Vibration Serviceability of Pultruded GFRP Sandwich Panel Floor Systems

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B.Sc in Civil Engineering

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Department of Civil Engineering
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Abstract

Glass fibre-reinforced polymer (GFRP) composites have emerged as a potential structural material for building floor structures. Pultruded GFRP (pGFRP) sections are easily manufactured in a wide range of common structural section shapes and sizes. A pGFRP floor system offers advantages such as lower floor mass, efficient structural performance and cost-effective construction process. Recently, at Monash University, a novel modular pGFRP sandwich panel has been proposed for building floor applications. To date, the studies on the bespoke pGFRP sandwich panels have only pertained to the static behaviour – knowledge on its dynamic behaviour, especially under human-induced loadings remains limited. This is a concern because, due to the relatively lower weight and stiffness, pGFRP sandwich panel floor systems could be susceptible to excessive vibrations under service loads. Furthermore, current vibration serviceability design rules are based on experience of heavier and stiffer structures made from steel and concrete. The lack of an appropriate design guide means that current practice of pGFRP floors could be conservative, limiting the benefits for using pGFRP in the first place.

The aim of this thesis is to investigate the vibration serviceability of pGFRP sandwich panel floor systems. To achieve this aim, a research methodology involving numerical study and experimental study is considered to assess the vibration serviceability performance of pGFRP sandwich panels. A representative pGFRP sandwich structure is considered for experimental studies. The representative structure comprises a pGFRP sandwich panel footbridge – its boundary condition provides a good validation basis as a one-way spanning pGFRP floor system. The numerical framework is developed and validated to analyse the vibration performance of the pGFRP sandwich panel floors. The numerical framework includes representations of human-structure system to account the interactions of human body to the structure during walking. Additionally, model updating procedures are considered in this thesis to reconcile results of numerical models with experimental measurements. Following the validation works, the numerical framework is deemed reliable to predict vibration responses of pGFRP sandwich panel floors. Following this, a number of pGFRP sandwich floors are
first designed to satisfy static design rules prior to vibration analysis. Then, the vibration assessment is performed for the floors using the developed numerical framework. In addition, the vibration assessments are performed using current design practices to draw comparisons. Collectively the findings of this thesis make recommendations on the vibration serviceability performance of pGFRP sandwich panel floors. The outputs of this research can be used in planning, design, and evaluation of vibration performance for pGFRP floor structures.
Declaration
This thesis contains no material which has been accepted for the award of any other degree or diploma at any university or equivalent institution and that, to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

Signature: …………..

Print Name: …Jun Wei Ngan………………..

Date: ………07/06/2019…………………..
Publications during enrolment

The publications by the candidate are listed as follows. This includes a journal published in an ISI quarter 1(Q1) peer reviewed journal. The list also includes conference proceedings published throughout the candidature.

Published journal paper


Journal Papers Ready to Submit


Conference proceedings


Thesis including published works declaration

I hereby declare that this thesis contains no material which has been accepted for the award of any other degree or diploma at any university or equivalent institution and that, to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

This thesis includes one original paper published in peer reviewed journals and four submitted publications. The core theme of the thesis is vibration serviceability and dynamic behavior of civil structures. The ideas, development and writing up of all the papers in the thesis were the principal responsibility of myself, the student, working within the Department of Civil Engineering under the supervision of Dr. Colin Caprani.

In the case of Chapters 2 to 6, my contribution to the work involved the following:

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<td>3</td>
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<td>Colin Caprani: (30%) Conceptual development, manuscript revisions.</td>
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<td>1) Colin Caprani: (20%) Bridge design, data collection, supervise edits of manuscript; 2) Ehsan Ahmadi (20%) dynamic test and construction 3) Sindu Satasivam (20%) - static tests and construction</td>
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I have renumbered sections of submitted or published papers in order to generate a consistent presentation within the thesis.

**Student signature:**

**Date:**

The undersigned hereby certify that the above declaration correctly reflects the nature and extent of the student’s and co-authors’ contributions to this work. In instances where I am not the responsible author I have consulted with the responsible author to agree on the respective contributions of the authors.

**Main Supervisor signature:**

**Date:**
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Chapter 1: Introduction

1.1. Background

Steel and concrete are the most widely used construction material for civil engineering structures. However, these traditional materials are vulnerable to corrosion and deterioration. In turn, these civil structures require tremendous maintenance, which can be costly and difficult to perform. As a result, there is an increasing need for more durable materials to alleviate the impacts of corrosion in civil structures.

Recently, glass fibre-reinforced polymer (GFRP) composites have shown great potential as construction materials of civil structures. This stems from the excellent properties of GFRP: its lightweight allows offsite manufacturing potentials, ease of transportation and rapid installations; while its excellent corrosion resistance alleviates maintenance cost of civil structures. These properties can lead to overall reduction in construction and maintenance costs of structures. Furthermore, the production cost of pultruded GFRP has been reduced considerably due to advances in manufacturing GFRP sections, e.g. the pultrusion process. Initially used in retrofitting and rehabilitation of existing civil structures [1], GFRP composites have seen increasing applications as complete load-bearing members of civil structures. This has been seen in many real world applications, including floors [2], roofs [3], wall panels [4], houses [5, 6] and bridge superstructures [7-12].

1.2. Pultruded GFRP sandwich panels floors

In an effort to introduce new applications of pultruded GFRP composites (denoted as pGFRP), a novel pGFRP sandwich panel floor system has been introduced for building structures [13]. Figure 1.1 illustrates the proposed system for building floor frames, which comprised of a modular GFRP sandwich panel that replaces the concrete slabs of floor frame systems. The pGFRP sandwich panel is assembled from individual pGFRP box profiles that are incorporated in-between two pGFRP flat panels using either adhesive bonding or mechanical bolts (Figure 1.2). The modular construction method is beneficial for manufacturing works in terms of quality control as well as offering quick on-
site assembly. Additionally, the lightweight of components allows easy handling and quick on-site assembly.

Figure 1.1. Typical building floor frame with the proposed GFRP sandwich panel system (after [13]).

Figure 1.2. Modular GFRP sandwich panel with adhesively bonded individual components, with a bidirectional pultrusion orientation (after [13]).

1.2.1. Mechanical performance

To date, the mechanical performance of the bespoke pGFRP sandwich panel has been investigated through experimental studies on sub-scale specimens including sandwich beams [14], two-way sandwich slabs [15], and GFRP–steel composite beams [16]. For pGFRP, the main strength of the
material lies in direction of fibres (longitudinal). Consequently, the pGFRP sandwich panel has orthotropic properties and is prone to cracking in the weaker (transverse) direction. To alleviate this, the a bidirectional pultrusion direction is adopted, whereby the pultrusion direction of flat panels are aligned perpendicular to pultrusion direction of box profiles (Figure 1.2). In addition, bidirectional orientation of components was shown to provide greater bending stiffness in pGFRP – steel composite beams than the traditional GFRP decks which have fibres aligned in transverse direction of the beam. The bidirectional feature is not achievable in many existing pultruded decks which have its fibre pultrusion direction fixed to a single direction. According to [16], the addition of foam cores between spacings of box profiles can further enhance the bending stiffness in the transverse direction (along box profile).

For structural connections, it was shown that adhesive bonding provides full-composite actions between components. In contrary, mechanical bolts offer varying degree of composite actions which depends on the bolt spacing, type of bolt, and longitudinal shear force at interfaces. Blind bolts can be used over conventional through-bolts, allowing components to be connected from one face of the sandwich panel (e.g. flat panels and box profiles). Between both connection methods, mechanical bolts are easier to operate over adhesive bonding. This is because adhesive bonding requires high degree of controls and preparations to ensure quality of the bonds. For example, sufficient clamping pressures must be applied onto bonded layers during curing period of adhesive bonds. For this reason, it is recommended to consider adhesive bonding for off-site assembly of sandwich panels, where quality of bonds can be ensured through a factory setting. Overall, both adhesive bonding and mechanical bolts are both viable options for the structural connections of the pGFRP sandwich panels and pGFRP – steel composite beams.

### 1.2.2. Vibration serviceability performance
Chapter 1: Introduction

To date, research into dynamic behaviour of sandwich panels have been performed. Specifically, dynamic behaviour under blast [17-19] and impact loading [20-23] have been studied extensively on small-scale sandwich panels. Within these studies, only a handful are pertinent to pGFRP sandwich panels [19]. Some studies perform vibration analysis on sandwich panels (e.g. [24]). However, small-scale sandwich panel specimens could not directly describe its behaviour as a full-scale structure. Interestingly, there are currently no studies that evaluate dynamic performance of pGFRP sandwich panels under human-induced vibrations (known as vibration serviceability). In regards to full-scale structures, although studies of vibration serviceability have been performed considering pGFRP sandwich panels in bridges [25], these pGFRP sandwich panels are unidirectional – having only one pultrusion direction. Consequently, existing research outcomes cannot be applied directly to describe dynamic behaviour of the bespoke pGFRP sandwich panels with bidirectional fibre orientations. Furthermore, outcomes pertinent to pGFRP sandwich panel bridge structures cannot be directly applied to describe its behaviour as floor structures without further validation studies.

Compared to traditional materials, pGFRP is associated with lower mass and lower damping. In turn, structures made from pGFRP can have higher accelerance (acceleration per unit harmonic force) than comparable structures made of traditional materials [26]. In turn, vibration serviceability can govern the designs of pGFRP sandwich panel floor systems. Currently, research into sandwich panel structures have focused on static performance (e.g. [27, 28]). In contrast, there is lack of dynamic performance studies in the literature. Consequently, the lack of dynamic performance data means that current pGFRP floor systems are limited to short span applications [29]. In addition, there is lack of universally-accepted vibration serviceability design guideline for pGFRP structures. Notably, the current available design guidelines of pGFRP have based on existing knowledge of steel and concrete. With different properties of steel and concrete, current design guidelines for vibration serviceability of pGFRP structures can be conservative. This can result in overly-large pGFRP sections, defeating
the purpose of using lightweight pGFRP composites in the first place. Hence, quantification of vibration serviceability performance for pGFRP sandwich panel floor systems is needed.

1.3. Research aim and scope

The main aim of the research is to investigate the vibration serviceability of pGFRP sandwich panel floor system. Notably, the vibration serviceability of floor systems is required to ensure comfort of floor occupants as well as the tolerance limits for vibration sensitive equipment. This thesis focuses on the former requirement of vibration serviceability, which relates to human-induced vibrations from activities floor occupants such as walking or jumping. In this thesis, other aspects of dynamic performance - such as impact, wind, and seismic - are not investigated. It should be noted that this thesis regards vibration serviceability in the vertical direction – vibration serviceability for lateral vibrations are not covered.

1.4. Research methodology

The research methodology of the thesis is shown in Figure 1.3. As will be required later, the chapters relevant to each research process are outlined alongside Figure 1.3. In essence, the research methodology involves several processes that collectively work towards vibration assessments of pGFRP sandwich panel floors. The research methodology is multidisciplinary in nature, involving the field of GFRP material and human-induced vibrations (Figure 1.3).

First, a pGFRP sandwich structure is considered in the experimental testing framework of the thesis (Chapter 2). Experimental testing allows relevant experimental data to be acquired i.e. dynamic properties and responses of test structure. Following, a numerical modelling framework developed for vibration serviceability assessment of pGFRP sandwich panel floors (Chapter 4). The numerical framework is applied to the experimental test structure to validate its use for vibration analysis of pGFRP sandwich panel floors. A model updating framework is considered, allowing improvements
Chapter 1: Introduction

to numerical models to match numerical predictions with experimental results (Chapter 3). Note that model updating framework is first presented in Chapter 3 as it will be used in Chapter 4. Following, prototypical floors - with pGFRP sandwich panel as the slab solution - are considered for vibration assessment using the established numerical framework. Prior to numerical modelling, the static design of pGFRP sandwich panel floors is performed to establish feasible pGFRP sandwich panel floors for vibration assessments (Chapter 5). Finally, vibration serviceability assessments of pGFRP sandwich panel floors are performed (Chapter 6) and the thesis concludes the outcomes of the studies.

Figure 1.3. Schematic diagram of research methodology, highlighting corresponding chapters of the processes. The disciplinary fields of each framework are also mapped onto the processes.
1.5. Structure of the dissertation

The thesis comprises of five chapter-papers (Chapters 2 to 6). Each chapter were originally prepared as self-contained papers for publications. In turn, there will be cross-referencing of materials (e.g. figures, tables, and results) between the chapters to ensure smooth flow between thesis chapters. The chapter-papers have been devised to address knowledge gaps associated with pGFRP structures within the research methodology of Figure 1.3. A more comprehensive review of the knowledge gaps is provided in the introduction of each following chapters hereafter. Only after Chapters 2 to 5 can the vibration serviceability of pGFRP sandwich panel floors (Chapter 6) be achieved. An overview and linkage of the chapters with respect to the research aim is provided as a preface before each chapter.

References

Chapter 1: Introduction


Chapter 2.
Design, Construction, and Performance of the Monash pultruded glass fibre reinforced polymer footbridge
Preface

Chapter 2 presents the experimental studies of the thesis. This chapter presents a pGFRP sandwich panel footbridge, covering the details of design, construction, and experimental testing. This pGFRP footbridge is considered as a representative pGFRP sandwich panel structure to provide the validation basis for the numerical framework of this thesis. Specifically, the numerical framework is applied to model the pGFRP sandwich footbridge and predict its responses from walking experiments (described later in Chapter 4). Only after this can the numerical framework be validated as reliable for vibration predictions and assessments for pGFRP sandwich panel floors (Chapter 6). Furthermore, the outcomes of Chapter 2 have made recommendation for studies of Chapters 3 and 4. The experimental outputs from Chapter 2 make up the inputs for Chapters 3 and 4.

This chapter-paper is prepared for the following publication:


The contents of this chapter have been modified slightly from the publication’s version to produce a smooth flow of the thesis.
Chapter 2. Design, construction and performance of the GFRP sandwich panel footbridge

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Chapter 2. Design, construction and performance of the GFRP sandwich panel footbridge

Statement of Contribution

This achievement of this chapter involved the contributions of multiple authors: Dr. Colin Caprani, Prof. Yu Bai, Dr. Sindu Satasivam, Dr. Shao Hua Zhang, and Dr. Ehsan Ahmadi. Dr. Colin Caprani devised the projects and the conceptual ideas with input from Prof. Yu Bai. The author (Jun Wei Ngan), Dr. Colin Caprani, and Dr. Sindu Satasivam completed the design aspects of the project. All authors were involved in the construction of the bridge. Dr. Sindu Satasivam and Dr. Shao Hua Zhang directed the static testing experiments. The author and Dr. Ehsan Ahmadi completed the dynamic testing experiments. All authors discussed and interpreted the results. The author prepared this paper.
Abstract

This chapter describes the design, construction, and experimental testing of a full-scale epoxy-bonded pultruded glass fibre-reinforced polymer (pGFRP) footbridge with an orthotropic sandwich deck. The structure is lightweight and well-suited to modular construction. The footbridge was constructed in the civil engineering laboratory at Monash University. The design of the footbridge was facilitated by experimental data on small-scale specimens and a numerical model. This chapter documents the steps and details its construction, from which lessons are learned that may be relevant to similar structures of this kind. Finally, this chapter reports on the structural performance obtained from experimental static and dynamic testing. This pGFRP footbridge is shown to be a viable solution for new and replacement footbridges. Its light weight and good strength make it easy to transport, making it suitable for short span footbridge constructions.

KEYWORDS

Pultruded Glass Fibre-reinforced polymer; Sandwich panel; Footbridge; Design; Construction process; Finite element; Static load test; Experimental modal analysis
Chapter 2: Design, construction and performance of the GFRP sandwich panel footbridge

2.1. Introduction

2.1.1. Background

In the last two decades, glass fibre-reinforced polymer (GFRP) composites have been increasingly applied in footbridge constructions. This stems from its advantages: its light weight yields benefits such as rapid construction, minimal disruption as well as low labour costs [1]; while its excellent durability and corrosion resistance permits lower maintenance cost, making it particularly well-suited for the replacement of deteriorated footbridges [2]. However, there is still some reluctance for the use of GFRP in bridge construction, due to lack of structural performance data [3, 4]. Regardless, an increasing number of GFRP footbridges have been built around the world as there is a pressing need for the renewal of deteriorating bridges with more durable materials [5]. The first FRP Bridge was constructed in Miyun, Beijing in 1982. In the United States, No-Name Creek Bridge is the first all-FRP honeycomb core sandwich panel bridge [6]. The Alberfeldy Bridge in Scotland is the world’s longest GFRP footbridge – with a main span of 63 m [7]. Over the last decade, significant studies have been performed on footbridges with GFRP superstructure. The majority of performance studies have focused on static performance [8]. In contrast, only a few studies focus on the dynamic performance of existing GFRP footbridges [7, 9-11].

Currently, there are very few design guides for GFRP footbridges and even those often use design approaches developed for traditional steel and concrete structures. As a pertinent example, the AASHTO GFRP Bridge Design Guide [12] and UK publication BD 90/05 [13] specify the 5 Hz ‘rule-of-thumb’ in which vibration problems are deemed unlikely as long as the fundamental natural frequency of GFRP footbridges is at least 5 Hz. This assumes that the higher harmonics of human-induced forces will not cause vibration problems, and this rule-of-thumb has served well in the past for much heavier concrete and steel footbridges. But with the different properties of GFRP bridges—the strength of steel, stiffness of concrete, and weight of dense timber—the solutions from these design rules may not be optimal for GFRP footbridges. Specifically, GFRP structures have a higher
accelerance (acceleration per unit harmonic force) \([14]\), making them potentially more susceptible to human-induced vibrations.

Pultruded GFRP (pGFRP) is a means of manufacturing regular structural section shapes from GFRP. It is an efficient manufacturing process, and facilitates much cheaper construction than laid-up laminated GFRP. In pGFRP, besides the chopped strand mat surface veils, the main structural fibres are oriented in one direction (longitudinal). As a consequence, pGFRP is an orthotropic material that is prone to cracking in its transverse direction \([15]\). To maintain the advantages of GFRP, and remove the disadvantage of pGFRP, a novel, orthotropic sandwich assembly has been proposed \([15]\). This modular sandwich panel is comprised of pGFRP box or I-profiles incorporated between pGFRP flat panels—see Figure 2.1. Previously, the mechanical properties of this sandwich panel have been investigated by constructing and testing small-scale sandwich specimens including sandwich beams \([16, 17]\), two-way spanning slabs \([18]\), and a GFRP-steel composite beam \([19]\).

![Pultrusion direction of flat panels](image)

![Pultrusion direction of box profiles](image)

Adhesive

Figure 2.1. GFRP sandwich panel system, showing fibre directions of components after \([18]\).

2.1.2. Contributions

The benefits of a light, strong, and durable material are significant, in terms of off-site fabrication, easy transport and erection, and low maintenance costs. Pultruded GFRP footbridges comprised of
the orthotropic sandwich deck will meet this demand. However, as has been seen, most of the published work relates to laminated GFRP and knowledge of their in-service vibration performance under human-induced loads remains very limited [14, 20, 21]. This work aims to address the gaps in current knowledge on:

1. Relevant manufacturing issues of epoxy-bonded pGFRP structures;
2. The static performance of a full-scale orthotropic sandwich deck epoxy-bonded pGFRP bridge;

To achieve these aims, a full-scale orthotropic sandwich deck epoxy-bonded pGFRP laboratory footbridge has been constructed at Monash University. This chapter presents the design, construction and performance testing of the novel Monash Bridge (MB).

2.1.3. Description of the MB

The MB is a 9 m long, twin girder footbridge as illustrated in Figure 2.2. The MB has a mass per unit length of 92.56 kg/m, making it a very lightweight footbridge. A comparable steel-concrete footbridge—the Warwick University footbridge—has a linear mass of 829 kg/m [22]. Epoxy bonding was used for all structural connections of the MB. To reduce extraneous sources of uncertainties in experimental tests, the MB has no external attachments such as handrails for example. The deck of the MB is a modular sandwich panel made from individual pGFRP box profiles and flat panels which spans transversely between two pGFRP I-beam girders, which are in turn supported at both ends. The bridge span can be changed by moving the supports.
Figure 2.2. Overview of the MB: (a) photograph view; (b) composite section showing fibre orientations of different components; (c) photograph of sideview.

The MB is constructed from individual pGFRP components as shown in Figure 2.3. Unlike the Advanced Composites Component system [23], where pGFRP decks consist of prefabricated composite building parts, the sandwich deck is constructed from standard pGFRP sections e.g. flat panels and box profiles. The individual pGFRP flat panels comes in length of either 1.5 m or 3 m, which are then constructed in a staggered configuration to form the entire deck surface of the MB - see Figure 2.3a. To ensure structural continuity in the longitudinal direction, the flat panels are connected to adjacent panels using 6 mm-thick pGFRP connecting plates bonded from within sandwich panels. Individual pGFRP box profiles with dimensions of $76 \times 76 \times 9.5$ mm spans in the transverse direction, forming the core layer of the sandwich panel. The girders consist of pGFRP I-beam with dimensions of $203 \times 203 \times 9$ mm. In addition, five pGFRP T-beam sections were incorporated between the two girders to add transverse stiffness and stability against distortion. These
stiffeners also serve to prevent unwanted localized vibrational modes pertinent to I-beams that would pollute the responses from global bending and torsional modes. The maximum available length of the individual I-beams is 6 m. In turn, two segmented I-beams (a 3 m and a 6 m length) were connected by pGFRP connecting plates along the webs and flanges.

Figure 2.3. Dimensions of the MB showing built-up of individual components: a) plan view; and b) typical cross-section (units: mm).

2.2. Structural design

2.2.1. Design requirements

From the onset, the purpose of constructing a footbridge specimen stems from interests in multiple research studies pertinent to structural dynamics and pGFRP structures. The research intentions are as follows (ranked according to interest):

- **Study 1**: Human-structure interactions (HSI) in lightweight and lightly-damped structures.
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- **Study 2**: Evaluation of current vibration rules for pGFRP footbridges.
- **Study 3**: Static and dynamic performance of full-GFRP structures.
- **Study 4**: Explore epoxy bonding techniques for modular constructions pGFRP structures.

Consequently, a footbridge made entirely of pGFRP composites complements requirements for Study 1 (lightweight of pGFRP), Study 3 (GFRP testbed for experiments) and Study 4 (considering a full-bonded construction).

The requirement for Study 2 is achieved by designing the dynamic behaviour of the MB. In context of accelerance, the targeted dynamic behaviour of the MB shown in Figure 2.4. For comparisons, the accelerance of several comparable footbridges from Živanović [14] are also shown in Figure 2.4. The harmonic ranges of common walking frequencies (first harmonic from 1.8 Hz to 2.2 Hz [24]) are shown shaded, along with the common 5 Hz rule – as a dashed line in Figure 2.4. As the common 5 Hz rule is based on assumption that excitations of higher harmonics (more than two) are negligible, the performance of the 5 Hz rule can be assessed by checking the vibrations levels of the MB due to excitations from higher walking harmonics. To this, the MB is designed with a first natural frequency, $f_1$ within the third harmonic range of walking frequencies (between 5.4 Hz to 6.6 Hz).
Figure 2.4. Relationship of first mode FRFs (accelerance) between different footbridges; typical walking harmonics (shaded grey), and the 5 Hz rule (dashed line). AB – Aberfeldy Footbridge (GFRP); PB – Podgoricia Bridge (Steel); WB – Warwick Bridge (Steel-Concrete Composite); SB – Sheffield Bridge (Concrete); EB – EMPA Bridge (GFRP deck); MB – Monash Bridge (GFRP) (some data from [14]).

### 2.2.2. Design checks using numerical analysis

Apart for structural requirements, the MB is designed to conform allowable stress design (ASD) and deflection check according to the AASHTO GFRP Bridge Design Guide [12]. A nominal uniformly distributed load of 4.07 kN/m² (AASHTO’s [12] requirement for serviceability) is considered for the design checks. The maximum allowable stress of sections under checking must not exceed 20% of corresponding strength capacity. For deflection checks, the maximum deflection under the design service load is limited to 1/500 of the bridge’s span, which corresponds to 29.2 mm for the 9 m span MB.

The structural design of the MB is supported with numerical analysis. A finite element (FE) model of the MB is developed in LUSAS software [25] – see Figure 2.5. All structural components, including flat panels, box sections, and bottom I-beam girders were modelled using eight-node...
quadrilateral shell elements (QTS8). The shell element has six degree of freedom at each node: translation in nodal $x$, $y$, and $z$ directions and also rotation about nodal $x$, $y$, and $z$ axes. All connected pGFRP components (e.g. between beam flange and flat panel) were modelled as an equivalent shell element with combined thickness. Anisotropic properties of pGFRP were defined for the shell elements in the fibre direction of components accordingly. Pinned supports were considered at both ends of the I-beam girders. For dynamic behaviour analysis, vertical spring elements have been considered instead to represent support flexibility. Horizontal restraints were implemented at the support nodes to limit the responses (i.e. deflections and accelerations) to the vertical direction.

The FE model is used to check design requirements for a 6 m and 9 m span configuration of the MB. According to static load simulations, the 6 m span of the MB conforms to stress limits and deflection check, while the 9 m span only failed deflection check. Close inspection reveal that the deflection criteria of the 9 m span is compromised to achieve the targeted dynamic behaviour – first natural frequency of about 6.2 Hz. Despite none-conformance to deflection limits, the performance of the 9 m span can be justified for the evaluation of the vibration serviceability design rule i.e. the 5 Hz requirement.
2.2.3. Material Characterization

To improve fidelity of the FE model, the input properties of pGFRP were obtained through tensile testing of pGFRP coupons cut from sacrificial sheets and members corresponding to those in the MB [26]. The coupons were taken for two thicknesses - 6 mm (flat panel, connecting plates, and I-beams) and 9 mm (box profiles). A total of ten specimens were tested for each fibre directions, namely longitudinal and transverse pultrusion directions. The tests were performed using a 100 kN Instron Universal Testing Machine in accordance to ASTM 3039 [27]. In addition, 6 mm-thick pGFRP coupons of size 250x25 mm was extracted on a 10° off-axis angle cut and tested to determine the in-plane shear modulus, \( G_{LT} \). Preliminary burn-off tests shows the fibre volume fraction (FVF) of the pGFRP materials is around 42.1 ± 0.3% [26].
Table 2.1 summarises the mechanical properties of pGFRP, i.e. longitudinal and transverse elastic modulus in tension, longitudinal tensile strength, and the in-plane shear modulus. These properties were taken from averages of ten specimens. The elastic moduli and Poisson’s ratios for both thicknesses were found based on Hooke’s law.

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal elastic modulus, $E_L^*$</td>
<td>GPa</td>
<td>22.99  24.63</td>
</tr>
<tr>
<td>Transverse elastic modulus, $E_T^*$</td>
<td>GPa</td>
<td>10.32  10.03</td>
</tr>
<tr>
<td>Major Poisson’s ratio, $\nu_{LT}$</td>
<td>-</td>
<td>0.30    0.31</td>
</tr>
<tr>
<td>Minor Poisson’s ratio, $\nu_{TL}$</td>
<td>-</td>
<td>0.15    0.14</td>
</tr>
<tr>
<td>In-plane shear modulus, $G_{LT}$</td>
<td>GPa</td>
<td>4.45    -</td>
</tr>
</tbody>
</table>

2.3. Construction

2.3.1. Bonding sequence

A two-part epoxy (R180 epoxy resin and H180 hardener supplied by Fibre Glass International) was mixed proportionally to adhesively-bond all pGFRP components. A total of 83 pGFRP components are to be bonded during the assembly process. The lightweight nature of pGFRP components allows the MB to be constructed in an upside-down sequence – building the sandwich deck first then bonding the I-beams on top of the deck to form the underside of the MB. This construction sequence complements various aspects of construction, such as the handling of epoxy, precision in aligning pGFRP components, and instrumentations (described in detail later).

The entire bonding operation of the MB is shown in Figure 2.6 (a – i). Prior to bonding operations, all bonding interfaces were roughened to improve the adherence of epoxy to pGFRP components. All
bonding surfaces were cleaned using isopropanol to remove dust and improve adherence. The sandwich deck is constructed in segments, where the bonded modules are left for epoxy to dry while simultaneously constructing other modules. This segmented bonding procedure allowed efficient time management between bonding and drying period of different segments. The sandwich panel is constructed into five segments, which are joined together forming the entire deck. The flat panels on the upper side are only bonded following instrumentations of sensors within the deck (later described). Thereafter, the top flat panels were bonded to form the completed sandwich deck. The I-beams and by T-sections were subsequently bonded onto the sandwich deck. Wooden spacers were used to help align the I-beams during bonding procedures. The completed MB was then rotated using an overhead crane. Finally, the temporary construction platform was removed and the MB was lifted to its final position.
2.3.2. Instrumentations

During construction, a total of 70 strain gauges (FRA-10 supplied by TML, Japan) were simultaneously installed along different positions of the MB. The measurement positions were selected to plot strain distributions along the depth and width of the composite section. Each strain gauge has a gauge factor of 2.11 ± 1 %, and measures strains to a maximum of 5%. All strain gauges were recessed to the external surfaces (approximately 2 mm from the surface) and protected with a clear epoxy filling – see Figure 2.7a. The recessed feature of strain gauges allows the application of
clear epoxy coating to prevent damage and ensure adherence of strain gauges onto the structure, which is suitable for long term data recording. In turn, the strain analysis can be adjusted to account for the recessed characteristics. The strain gauges are installed in two configurations, namely as a single gauge or a strain rosette (Figure 2.7). The proposed construction sequence allowed easy access various regions (e.g. beneath deck). The cables of strain gauges that are mounted on the sandwich deck (top and bottom flat panel) were draw from within the sandwich panels and in turn, directed out to the data acquisition point.

Figure 2.7. Strain gauges: (a) showing recessed properties and through panel wiring, (b) single strain gauge sealed with clear epoxy, and (c) strain rosettes.

2.3.3. Key challenge and solutions

The key manufacturing issue is the handling of epoxy during bonding operations. Due to high fluidity of epoxy, the running epoxy is a major concern. Furthermore, the epoxy generally took over 24 hours for sufficient hardening to occur, which required substantial measures to control excess epoxy during the long curing time. During construction, running epoxy resulted from excessive application of epoxy onto bonding interfaces.

Figure 2.8 shows sections of the MB that are consequences of excess epoxy, with a description following each image. In a few instances of poor epoxy control, several connecting plates have been covered in hardened epoxy as shown in Figure 2.8a. Several affected connecting plates have strain gauges that needs to be installed, which in turn required additional work in removing the hardened
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epoxy (Figure 2.8b). Some coverage of hardened epoxy was found at the deck surface of the completed MB (Figure 2.8c) upon rotation. This occurred during the bonding of adjacent flat panels, when the epoxy flowed between gaps of adjacent flat panels onto the underside of the sandwich panel. Similar to the connecting plates, extra work was needed to remove excess epoxy due to instrumentation positions of strain gauges as seen in Figure 2.8c. The control of excess epoxy was complicated for bonding interfaces that are vertical (e.g. bonding of connecting plates onto underside of beam flange in Figure 2.8d) and for bonding regions with multiple components (e.g. the section between T-section and beam flange in Figure 2.8e). Consequently, a clean bonding surface was difficult to achieve at the aforementioned regions. Throughout construction, taping along bonding lines was one of the methods of controlling running epoxy (e.g. see Figure 2.8f). The masking tape remained in place until the end of drying period.

Figure 2.8. Issues related to excess running epoxy.

Despite the issues with running epoxy, the following features of the construction process has shown to alleviate majority of the effects of running epoxy:
Chapter 2: Design, construction and performance of the GFRP sandwich panel footbridge

- The levelled construction platform negates any uneven surfaces of components (Figure 2.9), in turn improving the control on excess epoxy. Additionally, the levelled platform was covered with large sheets polyethylene paper, which prevents pGFRP components sticking to the underlying levelling platform.

- Masking tape along bonding lines serve as an easily removed solution for excess epoxy. The tape can be removed (along with running epoxy) at the end of construction day during which the epoxy has begun to dry up (lower fluidity).

- To ensure uniform epoxy bond thickness, 5 mm-long wire spacers of 0.7 mm thickness were placed at regular intervals along the adherence surfaces to ensure a uniform thickness of all bond lines. The spacers provide gaps in between bonding components, which helps ensure a thin layer of epoxy the instance clamping pressures were applied onto bond interfaces.

![Figure 2.9. Bond control operations: (a) cleaning and taping, (b) spacer wires along bond lines, and (c) clamping means through weights and G-clamps.](image)

2.4. Static performance

2.4.1. Static load test

Static load tests were performed to determine the vertical deflections and strain distributions of the MB under service load (Figure 2.10). The MB is tested for three span configurations, namely an 8.785 m single-span (Test 1), a 6 m single-span (Test 2) and 4.39 m two-span (Test 3). The MB was loaded with lead weights and concrete blocks up to a uniformly distributed load level of 4.07 kN/m².
(following the AASHTO[12]). For Test 3, loading was performed on one span to simulate a continuous-span behaviour. To ensure static response measurement, loading operations were conducted in 10 incremental steps with a maximum data collection period of 5 minutes between each step. All loading operations were performed as fast as possible to minimise creep effects. During loading operations, load cell readings were simultaneously monitored to ensure truly symmetric loading.

The acquisition system is shown in Figure 2.11. Vertical deflections were measured using Linear Voltage Displacement Transducers (LVDTs) placed in a measuring grid as shown in Figure 2.12. In addition, four C10 HBM load cells were placed at the four support ends of the pGFRP I-beam girders. The load cells are capable of measuring static and dynamic forces up to 25 kN with accuracy class of 0.04% (i.e. maximum load cell deviation specified as percentage). For each test, the LVDTs were repositioned according to the measurement layout shown in Figure 2.12. For Test 3, two additional 50 kN load cells were considered as the middle support (LC 5 and LC 6 in Figure 2.12).
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Figure 2.10. Static Test: (a) Test 1 – 9 m single-span, (b) Test 2 – 6 m single-span, and (c) Test 3 – 9 m single-span

Figure 2.11. Measuring devices in static tests: (a) Data Acquisition device connecting all strain gauges, (b) Load cells, and (c) Linear voltage displacement transducers (LVDTs).
2.4.2. Load-deflection responses

Figure 2.13 shows the load-deflection responses (Test 1, 2 and 3). The MB displayed linear-elastic load-deflection responses up to the maximum applied load (4.07 kPa). To evaluate the difference in deflections, the bending stiffness, $EI$, of the MB cross-section was calculated from the load-displacement curves using Euler beam theory. The bending stiffness was found to be $6.61 \times 10^{12} \text{ Nmm}^2$ for Test 1 and $6.02 \times 10^{12} \text{ Nmm}^2$ for Test 2 (with a difference less than 10%). However, the bending stiffness for Test 3 was $2.58 \times 10^{12} \text{ Nmm}^2$, which was about 60% lower than those calculated for Tests...
1 and 2. This significant difference is considered to be due to the substantial shear deformations in the shorter 4.39 m span, which is not accounted for in Euler beam theory.

To account for shear stiffness, the bending stiffness is instead evaluated using Timoshenko beam theory for this shortest span configuration. The MB in Test 3 can be considered as a single span propped cantilever continuous beam. The new bending stiffness for Test 3 was $5.5 \times 10^{12} \text{ N mm}^2$, which was only 9% less than that found using Euler beam theory. This observation is as expected: that shear deformation is significant for the shortest span, contributing to 51% of the overall deflection. Therefore, Timoshenko beam theory should be considered for serviceability checks of lower span-to-depth ratio of the MB, which in fact is far higher for comparable footbridges of traditional materials (e.g. steel and concrete).

Figure 2.13. Load vs maximum midspan deflections for Test 1, 2 and 3 of the static tests, after [28].

2.4.3. Composite behaviour of cross section

The strain distributions across the cross section were measured at midspan (see Figure 2.14). Since bending moments were low in the negative bending region, small strain values were obtained at
position CS 4-4 (see in Figure 2.12) for Test 3. Consequently, only the positive bending region (CS 1-1) was considered for Test 3. It is clear from Figure 2.14 that the longitudinal strain distributions present a linear trend in two ranges below 200 mm (from the lower flat panel to lower flange of I-beam). For depth within the sandwich deck, it is difficult (at this stage) to classify the distributions as linear - this requires further studies. Further, there were compressive strains above the 200 mm depth (from the lower flat plate to upper flat plate), and tensile strains below this level. There is continuity in the strain profile at the interface between the sandwich deck and I-beam indicating that full composite action was provided by the adhesive bonding.

The transverse web of the deck provided full composite which can be seen by the approximately straight continuation of the strain profile (as opposed to a more uniform axial-force-only strain profile). This enhanced composite action may be due to the higher web thickness of the footbridge sandwich deck, which used 9.5 mm-thick box-profiles while the previous study used 6 mm thickness [16]. Overall then, it is found that full composite action across the bridge deck can be achieved by combining the use of epoxy bond and thicker box profiles.
2.5. Dynamic performance

2.5.1. Experimental modal analysis

Experimental modal analysis (EMA) was carried out to identify the dynamic properties i.e. natural frequencies, mode shapes and damping ratios of the MB. The EMA was performed for the 8.785 m span configuration (Test 1). The MB is excited using an electrodynamic shaker (denoted as shaker test). Vertical accelerations were measured using 10 piezoelectric accelerometers in a measurement grid as shown in Figure 2.16a. The position of the shaker was placed offset the symmetric line of the MB, at quarter spans, in order to excite a range of bending and torsional modes.

The shaker mass is considered to be significant to the mass of the bridge (about 5.6%) and will contribute mass to its overall vibrating system. To check this, an impact hammer test was performed with three accelerometers placed along the centre line of the bridge (Figure 2.16b). In the hammer test, three accelerometers were placed along the MB’s centreline to measure and compare the bending mode shapes with those from the shaker test. A sampling period of 15 seconds was considered to
allow free vibration responses from impact to decay. It should be noted that the hammer test is performed with the hammer operator standing off the bridge (on the ground) to avoid mass loading due to operator’s mass.

Figure 2.15. Experimental modal analysis setup with electrodynamic shaker and accelerometers (after[29])

Figure 2.16. Measurement grid for: (a) shaker tests, and; (b) impact hammer tests.

2.5.2. Modal properties
Chapter 2: Design, construction and performance of the GFRP sandwich panel footbridge

The first three mode shapes from the shaker test and the FE model are shown in Figure 2.17 and Figure 2.18 respectively. The natural frequencies of the first six vibrational modes from the tests and the FE model are summarized in Table 2.2. The MB has a first natural frequency, $f_1$ of 5.86 Hz. The uniformly lower natural frequencies from the shaker test clearly indicates the mass loading of the shaker. From the hammer test, the first natural frequency of the MB, $f_1$ is 6.17 Hz, which is close to the natural frequency from the numerical model without the shaker mass (6.2 Hz). The modal damping for all vibrational modes is generally low (less than 1%), which is expected for pGFRP which inherit low material damping. Notably, the damping ratios are relatively lower compared to other GFRP footbridges reported in the literature [21, 30, 31] (ranging from 1% to 2%). This is most likely due to the simple geometry of the MB, having no complicated attachments such as handrails for example. In contrast, the predictions of higher natural frequency (mode 3 onwards) were closer in the 3-D detailed model than the 1-D and 2-D model. The lower accuracy of higher natural frequencies in both 1-D and 2-D model is likely due to the influence of shear deformation in various component of the MB, which the 3-D FE model considered in its representation. For example, the intermediate T-sections along the I-beams are modelled in the 3-D FE model, where its shear and torsional stiffness are included in the modal analysis can affect the natural frequencies of torsional modes for example. Despite differences in higher natural frequencies, the order of mode shapes in the 2-D FE models are identical to those measured, which provide confidence in the numerical results.

![First three modes of vibration obtained from shaker test: (a) $f_1$ = 5.86 Hz, (b) $f_2$ = 10.02, (c) $f_3$ = 18.14 Hz Contour is given to highlight the region of maxima and minima within mode shape feature in regards to the vertical direction.](image)
Figure 2.18. First three modes of vibration from FE model: (a) $f_1 = 5.95$ Hz; (b) $f_2 = 9.62$ Hz; (c) $f_3 = 20.27$ Hz. Contour is given to highlight the global maxima and minima of mode shape feature.

Table 2.2. Comparison between natural frequencies from experiment and FE model. Damping ratios are from shaker test.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Shaker test</th>
<th>Hammer test</th>
<th>FE model</th>
<th>Diff (%)</th>
<th>(%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.86</td>
<td>6.17</td>
<td>5.95</td>
<td>1.5</td>
<td>0.59</td>
<td>0.98</td>
</tr>
<tr>
<td>2</td>
<td>10.02</td>
<td>-</td>
<td>9.62</td>
<td>4.0</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td>3</td>
<td>18.14</td>
<td>19.60</td>
<td>20.27</td>
<td>11.7</td>
<td>0.61</td>
<td>0.98</td>
</tr>
<tr>
<td>4</td>
<td>20.60</td>
<td>-</td>
<td>23.87</td>
<td>15.9</td>
<td>1.65</td>
<td>0.64</td>
</tr>
<tr>
<td>5</td>
<td>25.60</td>
<td>-</td>
<td>28.85</td>
<td>12.7</td>
<td>1.33</td>
<td>0.66</td>
</tr>
<tr>
<td>6</td>
<td>37.54</td>
<td>38.30</td>
<td>39.52</td>
<td>5.3</td>
<td>0.92</td>
<td>0.85</td>
</tr>
</tbody>
</table>

1Against shaker test; 2Modal assurance criterion

As can be seen in Table 2.2, the first and second natural frequencies of the FE model are in good agreement with the measured natural frequencies. However, larger natural frequency differences are observed for higher vibrational modes. This is most probably due to the mass loading of the shaker in the experiment, since the FE model did not consider the shaker mass. Modal Assurance Criterion (MAC) is used to compare the correlation of mode shapes between FE model and shaker test [32]. From Table 2.2 the MAC indicates good correlation for first three modes shapes. The lower MAC for higher vibrational modes is due to the lack of measurement points which are limited to the number of accelerometers used in EMA. Overall, the dynamic properties predicted from FE model are reasonable despite the lack of mass loading representation in the FE model.
A close inspection of the measured mode shapes reveals asymmetric modal behaviour in mode 1 and mode 3. Interestingly this phenomenon is still observed even after several verification were performed \[33\]. Since the shaker does not explicitly contribute stiffness to the over stiffness of the system, the asymmetric modal behaviour may stem from the shaker mass balancing out some anomaly in longitudinal stiffness. Possible sources leading to deviation of longitudinal stiffness can be due to (i) geometric deviations introduced during construction, or (ii) material properties deviations (these are further assessed in \[33\]). Detailed study on this interesting phenomenon is beyond the scope of this chapter and has been performed elsewhere \[34\].

### 2.5.3. Walking trials

Limited walking trails were carried out to measure resonance responses of the first natural frequency due to third harmonic excitations of walking. The walking trails were performed based on controlling pacing frequencies \(f_p\). Acceleration responses were measured and averaged from two accelerometers placed at both sides at the mid span of the MB. Three test subjects (TS) participated in this walking trial, with properties summarized in Table 2.3. A metronome is used to regulate the pacing frequencies of test subjects. Five acceptable (in terms of realized pacing frequencies) were performed for each test subject at pacing frequencies \(f_p = 1.95\) Hz, 1.8 Hz, and 2.1 Hz, intended to bracket the third harmonic excitation. Vibrations are allowed to decay before commencing each subsequent walk.

<table>
<thead>
<tr>
<th>Test Subject</th>
<th>Gender</th>
<th>Weight (N)</th>
<th>Height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TS1</td>
<td>Male</td>
<td>624</td>
<td>170</td>
</tr>
<tr>
<td>TS2</td>
<td>Male</td>
<td>706</td>
<td>178</td>
</tr>
<tr>
<td>TS3</td>
<td>Male</td>
<td>1060</td>
<td>183</td>
</tr>
</tbody>
</table>
An initial attempt of numerical simulation is performed to predict the dominant response i.e. the first bending mode of the MB. First, a one-dimensional Euler Bernoulli beam model is used to model the MB. In this model, the first bending mode is replicated to match the experimental measurements. Two natural frequencies of the first bending mode are considered, namely from shaker test (accounting possible mass loading), and from the impact hammer test. These are achieved by manually-tuning the properties of the beam model (i.e. the stiffness parameter). Additionally, the damping of the beam model is defined as amplitude-dependant, which are obtained for each cycle of the accelerations from the free vibration portion of the response.

For this initial simulation, a moving force (MF) model is considered as shown in Figure 2.19. The MF model is chosen for this simulation due to its simplicity, allowing quick assessments of acceleration responses. The MF model has a concentrated force which varies over time, \( F(t) \). The magnitude of \( F(t) \) is represented by a Fourier series as:

\[
G(t) = W_p \sum_{k=0}^{\infty} \eta_k \cos (2\pi k f_w t + \phi_k)
\]

where \( W_p = m_p g \) is the subject’s weight; \( m_p \) and \( g \) are the walker mass and gravitational acceleration respectively; \( f_w \) is the pacing frequency; \( t \) is time; and \( \eta_k \) is the dynamic load factor (DLF) for the \( k \)th harmonic. For this study, Young’s dynamic load factors (DLFs) [35] up to four harmonics are considered.

![Figure 2.19. Moving force model to simulate walking responses of the test subjects](image-url)
Figure 2.20 shows the time series of mid-span accelerations from the MF model and experimental measurements. The maximum mid-span acceleration responses from numerical simulation and measurements are summarized in Table 2.4, from which it is clear that the walking model with footbridge frequency \( f_{b1} \) of 5.86 Hz presents the resonant response of the MB (evident by the large responses of all TS). The predicted accelerations were overestimated for TS1 and TS2 but underestimated for TS3, who has a larger mass. This is presumably due to the phenomenon of the human-structure system that is not captured by the relatively simple moving force model. Similar to the effect of the shaker mass, the pedestrian mass may have an effect on the dynamic properties of the MB. This is apparent in the walking trials of TS3. Such accelerations are perceivable, and consequently may influence walking behaviour of test subjects. Overall, this observation warrants human-structure representations in numerical simulations of the walking trials. This can be achieved through the use of human interactive models (e.g. spring-mass-damper model).

The observed responses in Table 2.4 can be compared with the limits in the Sétra guideline [11], as shown in Table 2.4. Maximum mid-span accelerations from measurements and moving force model.

<table>
<thead>
<tr>
<th>TS</th>
<th>( f_p ) (Hz)</th>
<th>( f_{b1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td>MF Model</td>
</tr>
<tr>
<td></td>
<td>1.80</td>
<td>1.95</td>
</tr>
<tr>
<td>1 (624 N)</td>
<td>0.38</td>
<td>0.50</td>
</tr>
<tr>
<td>2 (706 N)</td>
<td>0.32</td>
<td>0.57</td>
</tr>
<tr>
<td>3 (1060 N)</td>
<td>1.16</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Table 2.5. The MB attained high levels of accelerations from the third harmonic of walking frequencies even though the footbridge has a natural frequency of 6.1 Hz (i.e. conform to the 5 Hz rule). Clearly, the responses from walking trials reaches the CL4 comfort level (unacceptable discomfort). Therefore, this shows that the 5 Hz rule from AASHTO [12] is not suited for the design of the MB, indicating that current rules not be generally applicable for GFRP footbridges.
Figure 2.20. Measured and simulated vibration response for TS3 and pacing frequency of: (a) 1.8 Hz (b) 1.95 Hz (c) 2.1 Hz. (after[29]).

Table 2.4. Maximum mid-span accelerations from measurements and moving force model.

<table>
<thead>
<tr>
<th>TS</th>
<th>$f_p$ (Hz)</th>
<th>Measured</th>
<th>MF Model $f_{bl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.80</td>
<td>1.95</td>
<td>2.10</td>
</tr>
<tr>
<td>1 (624 N)</td>
<td>5.86 6.10</td>
<td>5.86 6.10</td>
<td>5.86 6.10</td>
</tr>
<tr>
<td>2 (706 N)</td>
<td>0.38 0.50 0.35 0.74</td>
<td>0.66 0.68 1.36</td>
<td></td>
</tr>
<tr>
<td>3 (1060 N)</td>
<td>1.16 0.85 0.60 2.26</td>
<td>3.27 1.71 2.86 1.16 2.17</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.5. Comfort levels from Setra guidelines[36].

<table>
<thead>
<tr>
<th>Comfort Level</th>
<th>Degree of comfort</th>
<th>Vertical acceleration limits (m/s^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL 1</td>
<td>Maximum</td>
<td>&lt; 0.5</td>
</tr>
<tr>
<td>CL 2</td>
<td>Medium</td>
<td>0.5 – 1.0</td>
</tr>
<tr>
<td>CL 3</td>
<td>Minimum</td>
<td>1.0 – 2.5</td>
</tr>
</tbody>
</table>
2.6. **Summary and Conclusions**

This chapter summarized the relevant aspects of the design, construction and performance testing of a 9 m long, epoxy-bonded pGFRP footbridge (MB) with a 1.5 m wide orthotropic sandwich deck. A novel pultruded sandwich deck constructed from individual pGFRP profiles and are adhesively bonded throughout, i.e. no mechanical bolts were used. The MB was designed to be a research tool in both areas of structural dynamics and pGFRP structures. The lightweight of MB render construction possible without any heavy machinery or tools. The construction process demonstrates the potential of epoxy-bonding for practical construction of similar footbridges.

Static tests were performed for several span configuration of the MB. From these tests, it is found that the deflection response is linear for all test scenarios. However, shear deformation has a significant influence on the bending stiffness for shorter spans (e.g. 4.39 m), accounting for 51% of the overall deflection. In contrast, they are negligible in the longer span tests. Consequently, it is concluded that the design of such structures should consider shear deformations for serviceability checks: they have far more significance for pGFRP structures than for structures of traditional materials.

Experimental modal analyses were performed to determine the modal properties of the MB. The damping ratios were estimated between 0.6% and 1.65%, which are relatively lower than comparable GFRP footbridges. This is most likely due to the simple geometry of the MB, having no complicated attachments such as handrails for example. The numerical model was able to predict the mode shapes and natural frequencies of the MB with reasonable accuracy. The first bending mode of the footbridge is more than 5 Hz and lies in the third harmonic range of human walking frequency. Comparison of mode shapes between numerical model and measurements reveal a clear asymmetry behaviour in the
measured bending modes (i.e. mode 1 and mode 3). Verifications showed that the mass of the shaker is somewhat balancing out anomalies in structural stiffness.

Overall, the experience gained from the design and construction of the MB presented in this chapter indicates the many significant advantages compared to more conventional forms of construction. The lightweight and good performance makes it easy to transport, but suited to common short footbridge spans. Furthermore, the design procedure showed that current numerical and analytical methods can be readily applied for design of such structure.

**Reference**


Chapter 2: Design, construction and performance of the GFRP sandwich panel footbridge


Chapter 3.

Full-field finite element model updating using Zernike Moment Descriptors for structures exhibiting localized mode shape features
Preface

Chapter 3 investigates the use of shape descriptors, in particular Zernike Moment Descriptors, in model updating procedures. The goal of Chapter 3 is to address shortcomings of the conventional model updating techniques i.e. correlation of mode shapes using the Modal Assurance Criterion (MAC). This was observed during model updating attempts for the pGFRP footbridge in Chapter 2, which results in the updated numerical models failing to capture the structural behaviour from measurements. Chapter 3 subjects the pGFRP footbridge in Chapter 2 as a reference structure to demonstrate the use of the proposed model updating framework. Overall, the model updating framework of the thesis is established in Chapter 3 and will be considered in Chapter 4 for validation of numerical framework.

The contents of this chapter-paper have been published in the journal Mechanical Systems and Signal Processing. The contents of this chapter-paper has been modified slightly, adding cross-referencing to Chapter 2, to allow smooth flow between chapters of the thesis.
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Abstract

The Modal Assurance Criterion (MAC) is a simple and powerful indicator to correlate experimental and analytical mode shapes. However, the MAC is not optimal for use in model updating of structures with localized mode shape features, since it is a global single index. Consequently, an alternative to the MAC is needed to optimally update numerical models of such structures for improved representation of structural dynamic behaviour. Developed from techniques in image processing and pattern recognition, Zernike moment descriptors (ZMDs) have been proposed as alternative correlation indicator in this study. They are highly sensitive to image features and so offer good promise to correlate mode shapes of relevant structures. This chapter presents a framework for the use of ZMDs in model updating of structures exhibiting localized mode shapes. A particular example of such structures increasingly being used in civil engineering are glass fibre-reinforced polymer (GFRP) structures, such as buildings and footbridges. The proposed approach is applied to a full-scale pultruded GFRP (pGFRP) footbridge with localized features to produce a more faithful numerical model. The effectiveness of both MAC and ZMD are considered. It is found that the use of ZMDs as the target response metrics improves mode shape correlation, giving improved FE representation of the pGFRP footbridge. This chapter informs on the applicability of ZMDs in FE model updating of structures with localized mode shapes and has recommendations for their use in practice.

KEYWORDS

Model updating; Finite element; GFRP, Zernike Moment Descriptors; Localized mode shape features.
3.1. Introduction

3.1.1. Motivation

Finite element (FE) analysis is a powerful analysis tool for structures. Discrepancies between measured and FE-predicted structural responses are often inevitable because of simplifications and uncertainties in FE modelling. For this reason, model updating is carried out to reduce differences between measured and numerical results, rendering an improved model for prediction of structural behaviour. Model updating optimises parameters in an FE model to best correlate experimental with numerical outputs. For structural dynamics, these outputs are typically natural frequencies and mode shapes, and are matched to the experimental results using an indicator\[1\]. The most widely used indicator in model updating is the Modal Assurance Criterion (MAC) \[2-4\].

The MAC provides a simple and powerful tool to evaluate the correlation of two mode shapes or modal vectors. The MAC is a single number that takes values between 0 (indicating no correlation) and 1 (perfect correlation). Since it was first presented in 1980 by Allemang \[3\], it has been used extensively to correlate mode shapes in model updating due to its simplicity—a single number. However, this means all information regarding mode shape difference is condensed into the scalar index of the MAC\[5\]. Consequently, it is difficult to detect localized mode shape differences from the MAC. Even though this drawback can be remedied somewhat by increasing the coverage of measurement points for mode shapes, such arrangements can be time consuming and difficult, especially in large and complex structures. Moreover, the scalar index MAC is not sensitive to small changes in mode shape \[4, 6\]. Together, these properties of the MAC imply that model updating using it for structures that may exhibit localized mode shape features could be non-optimal.

Although many forms of structure can exhibit localised mode shapes \[7-11\], a particular example of such structures being increasingly used in civil engineering are glass fibre-reinforced polymer (GFRP) structures. For GFRP structures, the use of GFRP sections is typically centred on structural
elements of buildings and footbridges for example. Pultruded GFRP (pGFRP) sections have glass fibre reinforcements that run in one direction only (the pultrusion direction)[12], making it an orthotropic material. The mechanical properties of pGFRP are highly dependent on properties related to the matrix of the composite such as stacking sequence, number of layers, mass and volume fraction of fibre, and the chopped strand mat (CSM)[13, 14]. Such properties, especially the CSM, impose randomness on the content of pGFRP material. As a result, pGFRP materials can have variations in mechanical properties among samples, even in a same manufacturing run[14]. Further to material variations, pGFRP structures are assembled in a variety of connection methods[12]. Bolted and adhesive bonding connection are most common [12, 15] but both types of connection can have non-ideal behaviour[16]. This can be caused by poorly tightened bolts or poor bonding, for example. By its nature then, the variation in material properties and connections of pGFRP structures means that assumptions about ideal structural behaviour may not be realized, especially in more complex designs [17, 18]. Such effects can result in localized changes in mode shapes[8]. Consequently, model updating of pGFRP structures using the scalar index MAC may no longer be optimal.

3.1.2. Shape descriptors

Shape descriptors are feature representations of an image, somewhat analogous to a Fourier series decomposition of a periodic signal into sine and/or cosine parts. For this study, images of mode shapes can be categorised into small numbers of descriptors, each highlighting the degree of similarity/dissimilarity/resemblance of corresponding shape features within. This nature of shape descriptors improves characterisation of mode shape features, encapsulating more information than the MAC. Because of this, shape descriptors are highly useful as an alternative indicator to correlate mode shapes. Several types of shape descriptors have already been proposed in the literature[19].

In the context of model updating, the use of shape descriptors in model updating procedures has been well-considered, most notably by Wang, Mottershead and their team[20]. They considered shape
Chapter 3: Full-field finite element model updating using Zernike Moment Descriptors

descriptors to correlate full field mode shapes [19, 21, 22] and strain field data[23-25], leading to updated FE models. However, the applications of shape descriptors in model updating are still limited and there are none for civil engineering structures such as building and footbridges. Consequently, the full potential of shape descriptors in model updating are not well understood, especially as they can be related to the growing area of pGFRP structures.

3.1.3. Contribution

This chapter introduces a model updating approach for civil structures exhibiting localised mode shapes using Zernike Moment Descriptors (ZMDs) as indicators for mode shapes. ZMDs have many advantageous properties as a shape descriptor[21]. One of the properties of these descriptors is its orthogonality, which originate from the orthogonality nature of ZMDs. This property allows for quick and effective feature characterization of images e.g. mode shapes. In turn model updating can then be carried out by correlating these descriptors. Based on this, the contributions of this work are: (1) to consider the effectiveness of ZMDs for model updating; (2) to evaluate the use of approximated full-field mode shapes using interpolation of discrete experimental mode shape data; (3) to provide a benchmark study for comparison between model updating approaches using both the MAC and ZMDs, and; (4) to make recommendations for the robust use of ZMDs in model updating.

3.2. ZMD-based model updating

The model updating framework is an iterative method similar to that described in [26]. In this work, the experimentally identified natural frequencies and mode shapes (expressed as ZMDs) are used to update FE models iteratively.

3.2.1. Sensitivity method

The iterative method uses a sensitivity method to update FE model parameters, as described in[2]. In each iteration, sensitivities of FE model parameters to proportional changes in outputs (i.e. ZMDs
and natural frequencies) are computed[27]. This essentially linearizes non-linear model updating problems. For this, a sensitivity matrix, $S^*$, is calculated for iteration $k$ as[28]:

$$
S_i^* = [S_{ij}^*] = \left[ \frac{\delta R_i}{\delta P_j} \right]
$$

(3.2)

where $\delta R$ is the difference in measurable outputs of the FE model due to an increment in model parameters $\delta P$, for response $i$, and parameter $j$. Thus far, the quantities are absolute. To compare sensitivities of different model parameters (natural frequencies and ZMDs), which may have different units or orders of magnitudes, a normalised sensitivity matrix, $S$, is used where differences in parameter changes and responses are divided by the initial values of the parameters and responses respectively. The resulting normalised sensitivity matrix is a matrix with dimensionless numbers and can thus be written as[28]:

$$
S_k = [S_{ij}] = \left[ \frac{\delta R_i}{\delta P_j} \right] [P_{ij}]
$$

(3.3)

where $P_{ij}$ represents a diagonal square matrix containing the updating parameter values. For all model updating in this work, a parameter increment of 0.1% was determined to ensure the ZMD changes are small enough to avoid non-linear increment effects in the sensitivity calculations. The iterative updating equation is [14]:

$$
\theta_{k+1} = \theta_k + \left[ S_i^T S_i \right]^{-1} S_i^T (R_m - R_k)
$$

(3.4)

where $\theta_k$ and $\theta_{k+1}$ are vectors containing the FE model parameter of current and next iterative steps; $R_m$ is the measured metric and $R_k$ is the output metric from the current iterative step. The final parameters are found when the Euclidian norm of the parameter changes between increments is sufficiently small ($< 1\%$):

$$
\varepsilon = \left\| \frac{R_m - R_k}{R_k} \right\| \leq 0.01
$$

(3.5)

In this study, the metric vectors $R_m$ and $R_k$ contain both the natural frequencies and ZMDs of the experimental and FE mode shapes respectively.
3.2.2. Zernike moment descriptors

Originally developed by Zernike [29] in 1934, ZMDs of an image are obtained from image decomposition using orthogonal polynomials as kernel functions. The ZMDs of a greyscale image, \( I(x,y) \), can be expressed as:

\[
Z_{n,m} = \frac{n+1}{\pi} \iint_{0 \leq x^2 + y^2 \leq 1} I(x,y) V_{n,m}^*(x,y) \, dx \, dy
\]  
(3.6)

where \( V_{n,m}(x,y) \) is the complete set of orthogonal polynomials introduced by Zernike [29] as:

\[
V_{n,m}(x,y) = V_{n,m}(\rho, \theta) = R_{n,m}(\rho) e^{i\theta}
\]  
(3.7)

where \( i = \sqrt{-1} \); \( n \) is a non-negative integer, representing the order of the radial polynomial; \( m \) is an integer subjected to constraints \( n \mid m \mid \leq n \); \( \rho \) is the vector length from the origin to \( (x,y) \); \( \theta \) the angle between vector \( \rho \) and x-axis; \( R_{n,m} \) is a radial polynomial defined as:

\[
R_{n,m}(\rho) = \sum_{s=0}^{(n-|m|)/2} (-1)^s \frac{(n-s)!}{s! \left( \frac{n+|m|}{2} - 2 \right)! \left( \frac{n-|m|}{2} - 2 \right)!} \rho^{n-2s}
\]  
(3.8)

and finally the asterisk (*) denotes the complex conjugate. Detailed derivations of ZMDs are given in Zernike [29] & Wang[21]. Each orthogonal polynomial represents a feature of the image as illustrated in Figure 3.1, where the degree of feature resemblance within the image is indicated by a corresponding descriptor, i.e. ZMD. Note that the sequential indexing of ZMDs used in this study follows those from Wang[21]; other forms of indexing also appear in different studies.

![Zernike polynomials and features](image-url)

Figure 3.1. Zernike polynomials and features up to 4th order (\( n = 4 \) according to Equation 5).
3.2.3. Approximation of full-field mode shape

According to Wang [25, 30], the use of shape descriptors are usually paired with mode shape data obtained using Digital Image correlation (DIC) measurements. DIC measurements provide full-field, continuous measurement of mode shapes, equivalent to having infinite sets of sensors on a surface of the structure—an ideal basis for using shape descriptors to describe mode shapes. However, DIC equipment is expensive, and often may not be practically feasible for some applications; for example, very large or complex structures. Furthermore, DIC measurements require a controlled environment to reduce noise effects on data[31], which is difficult to achieve for many civil structures. For example, conditions that lead to movement of DIC scanning devices introduce noise that can affect the quality of measured modal properties. Such issues outline the difficulty in obtaining full-field mode shapes for civil engineering structures.

For large-scale civil engineering structures, mode shapes are commonly obtained through discrete measurements from discrete instrumentation, typically accelerometers. Using this data, full-field mode shapes can be approximated. In this study, discrete mode shape measurements are converted into continuous mode shape images using interpolation over the unmeasured regions within the discrete measurement grid. The precision of the interpolated mode shapes can be controlled by the choice of image interpolation method. For this study, biharmonic spline interpolation is used to generate smooth mode shape surfaces, through the MATLAB function griddata [32]. The performance of mode shape images generated through interpolation for model updating will be evaluated later.

3.2.4. Image circle mapping

As can be seen from equations (2) and (3), ZMDs are defined on a unit circle. To obtain the ZMDs of a (more typical) non-circular (but plane) mode shape image, scaling and transformation are performed so that mode shape images are mapped into the domain of the Zernike polynomials. For
this, image mapping is carried out using an appropriate square-to-circle mapping. Two common metrics for image mapping are conformity (angle-preserving) and equiareal (area-preserving)[33]. The conformity metric, also known as shape preservation, is especially important to ensure that the features are retained and distortion is reduced as much as possible through the mapping transformation. Therefore, a conformal mapping technique, the Schwarz-Christoffel mapping, is used for the image transformations. The full mapping of a rectangular/square image with coordinates \( I(x,y) \) to a disc with coordinates \( I(u,v) \) is given by the Schwarz-Christoffel mapping as:

\[
\begin{align*}
    u &= \text{Re} \left( \frac{1-i}{\sqrt{K_e}} cn \left( K_e \frac{1+i}{2} (x+iy) - K_e, \frac{1}{\sqrt{2}} \right) \right) \\
    v &= \text{Im} \left( \frac{1-i}{\sqrt{K_e}} cn \left( K_e \frac{1+i}{2} (x+iy) - K_e, \frac{1}{\sqrt{2}} \right) \right)
\end{align*}
\] (3.9)

(3.10)

where \( x \) and \( y \) are the pixel coordinate of the original image; \( u \) and \( v \) the pixel coordinate after mapping; \( cn \) is the complex Jacobi elliptical function and \( K_e \) is defined as[33]:

\[
K_e = \int_0^{\pi/2} \frac{dt}{\sqrt{1 - \frac{1}{2} \sin^2(t)}} \approx 1.853
\] (3.11)

For consistency across all model updating operations in this study, circular mode shapes from image transformations are output at 400×400-pixel resolution. To illustrate, the transformed mode shape images are presented later in Figure 3.9.

It should be noted that other orthogonal polynomials are defined on a rectangular domain, such as Tchebichef polynomials. A reasonable consideration is the potential for numerical error of the ZMD-based model updating algorithm caused by the mapping from the rectangular (structure) domain to the circular (Zernike polynomials) domain. To address this aspect, Tchebichef polynomials are also considered later.
3.3. Demonstration structure

3.3.1. Structural description

The pGFRP footbridge in Chapter 2 is considered to demonstrate the performance of the model updating procedure. The footbridge is a 1.5 m wide 9 m long twin girder footbridge (Figure 3.2a). The footbridge has an average mass per unit length of 92.56 kg/m. The deck is an orthotropic sandwich panel made up of pultruded box sections sandwiched between flat sheets. The sandwich panel therefore has two fibre orientations where the fibres of flat sheets is aligned perpendicular to the fibres of the box profiles (Figure 3.2b). The deck is supported by two pGFRP I-beam girders. Both pultruded pGFRP I-beam girders are stiffened by transverse web stiffeners at intervals along its length. All pGFRP components are joined using epoxy adhesive bonding to offer full-composite action. No bolted connections or steel components were used. The structure is supported on four load cells placed at each end of the I-beam girders. The supports of the footbridge are adjusted to make the bridge level.

![Figure 3.2](image1.png)

Figure 3.2. The Monash University pGFRP sandwich footbridge: (a) photographic view; (b) Fibre orientation of the pultruded sections.

3.3.2. Numerical Finite Element model

A FE model of the footbridge was developed using LUSAS software[34]. All components of the footbridge including flat sheets, box profiles, and I-beam girders are modelled using 8-node
quadrilateral shell element (QT38). The element has six degrees of freedom at each node: translation in x, y, and z directions and rotation about nodal x, y, and z axes. Orthotropic pGFRP material properties are incorporated in this model. The material orientations of orthotropic pGFRP were assigned according to the fibre directions in the footbridge as shown in Figure 3.2b. Spring-supported boundary conditions have been incorporated at each end of both I-beam girders to allow for slight vertical movement of the load cells.

The stiffness of the vertical springs supports of the FE model is a key parameter influencing the dynamic characteristics of the model. Horizontal springs are not considered since only the vertical modes are of interest. All vertical spring support boundary were assumed to have the same stiffness. A manual calculation was performed to identify the spring stiffness values that provide reasonable match of the calculated natural frequencies to the experimental natural frequencies. A reasonable match was achieved when the order of natural frequencies from the FE model and experimental data are close (<0.01). The best match between FE model and experimental data was found for a vertical spring stiffness of $417 \times 10^6$ N/m. This value was taken as the sum of the stiffnesses of each support components (i.e. steel plate, load cell, and underlying timber planks).
3.3.3. Experimental results

Experimental modal analysis (EMA) was carried out to determine the dynamic properties of the footbridge, and to provide modal data for FE model updating. Both modal hammer and electrodynamic shaker excitations are used. However, just the shaker results are considered since, as will be seen, the mass loading effect is found to be useful in the study. The shaker is placed at the location shown in Figure 3.4 to ensure excitation of at least the first five vibrational modes of the footbridge. The footbridge was instrumented using 10 accelerometers in a measurement grid shown in Figure 3.4 (A1 to A10). System identification was performed on the acquired acceleration data using combined deterministic-stochastic subspace identification (CSI)[35], implemented in the MATLAB toolbox MACEC[36].
The first three mode shapes identified from the EMA are shown in Figure 3.5. The first six vibrational modes of the initial FE model were obtained from eigenvalue analysis, and the first three mode shapes are given in Figure 3.6. It can be seen from Figure 3.5 and Figure 3.6 that the mode shapes match very well. Indeed, the ordering of modes with regards to bending or torsion, are found to be the same for the first six modes. This is to be expected as the footbridge is built to closely resemble its FE model, without complicating attachments such as handrails, for example.

Figure 3.5. First three modes of vibration obtained from EMA: (a) $f = 5.86$ Hz; (b) $f = 10.02$ Hz; (c) $f = 18.14$ Hz. Contour is given to highlight the region of maxima and minima within mode shape feature in regards to the vertical direction.

Figure 3.6. First three modes of vibration from FE model: (a) $f = 5.95$ Hz; (b) $f = 9.62$ Hz; (c) $f = 20.27$ Hz. Contour is given to highlight the region of maxima and minima within mode shape feature.
Table 3.1 gives more detailed outputs of the EMA and FE model, along with comparisons. It can be seen that the initial FE model consistently overestimates the natural frequencies, and that the damping of the bridge is low. The MAC of each mode shape is also given in Table 3.1. Generally, a MAC value of more than 0.8 indicates high consistency between two mode shapes[37]. Overall the MAC values for modes 1, 2, 3, and 6 are high. The MAC of modes 4 and 5 are much lower, and these are primarily torsional modes. A close inspection of these modes from the initial FE model shows that these torsional modes are coupled modes, with horizontal sway motion in each mode. Since the acceleration data were sampled only in the vertical direction, the data is therefore unable to identify these modes accurately. Further studies are being conducted to verify these vibration modes, but because these modes are not accurately characterized they are excluded in the following updating process.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Type</th>
<th>Experiment Natural Frequency, $f$ (Hz)</th>
<th>Damping, $\xi$ (%)</th>
<th>Initial FE model Natural Frequency, $f$ (Hz)</th>
<th>Frequency Difference (%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1V</td>
<td>5.86</td>
<td>0.59</td>
<td>5.95</td>
<td>1.5</td>
<td>0.98</td>
</tr>
<tr>
<td>2</td>
<td>1T</td>
<td>10.02</td>
<td>0.96</td>
<td>9.62</td>
<td>4.0</td>
<td>0.97</td>
</tr>
<tr>
<td>3</td>
<td>2V</td>
<td>18.14</td>
<td>0.61</td>
<td>20.27</td>
<td>11.7</td>
<td>0.98</td>
</tr>
<tr>
<td>4</td>
<td>1C</td>
<td>20.60</td>
<td>1.65</td>
<td>23.87</td>
<td>15.9</td>
<td>0.64</td>
</tr>
<tr>
<td>5</td>
<td>2C</td>
<td>25.60</td>
<td>1.33</td>
<td>28.85</td>
<td>12.7</td>
<td>0.66</td>
</tr>
<tr>
<td>6</td>
<td>3V</td>
<td>37.54</td>
<td>0.92</td>
<td>39.52</td>
<td>5.3</td>
<td>0.85</td>
</tr>
</tbody>
</table>

3.3.4. Localized mode shape features

In structural dynamics, the term ‘mode localization’ refers to vibration confinement, i.e. where the magnitude of a specific part of the free-vibrational modes is relatively large relative to the rest of the mode[38]. Close examination of the experimental mode shapes in Figure 3.5 shows that (mostly) the first and third modes exhibit some localized features. For example, the highest displacement in mode shape 1 (Figure 3.5a) is localized off centreline in the longitudinal direction of the footbridge. This
observation was initially identified as the shaker mass loading since the shaker is placed off the
centreline in the longitudinal direction; the mass ratio of the bridge to shaker is about 17:1, and it was
postulated that the localized mode shape features were due to the shaker mass.

To examine the shaker mass loading in detail, five separate EMA were conducted by varying the
shaker to different positions, as shown in Figure 3.7 (positions 2 to 6). The first mode shape identified
from each EMA are also shown. It can be seen that most mode shapes are asymmetric, exhibiting
severe localized features, especially in positions 1 (the initial EMA setup) to 5 in Figure 3.7. Interestingly, as the shaker is placed towards positions 5 and 6 in Figure 3.7, the localized behaviour
decreases. Eventually, when the shaker is at position 6 (Figure 3.7) the localized behaviour has almost
disappeared. This indicates that the shaker’s mass is effectively ‘balancing out’ some asymmetry in
structural stiffness (since there are no obvious mass asymmetries). The results suggest that the south
side of the bridge (position 5 and 6 with reference to Figure 3.7) has higher stiffness than the north
side (position 1 and 4 with reference to Figure 3.7). Since the longitudinal stiffness is mainly
contributed by the I-beam girders, the imbalanced stiffness surely originates from properties related
to the composite behaviour of the I-beam and deck. Two possible explanations for this are: (1) poor
bond quality between the I-beam girder and deck and/or within the deck components (see Figure 3.2),
leading to reduced composite actions, and; (2) different material properties of the two longitudinal I-
beams, perhaps due to different manufacturing runs of the sections. Both potential causes can be
represented as a lowered elastic modulus for the affected composite beam which is referred to as
Beam 1 for the rest of this chapter.
3.4. Application of framework

The model updating framework presented earlier is applied to the demonstration footbridge to calibrate and improve the FE model; specifically, to capture the reduced longitudinal stiffness and improve natural frequency predictions. By identifying the FE model parameters that give the closest match, the structural origins of the observed localized mode shape features can be better understood. In practice, such knowledge assist damage detection and maintenance actions, for example. Several updating cases are considered in which the proposed ZMD-based updating framework is compared against conventional MAC-based updating. This allows comparison of the performances between full-field and scalar indicators, given later in Section 3.5.

3.4.1. FE modification and parameterization

Since the shaker mass has a large effect on the EMA mode shapes, the initial FE model is modified to include the shaker mass. This FE model is denoted as the modified FE model. The 47 kg mass is incorporated as a non-structural mass (i.e. does not contribute structural stiffness) on the footbridge deck as shown in Figure 3.8. The natural frequencies and MAC obtained for the modified FE model are given in Table 3.4 and Table 3.5 respectively: natural frequencies are found to be lowered of course, and the MAC has slightly improved.
The reduced longitudinal stiffness of the footbridge along Beam 1 can be represented with a lower elastic modulus, regardless of the precise source of the stiffness reduction. Consequently, the elastic modulus of pGFRP in the FE model are parameterized into three regions; a region for each I-beam and one for the deck, denoted as $E_{B1}$, $E_{B2}$ and $E_D$ respectively (Figure 3.8). Since the pGFRP’s fibre orientation of the I-beams runs in the longitudinal direction, only the longitudinal elastic modulus is adjusted ($E_{B1}$, $E_{B2}$). Parameter $E_D$ is the longitudinal elastic modulus of the deck structural elements shown in Figure 3.2b. Since most longitudinal stiffness is contributed by the I-beam girders, the contribution of $E_D$ is assumed to be minimal and is thus excluded in the model updating. Therefore, the parameters for the updating process are only the longitudinal elastic modulus of Beam 1 and Beam 2 ($E_{B1}$, $E_{B2}$). The elastic modulus of Beam 1 shall correspond to the beam with reduced longitudinal stiffness as shown in Figure 3.7. Material tests for the pGFRP components of the footbridge suggest a longitudinal elastic modulus of 24.07 GPa, and so this is used for $E_D$ and as the starting parameter value for $E_{B2}$; a starting reduced stiffness of 22 GPa is postulated for $E_{B1}$. It should be emphasized that the parameterization of elastic modulus acts as a surrogate to represent the reduced stiffness, which could be due to several sources mentioned previously.

Figure 3.8. FE model with added shaker mass and parameterized elastic modulus regions.
3.4.2. ZMD of mode shapes

The extraction of ZMDs from the experimental and FE mode shapes is next described. For the experimental mode shapes, full-field mode shapes are approximated as explained in Section 2.3. Mode shapes from the modified FE model are extracted from LUSAS as monochrome images. Both experiment-approximated and FE mode shape images have a resolution of 400×800 pixels to maintain consistent ZMD scaling which are affected by different image resolutions. As an illustration, the FE mode shapes are shown in Figure 3.9, along with the corresponding image obtained from Schwarz-Christoffel mapping. All circle-mapped images are output with resolution of 400×400 pixels.

Twenty-five terms of ZMDs corresponding to Zernike polynomials of 8th order are used to describe the features of the mode shape images. Because the values of ZMDs can be complex quantities, the magnitude of the ZMDs are used for comparison (termed ZMD amplitude). The ZMD amplitudes indicate the degree of resemblance of corresponding shape features (see Figure 3.10) within the mode shape contour image. The plots of ZMD amplitude decompositions for the experimental and FE mode shapes are shown in Figure 3.10 (a) and (b) respectively. The ZMD plots show that the twenty-five ZMD terms are sufficient to describe shape features of the first three mode shapes. From the ZMD decompositions, the localized mode shapes are successfully characterized: a tilt feature, ZMD 2 (see Figure 3.1), is represented in mode 1 for example.

Notably, a majority of significant ZMD amplitudes in both experimental and FE mode shapes are of similar magnitude and ranking. This gives confidence in the ability of ZMDs to characterize mode shape features. Similar to[20], each mode shape can be sufficiently described by retaining significant ZMDs. The ZMDs selected as the model updating target metric $R_m$ for Eq. (3) are marked in Figure 3.10 (as ranked by amplitude). These significant ZMDs can be used to better understand the origins of the structural asymmetry in each mode, shape such as the tilt features of mode shape 1 (represented
Further information can be obtained by analysing the sensitivities of these ZMDs towards each parameter change, which is conducted at the start of the model updating. For the significant ZMDs in Mode 1 (Figure 3.10), the magnitudes of ZMD 3 and 4 are interchanged for experimental and FE mode shapes. Close inspection indicates that the combination of ZMD 3 and 4 (see Figure 3.1) make up for the main shape of Mode 1, suggesting that the differences can be used to understand the features.

Figure 3.9. Monochrome mode shapes of FE model after circular mapping.

(a)
Close inspection of ZMD amplitudes in Figure 3.10 indicates different scaling of ZMD amplitudes between ranked pairs in each mode. For example, it can be observed that the differences in ZMD amplitudes for mode 3 are far larger than those of modes 1 and 2, even though the ranking of significant ZMD terms are similar. This highlights the scaling problem which occurs because the mode shape images are generated from different sources (i.e. from FE software and MATLAB). Intuitively, images from different software source will have different colour scaling, resulting in different scaling of ZMD calculations. With this, solely comparing the ZMDs from the two sources will result in non-meaningful comparison. To resolve this, the shape feature scale factor introduced by Wang et al [30] is implemented to scale the ZMD of the FE mode shapes to the experimental ZMDs. This feature scale factor is introduced to the $k$th FE mode shape images, given as:

$$SF_k = \frac{f_k^F f_{m,k}^F}{f_{m,k}^E f_{m,k}^F}$$  \hspace{1cm} (3.12)$$

where $f$ and $f_m$ are vectors containing the updating ZMD terms of the $k$th mode shape from FE model and experiment (‘measured’) respectively.

### 3.4.3. Tchebicief moment descriptors
As mentioned earlier, Zernike polynomials are based on a circular domain and the mapping of rectangular domain mode shape images to the circular domain may introduce errors. To examine this, Tchebichef Moment Descriptors (TMDs) which are defined on a rectangular domain are also used to perform model updating for comparison with the ZMD-based updating[23]. The framework for TMD-based model updating is similar to that of the ZMD-based model updating; only the shape descriptor differs. The updating results using TMDs are presented later in Section 5 and compared with those from the ZMD-based approach.

Detailed derivation of TMDs have been summarized in[23], [39], and[40]. Only a brief description is given here. The TMDs of a two dimensional intensity function i.e. mode shape image, \(f(x,y)\) can be obtained as the product of two one-dimensional Tchebichef polynomial given by:

\[
T_{nm} = \sum_{x=0}^{N_x-1} \sum_{y=0}^{N_y-1} \tilde{t}_n(x) \tilde{t}_m(y) f(x, y) \tag{3.13}
\]

where \(N_x\) and \(N_y\) are the total number of image pixel in the \(x\) and \(y\) dimensions of the image in which the scaled Tchebichef polynomials is calculated, \(n\) and \(m\) denote the Tchebichef polynomials of the \(x\) and \(y\) dimension respectively, and \(\tilde{t}_n(x)\) is the scaled Tchebichef polynomial in the \(x\) dimension defined as:

\[
\tilde{t}_n(x) = \frac{t_n(x)}{\beta(n, N)} \tag{3.14}
\]

where \(\beta(n,N)\) is the scaling constant independent of \(x\) with definition outlined in[41]. The discrete Tchebichef polynomial, \(t_n(x)\) up to order of \(N\) is defined as[41]:

\[
t_n(x) = (1-N)_n {}_3F_2 \left(-n,-x,1+n;1,1-N;1 \right) \tag{3.15}
\]

where \( {}_3F_2 \) is the generalized hypergeometric function outlined in [41] and[39].
For TMD calculations, each mode shape images were decomposed into thirty descriptors: the first six Tchebichef features are shown in Figure 3.11. The TMD decompositions of the first three FE mode shapes are shown in Figure 3.12. Similar to ZMDs, TMDs are also effective in characterizing the features of the mode shapes which are distinctive from ZMDs based on the shape feature each TMDs entails (Figure 3.11). For the TMD-based updating, the first six maximum TMDs are selected as the target response metrics, $R_m$.

![Figure 3.11](image1.png)

Figure 3.11. First six Tchebichef polynomials with corresponding features mapped onto footbridge’s domain.

![Figure 3.12](image2.png)

Figure 3.12. TMD amplitudes of the first three mode shapes FE model.

### 3.4.4. Validation of framework

Before applying the experimental mode shape results to update the FE model, the framework is first validated through a simulated model updating problem, based on results from the modified FE model. As a pseudo-experimental result, parameters $E_{B1}$ and $E_{B2}$ in the FE model were set as 15 GPa and 24 GPa respectively to obtain the target ZMDs of mode shapes. Then, model updating is started with initial parameter values of 22 GPa and 24 GPa for $E_{B1}$, $E_{B2}$ respectively. The parameter updating
history is given in Figure 3.13 for both ZMDs and TMDs: both parameters converged to the prescribed values within 10 iterations. The initial and final updated ZMD amplitudes are shown in Figure 3.14: all ZMD amplitudes converged to the target ZMD amplitudes. Further, both ZMDs and TMDs yield consistent convergence (Figure 3.13). This indicates that the rectangular to circular mapping required for ZMDs has little influence in this case.

Figure 3.13. Parameter updating history of $E_{B1}$ and $E_{B2}$ using ZMDs and TMDs for updating (target 15 GPa and 24 GPa).

Figure 3.14. ZMD amplitudes of FE mode shapes considered (numbered according to Figure 3.10) at start and end of simulated updating. The ZMD rankings are grouped by modes 1 to 3, from left to right.
Chapter 3: Full-field finite element model updating using Zernike Moment Descriptors

3.4.5. Model updating cases

Model updating cases considered in this study are as listed in Table 3.2. In each case the ZMD-based framework is conducted (sub-cases B) along with the conventional MAC (sub-cases A). Case 1 considers just a single parameter $E_{B1}$ for model updating which implies perfect knowledge of the elastic modulus for the remainder of the footbridge ($E = 24.07$ GPa). Case 2 represents a more realistic model updating, considering the elastic moduli for the two parameterized regions, $E_{B1}$ and $E_{B2}$, as described earlier. For all cases, similar starting values for parameters $E_{B1}$ and $E_{B2}$ were as given previously. In both Cases 1 and 2, only the ZMD/MAC indicators are used for updating. Case 3 extends Case 2 to include the natural frequencies in the response vector, and so should lead to improved natural frequency match. In addition, the TMD-based updating (sub-cases C) is also considered for Case 2 and 3.

Table 3.2. Comparison of model updating cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Sub-case</th>
<th>Updating Parameter(s)</th>
<th>Indicator(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>$E_{B1}$</td>
<td>MAC</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$E_{B1}$</td>
<td>ZMD</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>$E_{B1}$ and $E_{B2}$</td>
<td>MAC</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$E_{B1}$ and $E_{B2}$</td>
<td>ZMD</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>$E_{B1}$ and $E_{B2}$</td>
<td>TMD</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>$E_{B1}$ and $E_{B2}$</td>
<td>MAC + nat. freq.</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$E_{B1}$ and $E_{B2}$</td>
<td>ZMD + nat. freq.</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>$E_{B1}$ and $E_{B2}$</td>
<td>TMD + nat. freq.</td>
</tr>
</tbody>
</table>

For all updating sub-cases, the responses (ZMD, MAC, TMD, and natural frequencies) of the first three modes are considered for the target response metrics, $R_m$ of the updating process. Lower and upper bounds of parameter changes are not enforced in this study purely to compare the performance of the indicators (the MAC and ZMDs). Nevertheless, the physical meaning of the updated parameters will be evaluated later in Section 5.
3.5. Model updating results

3.5.1. Summary of results

Model updating results for the three cases (each with sub-cases A, B, and C) are given in Table 3.3 and Table 3.4. In each case the first three mode shapes and frequencies are considered. As will be discussed below, in Case 2B a clear convergence was not achieved; in such cases, the values about which oscillations occur are given instead, and marked with an asterisk (*). Table 3.5 shows the starting and final values of the response metrics (the MAC and ZMDs) for Case 3, as an example of the model updating optimization.

Table 3.3. Model updating results for the three cases (and sub-cases) considered (see Table 3.2). Parameter values marked with an asterisk (*) did not fully converge and are approximated by the value about which oscillations occur.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Modified</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1A</td>
</tr>
<tr>
<td>$E_{B1}$ (GPa)</td>
<td>22.00</td>
<td>24.8</td>
</tr>
<tr>
<td>$E_{B2}$ (GPa)</td>
<td>24.07</td>
<td>-</td>
</tr>
<tr>
<td>$f_1$ (Hz)</td>
<td>5.42</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 3.4. Model updated natural frequencies (Hz) for the three cases (and sub-cases) considered (see Table 3.3), showing percentage errors in brackets.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Exp.</th>
<th>Initial</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1A</td>
<td>1B</td>
</tr>
<tr>
<td>1</td>
<td>5.86</td>
<td>5.42 (7.5)</td>
<td>5.44 (7.2)</td>
</tr>
<tr>
<td>2</td>
<td>10.02</td>
<td>9.31 (7.1)</td>
<td>9.32 (7.0)</td>
</tr>
<tr>
<td>3</td>
<td>18.14</td>
<td>17.21 (5.1)</td>
<td>17.3 (4.6)</td>
</tr>
<tr>
<td>4</td>
<td>20.60</td>
<td>21.05 (2.2)</td>
<td>21.1 (2.4)</td>
</tr>
<tr>
<td>5</td>
<td>25.60</td>
<td>24.25 (5.3)</td>
<td>24.3 (5.1)</td>
</tr>
<tr>
<td>6</td>
<td>37.54</td>
<td>34.52 (8.0)</td>
<td>34.7 (7.6)</td>
</tr>
</tbody>
</table>

1 FE model with added shaker mass  
*Natural frequencies from approximated parameter values in Table 3.3

Table 3.5. Comparison of response metrics (the MAC and ZMDs) in model updating case 3A and 3B.

<table>
<thead>
<tr>
<th>Case</th>
<th>Mode</th>
<th>Feature Number</th>
<th>Starting Value</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 3A</td>
<td>1</td>
<td>-</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>MAC</td>
<td>2</td>
<td></td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>9648</td>
<td>9702</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>9523</td>
<td>9576</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>1378</td>
<td>2179</td>
</tr>
<tr>
<td>Case 3B</td>
<td>1</td>
<td></td>
<td>9700</td>
<td>9819</td>
</tr>
<tr>
<td>ZMD number* &amp; amplitude</td>
<td>2</td>
<td>2</td>
<td>2076</td>
<td>2078</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
<td>1803</td>
<td>1808</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>8072</td>
<td>8060</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>3368</td>
<td>3351</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>2883</td>
<td>2873</td>
</tr>
</tbody>
</table>

*as ranked according to Figure 3.14
3.5.2. Analysis of results

For Case 1, the model updating iterations are shown in Figure 3.15. Parameter $E_{B1}$ converged after 19 iterations for Case 1A (MAC), and after just six iterations for Case 1B (ZMD). Clearly both the speed of convergence and final values are significantly different (see Figure 3.15 and Table 3.3). Indeed, for the MAC (Case 1A), the updated parameter is not physically valid because the elastic modulus should be lower than the initial value, corresponding to the observed reduced longitudinal stiffness from the EMA. Conversely, for ZMD-based updating (Case 1B), the result clearly describes the reduced longitudinal stiffness, and is significant—about 20% reduction in stiffness. However, the natural frequencies for either Case 1A or Case 1B are not particularly good. This is unsurprising, given that natural frequency was not included in the response metric, $R_m$ of the updating process. Omission of natural frequencies does give the model updating algorithm more freedom to match mode shapes more accurately, and so this case is nonetheless a useful indicator of the footbridge’s behaviour.

![Figure 3.15. Case 1A (MAC) and 1B (ZMD) parameter updating history ($E_{B1}$).](image_url)

For Case 2, the parameter updating histories for $E_{B1}$ and $E_{B2}$ from both sub-cases are shown in Figure 3.16. The results show that the parameters in the MAC-based updating (Case 2A) achieved
convergence of values. In contrast, convergence of parameters was not observed for the ZMD-based updating (Case 2B). Instead, these parameters oscillate about values lower than their starting values. Interestingly, the local oscillations of both parameters are of similar amplitude. The oscillations could be due to several reasons which are examined later. Nevertheless, the values about which the oscillations occur (shown by * in Table 3.3) indicate that indeed $E_{B1}$ is less than $E_{B2}$. Again, similar to Case 1, since the natural frequencies are not a response metric the values are not well-matched to the EMA results (see Table 3.4). It is interesting that the TMD results (Case 2C) converge to quite different values compared to MAC and ZMD. It suggests that TMDs may not be quite as sensitive as ZMDs to localized mode shape features.

![Figure 3.16. Case 2A parameter updating history ($E_{B1}$ and $E_{B2}$).](image)

To improve the natural frequency estimation, Case 3 extends Case 2 to include natural frequency of the first three modes as the response metric, $R_m$. The parameter updating histories of Case 3A and 3B are shown in Figure 3.17: convergence of parameters $E_{B1}$ and $E_{B2}$ was achieved in both sub-cases. The natural frequencies from the updated FE model of both cases are given in Table 3.4. The majority of natural frequencies are well-matched in addition to the improved structural behaviour representation (reduced longitudinal stiffness of $E_{B1}$). The convergence of response metrics (the MAC
and ZMDs) of Case 3 are summarised in Table 3.5. The results show that the mode shape objectives (the MAC) in Case 2A is near to 1, indicating that the mode shapes are very close to optimal. It can be seen that although the MAC can sufficiently update the FE model (i.e. with the reduced longitudinal stiffness), the proposed ZMD-based updating framework results in a closer matched frequency. Furthermore, the performance of TMDs as a mode shape indicator is now (compared with Case 2C) similar to the other measures. The results in Table 3.5 indicates that the ZMDs are more sensitive than the MAC as mode shape indicators and provides more sensitivity when updating natural frequency objectives, resulting in closer match of frequencies. This can be very beneficial in updating more complex pGFRP structures considering more updating parameters.

It should be noted that the final values of the updating parameters ($E_{B1}$ and $E_{B2}$) are beyond physically-meaningful ranges of the pGFRP materials. Material tests for the pGFRP components of the footbridge suggest a longitudinal elastic modulus of 24.07 GPa with variation of ±20 %. This means there is likely another phenomenon contributing to the overall longitudinal stiffness of the footbridge, not explicitly considered in the updating process. Consequently, the parameters $E_{B1}$ and $E_{B2}$ are updated to physically non-meaningful values to implicitly account for other sources of stiffness changes.

![Figure 3.17. Case 3B parameter updating history ($E_{B1}$ and $E_{B2}$).](image)
3.5.3. Implementation Issues

From the demonstrated application of the ZMD-based model updating, several aspects of note for implementation to practice are noted. Of course, these may be related solely to the nature of the demonstration footbridge and may not be applicable for other updating cases. Nevertheless, these issues could prove useful to know for practical application of ZMD-based updating for civil engineering structures.

In Case 2B (ZMD-based model updating), it was observed that the parameter changes exhibited oscillations. Further examination of the cause reveals several close ZMD amplitudes for many combinations of $E_{B1}$ and $E_{B2}$ around the vicinity of the oscillating values. Because of this, there can be many similar sets of parameters that result in suitable ZMD amplitudes. Consequently, the updating algorithm oscillates between suitable sets of parameters. This problem is not apparent when natural frequencies are included in the metric since the updating parameters must fit the natural frequency objective.

An issue was identified from an attempt of model updating using mode shapes from EMAs of different shaker positions, as described in Section 3.4. Due to the circular basis of ZMDs, the ZMD amplitudes are rotationally invariant with respect to a rotation of the image (see Figure 3.1)[21]. Hence ZMD-based updating cannot distinguish between two images, when one is a rotated version of the other. To give a more practical example from the footbridge, Figure 3.18 shows two FE mode shapes which have similar localized effect, but on opposite sides: one is the flipped version of the other. This set of mode shapes were obtained from model updating checks using shaker position at the stiffer side of the footbridge (refer to position 6 in Figure 3.7). Due to the rotational invariance of ZMDs, the ZMD amplitudes calculated for both mode shapes are identical. For the footbridge model, this means that ZMD-based updating cannot distinguish when parameter values are swapped.
between Beam 1 and Beam 2. Thus, the updating history can exhibit ‘swapping’ of values between parameters that influence the mode shape such that it exhibits rotational symmetry. For practical applications, it is imperative to ensure that the updating FE models have no rotationally symmetric mode shapes which results from certain combination of model parameters in order to avoid non-convergence due to this rotational invariance property of ZMDs.

The need for mapping between the circular ZMD domain and the rectangular structure domain does not appear to negatively influence the results. This is based on the comparative performance of TMDs as a descriptor, which do not require mapping between domains. Interestingly, the ZMDs appear more sensitive than TMDs in spite of the additional need for mapping (compare Case 2 with Case 3).

Lastly, the selected interpolation method (i.e. smooth interpolation) to generate the interpolated mode shapes from discrete measurement points may not be fully justified in all types of structures. For example, several structures may have more prominent localised effect where the smooth assumption may be too strong in that regard. However, it is commonly assumed for full-scale civil engineering structures that mode shapes can be sufficiently interpolated from discrete measurement points[42-44]. Nevertheless, validation checks are recommended when the presence of highly localized mode shape feature is suspected.
3.6. Summary

This chapter presents a model updating framework for civil engineering structures exhibiting localized mode shape features. The framework uses ZMDs to correlate mode shapes, which are shown to be very effective and sensitive to mode shapes features. Further, it is proposed that full-field mode shape images are interpolated from discrete experimental measurement points. In addition, this model updating framework utilizes interpolated mode shape images generated from discrete FRF measurements over DIC mode shape measurements, which are typically more ideal for ZMD correlation. The use of interpolated mode shapes is an effective alternative to DIC measurements, of which can be costly and not practical for large and complex structures.

The framework is applied to a pGFRP footbridge which exhibits localized mode shapes. These localized mode shapes were indistinguishable by the MAC, leading to unsatisfactory model updating. ZMD-based updating allowed characterization of mode shapes into a small number of descriptors, each describing mode shape resemblance to corresponding shape features. Using this property, model updating was conducted by evaluating the significant ZMDs in each mode shape. The ZMD-based model updating performance was evaluated against the MAC-based model updating. For this demonstration structure, the results show that the MAC is unable to update the parameters optimally, while it is successfully achieved using ZMDs. The use of ZMDs paired with natural frequencies for the target objective is more successful in updating the FE model. Although the final updated model parameters ($E_{B1}$ and $E_{B2}$) are not within reasonable bounds, it captures other factors influencing the structural behaviour of the footbridge.

However, for practical application, it is recommended that the ZMDs to be carefully examined to check for instances where rotational invariance of ZMDs leads to non-optimal convergences. Further, future work is needed to evaluate the performance of the interpolated mode shapes by comparing with DIC measurements. Overall, the proposed model updating framework is found to be well-suited to
model updating of structures which may exhibit localized mode shape features. As such, it should find relevance for those involved in structural identification of civil structures, including those specialized in the growing field of pGFRP structures.

Reference

Chapter 3: Full-field finite element model updating using Zernike Moment Descriptors


Chapter 3: Full-field finite element model updating using Zernike Moment Descriptors


Chapter 4. Effect of structural representation on the parameter identification of the human spring-mass-damper model
Chapter 4: Effect of structural representation of parameter identification of SMD model

Preface

Chapter 2 presented evidence of human-structure interactions (HSI) towards the vibrating system of lightweight pGFRP sandwich structure. This observation warrants the need for HSI representations within the numerical framework of the thesis in order to achieve more reliable vibration serviceability assessments. Lack of HSI representation can lead to overestimation of numerical predictions (as seen in Chapter 2). In turn, the vibration assessment of structures can result in unsatisfactory outcomes, where in fact it is serviceable.

The goal of Chapter 4 is to present the numerical framework of the thesis. The framework comprises a simple human interactive model (i.e. the moving spring-mass-damper model) to represent human walking. The paper in Chapter 4 utilises the numerical framework in order to investigate a research hypothesis pertinent to HSI and human interactive models: whether the explicitly-proposed human model parameters are inherent to the type of structural representations adopted during its parameter identification procedure. This chapter-paper subjects the pGFRP footbridge in Chapter 2 with the numerical framework for the study. Chapter 4 also adopts the model updating framework from Chapter 3 as part of the study. The content of this chapter-paper has been modified to ensure a smooth flow between thesis chapters.

This chapter-paper is prepared for the following publication:

Chapter 4: Effect of structural representation of parameter identification of SMD model

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Chapter 4: Effect of structural representation of parameter identification of SMD model

Abstract

The simple spring-mass-damper (SMD) model has recently become more widely used to represent human-structure interaction (HSI) in vibration analysis. To use the SMD model, parameters have been proposed in the literature to simulate walking responses of finite element (FE) models. However, the explicitly-proposed parameters have been based on a specific representation of structures. Specifically, most studies adopted an Euler-Bernoulli beam model to represent structure under study. Thus, the question arises, as to whether the determined parameters of SMD model are consistent for other structural representations of the same structure. This chapter presents an experimental-numerical framework to investigate dependency of calibrated SMD parameters for different representations of a structure. Thus, a reference structure is numerically modelled using two different representations: a one-dimensional (1-D), and two-dimensional (2-D) finite element model. From both numerical models, SMD parameters are identified by optimizing the acceleration predictions from numerical simulations with the experimental measurements. Both structural models are first updated to predict accurately the dynamic properties of the reference structure. Two different sets of SMD parameter were obtained from identification using the two distinctive structural models of the reference structure. From the results, it is concluded that the calibrated SMD parameters are not invariant to the structural representation and so predictions of the response of real structures is inherently biased. Discussions on the results and its implications towards practical HSI analysis are presented.

KEYWORDS

Human-structure Interaction; Spring-mass-damper model; Finite element; human-induced vibrations.
4.1. Introduction

4.1.1. Background

Human occupants of civil engineering structures induce dynamic forces onto occupied structures due to various activities, such as walking or running. In addition to generating load, the presence of humans can affect the dynamic behaviour of the structure they occupy; this effect is termed human-structure interaction (HSI). For example, the presence of human adds sources of damping to the structure which in turn reduces vibration responses [1, 2]. Furthermore, the effects of HSI are more prominent when the mass of occupants are comparable to the structure mass [3]. This is important for the vibration design of lightweight structures, whereby the effects of HSI need to be accounted to enable safe and economical designs of civil engineering structures. Currently, design guidelines for vibrations (e.g. Setra [4], HIVOSS [5], Eurocode 5 [6], and BS 5400 [7]) are based on the deterministic moving force (MF) model which does not account for HSI [8]. Consequently, the MF model can lead to overestimations of responses [8-10] and designs may fail serviceability check when in fact it is serviceable. Recognizing this matter, an increase in HSI representation for vibration analysis of civil engineering structures has recently developed [11-18].

Instead of the MF model, it is more realistic to model a walking person is using a single-degree-of-freedom (SDOF), moving spring mass damper (SMD) model. The SMD model consist of a single mass, stiffness, and damping parameter that represent the behaviour of the walking person in the vertical direction [18]. Worth nothing, the SDOF parameters of the SMD model are surrogate to the complex system of the human body, which comprises of various mass, stiffness, and damping properties for different parts of the body [19]. The SMD model is coupled to the structure model and maintains contact throughout its motion. In addition, the impose ground reaction force (GRF) from walking motion are acted at the contact point of the
SMD model with the structure model to simulate the active excitation of the SMD model in the vertical direction. Worth noting, there are other types of human models, such as the bipedal and inverted pendulum representations - this work only considers the SDOF variant of the moving SMD model. To date, the moving SMD model has been used in numerous HSI studies (e.g. [8, 11, 21-23]).

To use the SMD model, accurate parameters of mass, stiffness, and damping are required to predict vibration responses. To date, a wide range of SMD parameters have been proposed to represent the human body. Within the literature, majority of parameter identification studies stems from the field of biomechanics (e.g. study of human gait), which are reported in literature reviews by Elis et al. [1], Jones et al. [20], Caprani et al. [18], Shahabpoor et al. [24], and Zhang et al. [25]. However, many HSI studies adopts SMD parameters from the biomechanics field, where majority parameters have been identified from measurements (accelerations and forces) of walking on rigid surface (e.g. [8, 15, 22, 26, 27]). Specifically, SMD parameters should be calibrated on vibrating surfaces of structures for application in vibration serviceability design and assessment purposes.

Recent studies in the field of human-induced vibrations have identified SMD parameters to represents human walking in vibration analysis. Studies of Toso et al. [28], Ahmadi et al. [29], Silva and Pimentel [30], Zhang et al. [31], and Shabahpoor et al. [32, 33] are few examples that have proposed SMD parameters that are aimed towards vibration serviceability purposes. Silva and Pimentel [30] proposed SMD parameters based on empirical formula which are obtained from curve fitting on experimental walking data. In regards to explicitly proposed SMD parameters, a damping ratio of 0.23 - 0.39 and natural frequency of 1.78-1.92 Hz has been reported by Zhang et al. [31]. In addition, Toso et al. [28] reported damping ratio of 0.35 to 0.57 and natural frequency of 1.35 - 2.12 Hz. As can be seen, the SMD parameters can have
In regards to numerical modelling, the supporting structure can be modelled using different structural representations. The representations are based on the degrees-of-freedom (DOFs) of structural models. This is the case for finite element (FE) methods, where the DOFs of structural models are dependent on the element type e.g. beam, bar, or shell elements. The appropriate structural representation is justified by the level of refinement required for the structural analysis. For example, a one-dimensional (1-D) representation using beam elements would be reasonable to model a bridge structure since its transverse width (and behaviours) is relatively small in comparison to its longitudinal length [46]. Conversely, a structure with transverse width that is comparable to its length (e.g. a floor) can be represented with a two-dimensional (2-D) representation in order to account for behaviours in the transverse direction (e.g. torsion).

It should be emphasized that almost all HSI studies use a 1-D Euler-Bernoulli beam model with uniform cross section to represent the supporting structure (e.g. [8, 15, 20, 22, 26, 27, 32-35]). In contrast, only a handful of studies (following [26, 36]) considered more elaborative representations, i.e. 2-D representation. Furthermore, all parameter identification studies mentioned previously have based the structure under study with the 1-D beam representation. In other words, there are no parameter identification studies that consider more elaborate structural representations beyond the 1-D representation. This leaves the possibility that the identified SMD parameters could be different when considering a different structural representation. In turn, the question arises whether the explicitly-proposed SMD parameters are suitable to be used for different structural representations. In other words, will the SMD models yield results that are invariant of the structural representations under consideration?
Chapter 4: Effect of structural representation of parameter identification of SMD model

Therefore, a precise study of this phenomenon needs to be conducted on parameter identification procedure.

4.1.2. Approach of this work

Drawing from the review in Section 4.1.1, the explicitly-proposed SMD parameters have been based of a specific structural representation of the subjected structure (i.e. 1-D representation). In turn, it is postulated that during parameter identification processes, the fitted SMD parameters inherit information regarding the structure representation while matching the target metrics of experimental data. This information regarding structural representation can include the number of DOFs, boundary condition modelling, and type of elements (underlying theory) for example. With this, it is also postulated that the proposed SMD parameters are inherent of any modelling errors. Therefore, the hypothesis of this chapter is that the parameters of an SMD model is a function of, not only the GRFs (i.e. induced forces), but also the representation of structure. If the hypothesis is true, it means that parameter identification studies should consider various structural representations when proposing SMD parameters for applications. The outcome of this hypothesis is fundamental for SMD models, furthering the knowledge on its applicability in numerical modelling.

In this chapter, a numerical-experimental framework is adopted to test the hypothesis. The framework entails parameter identification of SMD models for different structural representations of a reference structure. An overview of the SMD parameter identification procedure and evaluation process is shown in Figure 4.1. A moving SMD model paired with the GRFs is used to simulate HSI in a walking experiment of a reference test structure. The structure is modelled using FE method into two different representations, i.e. 1-D and 2-D. Prior to parameter identification, both models are first updated to predict accurately the modal properties based on the experimental measurements of modal properties. Subsequently, the
SMD parameters are identified using both structural representation models by fitting the simulated responses to the measurements. From the identification procedure in Figure 4.1, any differences of the identified SMD parameters, $\varepsilon$, from the different structural representations is the evidence that the SMD parameters are influenced by the structural representations.

![Diagram](image)

**Figure 4.1.** Schematic overview of the parameter identification procedure, showing the hypothesized differences, $\varepsilon$, between SMD parameters from different FE models.

### 4.2. Experimental Programme

#### 4.2.1. Structural description

The pultruded GFRP footbridge in Chapter 2 (denoted as the MB) is considered as the benchmark structure for this study (Figure 4.2). In this chapter, the MB has included a force plate located at midspan and a stiff foam layer which provides the flushed surface for the 75 mm height of the force plate. This flushed surface was needed for walking experiments – to allow test subjects to not perceive the force plate during walking trials. In turn, the covered MB has a mass per unit length of 100.1 kg/m: the mass of the bare structure is 92.56 kg/m [37].
From Chapter 2, experimental modal analysis (EMA) was carried out to identify the modal i.e. natural frequencies, mode shapes and damping ratios of the MB. The first mode of the MB is a bending mode having a natural frequency and damping ratio of 6.1 Hz and 0.6 % respectively (following hammer tests). The second mode is a torsional mode with natural frequency and damping ratio of 10.0 Hz and 1.0 % respectively (following shaker tests). Accordingly, the MB is excitable by the third harmonic component of common walking frequencies (e.g. 1.6-2.2 Hz).

4.2.2. Walking experiments

Following EMA, walking experiments were performed to evaluate the MB responses under pedestrian excitations [39]. The walking experiments is based on a larger scale research project, which details of the instrumentations and procedures are presented in [39], herein only the key information summarized for brevity. As mentioned, the MB is covered with the stiff foam layer as shown in Figure 4.3. The force plate was used in [39]: in this study the measured forces from force plate is not considered. A total of 18 test subjects, which covers a range of mass from 40 kg up to 120 kg, participated in the walking experiments. In each walk, test subjects traversed the MB at selected pacing frequencies to excite the MB. Each test subject performed 15 acceptable walking trials. During each trial, vertical acceleration responses at mid span of
the MB were measured using accelerometers. Tekscan F-scan in-shoe pressure sensor were used to the time series of footfall forces [39]. From this, a continuous measured walking force is obtained by adding the footfall time series of both feet.

![Figure 4.3. Setup for walking experiments. (after [39]).](image)

### 4.3. Numerical formulations

#### 4.3.1. Finite element representations

The MB was idealized into a 1-D FE model and a 2-D FE model. For the 1-D FE model, ten 1-D Euler-Bernoulli beam elements were used to represent the entire MB. Figure 5 illustrates the 1-D FE model of the MB with the simply-supported boundary conditions, which was implemented using MATLAB scripts. As it will be described later, a moving SMD model is also depicted in Figure 4.5.
Chapter 4: Effect of structural representation of parameter identification of SMD model

The 2-D FE model of the MB is shown in Figure 4.5. The pGFRP sandwich panel deck was modelled using a single plate element layer. First-order shear-deformable theory is considered for the plate elements to account the large shear deformation of the pGFRP sandwich panel [40]. Each plate element has four nodes, each with three degree-of-freedoms (DOFs) – one vertical translation and two rotational. Representation of the entire deck (box and flat panels) was achieved by defining equivalent orthotropic properties to the plate elements. The supporting I-beams were modelled with 1-D Timoshenko beam elements which are incorporated in-between plate elements. Element offset properties (e.g. second moment of area, \( I_{xx} \)) were considered to represent the composite slab-beam section. Simply-support condition is considered at along the edge at both ends of the model. This simplification is justified by the significantly larger transverse stiffness (shorter span combined with intermediate T-sections between I-beams) compared to the longitudinal stiffness. The additional mass from stiff foam layer and force plates are considered in the FE model as a non-structural mass onto each plate element.

Figure 4.4. 1-D FE representation of the MB with a moving SMD model of a test subject (after [12]).
4.3.2. Formulation of moving SMD model

To date, formulations of interactive human models on 2-D representation of structures have been provided only by Emus [36] and Mulas et al. [26]. Mulas et al. [26] provided analytical formulations for a bipedal model, which however, the structure model of the as studied footbridge is constructed from beam and link elements. Emus [36] provided FE formulations for a moving mass model on a 2-D plate/shell model. However, the moving mass model is fundamentally different to a SMD model as it does not include stiffness and damping parameters. As discussed earlier, there is no study present in the literature that details formulation of SMD models on 2-D FE representation of structures, which is in essence the proposed model in this chapter.

In this section, the formulations of the SMD model generalized for both 1-D and 2-D FE representations of a structure. The human body is a complex system which comprises various mass, stiffness and damping properties for different parts of the body [41]. The equation of motion of the FE model for dynamic analysis is given as:

\[
M_b \ddot{\mathbf{d}} + C_b \dot{\mathbf{d}} + K_b \mathbf{d} = \mathbf{N}^T f(x, y, t)
\]  
(4.1)
Where $M_b, C_b$ and $K_b$ are the mass, damping and stiffness matrices of the structure respectively, $d$ represents the DOFs of the plate model, $N$ is the global shape functions of the plate model. As will be needed later, the first derivative of the shape functions with respect to the displacement (denoted by subscripts $x$ and $y$) are given as:

$$N = p(x) q(y)^T \quad (4.2)$$

$$N_x = p'(x) q(y)^T \quad (4.3)$$

$$N_y = p(x) q'(y)^T \quad (4.4)$$

where $p(x)$ and $q(y)$ are the Hermite polynomials that represent shape functions of the plate (derivatives denoted with $'$ superscript) in $x$ and $y$-directions respectively. In the 1-D FE model, the Hermite polynomials in the $y$-directions is excluded, which leaves the 1-D shape functions [18].

The moving SMD model has a point contact with the FE model that is maintained throughout its movement. The interaction force at the point of contact, $f(x,y,t)$, is described as a sum of two forces: (1) the walking GRF of that person from a non-vibrating surface, and (2) the GRF generated by the person’s SMD model when excited by the structure’s vibration [15]. The $f(x,y,t)$ can be written as [18]:

$$f(x, y, t) = G(t) + c_p [\ddot{z} - \dot{w}] + k_p [z - w] \quad (4.5)$$

Where $w$ and $\dot{w}$ are the displacement and velocity for the 2-D model respectively, $z$ and $\dot{z}$ are the displacement and velocity of the SMD model respectively. The magnitude of the non-interactive force component, $G(t)$, can be described using a harmonic force to represent the near-period nature of walking forces [44]. For a harmonic representation, the magnitude of $G(t)$ can be represented by a Fourier series expressed as:

$$G(t) = W_p \sum_{k=0}^{\infty} \eta_k \cos (2\pi k f_n t + \phi_k) \quad (4.6)$$
Chapter 4: Effect of structural representation of parameter identification of SMD model

where \( W_p = m_p g \) is the subject’s weight; \( m_p \) and \( g \) are the walker mass and gravitational acceleration respectively; \( f_w \) is the pacing frequency; \( t \) is time; and \( \eta_k \) is the dynamic load factor (DLF) for the \( k \)th harmonic. For this study, Young’s dynamic load factors (DLFs) [42] for first four harmonic of human walking frequencies are used. The phase angle, \( \phi_k \) of each \( k \)th harmonic is taken as zero. When only the non-interactive force is considered the model is in essence the conventional moving force (MF) model. Time-step analysis is performed using Newmark-\( \beta \) integration method. Worth noting, the MF model is the non-interactive component of Equation 4.8.

The displacement and velocity of the FE model in Equation 4.5 can be expressed using shape functions in Equation 4.2:

\[
w(x, y, t) = \mathbf{N} \mathbf{d}
\]  

(4.7)

From which the velocity is obtained by differentiation, given as:

\[
\dot{w}(x, y, t) = \frac{\partial w}{\partial x} v_x + \frac{\partial w}{\partial y} v_y + \frac{\partial w}{\partial t}
\]  

(4.8)

where \( v_x \) and \( v_y \) are the velocity of the SMD model in the \( x \) and \( y \) directions respectively. The terms of the partial derivatives are given based on shape functions and deflections, which are written as:

\[
\frac{\partial w}{\partial x} = \mathbf{N}_x \mathbf{d} ; \quad \frac{\partial w}{\partial y} = \mathbf{N}_y \mathbf{d} ; \quad \frac{\partial w}{\partial t} = \mathbf{N} \dot{\mathbf{d}}
\]  

(4.9)

Using the derivatives defined in Equations (4.3) and (4.4), each term within the interactive force in Equation (4.5) can be expanded. Then, Equation (4.9) is substituted into Equation (4.5), which is written as:

\[
f(x, y, t) = G(t) + c_p [\dot{z} - (\mathbf{N}_x v_x \mathbf{d} + \mathbf{N}_y v_y \mathbf{d} + \mathbf{N} \dot{\mathbf{d}})] + k_p [z - \mathbf{N} \mathbf{d}]
\]  

(4.10)

Substituting Equation (4.10) into Equation (4.1) yields the equation of motion of the 2-D FE model, which can be written as:
Chapter 4: Effect of structural representation of parameter identification of SMD model

\[
\mathbf{M}_b \ddot{\mathbf{d}} + \left[ \mathbf{C}_b + c_p \mathbf{N}^T \mathbf{N} \right] \mathbf{d} - \mathbf{N}^T c_p \ddot{\mathbf{z}} - k_p \dot{\mathbf{z}} + \left[ \mathbf{K}_b + c_p \mathbf{N}^T \mathbf{N} \mathbf{v}_x + c_p \mathbf{N}^T \mathbf{N} \mathbf{v}_y + k_p \mathbf{N} \right] \mathbf{d} = \mathbf{N}^T \mathbf{G}(t)
\]  

(4.11)

Note the terms with respect to \( \mathbf{d} \) and its derivatives can be collected. Similarly, the equation of motion of the SMD model - analogous to Equation (4.1) and following Filho [47] and Rieker [48], becomes:

\[
m_p \ddot{\mathbf{z}} + c_p \dot{\mathbf{z}} + k_p \mathbf{z} - c_p \mathbf{N} \ddot{\mathbf{d}} - (c_p \mathbf{N} \mathbf{v}_x + c_p \mathbf{N} \mathbf{v}_y + k_p \mathbf{N}) \mathbf{d} = 0
\]

(4.12)

Both Equations (4.11) and (4.12) can be coupled so can be better expressed as follows:

\[
\begin{bmatrix}
\mathbf{M}_b & 0 \\
0 & m_p
\end{bmatrix}
\begin{bmatrix}
\dot{\mathbf{d}} \\
\dot{\mathbf{z}}
\end{bmatrix}
+ \begin{bmatrix}
\mathbf{C}_b + c_p \mathbf{N}^T \mathbf{N} & -\mathbf{N}^T c_p \\
-c_p \mathbf{N} & c_p
\end{bmatrix}
\begin{bmatrix}
\mathbf{d} \\
\mathbf{z}
\end{bmatrix}
+ \cdots
+ \begin{bmatrix}
\mathbf{K}_b + c_p \mathbf{N}^T \mathbf{N} \mathbf{v}_x + c_p \mathbf{N}^T \mathbf{N} \mathbf{v}_y + k_p \mathbf{N} \\
-c_p \mathbf{N} \mathbf{v}_x - c_p \mathbf{N} \mathbf{v}_y - k_p \mathbf{N}
\end{bmatrix}
\begin{bmatrix}
\dot{\mathbf{d}} \\
\dot{\mathbf{z}}
\end{bmatrix}
= \begin{bmatrix}
\mathbf{N}^T \mathbf{G}(t) \\
0
\end{bmatrix}
\]

(4.13)

When the person leaves the structure, the global shape functions and derivatives (\( \mathbf{N}, \mathbf{N}_x \) and \( \mathbf{N}_y \)) are populated by zeros and the Equation (4.13) reduces to free vibration problem of the 2-D structural model. For the 1-D FE model, the Equation (4.13) reduces to a 1-D representation when the terms with subscript \( y \) are populated by zeros.

For a SDOF system, the SMD model represents humans with properties i.e. mass, \( m_p \), damping, \( c_p \), and stiffness, \( k_p \) that are coupled to the structure’s mass, damping and stiffness matrix respectively. For this work, SMD parameters are expressed as the natural frequency, \( f_h \) and damping ratio, \( \xi_p \), of SMD model which are related to the SDOF stiffness, \( k_p \) and damping, \( c_p \) respectively. These are calculated as:

\[
c_p = 4\pi m_p f_p \xi_p
\]

(4.14)

\[
k_p = 4\pi^2 m_p f_p
\]

(4.15)

To verify the correct implementations of the numerical formulations, a 1-D virtual bridge model from [18] is replicated using the 2-D FE model. With the available bridge properties,
Chapter 4: Effect of structural representation of parameter identification of SMD model

The first natural frequency of the 50 m spanning bridge (2 Hz) was replicated using the 2-D FE model using ten and two elements modelled along the length and width (a unit metre width) respectively. Subsequently, a single pedestrian walking was simulated as the moving SMD model on the 2-D FE model of the virtual bridge. Using the identical SMD parameters in [18] \( (m_p = 73.85 \text{ kg}, k_p = 14.11 \text{ kN/m}, \text{ and } c_p = 612.5 \text{ N s/m}) \), identical acceleration responses was achieved, thereby verifying the accurate implementations of the SMD formulations.

4.3.3. Updating of FE models

Before parameter identification procedures, the modal properties of both 1-D and 2-D FE model are reconciled to accurately match the measured natural frequencies. For the MB, the first mode is the dominant response mode, which is excitable by the third harmonic of normal pacing frequency ranges (1.2 – 2.2 Hz) [41]. Since the 1-D FE model does not capture torsional modes, it is reasonable to only consider the first natural frequency to have exact match – the response of higher natural frequencies will not significantly affect the results of parameter identification procedure. To this end, the stiffness parameters of both models are adjusted to match the first natural frequency of 5.62 Hz obtained from Chapter 2. The mass and damping parameters of both models are not adjusted simply because they follow experimental measurements.

For the 1-D FE model, the bending stiffness (EI) was updated, using the natural frequency as the target of updating. It was found that the bending stiffness had to increase from 7.39 MNm\(^2\) to 7.69 MNm\(^2\) to give the matching natural frequency of 5.62 Hz. For the 2-D FE model, shape descriptor-based model updating is employed to reconcile its stiffness parameter. According to [48, 49], it was found that the MB exhibits an asymmetric longitudinal stiffness between both sides of the MB which in turn resulted in asymmetric mode shape features. This localized mode shape behaviour was not captured via the conventional updating approach that was based on
the Modal Assurance Criterion. For this reason, the 2-D FE model was updated to using the shape-descriptor method in Chapter 3. Since the longitudinal bending stiffness of the MB is mostly contributed by the stiffness I-beam – about 90% of the transformed second moment of area - the second moment of area, I, of the two I-beams is updated to match the natural frequencies. On matching the first natural frequency, the starting value of second moment of area for both I-beams are $165 \times 10^6$ mm$^4$, which are updated to values of $140.3 \times 10^6$ mm$^4$ and $256.3 \times 10^6$ mm$^4$ for each I-beam respectively. This is consistent with results from preceding study, which considered the elastic modulus as the surrogate to stiffness to update the 3-D FE model of the MB [48, 49].

4.4. Application of Numerical formulations

4.4.1. Walking simulations

Figure 4.6 shows a typical acceleration response from the 2-D FE model simulated using the moving SMD model. The following features of the numerical models were considered in the walking simulations:

- The mass parameter of the SMD model, $m_p$, was set equal to the full mass of the test subjects (following [13, 24]).
- Damping of both FE models is considered Rayleigh damping for damping ratio of 0.6% (following measurements from Chapter 2).
- The traversing speed of the SMD model ($v$ in Figure 4.4) is taken as the average speed from the 15 acceptable walks, following [40].
- The SMD parameters (natural frequency $f_h$ and damping ratio $\xi_p$) for test subjects are taken based on [37].

The rolling 1-s root mean square (RMS) accelerations are shown in Figure 4.6 as envelope lines alongside the acceleration responses. As can be seen in Figure 8, the presence of the SMD
The SMD model reduces the acceleration responses of the MB, making it closer to the measurements compared than those from the MF model.

Figure 4.6. Typical acceleration time history from numerical simulations on 2-D FE model (Test subject 1). The 1-s RMS acceleration envelope for both models are marked by bolded envelope line.

### 4.4.2. Analysis of results

Table 4.1 and Table 4.2 summarizes the maximum 1-s RMS acceleration for the combinations with the Fourier series force and measured walking forces respectively. The maximum 1-s RMS accelerations from measurements are reported as the average among the 15 acceptable walks. The difference between the maximum 1-s RMS accelerations, \( \Delta \), are given as ratio of numerical simulations to the measured acceleration.

As can be seen in Table 4.1 and Table 4.2, the SMD model give lower responses than the MF model and are in closer agreement to measurements. Interestingly, the acceleration responses of the 2-D FE model of the MB is uniformly lower than the 1-D FE model – indicated by the lower mean difference to the 1-D FE model. Since the number of DOFs between both FE models are different, this means that the mass and stiffness matrices of each model are
Chapter 4: Effect of structural representation of parameter identification of SMD model

inherently different in order to achieve the identical natural frequency (5.6). As Rayleigh

damping is considered in both models, the proportional damping matrices in the 2-D plate

model are different due to different number of DOFs of the mass and stiffness matrices. In turn,

the representation of the damping ratios (e.g. 0.6 % for first mode) in both models are explicitly
different. In addition, the model combinations that uses the measured walking forces as the

GRFs gives lower responses than those using the Fourier series harmonic forces. This is likely
due to the different DLFs of the harmonics of the measured walking forces compared to the

Fourier series harmonic force (using Young’s DLF).

Table 4.1. Comparison of maximum 1-s RMS accelerations between simulated and measured
results for all test subjects –using a Fourier series force as GRFs (up to four harmonics, Units in m/s²)

<table>
<thead>
<tr>
<th>TS</th>
<th>Measured</th>
<th>1-D model</th>
<th>2-D model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MF</td>
<td>(\Delta_{1,D,MF})</td>
<td>SMD</td>
</tr>
<tr>
<td>1</td>
<td>0.72</td>
<td>2.68</td>
<td>3.70</td>
</tr>
<tr>
<td>2</td>
<td>0.93</td>
<td>2.27</td>
<td>2.43</td>
</tr>
<tr>
<td>3</td>
<td>0.45</td>
<td>2.03</td>
<td>4.57</td>
</tr>
<tr>
<td>4</td>
<td>0.40</td>
<td>1.39</td>
<td>3.44</td>
</tr>
<tr>
<td>5</td>
<td>1.14</td>
<td>2.12</td>
<td>1.86</td>
</tr>
<tr>
<td>6</td>
<td>1.42</td>
<td>2.65</td>
<td>1.87</td>
</tr>
<tr>
<td>7</td>
<td>0.55</td>
<td>2.35</td>
<td>4.29</td>
</tr>
<tr>
<td>8</td>
<td>1.03</td>
<td>3.07</td>
<td>2.99</td>
</tr>
<tr>
<td>9</td>
<td>0.93</td>
<td>1.64</td>
<td>1.78</td>
</tr>
<tr>
<td>10</td>
<td>2.17</td>
<td>3.07</td>
<td>1.41</td>
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<tr>
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<td>1.08</td>
<td>1.87</td>
<td>1.73</td>
</tr>
<tr>
<td>12</td>
<td>0.87</td>
<td>2.28</td>
<td>2.61</td>
</tr>
<tr>
<td>13</td>
<td>0.81</td>
<td>1.55</td>
<td>1.91</td>
</tr>
<tr>
<td>14</td>
<td>0.74</td>
<td>1.83</td>
<td>2.47</td>
</tr>
<tr>
<td>15</td>
<td>0.56</td>
<td>1.55</td>
<td>2.76</td>
</tr>
<tr>
<td>16</td>
<td>0.90</td>
<td>2.23</td>
<td>2.48</td>
</tr>
<tr>
<td>17</td>
<td>1.74</td>
<td>4.66</td>
<td>2.67</td>
</tr>
<tr>
<td>18</td>
<td>1.35</td>
<td>3.30</td>
<td>2.44</td>
</tr>
</tbody>
</table>

**| Mean Δ | \(\Delta_{1,D,MF}\) | \(\Delta_{1,D,SMD}\) | \(\Delta_{2,D,MF}\) | \(\Delta_{2,D,SMD}\) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Δ</td>
<td><strong>2.63</strong></td>
<td><strong>1.49</strong></td>
<td><strong>2.47</strong></td>
<td><strong>1.39</strong></td>
<td></td>
</tr>
<tr>
<td>COV*</td>
<td>0.34</td>
<td>0.51</td>
<td>0.35</td>
<td>0.49</td>
<td></td>
</tr>
</tbody>
</table>

*: coefficient of variation, ratio of standard deviation to mean.
Chapter 4: Effect of structural representation of parameter identification of SMD model

Table 4.2. Comparison of 1-s RMS accelerations between simulated and measured results for all test subjects—using full-time measured walking forces from [39] (Units in m/s²)

<table>
<thead>
<tr>
<th>TS</th>
<th>Measured</th>
<th>1-D</th>
<th>1-D,MF</th>
<th>SMD</th>
<th>2-D</th>
<th>2-D,MF</th>
<th>SMD</th>
<th>2-D,SMD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MF</td>
<td>Δ₁,D,MF</td>
<td>SMD</td>
<td>Δ₁,D,SMD</td>
<td>MF</td>
<td>Δ₂,D,MF</td>
<td>SMD</td>
</tr>
<tr>
<td>1</td>
<td>0.72</td>
<td>1.01</td>
<td>1.39</td>
<td>0.77</td>
<td>1.06</td>
<td>1.01</td>
<td>1.40</td>
<td>0.76</td>
</tr>
<tr>
<td>2</td>
<td>0.93</td>
<td>1.88</td>
<td>2.02</td>
<td>1.11</td>
<td>1.19</td>
<td>1.81</td>
<td>1.94</td>
<td>1.10</td>
</tr>
<tr>
<td>3</td>
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<td>0.65</td>
<td>1.47</td>
<td>0.47</td>
<td>1.05</td>
<td>0.61</td>
<td>1.37</td>
<td>0.45</td>
</tr>
<tr>
<td>4</td>
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<td>0.46</td>
<td>1.13</td>
<td>0.43</td>
<td>1.07</td>
<td>0.42</td>
<td>1.05</td>
<td>0.40</td>
</tr>
<tr>
<td>5</td>
<td>1.14</td>
<td>1.52</td>
<td>1.33</td>
<td>0.97</td>
<td>0.86</td>
<td>1.30</td>
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<td>0.89</td>
</tr>
<tr>
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<td>0.51</td>
<td>0.94</td>
<td>0.82</td>
<td>1.50</td>
<td>0.52</td>
</tr>
<tr>
<td>8</td>
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<td>1.05</td>
<td>1.03</td>
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<td>0.61</td>
<td>0.98</td>
<td>0.95</td>
<td>0.57</td>
</tr>
<tr>
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<td>1.31</td>
<td>1.42</td>
<td>1.17</td>
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</tr>
<tr>
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<td>1.10</td>
<td>0.72</td>
<td>0.82</td>
<td>0.88</td>
<td>1.01</td>
<td>0.67</td>
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<td>0.97</td>
<td>0.83</td>
<td>1.02</td>
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<td>0.74</td>
<td>0.54</td>
<td>0.73</td>
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<td>0.61</td>
<td>0.47</td>
<td>0.64</td>
<td>0.40</td>
</tr>
<tr>
<td>15</td>
<td>0.56</td>
<td>0.65</td>
<td>1.16</td>
<td>0.60</td>
<td>1.07</td>
<td>0.62</td>
<td>1.10</td>
<td>0.55</td>
</tr>
<tr>
<td>16</td>
<td>0.90</td>
<td>1.53</td>
<td>1.69</td>
<td>1.00</td>
<td>1.11</td>
<td>1.49</td>
<td>1.66</td>
<td>0.97</td>
</tr>
<tr>
<td>17</td>
<td>1.74</td>
<td>4.12</td>
<td>2.36</td>
<td>1.23</td>
<td>0.71</td>
<td>3.97</td>
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<td>1.55</td>
<td>1.15</td>
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<td>0.50</td>
<td>1.43</td>
<td>1.06</td>
<td>0.68</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>1.37</td>
<td>0.88</td>
<td>1.29</td>
<td>0.84</td>
<td>0.31</td>
<td>0.28</td>
<td>0.33</td>
</tr>
<tr>
<td>COV*</td>
<td></td>
<td>0.31</td>
<td>0.28</td>
<td>0.33</td>
<td>0.29</td>
<td>0.31</td>
<td>0.28</td>
<td>0.33</td>
</tr>
</tbody>
</table>

*: coefficient of variation, ratio of standard deviation to mean

The results in Table 4.2 show that the SMD model gives overall closest acceleration predictions to the measured responses when considering the measured walking force as the GRFs (mean Δ closest to unity). One possible reason is that the adopted DLFs of the harmonic forces can be different (Young’s DLF) since the vibrating surface of the MB can lower DLFs for example (this is second phenomenon of HSI known as structure-to-human interaction, S2HI).

Interestingly, the difference of mean Δ between 1-D and 2-D models (evaluation within table) are smaller than those between different GRFs (evaluation between tables). This indicate that the differences in walking simulation responses (for the same GRFs) are not significant in both FE representations.
Chapter 4: Effect of structural representation of parameter identification of SMD model

4.5. Identification of SMD parameters

4.5.1. Identification procedure

The parameter identification procedure is an iterative procedure that calibrates the SMD parameters while minimizing some target metric. For this work, the target metric is taken as the least square error (LSE) of the 1-s RMS acceleration envelopes between measured and simulated acceleration responses. As a comparison, other studies considered different target metrics in their identification procedure e.g. the acceleration responses of the pelvis [30], frequency response functions of structure [33], and displacement of the body’s centre-of-mass [31]. The LSE of 1-s RMS acceleration responses is calculated as:

$$LSE = \frac{1}{n} \sum_{i=0}^{n} (a_{exp}(t_i) - a_{sim}(t_i))^2$$  (4.16)

where $a_{exp}$ and $a_{sim}$ are the time histories of the 1-s RMS acceleration from measurements and simulations respectively, $n$ is the number of points through the time history series of the envelopes, taken as constant (for paired measurements and simulations) based on the traveling speed of the test subject. From the 15 acceptable walks recorded for each test subject, an average envelope of the 1-s RMS acceleration time history is taken as $a_{exp}$ in Equation (4.16).

To ensure convergence of SMD parameters, a constrained range is considered to allow parameters to vary. The ranges for $f_p$ and $\xi_p$ are set to 0.5 to 5 Hz and 15 % to 70 % respectively. The maximum ranges of the parameter are based on values proposed in the literature. For example, the value of $f_p$ and $\xi_p$ can be up to 5.74 Hz [50], and 55% [9, 41, 50] respectively.

The walking simulation is performed in each iterative step of the parameter identification procedure. The assumptions of walking simulations are analogous to those outlined in Section 3 and 4. The walking simulations and iterative procedure are written in MATLAB scripts. During each step, the fmincon function is used for the constrained optimization. The optimization completes when the increment of $f_p$ and $\xi_p$ in subsequent steps are reasonably
An initial sensitivity trial of starting parameters (i.e. increasing and decreasing the starting values intentionally) is performed prior to each identification attempts to ensure a global convergence solution is achieved. The sensitivity trial showed that the identification procedure for all test subjects can achieve convergence (i.e. solution for $f_p$ and $\zeta_p$ are unique for each test subject).

Worth noting, the identification procedure can include other parameters, e.g. the traversing speed and mass parameter. While it is likely that increasing the number of varying parameters will better optimize the target metric, increasing the number of variables will affect the information entailed by $f_p$ and $\zeta_p$ (e.g. natural frequency is dependent on both mass and stiffness parameter). Without increasing the complexity of the optimization problem, the identification of $f_p$ and $\zeta_p$ are sufficient for the basis of HSI (i.e. adopting Einstein’s Razor).

4.5.2. Results of identified SMD parameters

The identification procedure is performed using both 1-D and 2-D FE model of the MB, with the Fourier series harmonic force as the GRFs of the SMD model. Figure 4.7 shows the 1-s RMS acceleration envelopes of SMD model with the starting (initial) and identified (final) parameters for the 2-D representation of the MB. As can be seen in Figure 4.7 the amplitude of the envelopes matched closely with the measured envelopes using the identified SMD parameters compared to the starting SMD parameters, e.g. test subject 3. However, close inspection reveals difference in the time of peaks between the measured and simulated (final) envelopes - e.g. see responses for test subject 6 and 18. The mismatched peak time is affected by the traveling speed of the SMD model. The identification procedure can include the walking speed to match the peak time of the envelopes, where however, this is not considered for the aim of this work.
Figure 4.7. Comparison of the 1-s RMS acceleration response envelope for 2-D FE model of the MB, using SMD model with initial (green line) and final (blue line) SMD parameters. The measured envelope of each subject (red line) is obtained from the mean of the 15 individual envelopes from the 15 walks (black dashed lines), with spread of envelopes shown as grey region.
The root-mean-square error (RMSE) that corresponds to the optimal SMD parameters from the identification procedure are presented in Table 4.3. As can be seen, a smaller mean RMSE is observed for parameters identified using the 2-D FE model, indicating that the SMD model with the 2-D FE model is a better representation of the walking tests.

Table 4.3. Root mean square errors of the 1-s RMS acceleration responses from 1-D and 2-D FE model using SMD models.

<table>
<thead>
<tr>
<th>Test Subject</th>
<th>Mass (N)</th>
<th>FE model</th>
<th>1-D</th>
<th>2-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>850</td>
<td></td>
<td>0.14</td>
<td>0.09</td>
</tr>
<tr>
<td>2</td>
<td>718</td>
<td></td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td>3</td>
<td>674</td>
<td></td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>444</td>
<td></td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td>678</td>
<td></td>
<td>0.14</td>
<td>0.08</td>
</tr>
<tr>
<td>6</td>
<td>862</td>
<td></td>
<td>0.36</td>
<td>0.27</td>
</tr>
<tr>
<td>7</td>
<td>717</td>
<td></td>
<td>0.08</td>
<td>0.04</td>
</tr>
<tr>
<td>8</td>
<td>970</td>
<td></td>
<td>0.17</td>
<td>0.14</td>
</tr>
<tr>
<td>9</td>
<td>522</td>
<td></td>
<td>0.11</td>
<td>0.06</td>
</tr>
<tr>
<td>10</td>
<td>1063</td>
<td></td>
<td>0.28</td>
<td>0.22</td>
</tr>
<tr>
<td>11</td>
<td>647</td>
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<td>0.15</td>
<td>0.11</td>
</tr>
<tr>
<td>12</td>
<td>773</td>
<td></td>
<td>0.18</td>
<td>0.12</td>
</tr>
<tr>
<td>13</td>
<td>495</td>
<td></td>
<td>0.10</td>
<td>0.05</td>
</tr>
<tr>
<td>14</td>
<td>609</td>
<td></td>
<td>0.11</td>
<td>0.07</td>
</tr>
<tr>
<td>15</td>
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<td>0.07</td>
</tr>
<tr>
<td>16</td>
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<td></td>
<td>0.14</td>
<td>0.10</td>
</tr>
<tr>
<td>17</td>
<td>1489</td>
<td></td>
<td>0.19</td>
<td>0.09</td>
</tr>
<tr>
<td>18</td>
<td>1112</td>
<td></td>
<td>0.27</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td></td>
<td></td>
<td><strong>0.15</strong></td>
<td><strong>0.10</strong></td>
</tr>
</tbody>
</table>

The identified SMD parameters based on the 1-D and 2-D FE model of the MB are summarized in Table 4.4. As evident in Table 4.4, the identified SMD parameters $f_h$ and $\xi_p$ are different between the 1-D FE model and the 2-D FE model. The ratio of SMD parameters identified from 2-D FE model to 1-D FE model are calculated (denote as difference ratio, $\Delta_{2-D/1-D}$). From the average of the difference ratio, it can be seen that the natural frequency $f_h$ from the 2-D FE model is on average higher than those identified from the 1-D FE model (average less than 1). Contrary is observed for damping ratio, $\xi_p$, where the average of difference ratios is greater than 1 – with the exclusion of the outlier i.e. Test Subject 4.
Table 4.4. Parameters of SMD model identified after optimization, based on 1-D and 2-D FE models of the MB.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Mass (N)</th>
<th>From [37]</th>
<th>1-D</th>
<th>2-D</th>
<th>Δ2-D/1-D</th>
<th>From [37]</th>
<th>1-D</th>
<th>2-D</th>
<th>Δ2-D/1-D</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.45</td>
<td>0.49</td>
<td>1.09</td>
</tr>
<tr>
<td>2</td>
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<td>2.13</td>
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<td>0.24</td>
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</tr>
<tr>
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<td>0.70</td>
<td>23.33</td>
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<tr>
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<td>4.14</td>
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<td>0.50</td>
<td>0.53</td>
<td>1.06</td>
</tr>
<tr>
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<td>1.67</td>
<td>0.91</td>
<td>0.54</td>
<td>0.18</td>
<td>0.31</td>
<td>0.69</td>
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</tr>
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<td>0.75</td>
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<tr>
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<td>0.25</td>
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</tr>
<tr>
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<td>2.69</td>
<td>0.92</td>
<td>0.18</td>
<td>0.20</td>
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</tr>
<tr>
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<td>1112</td>
<td>1.87</td>
<td>2.19</td>
<td>2.06</td>
<td>0.94</td>
<td>0.37</td>
<td>0.23</td>
<td>0.25</td>
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</tr>
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<table>
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<tr>
<th></th>
<th>Mean</th>
<th>COV1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequency, $f_h$</td>
<td>0.95</td>
<td>0.19</td>
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<tr>
<td>Damping ratio, $\xi_p$</td>
<td>0.36</td>
<td></td>
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</tbody>
</table>

¹: coefficient of variation, ratio of standard deviation to mean
²: Excluded outlier – Test subject 4.

To check the influence of the parameter differences on the response predictions, the SMD parameters of Table 4.4 are used for walking simulations in different structural representations of which it was originally derived. Namely, the identified SMD parameters from the 1-D FE model are adopted in the walking simulations onto 2-D FE model, and vice versa. From these walking simulations, the maximum 1-s RMS acceleration responses are summarized in Table 4.5. The ratio of maximum 1-s RMS acceleration responses between the 1-D and 2-D derived SMD parameters are calculated, which are then plotted in Figure 4.8 for illustration. From the results of Table 4.5 and Figure 4.8, the non-unity of acceleration ratios are indicative of the prediction errors when using SMD parameters from the different structural representations. As it stands, this highlights the potential application error when adopting such SMD parameters into different structural representation of the same structure.
Chapter 4: Effect of structural representation of parameter identification of SMD model

Collectively, the results in Section 4.5 provide evidence that the fitted SMD model parameters are acting as a surrogate for the difference related to the structural representations.

Table 4.5. Maximum 1-s RMS acceleration from simulations considering the identified SMD parameters into different structural representations. (Units in m/s²)

<table>
<thead>
<tr>
<th>Test Subject Mass (N)</th>
<th>1-D FE model</th>
<th>2-D FE model</th>
<th>Ratio*</th>
<th>1-D parameters</th>
<th>2-D parameters</th>
<th>Ratio*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2-D parameters</td>
<td>1-D parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.57</td>
<td>0.94</td>
<td>0.64</td>
<td>0.67</td>
</tr>
<tr>
<td>2</td>
<td>718</td>
<td>0.72</td>
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<td>0.90</td>
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<td>0.84</td>
</tr>
<tr>
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<td>674</td>
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<td>0.80</td>
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<td>0.34</td>
</tr>
<tr>
<td>4</td>
<td>444</td>
<td>0.30</td>
<td>0.33</td>
<td>0.91</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
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<td>0.98</td>
<td>0.88</td>
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<td>1.06</td>
</tr>
<tr>
<td>6</td>
<td>862</td>
<td>1.45</td>
<td>1.24</td>
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<td>1.29</td>
</tr>
<tr>
<td>7</td>
<td>717</td>
<td>0.44</td>
<td>0.49</td>
<td>0.90</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>8</td>
<td>970</td>
<td>0.84</td>
<td>0.85</td>
<td>0.99</td>
<td>0.86</td>
<td>0.96</td>
</tr>
<tr>
<td>9</td>
<td>522</td>
<td>0.76</td>
<td>0.80</td>
<td>0.95</td>
<td>0.83</td>
<td>0.91</td>
</tr>
<tr>
<td>10</td>
<td>1063</td>
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<td>2.00</td>
<td>1.07</td>
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<td>2.00</td>
</tr>
<tr>
<td>11</td>
<td>647</td>
<td>1.01</td>
<td>0.98</td>
<td>1.03</td>
<td>0.92</td>
<td>0.99</td>
</tr>
<tr>
<td>12</td>
<td>773</td>
<td>0.67</td>
<td>0.70</td>
<td>0.95</td>
<td>0.75</td>
<td>0.77</td>
</tr>
<tr>
<td>13</td>
<td>495</td>
<td>0.60</td>
<td>0.68</td>
<td>0.88</td>
<td>0.75</td>
<td>0.70</td>
</tr>
<tr>
<td>14</td>
<td>609</td>
<td>0.59</td>
<td>0.68</td>
<td>0.87</td>
<td>0.69</td>
<td>0.66</td>
</tr>
<tr>
<td>15</td>
<td>509</td>
<td>0.42</td>
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<td>0.47</td>
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<tr>
<td>16</td>
<td>683</td>
<td>0.69</td>
<td>0.75</td>
<td>0.92</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>17</td>
<td>1489</td>
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<td>1.60</td>
<td>0.93</td>
<td>1.61</td>
<td>1.60</td>
</tr>
<tr>
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<td>1.16</td>
<td>0.98</td>
<td>1.22</td>
<td>1.28</td>
</tr>
</tbody>
</table>

Mean | 0.95 | 0.97 |
COV  | 0.09 | 0.06 |

*: Adopted parameters against original parameters of FE model.
Figure 4.8. Results of walking simulations for all test subject, adopting SMD parameters derived from different structural representation. Ratios are based on comparisons of adopted parameters against base parameters – see Table 4.5.

4.6. Conclusions

This chapter investigates the effects of different structure representations towards the identification of SMD parameters. A representative footbridge structure (denoted as the MB) is subjected to two different FE representations, using a 1-D and 2-D FE model. Both FE models are verified and validated to predict accurately the first mode response of the MB. Walking-responses of the MB were obtained by simulating the SMD model onto both FE models of the MB. The parameters of SMD model (natural frequency and damping ratio) are identified by fitting the envelopes of 1-s RMS acceleration to the envelopes from measurements. The identification procedure was conducted on both structural models (1-D and 2-D FE models) and the identified SMD parameters are compared.

The results showed that the identified SMD parameters are different between both FE models. The walking simulations verified that the difference in response prediction is related to the structural representations of the FE model. Consequently, the results are evidence to support hypothesis that the SMD parameters are a function of the structural representations. Overall, the work is instructive to
users of SMD model in adopting explicitly-proposed SMD parameters in the literature. The approach of this work can serve as a starting point to determine fundamental data of SMD models for its application on different structural representation. The proposed framework can be used for other structures, albeit this chapter is focused on pGFRP structures.

Reference

Chapter 4: Effect of structural representation of parameter identification of SMD model


Chapter 4: Effect of structural representation of parameter identification of SMD model


Chapter 5. Design of statically-feasible pultruded GFRP sandwich panel floor systems
Chapter 5: Design of statically–feasible pGFRP sandwich panel floors

Preface

Chapter 5 explores feasible design of pGFRP sandwich panel floors through the application the current state-of-the-art design rules for pGFRP sandwich panel. Chapter 5 is needed in order to address the knowledge gap in feasible designs of pGFRP sandwich panel floor systems which vibration serviceability assessment then follows.

In this chapter-paper, the static design rules are developed into easy-to-use design charts, allowing quick evaluation of pGFRP sandwich panel designs. This chapter-paper refers to the static design rules published in the preceding research, which are referenced accordingly. The design charts include load-span charts (individual sandwich panel and composite beam-panel section) as well as failure maps. Based on the design charts, a simple design framework is presented. The design method is demonstrated in this chapter, which is later considered in Chapter 6 to design several feasible prototypical pGFRP sandwich panel floors for vibration assessments. It should be emphasized that the design charts are developed based on design practices of Australian Standards (e.g. load factors).

This chapter-paper is prepared as the following publication:


The publication has extended the design charts to enable broader application in other design practice i.e. the Eurocode design guidelines.
Chapter 5: Design of statically-feasible pGFRP sandwich panel floors

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Chapter 5: Design of statically –feasible pGFRP sandwich panel floors

Abstract

Pultruded glass fibre-reinforced polymer (pGFRP) sandwich panel has been increasingly considered in the construction of civil structures. Recently, a novel pGFRP sandwich panel floor system has been proposed for building floor applications. Despite the established state-of-the-art design rules for the static performance of a bespoke pGFRP sandwich panel, the design for full-floor systems remains unexplored. This chapter presents design aids based on the static design rules to allow easy and efficient evaluation of the static performance of pGFRP sandwich panel floor systems. Failure maps and load-span charts have been provided to evaluate feasible geometries of pGFRP sandwich panels. Design charts for pGFRP-steel composite beams have been provided. From the design charts, a simple design procedure is proposed. The design procedure is demonstrated on an example floor structure to design a feasible pGFRP sandwich panel floor system. This work should find relevant for designers of pGFRP sandwich panels and those involved in the field of GFRP.

KEYWORDS

Glass fibre-reinforced polymers; Sandwich Panels; Adhesive bonding; Mechanical bolts; Office floor system; Design charts;
Chapter 5: Design of statically feasible pGFRP sandwich panel floors

5.1. Introduction

5.1.1. Background

Over the last decade, pultruded glass fibre-reinforced polymer (pGFRP) composites have been increasingly introduced in construction of civil structures. Their lightweight and high durability are beneficial in terms of easy transportations, rapid constructions, and lower maintenance cost [1, 2]. Furthermore, the production cost of pGFRP has been reduced considerably due to advances in manufacturing GFRP sections, i.e. the pultrusion process. Recognizing the advantages pGFRP offers over traditional construction materials (steel and concrete), new designs and applications of GFRP sandwich panels are actively investigated and introduced in practice [3]. However, the application of pGFRP remains limited, citing the lack of internationally-accepted design guidelines as the main reason. To address this issue, recent developments from research sector have proposed GFRP composites as load-bearing members in floors [4], roofs [5], wall panels [6], houses [7, 8] and bridge superstructures [9-14].

In an effort to introduce new applications of pGFRP, a novel pGFRP sandwich panel has been proposed for building floor applications [15]. The pGFRP sandwich panel is a modular web-flange sandwich panel which comprised of individual pultruded pGFRP box profiles sandwiched between two pGFRP flat panels [16]. To date, the mechanical properties of the pGFRP sandwich panel have been investigated by constructing and testing small-scale sandwich specimens, including sandwich beams [17, 18], two-way spanning slabs [19], and pGFRP-steel composite beam [20, 21]. From these studies, static design rules have been developed and validated for the bespoke pGFRP sandwich panels. Furthermore, a full-scale application of the bespoke pGFRP sandwich panel has been realized in the deck solution for a pGFRP sandwich footbridge (Chapter 2) [22-24]. Collectively, these studies have led to the development of static design rules for the bespoke pGFRP sandwich panels. However, the designs of feasible pGFRP sandwich panel floors remain unexplored. This is significant as static requirements of floors (i.e. strength and deflection criteria) precedes requirements such as vibration,
Chapter 5: Design of statically-feasible pGFRP sandwich panel floors

Acoustic and seismic performance for example. Consequently, there is a need to evaluate designs of feasible pGFRP floor systems prior to performing vibration assessment. A comprehensive design approach for the bespoke pGFRP sandwich panels can fulfil this need.

Naturally, design aids (e.g. design charts and tables) have been adopted in design procedures of civil engineering structures. In particular, design charts provide tangible and efficient assessment of designs through the inherit visual representations. This makes design charts advantageous for both preliminary and detailed design - where end-user can verify numerical solutions which are often treated as “black-box” problems with the analytical solutions e.g. sandwich theory [25, 26]. Consequently, many studies have establish design charts of sandwich panels based on the analytical formulations of sandwich theory (following [4, 27-30]). For instance, Betts et al. [30] have verified design-oriented models (i.e. stress-strain, moment curvature and load-deflection models) for foam core sandwich beams and proposed a simple design procedure that is readily used by designers. In addition, failure maps have also been developed to assess failure modes of sandwich panels [31-35]. Furthermore, some studies have combined optimization procedures of sandwich panel with the visual enhancements of design charts [31, 36, 37]. Overall, design charts are versatile tools in comprehensive design approach for sandwich structures.

5.1.2. Contribution

The proposed pGFRP sandwich panel floor system has numerous benefits that make it highly competitive to traditional floor systems. Although state-of-the-art static design rules have been established from the preceding research works, its application in practice remains unexplored. In turn, feasible designs of pGFRP sandwich panel floors has not been explored. Hence, this is the knowledge gap that needs to be addressed prior to the subsequent analysis (e.g. vibration, acoustics, seismic etc.) of the bespoke pGFRP sandwich panel.
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This chapter addresses the knowledge gap by investigating statically-feasible pGFRP sandwich panel floors. It should be mentioned that the design charts and procedure of this chapter is limited to the bespoke pGFRP sandwich panel design - other types of GFRP sandwich panel is not included. First, a summary of the state-of-the-art static design rules of pGFRP sandwich panel floor components is provided. Design charts are developed based on the static design rules. In turn, a simple design procedure based on the developed design charts is presented. To demonstrate application, the proposed design procedure is applied to design a pGFRP sandwich panel floor system for a prototypical building floor frame. The applicability of the design charts and the recommendations for future improvements are discussed. The design charts of this chapter are aimed towards preliminary designs of pGFRP sandwich panel floor system (i.e. satisfying strength requirements). At a more advanced stage in design, these preliminary floor designs should still be checked with more refined analysis (e.g. numerical methods). It should be emphasize that the design charts and procedure presented in this work are limited to the pGFRP sandwich panel floors only.

5.2. Proposed pultruded GFRP sandwich panel floor systems

5.2.1. Structural members

The proposed pGFRP sandwich panel system is shown in Figure 5.1. The pGFRP sandwich panel acts as a one-way-spanning floor slab which spans between secondary beams. An additional foam core can be incorporated between box profiles to increase the bending stiffness, damping, and thermal insulation [15]. The pultrusion directions of the flat panels (longitudinal) are aligned perpendicular to the pultrusion direction of the box profiles (transverse), forming an orthotropic sandwich panel with bidirectional pultrusion directions. Conversely, current pGFRP cellular decks have fibre directions in only one direction (longitudinal). The pGFRP sandwich panel is assembled transversely onto supporting beams through either adhesive bonding or bolted connections which in turn forms a pGFRP-steel composite beam.
5.2.2. Design approach and assumptions

The design of feasible pGFRP sandwich panel system is explored for typical floor systems in office and commercial buildings (i.e. having slab and supporting beams). The design load considers a uniformly distributed load, w (UDL) which has three components: (1) dead load from self-weight; and (2) a superimposed dead load of 1 kPa representing services and finishes; and (3) live load of 3.5 kPa for general office occupancy (following AS1170.1[38]). For this work, the analysis and design procedures are based on partial factor design. The load factors are defined following AS1170.1 [38] for the limit state design: 1.2 and 1.5 for ultimate limit state (ULS); 1.0 and 0.7 for serviceability limit state (SLS). From hereafter, the experimental specimens from the literature [17-21] are considered to explain the design methods and results.

With reference to previous studies, the following assumptions are considered for the design and analysis of the pGFRP sandwich panel and pGFRP-steel composite beams in this study:

- Consideration of adhesive bonding allows the assumption of full composite action between components within the pGFRP sandwich panel [15] and between flat panels and I-beams. In
contrast, mechanical bolts provide partial composite actions between sandwich panel and steel beams [15]. Full composite action can be assumed for bolted connections, provided that sufficient shear connectors are designed to resist the longitudinal shear forces at the panel-beam interface.

- For pGFRP-steel composite beam, full composite action between pGFRP components (e.g. between top and bottom flat panels) can be achieved with a sufficiently large box profile thickness. This is demonstrated in linear strain distributions along the composite section depth of the full-scale pGFRP footbridge specimen of Chapter 2 and in [22-24], which used 9.5 mm-thick box-profiles while the composite-beam specimens in [17] used thicknesses of 6 mm.

- The study of [39] showed that the effective width of a pGFRP-steel deck-beam system is similar to those in steel deck-beam systems with similar geometries. This indicates that pGFRP-steel composite beams can be analysed similarly to a steel composite beam without changing the considerations of shear lag, for example.

- The bidirectional pultrusion of sandwich panel optimizes strength in both longitudinal and transverse directions. In turn, this increases the strength in the transverse direction and prevents premature cracking of webs along the otherwise weaker transverse direction when considering a unidirectional configuration [19]. Therefore, cracking along transverse direction of sandwich panel can be ignored in the design.

- According to [17], failure at connections (both adhesive bond and bolts) were not observed, whereby any connection failure were only observed as a secondary failure mode after buckling or shear failure. For this work then, connection failure is not considered as critical since the strength of adhesive (epoxy) and bolts (steel) are higher than pGFRP material.

- Due to the bidirectional orientations, the pultrusion direction of the flat panels are parallel to the longitudinal span of I-beams while those of the box profiles are perpendicular to the beam (see Figure 5.1.). Thus, box profiles can be ignored when evaluating the longitudinal stiffness of composite beam as they are discontinuous in that direction.
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- The material properties of the pGFRP components are given in Table 5.1. The yield strength of steel is taken as 320 MPa (mild steel).

- For this work, the compressive strength of pGFRP is taken as equal to its tensile strength, following the specification provided by the manufacturer (Excel composites) [40].

- Panel and box (webs) thickness are taken as 10 mm, following the previous study on pGFRP sandwich panels [17-19].

Table 5.1. Elastic and strength properties of pGFRP composites after [40].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength, ( \sigma_t )</td>
<td>300</td>
<td>MPa</td>
</tr>
<tr>
<td>Tensile modulus in fibre direction, ( E_x )</td>
<td>24.07</td>
<td>GPa</td>
</tr>
<tr>
<td>Tensile modulus in transverse direction, ( E_y )</td>
<td>4.5</td>
<td>GPa</td>
</tr>
<tr>
<td>Interlaminar shear strength, ( f_{sh} )</td>
<td>28.2</td>
<td>MPa</td>
</tr>
<tr>
<td>Shear modulus, ( G )</td>
<td>3.5</td>
<td>GPa</td>
</tr>
<tr>
<td>Density, ( \rho )</td>
<td>1800</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.3</td>
<td>-</td>
</tr>
</tbody>
</table>

5.3. Static design rules

5.3.1. GFRP sandwich panels with bidirectional fibre orientations

For analysis, the bidirectional pGFRP sandwich panel can be treated as a one-way sandwich beam of unit width (1 m). Adopting the partial factor design method, the sandwich beam is required to satisfy the following limit:

\[
\frac{w_{d,F}}{w_p} \leq 1
\]  

(5.17)

where \( w_{d,F} \) is the factored design load (load factors applied); \( w_p \) is the capacity load of pGFRP sandwich panels which corresponds design checks for failure and deflections, which are next presented.

In addition to usual failure modes, and with reference to experimental studies [18, 19], two possible modes have been identified as particular to pGFRP sandwich panels, namely shear failure at webs of
box profile and out-of-plane buckling of upper flat panel. The latter is a local failure mode that occurs within upper flat panels between box profiles. A global compressive failure of upper panel is also included in the analysis as a possible failure mechanism under bending moments. Since the top and bottom panels experiences maximum stresses under bending, the tensile strength of bottom panels is indirectly checked when checking for compressive failure.

The shear failure load can be calculated by checking the in-plane shear strength of box profiles. Failure load correspond to shear failure, \( w_{p,s} \), is estimated using simple beam theory:

\[
    w_{p,s} = f_{s,B} \frac{2b_w I_b}{L_p Q} \tag{5.18}
\]

Where \( I \) is the second moment of area of the sandwich panel, \( b_w \) is the total width of the webs in the evaluated section, \( L_p \) is the span of sandwich panel, \( Q \) is the shear flow of the sandwich panel evaluated at mid-section (correspond to maximum shear stress) and \( f_{s,B} \) is the shear strength of box profile webs.

The buckling failure load is predicted by assuming the upper panel (between box profiles) acting as a thin orthotropic plate subjected to a uniformly distributed compressive load \( N_{cr} \) [18]. Then, the failure load due to buckling, \( w_{p,buck} \), can be determined from \( N_{cr} \) by considering force and moment equilibrium between top and bottom flat panels:

\[
    w_{p,buck} = N_{cr} \frac{8d_p}{L_p^2} \tag{5.19}
\]

\[
    N_{cr} = \frac{24}{b_{cr}^2} \left[ 1.871 \sqrt{D_{11} D_{22}} + D_{12} + 2D_{66} \right] \tag{5.20}
\]

\[
    D_{11} = \frac{E_{11} t_f^3}{12 \left( 1 - v_{12}^2 \frac{E_{22}}{E_{11}} \right)} \tag{5.21}
\]
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\[
D_{22} = \frac{E_{22} I_f^3}{12 \left(1 - \nu_{12}^2 \frac{E_{22}}{E_{11}}\right)} \quad (5.22)
\]

\[
D_{12} = \nu_{12} D_{22} \quad (5.23)
\]

\[
D_{66} = \frac{G_{12} I_f^3}{12} \quad (5.24)
\]

where \( b_{cr} \) is the buckling plate width (taken as the gap distance between adjacent box profiles), \( d_b \) is the depth of box profiles; \( D_{11}, D_{22}, D_{12}, \) and \( D_{66} \) are the bending stiffness coefficients where \( E_{11} \) and \( E_{22} \) are the elastic moduli of flat panels in the longitudinal and transverse pultrusion direction, respectively; \( \nu_{12} \) is the Poisson’s ratio; \( G_{12} \) is the shear modulus of pGFRP (see Table 5.1). The compressive failure load, \( w_{p,bend} \), is checked against the compressive strength of pGFRP flat panels using simple beam theory:

\[
w_{p,bend} = f_{c,p} \frac{8 I}{L_p^2 y} \quad (5.25)
\]

where \( f_{c,p} \) is the compressive strength (equal to tensile strength in Table 5.1) of pGFRP flat panels, and \( y \) is the distance from neutral axis to the centre of flat panels. The neutral axis of pGFRP sandwich panel is taken at the mid-section due to symmetry.

The deflection of pGFRP sandwich panels require the consideration of shear deformations because pGFRP has a relatively low in-plane shear modulus compared to its longitudinal elastic modulus. From [18, 19], shear deformations are influential towards the total deflections of pGFRP sandwich panels (up to 40% of total deflection), particularly when short spans were considered. To this, Timoshenko beam theory [41] – which considers shear deformation - is adopted over the Euler Bernoulli beam theory to evaluate deflections. The midspan deflection \( \delta_p \) of the sandwich panel can be expressed as [25]:

\[
\delta_p = \frac{5 w_{p,def} L_p^4}{384 EI} + \frac{w_{p,def} L_p^2}{8 A_G} \leq \frac{L_p}{300} \quad (5.26)
\]
where $EI$ is the bending stiffness of the sandwich panel, $A_w$ is the cross section of box web and $G$ is the in-plane shear modulus of the box profiles. The first term in the right-hand side of Equation (5.26) is the deflection due to bending and the second term is due to shear deformation. To determine the criteria load for deflection limits, Equation (5.26) can be rearranged by considering the deflection as the prescribed deflection limit (e.g. $L_p/300$), thus expressed as:

$$w_{p,\text{def}} = \frac{L_p}{300} \left( \frac{5L_p^4}{384EI} + \frac{L_p^2}{8A_wG} \right)^{-1}$$

(5.27)

where $w_{p,\text{def}}$ is the UDL that correspond to the deflection limit of $L_p/300$ for example. The bending stiffness of pGFRP sandwich panel can be evaluated using section transformation of second moment of area for each component (i.e. box profile and flat panels). Accordingly, the bending stiffness of sandwich panel can be expressed as:

$$EI = E_f b t_f \left( \frac{t_f + d_b}{2} \right)^2 + E_c b t_b \frac{3}{6} + n \left( \frac{E_c d_b^4}{12} - \frac{E_f (d_p - 2t_b)^4}{12} \right)$$

(5.28)

where $b$ is the width of sandwich panel (per metre width), $t_f$ and $t_b$ are the thickness of flat panels and box profiles respectively; $E_f$ and $E_c$ are the elastic modulus of flat panels and box profiles respectively, evaluated in the span direction of sandwich panel, $n$ is the number of box profiles per unit width of sandwich panel.

### 5.3.2. GFRP-steel composite beams with bidirectional sandwich panel

For composite beams with bidirectional pGFRP sandwich panels, failure modes can occur within both flat panels and the web of box profiles [20]. With reference to experiments in [20], failure modes pertinent to pGFRP sandwich panel were observed at load levels beyond those correspond to yielding of steel beam. The yield failure of steel beams provides the ductility of the composite beam, ensuring sufficient warnings of structural failure prior to the brittle failure of pGFRP components. As such, the ultimate design of composite beam is achieved by ensuring the capacities of pGFRP failure modes.
are higher than the yielding capacity of steel beams. For composite beams under bending, the ultimate design of composite is checked by evaluating the bending moment limits, given as:

\[ M_y < M_u \]  

(5.29)

where \( M_y \) is the yield moment of steel beams and \( M_u \) is the bending moment corresponding to failure capacities of pGFRP components. Yielding of steel beam occurs when the maximum stresses at outermost tension fibre steel beams reaches its yield strength. The yield moment of steel beams can be calculated from:

\[ M_y = \frac{f_y I_{comp}}{y_{st}} \]  

(5.30)

where \( y_{st} \) is the distance between extreme tension fibre of steel and the neutral axis of composite beam. \( I_{comp} \) is the second moment of area of composite section, and \( f_y \) is the yield strength of steel. Accordingly, the neutral axis and second moment of area of composite beam can be evaluated using section transformation.

According to [20], the stress distributions within the top and bottom flat panels of the sandwich panel under bending are equal due to partial composite action. In turn, the compressive forces of top and bottom flat panels can be regarded as equal [21]:

\[ N_{fu} = N_{fb} \]  

(5.31)

When assuming full composite action, the terms in Equation (5.15) will not equate. In turn, \( N_{fu} \) and can be determined assuming linear strain distributions along the section depth. The bending moment resulting in failure stresses experience by top and bottom panels can be calculated based on force moment equilibrium analysis [21]. The bending moment can be determined by summation of moments from each components of the cross-section as [21]:

\[ M_u = N_{fu} \cdot (1.5t_p + d_b + t_{gf} - x) + N_{fb} \left(0.5t_p + t_{gf} - x\right)\ldots \]

\[ + h_w t_{gw} \left(0.5h_w + x\right) + b_{gf} t_{gf} \left(h_w + t_{gf}\right) f_y + b_{gf} f_y x^2 \]

(5.32)
where \( x \) is the distance from neutral axis to the top edge of the web of steel beam, \( b_{gf} \) and \( t_{gf} \) are the width and thickness of beam flange, \( h_w \) and \( t_{gf} \) are the height and thickness of beam web. The position of \( x \) is calculated by summation of forces from each component:

\[
x = \frac{N_{fu} + N_{fl} - h_w t_{gw} f_y}{2w_{gf} f_y}
\]

(5.33)

From Equation (5.16), the bending moment corresponding to failure of pGFRP components can be determined by identifying the corresponding compressive force \( N_f \). The comprehensive work of [21] has identified the critical compressive forces that corresponds to various failure in pGFRP sandwich panel, including flexural failure (compression and buckling of upper panels), shear failure (web of box profiles) and horizontal shear capacities at interfaces (e.g. between I-beam and flat panels). Subsequently, each calculated failure moment is then compared with the yielding moment according to Equation (5.13) to ensure greater capacities. The critical compressive force \( N_f \), for each failure modes pertinent to pGFRP components can be found in [21] and herein are not presented for brevity.

### 5.4. Design charts

#### 5.4.1. Parameters of design charts

Figure 5.2 shows the various geometry of the pGFRP sandwich panel system which comprise of: (1) depth of box profile \( d_b \), (2) spacing of box profiles \( S_b \), and (3) panel thickness \( t_p \). These parameters are used to represent the design charts in this section. As will be seen later, design charts consider a box spacing ratio, i.e. the spacing of box profile to box depth, \( S_b/d_b \). A box spacing ratio of 1 means that there are no gaps between adjacent box profiles within the sandwich panels. In turn, the pGFRP sandwich panel spans between secondary beams as shown in Figure 5.3. As will be needed later, the design charts of composite beams consider the length (L) and width (B) of the composite beam (Figure 5.3).
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5.4.2. Load span charts of sandwich panels

Load span charts for pGFRP sandwich panels are developed to check the allowable live load for various design spans. Figure 5.4 shows examples of load span charts that are generated for box spacing ratios of 1 and 3. Five design curves are plotted in each load span chart corresponding to a
range of box profile depth, $d_b$, (between 50 mm and 150 mm). Each design space along design curve represents the envelope of both strength and deflection criteria calculated using Equations (5.2), (5.3), (5.9), and (5.11). In turn, the design space below each load-span chart corresponds to the feasible sizing of panels that satisfy both ULS and SLS. From Figure 5.4b, a transition point exists for the design curves $d_b = 100$ mm and $d_b = 150$ mm, where strength criteria (buckling) governs the envelope of design criteria. When interpreting the load span charts, designers can ensure the design geometry of sandwich panel is below the prescribed design curve. For example, in Figure 5.4a, a pGFRP sandwich panel with $d_b = 100$ mm is feasible up to 3.5 m span for a design live load of 3.5 kPa (office occupancy according to [38]).

![Diagram of load span charts](image-url)
Figure 5.4. Load span charts of bidirectional pGFRP sandwich panels: (a) $S_b/d_b = 1.5$; (b) $S_b/d_b = 3$. ULS criteria where transition of SLS to ULS of the design curves are shown alongside relevant design curves. Panel thickness and box thickness = 10 mm.

5.4.3. Failure map of sandwich panels

From Equation (5.2), (5.3) and (5.9), failure maps of pGFRP sandwich panels are developed to evaluate the critical failure mode for a given design geometry. Figure 5.5 shows an example of failure maps. The failure maps are a function box spacing ratio, $S_b/d_b$ against span-to-depth ratio of the sandwich panel. The design space represents critical failure mode - either shear, buckling or compression failure - which is the minimum capacities among Equation (5.2), (5.3) and (5.9). Accordingly, the design space forms different failure regions for pGFRP sandwich panel. The contour of failure load is also presented alongside the failure maps. The x-axis of Figure 5.5 is adjusted to clearly illustrate the three failure regions (shear, buckling and compression). The failure region is reproduced for different span of sandwich panels (i.e. 2 m, 3 m and 4 m).
To demonstrate its application, the failure maps are used to evaluate the failure modes of the experimental sandwich beam specimens AB – 1.65 and AB – 2.7 from [18]. Both sandwich specimens are adhesively-bonded with bidirectional pultrusion direction, with the span denoted by the number in the specimen designation. Due to different parameters, a separate failure map is generated for both specimens. Specifically, AB – 1.65 has a shorter span (1.65 m) and a thinner box profile thickness of 5 mm (10 mm for AB – 2.7). The failure maps of specimens AB – 1.65 and AB – 2.7 are presented in Figure 5.6a, and Figure 5.6b respectively. The location of both specimens is presented alongside the respective failure maps. For AB – 2.7, the x-axis of its failure map is extended to illustrate regions of compressive failure. As can be seen Figure 5.6, the predicted failure modes from are in good accordance with the observed failure modes in [18]. From Figure 5.6a, the failure map shows that specimen AB – 1.65 is positioned very close to the border between buckling and shear failure. Based on experimental testing, the failure mode observed for AB - 1.65 first experience shear cracking.

Figure 5.5. Failure map of pGFRP sandwich panel, including the envelope borders for different spans. Panel thickness and box thickness = 10 mm. (Contour units are in kPa)
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within webs of box profile and simultaneous buckling of upper panel [18]. Intuitively, the position of the specimen AB – 1.65 captures the experimental observations of closely-spaced failure modes.

Figure 5.6. Failure maps for experimental specimens with $t_f = 6$ mm: (a) for AB – 1.65: $t_b = 5$ mm, $L = 1.65$ m, and; (b) for AB – 2.7: $t_b = 10$ mm, $L = 2.7$ m. Contour of failure load are also presented, units in kPa.
From the application of the failure maps to experimental specimens, the following conclusions can be drawn regarding the design of pGFRP sandwich panels:

- The two possible failure modes (i.e. shear of webs and buckling of upper panel) are apparent for the common ranges of span-to-depth ratio found in contemporary floor systems (20 to 30).
- Buckling failure is a function of box spacing and span. The region of buckling failure is proportional to increases in spans and box spacing – e.g. a larger buckling failure region is observed for increasing spans in Figure 5.5.
- According to Figure 5.6, a span-to-depth ratio greater than 40 combined with a box spacing ratio of less than 1.5 would result in compressive failure. From this, compression failure is unlikely to govern as such geometries of sandwich panel are often not feasible to meet other requirements such as stiffness for example.
- The capacities of shear failure (denoted by the contour in Figures 5.4 and Figure 5.5) are generally higher than buckling failure. Hence, it can be worth designing sandwich panel to have shear failure as the primary failure mode to ensure high strength capacities.

5.4.4. Design charts for pultruded GFRP-steel composite beams

The design charts for pGFRP-steel composite beams are presented in Figure 5.7 and Figure 5.8, which corresponds to the design live load of 3.5 kPa and 5 kPa respectively. In each design chart, design curves are presented as a function of span (L), panel width (B). The region below the design curves represents the design space that satisfy the following design requirements for composite beam:

- **Requirement 1**: Bending moments correspond to failure of pGFRP components should be less than the yield moment of steel beam.
- **Requirement 2**: The maximum allowable deflection of composite beam is limited to span/300 (following AS1170.1[38]).
**Requirement 3**: Bending moment under a factored design load should be less than the yield moment of steel beam.

The design spaces considered a range of box profile depth (100 mm, 125 mm or 150 mm), and I-beam sizing (between 300 to 600 mm with designations based on British standards [42]). All design curves are developed based on factored design loads (according to load factors shown in Figure 5.7 and Figure 5.8). To interpret the design curve, the design point based on the geometry of the composite beam and the corresponding design curve are identified. A composite beam design is feasible when it lies below the corresponding design curve. When interpreting the design charts, the panel width ($B$) can be regarded as the effective width for shear lag considerations (Figure 5.3).

With reference to Figure 5.7 and Figure 5.8, the following geometries of composite beams are noted:

- pGFRP sandwich panel with box profile depth less than 75 mm are unable to satisfy all design requirements. Due to the relatively small depth, the buckling and compression moment capacities of the pGFRP flat panels are generally lower than yielding moment, which results in pGFRP failure prior to yielding of steel beam (Requirement 1).

- Composite beams with panel width less than 2 m failed Requirement 1. Specifically, these composite beams failed requirement 1, having lower buckling and compressive capacities of pGFRP components than the yielding moment of steel beam. Since compressive and buckling capacities of flat panels are proportional to the panel width, $B$, a larger panel thickness (e.g. 20 mm) can be adopted to increase buckling and compressive capacities of pGFRP flat panels.

- Composite beams with relatively small steel beam sections and large panel width fail Requirement 2. For example, the locus points for design curves (beam depth of 300 mm and 350 mm, box profile depth $d_b = 100$ mm) have a maximum panel width of 4 m. To achieve larger panel widths (satisfy deflection limits for this case), designers can either select a larger panel depth or beam size.
Figure 5.7. Design chart for pGFRP-steel composite beam for different box profile depth and beam sizing. Live load, $Q = 3.5$ kPa.
5.5. Design procedure and example

5.5.1. Overview

Based on the design charts developed in Section 5.4, a simple design procedure for pGFRP sandwich panel floor system is presented in Figure 5.9. The procedure starts with the design of pGFRP sandwich panel by ensuring the allowable load capacity is greater than the design live load. Subsequently, the selected pGFRP sandwich panel forms the pGFRP - steel composite beam which
are checked for its geometry (e.g. $L$, $B$, beam size, and $b_d$). It should be mentioned that the simplified design approach entails designs for pGFRP sandwich panel slab and pGFRP-steel secondary composite beams – the design of primary beam is later discussed in the following section.

Figure 5.9. Design procedure for pGFRP sandwich panel floor systems – pGFRP sandwich panel and pGFRP-steel secondary composite beams.
5.5.2. Example application

The design procedure in Figure 5.9 is demonstrated on a prototype floor of a building floor frame shown in Figure 5.10. The prototype floor consists of a column grid of 12 m by 9 m. The secondary beam (B2) spans along the 12 m length and are spaced at thirds of the primary beam’s span. In turn, pGFRP sandwich panel is required to span 3 m between secondary beams.

Assuming office occupancy, the design live load of 3.5 kPa is considered. From this design load, the feasible box profile depth of the pGFRP sandwich panel are identified using the load span charts Figure 5.4. Assuming the box profile spacing ratio of 1.5, the load span chart in Figure 5.4a indicates that the box profile depth requires a minimum 100 mm for a 3 m span. The next increment sizing, should be selected, i.e. 125 mm. From the selected box profile depth, the failure mode for the sandwich panel geometry considered is identified using the failure map in Figure 5.5. Assuming a panel thickness of 10 mm, the 120 mm deep pGFRP sandwich panel corresponds to a span-to-depth ratio of 25. According to Figure 5.5, the geometry of the sandwich panel (span-to-depth ratio and box
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Spacing ratio lies within the shear failure region. From here, designers can calculate the capacities according to shear failure capacity according to Equation (5.3). For the considered box profile depth (100 mm), the composite beam is checked using the design charts in Figure 5.7 (for live load of 3.5 kPa). Assuming the span of the pGFRP sandwich panel is equal to the full panel width (3 m), the feasible design space of composite beam for the floor bay dimensions (12 × 9 m) is located under the design curve for a box depth of 100 mm and beam sizing of 457×191×98 (Figure 5.7). This indicates that the beam size of 457×191×98 is the minimum feasible beam size for the 12 m secondary beam.

To date, the design of composite primary beams (with box profile aligned parallel to beam span) has not been studied. For this reason, a conservative approach is adopted to predict the sizing of the primary beams. Specifically, the primary beam is designed as individual I-beams which are non-composite to the supporting pGFRP sandwich panels. In turn, the capacities (moment and shear) are calculated, considering the loading scenario as shown in Figure 5.11. Two-point loads, \( P_{B2} \) corresponds to the combined load from self-weight of secondary beam (457×191×98), self-weight of sandwich panel (\( D = 120 \) mm, and \( B = 3 \) m), and the design loads (superimposed dead load and design live load). In addition, the self-weight of a metre width of the sandwich panel is considered as a uniform distributed load, \( w \), acting on the primary beam (see Figure 5.11). The checks are performed based on simple beam theory. Based on the checks, a beam sizing of 533×210×122 is sufficient to resist the design load from secondary beams (\( P_{B2} \)).

![Figure 5.11. Schematic view of the loading condition of primary beam.](image-url)
Chapter 5: Design of statically–feasible pGFRP sandwich panel floors

Going a step further, the above design is compared to conventional composite floor system, with analogous beam layout shown in Figure 5.10. The concrete-steel composite floor is based on the example in [43] which has a 120 mm deep concrete floor slab, which is comparable to the example pGFRP floor in this section. For the conventional floor, the secondary and primary have designations of 533×312×150 and 610×229×140 respectively, which are close to double of those in the example pGFRP sandwich panel floor. Based on the similar steel framing in Figure 5.10, the mass per square metre of steel work between the example pGFRP floor and the concrete-steel floor are approximately 5.89 kg/m$^2$ and 8.17 kg/m$^2$ respectively. This combined with the smaller self-weight of the pGFRP sandwich panel (about 68.53 kg/m$^3$) can be significant for construction, giving cheaper foundation design due to overall lower building mass. Overall, the example design showed that feasible pGFRP sandwich panel floors can give competitive solutions to conventional composite floor systems.

5.6. Conclusions and Future works

This chapter investigates the design of feasible pGFRP sandwich panel floor systems that satisfy static requirements for strength and deflections. A summary of the current state-of-the-art of static design has been presented. Based on the static design rules, the design charts for pGFRP sandwich panel and its constituent pGFRP-steel composite beam are presented. The design charts were developed to allow simple and efficient evaluation of feasible geometries. Failure maps and load span charts were developed for pGFRP sandwich panel assuming one-way spanning conditions. Design charts for the pGFRP-steel composite beam is developed by considering the composite beam as a secondary beam of the floor framing system. Based on the design charts, a simple design procedure for pGFRP sandwich panel floor systems have been devised. The design procedure is demonstrated on an example building composite floor. The pGFRP solution of the example floor gives lower floor mass than a comparable concrete-steel composite floor. Overall, the design charts and the simple design procedure of this work can be applicable to explore designs of statically-feasible pGFRP sandwich panel floor systems.
Chapter 5: Design of statically-feasible pGFRP sandwich panel floors

It should be mentioned that the design charts in this work considers only static design rules. Future work should extend the design charts to include other design criteria, e.g. vibration serviceability designs. In addition, future work in this area should extend experimental testing of pGFRP sandwich panels to further validate the proposed design charts. The design charts of this work can be used as a starting basis to design preliminary experiment specimens. Furthermore, it should be emphasized that the applicability of design charts remains limited to the specific dimensions of pGFRP sandwich panel in this work. Further work can be done to extend the applicability of design charts to various dimensions (e.g. different box profile thickness). Overall, this work should find relevance for designers of pGFRP sandwich panels, including those specialized in the growing field of GFRP composites.
Chapter 5: Design of statically –feasible pGFRP sandwich panel floors

Reference

Chapter 5: Design of statically feasible pGFRP sandwich panel floors


Chapter 5: Design of statically feasible pGFRP sandwich panel floors

Chapter 6.

Vibration serviceability performance of pultruded GFRP sandwich panel floor systems
Preface

Chapter 6 presents the application of the research methodology in order to obtain the main outputs of the thesis. The vibration serviceability performance of pGFRP sandwich panel floors is investigated using the numerical framework presented in Chapter 4. Collectively, the application of the numerical framework to the representative pGFRP sandwich footbridge (Chapters 2 to 4) validates the numerical framework as reliable to predict vibration responses of pGFRP sandwich panel floors under human walking. The simple design procedure from Chapter 5 is used to develop a number of prototypical pGFRP sandwich panel floors for vibration serviceability assessment.

The chapter-paper refers to contents from previous chapters of this thesis. As such, there will be cross-referencing of contents from previous chapters. In addition, the contents have been modified slightly to ensure smooth flow from previous chapters. The chapter-paper is prepared for the following publication.

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Chapter 6: Vibration serviceability performance of pGFRP sandwich floors

Abstract

A novel pultruded glass-fibre-reinforced polymer (pGFRP) sandwich panel has recently been proposed as an innovative alternative to traditional steel-concrete floor slabs. However, pGFRP structures have lower damping and higher accelerance (response per unit force), which render vibration serviceability an important element to consider in design. Current vibration design guidelines were established from experience with traditional steel and concrete structures which have different properties compared to pGFRP – e.g. higher mass and damping. Furthermore, the interaction between humans and structures – termed human-structure interaction (HSI) – is not yet recognized in many design guidelines. Consequently, designs using current practice can result in over-conservative designs of pGFRP sandwich panel floors. This chapter investigates the vibration serviceability performance of pGFRP sandwich panel floors using an experimentally calibrated human spring-mass-damper (SMD) model to account for HSI. Several statically feasible prototype pGFRP sandwich panel floors are considered for vibration serviceability assessment. For comparison, the assessments are performed using current practice, which includes (1) general approaches from design guidelines; and (2) current practice of numerical simulation. The assessment results are compared statistically. The results show that the floors can be achieve adequate vibration serviceability when considering mitigating HSI effects. In contrast, the conservativeness of current practice in vibration assessment of pGFRP sandwich panel floors, means that some floors are deemed unserviceable. The weight penalty to achieve conservative design rule requirements is quantified. Overall, the outcomes of the work should instruct on the applicability of current vibration assessment rules for pGFRP floor structures in general.

KEYWORDS

GFRP; Sandwich Panel; Vibration Serviceability Assessment; Finite Element; Human-structure Interactions; Moving Spring-Mass-Damper Model.
6.1. Introduction

6.1.1. Background

Pultruded glass fibre-reinforced polymer (pGFRP) sandwich panels are seeing increasing applications in constructions of civil engineering structures. This stems from the favourable properties of pGFRP material: its lightweight is significant in terms of transportation and installations [1, 2]; while its excellent corrosion resistance provides service life benefits due to lower maintenance requirements [3]. A pGFRP sandwich panel have the merits of both pGFRP properties (lightweight and durability) and the improved mechanical properties from sandwich constructions, i.e. increased bending stiffness due to increase in second moment of area. Recently, pGFRP sandwich panels are seen as a potential alternative to the conventional concrete-steel deck in building floor systems [4-6]. Despite the tremendous potential, the lower mass, lower stiffness (one fifth of steel [7]) and relatively low damping (damping ratio of 1-2% [8]) of pGFRP means that pGFRP floor systems could be more susceptible to vibrations caused by human activities. Currently, there is lack of performance data on vibration serviceability of pGFRP sandwich panels. Available studies on dynamic performance only concerned free vibration behaviour of small-scale sandwich panel specimens of roof frames [9]. Consequently, the vibration performance of pGFRP sandwich panel floor systems remain unexplored.

A number of practical design methods pertinent to vibration serviceability of floor structure have been presented in contemporary design guidelines for concrete and steel structures. With the different properties - the strength of steel, stiffness of concrete, and weight of dense timber—the solutions from current design rules may be over-conservative for lightweight and lightly-damped pGFRP floor systems. Specifically, GFRP structures have higher accelerance (acceleration per unit harmonic force) [8] which makes them more sensitive to dynamic loads than a comparable traditional structure. Consequently, any conservatism in the current vibration serviceability assessment approaches can lead to additional cost of construction in terms of both materials and labour.
In addition to generating loads, floor occupants can also interact with the floor structure. The mechanical interactions of the human body can add damping, mass, and stiffness to overall vibrating system of floors, which in turn alters the dynamic behaviour; such phenomenon is known as human-structure interaction (HSI) [10, 11]. To date, there are convincing evidences for HSI towards dynamic behaviour of civil structures [12-14]. In particular, the effect of HSI is more significant in lightweight structures, where the ratio of human mass to structure mass is considerable [15, 16]. Current state of the art in numerical simulation uses the moving force (MF) model which ignores HSI effects. In turn, the MF model can result in large overestimation of vibrations [17-19]. As a result, a design may fail vibration serviceability checks when in fact it is serviceable. Consequently, this study considers both current design guidelines and a state-of-the-art assessment method which includes HSI for the vibration serviceability assessment of pGFRP sandwich panel floor systems.

6.1.2. Contribution

With regards to the preceding motivations, the specific aims of this study are to:

1) Investigate vibration serviceability of feasible pGFRP sandwich panel floors systems using state-of-the-art methods, including HSI.
2) Ascertain any weight penalty for designs conforming to current vibration serviceability assessment approaches.

Figure 6.1. shows the approach of this work to assess vibration serviceability of feasible pGFRP sandwich panel floor which designed based on Chapter 5. The vibration assessment is conducted for several prototypical pGFRP sandwich panel floors by subjecting to three different prediction methods outlined in Figure 6.1. The first method comprises the general approaches from current design guidelines. The second method is the current practice in numerical modelling of structures for vibration assessments. The third method is proposed as the best-practice high-fidelity modelling approach, validated for pGFRP sandwich panel floor structures. From the three methods, acceleration predictions are compared with the vibration comfort limits from guidelines to assess vibration
Chapter 6: Vibration serviceability performance of pGFRP sandwich floors

serviceability. Finally, the assessment results are drawn together to draw conclusions, addressing the aim of the thesis.

It should be emphasized that this study evaluates vibration performances of pGFRP sandwich panel floors by considering a single person excitation as the most relevant loading scenario. In reality, floor structures are subjected to multiple human occupants, which can induce larger vibration levels, which however, is not covered in this study. Furthermore, this study focuses on vibration assessment pertinent to human perceptions – serviceability towards vibration sensitive equipment on floor structure is not covered in this work.

Figure 6.1. Vibration serviceability assessment of this work, showing the different assessment methods.

6.2. Overview of current vibration assessment approaches

6.2.1. Vibration comfort limits

In the vibration serviceability assessment of floors, the predicted response is checked against established acceptance criteria to evaluate the adequacy for human comfort. Floor responses can be expressed as peak acceleration, root-mean-square (RMS) acceleration or RMS velocity. The International Standard ISO 2631-2:2003 [20] and ISO 10137:2007 [21] have proposed limits based on multipliers of a baseline curve for vertical vibrations for which the probability of adverse comment
is low. For offices, a baseline value of 0.005 m/s² is suggested for floors with first natural frequency between 4 Hz to 8 Hz. Table 6.1 summarize the human comfort limits as suggested in contemporary design guidelines for office occupancy. For example, the RMS acceleration limits from the UK Concrete Centre publication, CCIP-016, and Steel construction Institute, SCI P354, corresponds to a multiplier of 8 to the baseline curve of ISO 10137 (0.04 m/s²), while the AISC DG 11 suggests a peak acceleration based on a multiplier of 10 (0.05 m/s²). The RMS acceleration is considered as more meaningful than peak accelerations as it is not influenced by one unrepresentative peak in the response time history [22].

Table 6.1. Recommended acceleration response limits for office occupancy according to contemporary design guidelines.

<table>
<thead>
<tr>
<th>Code/guidelines</th>
<th>Response metric</th>
<th>Limit</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AISC DG 11</td>
<td>Peak acceleration, (a_{\text{peak}}) (m/s²)</td>
<td>0.05</td>
<td>[23]</td>
</tr>
<tr>
<td>SCI P354</td>
<td>RMS acceleration, (a_{\text{rms}}) (m/s²)</td>
<td>0.04</td>
<td>[22]</td>
</tr>
<tr>
<td>CCIP – 016</td>
<td></td>
<td>0.04</td>
<td>[24]</td>
</tr>
<tr>
<td>ISO 10137</td>
<td></td>
<td>0.02</td>
<td>[21]</td>
</tr>
</tbody>
</table>

6.2.2. Numerical methods

For floors with irregular plan layouts, FE modelling presents a better alternative to analytical methods for performing vibration analysis. Since the early 1980s, the increasing use of numerical models for vibration serviceability assessment stems from its ability to analyse large and complex structures [25]. An example review from the literature on FE techniques for vibration analysis of floor systems can be found in [26]. In terms of vibration serviceability assessment, FE techniques involve building FE models of the analysed structure which are subsequently used to perform modal analysis and simulation of dynamic loads. The predicted responses are then compared with acceptance limit (e.g. Table 6.1).

Floor structures are categorized into low frequency floors where vibration amplification results from resonance, and high frequency floors where transient responses are more predominant over resonant
Chapter 6: Vibration serviceability performance of pGFRP sandwich floors

responses. This artificial characterization of floor responses is presented as a threshold natural frequency. Different threshold values of natural frequencies have been reported. For example, threshold is taken as 10 Hz as per SCI P354 [22] while the AISC DG 11 reported 9 Hz [23]. In some circumstances, some floors may exhibit both low and high frequency response characteristics. From this, vibration analysis of floors is performed based on the respective vibration types (resonance, transient, or a combination of both). Currently, there is no single model of the walking force that represents full response of both low and high frequency floor responses. Instead, the representation of walking force is segregated into either resonant (periodic force models) or transient (impulse).

Transient responses of floors result in a series of impulses corresponding to each footfall. The impulses have an initial peak followed by a decaying vibration before the next footfall. This can be simulated either using force measured from a heel-drop test or an effective impulse (following method in [24]).

Resonance occurred as a result of repeated footfalls. For this, walking forces are typically modelled as a moving concentrated force acting onto structural models – commonly referred as the moving force (MF) model. As the repeated footfalls of humans are generally periodic in nature, the MF model can represent the magnitude of human force as a Fourier series with several harmonic components. The general form of the MB model can be expressed as:

\[ G(t) = W_p \sum_{i=0}^{h} \alpha_i \cos (2\pi f_w t + \phi_i) \]  

(6.1)

where \( W_p = m_p g \) is the subject’s weight (often assumed between 700 N and 800 N); \( m_p \) and \( g \) are the walker mass and gravitational acceleration respectively; \( f_w \) is the pacing frequency; \( t \) is time; and \( \alpha_i \) is the dynamic load factors (DLFs) for the \( i \)th harmonic component of the walking excitation. The MF model has been suggested by a number of researchers and is adopted in current design guidelines [21]. Despite widespread adoption, many different DLFs, \( \alpha \), and phases, \( \phi \), have been suggested by various authors. Table 6.2 summarizes the different representations of the Fourier series force [27].
Chapter 6: Vibration serviceability performance of pGFRP sandwich floors

For example, the AISC DG 11 ignores the phase angle $\phi$ and does not consider the static force component of the walker, $W_p$.

Table 6.2. Dynamic load factors of MF models as suggested in design guidelines.

<table>
<thead>
<tr>
<th>Harmonic, $i$</th>
<th>AISC DG 11</th>
<th>CCIP-016</th>
<th>SCI P354</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_i$</td>
<td>$\alpha_i$</td>
<td>$\alpha_i$</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.41($if_w - 0.95) \leq 0.56$</td>
<td>0.436($if_w - 0.95$)</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.069 + 0.0056$if_w$</td>
<td>0.006($if_w + 12.3$)</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>0.033 + 0.0064$if_w$</td>
<td>0.007($if_w + 5.2$)</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>0.013 + 0.0065$if_w$</td>
<td>0.007($if_w + 2$)</td>
</tr>
</tbody>
</table>

In vibration analysis, it is important to specify a critical pacing frequency to simulate walking forces. This is important for resonance responses, where the aim is to select a pacing frequency such that one or more of its harmonics matches closely to the floor’s natural frequencies. Contrary for high frequency floors, a pacing frequency is not needed since responses are transient rather than steady state.

6.2.3. General approaches in design guidelines

Contemporary design guidelines propose a number of analytical methods to predict the dynamic properties and responses of floor structures. Analytical methods are simpler and more convenient to perform quick evaluation of floor responses. Similar to numerical method, analytical methods are characterized based on the dominant responses of the floors: resonant for low frequency floors and impulse for high frequency floors. As mentioned, the threshold natural frequency that demarcate between high and low frequency floors are about 10 Hz. Resonant calculations are based on the MF model in Equation (6.1) which utilize either one or multiple harmonic excitations to calculate the responses. For example, the AISC DG 11 is based on single harmonic excitation [23]. In contrast to resonant responses, Impulse response calculations regard excitation due to a single footfall, where the maximum velocity due to the single footfall is typically compared with prescribed velocity limits [24].
Analytical methods can be paired with the use of a finite element (FE) model, which can be denoted as semi-FE method. In general, analytical expressions of semi-FE methods use inputs of mode shape magnitude and natural frequency from an FE model. The analytical methods of the CCIP-016 and the SCI P354 are semi-FE methods. The MF model is then used on the FE-generated mode shape and frequency, and the responses are calculated. For this work, the semi-FE methods and analytical method of SCI 354, CCIP-016, and AISC DG 11 are considered as the general approaches.

6.3. Numerical modelling

6.3.1. Overview

In this chapter, the numerical framework from Chapter 4 is adopted to study the vibration serviceability of pGFRP sandwich panel floors. Given the light weight of pGFRP sandwich panel structures, the effects of HSI can be more significant in pGFRP sandwich panel floors due to the larger ratio of human mass to floor structure modal mass \([28, 29]\). The framework utilizes interactive models that account for interaction of the human body (in terms of damping and stiffness) with the structural models. This representation of human forces is more realistic \([30-33]\), giving lower and closer responses to the actual measurements than the current practices that ignores HSI. Herein, the framework is validated as best-practice and high-fidelity to predict vibration responses of pGFRP sandwich structures.

6.3.2. Human-structure model

Pultruded GFRP sandwich panel floors are modelled using a simplified 2-D FE model shown in Figure 6.2. The model uses 2-D plate elements with consideration of first-order-shear-deformation theory to account for shear deformation when spans are short, which is important for pGFRP structures \([34]\). The supporting beams of the floor are modelled using 1-D Timoshenko beam elements which also accounts for shear deformations. Offset properties are defined for the beam
elements to allow for eccentricity between structural members (deck and beam). The modelling of beam and plate elements assumes full-composite action between floor panel and beams, which is reasonable for dynamic analysis since vibrational strains are generally lower than those under large static loading and most fixing methods (bolting or adhesive) achieve full composite action. Orthotropic properties were defined for the plate elements based on equivalent properties of the entire sandwich panel cross-section.

To account for HSI, an interactive human model is considered. The mass, stiffness and damping contribution of the human body are represented with a single degree-of-freedom (SDOF) moving spring-mass-damper (SMD) model. To date, this approach has begun to emerge recently in the literature in explicit studies of this [35, 36]. To use the SMD model, the parameters of SMD model of the human body are required (i.e. natural frequency \( f_p \), and damping ratio, \( \zeta_p \)). A large number of studies have proposed SMD parameters for vibration serviceability of structures (hereafter the assessment approach using this HSI framework is denoted as SMD model). However, extensive human-walking trials have been used for this work to inform the suitable parameters, as is explained later.

![Figure 6.2. Formulation of a moving SMD model on simple 2-D FE model of a pGFRP sandwich panel floor (shown without boundary conditions).](image-url)
6.3.3. Validation

The numerical framework is validated to reliably-predict the vibration serviceability of pGFRP sandwich panel systems. For this, the framework is applied to model and predict the vibration performance of a representative pGFRP sandwich footbridge (denoted as the MB). The representative structure adopts the bespoke pGFRP sandwich panel as the deck solution [37]. The MB consists of a deck supported with underlying I-beams, which is representative of a one-way spanning floor unit. For this work, a reference test subject described in Section 6.3.3 is considered as representative subject of for vibration assessment. The reference test subject has a mass (718 N) that is close to the average person mass as suggested by design guidelines for vibration analysis [24].

The MB is modelled as a 2-D FE model. The mass and stiffness properties of the FE model are determined based on the physical geometry of the MB. The mode shapes and natural frequencies of the MB are obtained through modal analysis of the FE model – see Chapter 2. According to experimental measurements, the first mode (flexural) and second mode (torsional) have natural frequency of 5.6 Hz and 10.0 Hz respectively [29]. According to Chapter 4, the FE model gives reasonable predictions of the natural frequencies from the simplified representation of sectional properties (e.g. sandwich panel and beam members). The comparison of modal properties validates the assumptions and element choices made for pGFRP sandwich panel floor models.

Then, the moving SMD model is used to model a single person walking and obtain the acceleration responses of the MB. The parameters of the SMD model is obtained based on the parameter identification method devised in Chapter 4. The parameters are obtained by matching the simulated acceleration responses with those from measurements. The SMD parameters includes the natural frequency, $f_p$, and damping ratio, $\zeta_p$. From the initial SMD parameters proposed in [28] ($f_p = 1.87$ Hz, and $\zeta_p = 0.2$), the fitted natural frequency and damping ratio are 2.03 Hz and 0.4 respectively.
Accordingly, the fitted parameters correspond to the SDOF stiffness, \( k_p \), and damping, \( c_p \), of 13.11 kN/m and 783.6 N s/m respectively, which are considered in the formulation of the SMD model. The acceleration responses from the initial parameters and the fitted parameters for the reference test subject are shown in Figure 6.3. As can be seen, the fitted parameters give closer agreement of the measured acceleration envelopes compared to those using the initial SMD parameters.

![Acceleration responses comparison](image)

**Figure 6.3.** Positive envelope of rolling 1-s RMS acceleration responses using SMD model with initial parameters \( (f_p = 1.87 \text{ Hz}, \ zeta_p = 0.2) \) and fitted parameters \( (f_p = 2.03 \text{ Hz}, \ zeta_p = 0.4) \) for the reference subject \( (m_p = 73.2 \text{ kg}) \).

### 6.4. Application

#### 6.4.1. Investigated pultruded GFRP sandwich panel floors

In this study, a prototype pGFRP sandwich panel floor is considered for vibration serviceability assessment. This multi-bay floor consists of column grids with steel beam frames as depicted in Figure 6.4. Each column grid consists of secondary beams (B2) at third spans of the primary beams (B2). Table 6.3 summarizes the different parameters of each floor, which includes the dimensions of...
Chapter 6: Vibration serviceability performance of pGFRP sandwich floors
column grid, depth of pGFRP sandwich panel and beam sizes. The primary and secondary beam
designations are based on British standards [38]. The floor parameters are designed in the previous
study to satisfy static requirements, i.e. strength and deflection. According to Chapter 5, it is shown
that the deflection limits (serviceability limit state) tends to govern the design of the prototypes.
Specifically, the pGFRP sandwich panels and composite beams are designed for a deflection limit of
1/300 of the span length – following Chapter 5.

![Figure 6.4. Structural plan of a generic floor framing system (dimensions corresponds to Floor 3). The bay of interest for vibration assessment has been highlighted in red.](image)

<table>
<thead>
<tr>
<th>Floor ID</th>
<th>Column grid (m × m)</th>
<th>Depth of sandwich panel, D (mm)</th>
<th>Beam designation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>9 × 7.5</td>
<td>100</td>
<td>305×165×54</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>356×171×67</td>
</tr>
<tr>
<td>2</td>
<td>9 × 9</td>
<td>120</td>
<td>356×171×67</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>406×178×67</td>
</tr>
<tr>
<td>3</td>
<td>12 × 9</td>
<td>120</td>
<td>457×191×98</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>533×210×122</td>
</tr>
<tr>
<td>4</td>
<td>16 × 9</td>
<td>120</td>
<td>533×210×122</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>610×229×140</td>
</tr>
</tbody>
</table>
Chapter 6: Vibration serviceability performance of pGFRP sandwich floors

As the statically-conforming designs (prototype) did not include vibration analysis, it is anticipated that some of the prototype floors may be inadequate for vibration serviceability. For this, an additional three variants are considered for each prototype in Table 6.3 (Floor ID 1 to 4) have been considered - each having a specific floor parameter enhanced over its base prototype variant. These variants can be regarded as stiffening schemes, i.e. increasing sandwich panel depth, D, and stiffening beam members. As a result, a total of twelve pGFRP sandwich floors variants are added for assessment - the floor parameters are given in Table 6.4 (denoted as variants of the prototype from hereafter). The variants are designated with letters “B” and “P” to represent the increase to beam sizing (both B1 and B2) and sandwich panel depth respectively. A “B-P” variant represents a prototype floor with increase to both beam size and sandwich panel depth. The increments of floor parameters are based on the member dimensions considered in Chapter 5. This includes designations for beam sizes (both primary and secondary), depth of sandwich panel, and column grids.

Table 6.4. Variants of prototype pGFRP sandwich panel floors.

<table>
<thead>
<tr>
<th>Floor ID</th>
<th>Column grid (m × m)</th>
<th>Depth of sandwich panel, D (mm)</th>
<th>Beam designation</th>
<th>B1</th>
<th>B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-B</td>
<td>9 × 7.5</td>
<td>100</td>
<td></td>
<td>356×171×67</td>
<td>406×178×67</td>
</tr>
<tr>
<td>2-B</td>
<td>9 × 9</td>
<td>120</td>
<td></td>
<td>406×178×67</td>
<td>457×191×98</td>
</tr>
<tr>
<td>3-B</td>
<td>12 × 9</td>
<td>120</td>
<td></td>
<td>533×210×122</td>
<td>610×229×140</td>
</tr>
<tr>
<td>4-B</td>
<td>16 × 9</td>
<td>120</td>
<td></td>
<td>610×229×140</td>
<td>686×254×170</td>
</tr>
<tr>
<td>1-P</td>
<td>9 × 7.5</td>
<td>120</td>
<td></td>
<td>305×165×54</td>
<td>356×171×67</td>
</tr>
<tr>
<td>2-P</td>
<td>9 × 9</td>
<td>150</td>
<td></td>
<td>356×171×67</td>
<td>406×178×67</td>
</tr>
<tr>
<td>3-P</td>
<td>12 × 9</td>
<td>150</td>
<td></td>
<td>457×191×98</td>
<td>533×210×122</td>
</tr>
<tr>
<td>4-P</td>
<td>16 × 9</td>
<td>150</td>
<td></td>
<td>533×210×122</td>
<td>610×229×140</td>
</tr>
<tr>
<td>1-B-P</td>
<td>9 × 7.5</td>
<td>120</td>
<td></td>
<td>356×171×67</td>
<td>406×178×67</td>
</tr>
<tr>
<td>2-B-P</td>
<td>9 × 9</td>
<td>150</td>
<td></td>
<td>406×178×67</td>
<td>457×191×98</td>
</tr>
<tr>
<td>3-B-P</td>
<td>12 × 9</td>
<td>150</td>
<td></td>
<td>533×210×122</td>
<td>610×229×140</td>
</tr>
<tr>
<td>4-B-P</td>
<td>16 × 9</td>
<td>150</td>
<td></td>
<td>610×229×140</td>
<td>686×254×170</td>
</tr>
</tbody>
</table>
6.4.2. Numerical simulation of walking responses

The numerical framework (Chapter 4) is used to simulate the acceleration responses of the prototype floors in Table 6.3 and Table 6.4. First, the floors are modelled as a 2-D FE plate model as described in Section 6.3. Figure 6.5 shows the 2-D FE model of the multi-bay floor framing system, along with the column boundary conditions. For this work, columns are modelled as an ideal hinge (restrained in vertical translation but allowing rotations). The columns can be modelled as flexible elements with rotational stiffness provided by the column sections, however this is not considered for this work with the assumption that the columns have sufficient rigidity [39]. Presence of partitions on floor structure can affect the dynamic properties of floors [40], however, these effects are not covered in the simulations.

![2-D FE model of prototype floor](image)

Figure 6.5. 2-D FE model of prototype floor (no dimensions for illustration only), showing the simulated walking path of MF and SMD models.

Walking path of the SMD model is selected to produce the maximum vibration responses of the interior floor bay of interest. An example walking path of a single person is shown in Figure 6.5. The walking path for each floor model is selected based on two criteria. First, the direction walking path is selected as parallel to the longest dimension of the floor bay to allow the longest excitation possible.
onto the inner floor bay. Secondly, the walking path must cross the largest modal displacement of the dominant vibration mode for the floors (described later in Section 6.4.3).

A number of FE modelling features are considered. Time-step analysis is performed using Newmark-beta method. Damping of all FE models considers Rayleigh damping as the sum of all damping contributions from both structural and non-structural components. For this work, a damping ratio of 3% is selected, which assumes structural damping of pGFRP sandwich floor system of 1% (with reference to modal damping identified in experiment [29]), damping due to finishing of 1% [41] and damping due to raised floors of 1%. An additional non-structural mass of 90 kg/m² is considered in the FE models during numerical simulations to represent the mass due to non-structural finishes. Similar to common practice, a portion of the live load is considered to the overall mass of the FE model. For this, addition live load is considered as 10% of the design live load (0.35 kPa), based on the rule of thumb from SCI P354 [22].

6.4.3. Modal properties and critical pacing frequency

Figure 6.6 shows natural frequencies and mode shapes of Floor 1 obtained from the FE model. As will be needed later, the mode shapes of Floor 1 that results in large modal displacement of the interior bay have been illustrated in Figure 6.6. The natural frequencies of Floor 1 and its variants are presented in Table 6.5. Also presented are the percentage difference in natural frequencies between each floor variant. Of course, the variants that have been stiffened have higher natural frequencies. Notably, the mode shapes order did not change, with the first and eighth vibrational mode giving maximum modal displacement at the interior bay. For brevity, the results for Floors 2, 3 and 4 are presented in the appendix of this chapter.
Table 6.5. Comparison of natural frequencies between variants of Floor 1. Natural frequencies of modes that result in large modal displacement in the interior bay are given in bold.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Floor 1</th>
<th>Variants</th>
<th>Percentage change in natural frequency, %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B¹</td>
<td>P²</td>
</tr>
<tr>
<td>1</td>
<td>6.32</td>
<td>6.87</td>
<td>6.91</td>
</tr>
<tr>
<td>2</td>
<td>6.89</td>
<td>7.51</td>
<td>7.61</td>
</tr>
<tr>
<td>3</td>
<td>6.96</td>
<td>7.63</td>
<td>7.62</td>
</tr>
<tr>
<td>4</td>
<td>7.24</td>
<td>8.07</td>
<td>7.76</td>
</tr>
<tr>
<td>5</td>
<td>7.27</td>
<td>8.34</td>
<td>7.99</td>
</tr>
<tr>
<td>6</td>
<td>7.57</td>
<td>8.50</td>
<td>8.21</td>
</tr>
<tr>
<td>7</td>
<td>7.67</td>
<td>8.88</td>
<td>8.33</td>
</tr>
<tr>
<td>8</td>
<td>7.95</td>
<td>8.91</td>
<td>8.74</td>
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<tr>
<td>9</td>
<td>8.16</td>
<td>9.15</td>
<td>8.98</td>
</tr>
<tr>
<td>10</td>
<td>9.49</td>
<td>10.84</td>
<td>10.68</td>
</tr>
</tbody>
</table>

¹: Increasing beam sizes
²: Increasing sandwich panel depth
³: Increasing both beam and sandwich panel

For all simulations, a pacing frequency from the common range of walking frequencies (1.6-2.2 Hz) is selected to excite the floors to resonant responses, namely the largest responses in the interior bay. As there are several modes that result in large modal displacement in the bay of interest (shown in Figure 6.6), the first fundamental mode is not necessarily the most responsive mode of the interior bay. Furthermore, the presence of the reference subject at the middle of the floor bay may alter the natural frequency of the structure during walking. To identify the critical pacing frequency, $f_w$, a simulated heel-drop impulse test is performed at the centre of the interior bay to identify which gives the largest response for the interior bay. In addition, a stationary SMD model of the reference subject is modelled in the middle of the floor bay to include its influence towards the floor responses. The heel-drop force is approximated by a ramp function of which the maximum amplitude is 1000 N and decreases to zeros within a duration of 40 milliseconds.

The time histories of floor responses due to the simulated impulse test are transformed to frequency domain using Fast Fourier Transform (FFT). Figure 6.7 shows the frequency domain of responses from Floor 1 due to the impulse induced in the interior bay and in the adjacent edge bay. As can be
seen, the maximum responses occur at approximately 8 Hz, which is very close to fourth harmonic excitation of the 8th mode in Figure 6.6. The response due to the mode 1 is evident in Figure 6.7. (approximately 6 Hz), albeit smaller than the response from mode 8. This can be due to the low modal mass of mode 8 (about 6200 kg) in comparison to modal mass of mode 1 (about 30000 kg). For all floors, the simulated impact test is performed prior to numerical simulations of SMD model.

Figure 6.6. Resonant modes of Floor 1 for interior and edge bays: (a) Mode 1, $f = 6.32$ Hz, (b) Mode 2, $f = 6.89$, (c) Mode 4, $f = 7.24$ Hz, and (d) Mode 8, $f = 7.95$ Hz.

Figure 6.7. Frequency domain of responses at middle of interior and edge floor bay for Floor 1 due to impulse excitation.
6.5. Vibration serviceability assessment

6.5.1. Response from numerical simulations

As an example, the acceleration responses of Floor 1 obtained from simulations of the MF and SMD model is presented in Figure 6.8. The acceleration responses are obtained by simulating a pacing frequency of 2 Hz – the critical pacing frequency identified in Section 6.4.3. The time histories of acceleration are obtained from two points of the structure, namely the midpoint of the interior bay and edge bays (as described in Section 6.4.3). In Figure 6.8, the 1-s RMS acceleration of the time histories are also plotted alongside the acceleration time histories. For both human model (MF and SMD model), vibration assessment involves extracting the maximum 1-s RMS acceleration of the interior bay responses.

Figure 6.8. Acceleration time histories of Floor 1 under critical walking frequency (2 Hz) using the: (a) MF model, and (b) SMD model.
6.5.2. Summary of results

Figure 6.9 presents the summary of assessment results using both the general approaches form design guidelines and the numerical simulations described previously. The methodologies used in the contemporary design guidelines have been summarized in the literature, (e.g. [42]) and so are not presented for brevity. The response metric using SCI P354 and CCIP-016, as well as both FE approaches (MF and SMD models) are reported as the maximum 1-s RMS acceleration - indicated in Figure 6.8. For the AISC DG 11 approach, the peak accelerations are given alongside the same axis for RMS accelerations in Figure 6.9. The respective limits of peak and RMS acceleration are also plotted alongside in Figure 6.9. To interpret the assessment results, floors with responses (peak or RMS) above the respective limits fails vibration serviceability. In addition, the maximum 1-s RMS acceleration responses from Figure 6.8 (Floor 1) are also noted in Figure 6.9a.

![Figure 6.9](image)

(a) Floor 1; (b) Floor 2; (c) Floor 3; (d) Floor 4;

Figure 6.9. Acceleration response predictions for prototype and variants of Floors 1 to 4 (a – d).
6.5.3. Statistical comparisons

Table 6.6 presents the ratio of predicted responses from both current practices (general and numerical approaches) with those using the best-practice approach (SMD model). The mean of the ratios, $\mu$ are obtained for the current practices in Table 6.6. As can be seen, the mean of ratios is different to those of the SMD model, with values greater than 1 indicating the overestimations of current practice.

<table>
<thead>
<tr>
<th>Floor ID</th>
<th>$f_1$ (Hz)</th>
<th>$f_w$ (Hz)</th>
<th>AISC(^1)</th>
<th>SCI</th>
<th>CCIP</th>
<th>MF</th>
<th>SMD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_{ASC} / a_{SMD}$</td>
<td>$a_{SCI} / a_{SMD}$</td>
<td>$a_{CCIP} / a_{SMD}$</td>
<td>$a_{MF} / a_{SMD}$</td>
<td>$a_{SMD} / a_{SMD}$</td>
</tr>
<tr>
<td>1</td>
<td>6.32</td>
<td>2.00</td>
<td>2.22</td>
<td>1.16</td>
<td>1.17</td>
<td>1.18</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>5.71</td>
<td>2.17</td>
<td>3.77</td>
<td>1.38</td>
<td>1.29</td>
<td>1.31</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>6.53</td>
<td>1.97</td>
<td>1.90</td>
<td>1.18</td>
<td>1.10</td>
<td>1.10</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>5.03</td>
<td>2.26</td>
<td>3.05</td>
<td>1.22</td>
<td>1.08</td>
<td>1.14</td>
<td>1.00</td>
</tr>
<tr>
<td>1-B-F</td>
<td>7.44</td>
<td>2.05</td>
<td>3.41</td>
<td>2.24</td>
<td>2.24</td>
<td>1.24</td>
<td>1.00</td>
</tr>
<tr>
<td>2-B-F</td>
<td>7.32</td>
<td>2.17</td>
<td>4.90</td>
<td>1.65</td>
<td>1.35</td>
<td>1.25</td>
<td>1.00</td>
</tr>
<tr>
<td>3-B-F</td>
<td>7.83</td>
<td>1.97</td>
<td>1.97</td>
<td>1.67</td>
<td>1.37</td>
<td>1.10</td>
<td>1.00</td>
</tr>
<tr>
<td>4-B-F</td>
<td>6.03</td>
<td>2.26</td>
<td>2.15</td>
<td>1.21</td>
<td>1.06</td>
<td>1.09</td>
<td>1.00</td>
</tr>
<tr>
<td>1-P</td>
<td>6.86</td>
<td>2.40</td>
<td>2.01</td>
<td>1.26</td>
<td>1.23</td>
<td>1.32</td>
<td>1.00</td>
</tr>
<tr>
<td>1-P</td>
<td>6.64</td>
<td>2.40</td>
<td>3.82</td>
<td>1.79</td>
<td>1.65</td>
<td>1.26</td>
<td>1.00</td>
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<tr>
<td>1-P</td>
<td>7.1</td>
<td>2.40</td>
<td>1.80</td>
<td>1.20</td>
<td>1.07</td>
<td>1.11</td>
<td>1.00</td>
</tr>
<tr>
<td>4-P</td>
<td>5.71</td>
<td>2.04</td>
<td>2.69</td>
<td>1.31</td>
<td>1.09</td>
<td>1.09</td>
<td>1.00</td>
</tr>
<tr>
<td>1-B</td>
<td>6.91</td>
<td>2.28</td>
<td>1.88</td>
<td>1.23</td>
<td>1.31</td>
<td>1.37</td>
<td>1.00</td>
</tr>
<tr>
<td>2-B</td>
<td>6.39</td>
<td>2.4</td>
<td>4.09</td>
<td>1.76</td>
<td>1.53</td>
<td>1.06</td>
<td>1.00</td>
</tr>
<tr>
<td>3-B</td>
<td>7.25</td>
<td>2.22</td>
<td>1.12</td>
<td>1.09</td>
<td>1.05</td>
<td>1.11</td>
<td>1.00</td>
</tr>
<tr>
<td>4-B</td>
<td>5.36</td>
<td>2.4</td>
<td>3.13</td>
<td>1.23</td>
<td>1.07</td>
<td>1.10</td>
<td>1.00</td>
</tr>
<tr>
<td>Mean $\mu$</td>
<td>1.47</td>
<td>1.39</td>
<td>1.27</td>
<td>1.17</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^1\): Peak acceleration
\(^2\): using peak acceleration from responses using SMD model.

From the difference of assessment results in Table 6.6, two questions arise as to whether the mean of each assessment approach are statistically the same, namely: (1) between general approaches and the MF model, and (2) between current practice and the SMD model. That is, are the fundamental phenomena of the problem (vibration performance of pGFRP sandwich panel floors) adequately
captured through the current assessment approaches? To check this, statistical hypothesis testing is used. The two-sided Student’s $t$-test with equal variance is used to compare the assessments between general approaches, the MF model and the proposed SMD model approach. For the two comparisons, two sets of hypothesis testing are conducted. The null, $H_0$, and alternative hypothesis, $H_1$ for both tests are given as:

1) The assessment results between general approaches and MF model

\[
H_0 : \mu_{DG} = \mu_{MF} \\
H_1 : \mu_{DG} \neq \mu_{MF}
\]  

(6.2)

2) The assessment results between current practice and SMD model

\[
H_0 : \mu_{DG/MF} = \mu_{SMD} \\
H_1 : \mu_{DG/MF} \neq \mu_{SMD}
\]  

(6.3)

where the subscript $\mu$ denoted the assessment approaches (e.g. DG stands for general approach). If the null hypothesis is rejected, it means that the means of two assessment results statistically different; otherwise, the means of two assessment approaches are statistically similar. For each hypothesis testing, the $p$-value that indicates the probability level of two assessment rules are equal, is calculated. Typically, a significance value, $\alpha$, is assigned to represent the probability to accept the null hypothesis (a $p$-value smaller than $\alpha$ means reject). Table 6.7 presents the mean ratios of each tests represented by Equations (6.2) and (6.3). For the tests, the means of the general assessments are checked with the unity means.

<table>
<thead>
<tr>
<th>$t$-test</th>
<th>$\mu_{DG}$</th>
<th>$\mu_{MF/SMD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AISC DG 11</td>
<td>SCI P354</td>
</tr>
<tr>
<td>MF vs general approaches</td>
<td>1.32</td>
<td>1.20</td>
</tr>
<tr>
<td>SMD vs current practice</td>
<td>1.47</td>
<td>1.39</td>
</tr>
</tbody>
</table>

Table 6.7. Mean of ratios considered for the $t$-tests. The means are calculated from Table 6.6 and Table 6.13.
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The results of the $t$-tests are summarized in Table 6.8. Small $p$-values (close to zero) are obtained for comparison of general approaches with SMD model. In contrast, larger $p$-values are observed for comparison of general approaches with MF model. This can be expected since the general approaches are based on the MF model. Further, the $p$-value between MF model and the CCIP-016 approach are fairly similar, which can be expected since both method uses the same DLFs (Young [27]). Overall, the $t$-tests showed that the mean of the assessment approaches are statistically significantly different to the SMD model. This difference in the means indicates that there is a phenomenon not captured in the current practices, supporting the hypothesis that consideration of HSI is important to consider in vibration serviceability assessment of pGFRP sandwich panel floors.

Table 6.8. Hypothesis testing results ($p$ – values) for serviceability assessment for all cases

<table>
<thead>
<tr>
<th>$t$-test</th>
<th>AISC DG 11</th>
<th>SCI P354</th>
<th>CCIP-0016</th>
<th>MF model</th>
<th>SMD model</th>
</tr>
</thead>
<tbody>
<tr>
<td>MF vs general approaches</td>
<td>0.02</td>
<td>0.01</td>
<td>0.13</td>
<td>-</td>
<td>0.00</td>
</tr>
<tr>
<td>SMD vs current practice</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
</tbody>
</table>

6.6. Discussion and remarks

6.6.1. Vibration performance of pultruded GFRP sandwich panel floors

As can be seen in Figure 6.9, all floors responded differently to their critical pacing frequencies that gives the largest responses in the interior bay of interest. Among the four prototypes, Floor 1 was the most responsive, with acceleration levels generally above the prescribed human comfort limits (e.g. 0.04 m/s$^2$ for RMS acceleration). Interestingly, the levels of vibration responses decrease with increasing floor spans, e.g. Floor 4 in comparison to Floor 1. This can be explained by the relatively larger mass of floors due to larger floor area and longer steel beam sections. In turn a larger effective weight is evident for vibration modes of Floor 4 (mode 1 about 80 t) compared to floor 1 (mode 1 about 30 t), which results in lower responses in Floor 4.
According to the results in Figure 6.9 all current practices overestimated the acceleration responses compared to the benchmark calibrated SMD model. As a result, current practice shows Floors 1, 2, and 3 (both prototype and variants) failing vibration serviceability, where instead they (the floor variants) are shown to be serviceable from the simulations with SMD model. Interestingly, the influence of the SMD model (reducing the response) is less prominent in Floors 3 and 4 compared to Floor 1. This can be again explained by the relatively larger mass of Floor 3 and 4 (larger floor area and longer spans of beam members) which means that the effects of the SMD model is smaller, since it is proportional to the ratio of human mass to floor structure mass.

6.6.2. The cost of conservative assessments

The assessment results in Figure 6.9 showed that stiffening floor members in all floor prototype can reduce the responses of floors to serviceable levels. However, increasing the sizes of floor members for adequate vibration serviceability places a weight penalty onto the floor structure. To evaluate the weight penalty for vibration serviceability, the mass per unit area of the pGFRP sandwich panel floors are compared between static conforming designs (prototype) and designs that conforms vibration comfort. The total mass of floors, $M_{total}$, comprise of the mass per unit area of the pGFRP sandwich panels, $M_{panel}$ (density taken as 1800 kg/m$^3$), the equivalent mass per unit area of steel beams, $M_{steel}$. Since non-structural mass are constant for all floors (90 kg/m$^2$) they are excluded from comparisons.

For the comparison of weight penalty, the feasible floor variants that satisfy vibration comfort based on CCIP-016 and SMD model are considered. From the results in Figure 6.9, it was found that the depth of the sandwich panel needs to be at least doubled in order to satisfy vibration serviceability according to CCIP-016. In contrast, the “P” variants of the prototype floors are shown to be serviceable according to the SMD model. Table 6.9 shows the mass breakdown of floor variants that satisfy vibration comfort according to assessment based on general approaches (CCIP -016) and the SMD model. As can be seen, a lower average weight penalty for vibration conformance is obtained.
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Based on assessment using SMD model. Overall, the results show the significance of HSI considerations as measured by the weight penalty of conservative vibration serviceability assessments for pGFRP sandwich panel floors.

Table 6.9. Comparison of mass per unit area between feasible prototype floors from general approaches and the SMD model. (Units in kg/m²)

<table>
<thead>
<tr>
<th>Floor</th>
<th>Prototype</th>
<th>CCIP - 016&lt;sup&gt;1a,b&lt;/sup&gt;</th>
<th>SMD model&lt;sup&gt;2&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M&lt;sub&gt;panel&lt;/sub&gt;</td>
<td>M&lt;sub&gt;steel&lt;/sub&gt;</td>
<td>M&lt;sub&gt;total&lt;/sub&gt;</td>
</tr>
<tr>
<td>1</td>
<td>67.59</td>
<td>33.94</td>
<td>101.53</td>
</tr>
<tr>
<td>2</td>
<td>68.52</td>
<td>35.84</td>
<td>104.36</td>
</tr>
<tr>
<td>3</td>
<td>68.52</td>
<td>49.96</td>
<td>118.48</td>
</tr>
<tr>
<td>4</td>
<td>68.52</td>
<td>56.84</td>
<td>125.36</td>
</tr>
</tbody>
</table>

<sup>1a</sup>: with sandwich panel depth doubled and stiffened beams  
<sup>1b</sup>: assessment based on CCIP - 016  
<sup>2</sup>: with sandwich panel depth increased (but less than doubled) and stiffened beams (required for Floor 1 only)  
<sup>*</sup>: difference of M<sub>total</sub> between prototype and variants that satisfy the considered assessment method.

Increasing the depth of floor members can result in an overall increase of total floor depth, potentially increasing the overall height and cost of the building. As an alternative to stiffening schemes, additional floor damping can be incorporated to reduce the floor responses [43]. It is possible to introduce additional elements/sources of damping to the floor system: for example, a polyurethane foam core can be incorporated between voids of sandwich panel to increase structural damping.

To evaluate the effects of higher damping on assessment results, the assessments for Floor 1 are again conducted with a higher damping ratio of 6%. The results are presented in Figure 6.10 which shows that Floor 1 can be serviceable when higher floor damping is considered. This is an attractive option to reduce vibration responses. However, the level of damping to be attained must be determined in advance, and may be difficult to explicitly-determine in practice.
6.6.3. **Inclusion of HSI in future vibration serviceability assessment**

Based on the assessment results in Section 6.5, the consideration of the human body mechanical system has a positive impact on the vibration performances of pGFRP sandwich panel floors. Specifically, the results in this study showed that humans can act as vibration absorbers in reducing the vibration level. However, there is little difference in the response predicted between the SMD model and current practices when the floor’s mass is substantially large (e.g. Floor 3 and Floor 4).

Figure 6.11 shows a plot of ratio between responses of MF and SMD model against mass ratio – i.e. the ratio of reference subject mass (73.2 kg) to the total mass of the prototype and variant floors (Table 6.3 and Table 6.4). In addition, a linear trend line is presented alongside the data points of Figure 6.11. As can be seen, the ratio between MF and SMD model responses increases with increasing mass ratio. Notably, a larger ratio between MF and SMD model responses is observed for Floors 1 and 2, indicating that the HSI effect is more significant in these floors compared to Floors 3 and 4. For this reason, it is tentatively proposed that the inclusion of HSI should be considered in...
vibration assessment of pGFRP sandwich panel floors with relatively larger ratio of human mass to structure mass.

Figure 6.11. Ratio of acceleration responses from MF and SMD model against ratio of the mass of reference subject to the total mass of the floor (Given as a percentage).

The inclusion of HSI in design can be beneficial towards floor requirement for vibration serviceability. Specifically, HSI can lower floor responses to serviceability level. This can be beneficial for floors where stiffening schemes or damping incorporation is difficult to implement. To date, the inclusion of HSI in vibration serviceability assessments has been proposed for other structures [30, 44]. However, the work of Chapter 4 has shown that the parameters for SMD models for vibration assessments are inherent to the structural representation used to identify those parameters. For this reason, further experimental validation of the SMD parameters is required for pGFRP sandwich floors. Nevertheless, it is clear that accurate vibration serviceability assessment of pGFRP sandwich panel floors requires consideration of HSI.
6.7. Conclusions and future work

This chapter investigates vibration serviceability performance of pGFRP sandwich panel floors. Several methods for evaluating the vibration performance, including general approaches from current guidelines, conventional numerical simulation, and a validated best-practice numerical approach were considered. Vibration assessment is performed for four prototypical pGFRP sandwich panel floor systems which are optimally designed to conform to static requirements. In addition, additional variants of the prototype floors are also considered to investigate their vibration performance.

The vibration assessment showed that pGFRP sandwich panel floors that conform to static requirements fail vibration serviceability assessments. Vibration serviceability can be achieved through stiffening. As demonstrated in this work, the floors with increased beam and panel sizing can pass vibration serviceability checks. Furthermore, the consideration of HSI has a significant influence on the responses of floors, even reducing responses for some of the floors to serviceable levels. On this basis then, designs that conform to current general approaches can be overly-conservative which is significant for the economics of pGFRP buildings.

This initial study on the vibration performance of pGFRP sandwich panel floors can be supplemented with much further work. Full-scale testing of a pGFRP sandwich panel floor systems would help to validate the results herein. Specifically, experimental testing is needed to determine the damping levels that can be achieved in pGFRP sandwich panel floors. Further, the numerical framework of this work can also be adopted for vibration serviceability assessment, including HSI, of other floor system types. Overall, the study of vibration serviceability requires a much larger body of data (experimental, analytical and numerical) for reliable assessment.
Appendix

This section presents the natural frequencies of prototypical floors. The tables are given for Floors 2, 3 and 4, which are presented analogous to Table 6.5. The assessment results reported in Table 6.7 are obtained from the ratio of predicted responses between current practice (general approach and MF model) and SMD model – these are presented in Table 6.6.

Table 6.10. Comparison of natural frequencies between variants of Floor 2.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Floor 2</th>
<th>Variants</th>
<th>Percentage change in natural frequency, %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B¹</td>
<td>P²</td>
</tr>
<tr>
<td>1</td>
<td>5.72</td>
<td>6.64</td>
<td>6.39</td>
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<tr>
<td>2</td>
<td>6.47</td>
<td>7.33</td>
<td>7.28</td>
</tr>
<tr>
<td>3</td>
<td>6.74</td>
<td>7.68</td>
<td>7.60</td>
</tr>
<tr>
<td>4</td>
<td>7.35</td>
<td>8.21</td>
<td>8.30</td>
</tr>
<tr>
<td>5</td>
<td>7.63</td>
<td>8.41</td>
<td>8.56</td>
</tr>
<tr>
<td>6</td>
<td>7.96</td>
<td>8.75</td>
<td>8.92</td>
</tr>
<tr>
<td>7</td>
<td>8.50</td>
<td>9.33</td>
<td>9.38</td>
</tr>
<tr>
<td>8</td>
<td>8.69</td>
<td>9.58</td>
<td>9.67</td>
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<tr>
<td>9</td>
<td>8.72</td>
<td>9.72</td>
<td>9.84</td>
</tr>
<tr>
<td>10</td>
<td>10.87</td>
<td>11.88</td>
<td>12.50</td>
</tr>
</tbody>
</table>

¹: Increasing beam sizes  
²: Increasing sandwich panel depth  
³: Increasing both beam and sandwich panel

Table 6.11. Comparison of natural frequencies between variants of Floor 3.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Floor 3</th>
<th>Variants</th>
<th>Percentage change in natural frequency, %</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>B¹</td>
<td>P²</td>
</tr>
<tr>
<td>1</td>
<td>6.53</td>
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<td>2</td>
<td>6.87</td>
<td>7.53</td>
<td>7.38</td>
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<tr>
<td>3</td>
<td>6.90</td>
<td>7.64</td>
<td>7.67</td>
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<tr>
<td>4</td>
<td>6.94</td>
<td>7.86</td>
<td>7.69</td>
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<td>5</td>
<td>7.08</td>
<td>7.95</td>
<td>7.87</td>
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<td>6</td>
<td>7.21</td>
<td>8.07</td>
<td>7.92</td>
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<td>10</td>
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<td>10.35</td>
<td>10.36</td>
</tr>
</tbody>
</table>
Chapter 6: Vibration serviceability performance of pGFRP sandwich floors

1. Increasing beam sizes
2. Increasing sandwich panel depth
3. Increasing both beam and sandwich panel

Table 6.12. Comparison of natural frequencies between variants of Floor 4.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Floor 4</th>
<th>Variants</th>
<th>Percentage change in natural frequency, %</th>
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</thead>
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<td></td>
<td></td>
<td>B¹</td>
<td>P²</td>
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<td>5.35</td>
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<td>7.10</td>
<td>7.77</td>
<td>8.13</td>
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Table 6.13. Ratio of predicted responses from general approaches against MF model.

<table>
<thead>
<tr>
<th>Floor ID</th>
<th>( f_i ) (Hz)</th>
<th>( f_w ) (Hz)</th>
<th>AISC¹</th>
<th>SCI</th>
<th>CCIP</th>
<th>MF</th>
<th>SMD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a_{ASC} ) / ( a_{SMD} )</td>
<td>( a_{SCI} ) / ( a_{SMD} )</td>
<td>( a_{CCIP} ) / ( a_{SMD} )</td>
<td>( a_{MF} ) / ( a_{SMD} )</td>
<td>( a_{SMD} ) / ( a_{SMD} )</td>
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<td></td>
</tr>
<tr>
<td>1</td>
<td>6.32</td>
<td>2.00</td>
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<td>0.98</td>
<td>1.00</td>
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<td>1.00</td>
<td>1.32</td>
</tr>
<tr>
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<td>6.53</td>
<td>1.97</td>
<td>1.04</td>
<td>1.06</td>
<td>1.00</td>
<td>1.00</td>
<td>1.10</td>
</tr>
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<td>5.03</td>
<td>2.26</td>
<td>1.46</td>
<td>1.07</td>
<td>0.96</td>
<td>1.00</td>
<td>1.12</td>
</tr>
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<td>1-B-F</td>
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<td>2.05</td>
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<td>1.81</td>
<td>1.82</td>
<td>1.00</td>
<td>1.08</td>
</tr>
<tr>
<td>2-B-F</td>
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<td>2.17</td>
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<td>1.08</td>
<td>1.00</td>
<td>1.24</td>
</tr>
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<td>3-B-F</td>
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<td>1.00</td>
<td>1.53</td>
<td>1.24</td>
<td>1.00</td>
<td>1.08</td>
</tr>
<tr>
<td>4-B-F</td>
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<td>2.26</td>
<td>1.08</td>
<td>1.10</td>
<td>0.97</td>
<td>1.00</td>
<td>1.09</td>
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<td>0.93</td>
<td>1.00</td>
<td>1.32</td>
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<td>1.69</td>
<td>1.42</td>
<td>1.29</td>
<td>1.00</td>
<td>1.26</td>
</tr>
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<td>0.93</td>
<td>1.09</td>
<td>0.96</td>
<td>1.00</td>
<td>1.10</td>
</tr>
<tr>
<td>4-P</td>
<td>5.71</td>
<td>2.04</td>
<td>1.37</td>
<td>1.19</td>
<td>1.00</td>
<td>1.00</td>
<td>1.10</td>
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<td>1.36</td>
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<td>6.39</td>
<td>2.4</td>
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<td>1.63</td>
<td>1.41</td>
<td>1.00</td>
<td>1.07</td>
</tr>
<tr>
<td>3-B</td>
<td>7.25</td>
<td>2.22</td>
<td>0.61</td>
<td>0.98</td>
<td>0.96</td>
<td>1.00</td>
<td>1.10</td>
</tr>
<tr>
<td>4-B</td>
<td>5.36</td>
<td>2.4</td>
<td>1.70</td>
<td>1.10</td>
<td>0.97</td>
<td>1.00</td>
<td>1.11</td>
</tr>
<tr>
<td>Mean ( \mu )</td>
<td>1.32</td>
<td>1.20</td>
<td>1.10</td>
<td>1</td>
<td>1.17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 6: Vibration serviceability performance of pGFRP sandwich floors

1: Peak acceleration
2: using peak acceleration from responses using SMD model.

References


N. Haritos, E. F. Gad, and J. L. Wilson, "Human-induced vibration of floor systems and stadia: some practical observations from field measurements," in Proceedings of the 19th
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Chapter 7. Conclusions
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7.1. Overview

The aim of this research is to investigate the dynamic behaviour of pGFRP sandwich panel floor systems. To achieve this, an experimental – numerical framework is considered to predict vibration performance. Numerical modelling is used to model and simulate walking responses of pGFRP sandwich panel floors. Experimental results from a representative pGFRP sandwich panel floor structure is used to validate the numerical modelling approach. Model updating procedures are performed to improve the predictions of numerical models with experimental results. The consideration of human-structure interaction (HSI) in the numerical framework was motivated by the observable effects of HSI in the experiments. For this, an interactive human model is considered to better represent the effects of human walking on pGFRP sandwich panel floors. Prior to vibration analysis, the design of feasible pGFRP sandwich panel floor systems are investigated. Collectively, the feasible designs are assessed for vibration serviceability performance. Overall, this research has devised a number of numerical tools for vibration serviceability of lightweight pGFRP sandwich panel floors. The findings and outcomes of the research project are next described.

7.2. Research outcomes

7.2.1. Performance of full-bonded GFRP sandwich structures

A novel full-pGFRP sandwich footbridge (denoted as the MB) was designed and constructed as part of the experimental studies of the research project. The MB is 9 m long twin-girder footbridge with a 1.5 m wide orthotropic pGFRP sandwich panel as the deck solution. The composite section and boundary conditions of the MB render a good validation basis as a one-way spanning pGFRP floor unit. The MB adopts a novel adhesive bonding technique for all connections, i.e. no mechanical bolting was used.

The design and construction of the MB has demonstrated the potential of full-bonded pGFRP structures. Its light weight and good strength make it easy to transport. When combined with modular
constructions, the lightweight of pGFRP members negate the need for heavy lifting equipment. This part of the research has discussed the steps and details the construction, from which lessons learned may be relevant to design and constructions of similar structures.

The static and dynamic testing were performed to assess the performance of the MB. The results of static tests showed that full composite action was achieved in the MB from epoxy bonding. From the walking experiments, the results showed that the MB attains unacceptable acceleration responses despite conforming to current design rules. The poor performance of current design rules that are based on traditional structural forms indicate the need for state-of-the-art design rules for pGFRP sandwich structures. Overall, the performance data of the MB provides impetus for the development of new guidelines for pGFRP sandwich panel structures.

7.2.2. Model updating of pultruded GFRP structures with localized mode shapes

The mode shapes of the MB exhibit asymmetric behaviours, especially in the longitudinal bending modes. Experimental verification showed that the asymmetric behaviour of mode shapes is due to an imbalance longitudinal stiffness within the MB. This can be due to construction of the MB as a result imperfect bonding quality for example. For this reason, model updating of the numerical model is performed to capture the imbalance longitudinal stiffness by correlating mode shapes between numerical and measurements. However, the conventional mode shape metric, the Modal Assurance Criterion (MAC), uses a single index to correlate mode shapes. Consequently, the single index of the MAC was unable to distinguish the asymmetric shape feature in model updating procedures.

An alternative to the MAC was sought by using shape descriptors to correlate mode shapes. Specifically, Zernike Moment Descriptors (ZMD) were adopted because they can represent an image of mode shape into multiple indices, which enables the asymmetric behaviour of the mode shapes to be represented. In turn, the numerical model of the MB was successfully updated to capture the
asymmetric longitudinal behaviour by using the ZMDs as correlation indicators of mode shapes. The procedures and suggestion of uses of the ZMD-based model updating procedure has been investigated as part of this research. The proposed model updating procedure can find uses in model updating of structures in general, and of GFRP structure in particular, which exhibit localized mode shape features that are indistinguishable by the conventional MAC.

7.2.3. Static design of pultruded GFRP sandwich panel floors

Prior to this research project, the static performance of pGFRP sandwich panels (i.e. strength and deflections) had been investigated in previous research. Although static design rules were proposed, the design of pGFRP sandwich panels as feasible floor systems had not yet been investigated. This is identified as a knowledge gap between the previous and present study, where the design of feasible floors is significant for the evaluation of dynamic performance.

A part of the research addresses the knowledge gap by investigating designs of feasible pGFRP sandwich panel floor systems. This part of the research summarizes the previous static design rules for various aspects of pGFRP sandwich panel floors (e.g. slab, composite beam etc.) and applying them to design a full-scale floor system. From the static design rules, a number of design charts have been developed to aid designers in selecting feasible geometries of pGFRP sandwich panel floor members. In turn, a simple design procedure is developed based on the design charts. The design procedure is demonstrated on a prototype floor frame. Overall, the design charts should find use in practice to for preliminary evaluation of feasible geometries of pGFRP sandwich panel floor systems.

7.2.4. Human-structure interactions in pultruded GFRP sandwich structures

Human occupants can interact with the occupied floor structure, where presence of humans can change the dynamic behaviour of the structure (i.e. HSI). HSI effects were observed in the walking experiments of the MB. The moving force (MF) model was adopted to simulate walking responses
Chapter 7: Conclusions

of the MB, as a commonly-used model in design practice that ignores HSI. It was found that the MF model highly overestimated the walking responses of the MB, especially at resonant walking. Consequently, this observation strengthens the need for HSI considerations in vibration analysis of pGFRP sandwich panel floors.

To account for HSI, a moving spring-mass-damper model has been considered into the numerical framework for pGFRP sandwich panel floors. The SMD model is simulated on the simple 2-D plate model of the MB to predict the responses. Using the SMD model, lower responses of the MB are obtained, which are closer to the measurements. To improve the results, the numerical framework is used to identify parameters of SMD model (mass, stiffness, and damping) that gives matching responses of walking participants. Overall, the numerical framework that focuses on HSI can be readily applied to practice.

Going a step further, the numerical framework is used to investigate the effects of different structural representations on the identification of SMD parameters. The study is presented in Chapter 4, where the representation of the proposed framework (2-D plate model) is compared against the common structure representation in the literature (1-D beam model). The results of Chapter 4 showed that different structural representation of the same structure gives different values of SMD parameters. The findings can be incorporated in development of new guidelines with HSI considerations, in regards to SMD parameters for different representations of the same structure.

7.2.5. Vibration serviceability of feasible pultruded GFRP sandwich panel floor system

Vibration serviceability performance of pGFRP sandwich panel floor systems are investigated using the proposed numerical framework. Specifically, the moving SMD model has been validated as the best practice high-fidelity approach to predict walking responses of pGFRP sandwich panel floors.
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From this approach, the predicted floor responses are compared to human comfort limits to assess the floors’ serviceability. Vibration assessment is performed for a number of pGFRP sandwich panel floors which are optimally-designed based on the design aids presented Chapter 5. To draw comparisons, the assessments are also performed using current practices, which consist of analytical methods suggested in contemporary design guidelines as well as the MF model simulations.

From the results, the statically-conforming floors failed vibration serviceability from assessment using current practice (which ignores HSI). The weight penalty of pGFRP sandwich panel floors have been calculated for floors that conform to current practice and with considerations of HSI from the proposed SMD model. The results showed that a smaller weight penalty of pGFRP sandwich panel floors is obtained when HSI considerations is included in the vibration assessments. Overall, the assessment results showed that pGFRP sandwich panel floor systems are serviceable when accounting for HSI. It is suggested that HSI should be included in vibration serviceability assessment of pGFRP sandwich panel floor systems.

7.3. Future work

From this thesis, it was shown the vibration serviceability design of pGFRP sandwich panel floors would require the use of human interactive models to simulate vibration responses. This requires comprehensive experimental studies on different floor configurations, spans for thorough understanding on the vibration serviceability requirements. In this research, a representative pGFRP sandwich panel floor structure was adopted to perform, and from that specimen, a number of numerical tools have been developed that are reasonably applicable for practice. While the results of the research show promising performance of pGFRP sandwich panel floors, more experimental work is needed to establish understanding for development of future guidelines.

Based on the findings of this research, the following are recommended for future research:
1) The vibration performance of pGFRP sandwich panel floors was based on single person excitation. For future assessments, vibration assessment should be made for crowd dynamics covering other human activities, including running, jumping and rhythmic activities.

2) The numerical modelling approach of this research can be used for further studies pertinent to human-structure interactions. The validation of numerical models of this work are based on single pedestrian excitation on the representative structure. As human occupants are shown to reduce floor responses, the consideration of multiple occupant is worthwhile addition to the numerical framework in order to investigate effects of multiple occupants towards the dynamic behaviour of occupied floors.

3) The effect of different walking paths towards vibration responses of pGFRP sandwich panel floors can be further examined with the proposed numerical framework. For example, a vibration performance map of floor structure due to the various walking paths can be constructed.

4) The proposed SMD parameters should be validated for other GFRP structures. In addition, the parameter identification of this work can be extended to 3-D representations of structures.

5) Static design rules of pGFRP sandwich panels have been proposed as design charts. The design charts should be further validated with experimental designs.

6) Static design rules of pGFRP – steel composite beams as primary beams in floor systems has not been explored and should be further investigated.

7) An alternative model updating technique have been proposed in this study. It would be beneficial to explore the proposed updating procedure on other structures to further validate its application in practice.