Discordant Notes on Raphael's Slate

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In the Vatican there is a room that contains works of art; and the room, too, is a work of art, but it is an art object of a very different kind. There are abstract geometric and arithmetic relationships among the frescoes and other decorations in this room; those interlocking interrelationships collectively constitute a kind of 'abstract object' that can be grasped more fully by the intellect than by merely passive vision alone. This abstract object has its own mode of existence and its own distinctive species of aesthetic value.

This room is commonly called the *Stanza della Segnatura*, 'the Room of the Signature'. It contains four large frescoes, one on each of the four walls, with bas-relief and other decorations below and smaller frescoes and decorations above on the ceiling and in the upper corners of the room. These frescoes were painted by Raphael and his assistants between the years of 1508 and 1524. There is a lot here to evaluate and appreciate visually, emotionally and intellectually.

It is hard work verifying historical speculations about an abstract object like this one. Opinions should rest upon evidence, not imagination. Nevertheless, historical imagination is manifestly invited, indeed demanded, by the images on the walls and ceiling of this room. Even in historical enquiries there is nothing wrong with exercising the imagination − as long as guessing is followed by testing against documented evidence.

Imagine, then, that you are seated on a stool in a slight recess part-way along the South Wall of the room, with your back to a window; see Figure I. This window has shutters that are almost always closed to protect the contents of the room from weather and sun damage; but imagine that today the shutters have been opened and the early morning light is coming over your right shoulder and falling onto the pages of a book that you are reading. Imagine that the book you are reading is one of Plato's dialogues, the *Timaeus*.

This is a book that was widely available all over Europe, all the way through the Middle Ages, and it became especially influential in the Renaissance, around the time of Raphael. The *Stanza della Segnatura* was probably used as the Pope's library or reading room, and a library like this was very likely to contain a copy of Plato's *Timaeus*. Raphael painted a celebrated portrait of the Pope's Librarian, Inghirami; and Inghirami was the kind of person who would have been likely to read Plato's *Timaeus*, and to read it again and again; see Hall (1997), both Hall's Introduction and the chapter by Ingrid Rowland in Hall (1997, pp. 131-70). If Inghirami were to have read the *Timaeus* in this *Stanza*, a sensible place to sit would have been in front of the window in the South wall.

This dialogue opens with four characters, Socrates, Timaeus, Critias and Hermocrates who are assembling for a pre-arranged meeting. The first words in this dialogue are:

> *Socrates*: One, two, three; but where, my dear Timaeus, is the fourth of those who were yesterday my guests and are to be my entertainers today?
>
> *Timaeus*: He has been taken ill, Socrates; for he would not willingly have been absent from this gathering.

The very first words in this dialogue refer to *numbers*; and that is a foretaste of what is to come. This is not an easy book to read and, after a time, your attention begins to wander. You look up and cast your eye over the fresco on the East Wall, the one that is generally called

The School of Athens. One of the things you can see is an image that (you have been told) unambiguously represents Socrates. Evidently it is known that this image represents Socrates primarily because he is 'snub-nosed'. And Socrates appears to be counting on his fingers as he addresses four other figures. Socrates is #49 in Figure II; and see also Figure III.

In this fresco, one of the four people who are being addressed by Socrates is #45 in Figure 2. This is the only person in *The School of Athens* who is dressed as a warrior rather than a scholar. For that reason, Raphael had ample reason to anticipate that Alexander the Great might come to mind among some viewers of this fresco; and, indeed, over the years some commentators have offered that identification. But Alexander the Great was a student of Aristotle, not of Socrates.

Raphael also had reason to expect that some who view this painting would know that the favourite student of Socrates was Alcibiades. Consequently, and unsurprisingly, some commentators have said that #45 in Figure II is Alcibiades. But it is worth also considering the possibility that Alcibiades might alternatively be identified as #48. Famously, Socrates tried to teach virtue to Alcibiades, whom he loved; but this pupil became an incorrigible rogue, fighting at one stage for the Athenians, at another time against them, making many enemies and ending his days in exile − and that could be taken as an apt reason why Raphael represents the lovely young man, #48, as evidently not paying attention to what Socrates is saying.

In memory-games like these there does not always need to be just one right answer. Raphael could reasonably have expected that almost anyone who made any guess at all about the identity of #49 would guess that it was Socrates. But the clues he left were far too scanty to justify any confidence that well-informed commentators would reliably guess the identities of the others that Socrates is addressing: 45, 46, 47 and 48.

Nevertheless, #49 does clearly represent Socrates. And he is clearly counting on his fingers. And Raphael had good reason to anticipate that, when viewing this fresco, on seeing an image of Socrates counting on his fingers, at least some of his friends would have been quite likely to be reminded of the *Timaeus*. Hermocrates was one of the ones that Socrates was counting when he said, 'One, two, three …'. He was also a famous general and images of him have survived from ancient times, representing him as wearing a Greek military costume − with a helmet very like the one in Raphael's image of #45 in Figure II. Consequently, there are reasons to wonder whether #45 is Hermocrates and #48 is Alcibiades. And that would imply that #46 and #47 are Critias and Timaeus.

Imagine now that, while still seated by the South window in the *Stanza della Segnatura*, you turn your eyes away from the fresco on the East wall and continue your reading of the *Timaeus*. A little further on in the text, however, it becomes especially hard to sustain your concentration when you reach certain passages that describe, in detail, a kind of 'recipe' that is said to have guided a 'craftsman', called the Demiurge', during the creation of the material world.

In Plato's account of the creation of the material world, 'portions' are said to have been dished out in quantities measured by the numbers 1, 2, 3, 4, 9, 8, 27. And then further portions are placed in the 'double and triple intervals', and these intervening portions are measured in the ratios of 1:2, 2;3, 3;4. 8:9 and 256:243. It is very tempting to skim very lightly over this passage without trying to understand the details. (Why are ''9' and '8' listed in the 'wrong order'? Where did '256' and '243' come from?)

Most people do not expect to be asked to think, explicitly, about numerical ratios like (256:243) when reading poetry or novels, or when listening to music, or when surveying a tapestry or fresco − or, in general, when appreciating a work of art. Art is one thing; science,

crafts, technology and finance are different. Nevertheless, it is well to remember that in Raphael's time the Arts were not yet as far removed from the Sciences as they have come to be centuries later, in the wake of the Romantic alienation of 'feeling' from 'calculation'.

Consequently, imagine that you try hard to understand this ancient text − as hundreds of others have done before you over the past twenty-four centuries − in the first instance simply because you have been told that this book is very important. As you look up, for a second time, from your book, you recall that one of the things that has in fact assured you that this book is very important is Raphael's iconic image of *The School of Athens*.

In *The School of Athens,* Raphael prominently features a painted image of the very book you are reading. This image is situated very close to the visual centre, the perspectival 'vanishing point', of the entire fresco. The identification of many of the figures in this fresco is left thoroughly obscure but, as scholars regularly say, we can know for certain − just from the image itself − that #1 in Figure II unambiguously represents Plato. We can know this because the book he is carrying visibly bears the letters 'TIMEO', and that is the Italian title for Plato's *Timaeus*; see Figure IV. Raphael's Italian contemporaries did not even need to know Latin, let alone Greek, to understand the reference.

On the ceiling above *The School of Athens* is circular area, a tondo, and this contains a kind of 'title' for the fresco on the wall below it: 'COGNITIO CAUSARUM', 'the knowledge of causes'. This 'knowledge of causes' was something that evolved through the work of later intellectual giants like Galileo, and eventually matured into the modern mathematical sciences.

One of the first steps in these Italian Renaissance advances in scientific understanding lay in the geometry behind the law of perspective. And the geometry of the laws of perspective is very prominently visible in *The School of Athens*. Another of the spectacular early

Renaissance steps in scientific explanation included the rediscovery of the ancient Pythagorean theory of the frequency-ratios for the natural harmonics of musical notes. And this scientific theory, too, is prominently referenced in Raphael's fresco.

Imagine now that, as you sit in this room, a tour guide sits down beside you and tells you some of the lesser-known facts about the early Renaissance background behind this fresco. You are reminded that Raphael's Plato, as he advances towards us, is conversing with an image that (uncontroversially) represents Aristotle. For this reason, Raphael's depiction of Plato and Aristotle might well have recalled, to some of his contemporaries, historical memories of a relatively recent confrontation between Platonists and Aristotelians, which occurred during visit to Italy of a delegation from the Eastern Church in Byzantium.

This delegation from the Eastern Church came to Italy for a Council of Union. This Council met first in Ferrara in 1438 and then in Florence in 1439. The aim was to heal the aching schism that was developing between Eastern and Western Christendom.

Included in the delegation from the East there was an exciting orator called George Gemistos. And he was an enthusiastic Platonist. Like the image of Plato in Raphael's fresco, he was an old man, over eighty years of age. He had come to Italy from the East. He came bearing Greek texts. And he was energetically arguing against Aristotle and preaching Platonism; see Woodhouse (1986). After the Council of Union, Gemistos returned to Greece and continued to live, for the rest of his very long life, in the Greek city of Mistra, very near the ancient site of Sparta.

After his time in Florence in 1349, Gemistos began to identify himself under the pseudonym 'Plethon'. Many contemporaries suspected that this pseudonym 'insinuated a link to the soul of Plato'; see Woodhouse (1986, p. 187). One of the very influential thinkers of the Italian Renaissance was Marcillio Ficino; and he said that Gemistos took this pseudonym

to imply that he was a ‘second Plato’. Ficino also credits Gemistos with having inspired Cosimo de Medici to establish a Platonic Academy in Florence; see Woodhouse (1986, p. 156) and Ficino (1537, vol. ii). A former student of Gemistos, Cardinal Bessarion, founded a similar academy in Rome.

In Italy during the decades that followed the visit of Gemistos Plethon, there was a strikingly increased interest in Plato. Increased enthusiasm for Plato was naturally accompanied by an increased enthusiasm for mathematics − and, arguably, this may have been a factor in the emergence of the modern sciences in Renaissance Italy. One of the things Gemistos brought to Italy was an improved knowledge of ancient Greek geographers, like Strabo, and an improved understanding of the spherical geometry of a round Earth and this was indirectly instrumental in sparking the voyage of Columbus across the Atlantic Ocean in 1492; see Woodhouse (1986, p. 186). A keen interest in numbers played an important role in the emergence of the sciences in the Renaissance; and numbers play an important part in Plato’s *Timaeus*.

Following the Council of Union in 1438-9, the widespread rise of interest in Platonism in Italy was also accompanied by a passionate backlash from the Aristotelians who, in the Western Church, worked under the guidance of St. Thomas Aquinas. One strident enemy of Gemistos was another Greek, George Trapezuntius − who said he feared Gemistos as a ‘second Muhammad’; see Woodhouse (1986, p. 366).

The leading defender of Plethon and Plato against the attacks of Trapezuntius was Cardinal Bessarion, who had been a student of Gemistos Plethon. Bessarion wrote a major work, *In Calumniatorem Platonis*, which was one of the first thorough and sympathetic Latin expositions of Platonism to appear in the Western Church. Bessarion’s text was widely distributed; Copernicus is known to have owned a copy; see Woodhouse (1986, p. 376). In

this defence of Plato, Bessarion tones down the attacks that Gemistos launched against Aristotle and looks instead for common ground between Plato and Aristotle.

Mirroring Bessarion's spirit of reconciliation, Raphael's *School of Athens* suggests a harmony, rather than any irreconcilable conflict, between Plato and Aristotle. Aristotle is depicted as looking over his shoulder and back to Plato (who is pointing upwards). And Aristotle is gesturing downwards (thereby plausibly indicating the moral, political, legal and practical applications of Platonic abstractions). Raphael's Plato is − like Gemistos Plethon and Cardinal Bessarion − a Greek, a wise man, coming from the East and bearing a gift. And the gift he is bearing is not gold, frankincense or myrrh but his *Timaeus*.

Raphael could reasonably have trusted that many who saw this fresco would know by heart the memorable line from Virgil's *Aeneid*, in Latin: *Timeo Danaos et dona ferentes*, 'I fear Greeks, even those bearing gifts'. This line comes from the story of the Trojan War and the Wooden Horse. In Latin, 'TIMEO' means 'I FEAR'. Christian theologians had plenty of reason to fear that Plato's *Timaeus* might very well prove to be a Trojan Horse. It was perfectly reasonable for them to suspect that there might be something to fear, something heretical, hidden within the pages of the *Timaeus*.

The *Timaeus* is the primary European source for the Pythagorean doctrine of reincarnation, which was regarded by Christians as a heresy. Furthermore, the *Timaeus* also says that the material world was not created directly by the Father of All but by another, lesser, intelligent designer, a 'craftsman' or 'artist', called the Demiurge. And that doctrine too was a heresy. Some heretics were tempted to identify the 'Demiurge' with Satan.

Doctrines like these are close cousins of notorious heresies that were preached, centuries before Raphael, by the Cathars and related sects in France and Spain and by the Bogomils in the Balkans. Those heretics had been killed in very large numbers during some of the earliest

of the Crusades. Sometimes, in response to persecution, these heretics formed clandestine brotherhoods that met in secret; and the Church formed the Inquisition to root them out. It was at this time that both the Tarot and the Romany decks of cards ('the Devil's picture books') first appeared, almost simultaneously, all the way across Europe from Spain to the Balkans. Heresies like these are not to be sneezed at; yet here they are, enclosed between the covers of a book that is visually represented at the focus of one of the most famous images in Christendom, right at the heart of the Western Church in Rome.

Raphael's image of Plato also bears a much-remarked resemblance to a drawing that many commentators believe to be a self-portrait of Leonardo da Vinci in old age. Frescoes do not literally 'say' anything. And we can never be sure what the painter (or painters) had in mind, or consciously 'intended' to communicate, when they painted these images. Nevertheless, no matter what was in the mind of the painter, the image itself conveys messages. And there is a sense in which Raphael's fresco not only 'says' that Plato was the most important philosopher in Ancient Athens, but also pays homage to Leonardo by 'saying', in effect, that he was the most important artist in Renaissance Italy.

Gemistos Plethon died (probably) in about 1452; and Leonardo da Vinci was born in 1452. Let me be candid: I do not believe in reincarnation. But it is well to be mindful that some of the 'Platonists' around Raphael might well have taken reincarnation seriously, at least as a possibility. Anyone who did might well be struck by the fact that Raphael represents Plato almost as if he were being reincarnated as Leonardo.

Leonardo was reputed to have been a vegetarian; and this is something he had in common with many of the heretics in centuries past who had believed in reincarnation. Both historical memories and logical links between reincarnation, vegetarianism and Pythagoreanism,

survived in collective memory from the times of the Cathar and Bogomil heresies through to Raphael's time and well beyond.

The historical link between Pythagoras and reincarnation is mentioned by Shakespeare in *As You Like It* 3.2: 'I was never so berhymed since Pythagoras' time that I was an Irish rat, which I can hardly remember'. The logical link between reincarnation and vegetarianism is also clearly registered by Shakespeare in a comic portrayal of the methods of torture that had been employed by the Inquisition during the suppression of the Cathar and Bogomil heresies. In *Twelfth Night* 4.2 we find the lines: 'Thou shalt hold th' opinion of Pythagoras ere I will allow of thy wits, and fear to kill a woodcock lest thou dispossess the soul of thy grandam.' The setting for *Twelfth Night* was 'Illyria', which was the Greek name for a region within the Balkans − where the Bogomil heresies had been rife in earlier times. In a related vein, Shakespeare's *Love's Labours Lost* opens with the depiction of a Prince who gathers around him a small group of scholars − who vow chastity for a three year interval during which they will tirelessly seek 'forbidden knowledge'. This play is set in Navarre, which had been a stronghold of the Cathar heresies. Since historical memories of the Cathar and Bogomil heresies were evidently still alive in Shakespeare's time, they must surely have been even closer to the forefront of many people's minds at the time Raphael was painting *The School of Athens*.

Gemistos Plethon was evidently in fact guilty of holding this heretical belief in reincarnation. Late in his long career he wrote a *Summary of the Doctrines of Zoroaster and Plato*, which laid down a kind of 'creed'. In this creed he concisely listed twelve central doctrines. The tenth doctrine in this short list is that the soul 'is always attached to a mortal body, being sent by the gods from one body to another in the interests of the harmony and cohesion of the universe.'; see Woodhouse (1986, p. 319). Cardinal Bessarion, a student of

Gemistos, did not publicly endorse this heresy. Nevertheless, on receiving news of the death of Gemistos, Cardinal Bessarion wrote to the two surviving sons of Gemistos:

> So if one were to accept the doctrines of the Pythagoreans and Plato about the infinite ascend and descent of souls, I should not hesitate eve to add that the soul of Plato, having to obey the irrefragable decrees of Adrasteia [Fate] and to discharge the obligatory cycle, had come down to earth and assumed the frame and life of Gemistos.
>
> Woodhouse (1986, p. 13); Mohler iii (1923-42, pp. 468-9).

Bessarion was not a marginal or insignificant personage in Christendom. On two occasions he nearly became Pope; and he did become Patriarch of Constantinople. Bessarion represented a harmony, not an antagonism, between Plato and Aristotle. And this also represented a harmony, not an antagonism, between the Eastern and Western branches of Christianity. Raphael's visual image of a harmony between Plato and Aristotle is an expression of the peace-making spirit of Bessarion. And Raphael's Plato is depicted almost as if he has been reincarnated as Gemistos − or Leonardo − or both.

Consequently, we have here − at the focal point of Raphael's famous fresco in the Vatican − an image of a Greek book that expounds the heretical doctrine of reincarnation, held in the hand of an image that represents Plato but also was widely taken to 'co-represent' Leonardo da Vinci, who was reputed to have been a vegetarian. In the iconography of this fresco, Raphael is manifestly playing with fire.

Nevertheless, commentators never suggest that Raphael's fresco might be representing Leonardo as *literally* the reincarnation of Plato. It is one thing to speak of Renaissance Rome as a kind of 'rebirth' or 'reincarnation' of the glories of Ancient Athens. It is another thing to take such talk *literally*.

Anyone reading Plato's *Timaeus* attentively should be alive to the possibilities of taking many iconic stories as *figures of speech*. Plato's *Timaeus* recounts the story of Phaethon, who drove the chariot of the Sun and fell to earth, thereby creating the Sahara Desert. But Plato's text tells us that this story 'really' signifies the passage of a comet that periodically approaches too close to the Earth, destroying human civilisation and leaving only a small seed from which the entire historical cycle will begin all over again. If we are thinking about the iconography of Raphael's image of Plato, holding his *Timaeus*, then we should be alive to the possibility of taking Raphael's imagery as analogous to a figure of speech. His *School of Athens* represents the ancient Greek thinkers *as if* they were being reborn in Italy. Their clothes are Roman not Greek. The title on the book Plato is carrying is Italian not Greek. And the fantastic architecture in which they are standing is Roman or Italian, not Greek. The fresco is a metaphor for the Renaissance as a 'rebirth' of the spirit of the ancient Greeks.

There are many abstract arithmetical and geometrical patterns that can be observed in Raphael's frescoes. Some of these have been clearly described by Raphael scholars and have been invested with aesthetic and iconographic significance, analogous to taking them as figures of speech. Among these, one that is of special interest for present purposes was displayed by Marcia Hall (1997, p. 2) in a diagram that *numbers* the collection of 'great men of the ancient world' that are depicted in *The School of Athens*; see Figure II.

Diagrams of the kind Hall displays − numbering the various figures in a visual image − are used relatively frequently in art history, where their primary function is to facilitate concise and unambiguous textual identification and discussion of the various historical personages that are represented within complicated visual works of art and the various contemporaries of the artist who might have been used as 'models' for these images.

Nevertheless, in addition to its role in facilitating discussion of individual human figures in each fresco, Hall's diagram also reveals an interesting structural feature of *The School of Athens*. This diagram also appears to demonstrate that *The School of Athens* contains 56 human figures all together. And it demonstrates that they appear to be arranged in an exact, or nearly exact, numerical symmetry: 28 in a chain descending on the right-hand side and 28 in a second chain ascending back up the left-hand side.

This cyclic, numerical structure on the East Wall, in *The School of Athens*, is mirrored on the North Wall, in the fresco called the *Parnassus*. In the *Parnassus* there are 14 figures in a chain that begins with Apollo at the visual centre of the image and descends on the right-hand side of the fresco; and then there are 14 figures in another chain, which begins with Sappho at the bottom left, ascends on the left-hand side, and returns to Apollo. See Figure V.

It is aesthetically interesting that a *cyclic* pattern can be detected in these two frescoes on the East and North Walls, *The School of Athens* and *Parnassus*. It is worth asking whether the same cyclic pattern can be seen on the other two walls, the *Disputa* and *Jurisprudence*. One commentator says of the *Disputa* that

> The arrangement of this scene takes the form of a hemicycle, more pronounced than that employed in *The School of Athens*, by virtue of its repetition on the two levels of heavenly and earthy existence.
>
> Talvacchia (2007, p. 90).

Nevertheless, the 'hemicycle' to be found in the *Disputa* is not quite the same as the ones in *The School of Athens* and *Parnassus*. The *Disputa* can readily be seen as representing the Christian *linear* image of time, as having begun in the recent past and destined to end in the immanent future. There is a linear hierarchy, beginning with the congregation of the Christian

Church, standing on the Earth, and progressing up to the Heavens. There is no implication of anything like an 'Eternal Return'.

By contrast, there is a very credible implication of something analogous to an 'Eternal Return' in *The School of Athens* and *Parnassus*. And that aptly echoes the idea, clearly expressed in Plato's *Timaeus*, that civilisation is periodically destroyed by a passing comet, and that then history begins all over again in a new cycle. This 'cyclic' time is aptly embodied on the 'Pagan Walls' on the East and North, and not on the 'Christian Walls' to the South and West.

Plato's *Timaeus* emphasizes cyclic patterns both astronomically, in the Heavens, and in human history. But there is also an implicit allusion to a less obvious cyclic pattern – in Pythagorean music theory. This musical pattern is called the 'cycle of fifths'. This 'cycle' serves as a practical tuning system for musicians, and it is grounded in the fundamental musical consonances of octaves, fifths and fourths. And the mathematics behind those musical consonances is one of the important subject matters that is influentially summarized in Plato's *Timaeus*; and this musical theory was influentially preserved through the Middle Ages in the works of Boethius.

Imagine now that, while still seated in the *Stanza della Segnatura* reading the *Timaeus,* you were look up for a third time from Plato's text to survey the frescoes that surround you. This time you might well wonder whether Raphael's frescoes might visually embody any of the specifically musical patterns that Plato has described in his *Timaeus*. And you find that a very detailed reference to music theory is indeed visible in *The School of Athens*.

The Pythagorean theory of 'harmony' is succinctly and cleverly spelled out in an image of a slate that is depicted at the bottom left of *The School of Athens*; see Figure VI. Let us call this image 'Raphael's *Slate*'. This *Slate* is being held in place by someone who looks like an

angel (#32 in Figure II), and who is propping up this *Slate* in the line of sight of someone that is generally agreed to represent Pythagoras (#33). An early scholarly recognition of the importance of Raphael's *Slate* was registered by Emil Naumann (1879); see also Moyer (1992).

The heading that is written on this *Slate* is written in Greek lettering and it can be transliterated as 'EPOGDOÔN'. Even if you read Greek, this is a word you do not often meet when reading Homer or Aesop's Fables or other surviving texts from the ancient world − unsurprisingly, since it means 'nine-eighths' and that is not likely to be a common topic of conversation, or of any exciting narrative. Thus, the credibly 'intended audience' for this element in the fresco is extremely limited; but that limited audience might well have included some of the Pope's close advisors, such as his Librarian, Inghirami.

The word *epogdoôn* occurs in Plato's *Timaeus* 36*a* as part of the description of the mathematical structure of the 'World Soul'. This ratio of nine-eighths is the musical ratio of the whole tone interval, which is the backbone of the 'diatonic' or *sol-fa* division of the octave into seven distinct notes.

Immediately below the heading on Raphael's *Slate*, 'EPOGDOÔN', there is a series of numbers that prominently features the numbers nine and eight, written 'VIII' and 'VIIII':

VI VIII VIIII XII

Below these numbers, there is another pattern that is called the 'tetraktys', which was a standard symbol for Pythagorean or pseudo-Pythagorean 'brotherhoods':

I

I I

I I I

I I I I

X

The ‘X’ is the Roman numeral for ten, signifying that 1+2+3+4 = 10. This pattern echoes the opening words of Plato’s *Timaeus*, ‘One, two, three, …’. These are also the first four of the numbers 1, 2, 3, 4, 9, 8, 27, that are cited, later in the dialogue, in the description of the World Soul.

These specific numbers Raphael has represented, VI, VIII, VIIII, XII, are not explicitly mentioned in the *Timaeus*; but the Platonic significance of this group of numbers has been passed down through a long line of commentators on the *Timaeus*. Figure VII is an image of an explanatory diagram in the margins of a Latin translation of Plato’s *Timaeus*. This marginal image stands alongside the passage in the *Timaeus* 36*a* that describes the ratios that constitute the World Soul. This Latin translation derives from Calcidius (about 100 CE); and the manuscript copy in Figure VII dates from about 1100 CE. Raphael’s numbers are prominently featured at the top portion of this marginal image, on the left-hand side:

vi

viii *ix*

ix *xii*

xii *xviii*

… …

There was also a secondary source that was available to Raphael: a book about music theory by Franchino Gaffurio (1492), see Figure VIII. This work was dedicated to Ludovico Sforza, the Duke of Milan, who was also the patron of Leonardo da Vinci. The curved lines on Raphael's *Slate*, linking octaves, fifths and fourths, closely resemble curved lines that are similarly used in several of Gaffurio's diagrams.

The ratios 6, 8, 9, 12 cited on Raphael's *Slate* approximately (but not exactly) match the frequency ratios instantiated among the familiar '*sol-fa*' notes, ABCDEFG, as played by the white notes on a modern piano. Choose a starting note as the 'tonic', construct an ascending scale on that note, and you hear approximately what is called a 'mode'. The ancient Greek names were differently arranged, but the following is the way the modes were named in the Renaissance:

***Renaissance modes*:**

	6 8 9 12	
Dorian:	D EF GA BC D^{o}	
Phrygian:	E FG AB CD E^{o}	
Lydian:	F GA BC DE F^{o}	[anomaly: the semitone B-C is not in the ratio (8:9)]
Mixolydian:	G AB CD EF G^{o}	
Aeolian:	A BC DE FG A^{o}	
Locrian:	B CD EF GA B^{o}	[anomaly: the semitone E-F is not in the ratio (8:9)]
Ionian:	C DE FG AB C^{o}	
	… and now we return to the beginning again, like an Eternal Return.	

Here, 'D^{o}' is an abbreviation for 'an octave above the previous D', … , and so on.

The first of the two 'anomalies' that are noted above, the one found in the middle of the Lydian mode, arises because the semitone B-C falls right in the middle of the scale. And this logically motivates the very frequent introduction, within Medieval and Renaissance music,

of a precursor to the modern B-*flat*, thereby ensuring that the interval between B-*flat* and C is restored to the desired interval of a whole tone ratio (9:8).

The awkwardly-placed semitone E-F in the Locrian mode would, for similar reasons, motivate changing E to E-*flat* or F to F-*sharp*. But the Locrian mode was banished from Renaissance music because the 'mis-placed' F was a discordant interval above the tonic B.

This Pythagorean tuning system makes all the octaves, fifths and fourths very 'pure'. That is, the system minimizes interference beats in the natural harmonics. And in Renaissance Church music it is easy to experience this 'purity' as expressing a holy love of God. But this 'perfection' in the octaves, fifths and fourths does come at a cost. It entails the existence of serious discords in a handful of other intervals.

The moral to be drawn from this is that a handful of discords are essential in music. They are to be valued, and skilfully manipulated, but cannot be eliminated. Necessary evils face us in all walks of life. And the most central illustration of this inescapability of discords is found in the *epogdoôn*.

If two notes are separated by a Pythagorean whole tone interval (8:9), then if they are played loudly enough and sustained simultaneously for long enough, the resulting superposition of frequencies will create audible interference beats. The interference beats are even worse for the interval of a semitone. The same applies for intervals of a *seventh*, which *miss* the octave by either a semitone or whole tone interval. These interference beats are generally perceived as a kind of 'roughness', and they are associated with some degree of muscular tension in a hearer − sometimes even making the hearer *wince*.

Thus, Raphael's Slate concisely demonstrates that the attempt to create 'perfect harmonies' will inescapably also create the discordant frequency ratio of 'VIII' to 'VIIII', (8:9) the *epogdoôn*. In the Renaissance, this interval was experienced as a 'tension' that 'strains' to be

‘resolved’. Alternating ‘tensions’ and their ‘resolutions’ can be heard in standard cadences in Gregorian chants. These chants regularly close with the word ‘A-men’. Audible interference beats characteristically emerge in the natural harmonics of the notes that accompany the syllable ‘A- …’ − but then these interference beats almost completely vanish when we hear the notes accompanying the syllable ‘…-men’.

In the preamble to the creation story that is told in Plato’s *Timaeus*, the ratio of (8:9) is manifestly deployed with evident symbolic intent. An account is given of a conversation between Solon and Egyptian priests, during a visit of Solon to Egypt. The priests in Egypt are said to have told Solon that their civilisation was founded 8000 years ago − and also that an earlier civilisation in Athens had been founded 9000 years ago. The kind of ‘civilisation’ that is being described, for both Egypt and Athens, is one that possesses both a written language and monumental architecture. Neither the Bible nor Archaeology can credit the existence of any civilisation of that kind anywhere near as far back as 8000 or 9000 years before Solon. However, there is reason to suspect that the numbers may have been chosen here, in Plato’s text, not primarily for verisimilitude but because they echo the Pythagorean musical ratio of the whole tone.

With this musical background in place, we may look more closely at the numerical patterns that can be identified in Raphael’s frescoes in the *Stanza della Segnatura*:

There are (at least approximately) 56 figures in *The School of Athens* − Figure II.

There are (at least approximately) 63 figures in the *Disputa* − Figure IX.

56 = 7 times 8; and The ratio of 56 to 63 is equal to the ratio of 8 to 9. Hence, there is a whole tone musical ratio between the Pagans and the Christians − just like the ratio that Plato’s *Timaeus* placed between the Egyptians and the Greeks.

The *Disputa* is divided into three layers. At the top is God the Father, with three angels on his right and three more on his left, seven altogether. In the middle is God the Son, with seven figures on his left and seven on his right. Below the Son is a dove representing the Holy Spirit.

Presumably the Father and the Son should not be counted as two distinct figures − in addition to the Father − because the dominant Christian doctrine was that the Trinity of Father, Son and Holy Spirit constitutes 'three in one'. Consequently, we should count, altogether, 21 distinct figures hovering in the heavens, in the top portion of the fresco, sub-grouped as (7 + 14). And there is a harmonious (2:1) ratio of a musical octave between the 7 figures at the level of God the Father, and the 14 figures at the second level, with the Son.

Below the, Heavens, there appear to be 42, or at least approximately 42, salient figures on the Earth, judging by most relatively accurate reproductions of the *Disputa*. The chances that we have not overlooked a significant figure, or miscounted, is substantially reduced when we add the further evidence, that there is an echo of this number 42 in the opening words of the *New Testament,* in *The Gospel According to St Matthew*:

> 1. The book of the generation of Jesus Christ, the son of David, the son of Abraham.
>
> 2. Abraham begat Isaac; and Isaac begat Jacob; and …
>
> …
>
> 17. So all the generations from Abraham to David are fourteen generations; and from David until the carrying away into Babylon are fourteen generations; and from the carrying away into Babylon unto Christ are fourteen generations.
>
> *The Gospel According to St Matthew,* 1.1-17.

This same passage from the Bible is visibly on display in the *Disputa* immediately above the 42 figures in the fresco who are standing on the Earth. The four Gospels are held up before us, with the pages opened, by cherubs; and on the pages of the first of these Gospels we can read the opening words of *Matthew*, 1.1: … LIBER GENERATIONE IESU CHRISTI … . The Bible does not explicitly say so, but the three groups of fourteen, cited in the Gospel, do add up to 42 generations altogether.

Thus, the figures in the *Disputa* fall into groups numbering: 7, then 14, then 42. The *ratios* are therefore (7:14) = (1:2), a harmonious octave between the Father and the Son, and (14:42) = (1:3), an octave plus a perfect fifth between the Son and the Church. These two ratios are prominent in the World Soul of Plato's *Timaeus*. And they are also prominent on Raphael's *Slate*. Furthermore, in the objectively scientific terms of physics these ratios represent the first two of the natural harmonics of a musical note. These are paradigmatically harmonious frequency-ratios.

By contrast, the ratio between the Pagan world of *The School of Athens* and the Christian world of the *Disputa* is the *epogdoôn*, (8:9). Consequently, there is a whole tone 'discord' between the ancient Greek Pagans and the Renaissance Christians. But this is a discord that − the faithful will trust − is destined eventually to be resolved in the 'A-men' that will resound when Armageddon is followed by Christ's establishment of His Kingdom on Earth.

The existence of the numerical (8:9) ratio between *The School of Athens* and the *Disputa*, is also mirrored in an incarnation of that same ratio within the fresco of *Jurisprudence*, which lies between them on the South Wall (see Figure I).

Jurisprudence is broken into three parts. There is a panel on either side of the window, and an arched area above the window. The eastern panel is adjacent to *The School of Athens* and the western panel is adjacent to the *Disputa*. The eastern panel celebrates Justinian and the

establishment of Secular Law. The western panel celebrates the establishment of Ecclesiastic Law. The eastern panel includes 8 figures; and the western panel includes 9 figures. And there are 8 figures in the arched area above the window.

To back up these observational reports with an appeal to authority, Passavant (1879, p. 224) described this fresco. It is worth citing these observations by Passavant because they were recorded before various of the more recent conservation work had been completed. He counted 8 figures above the window and 8 on the panel to the left of the window. But, unaccountably, he also counted 8 in the panel to the right of the window. To support my count of 9, see Figure X.

The ratio of (8:9) is the Pythagorean 'ideal' frequency ratio for the musical whole tone interval. But it is worth being mindful of the fact that the frequency ratios between *actual* musical notes, as played by any actual musical instruments, will only ever achieve approximately the Pythagorean ideal ratio for the whole tone. And slight deviations from the Pythagorean ideal of a ratio of *precisely* (8:9) is not only inevitable, but also desirable. For one thing, there is expressive potential in dissonance. And for another thing, even if you do aim for a tuning system that maximizes perfect Pythagorean harmonies there is nevertheless still an inescapable necessity to allow some degree of deviation from the pure Pythagorean frequency-ratios. That is, there is a necessity to allow some degree of 'dissonance' in the Pythagorean sense that requires minimization of superposition 'beats' in the natural harmonics of two notes. It is mathematically impossible to minimize these 'beats' in the *fifths* without creating detectable 'beats' in other intervals: in the first place in the notorious *tritone* ('the devil's interval), and then also in the *thirds*. Any tuning system requires some degree of approximation, or compromise, at some point or other.

The necessity for compromise can be experienced by someone with a good ear who tries to tune a modern six-string guitar, with strings tuned to the notes: $E_oADGB^oE^o$ (using subscripts and superscripts to transpose a note down or up in octaves). The first four strings are traditionally tuned to successive intervals of a *fourth*. If these fourths are all tuned to minimize interference beats, then they will closely approximate the tuning described on Raphael's *Slate*. But the fifth string, B^o, is tuned to a *third* above the previous string, G; and the last string is tuned to a fourth above that. This 'sweetens' the triads GBD and CEG (it reduces superposition beats in the natural harmonics). But it draws the notes B and E out of the ambit of Pythagorean tuning. And that creates problems with the octaves, fifths and fourths.

Under this tuning system for the guitar, if all the strings are carefully tuned in such a way as to minimize audible interference beats, then someone with a good ear will be able to tell, by ear, that the first and the sixth strings, E_o and E^o, will *just miss* the harmonious interval of two octaves. And this creates a problem that is not easily fixed. Remedying the discord between the first and last string will inescapably damage the intermediate harmonies. Consequently, it is both a theoretical and a practical necessity that a compromise be reached, in which multiple (minimal) dissonances are distributed to locations where they will cause least offence. Of necessity, near enough must be good enough.

These practical problems in tuning an instrument were well portrayed by Shakespeare in *The Taming of the Shrew* 3.1:

> Hortensio: You'll leave his lecture when I am in tune?
>
> Lucentio: That will be never. Tune your instrument.
>
> …
>
> Hortensio: Madam, my instrument's in tune.

Bianca: Let's hear. O fie! The treble jars.

…

Hortensio: Madam, 'tis now in tune.

Bianca: All but the bass.

Poor Hortensio is a figure of fun. Nevertheless, given the familiar tuning system for a guitar − or a lute − it was *arithmetically impossible* for Hortensio to tune *all* the strings in a way that will minimize interference beats for *every* pair of strings.

Shakespeare was writing a century after Raphael; but the laws of arithmetic and the laws of natural harmonics in music are timeless. The same inescapable practical and theoretical musical problems of Pythagorean tuning were also clearly explained in a masterful book on organ-tuning, which was written by Arnolt Schlick at virtually the same time that Raphael was painting the *Slate* in his *School of Athens*. Schlick clearly explained that if you optimize perceived harmonies in the fourths and fifths, then the thirds will be perceived as 'too high, horrid, and hard'. His remedy for that is to improve the thirds at the cost of inflicting slight damage on some of the fifths: '… do not make it [the fifth] high enough, or completely pure, but hovering somewhat lower, as much as the ear can stand'; see Schlick (1511/1980, pp. 75-76).

Enlarging on the ubiquitous necessity for compromise, Plato's *Timaeus* alludes to several instructive Greek myths. These myths warn human artists and artisans that they should never aspire to the levels of 'mathematical perfection' that are personified in the gods. Thus, for instance, the *Timaeus* refers to the myth that Phaethon tried to drive the chariot of the Sun but fell to earth and perished. Near to this story of Phaethon, Plato also mentions Niobe, who boasted that because she had six sons and six daughters she was a better mother than Leto, who had only borne two, Artemis and Apollo. Consequently, with bow and arrows Artemis

killed all six of Niobe's daughters and Apollo killed all six of her sons. There is also a myth about a skilled weaver, Arachne, who boasted she could weave more skilfully than Athene, and she was turned into a spider. And there is a myth about a musician, Marsyas, who boasted that he could play better music than Apollo − and he was skinned alive. The story of Marsyas is prominently depicted in the North-West corner of the *Stanza della Segnatura*, in a triangular space, a spandrel, that lies between the *Parnassus* and the *Disputa*.

Cabalistic traditions advise that alphabetical and numerical 'codes' should always be 'smudged' a little by 'adding or subtracting one or two' as you complete the design. Relatedly, in many craft traditions in many cultures around the world, it has been considered good form to include at least one deliberate, minor imperfection in any work of art. Leonardo da Vinci pioneered a technique called *sfumato*, which created 'smoky' and deliberately imprecise boundaries to all the salient figures in an image.

It is not merely *ad hoc* wishful thinking to imagine that Raphael might have either deliberately or intuitively *approximated* − but might also have either deliberately or intuitively *deviated* from − the mathematical ratio of the *epogdoôn*. That ratio needs to be adjusted slightly if it is to fit perfectly against the salient observable ratios in Raphael's *Stanza*. This same ratio also needs to be adjusted slightly if it is to 'square the circle' with perfect accuracy. And it also needs to be adjusted slightly if we are to 'sweeten' the frequency ratio for musical thirds.

We can never know with certainty all the historical facts surrounding the frescoes in the *Stanza della Segnatura*. We can never know with certainty all the inner workings of Raphael's mind. We can never know whether all the mathematical patterns that can be observed in his frescoes were deliberate, or if they were intuitive, subliminal and unarticulated. We may never know whether Raphael was the one who invented these

patterns, or whether he was merely implementing arithmetical plans that had been imposed upon him by the Pope or the Pope's advisors. We may never know what, exactly, may have been going on in whatever artistic 'guilds' or esoteric 'brotherhoods' or Platonic 'reading groups' may have been operating privately behind the scenes in Rome between 1500 and 1520.

Nevertheless, we can establish without any significant room for doubt that the visual iconography in the *Stanza della Segnatura* does align, within a credible 'margin of error', against central features of the mathematical music theory that is described in Plato's *Timaeus* and teased out on Raphael's *Slate*. If Raphael's *Stanza della Segnatura* is indeed 'the Room of the Signature', then that 'Signature' is the musically harmonious 'World Soul' of Plato's *Timaeus*.

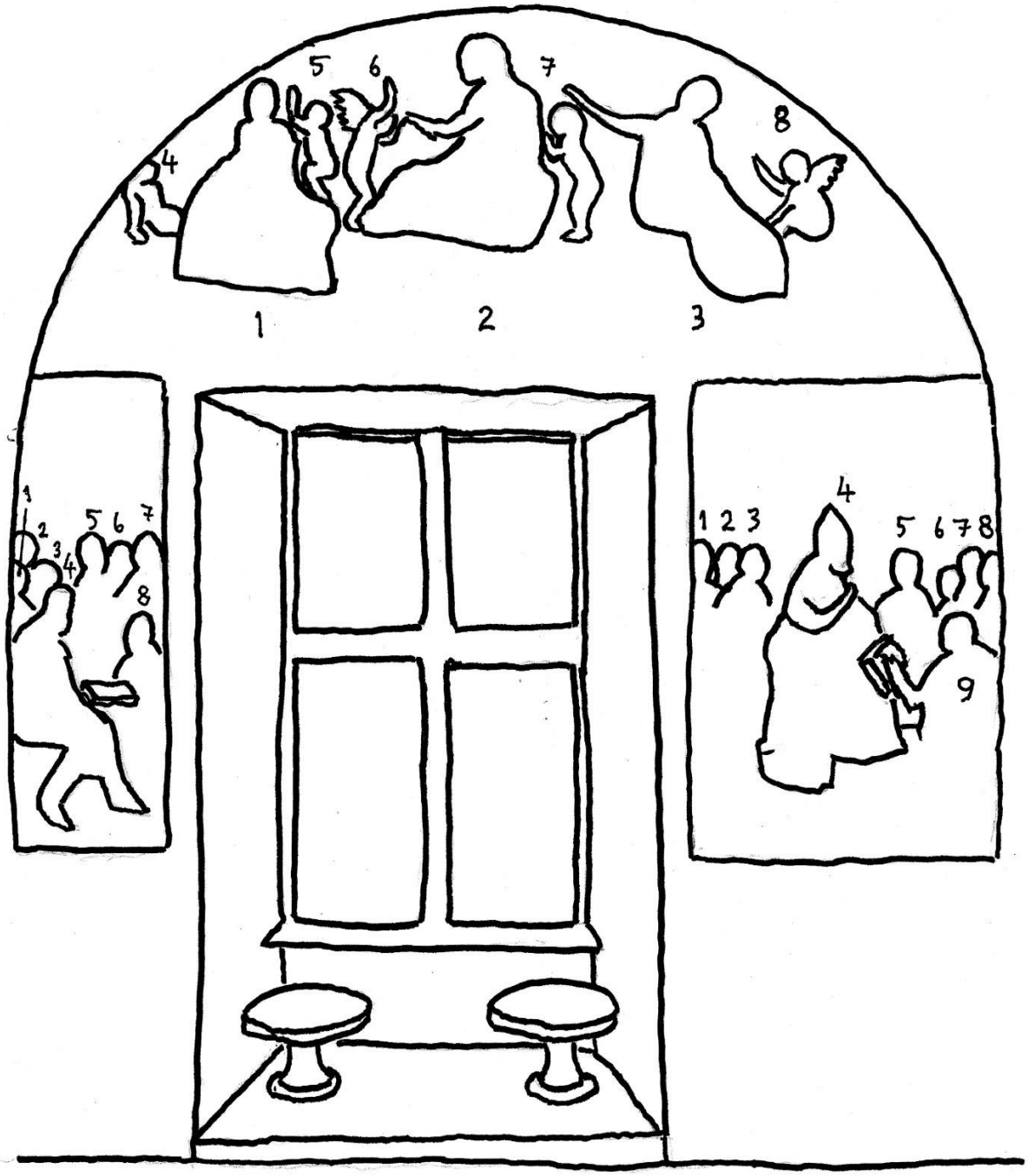

FIGURE I

South Wall: *Jurisprudence*

[traced from reproduction by Bruno del Priore, Vatican Polyglot Press, p. 137]

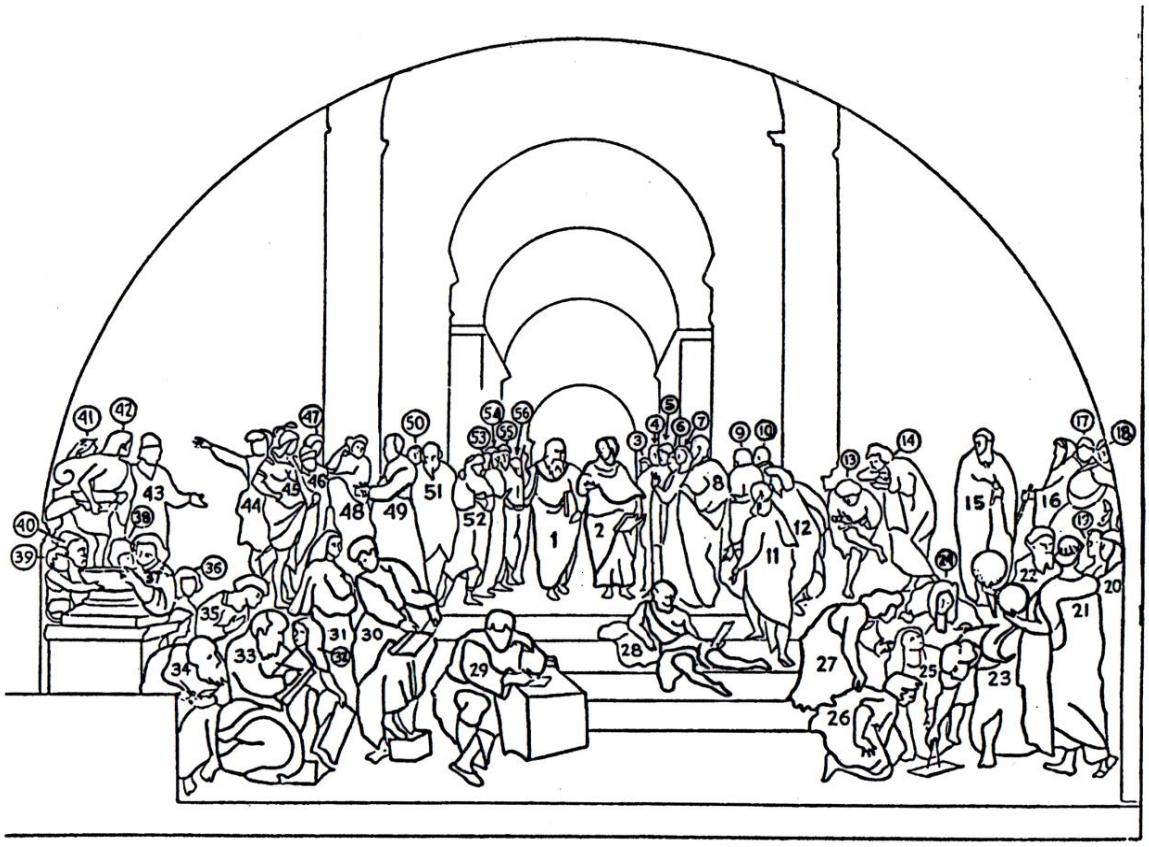

Figure 1. Diagram of *The School of Athens*.

FIGURE II

The School of Athens

['Diagram' from Hall (1997, p. 2)]

FIGURE III

Detail from The School of Athens

[photo credit, Bruno del Priore, Vatican Polyglot Press]

FIGURE IV

Plato's *Timaeus*

[detail from The School of Athens]

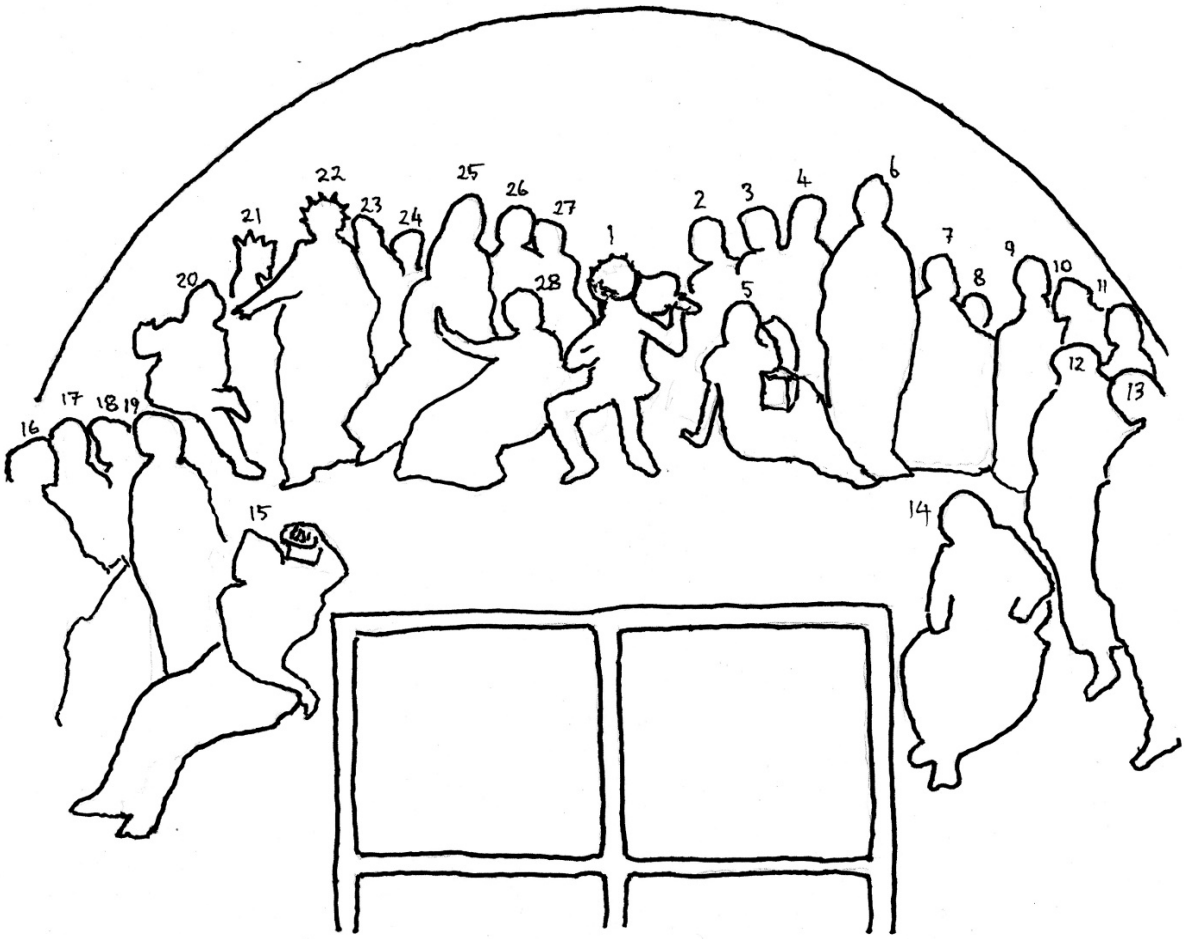

FIGURE V

Parnassus

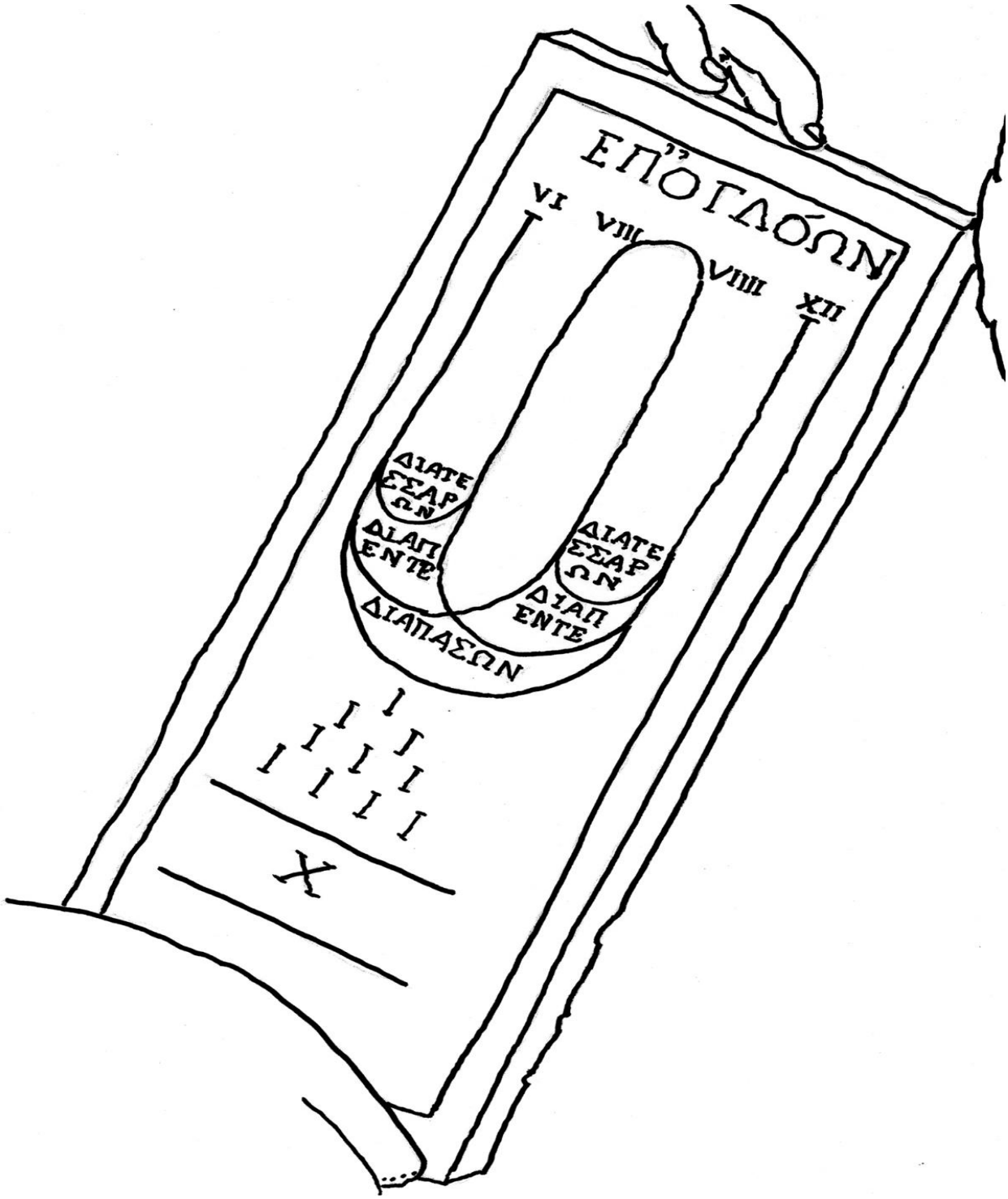

FIGURE VI

Raphael's *Slate* and the 'epogdoôn'

[detail from *The School of Athens*]

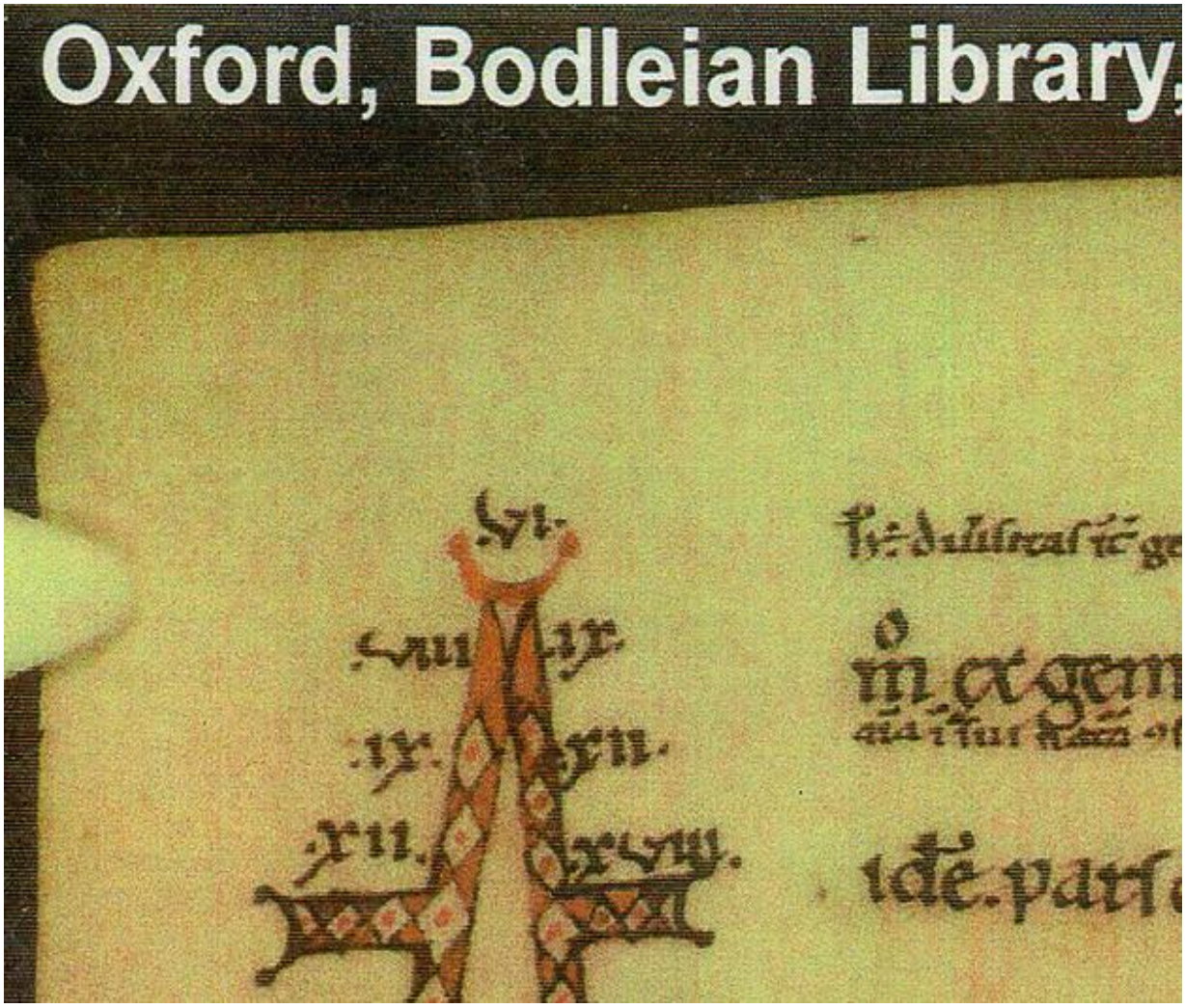

FIGURE VII

Raphael's numbers '*vi viii ix xii*',

appearing in Plato's 'World Soul' in MS ca 1100 CE

[image from Bodleian Library, MS. Digby23, fol.25v]

FIGURE VIII

Pythagoras and Jubal ('IVBAL') as a source for Raphael's numbers

[from Gaffurio (1993, p. 48)]

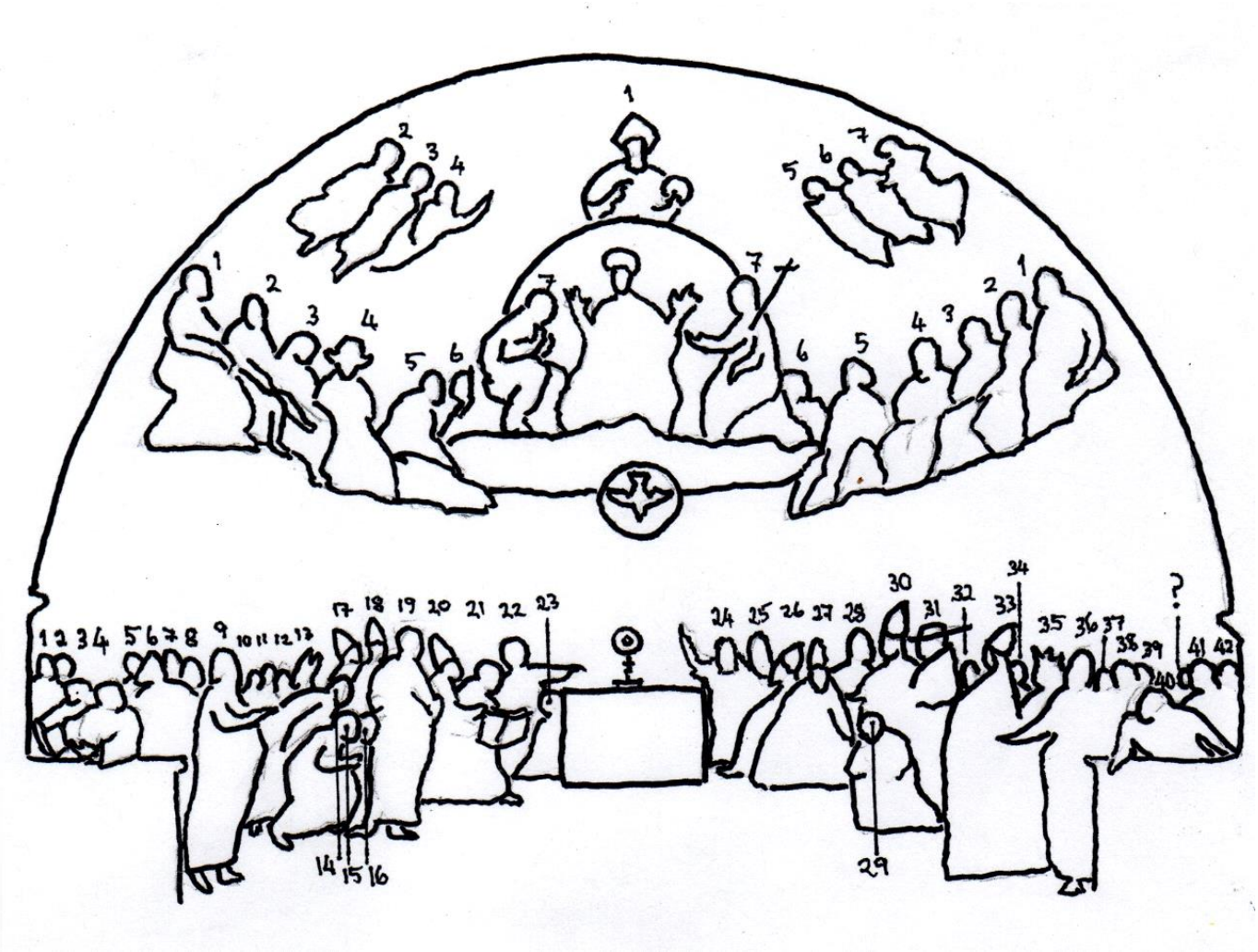

FIGURE IX

Disputa

FIGURE X

Right-hand panel of Jurisprudence

[photo credit, Bruno del Priore, Vatican Polyglot Press]

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