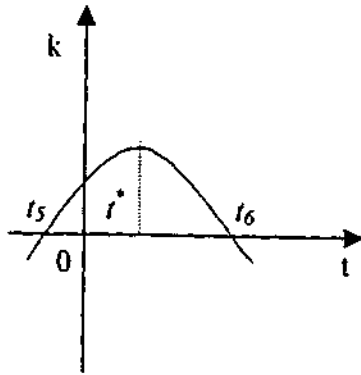
where  $\gamma \equiv 1 - 2k^{1.5} > 0$  iff.  $k < 2^{-2/3}$ ,  $\theta \equiv 2k(2k^{0.5} - 1) > 0$  iff.  $k > 1/4$ , which is always smaller than  $2^{-2/3}$ ,  $\rho \equiv 4k^2[1 - \alpha - (1 - 2\alpha)/k^{0.5}]$ , which is always positive when  $k > k_0$ . The shape of the graph of  $g(\alpha, k, t)$  in the coordinates  $g$ - $t$  is dependent on signs of  $\gamma$  and  $\theta$ . Hence, we consider three cases with various combinations of signs of  $\gamma$  and  $\theta$ .

(a) Assume that  $k > 2^{-2/3} > 1/4$  which implies  $\gamma < 0$  and  $\theta > 0$ . The graph of  $g(\alpha, k, t)$  is a concave inverted U-shape curve with the maximum point on the right side of the vertical axis since  $\partial g(\alpha, k, t)/\partial t < 0$  at  $t=0$  and  $\partial^2 g(\alpha, k, t)/\partial t^2 < 0$  within this region of parameter values. As shown in Figure 2.2a, suppose  $t_5$  and  $t_6$  are cutting points of the curve to the horizontal axis, given by  $g(\alpha, k, t) = 0$  and  $t_5 < t_6$ , then we can rule out the critical values of  $t_5$  and  $t_6$  as

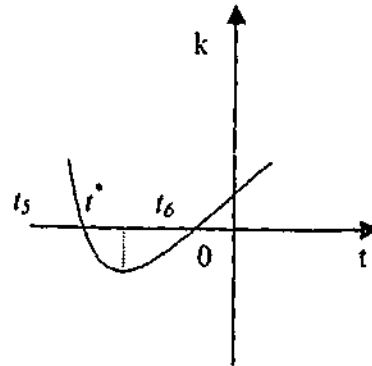
$$(2.6.7) \quad t_5 = \frac{k \cdot \left[ (1 - 2\sqrt{k}) - \sqrt{(1 - 2\sqrt{k})^2 - 4(1 - 2k^{3/2}) \cdot (1 - \alpha - \frac{1 - 2\alpha}{\sqrt{k}})} \right]}{1 - 2k^{3/2}}, \text{ and}$$

$$(2.6.11) \quad u_{DS} > u_{AS} \text{ iff. } t < t_5.$$

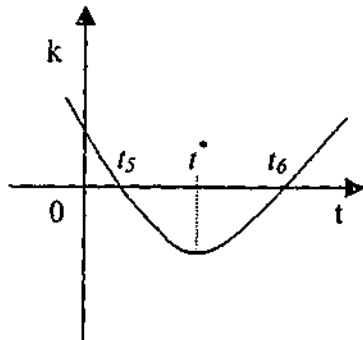
Figure 2.2 The Graph of  $g(\alpha, k, t)$  with Various Combinations of Signs of  $\gamma$  and  $\theta$



Case (a)



Case (b)



Case (c)

To summarize (a) to (c), the general equilibrium is the division of labor if stealing efficiency is sufficiently low and it is autarky if stealing efficiency is sufficiently high. The foregoing results can be summarized in the following proposition.

**Proposition 2.2**

For a given transportation efficiency, if stealing efficiency is high, the general equilibrium is autarky. As stealing efficiency is reduced, the general equilibrium

jumps from autarky to the division of labor. For a given stealing efficiency, as transportation efficiency improves, the general equilibrium discontinuously jumps from autarky to the division of labor.

Due to  $t \leq 2k$  which is required by a positive utility in  $D^S$ , it follows that  $t \cdot (t - 2k) < 0$ . The above inequality (2.6.6) holds only if,

$$(2.6.12) \quad 4k^2 \cdot (1 - \alpha) - 4k^{\frac{3}{2}} \cdot (1 - 2\alpha) + 2tk^{\frac{3}{2}} \cdot (2 - t) > 0 .$$

Here, this inequality holds only if it meets the following condition,

$$(2.6.13) \quad k > k_0 \equiv \left[ \frac{(1 - 2\alpha)}{(1 - \alpha)} - \frac{t(2 - t)}{2(1 - \alpha)} \right]^2 .$$

From this expression, it can be shown that  $\frac{\partial k_0}{\partial t} < 0$  when  $\alpha < \frac{1}{4}$ . Moreover, note that (2.6.12) is necessary but not sufficient for  $u_{DS} > u_{AS}$ . Alternatively, the inequality (2.6.6) yields the critical value of  $k$  for  $u_{DS} > u_{AS}$ . That is,  $u_{DS} > u_{AS}$ , iff.  $k > k_1$  which is given by  $g(\alpha, t, k_1) = 0$ . It can be shown that  $k_1 > k_0$ .

The stealing activity level of an individual in  $D^S$  is  $(\frac{t}{2k})^2$ , and the stealing activity level of an individual in structure  $A^S$  is  $l_x + l_y = \frac{t^2}{2}$ . Comparing the two levels of stealing activity in structure  $A^S$  and  $D^S$ , we can show that

$$(2.6.14) \quad \left(\frac{t}{2k}\right)^2 > \frac{t^2}{2}, \quad \text{iff. } k < \frac{1}{\sqrt{2}} .$$

Here, (2.6.14) implies when  $k \geq \frac{1}{\sqrt{2}}$ , the stealing level in structure  $D^S$  is lower than the one in structure  $A^S$ .

This analysis generates the following Table 2.2. This Table indicates that as trading efficiency is improved, the level of division of labor increases. But the equilibrium level of stealing activity may either increase or decrease. For instance, when  $k_1 > 1/\sqrt{2}$ , as  $k$  increases from a value smaller than  $k_1$  to a value larger than  $k_1$ , the equilibrium jumps from

autarky to the division of labor, while the stealing activity level decreases. When  $k_l < 1/\sqrt{2}$ , as  $k$  increases from a value smaller than  $k_l$  to a value larger than  $k_l$ , the equilibrium jumps from autarky to the division of labor, while the stealing activity level increases.

**Table 2.2 Relationships between the Level of Division of Labor and Stealing Level**

	$k_l > 1/\sqrt{2}$			$k_l < 1/\sqrt{2}$		
Trading efficiency $k$	$k < 1/\sqrt{2}$	$k \in [1/\sqrt{2}, k_l]$	$k > k_l$	$k < k_l$	$k \in [k_l, 1/\sqrt{2}]$	$k > 1/\sqrt{2}$
Equilibrium structure	A <sup>s</sup> with a stealing level lower than in D <sup>s</sup>	A <sup>s</sup> with a stealing level higher than in D <sup>s</sup>	D <sup>s</sup> with a stealing level lower than in A <sup>s</sup>	A <sup>s</sup> with a stealing level lower than in D <sup>s</sup>	D <sup>s</sup> with a stealing level higher than in A <sup>s</sup>	D <sup>s</sup> with a stealing level lower than in A <sup>s</sup>

This result shows that under a certain condition, the Hobbes conjecture (Hobbes, 1651) is correct that if the per capita real income is very low, people have more incentives to engage in stealing activities, while as the income increases, the incentives may reduce. But the conjecture is incorrect under other conditions. This is because not only stealing activity level is dependent on the income level, but also per capita real income is determined by the level of division of labor that is in turn dependent on the stealing activity level. There are infinite feedback loops between the level of division of labor, aggregate productivity, per capita real income, and stealing activity level, though feedback effects attenuate after each round of feedback in a general equilibrium mechanism. Hobbes considered only the first round interaction between income and stealing activity level. Hence, the whole picture of general equilibrium mechanism is much more complicated than he could appreciate. For  $k_l > 1/\sqrt{2}$ , as improvements in trading efficiency enlarges the scope for trading off benefit from specialized production against benefit from stealing, individuals have stronger incentive for production and weaker incentive for stealing, so that aggregate productivity increases while stealing activity level declines. For  $k_l < 1/\sqrt{2}$ , as improvements in trading efficiency enlarge the scope for trading off benefit from specialized production against benefit from stealing, individuals can afford a higher

stealing activity level, so that aggregate productivity and stealing activity level increase side by side.

Next, when comparing per capita real incomes of structure  $A$  and  $D^N$ , we can rule out  $k_2$  as a relevant critical value for  $u_{DN} > u_A$ , that is

$$(2.6.15) \quad u_{DN} > u_A \text{ iff. } k > k_2 \equiv \left(\frac{1-2\alpha}{1-\alpha}\right)^2.$$

Besides,  $g(\alpha, t, k)$  can be shown as the following,

$$(2.6.16) \quad g(\alpha, t, k) = \left[ u_{DN} - \frac{t(2k-t)}{8k^{\frac{3}{2}}} \right] - \left[ u_A - \frac{t(2-t)}{4} \right].$$

Plugging  $k = k_2$  into (2.6.16), which will always be negative if and only if the following inequality holds,

$$(2.6.17) \quad t < \bar{t} \equiv \frac{2k(1-2k^{\frac{1}{2}})}{1-2k^{\frac{3}{2}}} \text{ and } k < \left(\frac{1}{2}\right)^{\frac{2}{3}}.$$

Also, taking into account  $u_{DS} > u_{AS}$ , iff.  $k > k_1$  which is given by  $g(\alpha, t, k_1) = 0$ , it can be shown that  $k_1 > k_2$  under the condition in (2.6.17).

The temptation for stealing in a Nash game of choosing the stealing level, may generate endogenous transaction costs in a Hobbes' Jungle which result in an inefficient level of division of labor when  $k \in [k_2, k_1]$  and  $k < \left(\frac{1}{2}\right)^{\frac{2}{3}}$ . Within this parameter subspace, the equilibrium is autarky with stealing  $A^S$  and the Pareto optimum is associated with structure  $D^N$ . The endogenous transaction costs are mutually beneficial gains from the division of labor that cannot be exploited in a decentralized market.

We can use a table to further explore the welfare implications of equilibrium and endogenous transaction costs of two types.

**Table 2.3 General Equilibrium and Pareto Optimum**

2.3-1:  $k < (\frac{1}{2})^{\frac{2}{3}}$

Stealing efficiency $t$	Trading efficiency $k$	Equilibrium	Pareto Optimum
$t < \bar{t}$	$k < k_0$	Structure $A^S$	Structure $A$
	$k \in [k_0, k_2)$	Structure $A^S$	Structure $A$
	$k \in [k_2, k_1]$	Structure $A^S$	Structure $D^N$
	$k > k_1$	Structure $D^S$	Structure $D^N$
$t > \bar{t}$	$k < k_0$	Structure $A^S$	Structure $A$
	$k \in [k_0, k_1)$	Structure $A^S$	Structure $A$
	$k > k_1$	Structure $D^S$	Structure $D^N$

2.3-2:  $k > (\frac{1}{2})^{\frac{2}{3}}$

Trading efficiency $k$	Equilibrium	Pareto Optimum
$k < k_0$	Structure $A^S$	Structure $A$
$k \in [k_0, k_1)$	Structure $A^S$	Structure $A$
$k > k_1$	Structure $D^S$	Structure $D^N$

Table 2.3 shows that for  $k > k_1$ , stealing generates inefficient resource allocation (too much labor is allocated to theft), despite an efficient level of division of labor.

For  $k \in [k_2, k_1]$  and  $k < (\frac{1}{2})^{\frac{2}{3}}$ , stealing generates allocation inefficiency (too much labor is allocated to theft), as well as organization inefficiency (the equilibrium level of division of labor is inefficient). If  $k$  increases from a level smaller than  $k_0$  to a value greater than  $k_1$ , then equilibrium will jump from Pareto inefficient autarky in structure  $A^S$  to Pareto inefficient division of labor in structure  $D^S$ .

Since parameter  $t$  represents relative efficiency of stealing to self-protection-from-theft, a decrease in  $t$  can be interpreted as an increase in efficiency of self-protection-from-

theft. Proposition 2.1 and 2.2 show that the incentives for stealing create the "prisoners' dilemma" problem. Each individual will allocate resources to stealing activity despite the fact that theft behaviour makes everybody worse off. The distortions of resource allocation may cause an inefficient level of division of labour (endogenous transaction cost associated with organization inefficiency), if transportation efficiency is neither too high nor too low and stealing efficiency is low. Therefore, Hobbes' jungle may generate coordination failure of mutually beneficial division of labour, because of the trade-off between the positive network effects of the division of labour on aggregate productivity and endogenous transaction costs caused by theft and exogenous transaction costs. An increase in transportation efficiency and / or a decrease in stealing efficiency will raise the equilibrium level of division of labour, thereby enlarging the extent of the market, and increasing aggregate productivity and per capita real income.

This model offers an explanation why in developing countries, which have lower transaction efficiency and lower protection level for property rights, the network of division of labour is small and easier to break down. On the other hand, although developed countries have higher transaction efficiency and larger network size of division of labour, stealing activities still exist.

#### **2.4 Concluding Remarks**

In this chapter, we have developed a general equilibrium model with endogenous specialization and endogenous theft behaviour to investigate effects of theft on the equilibrium network size of division of labour, on aggregate productivity, and on per capita real income. If an individual can steal from her neighbours or her trade partners, then an increase in transportation efficiency or a decrease in stealing efficiency will increase the level of division of labour where each individual's resources allocated to theft may be either lower or higher than in autarky. This shows that conventional wisdom that "wealth reduces the desire for stealing and poverty stimulates theft" is not always consistent with a sophisticated general equilibrium analysis of interdependence between per capita real income and equilibrium levels of division of labor and stealing activity. An increase in transportation efficiency and / or a decrease in stealing efficiency will raise the equilibrium

level of division of labour, thereby enlarging the extent of the market, and increase aggregate productivity and per capita real income.



## Appendix 2.1 Mathematical Description of a mixed Nash - Walrasian Equilibrium Model with Endogenous Stealing and Endogenous Specialization

### A2.1.1. Introduction

It is well known that a property right environment may affect the economic decision-making of every individual in a society. In an economy without any effective self-protection and / or third party protection of property rights, illegal behaviour such as stealing or pirating may be encouraged, while incentives of carrying out a technical innovation or introducing new products may be suppressed. The issues of property rights have been addressed by quite a lot of studies. However, as far as we have seen, very few studies have related this problem directly to the general equilibrium analysis.

In Chapter 2, it is assumed that an economy with stealing is a mixed regime of Nash and Walras. As far as quantities traded and produced and prices are concerned, it is a Walrasian regime; but as far as stealing activity levels are concerned, it is a Nash regime. Each player makes her decision according to the price vector announced by the referee and her conjecture on the quantity of her products stolen by others. The total number of goods in the economy, the transaction efficiency for each good, and the stealing efficiency are assumed to be exogenously given and to be common knowledge. The production technology and the preference of each individual, are assumed to be ex ante identical.

Given the level of self-protection and/or the third party's protection of property rights, when any good is produced by any individual, part of it may be stolen by some other individual or individuals. The producer then can determine either to sell part of that left by the thieves in exchange for some other good produced by the other individuals or to consume all of that left for him. For simplicity, we assume that in this chapter the stolen good cannot be used for trading, and an individual can only steal the goods from her trade partners.

In this model, an exogenously given property rights environment will be denoted by a set of parameters. A better environment leads to lower returns to any stealing activity; on the other hand, a worse property rights environment increase the productivity of illegal activities. Thus, like most of the legal systems introduced in human society, in

our model the property rights system, instead of directly giving protection to producers, is mainly aimed at discouraging illegal behaviour. As a result, the equilibrium of the economy, as we will see, depends not only on self-protection and / or the legal system themselves, but also on the transportation efficiency of the economy.

With an exogenously given level of self-protection and / or the third party's protection of the property rights, together with a price vector for trading goods, an individual must first make a conjecture on what percentage of her produce may be stolen before she can make her decision on production or stealing. Note that, all these percentages are actually not determined until every individual *has made* her decision. A price vector together with a conjecture across the population induces a mixed Nash - Walrasian Equilibrium (NWE) if, when every individual maximizes her utility under her conjecture, the trading markets and the theft "markets" are all cleared, i.e., the ex ante conjectures of all the individuals concerning the production-theft environment coincide with ex post one generated by the individuals' utility maximization decisions.

We introduce the mathematical model in Section 2. Then we propose some simple results in Section 3 together with a simple example. Section 4 is devoted to the discussion of the existence of a NWE. In Section 5 we compute a NWE with complete division of labour. From our example one can see the not only the exogenously given property rights environment, but also the transaction efficiency in the trading markets may affect the theft activity.

### A2.1.2. The Mathematical Model

2.1. Consider a large economy with a continuum of individuals:

$$(A2.1.1) \quad E = [I, M, f, g, u].$$

Here  $I = [0, 1]$  is the set of *ex ante* identical individuals. Each individual is endowed with 1 unit of labour.  $M = \{1, \dots, m\}$  is the set of consumption goods.  $f = \{f_1, \dots, f_m\}$  is the set of production functions for each and every individual, i.e. a quantity of  $f_j(L_j)$  of good  $j$  will be produced if any individual allocated  $L_j$  units of labour for the production of good  $j$ .  $g = \{g_1, \dots, g_m\}$  is the set of transaction functions, i.e. if  $y_j$  units of good  $j$  is purchased by an individual from the market, then, because of the presence of

transaction costs, a quantity of  $g_j(y_j)$  is actually received by her for consumption. Finally  $u : R_+^m \rightarrow R$  is the utility function for every individual. We have assumed that the individual can only obtain utility from consumption of goods, i.e. the utility obtained from consumption of the vector of goods  $(z_1, \dots, z_m)$  is given by  $u(z_1, \dots, z_m)$ . In this chapter we assume that, while the knowledge  $I, M$  and  $g$  are common, the production function and the utility function of each individual is her private information.

In addition to the activity of production, we also consider the activity of *stealing* or theft activity. In this model, we consider a society with an exogenously given level of self-protection and / or the third party's protection of the property rights of every good  $j$ . We assume that the effectiveness of this self-protection and / or the third party's protection of the property rights is measured by  $r = (r_1, \dots, r_m)$  with  $r_j \in [0,1]$ . Thus a higher  $r_j$  value corresponds to a better property-rights environment for good  $j$ . On the other hand, we assume that every individual has a set of stealing functions  $\{s_1, \dots, s_m\}$ , i.e. a quantity up to  $(1-r_j) \cdot s_j(L_j^i)$  is stolen by her, where  $L_j^i$  is the amount of labour this individual allocates for stealing of good  $j$ . The function  $s_j$  is assumed to be strictly increasing and continuous. Thus we have assumed that, as far as the aggregate demand of stealing has not yet exceeded the aggregate quantity of good  $j$  being produced, the quantity of a good any individual can steal is computed by the above formula. We will see that, at any equilibrium, this case is always true. On the other hand, in case the total amount of good  $j$  produced,  $Q_j = \int_{s \in [0,1]} f_j(L_j^i)$ , is less than the total amount of good  $j$  people plan to steal,  $S(r, j) = (1-r_j) \int_{s \in [0,1]} s_j(L_j^i)$ , then the actual amount being stolen by  $i$  should be adjusted to  $\frac{Q_j}{S(r, j)} (1-r_j) s_j(L_j^i)$ .

From now on we will denote such an economy by  $E = [I, N, f, g, u; r]$ .  $r$  and the stealing function of every individual will be assumed as common knowledge.

2.2. We will impose some fundamental assumptions on the above mentioned functions.

**Assumptions:**

(i). For every  $j$ ,  $f_j : [0,1] \rightarrow R_+$  is continuous, non-decreasing, and convex or weakly convex; and  $f_j(1) > 0$ .

(ii). For every  $j$ ,  $g_j : R_+ \rightarrow R_+$  is strictly increasing and continuous, satisfying  $g_j(0) = 0$ ,  $g_j(y_j) \leq y_j$ , and  $\lim_{y_j \rightarrow \infty} g_j(y_j) = \infty$ .

(iii).  $u : R_+^m \rightarrow R$  is continuous, non-decreasing in each variable, and for any given  $j$  and  $(z_1, \dots, z_{j-1}, z_{j+1}, \dots, z_m) \gg 0$ , it holds that

$$\lim_{z_j \rightarrow \infty} u(z_1, \dots, z_{j-1}, z_{j+1}, \dots, z_m) = \infty.$$

(iv). For every  $j$ ,  $s_j : [0,1] \rightarrow R_+$  is continuous and strictly increasing.

2.3. With an exogenously given level of self-protection and / or the third party's protection of property rights, described by  $r$ , imagine that a price vector  $p = (p_1, \dots, p_m) \geq 0$  for the  $m$  goods is announced by the referee. When making a decision on allocating her labour, an individual has first to make a conjecture on the theft environment. A conjecture of an individual  $i$  is represented by two  $m$ -vectors:  $e^i = (e_1^i, \dots, e_m^i)$ , where  $e_j^i$  is the fraction of good  $j$  that will be stolen by other individuals from her produce. Of course a conjecture of any individual may depend on  $r$  and  $p$ . On the other hand, for simplicity, we assume that, when making a stealing decision on a good  $j$ , every one expects or believes that, an amount of  $(1 - r_j) \cdot s_j(L_j^i)$  will be stolen by her.

2.4. A decision plan of  $i$  is then

$$(A2.1.2) \quad d^i = (L_1^i, \dots, L_m^i; L_{s1}^i, \dots, L_{sm}^i; x_1^i, \dots, x_m^i; y_1^i, \dots, y_m^i),$$

where  $L_j^i$  is the amount of labour she allocates for good  $j$ 's production,  $L_{sj}^i$  is the amount of labour she allocates for stealing good  $j$ ,  $x_j^i$  is the amount of good  $j$  she sells, and  $y_j^i$  is the amount of good  $j$  she purchases. These quantities are subject to the following constraints:

$$(A2.1.3) \quad L_j^i \geq 0, L_{sj}^i \geq 0, \sum_j L_j^i + \sum_j L_{sj}^i \leq 1;$$

$$(A2.1.4) \quad 0 \leq x_j^i \leq (1 - e_j^i) f_j(L_j^i);$$

$$(A2.1.5) \quad y_j^i \geq 0, \sum_j p_j y_j^i \leq \sum_j p_j x_j^i.$$

The amounts of goods she expects to consume are given by

$$(A2.1.6) \quad z_j^i = (1 - e_j^i) f_j(L_j^i) - x_j^i + g_j(y_j^i) + (1 - r_j) s_j(L_j^i).$$

As a result, her utility is expected to be

$$(A2.1.7) \quad u^i = u(z_1^i, \dots, z_m^i)$$

2.5. Given  $r$  and  $p$ , denote the mapping  $i \mapsto e^i$  by  $e$ , and denote the mapping  $i \mapsto z^i = (z_1^i, \dots, z_m^i)$  by  $z$ , then  $\langle p, e, z \rangle$  is said to be a NEW of the above mentioned economy  $E$  if the following requirements are satisfied:

(i). For every  $j$ , the market for trading good  $j$  is cleared:

$$(A2.1.8) \quad \int_{i \in [0,1]} y_j^i = \int_{i \in [0,1]} x_j^i.$$

(ii). The ex ante system of conjecture  $e$  is consistent with the ex post production-theft environment, i.e. for every  $j$ :

$$(A2.1.9) \quad \int_{i \in [0,1]} (1 - r_j) s_j(L_j^i) = \int_{i \in [0,1]} e_j^i f_j(L_j^i).$$

(iii). The utility of every  $i$  is maximized at  $z^i$  given  $p$  subject to the constraints (A2.1.3), (A2.1.4), and (A2.1.5).

Note that, at any equilibrium, every individual must hold the same conjecture. As a result,  $e^i = e$ ,  $\forall i$ . Note also that, given the properties of the utility functions as described in 2.2. Assumption (iii), at any NWE we must have  $p \gg 0$ . Otherwise the aggregated quantities demanded for the *free goods* will be infinity and the markets can never be cleared.

### A2.1.3. Necessary Condition for Existence of a NWE, Examples

3.1. It is straight-forward to establish

#### Lemma A2.1.1.

Given the assumption that all individuals are ex ante identical, at any NWE, should it exist, every individual must achieve the same utility.

Proof. The argument is simple. Assume that at a NWE  $\langle p, e, z \rangle$  it holds that  $u^i < u^h$ , then by choosing individual  $h$ 's decision plan  $d^h$ , individual  $i$  can achieve a higher utility, which violates the requirement in 2.5 (iii).

3.2. In this subsection let us consider a simple example.

**Example A2.1.1.**

Consider a simple example with  $m = 2$ . Assume that  $r = (r_1, r_2) \gg 0$  is exogenously given. Assume that for every good  $j$ , we have

$$(A2.1.10) \quad u(x_1, x_2) = x_1 x_2 \quad ,$$

$$(A2.1.11) \quad f_j(L_j) = \max\{0, L_j - 0.5\} \quad ,$$

$$(A2.1.12) \quad s_j(L_{sj}) = L_{sj} \quad ,$$

$$(A2.1.13) \quad g_j(y_j) = k y_j, \quad k \in (0, 1] \quad .$$

Consider any price vector  $p$ . Let  $e$  be a system of conjectures with respect to the production-theft environment. Any individual  $i$  has five configurations for her decision options: (i). producing one good but neither trading nor stealing; (ii). producing one good and trading but not stealing; (iii). producing one good and stealing but not trading; (iv). producing one good and trading and stealing; and (v). not producing any good but stealing. Note that while configuration (v) does not generate any *symmetric structure*, it can be sustained at an equilibrium when each of the two goods is produced by a positive-measured subset of other individuals in the population.

**Observation A2.1.1.** With  $r = (1, 1)$ , a NWE of this economy is given by  $p = (1, 1)$ ,  $e^i = (0, 0)$ ,  $d^i = (1, 0; 0, 0; 0.25, 0; 0, 0.25)$ ,  $\forall i \in [0, 0.5)$ ;  $d^i = (0, 1; 0, 0; 0, 0.25; 0.25, 0)$ ,  $\forall i \in [0.5, 1]$ , and  $z^i = (0.25, 0.25k)$ ,  $i \in [0, 0.5]$ ;  $z^i = (0.25k, 0.25)$ ,  $i \in [0.5, 1]$ . It is easy to check that the maximal utility for every individual is  $0.0625 k$ .

Proof. Given  $r = (1, 1)$ , no one wants to waste labour for stealing. Given  $p = (1, 1)$  and the production functions, every individual will allocate labour to produce one good only. The above-mentioned NWE is achieved by complete specialization and trading.

**Observation A2.1.2.** For any value of  $k$ , at any NWE of this example, should it exist, the equilibrium utility for every individual is less than 0.0625.

**Proof.** We first establish the result that the equilibrium utility must be not *greater than* 0.0625 at any NWE. Assume that at a NWE, there are total amount of  $A$  individuals allocate labour for good 1 production, and  $B$  individuals allocate labour for good 2 production. Then the aggregated quantity of good 1 produced is  $Q_A \leq 0.5A$ , and that of good 2 is  $Q_B \leq 0.5B$ . As a result the sum of these two aggregated quantities is  $Q \leq 0.5(A+B) \leq 0.5$ . Assume that at this equilibrium every individual achieves a utility of  $U$ . Assume that individual  $i$  consumes  $z_1^i$  units of good 1. Then she must consume  $\frac{U}{z_1^i}$  units of good 2. We therefore have

$$(A2.1.14) \quad 0.5 \geq \int_{[0,1]} z_1^i + \int_{[0,1]} \frac{U}{z_1^i} = \int_{[0,1]} (z_1^i + \frac{U}{z_1^i}) \geq 2 \int_{[0,1]} U^{0.5} = 2U^{0.5},$$

from which we derive  $U \leq 0.0625$ .

**Observation A2.1.3.** With the  $r_j > 0$  being sufficiently small, for any value of  $k$ , in our example there does not exist any NWE.

**Proof.** On the one hand, from Observation 2, at any NWE, the equilibrium utility is not greater than 0.0625. On the other hand, by allocating all her labour for stealing, 0.5 units of labour for stealing each of the goods, an individual can consume  $0.5(1-r_j)$  units of good  $j$ , achieving a utility of  $0.25(1-r_1)(1-r_2)$ , which could be higher than 0.0625 for a sufficiently small  $r_j$ , provided her stealing plan is realized. Thus no NWE can exist.

**Remark.** Observation 3 reveals a new characteristic of our GE model with illegal behaviour. We see that, not every property rights system supports a NWE. The intuition behind this observation is that, when the property rights system is very poor and the expected returns to the illegal activities are very high, individuals are inclined to seize goods and services through illegal activities, and few people have incentive to engage in any production activity. As a result, the ex ante plans of "stealing" cannot be realized ex post.

An economy without a NWE is non-stable in the sense that individuals' decisions may change from one period to another. Given a very poor property rights system, when resources become extremely scarce, even professional thieves or bandits have to engage in production in order to survive. Once the resources become abundant, some of them may engage in theft activity again because it is more efficient for increasing their wealth.

#### A2.1.4. The Existence of a Nash - Walrasian Equilibrium

4.1. As we have seen in subsection 3.2, sometimes an economy with illegal behaviour may have no NWE. Here, however, we will try to derive some sufficient conditions for its existence.

Divide the population into  $m$  subsets:  $I = I_1 \cup \dots \cup I_m$ , and  $I_j$  consists of the individuals who sell only good  $j$  for  $j = 1, \dots, m$ .

For any given  $e$ , assume that every individual holds the same conjecture  $e^i = e$ . Let  $w_j(e)$  be the measure of the mass of  $I_j$ . We then have

##### Proposition A2.1.1.

For every given  $e$ , there exists a price vector  $p(e)$  and a weight vector  $w(e) = (w_1(e), \dots, w_m(e))'$  such that  $p(e)$  is an MVEPV and that  $w(e)$  clears the trading market.

The proof of Proposition 1 is very much similar to the GE existence proof in Sun-Yang-Yao (1999). Therefore we omit it.

Note that, while  $p(e)$  together with  $w(e)$  clears the trading market, it is not guaranteed that the theft "market" is also cleared. As a result, the *targeted allocation*  $z(e)$  may or may not be realized because the  $(1 - \tau_j) \cdot s_j(L_j)$  must be adjusted at the end.

Now we consider an economy  $E_\beta$  in which everything is the same as in  $E$ , except that the aggregate amount of good  $j$  is  $Q_j + \beta$  instead of  $Q_j$ , meaning, even if no production of good  $j$  is carried out, there is still some amount of good  $j$  there, say, dropping from heaven.



Now assume that all the goods in this economy are gross substitutes. Consider  $E_\beta$  first. For any given  $e$ , the price vector  $p(e)$ , the targeted allocation  $z(e)$  and the weight vector  $w(e)$ , as mentioned in Proposition 1, are uniquely determined by  $e$ . Let  $z'(e)$  be the actual allocation. Let  $e'$  be the vector of real percentages computed according to  $w(e)$  and  $z'(e)$ . We then have a mapping  $e \mapsto e'$ . It is a continuous mapping from a compact convex set into itself. By the Brouwer's Fixed Point Theorem there exists a  $e_\beta$  such that  $e_\beta \mapsto e_\beta$ . Then we have  $p_\beta = p(e_\beta)$  and  $w_\beta = w(e_\beta)$  which clear both the trading markets and the "theft markets".

Let  $e^*$  be a limit point of  $\{e_\beta : \beta > 0\}$  and let  $p^* = p(e^*)$  and  $w^* = w(e^*)$ . We then have

**Proposition A2.1.2.**

A Nash - Walrasian equilibrium exists if  $\{e_\beta : \beta > 0\}$  has a limit point  $e^* \ll (1, \dots, 1)$ .

4.2. With  $E = [I, M, f, g, u; r]$  given, an individual is said to be *professional thief* if she allocates all her labour for stealing. We first assume that her stealing plan could be completely realized, i.e. the amount of good  $j$  she could steal is precisely equal to  $(1-r_j) \cdot s_j(L'_y)$  for all  $j$ . The corresponding maximum utility she could achieve in such a way is denoted  $U^T$ . It is easy to demonstrate:

**Proposition A2.1.3.**

Given  $E = [I, M, f, g, u; r]$ , let  $E' = [I, M, f, g, u; (1, \dots, 1)]$  be the economy similar to  $E$  except that there is a perfect self-protection and / or the third party's protection of property rights, so that any theft activity results in nothing returned. Let  $U'$  be the maximal utility for every individual at a general equilibrium of  $E'$ . Then  $U' < U^T$  implies that there does not exist any NWE of  $E$ .

*Proof:* According to the First Welfare Theorem, if  $U$  is the equilibrium utility in  $E$ , we must have  $U' \leq U$ . But then  $U^T > U'$  gives a contradiction.

#### Proposition A2.1.4.

Following Proposition 3, we assume in addition that the model is completely symmetric, and that for any  $L_1, L_2$  such that  $1 \geq L_1 > L_2 > 0$  it holds that

$$(A2.1.15) \quad k[f_j(L_1) - f_j(L_2)] > (1 - r_j) \cdot s_j(L_1 - L_2).$$

Then  $U' > U^T$  implies the existence of a NWE with no active theft activity.

Proof: Actually the GE of  $E'$  is a NWE of  $E$  with  $e = (0, \dots, 0)$ . Given the equilibrium price  $p = (1, \dots, 1)$ . The inequality in (15) guarantees that trading is more profitable than stealing once you have determined to produce any good. On the other hand,  $U' > U^T$  guarantees that no one has the incentive to specialize as a thief.

#### A2.1.5. Continued Discussion of the Example.

Now we assume that, in the example we discussed in subsection 3.2,  $r_j = r < 1, j = 1, 2$ . We want to compute a NWE. We will compute a general equilibrium with complete specialization.

By symmetry we may assume that the equilibrium price vector is  $p = (1, 1)$ , and  $e = (a, a)$ . Now a good 1 producer will allocate all her labour on good 1 production, producing an amount of 0.5. With  $0.5a$  units of her produce being stolen, she has an amount of  $0.5(1 - a)$  left. For utility maximization she sells  $0.25(1 - a)$  units of the remaining product, and purchases  $0.25(1 - a)$  units of good 2 from good 2 producers. With the transaction efficiency equal to  $k$ , she consumes  $0.25(1 - a)$  units of good 1, and  $0.25k(1 - a)$  units of good 2, achieving a utility of  $U = 0.0625k(1 - a)^2$ . The incentive constraint for her not to allocate part of her labour for stealing<sup>2</sup> is given by  $k(1 - a) > 1 - r$ . We will always assume this constraint holds. Similarly the maximal utility for a good 2 producer is  $U = 0.0625k(1 - a)^2$ .

On the other hand, a professional thief will allocate 0.5 units of labour for stealing each of the two goods. He will consume  $0.5(1 - r)$  units of each good, achieving a utility of  $U^T = 0.25(1 - r)^2$ .

We now consider two cases:

Case 1.  $r < 1 - 0.5 \cdot \sqrt{k}$ .

In this case, we always have  $U < U^T$  even if all the producers are extremely optimistic, believing  $a = 0$ . As a result, there does not exist any NWE.

Case 2.  $r > 1 - 0.5 \cdot \sqrt{k}$ .

In this case we have multiple equilibria. If all producers are optimistic and believing that  $a = 0$ , then we have a NWE with half the population producing good 1 and another half producing good 2.

On the other hand, we can also determine an  $a$ , and  $a > 0$  which supports a NWE with professional thieves and complete specialization. What is required is

$$(A2.1.16) \quad r > 1 - k(1 - a), \quad r = 1 - 0.5(1 - a)\sqrt{k}.$$

From which one solves  $a = 1 - \frac{2 \cdot (1 - r)}{\sqrt{k}}$ . To determine the measure of the mass of each type of professionals, we may assume  $w_1 = w_2 = v$ , and that  $w_T = 1 - 2v$ . It is required that,

$$(A2.1.17) \quad (0.5a)v = 0.5(1 - r)(1 - 2v).$$

From which one solves  $v = \frac{1 - r}{2 \cdot (1 - r) + a}$ . It is easy to see that  $0 < v < 0.5$ .

From case 2, we see that given the same economy with the same property rights environment, the equilibrium of an economy not only depends on the transportation efficiency of the economy, but also depends on the protection of property rights.

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<sup>2</sup> If she had allocated any labor for stealing, she will first steal good 2.

## CHAPTER 3. Division of Labor, Specialization, and the Enforcement of Property Rights: A General Equilibrium Analysis

### 3.1 Introduction

Among others, Barzel (1997) and Buchanan (1975, 1991) develop the economics of state and constitutional economics respectively to explain the emergence and evolution of state and constitutional order. According to them legitimized and monopolized police violence that penalizes theft is essential for the emergence and evolution of private property rights. This chapter extends the model of endogenous theft and endogenous network of division of labor in the preceding chapter to formalize the economics of state and constitutional economics, as well as to investigate how new constitutional rules emerge and evolve.

Early economic analysis of the enforcement of property rights can be traced back to classical writers such as Montesquieu (1748), Beccaria (1767), Bentham (1789), Hobbes (1651) and Rosseau (1762). Hobbes and Rosseau, in particular, examined human society in its primordial "state of nature" where there is no third party protection of property rights. Hobbes argued that in the primordial state of nature, the "law of the jungle" prevailed in which individuals allocate part of their endowments to self-protect their own properties and steal goods from other parties. According to Hobbes, the "laws of nature" dictated the establishment of a property rights system with third party protection, which he called the "Civil Government", or "Commonwealth" (Hobbes, 1973, pp.64-66) which mandated punishment for those who violate the rights of others. Hobbes and Rosseau tried to explain how the Civil Government, or Commonwealth is erected, although they failed to tell us how the Commonwealth superpower endogenously evolves from economic development, or in their words, how the Man or the Assembly of Men emerges from the society.

...Therefore before the names of Just, and Unjust can have place, there must be some coercive Power, to compel men equally to the performance of their Covenants, by the terrour of some punishment, greater than the benefit they expect by the breach of their Covenant; and to make good that Propriety, which by mutual Contract men acquire, in recompence of the universal Right they abandon: and such power there is none before the erection of a Commonwealth... (Leviathan, p.74)

...if their actions be directed accounting to their particular judgments, and particular appetites, they can expect thereby no defense, nor protection, neither against a common enemy, nor against the injuries of one another. ...The only way to erect such a Common Power, ...as if every man should say to every man, *I Authorize and give up my Right of Governing my self, to this Man, or to this Assembly of men, on this condition, that thou give up thy Right to him, and Authorize all his Actions in like manner.* This done, the Multitude so united in one Person, is called a COMMON-WEALTH, ...This is the generation of that great LEVIATHAN... (Leviathan, p.89)

Establishing social order consists of a reduction in the resources devoted to productive activities, that is the costs of enforcement less that of attempting to steal benefit from others and self-protection (Rousseau, 1762). The social contract therefore should have the following components: the rules to establish property rights, a mechanism for enforcing the sanctions for violations of property rights, and the rules which specify the taxation system made by each individual to support this property rights system, like the "criminal justice system" (Skogh and Stuart, 1982). In recent decades, Becker (1968) sets up an economic approach for crime and punishment. Alchian (1983) emphasizes the implementing of property right system need the might from the state or the government. Buchanan (1975) analyzes the bases for a society where the people want to be free but who recognize the inherent limits that social interdependence places on them. Barzel (1997) applies contract theory to analyze property rights and the evolution of the state. Guth and Kliemt (1995) establish and apply an indirect evolutionary approach to show that institutions of enforceable adjudication in themselves may enable higher levels of contract compliance than could be obtained in their absence even though adjudicators are no better behaved than ordinary traders. Although Hobbes presented to us some aspects of the principal nature of the human society, they rarely happened or were only special cases at some periods in the human history (Demsetz, 1964).

During the same period of time, the pioneering work of Coase (1960, 1991), Williamson (1989), North (1987, 1990a) and others, has led to the recognition that various transaction costs are the primary reason why impersonal competitive markets do not function as effectively as might be suggested by the neoclassical benchmark of the Arrow-Debreu theory or the corresponding more general benchmark of the Aumann-Shapley core

in cooperative game theory, which forms the theoretical underpinning of the Coase theorem. That in turn explains the emergence of different mechanisms and institutions as devices that enable the participants to mitigate or to cope with the transaction costs. This mode of analysis has been developed and tested most fully in industrial economics and in economic history (Williamson 1989; North 1990a). North (1990b) studies the political process in the transaction-cost mode, and his main focus is on a particular facet of transaction costs, namely, a failure of "instrumental rationality" for participants in the process. As North states, the informational feedback is inadequate to convey to these participants the correct theory of how their world operates; this affects the individuals' decisions and in turn the outcome of the process and the information it generates. However, there are other aspects of transaction costs that are also prominent in industrial economics. They are to do with game-theoretic issues of asymmetric information and time-consistency of action, and they persist and affect outcomes even if there is full instrumental rationality, that is, even when every participant knows the correct theory of the world and can perfectly calculate his own optimal strategy. The problems arise in the strategic interaction between such individuals and the equilibrium of their game. Moreover, some formal modeling in political economy is close to this tradition of industrial economics in emphasizing transaction costs, but the connection does not appear to have been explicitly recognized or exploited. First, numerous analyses of time-consistency and commitment in fiscal and monetary policy derive from Kydland and Prescott's (1977) work on rules versus discretion. Difficulties with credibility of commitments constitute an important class of transaction costs (Maskin and Xu 1999). Second, the problems of agency in politics and administration have been studied by Banks (1991), Persson and Tabellini (1990), Tirole (1994), and Dixit (1996).

Through the process of institutional evolution, which is itself history-dependent and often quite slow, we should expect to see some systemic attempts to cope with transaction costs and to mitigate their ill-effects. In transaction-cost economics, an external enforcement agency, namely the legal institution governing the contract, is assumed to exist, although its performance is again constrained by the difficulties of verifying whether, or how well or badly, the parties have met the conditions of the contract, and it is recognized that sometimes bilateral private mechanisms of dispute resolution may

outperform external enforcement. Transaction-cost economics assumes that contracts are enforceable within the limits of the existing legal institutions and the available information. In an original position behind a veil of ignorance, everyone may voluntarily agree on a social contract, but the contract must include an explicit or implicit coercive mechanism to ensure continued participation after specific individual positions have been revealed. Olson (1993) puts such mechanisms with coercive capability at the center of his theory of the emergence and performance of governments. Although an intention of the constitution is that such force, or its threat, should serve the general interest, nothing can guarantee that once an agency is given the state's monopoly of force, it will not use this power in pursuit of its own interests. But it may be possible to structure the rulers' incentives in such a way that they will find it in their own interests to remain reasonably benevolent.

The Smithian perspective of economic development focuses on the roles of institutions in driving economic growth. Accordingly, markets are important in attracting private savings for capital accumulation and in promoting an increasing division of labor in markets, leading to the stimulation of growth in the industrial sector and agricultural sector. In a developed society, complex institutional structures are devised to constrain various opportunistic behavior such as cheating, and shirking and to reduce the uncertainty of social interaction. In general, the formation of institutions aims to ensure that transactions are not too costly and thus allow the potential productivity gains of the division of labor and improved technology to be realized. Various transaction costs directly affect the efficiency of resource allocation, which results from market failure or incomplete markets. In order to reduce transaction costs, such as various opportunistic behaviors, elaborate institutional structures must be devised that constrain the participants and so minimize the costly aspects. Formal contracts, legal guarantees, brand names, and other elaborate monitoring systems are developed to protect and enforce property rights. These institutions depend on a complex institutional structure that makes possible the specification and enforcement of property rights, which in turn allow transactions to occur and productivity gains from modern technology to be realized in order to achieve economic growth and development.

Yang and Ng (1993) formalize the Smithian perspective of economic development by taking transaction costs into account. They utilize a consumer-producer approach to

show that an exogenous improvement in transaction conditions leads to enhanced levels of individual specialization, increased degrees of roundaboutness of production, and an increased variety of intermediate goods. However, the above studies do not endogenize stealing and the enforcement of property rights from the third party in their models. Historical observation indicates that the demand for the enforcement of property rights from government, and the demand for transaction protection are significantly higher in developed countries than in developing countries, and also that violation of property rights occurs less in developed countries. However, the foregoing argument still lacks theoretical foundations.

Applying a general equilibrium model with economies of the division of labor through increased returns to specialization, this chapter assumes that in an economy with stealing a mixed regime of Nash and Walras prevails. As far as quantities traded and produced and prices are concerned, it is a Walrasian regime, but as far as stealing activity levels are concerned, it is a Nash regime. Each player makes her decision according to the price vector announced by the referee and her conjecture on the social environment. Meanwhile, this chapter will also examine the economies of the division of labor, effects of stealing on the network size of division of labor, effects of third party protection of property rights on the network size of division of labor.

In the first part of the model, the state can endogenously emerge from taxation that is used to finance the judicial system and enforcement of laws that penalize theft. The tradeoffs between positive network effects of the criminal laws and their enforcement and negative effects of taxation on the network size of division of labor, can be used to partition the parameter space into subspaces, in each of which a corner equilibrium in a structure is the general equilibrium. Particularly within certain parameter subspace, the third party, like the Monarch, emerges from a large network of division of labor. In the second part of the model, a Monarch is specified as the monopolist of legitimate violence, and has a trade-off between his consumption directly from tax revenue and his consumption indirectly from tax revenue via more effective enforcement of criminal laws. His optimum allocation of tax revenue and optimum tax rates efficiently balance the tradeoff. In the third part of the model, competition between two kingdoms, if the



populations are allowed free migration, is specified as a determinant of equilibrium allocation of tax revenues between Monarchs' consumption and law enforcement and equilibrium tax rates. This model may generate many predictions that reject conventional wisdom and support empirical observations. For instance, it may predict that per capita real income, the level of division of labor, productivity, and income share of tax revenue in a geo-political structure where many sovereigns of nearly the same size in the absence of overarching political power (such as in Europe in the 17<sup>th</sup> - 20<sup>th</sup> centuries), are higher than those in a geo-political structure with overarching political power (such as in China before the 19<sup>th</sup> century).

The geo-political structure in Western Europe and the North Atlantic were favorable for the evolution of competitive institutions and division of labor. Baechler (1976, p.79) states: "Fundamental springs of capitalist expansion are, on the one hand, the coexistence of several political units within the same cultural whole and on the other, political pluralism which frees the economy." McNeil (1974) also indicates,

...The political pluralism of early modern Europe was, I think, fundamental and distinctive. When all the rest of the civilized world reacted to the enhanced power cannon gave to a central authority by consolidating vast, imperial states, the effect in western and central Europe was to reinforce dozens of local sovereignties, each consciously competing with its neighbors both in peace and, most especially, in war. Such a political structure acted like a forced draft in a forge, fanning the flames of rival ideologies and nurturing any spark of technical innovation that promised some advantage in the competition among states... (The Shape of European History, p.125)

The driving force of the development of capitalist institutions is the absence of a single overarching political power in Europe and the rivalry between hostile sovereignties, which created the opportunity for social experiments with a great variety of institutions within a relatively short period of time. This rivalry also created great pressure for rulers to creatively mimic those institutions that enhance economic performance, and thereby, their power. These predictions reject conventional wisdom that a lower tax rate is always better than a higher one and confirms the conjecture proposed by many historians that checks and balances on the top level of political arena can increase income share of tax revenue allocated to law enforcement as well as aggregate

productivity, per capita real income and level of division of labor.<sup>1</sup>

The rest of this chapter is arranged as follow: Section 2 develops a general equilibrium model with two final goods, stealing and Monarch's taxation. Section 3 solves this equilibrium as well as the Monarch's optimal personal consumption and tax rate. Section 4 solves a mixed Nash-Walrasian general equilibrium, and explores implication of inframarginal comparative statics of general equilibrium. Then, another kingdom is introduced into the model to investigate effects of rivalry between Monarchs on the equilibrium level of division of labor; Section 5 concludes this chapter.

### 3.2 The Basic Model

Consider the same large economy with  $M$  ex ante identical consumer-producers and two final goods  $x$  and  $y$  where the set of individuals is assumed to be a continuum.

#### Assumption 3.1

**Each individual can only steal goods from parties with whom she trades.**

This assumption implies that in the absence of trade in the economy, there is no stealing occurs. Without this assumption, the calculation becomes intractable.

The specification of an individual's utility function and constraints, as well as the meanings of the terms, are the same as the ones in the preceding chapter except the specification of stealing efficiency coefficient and budget constraint. In this chapter, the stealing efficiency coefficient depends on the third party's enforcement for property rights. We assume that the level of self-protection that other parties have is exogenously given. Here, we use  $T$  to denote the stealing efficiency coefficient and  $t \in (0,1)$  to denote the stealing efficiency coefficient without third party protection. The relationship between  $t$  and  $T$  will be

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<sup>1</sup> The historians include Baechler (1976, p. 79), McNeil (1974, p.125), Hall (1987), Mokyr (1990), Jones (1981, pp. 226-35), Braudel (1984, pp. 128-29), Weber, quoted from MacFarlane (1988, pp. 186-87), and Laslett (1988, p. 235).

In contrast to Europe, the emperor of the continental Chinese empire dominated politics, the economy and religion within the empire. Compared to China, all other Asian countries were relatively small. Thus, the

$$(3.2.1) \quad T = \frac{t}{(1+s)^{\frac{1}{2}} \cdot [1+(1-\beta)]^{\frac{1}{2}}}$$

where the parameter  $s$  is the taxation rate imposed by the Monarch. The parameter  $\beta$  is that fraction of the total taxation revenue personally consumed by the Monarch and  $1-\beta$  is the percentage of the total taxation revenue that the Monarch devotes to the protection of property rights. In addition, we assume  $s \in (0,1)$  and  $\beta \in (0,1)$ , as well as  $\beta = 1$  when  $s \rightarrow 0$ . According to this formula,  $T = t$  if  $s = 0$  and  $\beta = 1$ .  $s = 0$  implies the absence of third party enforcement. With third party protection of property rights by the Monarch,  $T < t$ . The stealing efficiency coefficient in the absence of their party enforcement,  $t$ , relates to the other parties' self-protection which, as noted above, is exogenously given. From the stealing function, it is easy to show that when the taxation rate  $s$  and  $1-\beta$  increases, the stealing efficiency coefficient,  $T$ , decreases.

Strictly speaking, the parameter  $t$  represents the relative efficiency of an individual's stealing activities compared to others self-protection activities. Thus,  $t \cdot x'$  is the consumption of good  $x$  from stealing and  $t \cdot y'$  is the consumption of good  $y$  from stealing. Similarly, the parameter  $T$  represents the relative efficiency of an individual's stealing activities compared to others self-protection activities and the efficiency of third party protection of property rights. Thus, taking third party protection into account,  $T \cdot x'$  is the consumption of good  $x$  from stealing and  $T \cdot y'$  is the consumption of good  $y$  from stealing. In addition, because each person is assumed to be *ex ante* identical and other parties will also attempt to steal the individual's goods, we use  $x^T$  and  $y^T$  to denote quantities of  $x$  and  $y$  which other parties steal from the individual.

The budget constraint for the individual is different from the one in the preceding chapter,

$$(3.2.2) \quad p_x x^s + p_y y^s = (1+s) \cdot p_x x^d + (1+s) \cdot p_y y^d .$$

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competition between institutions within countries and between countries did not have the same opportunity to develop as in Europe.

The parameter  $s$  is the taxation rate imposed by the Monarch. The budget constraint implies that the Monarch can collect the tax revenue only when there are market transactions. Moreover, the model also exhibits that there is an endogenous institutional factor that determines the cost of establishing market transactions. A high value of  $T$  denotes an inefficient institution system, which encourages individuals to steal goods from others. This is because each individual must suffer high costs in order to maintain the transactional system, or to protect her property rights. That is, the development of institutional structures for securing exchanges permits individuals to take actions that involve complex relationships with other individuals. Furthermore, all the variables, parameters and coefficients are non-negative.

### 3.3 Configurations and Structures

Following the same procedure and definitions described in the preceding chapter, this chapter will examine each structure and derive the equilibrium solution with stealing and third party's protection.

Yang and Ng (1993) establish Lemma 1 in a multilateral bargaining game.

#### Lemma 3.1

**According to the Kuhn-Tucker condition, the optimal configuration entails each person selling at most one good and does not involve purchase and self-provision of the same good.**

Lemma 3.1 and Assumption 3.1, above, imply that theft never occurs in autarky and that an individual specializing in producing good  $x$  cannot steal from another individual specializing in producing good  $x$ . This is because individuals specializing in producing good  $x$  will only trade with individuals specializing in producing good  $y$ .

Taking into account Lemma 3.1 and Assumption 3.1, we first consider two alternative configurations; namely, Autarky, which we will call structure A, and the complete division of labor with stealing, which we will call structure  $D^S$ . There is a Monarch who provides third party protection of property rights in the  $D^S$  structure.

### 3.3.1 Autarky without Stealing (Structure A)

The optimal resource allocation in this situation of Autarky is as follows,

$$(3.3.1a) \quad l_x = l_y = \frac{1}{2}, \text{ and}$$

$$(3.3.1b) \quad u_A = \left(\frac{1}{2} - \alpha\right).$$

Here, the individual's maximum utility  $u_A$  equals the maximum per capita output level of the two final goods. Because each individual is only endowed with one unit of labor,  $u_A$  is the per capita real income as well as the maximum average labor productivity of the final goods in this structure.

### 3.3.2 The Complete Division of Labor with Stealing (Structure $D^S$ )

In this structure, all individuals specialize in producing one of the two final goods. Since stealing activity doesn't involve fixed learning costs and transaction costs, people still have an incentive to steal the others' possession as described in the preceding chapter. However, there is a Monarch in this economy who will provide protection for property rights in this case. Because the total taxation revenue is divided between Monarch's consumption and the enforcement of property rights, there is a trade-off between the Monarch's personal consumption and the enforcement of property rights for given tax revenue. The trade-off does not end here. One more trade-off is between positive effects of taxation that contributed to the enforcement of property rights and negative effects of taxation on the level of division of labor that determines aggregate productivity, transaction volume, and per capita real income. Feedback loops based on the trade-offs are more complicated. Tax revenue is dependent on the income level, aggregate productivity, and transaction volume which is dependent on the level of division of labor, which in turn depends on stealing efficiency, while stealing efficiency is determined by tax revenue allocated to the enforcement of property rights. There are two configurations in structure  $D^S$ ,  $(x^T / y^d y')$ , and  $(y^T / x^d x')$ , which characterize the choices of being a specialist in producing good  $x$  and being a specialist in producing good  $y$  respectively.

The good  $x$  specialist's decision problem is,

$$(3.3.2a) \quad \text{Max: } u_{(x^r, l_y, y^r)} = x^{\frac{1}{2}} \cdot (k \cdot y^d + T \cdot y^r)^{\frac{1}{2}}.$$

The constraints facing an individual who specializes in good  $x$  are:

$$(3.3.2b) \quad x + x^s = \max\{l_x - \alpha, 0\} \text{ and } \alpha \in (0, 1),$$

$$y^r = l_y^{\frac{1}{2}},$$

$$x = \max\{l_x - \alpha - x^s - x^r, 0\} \text{ and } \alpha \in (0, 1),$$

$$l_x + l_y = 1,$$

$$T = \frac{t}{(1+s)^{\frac{1}{2}} \cdot [1+(1-\beta)]^{\frac{1}{2}}},$$

$$p_x x^s = (1+s)p_y y^d.$$

The optimal solution for the individual in configuration  $(x^r / y^d y^r)$  is:

$$(3.3.2c) \quad l_y = \frac{t^2 \cdot (1+s)}{4k^2 \cdot p_y^2 \cdot (2-\beta)},$$

$$x^s = \frac{(1-\alpha - x^r)}{2} - \frac{3t^2}{8k^2 \cdot p_y^2 \cdot (2-\beta) \cdot (1+s)}.$$

Taking the symmetry into account, which yields  $x^r = y^r$ , then we get

$$(3.3.2d) \quad x^s = \frac{(1-\alpha)}{2} - \frac{\left[ 2kp_y \cdot t \cdot (1+s)^{\frac{1}{2}} \cdot (2-\beta)^{\frac{1}{2}} + 3t^2 \cdot (1+s) \right]}{8k^2 \cdot p_y^2 \cdot (2-\beta)}, \text{ and}$$

(3.3.2e)

$$u_{(x^r, l_y, y^r)} = \frac{\{4k^2 \cdot p_y^2 \cdot (1-\alpha) \cdot (2-\beta) - t \cdot (1+s)^{\frac{1}{2}} \cdot [2kp_y \cdot (2-\beta)^{\frac{1}{2}} - t(1+s)^{\frac{1}{2}}]\}}{8k^{\frac{3}{2}} \cdot p_y^{\frac{3}{2}} \cdot (1+s)^{\frac{1}{2}} \cdot (2-\beta)}.$$

Since all the individuals in the structure are ex ante identical, and the model is symmetrical, we can work out the optimal solution for configuration  $(y^r / x^d x^r)$  as well.

By the utility equalization condition and market clearing condition, we can obtain the relative prices and the number of individuals of the two different configurations in structure  $D^S$  as,

$$(3.3.2f) \quad \frac{p_x}{p_y} = 1 \quad \text{and}$$

$$M_{(x^r, y^d, y^r)} = M_{(y^r, x^d, x^r)} = \frac{M}{2}.$$

Here  $M_{(x^r, y^d, y^r)}$  and  $M_{(y^r, x^d, x^r)}$  denote the numbers of individuals in the different configurations respectively. The per capita real income in this structure is:

$$(3.3.2g) \quad u_{DS} = \frac{\{4k^2 \cdot (1-\alpha) \cdot (2-\beta) - t \cdot (1+s)^{\frac{1}{2}} \cdot [2k \cdot (2-\beta)^{\frac{1}{2}} - t(1+s)^{\frac{1}{2}}]\}}{8k^{\frac{3}{2}} \cdot (1+s)^{\frac{1}{2}} \cdot (2-\beta)}.$$

Because the endowment constraint of this configuration requires  $0 \leq l_x, l_y \leq 1$ ,

therefore a feasible corner equilibrium in  $D^S$  requires  $t \leq \frac{2k \cdot \sqrt{2-\beta}}{\sqrt{1+s}}$ , otherwise  $l_y \geq 1$ ,

which is infeasible. Moreover, since  $u_{(DS)} > 0$ , the following inequality must hold.

$$(3.3.2h) \quad r(\alpha, k, t) \equiv [k(2-\beta)^{\frac{1}{2}} - t(1+s)^{\frac{1}{2}}]^2 + k^2 \cdot (3-4\alpha) \cdot (2-\beta) > 0,$$

which requires the following conditions to be met,

$$(3.3.2i) \quad \alpha < \frac{3}{4}, \text{ or}$$

$$\alpha > \frac{3}{4} \text{ and } t < t_7, \text{ or}$$

$$\alpha > \frac{3}{4} \text{ and } t > t_8,$$

where  $t_7 = k \cdot (2-\beta)^{\frac{1}{2}} (1 - \sqrt{4\alpha - 3})$  and  $t_8 = k \cdot (2-\beta)^{\frac{1}{2}} (1 + \sqrt{4\alpha - 3})$ . Moreover, it can be

shown that  $x^s > 0$  if and only if

$$g(t, s, \beta) \equiv 3t^2(1+s) + 2kt \cdot (1+s)^{\frac{1}{2}} \cdot (2-\beta)^{\frac{1}{2}} - 4k^2(1-\alpha) \cdot (2-\beta) < 0, \text{ which holds if}$$

$$t \in (0, \frac{k(2-\beta)^{\frac{1}{2}} \cdot [\sqrt{1+12(1-\alpha)} - 1]}{3(1+s)}). \text{ Here, } g(t, s, \beta) \text{ is a convex U-shape curve with the}$$

minimum point on the left side of the vertical axis since  $\partial g(t, s, \beta) / \partial t > 0$  at  $t=0$  and

$\partial^2 g(t, s, \beta) / \partial t^2 > 0$  within this region of parameter values. However,

$$t_8 > \frac{k(2-\beta)^{\frac{1}{2}} \cdot [\sqrt{1+12(1-\alpha)} - 1]}{3(1+s)} \text{ when } \alpha > \frac{3}{4}, \text{ which implies that } t > t_8 \text{ is irrelevant for}$$

$$u_{(DS)} > 0 \text{ when } \alpha > \frac{3}{4}.$$

We now consider the decision problem of the Monarch in structure  $D^S$ , who imposes taxation and offers third party protection of property rights. We assume that when the Monarch imposes taxation, she faces a trade-off in terms of what to do with the revenue. The trade-off is between using the revenue for her own personal consumption and using the revenue to finance protection of property rights, which in turn will increase per capita income and related tax revenue through more effective enforcement of criminal laws. Hence, the decision problem of the Monarch is:

$$(3.3.3a) \quad \text{Max: } u_K = (x_k^d)^{\frac{1}{2}} \cdot (y_k^d)^{\frac{1}{2}},$$

subject to the budget constraint,

$$(3.3.3b) \quad p_x x_k^d + p_y y_k^d = s \cdot \beta \cdot (p_x x^d + p_y y^d) \cdot \frac{M}{2}.$$

Here,  $s \cdot p_x \cdot x^d$  is the tax revenue collected from a good  $y$  specialist and the number of good  $y$  specialists is  $\frac{M}{2}$  in equilibrium. Similarly,  $s \cdot p_y \cdot y^d$  is the tax revenue collected from a good  $x$  specialist and the number of good  $x$  specialists is  $\frac{M}{2}$  as well in equilibrium.

Plugging the budget constraint,  $p_{xy} = 1$  and

$$x^s = y^s = \frac{(1-\alpha)}{2(1+s)} \cdot \frac{\left[ 2kp_{xy} \cdot t \cdot (2-\beta)^{\frac{1}{2}} + 3t^2 \cdot (1+s)^{\frac{1}{2}} \right]}{8k^2 \cdot p_{xy}^2 \cdot (2-\beta) \cdot (1+s)^{\frac{1}{2}}} \text{ into the Monarch's utility}$$

function, the first order conditions of the Monarch's decision problem will be,

$$(3.3.3c) \quad \frac{\partial u_K}{\partial \beta} = 0 \quad \text{and} \quad \frac{\partial u_K}{\partial s} = 0.$$



This yields the following relationship between the taxation rate and the Monarch's personal consumption rate:

$$(3.3.3d) \quad s = \frac{\beta}{2 - \beta}.$$

The optimal value of the variable  $\beta$  is solved from the following equation,

$$(3.3.3e) \quad 4k^2(1 - \alpha)(2 - \beta)^3 - 2^{\frac{1}{2}}kt \cdot (2 - \beta)(4 - \beta) - 12t^2 = 0.$$

Then, the optimal value of Monarch's personal consumption rate is

$$(3.3.3f) \quad \beta^* \approx 2 - \left[ \frac{1.5t^2}{k^2 \cdot (1 - \alpha)} \right]^{\frac{1}{3}}.$$

Plugging  $\beta^*$  into (3.3.3d), we can rule out the optimal value of taxation rate, that is

$$(3.3.3g) \quad s^* = 2 \cdot \left[ \frac{k^2 \cdot (1 - \alpha)}{1.5t^2} \right]^{\frac{1}{3}} - 1.$$

From the above two equations, the optimal utility function of the Monarch and the individual in structure  $D^S$  are,

$$(3.3.3h) \quad u_K = \frac{M \cdot \left\{ 2 - \left[ \frac{1.5t^2}{k^2 \cdot (1 - \alpha)} \right]^{\frac{1}{3}} \right\}^2}{4 \cdot \left[ \frac{1.5t^2}{k^2 \cdot (1 - \alpha)} \right]^{\frac{2}{3}}} \cdot \left\{ (1 - \alpha) - \frac{2kt \cdot \left[ \frac{1.5t^2}{k^2 \cdot (1 - \alpha)} \right]^{\frac{1}{3}} + 3\sqrt{2}t^2}{4\sqrt{2}k^2} \right\}, \text{ and}$$

$$(3.3.3i) \quad u_{DS} = \frac{\sqrt{k} \cdot (1 - \alpha) \cdot (1.5t^2)^{\frac{1}{6}}}{2\sqrt{2} \cdot [k^2 \cdot (1 - \alpha)]^{\frac{1}{6}}} - \frac{t \cdot [k^2 \cdot (1 - \alpha)]^{\frac{1}{6}}}{4\sqrt{k} \cdot (1.5t^2)^{\frac{1}{6}}} + \frac{t^2 \cdot [k^2 \cdot (1 - \alpha)]^{\frac{1}{2}}}{4\sqrt{2}k^2 \cdot (1.5t^2)^{\frac{1}{2}}}.$$

Because it is assumed previously that this analysis is under the condition of a large economy, which means the population size  $M$  is very large, the Monarch's utility will always be higher than his subjects' given the conditions of  $t \leq \frac{2k \cdot \sqrt{2 - \beta}}{\sqrt{1 + s}}$  and (3.3.2h) in this economy.

$$(3.3.3j) \quad u_K \gg u_{DS}.$$

### 3.4 General Equilibrium and Comparative Statics

A comparison between  $u_{DS}$  in (3.3.2g) and  $u_A$  in (3.3.1b), yields  $u_{DS} > u_A$  if  $\alpha \in (0.5, 1)$ , which makes  $u_A < 0$  and  $u_{DS} > 0$ , or alternatively if  $\alpha \in (0, 0.5)$  and the following inequality holds,

(3.4.1)

$$(1.5t)^{\frac{1}{3}} \cdot (1-\alpha)^{\frac{5}{6}} - \frac{\sqrt{3}}{2} \cdot (1-\alpha)^{\frac{1}{6}} \cdot t^{\frac{2}{3}} \cdot k^{\frac{4}{3}} + \frac{k}{2} \cdot (1-\alpha)^{\frac{1}{2}} \cdot t^{\frac{2}{3}} - (1-2\alpha) \cdot k^{\frac{3}{2}} \cdot (1.5)^{\frac{1}{6}} > 0.$$

Here, this inequality holds if and only if it meets the following condition,

(3.4.2)

$$t < t_9 \equiv \left\{ \frac{(1.5)^{\frac{1}{3}}(1-\alpha)^{\frac{5}{6}} - \sqrt{(1.5)^{\frac{2}{3}}(1-\alpha)^{\frac{5}{3}} + [2\sqrt{3} \cdot (1-\alpha)^{\frac{1}{6}} \cdot k^{\frac{4}{3}} - 2k \cdot (1-\alpha)^{\frac{1}{2}}] \cdot (1-2\alpha) \cdot k^{\frac{3}{2}} \cdot (1.5)^{\frac{1}{6}}}}{\sqrt{3} \cdot (1-\alpha)^{\frac{1}{6}} \cdot k^{\frac{4}{3}} - k \cdot (1-\alpha)^{\frac{1}{2}}} \right\}^3,$$

or

$$t > t_{10} \equiv \left\{ \frac{(1.5)^{\frac{1}{3}}(1-\alpha)^{\frac{5}{6}} + \sqrt{(1.5)^{\frac{2}{3}}(1-\alpha)^{\frac{5}{3}} + [2\sqrt{3} \cdot (1-\alpha)^{\frac{1}{6}} \cdot k^{\frac{4}{3}} - 2k \cdot (1-\alpha)^{\frac{1}{2}}] \cdot (1-2\alpha) \cdot k^{\frac{3}{2}} \cdot (1.5)^{\frac{1}{6}}}}{\sqrt{3} \cdot (1-\alpha)^{\frac{1}{6}} \cdot k^{\frac{4}{3}} - k \cdot (1-\alpha)^{\frac{1}{2}}} \right\}^3.$$

If further assuming there are two kingdoms, country A and country B, we can examine two situations in the rest of this section. First, we will consider the case where one kingdom has the Monarch to protect property rights, while the other one is still in the situation of Hobbes' jungle. Second, we will investigate the case where both kingdoms have Monarchs, and they are ex ante identical. The only difference between them is in the arrangement of economic institutions. For simplicity without lose of generality, this chapter assumes the difference lies in the Monarch's personal consumption  $\beta$ , and  $\beta_A < \beta_B$ . In addition, people are free to migrate between these two kingdoms.

In the first case, we assume there is a Monarch in country A to offer the third party's protection of property rights, while country B doesn't have any third party's protection of the property rights. They will enjoy the same per capita real income in the autarky structure. However, there is crucial difference within the structure of complete division of labor with stealing,

$$(3.4.3) \quad u_{DSA} = \frac{\sqrt{k} \cdot (1-\alpha) \cdot (1.5t^2)^{\frac{1}{6}}}{2\sqrt{2} \cdot [k^2 \cdot (1-\alpha)]^{\frac{1}{6}}} - \frac{t \cdot [k^2 \cdot (1-\alpha)]^{\frac{1}{6}}}{4\sqrt{k} \cdot (1.5t^2)^{\frac{1}{6}}} + \frac{t^2 \cdot [k^2 \cdot (1-\alpha)]^{\frac{1}{2}}}{4\sqrt{2}k^{\frac{3}{2}} \cdot (1.5t^2)^{\frac{1}{2}}}, \text{ and}$$

$$u_{DSB} = \frac{[4k^2 \cdot (1-\alpha) - t \cdot (2k - t)]}{8k^{\frac{3}{2}}},$$

where  $u_{DSA}$  and  $u_{DSB}$  are the per capita real incomes in the division of labor with stealing in the two countries, while A is with the third party's protection of property rights and B is without it. Comparing the  $u_{DSA}$  and  $u_{DSB}$  with  $u_A$  respectively, we will solve two threshold values of the stealing efficiency coefficient.  $u_{DSB} > u_A$ , if and only if

$$(3.4.4) \quad t < t_{11} \equiv k - \sqrt{k^2 + 4k^{\frac{3}{2}} \cdot [(1-2\alpha) - k^{\frac{1}{2}} \cdot (1-\alpha)]}, \text{ or}$$

$$t > t_{12} \equiv k + \sqrt{k^2 + 4k^{\frac{3}{2}} \cdot [(1-2\alpha) - k^{\frac{1}{2}} \cdot (1-\alpha)]}.$$

Similarly,  $u_{DSA} > u_A$ , if and only if the inequalities in (3.4.1) hold.

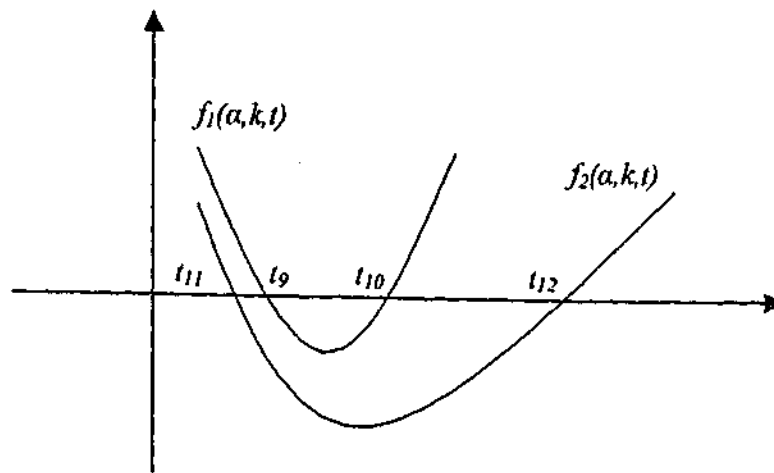
Here, all these  $t_i$  are under the domains for a feasible corner equilibrium in structure  $D^S$ . It is easy to show that if  $k > k_0 \equiv \left(\frac{1-2\alpha}{1-\alpha}\right)^2$ , or the division of labor is better off than autarky in the absence of stealing, the relationships among these  $t_i$  are,

$$(3.4.5) \quad t_9 > t_{11} \text{ and } t_{10} < t_{12}.$$

These two inequalities imply that the curve of  $u_{DSB} - u_A$  is higher than  $u_{DSA} - u_A$ , as shown in the following graph. This implies that  $u_{DSB} > u_{DSA}$ . In other words, if the equilibrium in the country with Monarch is the division of labor, then the equilibrium in the country with Monarch is always the division of labor. But if the equilibrium is the division of labour when in the country with Monarch, the equilibrium in the country without Monarch may not be the division of labour.

Let  $f_1(\alpha, k, t) = u_{DSA} - u_A$  and  $f_2(\alpha, k, t) = u_{DSB} - u_A$ , which are shown at Figure 3.1

Figure 3.1 The Graph of  $f_1(\alpha, k, t)$  and  $f_2(\alpha, k, t)$



These relationships indicate that the emergence of a Monarch, who offers third party protection of property rights, will make the division of labor more likely to occur in equilibrium compared with Hobbes' primordial state of nature. In other words, if  $t \in (t_{11}, t_9)$  or  $t \in (t_{10}, t_{12})$ , the equilibrium in country B is autarky while the equilibrium in country A is the division of labor.

**Proposition 3.1**

**If  $t \in (t_{11}, t_9)$  or  $t \in (t_{10}, t_{12})$ , a Monarch generates a higher level of division of labor, higher aggregate productivity and higher per capita real income compared with Hobbes primordial state of nature through an increase in the institutional efficiency of enforcement of property rights. The equilibrium level of division of labour in the presence of a Monarch is not lower than that in the absence of Monarch.**

Next, we examine the case where both countries have Monarchs, but their personal consumption of the tax rate differs. We assume that  $\beta_A < \beta_B$ . We also assume that there are no border controls preventing population flows between Countries A and B.

Taking equation (3.3.2k) into account, together with the Monarch's budget constraint and the endogenous values of  $x^d$  and  $y^d$ , we can derive the Monarch's and the individual's utility functions as functions of the Monarch's personal consumption rate,  $\beta$ .

$$(3.4.6) \quad u_{DS}(\beta) = \frac{\{4k^2 \cdot (1-\alpha) \cdot (2-\beta)^2 - t \cdot [2\sqrt{2}k(2-\beta) - 2t]\}}{8\sqrt{2} \cdot k^{\frac{3}{2}} \cdot (2-\beta)^{\frac{3}{2}}}, \text{ and}$$

$$u_K(\beta) = \frac{M \cdot \beta^2}{8} \cdot \left[ (1-\alpha) - \frac{2kt(2-\beta) + 3\sqrt{2}t^2}{2\sqrt{2}k^2 \cdot (2-\beta)^2} \right],$$

where  $u_K$  is King's utility and  $u_{DS}$  is a commoner's utility.

From the above two expressions, it is clear that  $\frac{\partial u_K(\beta)}{\partial \beta} > 0$ ,  $\frac{\partial u_K(\beta)}{\partial M} > 0$ , and

$\frac{\partial u_{DS}(\beta)}{\partial \beta} < 0$ . These relationships indicate that with no border controls preventing

population flows between these two kingdoms, all individuals will emigrate to the country with the lower  $\beta$ . This implies an increase in  $M$  in the country with a lower  $\beta$ , which will increase the Monarch's utility in the country with a lower  $\beta$  via a positive effect of increasing inflow population size on his utility, despite direct negative effects of a smaller  $\beta$  on Monarch's utility. Since as subjects run out of the kingdom with a larger  $\beta$ , the Monarch's utility tends to zero as  $M$  tends to 0 even if  $\beta$  is large. Hence, two Sovereigns will reduce the level of  $\beta$  to attract more people to emigrate to their Kingdoms, thereby increasing  $u_K$ . The process of reducing the level of  $\beta$  will continue until  $u_K = u_{DS}$ . We can derive the critical value of  $\beta$ , which is denoted as  $\beta_0$ , from  $u_K = u_{DS}$  which is equivalent to the following equation,

$$(3.4.7) \quad \sqrt{2}k^{\frac{3}{2}}(2-\beta)^{\frac{3}{2}} \cdot M\beta^2 \cdot [(1-\alpha) - \frac{2kt(2-\beta) + 3\sqrt{2}t^2}{2\sqrt{2}k^2 \cdot (2-\beta)^2}] = 4k^2 \cdot (1-\alpha) \cdot (2-\beta)^2 - t \cdot [2\sqrt{2}k(2-\beta) - 2t].$$

The critical value of  $\beta$  is:

$$(3.4.8) \quad \beta_0 \approx \frac{k(1-\alpha) \cdot (8M-1) - 4M \cdot t}{4(1-\alpha) \cdot k \cdot M}.$$

In (3.3.2m),  $\beta'$  is the equilibrium tax revenue share of Monarch's consumption in a single kingdom without competition threat from another kingdom. Since  $\frac{d\beta_0}{dM} > 0$  and

$\lim_{M \rightarrow \infty} \beta_0 = 2 - \frac{t}{k(1-\alpha)}$ , which is smaller than  $\beta^*$ , we have  $\beta_0 < \beta^*$ . Furthermore, comparing the per capita real income of the individuals in an economy with one Monarch, with those in an economy with two Monarchs, yields the following relationship, due to  $\frac{\partial u_{DS}(\beta)}{\partial \beta} < 0$ ,

$$(3.4.9) \quad u_{DS}(\beta_0) > u_{DS}(\beta^*).$$

Here,  $u_{DS}(\beta_0)$  denotes the per capita real income in an economy with two Monarchs in two kingdoms, and  $u_{DS}(\beta^*)$  denotes the per capita real income in an economy with one Monarch.

Because of (3.4.9), an economy with two Monarchs in two kingdoms has a higher utility within the structure  $D^S$  than one with just one. It follows that  $u_{DS} > u_A$  is more likely to hold in the economy with two sovereigns who are in competition for labor flows compared to the situation where there is just one sovereign within the  $D^S$  structure. Therefore, rivalry between the two sovereigns under pressure of free migration between the kingdoms will promote division of labor.

The results are summarized in the following Proposition,

**Proposition 3.2**

1. Monarch's function is to promote division of labor and to raise aggregate productivity and individuals' per capita real income through an increase in institutional efficiency of enforcement of property rights;
2. Competition between Monarchs generated by free migration between the two kingdoms will result in more effective third party protection for property rights. This, in turn, will expand the network size of the division of labor and productivity and reduce the income differential between the Monarch and her subjects compared to the situation where there is just one Monarch.

Basically, Proposition 3.2 is similar to Buchanan's (1975, 1991), Barzel's (1997) and Baechler's (1976) observations on economic history. These economists emphasized that: 1. The emergence and evolution of state and constitutional order are essential for economic development. According to them, legitimate and monopolized police violence that penalizes theft is essential for the emergence and evolution of a private property rights system. But monopolized police violence creates a scope for state opportunism which pursues Monarch's interest at the cost of social welfare despite the fact that monopolized police violence may generate higher per capita income than Hobbs jungle. This dilemma is detrimental to economic development; 2. Rivalry between Monarchs, generated by free migration, will solve this dilemma, promote division of labor, and raise aggregate productivity and per capita real income via improvement of enforcement of property rights. The rivalry will also reduce inequality of income between political elite and commoners, thereby creating momentum for democratic reforms. These results suggest that institutional efficiency in enforcing property rights is one of the most important driving forces behind economic development and structural transformation.

### 3.5 Conclusions

The previous sections have shown how an improvement in institutional efficiency from the third party's protection for property rights can promote the development of division of labor, specialization of workers and the enhancement of aggregate productivity and per capita real income. Contrary to the assumptions underlying ideas of technology and investment fundamentalism, this chapter stresses the importance of institutional factors, especially the enforcement of property rights that captures the relationship between structural transformation that occurs in an economy and the level of division of labor. In this model, parameter  $t$  represents technical efficiency of stealing in the absence of institutional protection of property rights, and variable  $T$  represents stealing efficiency when institutional protection of property rights is allowed. In this chapter, a Monarch's decision to collect tax and to allocate tax revenue between his consumption and enforcement of property rights is introduced to the model with endogenous stealing and endogenous specialization of Chapter 2. Then, this model is compared with the model of Hobbes' jungle in Chapter 2 to investigate the effects of the Monarch's power in enforcing

property rights on network size of division of labor and productivity. By comparing self-protection and the third party's protection, this chapter shows that the government can endogenously emerge from taxation that is used to finance the judicial system and enforcement of laws that penalize theft. This chapter has examined the trade off between positive network effects of the laws and their enforcement, and negative effects of taxation on the network size of division of labor. It is shown that the improvement in institutional efficiency expands the demand for transactions, which requires third party's protection. The market mechanism for goods as well as for the third party's protection, determines the selection of the protection system of property rights. In this model, aggregate productivity is determined by the network size of division of labor (extent of the market). The network size of division of labor is determined by the enforcement of property rights, which is dependent on tax revenue and in turn dependent on per capita real income, aggregate productivity, and network size of division of labor. Hence, the notion of general equilibrium is a powerful vehicle to investigate circular causation and related economics of state.



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## **CHAPTER 4. A General Equilibrium Model with Impersonal Networking Decisions and Bundling Sale**

### **4.1 Introduction**

The purpose of this chapter is to investigate the function of a particular type of bundling sale in exploiting network effects of the division of labor and in promoting productivity progress. We motivate this task from the following perspectives. First we compare it with the existing literature of bundling and tying sale. Then, we consider some common internet phenomena which cannot be predicted by the existing literature. Finally, we motivate the research of effects of bundling sale on the network size of division of labor by comparing our task with the literature of endogenous specialization and network effects of division of labor.

An extensive literature has been developed to investigate the role of bundling and tying sale (Bursten, 1960, Stigler, 1963, Adams and Yellen, 1976, Schmalensee, 1984, McAfee, McMillan, and Whinston, 1989, Whinston, 1990, Hanson and Martin, 1990, Eppen, Hanson, and Martin, 1991, Salinger, 1995, Varian, 1995, 1997, and Bakos and Brynjolfsson, 1999a, b). This literature focuses on bundling and tying that is associated with monopoly power. The following assumptions are made in this literature. Each consumer consumes at most one unit of a good and has constant valuation of the one unit of good. Resale of a good is not allowed. In addition, differentiated prices cannot be directly charged for individuals with differentiated valuations of goods because such valuations are not observable. The assumptions imply that utility is not specified as a function of amounts of all consumption goods and that no substitution between goods is allowed (so-called independent valuations). Hence, interesting interactions and feedback loops between consumption quantities, prices, income, production decisions, and substitution between goods, which are the focus of a standard general equilibrium analysis, are not investigated in this literature. With these quite special assumptions, it is easy to see that bundling can impose indirect price discrimination under a uniform price of a bundle of goods even if no monopoly power exists. Bakos and Brynjolfsson (1999a, b) have nicely presented the intuition about this function of bundling.

In this literature, research results on welfare effects of bundling are inconclusive. Adams and Yellen (1976) emphasize that adverse effects of bundling on welfare come from monopoly power rather than bundling itself. Bowman (1957), Blair and Kaserman (1978), Grimes (1994), DeLong (1998), Chae (1992), Fishburn, Odlyzko and Siders (1997), Varian (1995), Chuang (1999), and Bakos and Brynjolfsson (1997, 1999) pay more attention to positive welfare effects of bundling. Matutes and Regibeau (1992), Tirole (1989, pp. 146-48), and Martin (1999) pay more attention to adverse welfare effects of tie-in sale. Whinston (1990) shows that welfare effects of tying in an oligopoly regime are ambiguous.

As reviewers of some papers in the literature point out, many internet and e-business phenomena are inconsistent with the particular assumptions made in this literature. For instance, there are more than a thousand email or search engine providers and each of them bundles their services. Some of the services are charged positive prices (very likely lower than marginal cost) and others are provided free of charge. Also, resale of such services is possible, quantities of such services can be any integer numbers (for instance each person may get several email accounts from each of several providers), and substitution between services are not trivial (that is, a consumer's valuation of a service is not a constant, or a consumer's utility is a function of quantities of such services and other goods).

Bakos and Brynjolfsson, (1999b, p. 3) defend their position by arguing that bundling sale with zero prices of some services is a phenomenon of disequilibrium. We disagree. A good of zero price implicitly bundled with goods of positive prices can be a general equilibrium phenomenon. A conventional market for petrol and air pump services may illustrate our point. There are many petrol stations which sell petrol at a competitive price and provide air pump services free of charge. This market structure has been in place for long time. The bundling of petrol and air pump services must be a general equilibrium phenomenon. In this market, all consumers' preferences for petrol and air pump services might be very similar, so that the rationale for the type of bundling in the existing literature is irrelevant.<sup>1</sup> The intuition for this phenomenon is quite straightforward. Pricing of air pump services and collection of related payment involves transaction costs to consumers as well as to petrol stations (waiting time, inconvenience, and tangible resource cost for pricing

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<sup>1</sup> As shown by Bakos and Brynjolfsson (1999a), benefit of bundling disappears as consumers' evaluations converge to the same value.

and payment collection). If the production cost of such services can be added to the price of petrol which is complementary to air pump services, then such transaction cost can be avoided. Bundling sale may incur endogenous transaction costs that are the distortions caused by individuals who use air pump services but do not buy petrol from the same petrol station. But as long as reduction of exogenous transaction costs of pricing air pump services outweighs the increase in endogenous transaction costs, a competitive market will generate pressure to compel all petrol stations to implement such a bundling price structure. We call this phenomenon implicit bundling which charges a positive price for a good and zero price for another good without an explicit bundle. Implicit bundling is closer to mixed bundling than pure bundling investigated in the existing literature. Other implicit bundling cases include TV programs (TV shows are free of charge and associated advertisements are paid at positive prices by companies selling goods to viewers of TV programs) and an automobile company's marketing operation with positive prices of cars and free internet purchase services. Here, the key point is that competition pressure and prohibitively high pricing cost of some goods are essential for zero prices of goods bundled with goods of positive prices. Therefore, we need a model without monopoly power and with transaction costs and competitive (implicit) bundling. This paper will formalize this story using a general equilibrium model with well specified *ex ante* identical utility and production functions for all individuals.

We shall tell the story by formulating a trade-off between positive network effects of division of labor on aggregate productivity and transaction costs. As suggested by Allyn Young (1928), network effect is a notion of general equilibrium. Not only does the network size of division of labor depend on the extent of the market (the number of participants in the network of division of labor), but also the number of participants is determined by all individuals' participation decisions in the network of division of labor, which relates to their decisions of their levels of specialization. This circular causation, noted by Young, is of course an essential feature of general equilibrium, analogous to the circular causation between quantities and prices in the fixed point theorem (each individual's quantities demanded and supplied depend on prices, while the equilibrium prices are determined by all individuals' decisions as to quantities). Hence, a partial equilibrium model, such as those in the existing literature of bundling, does not work for our task.

Moreover, since we need an assumption of competitive market for investigating network effects of division of labor, we are not confined to the strategic networking decision that is associated with monopoly power. We need a general equilibrium model of impersonal networking decisions to investigate infinite feedback loops between network size of division of labor, each person's participation decision, prices, quantities, and different markets. Yang (2001) and Sun, Yang, and Yao (1999) have drawn the distinction between the strategic networking decision and the impersonal networking decision. For the latter, each decision maker is not concerned with whom she has a trade connection. She is concerned with how many goods she will trade and how many she will self-provide. Such concern is associated with the number of types of trading partners, which determines her trade network size and pattern. Impersonal networking decisions take place in a market where no body can manipulate prices, so that implicit bundling with zero prices of some goods may emerge from competitive pressure and free entry. Such impersonal networking decisions generate network effects of division of labor that are not network externalities since we assume that each individual is capable of conducting inframarginal analysis (total cost-benefit analysis across corner solutions in addition to marginal analysis of each corner solution). Inframarginal analysis means that each individual is capable of not only choosing locally optimum resource allocation for a given trade network pattern using standard marginal analysis, but also choosing a globally optimal trade network pattern by comparing several locally optimum values of objective functions. Formally, inframarginal analysis is non-linear programming. Coase (1946, 1960), Buchanan and Stubblebine (1962), and Yang (2001) have shown that a lot of so-called network externalities can be internalized by individuals' inframarginal decisions. They are considered externalities by many economists since these economists assume, naively, that individuals are incapable of doing inframarginal analysis. Many contributors to the literature of inframarginal analysis of network effects of division of labor and impersonal networking decision (see surveys of this literature by Yang and Ng, 1998 and by Yang, 2001, and references there in) have shown that marginal cost pricing does not work when individuals conduct inframarginal analysis. Hence, non-marginal cost pricing is compatible with a competitive market with increasing returns and impersonal networking decisions.

In this chapter, we will specify a general equilibrium model with a continuum of ex ante identical consumer-producers who prefer diverse consumption and

specialized production due to economies of specialization in production of three goods. There is a trade-off between transaction costs and positive network effects of division of labor on aggregate productivity. Hence, if the transaction cost coefficient for a unit of goods traded is very large, the positive network effect is outweighed by transaction costs. Therefore, individuals choose autarky where market transactions, the institution of the firm, and bundling sale do not occur. As the transaction cost coefficient decreases, the general equilibrium discontinuously jumps to a higher level of division of labor. Markets emerge from the division of labor. However, if the transaction cost coefficient for labor is smaller than that for goods, the institution of the firm and related labor market emerge from the division of labor. Otherwise, the markets for various goods will be used to organize the division of labor in the absence of the institution of the firm and related labor market. If the transaction cost coefficient for a good is extremely large and the equilibrium level of division of labor is sufficiently high, then this good will be implicitly bundled with other goods to avoid prohibitively high pricing cost, meanwhile getting this good involved in the large network size of division of labor and commercialized production.

Intuitively, this story can be told as follows. Suppose that an automobile manufacturer, such as General Motors, sells automobiles and internet services for purchasing cars online. Automobiles are tangible goods which are easy to price, but internet services are intangible and very difficult to price. General Motors can bundle two goods together by providing free internet services and by adding the operation cost of internet services to the price of automobiles. If such bundling can save consumers' transaction costs incurred in a purchase deal in excess the added cost to the price of automobiles, General Motors will have a competitive edge compared to other automobile manufacturers who do not provide such bundled deal. Then a competitive pressure in the market will force all manufacturers to provide such bundled deals. Here, monopoly power, constant and independent valuations of one unit of good, non-resale, and other peculiar assumptions are not needed. In addition, even if all individuals have ex ante identical utility function that allows substitution between goods, productivity gains from bundling may be generated by network effects of division of labor. Without bundling, involvement of the good with prohibitively high transaction cost coefficient in a high level of division of labor and avoidance of direct pricing cost of such a good cannot coexist. Hence, positive network effects of division of labor on aggregate productivity cannot be fully

exploited. With bundling, both of the tasks can be achieved at the same time. Therefore, the network effects can be fully exploited and aggregate productivity can be promoted by the bundling. It is interesting to see that bundling in a competitive market has very important productivity implications even if all individuals have ex ante identical utility function and substitution between different goods are non-trivial.

This chapter proceeds as follows. Section 2 is devoted to describing the model. Section 3 solves the equilibrium and its comparative statics and reports the main findings. The final section concludes the paper.

#### 4.2 A Model with Impersonal Networking Decisions and Bundling Sale

Consider an economy with a continuum of consumer-producers of mass  $M$ . This assumption implies that population size is very large. It avoids an integer problem associated with having numbers of different specialists, which may lead to non-existence of equilibrium with the division of labor (see Sun, Yang, and Zhou, 1998). Each consumer-producer has identical, non-satiated, continuous, and rational preference represented by the following utility function:

$$(4.2.1) \quad u = f(x^c, y^c),$$

where  $x^c \equiv (x + x^d)$  and  $y^c \equiv (y + y^d)$  are the amounts of the two final goods that are consumed,  $x$  and  $y$  are the amounts of the two goods that are self-provided,  $x^d$  and  $y^d$  are the amounts of the two goods that are purchased from the market, and  $f(\cdot)$  is continuously increasing and quasi-concave. For simplicity, it is assumed that  $f(\cdot) = (x^c)^\alpha \cdot (y^c)^{1-\alpha}$ .

Each consumer-producer's production functions are:

$$(4.2.2) \quad x^p = x + x^s = (z + z^d)^\beta \cdot l_x \quad \text{and} \quad \beta \in (0,1),$$

$$y^p = y + y^s = \text{Max}\{0, l_y - b\},$$

$$z^p = z + z^s = \text{Max}\{0, l_z - b\}, \quad \text{and} \quad b \in (0, 1).$$

where  $x^p$  and  $y^p$  are the amounts of the two final goods produced,  $z^p$  is the amount of the intermediate good produced,  $z^d$  is the amount of intermediate good purchased from the market,  $x^s$ ,  $y^s$  and  $z^s$  are the amounts of the three goods sold,  $b$  is a fixed learning and training cost in producing goods  $y$  and  $z$  and the parameter  $\beta$  represents the elasticity of output of good  $x$  with respect to the input level of intermediate good  $z$ .  $\beta + 1 > 1$  implies that there are increasing returns in producing the final good  $x$ . The endowment

constraint for each individual endowed with one unit of working time is given as follows:

$$(4.2.3) \quad l_x + l_y + l_z = 1,$$

where  $l_i$  is the amount of labor allocated to the production of good  $i$ . This system of production implies that each individual's labor productivity increases as she narrows down her range of production activities. As shown by Yang (2001, chapter 2), the aggregate production schedule for three individuals discontinuously jumps from a low profile to a high profile as each person jumps from producing three goods to a production pattern in which at least one person produces only one good (specialization). The difference between the two aggregate production profiles is considered as positive network effects of division of labor on aggregate productivity. This network effect implies that each person's decision as to her level of specialization, or gains from specialization, depends on the number of participants in a large network of division of labor, while this number is determined by all individuals' decisions in choosing their levels of specialization (the so-called Young theorem, see Young, 1928). Since economies of specialization is individual specific (learning by doing must be achieved through individual specific practice and cannot be transferred between individuals), a labor endowment constraint is specified for each individual, so that increasing returns are localized.

The budget constraint for an individual is,

$$(4.2.4) \quad k_x p_x x^s + k_y p_y y^s + k_z p_z z^s = p_x x^d + p_y y^d + p_z z^d, \text{ and } k_i \in (0,1),$$

where  $p_i$  is the price of good  $i$ . A fraction  $1-k_i$  of a good sold disappears in transit due to an iceberg transaction cost, or  $k_i$  is a trading efficiency coefficient, which represents the conditions governing transactions.<sup>2</sup>  $k_i$  relates to transportation conditions and the general institutional environment that affects trading efficiency. We assume that if labor trade occurs, a fraction  $1-g_i$  of the amount of labor employed to produce good  $i$  disappears in transit from the employee to the employer due to all kinds of transaction costs in labor trade (shirking, measurement cost of quantity and quality of labor, and anticipated moral hazard). Hence,  $g_i$  is the trading efficiency coefficient of labor employed to produce good  $i$ .

<sup>2</sup> The specification of such iceberg transaction cost is a common practice in the equilibrium models with the trade-off between increasing returns and transaction costs (see Krugman 1995). This specification avoids notoriously formidable index sets of destinations and origins of trade flows.



Due to the continuum number of individuals and the assumption of localized increasing returns in this large economy, a Walrasian regime prevails in this model. The specification of the model generates a trade-off between economies of division of labor and transaction costs. The decision problem for an individual involves deciding on what and how much to produce for self-consumption as well as to sell and to buy from the market. In other words, the individual chooses nine variables  $x_i, x_i^s, x_i^d, y_i, y_i^s, y_i^d, z_i, z_i^s, z_i^d \geq 0$ . Hence, there are  $2^9 = 512$  possible corner and interior solutions.

#### 4.3 Corner Solution in a Configuration and Corner Equilibrium in a Structure

Since corner solutions are allowed in our model, standard marginal analysis of interior solution does not work. We need a three-step inframarginal analysis. In the first step a set of candidates for an individual's optimum decision is identified by ruling out all inefficient interior and corner solutions. Possible network structures of division of labor and related transactions can then be identified as combinations of corner solutions. This first step of inframarginal analysis will be done in subsection 3.1. We then solve for all possible corner solutions and the local equilibrium in each market structure that is a combination of compatible corner solutions, using marginal analysis. The second step will be taken in subsection 3.2. Finally, we will use total cost-benefit analysis to figure out under what condition, which local equilibrium is a general equilibrium. This will be done in subsection 3.3.

##### 4.3.1 Configurations and Structures

The set of candidates for each individual's optimum decision includes many corner and interior solutions. In order to narrow down the list of the candidates, Yang and Ng (1993), and Wen (1998) used the Kuhn-Tucker conditions to establish the following lemma:

##### Lemma 1

**Each individual sells at most one good, but does not buy and sell the same good, nor buys and self-provides the same good at the same time.**

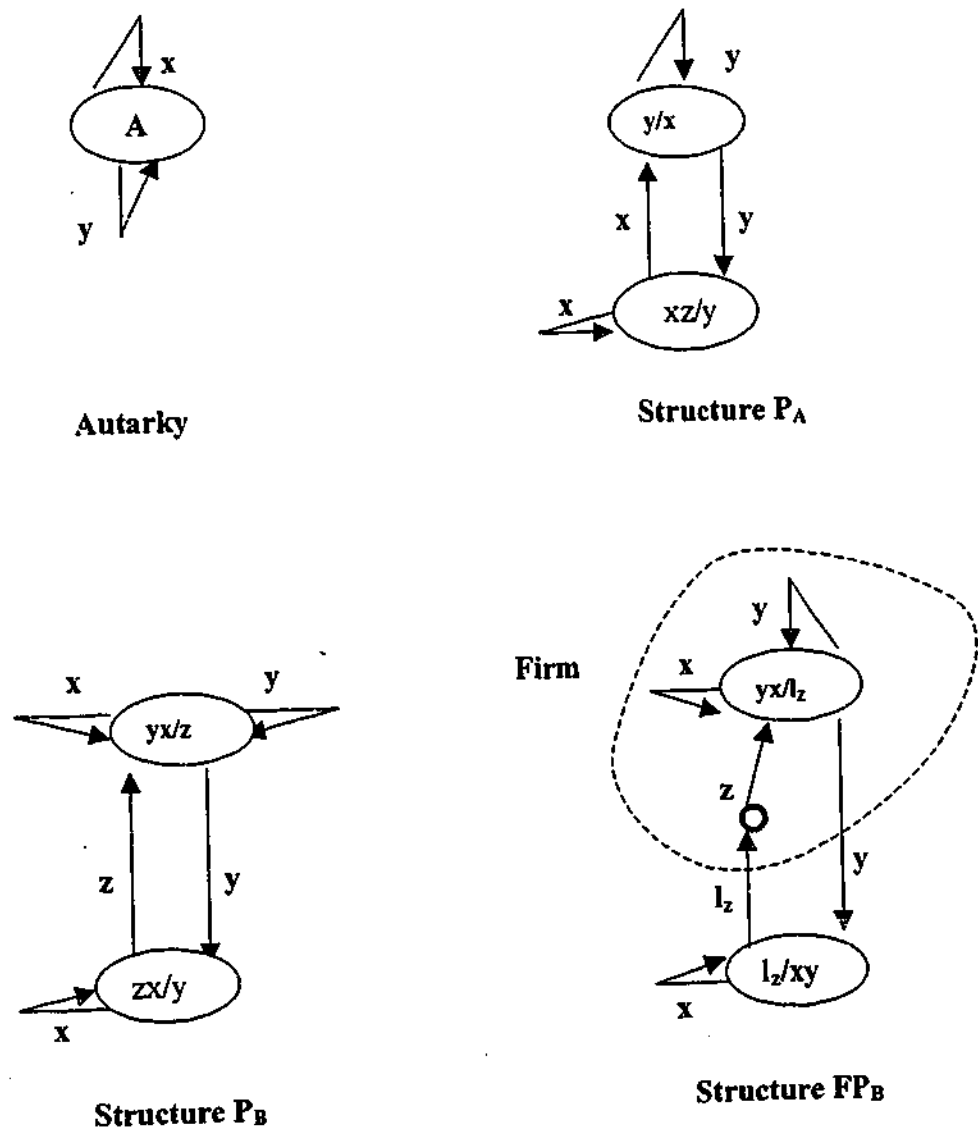
We define a *configuration* as a combination of zero and positive variables which are compatible with Lemma 1. When labor trade and bundling are allowed, there are 19 configurations from which the individuals can choose. A combination of all

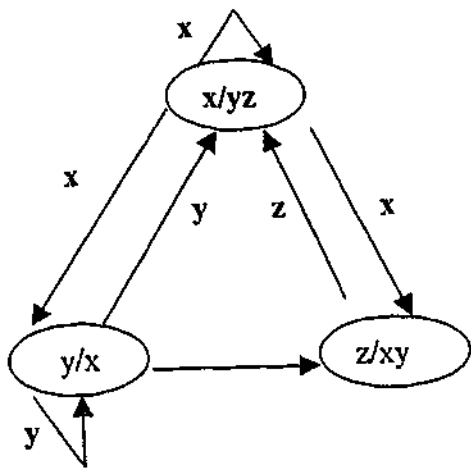
individual's configurations constitutes a *market structure*, or *structure* for short. Let us examine all structures that might occur in equilibrium.

**Structure A: Autarky**

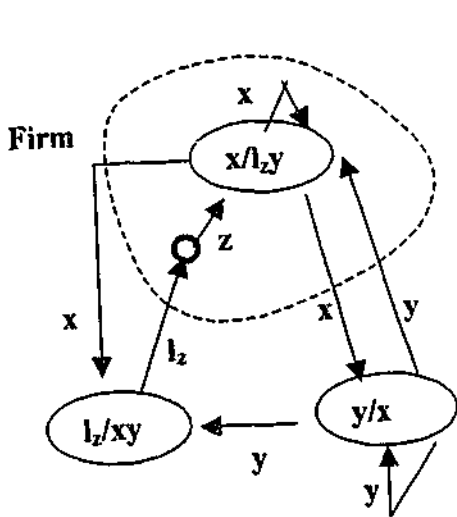
Structure A consists of all individuals choosing configuration A (self-sufficiency, or autarky), where an individual produces all the three goods for self-consumption. Configuration A is defined by  $x, y, z > 0$  and  $x^s = x^d = y^s = y^d = z^s = z^d = 0$ .

**Figure 4.1 Configurations and Structures**

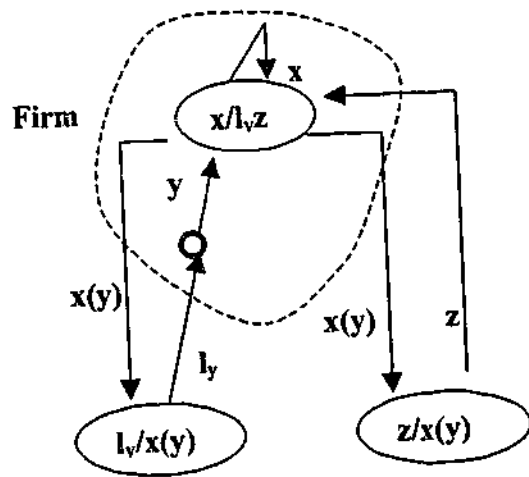




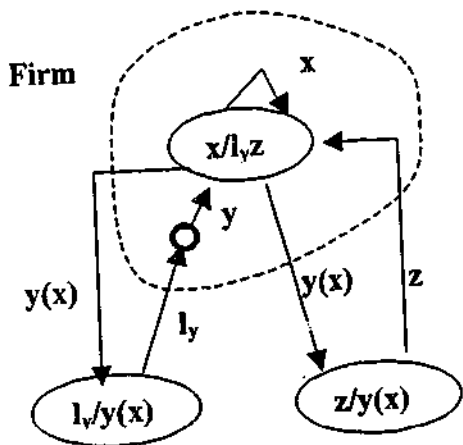
Structure CD



Structure FDA



Structure FT<sub>A</sub>



Structure FT<sub>B</sub>

**Structures with Partial Division of Labor:  $P_A$ ,  $P_B$  and  $FP_B$**

(1) Structure  $P_A$  is a division of the population between configurations  $(xz/y)$  and  $(y/x)$ .

A person choosing configuration  $(xz/y)$  produces goods  $x$  and  $z$ , buys good  $y$ , and sells good  $x$ . It is defined by  $x, x^s, z, y^d > 0, z^s = z^d = y = y^s = x^d = 0$ .

A person choosing configuration  $(y/z)$  produces good  $y$ , buys good  $x$ , and sells good  $y$ . It is defined by  $y, y^s, x^d > 0, x = x^s = z = z^s = z^d = y^d = 0$ .

Note that structure  $P_A$  involves trade of goods  $x$  and  $y$ , so that the trading efficiency coefficients  $k_x$  and  $k_y$  appear in this structure.

(2) Structure  $P_B$  is a division of the population between configuration  $(zx/y)$  and  $(yx/z)$ .

A person choosing configuration  $(zx/y)$  produces goods  $x$  and  $z$ , buys good  $y$ , and sells good  $z$ . It is defined by  $x, z, z^s, y^d > 0, x^s = x^d = y = y^s = z^d = 0$ .

A person choosing configuration  $(yx/z)$  produces goods  $x$  and  $y$ , buys good  $z$ , and sells good  $y$ . It is defined by  $x, y, y^s, z^d > 0, x^s = x^d = y^d = z = z^s = 0$ .

Note that structure  $P_B$  involves trade of goods  $z$  and  $y$ , so that trading efficiency coefficients  $k_z$  and  $k_y$  appear in this structure.

(3) Structure  $FP_B$  is a division of the population between configuration  $(l_zx/y)$  and  $(yx/l_z)$ .

An individual choosing configuration  $(l_zx/y)$  produces goods  $x$  and  $z$ , buys good  $y$ , and sells labor for producing an intermediate good  $z$ . It is defined by  $x, z, l_z, y^d > 0, x^s = x^d = y = y^s = z^s = z^d = 0$ .

A person choosing configuration  $(yx/l_z)$  produces goods  $x$  and  $y$ , sells good  $y$ , and employs labor to produce good  $z$ . It is defined by  $x, y, y^s, l_z > 0, x^s = x^d = y^d = z^s = z^d = 0$ .

Note that structure  $FP_B$  involves trade of good  $y$  and labor  $l_z$ , so that the trading efficiency coefficients  $k_y$  and  $g_z$  appear in this structure.

**Complete Division of Labor:**

**Structure CD with Complete Division of Labor and without the Institution of the Firm** is a division of the population among configurations  $(x/yz)$ ,  $(z/xy)$  and  $(y/x)$ .

An individual choosing configuration  $(x/yz)$  in structure CD produces and sells good  $x$  and buys goods  $y$  and  $z$ . It is defined by  $x, x^s, y^d, z^d > 0, x^d = y = y^s = z = z^s = 0$ .

An individual choosing configuration  $(y/x)$  in structure CD produces and sells good  $y$  and buys good  $x$ . It is defined by  $y, y^s, x^d > 0, x = x^s = y^d = z = z^s = z^d = 0$ .

An individual choosing configuration  $(z/xy)$  in structure CD produces and sells good  $z$  and buys goods  $x$  and  $y$ . It is defined by  $z, z^s, x^d, y^d > 0, z^d = x = x^s = y = y^s = 0$ .

Note that structure CD involves trade of goods  $x$ ,  $y$ , and  $z$ , so that trading efficiency coefficients  $k_x$ ,  $k_y$ , and  $k_z$  appear in this structure.

#### Complete Division of Labor with the Institution of the Firm: Structure $FD_A$

Structure  $FD_B$  is a division of the population among configurations  $(z/l_x y)$ ,  $(l_x/xy)$  and  $(y/x)$ .

An individual choosing  $(z/l_x y)$  produces and sells good  $z$ , hires labor to produce  $x$ , and buys good  $y$ . It is defined by  $z, y^d, l_x, x^s > 0, x^d = y = y^s = z^s = z^d = 0$ .

An individual choosing  $(l_x/xy)$  sells labor for producing  $x$  and buys goods  $x$  and  $y$ . It is defined by  $x^d, y^d, l_x > 0, x^s = y = y^s = z = z^s = z^d = 0$ .

Configuration  $(y/x)$  is the same as in structure CD.

Note that structure  $FD_B$  involves trade of goods  $x$ ,  $y$ , and labor  $l_x$ , so that the trading efficiency coefficients  $k_x$ ,  $k_y$ , and  $g_x$  appear in this structure.

#### Complete Division of Labor with Bundling Sale and the Institution of the Firm: Structures $FT_A$ , and $FT_B$

(1) Structure  $FT_A$  is a division of the population among configurations  $(x/l_y z)$ ,  $(l_y/x(y))$  and  $(z/x(y))$ .

An individual choosing  $(x/l_y z)$  produces good  $x$ , employs labor to produce  $y$ , and sells  $x$  that is bundled with  $y$ . It is defined by  $x, x^s, l_y, z^d, y^s > 0, x^d = y^d = z = z^s = 0$ .

An individual choosing  $(l_y/x(y))$  sells labor for producing  $y$ , buys good  $x$ , and gets the bundled good  $y$ . It is defined by  $x^d, l_y, y^d > 0, x = x^s = y = y^s = z = z^s = z^d = 0$ .

An individual choosing  $(z/x(y))$  produces and sells  $z$ , buys good  $x$ , and gets the bundled good  $y$ . It is defined by  $z, z^s, x^d, y^d > 0, x = x^s = y = y^s = z^d = 0$ .

Note that structure  $FT_A$  involves trade of goods  $x, z$ , and labor  $l_y$ , so that the trading efficiency coefficients  $k_x, k_z$ , and  $g_y$  appear in this structure. Good  $y$  is not directly priced though it is bundled with good  $x$ .

(2) Structure  $FT_B$  is a division of the population among configurations  $(x/l_yz)$ ,  $(l_y/y(x))$  and  $(z/y(x))$ .

Configuration  $(x/l_yz)$  in  $FT_B$  is symmetric to  $(x/l_yz)$  in structure  $FT_A$ . An individual choosing this configuration produces good  $x$ , hires labor for producing  $y$ , and sells  $y$ , which is bundled with good  $x$ . The difference between  $FT_A$  and  $FT_B$  is that good  $x$  is priced and good  $y$  is not in the former, while good  $y$  is priced and good  $x$  is not in the latter.

Configuration  $(l_y/y(x))$  is symmetric to  $(l_y/x(y))$  in structure  $FT_A$ , but good  $y$  is priced and good  $x$  is not.

Configuration  $(z/y(x))$  is symmetric to  $(z/x(y))$  in structure  $FT_A$ , but good  $y$  is priced and good  $x$  is not.

Note that structure  $FT_B$  involves trade of goods  $y, z$ , and labor  $l_y$ , so that trading efficiency coefficients  $k_y, k_z$ , and  $g_y$  appear in this structure. Good  $x$  is not directly priced though it is bundled with good  $y$ .

According to Sun, Yang, and Zhou (1998, see also Yang, 2001, chapter 13), a general equilibrium exists for a general class of the models of which the model in this paper is a special case under the assumptions that the set of individuals is a continuum, preferences are strictly increasing and rational; and both local increasing returns and constant returns are allowed in production and transactions. A general equilibrium in this model is defined as a set of relative prices of goods and all individuals' labor allocations and trade plans, such that, (1) Each individual maximizes her utility, that is, the consumption bundle generated by her labor allocation and trade plan maximizes her utility function for given prices; (2) All markets clear.

Since the optimum decision is always a corner solution and the interior solution is never optimal according to Lemma 1, we cannot use standard marginal analysis to

solve for a general equilibrium. We adopt a three-step approach to solving for a general equilibrium. The first step is to narrow down the set of candidates for the optimum decision and to identify configurations that have to be considered. We can identify structures from compatible combinations of configurations. In the second step, each individual's utility maximization decision is solved for a given structure. The utility equalization condition between individuals choosing different configurations and the market clearing conditions are used to solve for the relative price of traded goods and numbers (measure) of individuals choosing different configurations. The relative price and numbers, and associated resource allocation are referred to as corner equilibria for this structure. General equilibrium occurs in a structure where, given corner equilibrium relative prices in the structure, no individuals have an incentive to deviate from their chosen configurations. In the second step, we can substitute the corner equilibrium relative prices into the utility function for each constituent configuration in the given structure to compare the utility between this configuration and any alternative configurations. This comparison is called a total cost-benefit analysis. The total cost-benefit analysis yields the conditions under which the utility in each constituent configuration of this structure is not smaller than any alternative configuration. With the existence theorem of general equilibrium proved by Sun, Yang, and Zhou (1998), we can completely partition the parameter space into subspaces, within each of which the corner equilibrium in a structure is a general equilibrium. As parameter values shift between the subspaces, the general equilibrium will discontinuously jump between structures. The discontinuous jumps of structure and all endogenous variables are called inframarginal comparative statics of general equilibrium. The three steps constitute an inframarginal analysis.

The corner equilibria in the structures are solved in the following subsection.

#### **4.3.2 Corner Solution in a Configuration and Corner Equilibrium in a Structure**

In this subsection, we first use two examples to illustrate how marginal analysis can be conducted to solve for the corner solution in each configuration and for the corner equilibrium in each structure. The first example is the corner solution in configuration A that is the corner equilibrium in autarky structure A.

Autarky is a structure where each individual chooses configuration A. An individual's decision problem in A is:

$$(4.3.1a) \quad \text{Max: } u_A = x^\alpha \cdot y^{1-\alpha},$$

subject to the following constraints:

$$(4.3.1b) \quad \begin{aligned} x &= z^\beta \cdot l_x, \\ y &= l_y - b, \\ z &= l_z - b, \quad \text{and} \\ l_x + l_y + l_z &= 1. \end{aligned}$$

The optimal solution is:

$$(4.3.1c) \quad \begin{aligned} l_x &= \frac{\alpha \cdot (1-2b)}{1+\alpha\beta}, \\ l_y &= \frac{(1-\alpha) \cdot (1-2b) + b \cdot (1+\alpha\beta)}{1+\alpha\beta}, \\ l_z &= \frac{\alpha\beta \cdot (1-b) + b}{1+\alpha\beta}, \quad \text{and} \\ u_A &= \frac{\beta^{\alpha\beta} \cdot (1-\alpha)^{1-\alpha} \cdot (1-2b)^{1+\alpha\beta} \cdot \alpha^{\alpha(1+\beta)}}{(1+\alpha\beta)^{1+\alpha\beta}}, \end{aligned}$$

where  $u_A$  is per capita real income in structure  $A$ .

Next, we consider the corner equilibrium is structure  $FT_A$  with bundling sale and the institution of the firm. This structure involves the division of the population among configurations  $(x/l_y z)$ ,  $(l_y/x(y))$  and  $(z/x(y))$ . An individual choosing  $(x/l_y z)$  is the employer of a firm. She specializes in producing good  $x$ , and hires labor to produce a final good  $y$ . She sells good  $x$ , buys intermediate good  $z$  and labor, and bundles good  $y$  with good  $x$ , which means good  $y$  is not directly priced, and people can obtain some amount of good  $y$  when they buy good  $x$  from the market. The ratio of the amounts of the two goods bundled is chosen by the employer under competitive pressure in the market.

In structure  $FT_A$ , the decision problem for an individual choosing configuration  $(x/l_y z)$  is:

$$(4.3.2.a) \quad \text{Max: } u_{FTA} = x^\alpha \cdot y^{1-\alpha},$$

subject to the following constraints,

$$(4.3.2.b) \quad \begin{aligned} x + x^s &= (z^d)^\beta \cdot l_x \quad \text{and} \quad l_x = 1, \\ Y^s &= g_y \cdot L_y - b \quad \text{and} \quad L_y = 1, \end{aligned}$$



$$y^s = h \cdot x^s ,$$

$$y + y^s = N \cdot Y^s ,$$

$$k_x p_x x^s = p_z z^d + w \cdot N \cdot L_y ,$$

where  $g_y$  is again the transaction efficiency coefficient for labor hired to produce good  $y$ . Moreover,  $N$  is the number of workers hired by the employer to produce good  $y$ ,  $w$  is the wage rate, and  $h$  is the bundling ratio between goods  $y$  and  $x$ . In order to distinguish inter flow of goods from market trade flow, we use capitalized decision variables to denote internal flow. Hence,  $Y^s$  is internal transfer of good  $y$  produced by an employee to the employer and  $y^s$  is the amount of good  $y$  provided free of charge by the firm.

Here,  $x$  is priced and  $y$  is not. We assume  $h = e \cdot \frac{p_x}{w}$ . This implies that an individual selling  $x$ , buying labor, and bundling  $y$  with  $x$ , must choose the bundling ratio  $h = y/x$  according to  $p_x/w$ . For a small relative market price  $p_x/w$ , she must give away a small amount of  $y$  for each unit  $x$  sold. Otherwise, a small value of  $p_x/w$  may not be enough to cover the production cost of  $y$  which is not directly priced. Here,  $e$  is as given to the owner of the firm, while later based on the Yao Theorem (see Yang 2001, chapter 6, p.156), we can rule out the optimal bundling ratio of good  $x$  and  $y$  in this structure. In addition,  $l_x$  is the decision variable of the employer, while  $l_y$  is an employee's decision variable.  $u_{FTA1}$  is the utility for an  $x$  specialist-employer choosing  $(x/l_y z)$ .

The solution to the decision problem yields a demand function for labor and good  $z$ , a supply function of good  $x$ , and an indirect utility function for configuration  $(x/l_y z)$ .

Similarly, an employee choosing configuration  $(l_y/x(y))$  has the following decision problem,

$$(4.3.2c) \quad \text{Max: } u_{FTA2} = (x^d)^\alpha \cdot (y^d)^{1-\alpha} ,$$

subject to the following constraints,

$$(4.3.2d) \quad y^d = h \cdot x^d ,$$

$$L_y = 1 ,$$

$$w \cdot L_y = p_x \cdot x^d .$$

The solution of this problem yields demand for goods  $x$  and  $y$ , supply of labor, and an indirect utility function for configuration  $(l_y/x(y))$ .

An individual choosing configuration  $(z/x(y))$  has the following decision problem:

$$(4.3.2e) \quad \text{Max: } u_{FTA3} = (x^d)^\alpha \cdot (y^d)^{1-\alpha},$$

subject to the production function, endowment constraint, and budget constraint:

$$(4.3.2f) \quad \begin{aligned} z^s &= l_z - b, \\ l_z &= 1, \\ y^d &= h \cdot x^d, \\ k_z p_z z^s &= p_x x^d. \end{aligned}$$

The solution to this problem yields demand for goods  $x$  and  $y$ , supply of good  $z$ , and indirect utility function for configuration  $(z/x(y))$ .

The two utility equalization conditions across three configurations yield the corner equilibrium relative prices of goods  $x$  and  $z$  and labor.

$$(4.3.2g) \quad \frac{w}{p_z} = k_z(1-b), \quad \text{and}$$

$$\frac{w}{p_x} = \left[ \frac{k_x(g_y - b) \cdot k_z \cdot \beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right]^{\frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta) \cdot [(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta]^{(1-\alpha) + \frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}}.$$

Based on the Yao Theorem, maximizing utility with respect to  $e$ , yields the optimal value of  $e^*$  in this structure:

$$(4.3.2h) \quad e^* = \frac{k_x(1-\alpha)^2 \cdot (g_y - b)}{(1-\alpha) \cdot (1-\beta) + \alpha\beta}.$$

The two independent market clearing conditions for goods  $x$  and  $z$  (the other market clearing condition is not independent due to Walras' law) yield the corner equilibrium relative numbers of specialists producing goods  $x$ ,  $y$ , and  $z$ .

(4.3.2i)

$$\frac{M_x}{M_z} = \left\{ \left[ \frac{(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right]^{(1-\alpha) + \frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta) \cdot [(g_y - b) \cdot \beta]^{\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha} \cdot k_x^{\frac{1-2\beta}{1-\beta}}} \right\}^{\frac{1}{1-\beta}} \cdot \left( \frac{k_z^2}{1-b} \right)^{\frac{\beta}{1-\beta}},$$

and

$$\frac{M_y}{M_x} = \left[ \frac{(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right] \cdot \frac{(1-\alpha + \alpha\beta)}{\beta} \cdot \left[ \frac{k_x(g_y - b) \cdot k_z \cdot \beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right]^{\frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta) \cdot [(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta]^{(1-\alpha) + \frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}},$$

where  $M_x$  is the number of  $x$  specialist-employers choosing  $(x/y,z)$ ,  $M_z$  is the number of specialist producers of good  $z$  choosing  $(z/x(y))$ , and  $M_y$  is the number of employees choosing  $(y/x(y))$ . The relative numbers of specialists, together with population size identity  $M_x+M_z+M_y=M$ , yield the corner equilibrium numbers of different specialists. Plugging relative prices into an indirect utility function of any of three configurations yields the per capita real income in this structure:

(4.3.2))

$$u_{FTA} = \alpha^\alpha \cdot (1-\beta) \cdot (1-\alpha)^{1-\alpha} \cdot (k_z \cdot \beta)^{\frac{\beta}{1-\beta}} \cdot \left\{ \frac{k_x(g_y - b) \cdot [(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta]}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right\}^{(1-\alpha) \cdot \frac{\beta}{1-\beta}}$$

In Structure  $FT_A$ , a firm produces both good  $x$  and  $y$ , while selling  $x$  with good  $y$  bundled. The percentage  $h$  of good  $x$  and  $y$  is dependent on the relative price of good  $x$ , labor, and  $e$ . Note that good  $y$  is bundled through the purchase of good  $x$ , therefore transaction costs in directly pricing good  $y$  is avoided.

Following this procedure, we can solve for corner equilibria in all structures. Information about such solutions of corner equilibria in 11 structures is summarized in Tables 4.1 and 4.2.

**Table 4.1 Relative Price and Number of Specialists**

Structure	Relative Prices	Relative Number of Specialists
A	N/A	N/A
$P_A$	$\frac{p_x}{p_y} = \frac{k_y^\alpha \cdot (1+\beta)^{1+\beta}}{k_x^{1-\alpha} \cdot \beta^\beta \cdot (1-b)^\beta}$	$\frac{M_x}{M_y} = \frac{\alpha \cdot k_y^{1-\alpha}}{(1-\alpha) \cdot k_x^\alpha}$
$P_B$	$\frac{p_y}{p_z} = \left( \frac{k_z^{1-\alpha}}{k_y^{\alpha\beta}} \right)^{\frac{1}{1-\alpha+\alpha\beta}}$	$\frac{M_z}{M_y} = \frac{\alpha\beta \cdot k_y^{\frac{1-\alpha}{1-\alpha+\alpha\beta}}}{(1-\alpha) \cdot k_z^{\frac{\alpha\beta}{1-\alpha+\alpha\beta}}}$
$FP_B$	$\frac{p_y}{w} = P_{FPB}$	$\frac{M_z}{M_y} = M_{FPB}$

CD	$\frac{p_x}{p_z} = \left(\frac{1-b}{1-\beta}\right)^{1-\beta} \cdot \frac{k_z^{1-\beta}}{\beta^\beta \cdot k_x^{1-\alpha+\beta}}$ $\frac{p_z}{p_y} = \frac{k_y^\alpha}{k_z}$	$\frac{M_y}{M_x} = \frac{(1-\alpha) + \alpha\beta(1-k_x \cdot k_z \cdot k_y^{1-\alpha})}{\alpha \cdot (1-\beta) \cdot k_x^{1-\alpha}}$ $\frac{M_x}{M_z} = \frac{(1-\beta)}{\beta \cdot k_z \cdot k_x^\alpha}$
FD <sub>A</sub>	$\frac{w}{p_y} = k_y^\alpha \cdot (g_z - b)$ $\frac{w}{p_x} = k_x^{1-\alpha+\beta} \cdot (1-\beta)^{1-\beta} \cdot \beta^\beta \cdot (g_z - b)^\beta$	$\frac{M_x}{M_z} = \frac{g_z \cdot (1-\beta)}{\beta \cdot k_x^\alpha}$ $\frac{M_y}{M_x} = \frac{k_x^\alpha \cdot (1-\alpha)}{k_y^{1-\alpha} \cdot \alpha \cdot (1-\beta)}$
FT <sub>A</sub>	$\frac{w}{p_z} = k_z(1-b)$ $\frac{w}{p_x} = P_{FTA1}$	$\frac{M_x}{M_z} = M_{FTA1}$ $\frac{M_y}{M_x} = M_{FTA2}$
FT <sub>B</sub>	$\frac{w}{p_z} = k_z(1-b)$ $\frac{w}{p_y} = P_{FTB}$	$\frac{M_x}{M_z} = M_{FTB1}$ $\frac{M_y}{M_x} = M_{FTB2}$

Here,

$$P_{FPB} = \frac{p_y}{w} = \left\{ \frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot k_y \cdot [g_z(1-b) \cdot (1-\alpha) - b(1+\alpha\beta)]}{(\alpha\beta)^\alpha \cdot (1-\alpha) \cdot (1-b)^\alpha} \right\}^{\frac{\alpha\beta}{(1-\alpha)(1-\alpha+\beta)}}$$

$$M_{FPB} = \frac{M_z}{M_y} = \left\{ \frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot k_y \cdot [g_z(1-b) \cdot (1-\alpha) - b(1+\alpha\beta)]}{(\alpha\beta)^\alpha \cdot (1-\alpha) \cdot (1-b)^\alpha} \right\}^{\frac{\alpha\beta}{(1-\alpha)(1-\alpha+\beta)}} \cdot \left[ \frac{(1+\alpha\beta)^2}{(1-\alpha) \cdot (1-b)^2} \right]$$

$$P_{FTA1} = \frac{w}{p_x} = \left[ \frac{k_x(g_y - b) \cdot k_z \cdot \beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right]^{\frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta) \cdot [(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta]^{(1-\alpha) \cdot \frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}}$$

$$M_{FTA1} = \frac{M_x}{M_z} = \left\{ \left[ \frac{(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right]^{(1-\alpha) \cdot \frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta) \cdot [(g_y - b) \cdot \beta]^{\frac{\beta}{1-\beta}} \cdot \frac{1}{1-\beta} \cdot \left(\frac{k_z^2}{1-b}\right)^{\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha} \cdot k_x^{\frac{1-\beta}{1-\beta}}} \right\}$$

$$M_{FTA2} = \frac{M_y}{M_z} = \left[ \frac{(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right] \cdot \frac{(1-\alpha+\alpha\beta)}{\beta} \cdot \left[ \frac{k_x(g_y - b) \cdot k_z \cdot \beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right]^{\frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta) \cdot [(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta]^{(1-\alpha) \cdot \frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}}$$

$$P_{FTB1} \frac{w}{p_y} = \left[ \frac{k_y(g_y - b) \cdot k_z \cdot \beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right]^{\frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta) \cdot [(1-\alpha) \cdot (\alpha-\beta) + \alpha\beta]^{(1-\alpha) \cdot \frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}}$$

$$M_{FTB1} = \frac{M_x}{M_z} = \left\{ \left[ \frac{(1-\alpha) \cdot (\alpha-\beta) + \alpha\beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right]^{(1-\alpha) \cdot \frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta) \cdot [(g_y - b) \cdot \beta]^{1-\beta}}{(1-\alpha)^{1-\alpha} \cdot k_y^{1-\beta}} \right\}^{\frac{1}{1-\beta}} \cdot \left( \frac{k_z^2}{1-b} \right)^{\frac{\beta}{1-\beta}}$$

$$M_{FTB2} = \frac{M_y}{M_x} = \left[ \frac{(1-\alpha) \cdot (\alpha-\beta) + \alpha\beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right] \cdot \frac{(1-\alpha + \alpha\beta)}{\beta} \cdot \left[ \frac{k_y(g_y - b) \cdot k_z \cdot \beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right]^{\frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta) \cdot [(1-\alpha) \cdot (\alpha-\beta) + \alpha\beta]^{(1-\alpha) \cdot \frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}}$$

Table 4.2 Per-Capita Real Income in Different Structures

Structure	Per-Capita Real Income, $u$
A	$\frac{\beta^{\alpha\beta} \cdot (1-\alpha)^{1-\alpha} \cdot (1-2b)^{1+\alpha\beta} \cdot \alpha^{\alpha(1+\beta)}}{(1+\alpha\beta)^{1+\alpha\beta}}$
P <sub>A</sub>	$\frac{\alpha^\alpha \cdot (1-\alpha)^{1-\alpha} \cdot (1-b)^{1+\alpha\beta} \cdot \beta^{\alpha\beta} \cdot (k_x \cdot k_y)^{\alpha(1-\alpha)}}{(1+\beta)^{\alpha(1+\beta)}}$
P <sub>B</sub>	$u_{PB} = \frac{\alpha^\alpha \cdot (1-\alpha)^{1-\alpha} \cdot (1-b)^{1+\alpha\beta} \cdot (\alpha\beta)^{\alpha\beta} \cdot (k_x \cdot k_y)^{\frac{\alpha\beta(1-\alpha)}{1-\alpha+\alpha\beta}}}{(1+\alpha\beta)^{1+\alpha\beta}}$
FP <sub>B</sub>	$u_{FPB} = \left\{ \frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot k_y \cdot [g_z(1-b) \cdot (1-\alpha) - b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha) \cdot (1-b)^{\frac{1}{\alpha}}} \right\}^{\frac{\alpha\beta}{(1-\alpha)(1-\alpha+\alpha\beta)}} \cdot \left[ \frac{(1-\alpha)^{1-\alpha} \cdot \alpha^{\alpha+\beta} \cdot (1-b)^{1+\beta} \cdot \beta^\beta}{(1+\alpha\beta)^{1+\beta}} \right]$
CD	$\alpha^\alpha \cdot (1-\alpha)^{1-\alpha} \cdot (1-b)^{1-\alpha+\alpha\beta} \cdot \beta^{\alpha\beta} \cdot (1-\beta)^{\alpha(1-\beta)} \cdot k_z^{\alpha\beta} \cdot k_x^{\alpha(1-\alpha+\alpha\beta)} \cdot k_y^{\alpha(1-\alpha)}$
FD <sub>A</sub>	$\alpha^\alpha \cdot (1-\alpha)^{1-\alpha} \cdot (g_z - b)^{1-\alpha+\alpha\beta} \cdot \beta^{\alpha\beta} \cdot (1-\beta)^{\alpha(1-\beta)} \cdot k_y^{\alpha(1-\alpha)} \cdot k_x^{\alpha(1-\alpha+\alpha\beta)}$
FT <sub>A</sub>	$u_{FTA} = \alpha^\alpha \cdot (1-\beta) \cdot (1-\alpha)^{1-\alpha} \cdot (k_z \cdot \beta)^{\frac{\beta}{1-\beta}} \cdot \left\{ \frac{k_x(g_y - b) \cdot [(1-\alpha) \cdot (\alpha-\beta) + \alpha\beta]}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right\}^{(1-\alpha) \cdot \frac{\beta}{1-\beta}}$
FT <sub>B</sub>	$u_{FTB} = \alpha^\alpha \cdot (1-\beta) \cdot (1-\alpha)^{1-\alpha} \cdot (k_z \cdot \beta)^{\frac{\beta}{1-\beta}} \cdot \left\{ \frac{k_y(g_y - b) \cdot [(1-\alpha) \cdot (\alpha-\beta) + \alpha\beta]}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right\}^{(1-\alpha) \cdot \frac{\beta}{1-\beta}}$

### 4.3.3 General Equilibrium and Its Inframarginal Comparative Statics

We now consider the third step of inframarginal analysis. Based on the first two steps of the inframarginal analysis, we will partition the parameter space into subspaces within each of which a particular structure occurs in equilibrium.

For any given structure, each individual can plug the corner equilibrium prices into her indirect utility functions for all configurations including those that are not in this structure. She has no incentive to deviate from a constituent configuration in this structure if this configuration generates a utility value that is not lower than in any alternative configurations under the corner equilibrium values of prices in this structure. Each individual can conduct such total cost-benefit analysis across configurations. Let indirect utility in each constituent configuration not be smaller than in any alternative configurations under the corner equilibrium prices in this structure. We can obtain a system of semi-inequalities that involves only parameters. This system of semi-inequalities defines a parameter subspace within which the corner equilibrium in this structure is the general equilibrium. This total cost-benefit analysis is very tedious and cumbersome. Fortunately, the Yao Theorem (see Yang 2001, chapter 6, p.156) can be used to simplify this total cost-benefit analysis. It states that in an economy with a continuum of ex ante identical consumer-producers having rational and convex preferences and production functions displaying individual specific economies of specialization, a Walrasian general equilibrium exists and it is the Pareto optimum corner equilibrium. Here the Pareto optimum corner equilibrium is a corner equilibrium that generates the highest per capita real income. Since our model in this paper is a special case of the above-mentioned general class of models, the individuals have no incentive to deviate from their chosen constituent configurations in a structure if and only if individuals' corner equilibrium utility value in this structure is not lower than that in any other corner equilibria. With the Yao theorem, we can then compare corner equilibrium per capita real incomes across all structures, and the comparison partitions the five-dimension  $(\alpha, \beta, g, b, k)$  parameter space into several subspaces, within each of which one corner equilibrium is the general equilibrium. As parameter values shift between different subspaces, the general equilibrium discontinuously jumps between corner equilibria. This is referred to as inframarginal comparative statics of general equilibrium.

In order to obtain an analytical solution of the inframarginal comparative statics, we consider the economy with  $\alpha = \beta = 0.5$ . A close examination of per capita real incomes in different structures, given in Table 4.2, generates the results in the following table, in which trading efficiency coefficients in an entry positively correlate to per capita real income in a structure associated with the column.

**Table 4.3 Trading Efficiency Coefficients that Positively Affect Per capita Real Income in a Structure**

A	$P_A$	$P_B$	$FP_B$	CD	$FD_A$	$FT_A$	$FT_B$
n/a	$k_x, k_y$	$k_x, k_y$	$g_z, k_y$	$k_x, k_y, k_z$	$k_x, k_y, g_z$	$k_x, k_z, g_y$	$k_z, k_y, g_y$

From Table 4.2, we can see that as any trading efficiency coefficient in the second row tends to zero, the per capita real income in the corresponding structure in the first row in Table 4.3 goes to zero. For instance, per capita real income in structure  $FD_A$  positively depends on trading efficiencies of goods  $x$  and  $y$ ,  $k_x$ ,  $k_y$ , and trading efficiency of labor employed to produce  $x$ ,  $g_z$ . The per capita real income converges to zero as any of  $k_x$ ,  $k_y$ ,  $g_z$  goes to zero. Since per capita real income in autarky (structure A) is independent of the trading efficiency coefficients, if all trading efficiency coefficients are sufficiently close to zero, per capita real income in autarky will be greater than that in any other structures with trade. Also, we can see from Table 4.3 that a structure with partial division of labor ( $P_A$ ,  $P_B$ , or  $FP_B$ ) involves trading efficiency coefficients of two types of goods and/or labor, while a structure with the complete division of labor (three goods are involved in commercialized production) involves trading efficiency coefficients of three types of goods and/or labor. Hence, as trading efficiencies of more types of goods and labor are improved, the general equilibrium will discontinuously jump from autarky to partial division of labor, followed by the complete division of labor. Hence, the first conclusion from the total cost-benefit analysis of per capita real income in various structures is that trading efficiency determines the general equilibrium network size of division of labor.

The second conclusion from the third step of inframarginal analysis is that the institution of the firm is a way to replace trade of goods with trade of labor. As we can see from Table 4.3, all structures with the firm ( $FP_B$ ,  $FD_A$ ,  $FT_A$ ,  $FT_B$ ) involve the trading efficiency coefficient of labor employed to produce good  $i$ ,  $g_i$ . Per capita real

incomes in all structures without the firm ( $P_A$ ,  $P_B$ ,  $CD$ ) are independent of trading efficiency of labor. Hence, if the trading efficiency is higher for labor than that for goods, the institution of the firm and related labor market will be used to more efficiently organize the division of labor. Otherwise, the markets for goods will be used to organize the division of labor in the absence of the institution of the firm and related labor market. This formalizes the theory of Coase (1937) and Cheung (1983). This is consistent with the inframarginal analysis of the theory of the firm by Yang and Ng (1995) and the model formalizing the theory of irrelevance of the size of the firm developed by Liu and Yang (2000).

The third conclusion can be obtained by comparing structures with the firm and bundling ( $FT_i$ ,  $i = A, B$ ) and those with the firm and without bundling ( $FP_B$ ,  $FD_A$ ). A comparison between structures  $FD_A$  and  $FT_A$  shows that if trading efficiency is prohibitively low for good  $y$  ( $k_y$  tends to zero), then not only can a structure without the firm (such as structure  $CD$ ) not be used to coordinate the complete division of labor with three goods involved in commercialized production, but also structures with the firm ( $FD_A$ ,  $FD_B$ ) cannot be used to coordinate the complete division of labor in the absence of bundling. This is because structures  $CD$  and  $FD_A$  involve marketing and pricing of good  $y$ , while structure  $FT_A$  with bundling avoids direct pricing of good  $y$ , when it gets good  $y$  involved in commercialized production.

In order to make results more concrete, we explicitly solve for general equilibrium and its inframarginal comparative statics for some specific ranges of parameter values.

We first assume that the trading efficiency of good  $y$ ,  $k_y$ , is very close to zero. From Tables 4.2 and 4.3, we can see that this implies zero per capita real incomes in structures  $P_A$ ,  $P_B$ ,  $CD$ ,  $FD_A$ ,  $FT_B$ , since per capital real incomes in these structures are positively dependent on  $k_y$  and they go to zero as  $k_y$  tends to zero. Hence, the set of candidates for the equilibrium structure consists of structures  $A$  and  $FT_A$  in which per capita real incomes are independent of  $k_y$ . As shown in Tables 4.2 and 4.3, per capita real income in structure  $A$  is independent of trading efficiency, while per capita real income in structure  $FT_A$  depends on  $k_x$ ,  $k_z$ , and  $g_y$ . Therefore, when  $k_x$ ,  $k_z$ , and  $g_y$  are very small, the general equilibrium is the corner equilibrium in structure  $A$ . When  $k_x$  and  $k_z$  are large, the general equilibrium is the corner equilibrium in structure  $FT_A$ . The inframarginal comparative statics of general equilibrium are summarized in Table 4.4.



**Table 4.4 General Equilibrium and Its Inframarginal Comparative Statics**  
when  $k_y \rightarrow 0$

Trading efficiency of goods	$g_y$ , $k_x$ and $k_z$ are small	$g_y$ , $k_x$ and $k_z$ are large
Equilibrium structure	A	FT <sub>A</sub>

The inframarginal comparative statics in Table 4.4 indicate that as trading efficiencies increase from very small to very large values, the general equilibrium discontinuously jumps from autarky to the division of labor. Due to prohibitively low trading efficiency of good  $y$ , the division of labor must be organized via the institution of the firm that sells good  $x$  and provides good  $y$  free of charge. A particular structure with the firm and bundling can be used to avoid trade of a particular type of labor. Structure FT<sub>A</sub> can be used to avoid trade of labor employed to produce good  $x$ . Suppose that good  $y$  is an information good and  $x$  is hardware. Hence, the output and input of producing  $x$  are easy to measure, but the output and input of producing  $y$  is prohibitively expensive to measure. For instance, labor employed to produce good  $y$  is intellectual efforts put into thinking and research. The quantity and quality of such efforts are prohibitively expensive to measure. Under this circumstance, bundling in structure FT<sub>A</sub> is to avoid all direct pricing of output and input of the activity producing intangible good  $y$ .

As shown in Yang and Ng (1995), the institution of the firm can indirectly price intangible intellectual properties via claims to residual rights of the firm. But, the model in this paper shows that the institution of the firm coupled with bundling can enlarge the scope for such indirect pricing of intellectual properties. In the case of Table 4.4, the institution of the firm is not enough to indirectly price all input and output of the activity producing good  $y$  in the absence of implicit bundling. Hence, without implicit bundling, the division of labor and commercialized production of information goods becomes impossible, so that positive network effects of such commercialized production through specialization cannot be fully exploited.

In order to compare the roles of structures with and without bundling, we consider the case with  $k_x \rightarrow 0$ . The inframarginal comparative statics of general equilibrium within this range of parameter values are summarized in Table 4.5.

**Table 4.5 General Equilibrium and Its Inframarginal Comparative Statics**

when  $k_x \rightarrow 0$

Trading efficiency of good	$k_y, k_z$ are small	$k_y$ and $k_z$ are neither large nor small		$k_y$ and $k_z$ are large
Trading efficiency of labor	$g_y, g_z$ are small	$g_y, g_z$ are small	$g_z$ is neither large nor small	FT <sub>B</sub>
Equilibrium structure	A	P <sub>B</sub>	FP <sub>B</sub>	

The inframarginal comparative statics in Table 4.5 indicate that as trading efficiencies increase from very low to very high levels, the general equilibrium evolves from autarky first to the partial division of labor, then to the complete division of labor. The partial division of labor is coordinated by the institution of the firm and related labor market (FP<sub>B</sub>) if trading efficiency for labor is high. Otherwise, it is organized by the markets for goods in the absence of the firm and related labor market (P<sub>B</sub>). The complete division of labor can be organized only via the institution of the firm which sells good y with good x bundled due to prohibitively low trading efficiency of good x. A comparison between Tables 4.4 and 4.5 shows that direct pricing of a good (x or y) must be avoided via bundling if the trading efficiency of this good is extremely low.

Following Sun, Yang, and Yao (1999, see also Yang, 2001), it can be shown that a general equilibrium in our model is Pareto optimal. This first welfare theorem in our model with impersonal networking decisions and endogenous network size of division of labor implies that very function of the market is to coordinate impersonal networking decisions and to fully utilize network effects of division of labor on aggregate productivity, net of transaction costs. Bundling in a competitive market is an effective way to promote division of labor and productivity progress. This, together with the inframarginal comparative statics of equilibrium given in Tables 4.4 and 4.5, lead to the following proposition:

**Proposition 4.1**

**Absolute level of transaction efficiency of goods and labor determines the level of division of labor. As transaction efficiency is improved, the equilibrium level of division of labor increases. Relative level of transaction efficiency for labor to that**

for goods determines if the division of labor is organized by a labor market and the related institution of the firm. Bundling sale can be used to avoid direct pricing of output and input of the activity with the lowest transaction efficiency, meanwhile getting this activity involved in the division of labor, thereby promoting the division of labor and productivity progress. Bundling sale based on impersonal networking decisions in a competitive market has no adverse effects on welfare.

Proposition 4.1 implies that antitrust prosecutions should focus on the existence of an intention to block free-entry rather than on the bundling sale itself because according to Proposition 4.1 and the Yao theorem, bundling sale will promote the division of labor and increase aggregate productivity if it occurs in equilibrium. Bundling does not generate distortions in a competitive market.

Following Yang (2001), it is easy to prove that marginal cost price no longer holds in a structure with the division of labor and that the aggregate production schedule discontinuously jumps to a higher level as the network of division of labor expands. Due to the trade-off between transaction costs and positive network effects of division of labor on aggregate productivity, the equilibrium and Pareto optimum may be different from the PPF. As trading efficiency is improved, the equilibrium network size of division of labor enlarges, and the equilibrium and Pareto optimum become closer to the PPF.

#### 4.4 Concluding Remarks

This chapter develops a Walrasian general equilibrium model based on impersonal networking decisions to investigate the role of bundling sale in a competitive market and e-business. The following features distinguish our model of bundling from other models in the literature. In our model there is no monopoly power, substitution between different goods and resale of goods are allowed. An ex ante identical utility function is specified for all individuals whose valuations of each good are not a constant. Each individual can choose size and pattern of her trade network by choosing her level of specialization subject to impersonal prices. Hence, gains to each person's level of specialization depends on the number of participants in the network of division of labor, while the number of participants depends on each person's participation decision in the network, which is determined by her decision in choosing her level of specialization. Since individuals are capable of doing inframarginal analysis

in choosing a utility maximizing trading network from many possible corner solutions, the equilibrium network size and pattern of division of labor is Pareto efficient despite the existence of network effects of division of labor on aggregate productivity.

The function of the institution of the firm and bundling is to get the activity with the lowest trading efficiency involved in the division of labor and commercialized production, meanwhile avoiding direct pricing of the outputs and inputs of this activity. Implicit bundling coupled with the institution of the firm can provide a greater scope for indirectly pricing goods with the lowest trading efficiency than the institution of the firm alone can do. In our model, the complete division of labor can be organized by trade of three types of goods and labor. But there are six types of goods and labor:  $x$ ,  $y$ ,  $z$ ,  $l_x$ ,  $l_y$ ,  $l_z$ . Hence, a competitive market will find a three element combination from six elements to fully exploit total positive network effects of division of labor on aggregate productivity net of total transaction costs. Note that the total equilibrium value of transaction costs may increase as a consequence of evolution of division of labor caused by improvements in trading efficiency. For instance, as trading efficiency is improved, the general equilibrium jumps from autarky, where transaction costs are zero and aggregate productivity is lower than the PPF, to the division of labor where total transaction costs are positive and aggregate productivity is higher.

Since the general equilibrium in our model is always Pareto optimal as long as nobody can block free entry into any sector and nobody can manipulate relative prices and numbers of specialists, policy implications of our model are straightforward. Bundling in a competitive market is efficient and it ensures that network effects of division of labor can be fully exploited when some goods involved in the network of division of labor are associated with prohibitively high transaction costs. Hence, bundling in a competitive market can promote aggregate productivity by enlarging the scope for trading off network effects of the division of labor on aggregate productivity against transaction costs. Bundling itself is not a source of distortions in a competitive market. Bundling may generate distortions only if it is used in connection with monopoly power. Hence, in antitrust cases, such as in the case of Microsoft vs. the United States, attention should be placed on the existence of intention to block free entry in an attempt of gaining monopoly power rather than on alleged adverse effects of bundling itself on welfare. To business practitioners, our model suggests that successful bundling of intangible e-business with some tangible 'mortar-brick' business is a key for commercial viability of e-business companies.

A promising extension of our model is to assume that the seller of a bundle of goods cannot choose the bundling ratio. We may assume that each buyer of implicitly bundled goods must allocate resources to use those goods that are free of charge. Hence, it is the buyer rather than the seller who chooses the bundling ratio subject to her resource endowment constraint. When a firm sells information goods via a website, she usually cannot choose the bundling ratio of goods with positive prices and goods free of charge. We speculate that the extended model will confirm results in the current chapter with this assumption that is more relevant to real e-business.

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## Appendix 4.1 The Corner Equilibria of Different Market Structures

### A4.1.1 Partial Division of Labor: $P_A$

Structure  $P_A$  consists of two configurations,  $(xz/y)$  and  $(y/x)$ . In the structure  $P_A$ , given that  $x, x^s, z, y^d > 0, z^s = z^d = y = y^s = x^d = 0$ , an individual in configuration  $(xz/y)$  has the following decision problems,

$$(A4.1.1a) \quad \text{Max: } u_{PA1} = x^\alpha \cdot (y^d)^{1-\alpha},$$

subject to the following constraints:

$$(A4.1.1b) \quad x + x^s = z^\beta \cdot l_x \quad \text{and } \beta \in (0,1),$$

$$z = l_z - b \quad \text{and } b \in (0,1),$$

$$l_x + l_z = 1,$$

$$k_x \cdot p_x \cdot x^s = p_y \cdot y^d,$$

where  $u_{PA1}$  is the utility for an individual in configuration  $(xz/y)$ . The other equations represent an individual's budget constraint, endowment constraint, and the production function. Similarly, an individual in configuration  $(y/x)$  has the following decision problem:

$$(A4.1.1c) \quad \text{Max: } u_{PA2} = (x^d)^\alpha \cdot y^{1-\alpha},$$

subject to the following constraints:

$$(A4.1.1d) \quad y + y^s = l_y - b \quad \text{and } b \in (0,1),$$

$$l_y = 1,$$

$$k_y \cdot p_y \cdot y^s = p_x \cdot x^d,$$

where  $u_{PA2}$  is the utility for an individual in configuration  $(y/x)$ .

Based on the utility equalization condition and market clearing conditions, the price of good  $x$  in terms of good  $y$ , and the relative number of individual selling good  $x$  to individuals selling good  $y$  are given by:

$$(A4.1.1e) \quad \frac{p_x}{p_y} = \frac{k_y^\alpha \cdot (1 + \beta)^{1+\beta}}{k_x^{1-\alpha} \cdot \beta^\beta \cdot (1-b)^\beta}, \quad \text{and}$$

$$\frac{M_x}{M_y} = \frac{\alpha \cdot k_y^{1-\alpha}}{(1-\alpha) \cdot k_x^\alpha}.$$

The real per capita income in this structure is,

$$(A4.1.1f) \quad u_{PA} = \frac{\alpha^\alpha \cdot (1-\alpha)^{1-\alpha} \cdot (1-b)^{1+\alpha\beta} \cdot \beta^{\alpha\beta} \cdot (k_x \cdot k_y)^{\alpha(1-\alpha)}}{(1+\beta)^{\alpha(1+\beta)}}.$$

#### A4.1.2 Partial Division of Labor: P<sub>B</sub>

Similarly, in structure P<sub>B</sub> the decision problem for an individual with configuration (zx/y) is:

$$(A4.1.2a) \quad \text{Max: } u_{PB1} = x^\alpha \cdot (y^d)^{1-\alpha},$$

subject to the following constraints:

$$(A4.1.2b) \quad \begin{aligned} x &= z^\beta \cdot l_x \text{ and } \beta \in (0,1), \\ z + z^s &= l_z - b \text{ and } b \in (0,1), \\ l_x + l_z &= 1, \\ k_z \cdot p_z \cdot z^s &= p_y \cdot y^d, \end{aligned}$$

where  $u_{PB1}$  is the utility for an individual in configuration (zx/y). The equations of constraints state an individual's budget constraint, endowment constraint, and the production function.

An individual in configuration (yx/z) has the following decision problem:

$$(A4.1.2c) \quad \text{Max: } u_{PB2} = x^\alpha \cdot y^{1-\alpha}$$

subject to the following constraints:

$$(A4.1.2d) \quad \begin{aligned} x &= (z^d)^\beta \cdot l_x \text{ and } \beta \in (0,1), \\ y + y^s &= l_y - b \text{ and } b \in (0,1), \\ l_x + l_y &= 1, \\ k_y \cdot p_y \cdot y^s &= p_z \cdot z^d. \end{aligned}$$

The utility equalization and market clearing conditions yield a set of relative prices and relative number of specialists, and the per capita real income in this structure.

$$(A4.1.2e) \quad \begin{aligned} \frac{p_y}{p_z} &= \left( \frac{k_z^{1-\alpha}}{k_y^{\alpha\beta}} \right)^{\frac{1}{1-\alpha+\alpha\beta}}, \\ \frac{M_z}{M_y} &= \frac{\alpha\beta \cdot k_y^{\frac{1-\alpha}{1-\alpha+\alpha\beta}}}{(1-\alpha) \cdot k_z^{\frac{\alpha\beta}{1-\alpha+\alpha\beta}}}, \text{ and} \end{aligned}$$



$$u_{PB} = \frac{\alpha^\alpha \cdot (1-\alpha)^{1-\alpha} \cdot (1-b)^{1+\alpha\beta} \cdot (\alpha\beta)^{\alpha\beta} \cdot (k_z \cdot k_y)^{\frac{\alpha\beta(1-\alpha)}{1-\alpha+\alpha\beta}}}{(1+\alpha\beta)^{1+\alpha\beta}}$$

#### A4.1.3 Partial Division of Labor with the Institution of the Firm: Structure FP<sub>B</sub>

Structure FP<sub>B</sub> is a division of the population between configuration (l<sub>2</sub>x/y) and (yx/l<sub>2</sub>). Given that  $x, z, l_2, y^d > 0, x^s = x^d = y = y^s = z^s = z^d = 0$ , an individual in configuration (l<sub>2</sub>x/y) has the following decision problems,

$$(A4.1.3a) \quad \text{Max: } u_{FPB1} = x^\alpha \cdot (y^d)^{1-\alpha} .$$

subject to the following constraints:

$$(A4.1.3b) \quad x = z^\beta \cdot l_x \quad \text{and } \beta \in (0,1),$$

$$z = l_2 - b, \quad \text{and } b \in (0,1),$$

$$l_x + l_2 + L_2 = 1 ,$$

$$w \cdot L_2 = p_y \cdot y^d ,$$

Similarly, a person choosing configuration (yx/l<sub>2</sub>) produces goods x and y, sells good y, and employs labor to produce good z. It is defined by  $x, y, y^s, l_2 > 0, x^s = x^d = y^d = z^s = z^d = 0$ . An individual in configuration (yx/l<sub>2</sub>) has the following decision problems,

$$(A4.1.3c) \quad \text{Max: } u_{FPB2} = x^\alpha \cdot y^{1-\alpha} ,$$

subject to the following constraints:

$$(A4.1.3d) \quad y + y^s = l_y - b ,$$

$$x = (z^d)^\beta \cdot l_x \quad \text{and } \beta \in (0,1),$$

$$l_x + l_2 = 1 ,$$

$$z^d = N \cdot z^s ,$$

$$z^s = g_z \cdot L_2 - b , \quad \text{and}$$

$$k_y \cdot p_y \cdot y^s = w \cdot N \cdot L_2 .$$

The utility equalization and market clearing conditions, yield the price of good y in term of labor to produce good z, and the number of individuals selling good y relative to that of individuals selling labor to produce good z, are given by:

$$(A4.1.3e)$$

$$\frac{p_y}{w} = \left\{ \frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot k_y \cdot [g_z(1-b) \cdot (1-\alpha) - b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha) \cdot (1-b)^{\frac{1}{\alpha}}} \right\}^{\frac{\alpha\beta}{(1-\alpha)(1-\alpha+\alpha\beta)}}, \text{ and}$$

$$\frac{M_z}{M_y} = \left\{ \frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot k_y \cdot [g_z(1-b) \cdot (1-\alpha) - b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha) \cdot (1-b)^{\frac{1}{\alpha}}} \right\}^{\frac{\alpha\beta}{(1-\alpha)(1-\alpha+\alpha\beta)}} \cdot \left[ \frac{(1+\alpha\beta)^2}{(1-\alpha) \cdot (1-b)^2} \right].$$

The per capita real income in this structure is,

(A4.1.3f)

$$u_{FPB} = \left\{ \frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot k_y \cdot [g_z(1-b) \cdot (1-\alpha) - b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha) \cdot (1-b)^{\frac{1}{\alpha}}} \right\}^{\frac{\alpha\beta}{(1-\alpha)(1-\alpha+\alpha\beta)}} \cdot \left[ \frac{(1-\alpha)^{1-\alpha} \cdot \alpha^{\alpha+\beta} \cdot (1-b)^{1+\beta} \cdot \beta^\beta}{(1+\alpha\beta)^{1+\beta}} \right].$$

#### A4.1.4 Complete Division of Labor without the Institution of the Firm: Structure CD

There are three configurations ( $x/yz$ ), ( $z/xy$ ) and ( $y/x$ ) in this structure, where an individual produces only one of good  $x$ ,  $y$  or  $z$ , and sells them in exchange for others. The decision problems for the individuals under different configurations are given as below respectively,

In configuration ( $x/yz$ ):

$$(A4.1.4a) \quad \begin{aligned} \text{Max: } & u_{CD1} = x^\alpha \cdot (y^d)^{1-\alpha} \\ \text{s.t. } & x + x^s = (z^d)^\beta \cdot l_x \text{ and } \beta \in (0,1), \\ & l_x = 1, \\ & k_x \cdot p_x \cdot x^s = p_y \cdot y^d + p_z \cdot z^d. \end{aligned}$$

In configuration ( $z/xy$ ):

$$(A4.1.4b) \quad \begin{aligned} \text{Max: } & u_{CD2} = (x^d)^\alpha \cdot (y^d)^{1-\alpha} \\ \text{s.t. } & z^s = l_z - b \text{ and } b \in (0,1), \\ & l_z = 1, \\ & k_z \cdot p_z \cdot z^s = p_x \cdot x^d + p_y \cdot y^d. \end{aligned}$$

In configuration ( $y/x$ ):

$$(A4.1.4c) \quad \text{Max: } u_{CD3} = (x^d)^\alpha \cdot y^{1-\alpha}$$

$$\frac{p_y}{w} = \left\{ \frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot k_y \cdot [g_z(1-b) \cdot (1-\alpha) - b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha) \cdot (1-b)^{\frac{1}{\alpha}}} \right\}^{\frac{\alpha\beta}{(1-\alpha)(1-\alpha+\alpha\beta)}}, \text{ and}$$

$$\frac{M_z}{M_y} = \left\{ \frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot k_y \cdot [g_z(1-b) \cdot (1-\alpha) - b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha) \cdot (1-b)^{\frac{1}{\alpha}}} \right\}^{\frac{\alpha\beta}{(1-\alpha)(1-\alpha+\alpha\beta)}} \cdot \left[ \frac{(1+\alpha\beta)^2}{(1-\alpha) \cdot (1-b)^2} \right].$$

The per capita real income in this structure is,

(A4.1.3f)

$$u_{FPB} = \left\{ \frac{(1+\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot k_y \cdot [g_z(1-b) \cdot (1-\alpha) - b(1+\alpha\beta)]}{(\alpha\beta)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha) \cdot (1-b)^{\frac{1}{\alpha}}} \right\}^{\frac{\alpha\beta}{(1-\alpha)(1-\alpha+\alpha\beta)}} \cdot \left[ \frac{(1-\alpha)^{1-\alpha} \cdot \alpha^{\alpha+\beta} \cdot (1-b)^{1+\beta} \cdot \beta^\beta}{(1+\alpha\beta)^{1+\beta}} \right].$$

#### A4.1.4 Complete Division of Labor without the Institution of the Firm: Structure CD

There are three configurations ( $x/yz$ ), ( $z/xy$ ) and ( $y/x$ ) in this structure, where an individual produces only one of good  $x$ ,  $y$  or  $z$ , and sells them in exchange for others. The decision problems for the individuals under different configurations are given as below respectively,

In configuration ( $x/yz$ ):

$$\begin{aligned} \text{(A4.1.4a)} \quad & \text{Max: } u_{CD1} = x^\alpha \cdot (y^d)^{1-\alpha} \\ & \text{s.t. } x + x^s = (z^d)^\beta \cdot l_x \text{ and } \beta \in (0,1), \\ & l_x = 1, \\ & k_x \cdot p_x \cdot x^s = p_y \cdot y^d + p_z \cdot z^d. \end{aligned}$$

In configuration ( $z/xy$ ):

$$\begin{aligned} \text{(A4.1.4b)} \quad & \text{Max: } u_{CD2} = (x^d)^\alpha \cdot (y^d)^{1-\alpha} \\ & \text{s.t. } z^s = l_z - b \text{ and } b \in (0,1), \\ & l_z = 1, \\ & k_z \cdot p_z \cdot z^s = p_x \cdot x^d + p_y \cdot y^d. \end{aligned}$$

In configuration ( $y/x$ ):

$$\text{(A4.1.4c)} \quad \text{Max: } u_{CD3} = (x^d)^\alpha \cdot y^{1-\alpha}$$

$$\text{s.t. } y + y^s = l_y - b \quad \text{and } b \in (0,1),$$

$$l_y = 1,$$

$$k_y \cdot p_y \cdot y^s = p_x \cdot x^d.$$

The utility equalization condition and market clearing conditions, yield a set of relative prices and relative number of specialists,

$$(A4.1.4d) \quad \frac{p_z}{p_y} = \frac{k_y^\alpha}{k_z},$$

$$\frac{p_x}{p_z} = \left(\frac{1-b}{1-\beta}\right)^{1-\beta} \cdot \frac{k_z^{1-\beta}}{\beta^\beta \cdot k_x^{1-\alpha+\beta}},$$

$$\frac{M_y}{M_x} = \frac{(1-\alpha) + \alpha\beta(1-k_x \cdot k_z \cdot k_y^{1-\alpha})}{\alpha \cdot (1-\beta) \cdot k_x^{1-\alpha}}, \text{ and}$$

$$\frac{M_x}{M_z} = \frac{(1-\beta)}{\beta \cdot k_z \cdot k_x^\alpha}.$$

The per capita real income in this structure is:

$$(A4.1.4e)$$

$$u_{CD} = \alpha^\alpha \cdot (1-\alpha)^{1-\alpha} \cdot (1-b)^{1-\alpha+\alpha\beta} \cdot \beta^{\alpha\beta} \cdot (1-\beta)^{\alpha(1-\beta)} \cdot k_z^{\alpha\beta} \cdot k_x^{\alpha(1-\alpha+\beta)} \cdot k_y^{\alpha(1-\alpha)}.$$

#### A4.1.5 Complete Division of Labor with the Institution of the Firm: Structure $FD_A$

Structure  $FD_A$  consists of three individual configurations  $(x/l_zy)$ ,  $(l_z/xy)$  and  $(y/x)$ . Given that  $x, x^s, y^d, l_z > 0, x^d = y = y^s = z = z^s = 0$ , an individual in configuration  $(x/l_zy)$  has the following decision problems,

$$(A4.1.5a) \quad \text{Max: } u_{FDA1} = x^\alpha \cdot (y^d)^{1-\alpha}.$$

Her budget constraint and the production functions are,

$$(A4.1.5b) \quad x + x^s = (Z^d)^\beta \cdot l_x \quad \text{and } \beta \in (0,1),$$

$$l_x = 1,$$

$$z^s = g_z \cdot L_z - b, \quad g_z \in (0,1), \text{ and } b \in (0,1),$$

$$L_z = 1,$$

$$Z^d = N \cdot z^s,$$

$$k_x \cdot p_x \cdot x^s = p_y \cdot y^d + w \cdot N \cdot L_z,$$

where  $g_z$  is the transaction efficiency coefficient for labor hired to produce the intermediate good  $z$ . It encompasses all costs that relate to the measurement of the effects of efforts exerted for producing the intermediate good  $z$  in terms of quantity and quality. In essence, the measurement costs can be explained as pricing costs.  $N$  is the number of workers hired by the employer. In this configuration,  $l_x$  is the decision variable to the employer, while  $L_z$  is as given because it is bought from the labor market.  $u_{FDA1}$  is the utility for an individual in configuration  $(x/l_zy)$ , and she is the employer in this structure  $FD_A$ .

Similarly, an individual in configuration  $((l_z/xy))$  has the following decision problems,

$$(A4.1.5c) \quad \text{Max: } u_{FDA2} = (x^d)^\alpha \cdot (y^d)^{1-\alpha}.$$

The budget constraint and the production functions are,

$$(A4.1.5d) \quad L_z = 1,$$

$$w \cdot L_z = p_x \cdot x^d + p_y \cdot y^d.$$

The individual who chooses this configuration is the employee in this structure. Moreover, an individual in configuration  $(y/x)$  has the decision problem of,

$$(A4.1.5e) \quad \text{Max: } u_{FDA3} = (x^d)^\alpha \cdot y^{1-\alpha}.$$

The budget constraint and the production functions are,

$$(A4.1.5f) \quad y + y^s = l_y - b \text{ and } b \in (0,1),$$

$$l_y = 1,$$

$$k_y \cdot p_y \cdot y^s = p_x \cdot x^d.$$

The utility equalization and market clearing conditions, yield the set of prices of good  $x$  and  $y$  in terms of labor to produce good  $z$ ; and the number of individuals selling good  $x$ ,  $y$  relative to that of individuals selling labor to produce good  $z$ , are given by:

$$(A4.1.5g) \quad \frac{w}{p_y} = k_y^\alpha \cdot (g_z - b),$$

$$\frac{w}{p_x} = k_x^{1-\alpha+\alpha\beta} \cdot (1-\beta)^{1-\beta} \cdot \beta^\beta \cdot (g_z - b)^\beta,$$

$$\frac{M_x}{M_z} = \frac{g_z \cdot (1-\beta)}{\beta \cdot k_x^\alpha} \quad \text{and}$$

$$\frac{M_y}{M_x} = \frac{k_x^\alpha \cdot (1-\alpha)}{k_y^{1-\alpha} \cdot \alpha \cdot (1-\beta)}$$

The per capita real income in this structure is,

(A4.1.5h)

$$u_{FDA} = \alpha^\alpha \cdot (1-\alpha)^{1-\alpha} \cdot (g_z - b)^{1-\alpha+\alpha\beta} \cdot \beta^{\alpha\beta} \cdot (1-\beta)^{\alpha(1-\beta)} \cdot k_y^{\alpha(1-\alpha)} \cdot k_x^{\alpha(1-\alpha+\alpha\beta)}$$

#### A4.1.6 With Bundling Sale and the Institution of the Firm: Structure FT<sub>B</sub>

Structure FT<sub>B</sub> is with bundling sale and the institution of the firm, and involves the division of population among configurations (x/l<sub>y</sub>z), (l<sub>y</sub>/y(x)) and (z/y(x)). In Structure FT<sub>B</sub>, a firm specializes in producing good x, and also hires labor to produce another final good y. However, an owner of the firm only sells good y in exchange for intermediate good z and labor employed to produce good y; she bundles good x with good y, which means good x is not directly priced, and people can obtain some amount of good x when they buy good y from the market (This sentence is too long). The ratio of the amounts of the two goods is set up in a bundling sale.

In structure FT<sub>B</sub>, the decision problem for an individual in configuration (x/l<sub>y</sub>z) is as follow,

$$(A4.1.6a) \quad \text{Max: } u_{FTB1} = x^\alpha \cdot y^{1-\alpha},$$

subject to the following constraints,

$$(A4.1.6b) \quad x + x^s = (z^d)^\beta \cdot l_x \quad \text{and } \beta \in (0,1),$$

$$l_x = 1,$$

$$Y^s = g_y \cdot L_y - b, \quad g \in (0,1) \text{ and } b \in (0,1),$$

$$L_y = 1,$$

$$x^s = h \cdot y^s,$$

$$y + y^s = N \cdot Y^s,$$

$$k_y \cdot p_y \cdot y^s = p_z \cdot z^d + w \cdot N \cdot L_y,$$

where  $g_y$  is again the transaction efficiency coefficient for labor hired to produce good y. Moreover, N is the number of workers hired by the employer to produce good y. In order to distinguish inter flow of goods from market trade flow, we use capitalized

decision variables to denote internal flow. Hence,  $Y^s$  is internal transfer of good  $y$  produced by an employee to the employer and  $y^s$  is the amount of good  $y$  sold by the firm.  $h$  is the bundling ratio between the bundled good  $y$  and the final good  $x$  which is for sale. Here, we assume  $h = e \cdot \frac{p_y}{w}$ . This implies that an individual selling  $y$ , buying

labor, and bundling  $x$  with  $y$ , must choose the bundling ratio  $h = \frac{x}{y}$  according to  $\frac{p_y}{w}$ .

For a small market value of  $\frac{p_y}{w}$ , she must give away a small amount of  $y$  for each unit  $x$  sold. Otherwise, a small value of  $\frac{p_y}{w}$  may not be enough to cover the production cost of  $x$  which is bundled to the sale of good  $y$ . Here,  $e$  is as given to the owner of the firm, while later is based on the Yao Theorem (see Yang 2001, chapter 6, p.156), we can rule out the optimal bundling ratio of good  $y$  and  $x$  in this structure. In addition,  $l_x$  is the decision variable to the employer, while  $L_y$  is as given because it is bought from the labor market.  $u_{FTB1}$  is the utility for an individual in configuration  $(x/l_yz)$ , and she is the employer in this structure FT<sub>B</sub>.

The solution to the decision problem yields a demand function for labor and good  $z$ , a supply function of good  $x$ , and indirect utility function for configuration  $(x/l_yz)$ .

Similarly, an employee choosing configuration  $(l_y/y(x))$  has the following decision problem,

$$(A4.1.6c) \quad \text{Max: } u_{FTB2} = (x^d)^\alpha \cdot (y^d)^{1-\alpha} ,$$

subject to the following constraints,

$$(A4.1.6d) \quad \begin{aligned} x^d &= h \cdot y^d , \\ L_y &= 1 , \quad \text{and} \\ w \cdot L_y &= p_y \cdot y^d . \end{aligned}$$

The solution of this problem yields demand for goods  $x$  and  $y$ , supply of labor, and an indirect utility function for configuration  $(l_y/y(x))$ .

An individual choosing configuration  $(z/y(x))$  has the following decision problem:

$$(A4.1.6e) \quad \text{Max: } u_{FTB3} = (x^d)^\alpha \cdot (y^d)^{1-\alpha} ,$$

subject to the production function, endowment constraint, and budget constraint:

$$(A4.1.6f) \quad \begin{aligned} z^s &= l_z - b, \\ l_z &= 1, \\ x^d &= h \cdot y^d, \text{ and} \\ k_z p_z z^s &= p_y y^d. \end{aligned}$$

The solution to this problem yields demand for goods x and y, supply of good z, and an indirect utility function for configuration (z/y(x)).

The utility equalization conditions across three configurations yield the corner equilibrium relative prices of goods x and z and labor.

$$(A4.1.6g) \quad \frac{w}{p_z} = k_z(1-b), \text{ and}$$

$$\frac{w}{p_y} = \left[ \frac{k_y(g_y - b) \cdot k_z \cdot \beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right]^{\frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta) \cdot [(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta]^{(1-\alpha) + \frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}}.$$

Based on the Yao Theorem, maximizing utility with respect to  $e$ , yields the optimal value of  $e^*$  in this structure:

$$(A4.1.6h) \quad e^* = \frac{k_y(1-\alpha)^2 \cdot (g_y - b)}{(1-\alpha) \cdot (1-\beta) + \alpha\beta}.$$

The two independent market clearing conditions for goods x and z (the other market clearing condition is not independent due to Walras' law) yield the corner equilibrium relative numbers of specialists producing goods x, y, and z.

(A4.1.6i)

$$\frac{M_x}{M_z} = \left\{ \left[ \frac{(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right]^{(1-\alpha) + \frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta) \cdot [(g_y - b) \cdot \beta]^{\frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha} \cdot k_y^{\frac{1-2\beta}{1-\beta}}} \right\}^{\frac{1}{1-\beta}} \cdot \left( \frac{k_z}{1-b} \right)^{\frac{\beta}{1-\beta}},$$

and

$$\frac{M_y}{M_z} = \left[ \frac{(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right] \cdot \frac{(1-\alpha + \alpha\beta)}{\beta} \cdot \left[ \frac{k_y(g_y - b) \cdot k_z \cdot \beta}{(1-\alpha) \cdot (1-\beta) + \alpha\beta} \right]^{\frac{\beta}{1-\beta}} \cdot \frac{\alpha^\alpha \cdot (1-\beta) \cdot [(1-\alpha) \cdot (\alpha - \beta) + \alpha\beta]^{(1-\alpha) + \frac{\beta}{1-\beta}}}{(1-\alpha)^{1-\alpha}},$$

where  $M_x$  is the number of x specialist-employers choosing (x/l<sub>y</sub>z),  $M_z$  is the number of specialist producers of good z choosing (z/y(x)), and  $M_y$  is the number of employees choosing (l<sub>y</sub>/y(x)). The relative numbers of specialists, together with the



population size identity  $M_x + M_z + M_y = M$ , yield the corner equilibrium numbers of different specialists. Plugging relative prices into an indirect utility function of any of the three configurations yields the per capita real income in this structure:

(A4.1.6j)

$$u_{FTB} = \alpha^\alpha \cdot (1 - \beta) \cdot (1 - \alpha)^{1-\alpha} \cdot (k_z \cdot \beta)^{\frac{\beta}{1-\beta}} \cdot \left\{ \frac{k_y (g_y - b) \cdot [(1 - \alpha) \cdot (\alpha - \beta) + \alpha\beta]}{(1 - \alpha) \cdot (1 - \beta) + \alpha\beta} \right\}^{(1-\alpha) \cdot \frac{\beta}{1-\beta}}$$

In Structure FT<sub>B</sub>, a firm produces both good x and y, while selling y with good x bundled. The percentage  $h$  of good x and y is dependent on the relative price of good y and labor, and the wage rate  $w$  of labor hired to produce good y, and  $e$ . Note that obtaining good x is bundled through the purchase of good y, therefore we need not take the transaction costs of good x into account separately from good y. In other words, we suppose there is no extra transaction costs to obtain good x when good x is bundled with good y.

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