Airline Planning and Operation: towards planning under probability and recovery

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Abstract
Airlines are constantly facing uncertainties such as fluctuating passenger demand, volatile prices and operational disruptions such as reduced airport capacity, unexpected aircraft unavailability, and delayed or cancelled flights. In view of this, increasing numbers of publications have appeared over the last few years dealing with uncertainty in airline scheduling. This paper provides a review of the current state-of-the-art and possible future directions for airline planning and operation under uncertainty. Major approaches, namely robust planning/scheduling and disruption management and their challenges are presented. We explore the application of a novel approach, chance constrained programming (CCP), to deal with uncertainties in airline planning. This approach is extended to compute the expected airline revenue when the ticket price and the passenger demand are uncertain and correlated. We also propose the application of risk management(Conditional Value-at-Risk) to support more robust airline planning.
Finally, we discuss possible ways of integrating robust planning with disruption management and suggest an integrated approach which leverages the capabilities of major approaches.

Keywords: Robust Planning, Disruption Management, Chance Constrained Programming, *ROADEF* Challenge 2009

1. Introduction

The airline schedule development problem is to choose a departure time and plane type and the corresponding crew assignments for every flight leg in the schedule, so as to optimize the financial return (passenger revenue minus operation costs). Typically, airline schedule planning is completed several months prior to the actual day of operation and effected by solving several problems sequentially. *Schedule design* generates profit maximizing flight schedules (when and where to offer flights) by considering origin–destination (OD) market demands. *Fleet assignment* then assigns an aircraft type to each flight leg so at to minimize operational and spill costs. Then for each aircraft, *aircraft routing* determines the sequence of flight legs to be flown by each individual aircraft. *Crew pairing and crew scheduling* finally assigns crew members (cabin and cockpit crews) to each flight leg so that the crew costs are minimized. Airline schedule planning is a complex task as it involves aircraft maintenance rules, aircraft capacity, individual airport characteristics, and flying time regulations for crew members, etc. (Clausen et al. 2010). To date, most schedule development algorithms have assumed fixed flight durations and passenger demands (Barnhart et al., 2004; Lan et al., 2006). Airline plans are typically developed so as to maximize resource utilization. As a result, minimal turnaround between flights is usually suggested to reduce aircraft and crew costs. Therefore, there is very limited slack in an "optimal" airline plan. However, this optimistic scenario is rarely realized in practice because airlines are constantly facing operational disruptions such as reduced airport capacity because of bad weather or strikes, unexpected aircraft unavailability due to mechanical failures and delayed or cancelled flights. In fact, airlines have been suffering from increasing levels of disruption in the last decades, over and above the huge impact of 7/11 and its aftermath. In 2003, fuel shortage hit Sydney airport caused numerous flights delayed or cancelled affecting around 2,500 passengers (BBC news 2003). A recent computer glitch hit US flights with dozens of flights cancelled or delayed in Hartsfield-Jackson Atlanta International Airport alone (BBC news 2009). The Federal Aviation Administration (FAA) reported a 58% increase in delays from 1995 to 1999, and a 68% increase in flight cancellations over the same period (Schaefer 2005). These disruptions have imposed huge costs on airports and airlines. It was estimated that the total cost to Hartsfield airport due to cancellations was $250.9 million in 1999 (Schaefer 2005). The impacts of irregularities encountered by a single US major airline exceeded $400 million per year (Bratu et al. 2006).

A schedule built on a deterministic approach may prove to be hard to adjust in the light of major or minor disruptions and might even have to be completely abandoned, on the day of operation. Even though almost 50 years have passed since two seminal works on uncertainty appeared (Dantzig 1955, Beale 1955), Dantzig still considered planning under uncertainty as one of the most important open problems in optimization (Horner 1999). Even a fully integrated plan can be expected to be suboptimal in practice because plans are based on input data that is assumed to be known and fixed (Barnhart et al. 2004). As pointed out by some researchers (Lan et al. 2006), conventional models for airline schedule planning minimize planned costs, while airlines' ultimate goal is to minimize realized costs, that is, the sum of planned costs and the costs of delays and disruptions. However, how to estimate a priori the realized costs and include them in a planning model remains a challenging task.

Despite the importance of considering uncertainty in planning/scheduling models, the majority of airline research has focused until recently on developing efficient plans without considering uncertainty factors (Barnhart et al. 2004). In the last few years, however, the problem of effectively dealing with uncertainties in airline scheduling has attracted the attention of researchers and increasing numbers of publications have appeared in the literature. In practice, some airlines are now shifting towards ensuring that planned schedules are robust and allow for efficient recovery (Kohl et al. 2007).

In this paper, we summarize the perspectives and challenges facing airline planning under uncertainty. We focus on applying a novel approach, CCP, to airline planning. Applying CCP, we calculate the effect of flying time truncation on the optimal buffer time between connecting flights. To the best of our knowledge, this paper is an initial attempt to derive an analytical model to evaluate the influence of the truncation of flight flying times. A novel approach is then proposed to compute the expected airline revenue when the uncertain ticket price and the passenger demand are further correlated. The concept of risk management, i.e. Conditional Value-at-Risk, is discussed for possible applications in airline planning to provide more robust results. This paper is organized as follows. In section 2, we highlight the perspectives and challenges for robust airline planning/scheduling. In section 3, we briefly introduce two solution approaches, recourse models and CCP, and their corresponding literature review in stochastic programming. In section 4, CCP is applied into robust airline planning using an illustrative example. Analytical models to evaluate the effect of flying time truncation is derived. A novel approach is proposed to compute the expected airline revenue when the uncertain ticket price and the passenger demand are correlated. In section 5, we reviewed the reactive approach, disruption management, and its integration with the proactive approach. In section 6, discussions on determination of penalty terms in a recourse model, consequences of multiple measures of robustness and applying the concept of risk management to CCP are provided in the context of airline planning. In section 7, an integrated approach which leverages robust planning/scheduling and disruption management, is suggested. Finally, in section 8, we describe future research opportunities and conclusions.

2. Robust planning/scheduling

Robust planning/scheduling tries to anticipate unexpected events (delays, cancellations, aircraft unavailabilities) during the creation of schedule plans. In this proactive approach, flexibility or robustness is explicitly built into the schedules to enable improved recovery from disruptions. A robust recovery policy that considers potential changes or disruptions might provide a better solution than one that assumes the new flight schedule will operate as planned. Research areas on robust planning appeared in the literature include robust fleet assignment, robust aircraft routing, and robust crew scheduling (Gao et al. 2009).

2.1 Techniques to build robustness into schedules

Kohl et.al. (Kohl et al 2007) list some commonly used techniques to build flexibility or robustness into schedules in airline practice. Here is an extended version of their list and relevant literature.

- **Adding or re-distributing slacks in the plans**.
  Since deregulation in the 1970s, airlines have been operating near their optimal capacities, allowing little slack in flight durations in the hope that the airline schedule will operate as planned. However, this optimistic scenario is rarely achieved in practice. Thus, instead of operating at a minimum turn time, extra slack is incorporated into the plans such that each line of work has some degree of self-recovery. In airline practice, adding extra buffers after frequently delayed flights provides protection against disruptions. Ehrgott and Ryan (2002) optimize scheduled ground times between consecutive flights by defining a robustness objective for crew pairing problem to minimize the amount by which the expected delay of the incoming flight exceeds the remaining scheduled ground time before the departure of the outgoing flight. Ahmadbeygi et al. (2010) developed a linear model to re-distribute slacks between connecting flights. Flights are delayed discrete set of integer values in their model.
- **Crew follows each other and the aircraft**.
  This technique makes monitoring of operations easier. It also allows for a simple recovery strategy that preserves some of the properties of the original schedule.
- **Aircraft/crew swapping opportunities and short cycles**
  If an aircraft flies from a hub to a spoke and back to the same hub, these two flights can be cancelled without affecting the rest of the aircraft schedule. If the same crew is planned for these two flights, the cancellation will not affect the rest of the crew schedule either. A pool of resources available at a hub allows for replanning in an event of disruption. This technique can help to localize disruptions and reduce delay/cancellation propagations. Ageeva (2000) modifies the string based fleet assignment problem to reward opportunities for swapping aircraft. In general, Ageeva's approach tries to provide ways for more aircraft routes to intersect at different points, so that aircraft can switch strings if needed. Rosenberger et al. (2004) seek to develop solutions containing many short cycles of flights, allowing an airline to limit the number of flights cancelled when a cancellation is necessary. Shebalov and Klabjan (2006) try to maximizing the number of move-up crews, i.e., the crews that can potentially be swapped in operations, in their objective. The idea of crew re-linking, where another crew member takes over the line of work of a late crew member, is proposed to create schedules less sensitive to disruptions.
- **Station Purity**
  Station purity, which can also be called fleet purity, ensures that the number of fleet types serving a given station does not exceed a specified limit. Smith and Johnson (2006) impose station purity in their robust fleet assignment model. Gao et al. (2009) extend the station purity idea to both fleet purity and crew base purity so that the numbers of both fleet types and crew bases allowed to serve each airport are limited. Adding fleet purity can reduce planned crew costs, maintenance costs, and improve robustness (Gao et al.

2009). Imposing crew base purity can increase the opportunities of finding a move-up crew in crew recovery.

- **Regularity**
  Since regular solutions are easier to implement and manage and airline schedules repeats weekly, Klabjan et al. (2001) capture regularity in their model by adding a second goal of maximizing the repetition or regularity of itineraries over a weekly horizon. The authors show better solutions in FTC (Flight Time Credit), number of deadheads, and regularity from their model.
- **Adding stand by crew and aircraft**. Spare crew or aircraft can be very valuable in case of disruption though they are also costly.
- **Increasing cruise speed**. Aircraft typically operates at the most economic speed that always be lower than top speed. Increasing aircraft speed can absorb flight delays to some extent and avoid higher costs in itinerary repair for aircraft, crew and passengers.

2.2 Measure of robustness (MOR)

One of the major difficulties in robust planning is to define and quantify the robustness in the model. A number of measures in the literature have attempted to capture robustness. These attempts mostly focus on measures in a schedule that can be used while constructing schedules, as opposed to running the schedule on historical data, which can only be done when the schedule is complete. Table 1 lists measures of robustness (MORs) appeared in the literature.

Literature	MORs
Ageeva (2000)	Degree to which sequence of activities on swappable resources meet each other
Klabjan et al. (2001)	A regularity measure to capture robustness
Ehrgott and Ryan (2002)	Non-robustness measure for each crew pairing by considering each consecutive pair. If the scheduled ground time minus any ground duty time is less than the expected delay, a penalty will be incurred.
Shebalov and Klabjan (2006)	The number of move-up crews, i.e., the crews that can potentially be swapped in operations
Lan et al. (2006)	The expected total propagated delay. Airline historical data are used to determine the independent arrival delays which in turn are used to calculate the expected total propagated delay.
Schaefer et al. (2005)	The expected crew cost. Calculated using Monte Carlo simulator SimAir
Yen and Birge (2006)	The expected crew costs
Ahmadbeygi et al. (2010)	The expected total propagated delay

Table 1 MORs in the literature

2.3 Challenges in building robustness

Though a number of approaches have appeared in the literature for robust airline planning, however, there are a number of challenges still exist in building robustness into the airline schedule (Lan et al. 2006):

- **Robustness is difficult to define**. A robust plan might be a plan that yields the minimum cost for the worst case, the minimum expected cost, or minimizes costs given a required level of service.

- **Hard to capture complex operations resulting from disruptions**. Some disruptions, such as severe weather conditions, lead to very complex recovery operations which are very difficult to capture in a model which can be solved to optimality.
- **Computationally demanding**. Models capturing stochasticity require a great deal of computational resource to find even good upper bounds when applied to large-scale problems.

3. Summary of solution approaches to robust planning and scheduling

In this section, we briefly introduce the stochastic programming approach and show how stochastic programming can be applied to robust airline planning and scheduling. Stochastic programming deals with problems in which some parameters incorporated into the objective or constraints are uncertain (Prékopa 1995). This uncertainty is generally modeled in one of two ways (Li et al. 2004):

- as a finite set of alternatives each having a probability (a discrete probability distribution).
- as a continuous probability distribution.

Discrete alternatives can be handled by recourse models whereas continuous or discrete distribution can be handled by chance constrained programming.

3.1. Introduction to recourse models

There are many research studies that use recourse models for design/planning/scheduling under uncertainty. Recourse models use corrective actions (usually penalty functions) to compensate for the violation of constraints arising during the realization of uncertainty. The two-stage model is one of the main paradigms of recourse models. The two-stage model divides the decision variables into two stages. The first-stage variables are those that have to be decided right now before future realization of uncertain parameters. Then, the second-stage variables are those used as corrective measures or as recourse against any non-optimalities or infeasibilities arising during the realization of the uncertainty. In a design problem, the first-stage variables include capital investment, unit expansion, etc. After determining the unit capacities and the capital investments, further design or operational improvements can be achieved by selecting the values of the second-stage variables. This corrective action is known as recourse. In a planning problem, the first-stage variables are production rates, the amount of raw materials needed, aircraft cabin capacities etc. The second stage is to decide the amount of the product from other producers required to meet the actually realized product demand or the amount of raw material required from other suppliers to meet production requirements or the number of passengers transferred to other airlines due to short of aircraft seats.

Consider the simple example illustrated in Figure 1. Assume that the airline plans to purchase an aircraft which can be configured with any number of first class seats, P, from 0 to 30, under uncertain customer demand, d.

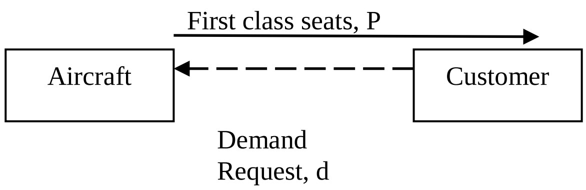

Figure 1: A simple example

As shown in Figure 1, the operation cost for one first class aircraft seat is $20,000. There are three seasonal demand scenarios: 1) D=10 units with probability 0.3; 2) D=20 units with probability 0.5; and 3) D=30 units with probability 0.2. Any unmet passenger demand is penalized by a seasonal loss of $30,000.

In terms of the two-stage model, the first-stage variable or the decision variable is the seat number, P, the second-stage variable is the unmet demand in this example. Two-stage programming divides the total cost into two categories: the first-stage cost, which includes the operation cost; the second-stage cost, which is the penalty cost for unmet demand. The objective of two-stage programming is then to find the optimal seat number, P, that will minimize the total cost. The two-stage model is formulated as follows:

$$\begin{aligned} &\min \quad \text{cost} = 20000P + \sum_{s=1}^{3} 30000\,\text{Pr}^{S} Y^{S} \\ &s.t. \quad Y^{S} \ge D^{S} - P \\ &\quad\; P \ge 0, \quad Y^{S} \ge 0 \end{aligned} \tag{1}$$

where P is the seat number. Pr^s is the probability of demand, D^s, in scenario s. Y^s is the unmet demand in scenario s. The cost of the first stage, 20,000P, is the airline operation cost. The expected second-stage cost is $\sum_{s=1}^{3} 30000\,\text{Pr}^{s} Y^{s}$. When the above model is solved, the optimal production rate, P, turns out to be 20 with a corresponding total cost of $460,000 (first stage cost $400,000 plus second stage cost $60,000).

3.2 Introduction to chance constrained programming (CCP)

Recourse models allow the violation of “soft” second-stage constraints by adding a penalty term (for example, the penalty of $30,000 for unmet demand in the simple example in Figure 1) in the objective function. That is, infeasibilities are allowed in the second stage provided that a certain penalty is imposed. However, the exact values of the penalty terms are difficult to determine since they include intangible components such as loss of goodwill, the costs of off-specification products or outsourcing of production requirements. Thus, in many cases of process operations, this penalty term is not available. This difficulty is avoided in CCP which deals with problems under probabilistic constraints as a decision model under uncertainty (Prékopa 1995). CCP, first introduced by Charnes et al. (1958), seeks to satisfy the constraints involved to a predetermined confidence level, using the known probability density/cumulative distribution of random variables. That is, rather than requiring constraints containing the uncertain parameters always to be satisfied, or imposing penalties for infeasibilities, a probability of the constraint being satisfied (usually called the confidence level) can be specified by the decision maker. CCP has been applied in many disciplines such as finance and management (Uryasev 2000). However, to the authors' knowledge, very few applications have been made in airline planning and operation under uncertainty.

The basic procedure through which CCP handles uncertainty is illustrated using the same example as illustrated in Figure 1. Since the penalty for unmet demand is difficult to know in reality, we do not apply a penalty term for unmet demand in the objective function. Instead, we require that the demand should be satisfied with a certain probability. In two-stage programming, we associate probabilities to three scenarios. In CCP, a continuous probability distribution is associated with the demand instead. Assuming that the demand conforms a normal distribution with mean θ=19 and standard deviation σ=2, the example is reformulated by CCP as follows:

$$\begin{aligned} &\min \quad \text{cost} = 20000P \\ &s.t. \quad \Pr\{P \geq \zeta\} \geq \alpha, \\ &\qquad P \geq 0, \end{aligned} \tag{2}$$

where ζ represents the random demand and α is the confidence level. Pr is the operator of the probability computation. For this simple example, it is easy to see from the standard reverse cumulative normal distribution, Φ^{-1}, that, to satisfy demand with a probability (or confidence level)α, the seat number *P* should be greater than $\theta + \sigma\Phi^{-1}(\alpha)$ i.e., the deterministic equivalent problem of (2) is:

$$\begin{aligned} &\min \quad \text{cost} = 20000P \\ &s.t. \quad P \geq \theta + \sigma\Phi^{-1}(\alpha), \\ &\qquad P \geq 0. \end{aligned} \tag{3}$$

If we set the confidence level, α, to 0.95, then $\theta + \sigma\Phi^{-1}(\alpha)$ =22.29.

After (3) is solved, the seat number, P, is 22.29 and the total cost is $445894. Note that rather than associating an additional cost to the potential spilled customers, we simply keep the potentiality of spillage “sufficiently” low.

Given a general model as follows:

$$\begin{aligned} &\min \quad c^T x \\ &s.t. \quad Ax \leq b \\ &\qquad x \geq 0, \end{aligned} \tag{4}$$

where $x \in R^n$ and $c \in R^n$ are vectors and $b \in R^m$ and $A \in R^{m \times n}$. Assume that c and A are deterministic parameters and b is an uncertain parameter. CCP requires that constraint $Ax \leq b$ is satisfied with a confidence level vector, $\alpha \in R^n$:

$$\Pr\{\sum_{j=1}^{m} A_{ij} x_i \leq b_i\} \geq \alpha_i, \qquad i = 1,2,\ldots\ldots,n. \tag{5}$$

By applying the cumulative distribution function, *F*, of b, constraint (5) can further be reformulated to:

$$\sum_{j=1}^{m} A_{ij} x_i \leq F_i^{-1}(1-\alpha_i), \qquad i = 1,2,\ldots\ldots,n. \tag{6}$$

In constraint (6), α_i and F_i^{-1} is known and thus the right hand side of (6) is known and (5) is reduced to deterministic linear constraints. Using constraint (6), the previous probabilistic model is then converted to an ordinary deterministic linear programming model.

In (5), there is an assigned confidence level, α_i, for each b_i. This implies that each b_i can be satisfied with different confidence level. In an airline fleet scheduling application, we could set different confidence levels to different flights, depending on the type of the flight (domestic/continental/intercontinental), the passengers on the flight and the aircraft (family/model/configuration) flying the flight. In case passengers are halfway through a travel itinerary, it is much more expensive than if they are at the beginning of a trip. We then should set higher confidence level to these flights. For problems in which meeting all the constraints as a whole is emphasized, all b_is are assigned one confidence level, λ. Constraint (5) is then replaced by a joint probabilistic constraint:

$$\Pr\{\sum_{j=1}^{m} A_{ij} x_i \leq b_i\} \geq \lambda. \tag{7}$$

When the uncertain b_is are independent random variables, the joint probabilistic constraint can be decomposed into the multiplication of the single probabilities:

$$\prod_{i=1}^{n}\left[1-F_i\left(\sum_{j=1}^{m}A_{ij}x_i\right)\right]\geq\lambda. \qquad (8)$$

The nonlinear constraint (8) transforms the model to a nonlinear programming model.
If the uncertain b_is are correlated, simultaneous integration of multivariate probability distributions is required. As we don't always have a closed-form expression for the joint probability distribution, some methods, such as Gaussian quadrature and Monte Carlo sampling, may be used to approximate the multivariate integration (Wenkai et al. 2005, 2006).

3.3 Summary of literature appeared to robust airline planning/scheduling

Despite its complexity, robust planning is a problem rich in opportunity and potential impact (Barnhart et al. 2004). Most work to date focuses on isolating causes of disruption and/or downstream effects, and incorporating these within the objective function. In most published work for airline planning, an extra term is added to reward desirable opportunities for robustness (e.g., the aircraft swapping opportunities in Ageeva 2000) or to penalize undesirable scenarios for robustness (e.g. the propagated delay in Lan et al. 2006). Hence most published research on airline scheduling under uncertainty can be cast into recourse models. They differ by defining different recourses and the way calculating recourses. The recourses in these works are essentially defined as the MORs as discussed in section 2.2. However, taking the expectation or the sum of the MORs in most works appeared implies that the probabilities for all desirable/undesirable opportunities for robustness or scenarios (e.g. swapping opportunities) are equal, which may not always the case in airline schedules.

Ageeva (2000) modifies the string based fleet assignment problem to reward opportunities for swapping aircraft. A string is a sequence of connected flight legs that begins and ends at maintenance stations (Barnhart et al. 1998). The measurement of robustness is based on identifying a schedule where strings meet each other as often as possible to create opportunities for aircraft swaps. This robustness measure is incorporated into the objective function. In general, this approach tries to provide ways for more aircraft routes to intersect at different points, so that aircraft can switch strings if needed. Their results show that in some cases the model provides an increase in robustness of up to 35% as compared to the original string model. Ehrgott and Ryan (2002) realize the importance of robust airline planning where disruptions are less likely to be propagated into the future. Due to the conflict between maximization of robustness and the minimization of cost of a crew pairing problem, they develop a generalized bi-criterion multi-objective set partition model. The multi-objective model is solved using ε-constraint method. The results show that at a small cost, robustness can be built into the generated rosters. Shebalov and Klabjan (2006) address the robustness in crew pairing. The objective of maximizing the number of move-up crews, i.e., the crews that can potentially be swapped in operations, is considered besides the objective of minimizing the crew cost. The idea of crew re-linking, where another crew member takes over the line of work of a late crew member, is proposed to create schedules less sensitive to disruptions. The paper shows that robustness leads to reduced operational crew cost and there is a trade-off between robustness and direct crew cost where robust solutions could produce significant annual savings. Yen and Birge (2006) apply the standard two-stage stochastic program to deal with crew planning under uncertainty, where the first stage is the conventional crew scheduling problem and the second stage involves the expected crew costs, i.e. MOR. The expected crew costs are approximated using crew delays multiplied by some penalties. Computational results for small problems are provided. The influence of different penalty values on crew scheduling is discussed. Ahmadbeygi et al. (2010) apply the two-stage stochastic program to re-allocate slacks between flights. In their model, the expected total propagated delay is defined as the MOR. However, the probability of each root delay is hard to determine in their model. Correlations of root delays are also not considered.

There is some published research applying CCP to airline planning and scheduling under uncertainty. Hsu et al. (2002) applied CCP to airline network design focusing on reliability evaluation. The reliability of the flight frequencies under fluctuating monthly OD demand is evaluated using CCP. A two-stage process is used to tackle the randomness of passenger demand. Marla (2010) studies robust aircraft routing problem using an extended CCP to reduce the need to iterate and re-solve the models arising from specifying the value of probability of each CCP constraint. Recently, Sohoni et al. (2011) model block-time (aircraft flying time) uncertainty in developing robust airline schedules. A FSL (flight service level) is defined as the probability that a particular flight is not delayed based on an acceptable arrival delay. With FSL incorporated into the model using CCP, they develop a stochastic integer programming model to maximize the expected profit, while ensuring the flight service levels.

4 Proposed approach to deal with uncertainties in airline scheduling using CCP

Recourse models discretize the distribution of uncertain parameters using discrete probabilities. The main advantage of the approach is that it can be applied to any distribution of uncertain parameters. However, there are two main difficulties exist in recourse models:

a). The problem size increases exponentially as the number of scenarios increases.

Yen and Birge (2006) sampled 100 scenarios to approximate a truncated gamma or lognormal distribution for the length of delays to match disruption data from Air New Zealand. Increasing the approximation accuracy to the distribution requires more sampling scenarios which will increase the size of the integer model exponentially.

b). The penalty term used in recourse is hard to determine.

Recourse models add a penalty term in the objective function to allow infeasibilities in the second stage. However, the exact values of the penalty terms are difficult to determine since they include intangible components, the costs of off-specification products or outsourcing of production requirements.

The above difficulties are avoided in CCP. CCP seeks to satisfy the constraints involved by a predetermined confidence level using the known probability density/cumulative distribution of random variables. A CCP model does not require complete knowledge of the distribution of the uncertain data. Knowledge of the quantile value of the distribution, is sufficient, which allows the user to approximate the distribution without requiring too much data (Marla 2010). In this section, we apply CCP to airline planning using uncertain flying time as the example. There are many uncertain parameters in airline scheduling such as flight departure/arrival times, customer demand, aircraft flying time, aircraft unavailability, etc. We can associate with each flight a probability distribution to describe the uncertain flight duration, and another to describe the passenger demand. Our scheduler assigns a plane type and departure time to each flight so as to meet customer demand and arrive “on time”, up to a certain confidence level (where “on time” means not more than a certain number of minutes late). Expected revenue minus cost are optimised.

4.1 CCP Methods

A

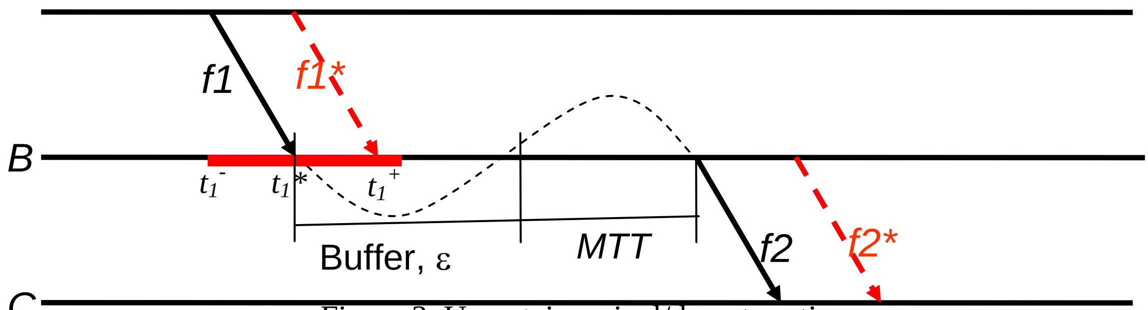

Figure 2. Uncertain arrival/departure times

In Figure 2, flight *f1* departs from airport A and is expected to arrive at airport B at time t_1^*. Due to the uncertain flying duration, its actual arrival time conforms to certain probability distribution. i.e., it may arrive earlier at time t_1^- or late at t_1^+ with certain probabilities. As there are many intangible factors influencing an aircraft flying time, we assume it conforms to a normal distribution with the expected arrival time (t_1^* in Figure 2) as its mean. The standard deviation can be obtained using historical flying data (e.g. the publicly accessible ASQP (Airline Service Quality Performance) database used in Lan et al. 2006). Normal distribution is widely used in scientific and statistical computing because it captures the essential features of variables in broad areas (Jensen et al. 1997). Furthermore, from *the Central Limit Theorem*, normal distribution can be used as an approximation to other distributions and provides the foundation for other statistical procedures because the distribution of non-normal average tends to be normal. Therefore, we focus on normally distributed demand and price in this paper.

The departure time of flight *f2* must be later than the arrival time of *f1* plus a minimum turnaround time (*MTT*) for the aircraft (i.e., the minimum time needed to prepare the aircraft for the subsequent flight: deplaning/boarding of passengers, cleaning, crew change, etc.). Thus, the arrival time of flight *f2* at airport C is correlated with that of flight *f1* at airport B, even though their durations may be assumed to be independent.

In airline practice, a "buffer",ε, is usually added onto the *MTT* which depends on the previous flight. The "official" flight duration is the mean flight time and the buffer is the extra time sufficient to exceed the duration of the previous flight with a given probability. If the preceding flight arrival time goes beyond the buffer, propagated delay will be incurred to its subsequent flight. In CCP, the constraint for a beginning arrival flight *j* must arrive less than ε minutes late with a probability greater than α can be expressed as follows:

$$\Pr\left\{t_j - t_j^* \le \varepsilon\right\} \ge \alpha \qquad (9)$$

where *Pr* is the operator of the probability computation, t_j is the random actual arrival time of flight *j*. α is the confidence level defined by the decision maker. 90% or 95% is commonly used in industry. Sohoni et al. (2011) derived a similar formula as (9) where α is defined as FSL.

Assuming the probability distribution is normal, (9) becomes the following applying CCP:

$$\varepsilon \ge \sigma_j \Phi^{-1}(\alpha) \qquad (10)$$

In the above constraint, ε is the decision variable, Φ^{-1} is the standard reverse cumulative distribution function, σ_j is the known standard deviation of flight arrival time. Thus, the right-hand side of the above constraint is a known value and the above constraint is reduced to a deterministic constraint. Constraint (10) requires that the buffer has to be bigger than certain threshold, $\sigma_j \Phi^{-1}(\alpha)$. It is easy to see that, the buffer size increases with the standard deviation of the flight flying time and the confidence level. Fuhr (2007) developed a stochastic analytic convolution model to evaluate the arrival delay of a flight in an aircraft rotation propagated from the departure delay and the flying time delay. It is assumed that the departure delay and block time (flying times) delays are independent. Fuhr (2007) then assumed an Erlang-Exp distribution to approximate random flying time and set a punctuality target for a flight in his schedule planning model. With the expected arrival delay calculated, the buffer size, ε, can also be readily obtained from Fuhr's model. The punctuality target used in Fuhr (2007) corresponds to the confidence level α used in this section.

4.2 Flying time truncation in CCP

Truncation can happen when a portion of data range is not attainable on physical grounds: for example flight duration cannot be less than zero or greater than the maximum time before the plane runs out of fuel. Thus, a truncated normal distribution should be used for flight durations. In constraint (10), non-truncated normal distributions are assumed for flight arrival times, the actual buffer size may significantly differ if we consider truncated normal distributions. To handle this, the formulae for truncated flight flying times are derived in this paper as follows. Assuming that the range of flight flying time is $[T_L, T_U]$, where $-\infty < T_L < T_U < +\infty$,

(9) becomes (we eliminate subscript *j* for the sake of simplicity):

$$\Pr\left\{t - t^* \le \varepsilon\right\} = \int_{T_L}^{t^*+\varepsilon} \rho_{BTN}(t)dt \ge \alpha$$

where $\rho_{BTN}(t)$ is the bi-truncated density function of flying time *t*:

$$\rho_{BTN}(t) = \frac{\phi(\frac{t-t^*}{\sigma})}{\sigma[\Phi(\frac{T_U - t^*}{\sigma}) - \Phi(\frac{T_L - t^*}{\sigma})]} = \frac{\phi(\frac{t-t^*}{\sigma})}{\sigma_{BTN}}.$$

Where $\sigma_{BTN} = \sigma[\Phi(Z_{XU}) - \Phi(Z_{XL})]$ and $Z_{XU} = \frac{T_U - t^*}{\sigma}$, $Z_{XL} = \frac{T_L - t^*}{\sigma}$. $\phi(t)$ is the standard normal density function.

Let $l = \frac{t-t^*}{\sigma}$ and define $Z_{XP} = \frac{\varepsilon}{\sigma}$, then

$$\int_{T_L}^{t^*+\varepsilon} \rho_{BTN}(t)dt = \frac{\sigma}{\sigma_{BTN}} \int_{Z_{XL}}^{Z_{XP}} \frac{1}{\sqrt{2\pi}} e^{\frac{-l^2}{2}} dl = \frac{\sigma}{\sigma_{BTN}}[\Phi(Z_{XP}) - \Phi(Z_{XL})] = \frac{\Phi(Z_{XP}) - \Phi(Z_{XL})}{\Phi(Z_{XU}) - \Phi(Z_{XL})}$$

Thus, (9) becomes

$\frac{\Phi(Z_{XP}) - \Phi(Z_{XL})}{\Phi(Z_{XU}) - \Phi(Z_{XL})} \ge \alpha$, or $\Phi(Z_{XP}) \ge \Phi(Z_{XL}) + \alpha(\Phi(Z_{XU}) - \Phi(Z_{XL}))$, i.e.,

$$\varepsilon \ge \sigma\Phi^{-1}(\Phi(Z_{XL}) + \alpha(\Phi(Z_{XU}) - \Phi(Z_{XL}))) \qquad (10a)$$

When flying time *t* conforms to non-truncated normal distribution, that is, $T_U \to +\infty$ and $Z_{XU} \to +\infty$, and $\Phi(Z_{XU}) \to 1$; $T_L \to -\infty$ and $Z_{XL} \to -\infty$ and $\Phi(Z_{XL}) \to 0$, (10a) reduces to (10).

4.3 Effect of flying time truncation

We use a simple example in this section to illustrate the effect of flying time truncation on buffer size. Assume that the flying time of flight *j* has a mean of 720 minutes and a standard deviation of 30 minutes. The confidence level α is fixed to 95%. If we assume non-truncated flying time, then the buffer size should be bigger than 49.35 minutes from (10). For truncated flying time, we assume a fixed T_L of 650 minutes. The minimum buffer sizes are listed in the Table 2 below with different T_U:

T_U, min	750	780	840	900
ε, min	25.224	44.025	49.479	49.488

Table 2 minimum buffer sizes at different T_U

It can be seen that, the minimum buffer size may be significantly smaller than the non-truncated buffer size when T_U is approaching mean flying time. For a one hour cut-off case, the minimum buffer size is 5.3 minutes shorter than the non-truncated case. For two/three hour cut-off cases, the minimum buffer size is near the non-truncated case. The minimum buffer size may also be larger than the non-truncated buffer size when T_U increases. Note that the minimum buffer size also depends on other parameters such as α, T_L and σ.

4.4 Airline expected revenue

Having dealt with uncertainty around flight duration and delays, constraint (11) calculates the expected revenue by considering uncertain passenger demand *x*:

$$\text{Revenue} = E[c \times \min(P, x)] \qquad (11)$$

where c is the ticket price, *P* is the airline seat capacity, *x* is the random demand for flights, and *E(x)* is the expectation of random variable *x*.

The above equation can be reformulated into (Li et al. 2004):

$$\text{Revenue} = c[\theta - \sigma L(z)] \qquad (12)$$

where *L(z)* is the standard loss function, $z = \frac{P-\theta}{\sigma}$ and θ is the demand mean. Underlying (12) is the assumption that demand and ticket price are independent and non-truncated. This assumption is rarely true in reality as the seasonal ticket price increases with the passenger demand: they are correlated. It is also obvious that the demand and price are both truncated in reality. Some researchers (Wenkai et al. 2005, 2006) have derived formulae for refinery revenue when crude oil demand and price are correlated and truncated. Those formulae can be applied to airline revenue calculation directly. Suppose that the range of demand is $[x_L, x_U]$, where $-\infty < x_L < x_U < +\infty$ and the range of price is $[c_L, c_U]$, where $-\infty < c_L < c_U < +\infty$. Then the pdf (probability density function) of BBTN (Bivariate Bi-Truncated Normal distribution) is:

$$f_{BBTN}(c,x) = \begin{cases} \dfrac{\varphi(c,x)}{F_{LU}}, & x_L \le x \le x_U \ \ and \ \ c_L \le c \le c_U \\ 0, & \text{otherwise} \end{cases}$$

where, $F_{LU} = \int_{c_L}^{c_U}\int_{x_L}^{x_U} \varphi(c,x)dxdc$ and $\varphi(c,x)$ is the pdf of the two-dimensional non-truncated normal distribution function:

$$\varphi(c,x) = \frac{1}{2\pi\sigma_c\sigma_x\sqrt{1-\rho^2}} e^{\frac{-1}{2(1-\rho^2)}\left[\frac{(c-\bar{c})^2}{\sigma_c^2} - \frac{2\rho(c-\bar{c})(x-\theta)}{\sigma_c\sigma_x} + \frac{(x-\theta)^2}{\sigma_x^2}\right]}$$

where, $\bar{c}$ is the mean of price, σ_c is the standard deviation of price. θ is the mean of demand and σ_x is the standard deviation of demand. ρ is the correlation coefficient.

(12a) is to calculate the revenue when the price and demand are correlated and truncated can be reduced to several single integrals (Wenkai et al. 2005, 2006):

$$\text{Revenue} = A1_T + A2_T + A3_T + A4_T + A5_T + C1_T + C2_T \qquad (12a)$$

Where single integrals in (12a) are listed in Table 3.

Approximating the single integrals in (12a) using simple polynomial functions, the revenue can then be calculated (Wenkai et al. 2005, 2006).

4.5 The optimization model

The optimization model integrating the revenue calculation and the confidence level constraint is shown as follows:

$$\begin{aligned} &\max \ \sum_{\mathrm{j}} Revenue_j \\ &\text{s.t.} \quad \text{Constraints for } Revenue_j \\ &\qquad \text{Constraints for confidence levels} \\ &\qquad \text{Operational constraints} \end{aligned} \tag{13}$$

$\mathrm{A1_T} = \frac{-\sqrt{1-\rho^2}\sigma_x\sigma_c}{2\pi F_{LU}} \int_{\frac{c_L-c}{\sigma_c}}^{\frac{c_U-c}{\sigma_c}} me^{-\frac{m^2}{2}}[e^{-\frac{\mathrm{U}^2}{2}} - e^{-\frac{\mathrm{L}^2}{2}}]dm$	$\mathrm{A2_T} = \frac{-\sqrt{1-\rho^2}\sigma_x\bar{c}}{2\pi F_{LU}} \int_{\frac{c_L-c}{\sigma_c}}^{\frac{c_U-c}{\sigma_c}} e^{-\frac{m^2}{2}}[e^{-\frac{\mathrm{U}^2}{2}} - e^{-\frac{\mathrm{L}^2}{2}}]dm$
$\mathrm{A3_T} = \frac{\rho\sigma_x\sigma_c}{\sqrt{2\pi}F_{LU}} \int_{\frac{c_L-c}{\sigma_c}}^{\frac{c_U-c}{\sigma_c}} m^2e^{-\frac{m^2}{2}}[\Phi(\mathrm{U})-\Phi(\mathrm{L})]dm$	$\mathrm{A4_T} = \frac{\sigma_c\theta+\rho\bar{c}\sigma_x}{\sqrt{2\pi}F_{LU}} \int_{\frac{c_L-c}{\sigma_c}}^{\frac{c_U-c}{\sigma_c}} me^{-\frac{m^2}{2}}[\Phi(\mathrm{U})-\Phi(\mathrm{L})]dm$
$\mathrm{A5_T} = \frac{\bar{c}\theta}{\sqrt{2\pi}F_{LU}} \int_{\frac{c_L-c}{\sigma_c}}^{\frac{c_U-c}{\sigma_c}} e^{-\frac{m^2}{2}}[\Phi(\mathrm{U})-\Phi(\mathrm{L})]dm$	$\mathrm{C1_T} = \frac{P\bar{c}}{\sqrt{2\pi}F_{LU}} \int_{m=\frac{c_L-\bar{c}}{\sigma_c}}^{\frac{c_U-\bar{c}}{\sigma_c}} e^{-\frac{m^2}{2}}[\Phi(\mathrm{U_U})-\Phi(\mathrm{U})]dm$
$\mathrm{C2_T} = \frac{P\sigma_c}{\sqrt{2\pi}F_{LU}} \int_{m=\frac{c_L-\bar{c}}{\sigma_c}}^{\frac{c_U-\bar{c}}{\sigma_c}} me^{-\frac{m^2}{2}}[\Phi(\mathrm{U_U})-\Phi(\mathrm{U})]dm$	$\frac{c-\bar{c}}{\sigma_c} = m$, $\frac{x-\theta}{\sigma_x} = n$
$\mathrm{U} = \frac{\frac{P-\theta}{\sigma_x} - \rho m}{\sqrt{1-\rho^2}}$, $\mathrm{L} = \frac{\frac{x_L-\theta}{\sigma_x} - \rho m}{\sqrt{1-\rho^2}}$	$\mathrm{U_U} = \frac{\frac{x_U-\theta}{\sigma_x} - \rho m}{\sqrt{1-\rho^2}}$

Table 3. Single integrals in (12a)

where $Revenue_j$ is the revenue from flight *j*. The model is to maximize the net profit of an airline. We use the formulae derived to calculate the actual revenue for each flight. The constraints for confidence levels and constraints for airline operations, such as flight cover/consistent constraints, should also be included. A scalable MILP solver such as IBM CPLEX can be used to solve the above model.

4.6 Risk management and CCP

The term risk plays a pervasive role in the literature on economic, political, social and technological issues. To measure risk, variance was first proposed by Markowitz to measure the risk associated with the return of assets. Value-at-Risk (VaR) was introduced in 1994 by the leading bank—JP Morgan (Morgan 1994). It is very popular in practice and has become part of financial regulations. VaR is introduced to answer the following questions: how much one can expect to lose in one day/week/year with a given probability? What is the percentage of the value of the investment that is at risk? For a given time horizon and a probability level k, 0 < k < 1, VaR_k, is the loss that is exceeded over this specified period with probability 1 - k, i.e. VaR_k is the maximum loss in a specified period with probability level k. The exact definition of the VaR_k of a random variable X is based on the k-quantile, i.e.

$$\text{VaR}_k = -F_x^{-1}(k) \tag{14}$$

F_x^{-1} denotes the inverse of the distribution function F_x .

However, VaR has been widely criticized mainly because it does not measure losses exceeding VaR (Szego 2005): huge loss may happen with low probability. Thus, Conditional Value-at-Risk (CVaR) has been proposed as a natural remedy for the deficiencies of VaR (Rockafeller et al. 2000). For continuous random variables, CVaR is the expected value of the losses exceeding VaR_k:

$$\text{CVaR}_k = \phi_k(x) = (1-k)^{-1} \int_{f(x,y)\geq\alpha_k(x)} f(x,y)p(y)dy \tag{15}$$

where *f(x,y)* is the loss associated with the decision vector *x* and the random vector *y*. The vector *y* stands for the uncertainties. *p(y)* is the underlying probability distribution of *y*. An analytical expression *p(y)* for the implementation of CVaR is not needed. It is enough to have an algorithm which generates random samples from *p(y)* (Rockafeller et al. 2000).

The ideas of risk management may be applied to generate more robust airline schedules. In CCP, constraint (5) is used to ensure that customer demand is satisfied (for example) at certain confidence level vector, $\alpha \in R^n$:

$$\Pr\{\sum_{j=1}^{m} A_{ij}x_i \leq b_i\} \geq \alpha_i, \qquad i = 1,2,\ldots,n. \tag{5}$$

Confidence level is essentially the counterpart of VaR in risk management. It is not safe enough because it cannot guarantee the amount of demand is satisfied at certain level. It is possible that, the scenario of huge amount of unmet demand fall in 5% (assume a 95% confidence level), which bring unacceptable overall unmet customer demand. To improve (5), we may need to minimize the expected value of unmet demand as is done in CVaR (constraint 15). In the example illustrated in Figure 2, (9) is used to guarantee that a preceding arrival flight *j* must arrive less than ε minutes late with a probability greater than α

$$\Pr\left\{t_j - t_j^* \leq \varepsilon\right\} \geq \alpha \tag{9}$$

(9) does not take the length of the lateness of flight *j* into account. As shown in Figure 3, flight *f1* may arrives far later than t_1^*, though with low probability, which causes a significant propagated delay to its subsequent flight *f3*. As a result, passengers connecting from flight leg *f3* to other flight legs will likely be disrupted. A major disruption of this nature is hard to be absorbed (e.g. by increasing aircraft speed of flight *f3* in an attempt to absorb flight delays) and take much longer time to recover and hence impose much higher cost to the airlines. To improve (9), the expectation of $t_j - t_j^*$, $E[t_j - t_j^*]$, should be considered so that with a confidence level α, the resulting schedules are free of major flight delay disruptions.

5. Disruption management

Closely related to the issue of robustness is the airline disruption management. Disruption management deals with how to recover from disruptions in an optimal manner. Though we can build flexibility or robustness into the airline schedules proactively, however robust the schedule is, disruption handling is essential and inevitable in airline operations. As resources are in most cases highly utilized and interdependent, upstream flight leg delays not only propagate to subsequent legs scheduled to use the same aircraft, but crew members and passengers scheduled to use these affected flight legs. Even the smallest disruptions can have cascading effects to airline schedules and may further lead to downstream disruptions.

5.1 Literature review on disruption management

Traditionally, airlines make recovery decisions manually with little decision support (Lettovsky 1997). Nowadays, the most technologically advanced airlines have implemented the information infrastructure needed to track all resources and accommodate real-time decision making (Barnhart et. al. 2004). This infrastructure, together with progress made to date, disruption management techniques have now emerged and are becoming essential for robust airline scheduling and operation (Kohl et al. 2007). The French Operational Research (OR) and Decision Support Society thus organized a worldwide competition in 2008/9 (*ROADEF 2009 Challenge*, referred to as *the Challenge* later in this paper (*ROADEF Challenge* 2009) for airline disruption management. The problem of *the Challenge* was to find the best aircraft routing and passenger re-accommodation solution to recover from a fixed set of disruptions within a specified period of time, with given regular operating constraints. The aim of *the Challenge* was to explore the problems encountered in real world airlines when disruptions happen and find efficient approaches to solve them. *The Challenge* provides two sets of problem instances (each with 10 instances), each set having a different size. Four types of disruptions were considered:

1. airport capacity: restrictions on the number of departures and landings (including closure) for a given period;
2. aircraft unavailability: a time and duration of an aircraft unavailability (i.e. due to an unserviceability or fault);
3. flight cancellations; and
4. flight delays.

A number of research works have appeared at addressing this problem, typically considering only a subset of the resources involved in the disruption. Teodorovic and Guberinic (1984) studied the aircraft recovery problem. They considered disruption due to aircraft unavailability with the objective of minimizing total passenger delays. Teodorovic et al. (1990) extended their work by considering airport curfews and using a greedy algorithm in which aircraft rotations were built one by one. Jarrah et al. (1993) developed a timeline network model to handle two kinds of flight disruptions: cancellation and re-timing. Talluri et al. (1996) built connection network models to precompute alternatives for swapping aircraft among flights when disruption happens. Lettovsky (1997) developed an integrated MIP model for crew, aircraft and passenger recovery. Due to the extremely large problem size, he applied Bender’s decomposition algorithm which involves a master problem and three sub-problems. Bard et al. (2001) proposed a time-band network to deal with disruptions. The model was a minimum cost flow network model with side constraints. Rosenberger et al. (2003) proposed an aircraft selection heuristic (ASH) to search a subset of other aircraft for potential swaps with a disrupted aircraft. Abdelghany *et al*. (2004) developed a decision support tool to automate crew recovery during irregular operations for large-scale commercial airlines. The tool proactively recovers crew problems ahead of time before their occurrence. When a flight delay happens, the tool tracks the effect of the delay and projects the possible future crew problems caused by the delay several hours before the scheduled departure time of the connecting flights. This projected future crew problems are recovered in their model using a rolling approach. The deviation from the originally planned schedule and the planned workload of the crew is minimized. Bratu et al. (2006) presented two models that address aircraft and crew recovery with a passenger-centric objective. Kohl et al. (2007) introduced the architecture of their disruption management system, called “Descartes” which included a dedicated passenger recovery solver, a dedicated aircraft recovery solver and a dedicated crew recovery solver.

5.2 Challenges and opportunties in disruption management

Barnhart et. al. (2004) reviewed current approaches and challenges facing airline scheduling. They pointed out that airline schedule recovery problems are particularly challenging, involving multiple highly-constrained resources and requiring a global view of the system. Optimal recovery decisions are hard to determine. Canceling a leg or rerouting a crew or a plane can have costly consequences throughout the airline’s system. This makes airline recovery difficult to model. In general, major challenges in airline disruption management include:

Need to be come up with a recovery solution as soon as possible

Typically, recovery decisions often need to be made in a matter of minutes (Barnhart et. al. 2004). Thus it is more important to be able to choose from a range of feasible solutions, rather than having one “best” solution.

Disruption management requires integration of several domains

These domains include aircraft routing, crew disruption, passenger itineraries, slot management, aircraft maintenance etc.

Define suitable objectives

Objectives of disruption management include three categories:

- Deliver the passengers and their luggage to their destination on time,
- Minimize the operation costs and
- Return to the original schedule as soon as possible.

It is hard to quantify the passengers dissatisfaction cost due to flight delays and cancellations and the penalty of not returning to the original schedule. These three categories are incorporated into one objective in *the challenge* using weighted sums of the categories.

Notwithstanding the significant improvement made in the literature, much important research still remains to be done in this area. Most work on disruption management in airlines has focused on resolving conflicts for a single resource at a time, and cooperation between airlines has not been covered (Kohl et al. 2007). Much research work remains to be done to address the needs of passengers, crews, and aircraft while determining low-cost solutions rapidly. The ultimate goal is to implement disruption management into airline OCC (operation control centers) so that recovery solutions can be obtained automatically using real-time data from airline information infrastructure.

6. An integrated approach: combining airline disruption management with robust planning

6.1 An introductory example

Robust planning/scheduling approach described in sections 2-4 and disruption management are closely connected: disruption management provides scenarios to be considered in recourse models.

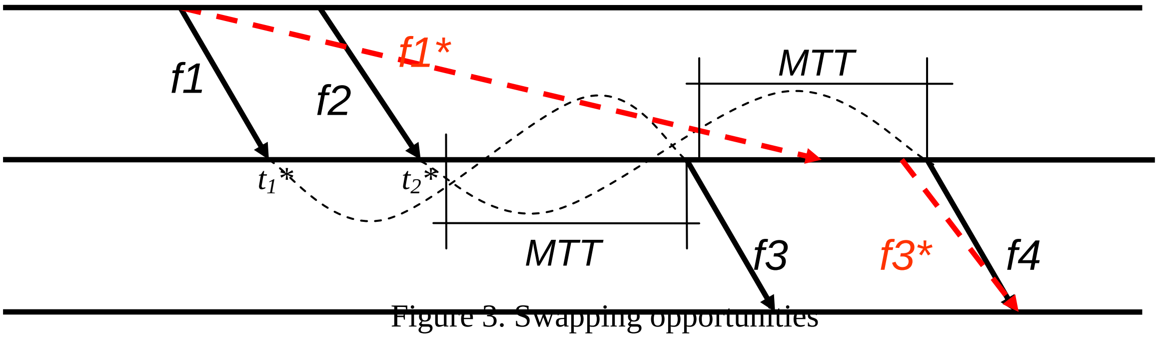

Figure 3. Swapping opportunities

Figure 3 is adapted from Lan et al. (2006). Assume that flight leg *f1* and flight leg *f3* are in the same route, and flight leg *f2* and flight leg *f4* are in the same route. If flight leg *f1* is delayed to the position of $f1^*$ (dotted red in Figure 3). This delay is longer than the buffer between flight leg *f1* and flight leg *f3*, causing delay to propagate from flight leg *f1* to *f3*. A swapping opportunity can be provided by disruption management models: swapping the second halves of the two routes so that flight leg *f1* and flight leg *f4* is in the same route, and flight leg *f2* and flight leg *f3* in another route. The effect is to add extra buffer after the delayed flight leg *f1* to mitigate the downstream effects of its delay. We say that a swapping opportunity identified in this way in disruption management can be added into recourse models in robust planning discussed in section 3.1 as one of their scenarios.

6.2. The challenge of integrating planning with disruption management

Disruption management, takes into account different types of disruptions in airline planning/scheduling and calculates their recovery costs. Though critical for airline operations, disruption management does not consider the probabilities of different disruptions and hence it is not easily integrated into robust airline planning/scheduling. Stochastic programming, including CCP and recourse models, considers the probabilities of different scenarios. However, stochastic programming lacks techniques to identify and calculate the recourses of different scenarios.

In the literature, Ahmadbeygi et al. (2010) apply a recourse model for robust airline planning. Though not considered in their model, they point out the potential impact of recovery decisions (i.e. disruption management) on how delays propagate and hence the results of the recourse model. Marla (2010) proposes a way of combining disruption management with flight planning models during operations. Flight planning is introduced as an enhanced disruption management tool. Two aspects of flight planning, i.e. speed changes and flight departure delay, are used for absorbing delays and complementing traditional disruption management approaches.

A general architecture for an integrated approach is proposed here. CCP can be used to handle scenarios with known probability distributions, recourse models can be used for scenarios with discrete probabilities and whose explicit probability distributions are hard to define. Disruption management techniques can then be used to provide stochastic programming with different types of scenarios (e.g. disruptions) and calculate the recourse at each scenario. We address this requirement in the remainder of this section.

6.2 Measures of robustness based on disruption handling

In a recourse model, the cost of recourse, or the MOR, has to be determined. The MOR, or "Measure of Robustness" was introduced in section 2.2 above. One of the major difficulties is to identify all possible scenarios to calculate a complete MOR. For example, Ageeva (2000) defines his MOR using the degree to which sequence of activities on swappable resources meet each other. Ageeva's approach tries to provide ways for more aircraft routes to intersect at different points. Besides the swapping opportunities identified by the author, there may exist more swapping opportunities in aircraft routes as shown in Figure 3. In general, disruption management models make a lot of efforts to identify every swapping opportunity so as to recover the airline schedules from disruptions. As these disruptions are essentially viewed as part of the uncertainties in robust models, the swapping opportunities associated with the disruptions identified in disruption management can be directly included as part of the MORs. In brief, with swapping opportunities identified in disruption management, one can define more complete MORs in the recourse models and provide more robust solutions. This integration of robust and disruption management models can be generally extended to other problems such as crew pairing problem. A robust crew pairing model as studied in Shebalov and Klabjan (2006) may be improved with the integration of crew pairing disruption management models so that more move-up crew opportunities (besides the crew re-linking in Shebalov and Klabjan 2006) can be identified.

6.3 Multiple MORs in recourse models

Most of the literature appeared define a single MOR as listed in Table 1. However, this may not be enough to capture all types of scenarios which could happen to airline operations. The opportunities for improving robustness, such as regularity and the number of move-up crews, could exist simultaneously and be combined to provide more robust solutions. A real world airline disruptions is not only originated from a single source, such as delays or cancellations, but also involves aircraft unavailability, airport capacity reduction as well as many side-constraints such as pre-assigned aircraft maintenance, minimum turnaround times, aircraft final location etc (Dickson et al. 2011). Defining multiple MORs in a recourse model may give more robust solutions as influences from multiple types of scenarios are captured. A recourse model with multiple MORs is more challenging, however. We could either i) define multiple objectives, with each MOR act as one objective, together with an economic objective in a recourse model and apply multiobjective algorithm such as ε-constraint method (Goicoechea et al. 1982), to solve the recourse model, or ii) weight all MORs and define a single "total" MOR. Together with other operations costs, the "total" MOR can then be defined as the single objective of a recourse model. Again, a disruption management tool will help to better understand the mechanisms of the interactions of different scenarios and determine the weights for MORs. A simulation model will be very helpful because it can evaluate the influences of disruptions on total costs. With the above approach, we could readily evaluate the influences of multiple sources of robustness such as the number of move-up crews, the total propagated delays, or the total cancellations, on the costs and robustness of airline operations. There also exist trade-offs among different methods. Combining two or more MORs into one model may give more robust solutions, but this method has to collect more data, design more sophisticated model and takes longer solution time than the method defining a single MOR in a recourse model.

6.4 Evaluation of penalty terms in recourse models

As stated in section 3.2, the exact values of the penalty terms in recourse models are difficult to determine. However, in some cases, these penalty terms can be evaluated with the understanding of airline operations and disruptions management. In this section, we illustrate how the values of the penalty terms can be estimated using airline operations data. *The Challenge* categorizes the costs incurred by flight delays into two types: operating costs and passenger inconvenience compensations, whose detailed data and calculation method have been provided by *the Challenge*. With these data, the penalty terms for flight delays in recourse models can then be estimated. For example, inspired from European Union regulations, the operating costs for delays are imposed depending on the length of delays and the initial planned durations. Drinks and meals (e.g. 15 euros per passenger) or lodging (e.g. 60 euros per passenger) should be provided for delayed passengers. The compensations to passenger inconvenience for a delay is assumed to be linear to the length of delay with a slope depending upon the passenger itinerary type and the itinerary's cabin class in *the Challenge*. The sum of the above two costs can be used directly as the penalty term for each delay because it measures the costs incurred by a scenario (in this case a delay) in the recourse model. The penalty term for the expected total propagated delay, the MOR used in Lan et al. (2006) and Ahmadbeygi et al. (2010), is then the sum of penalty terms of all delays. Similarly, the penalty term for non-robustness measure defined in Ehrgott and Ryan (2002) can also be estimated.

The operating and passenger inconvenience costs are also proposed for cancellations in *the Challenge*. These costs can be used to estimate the penalty terms of cancellation scenarios in a recourse model. Flight delays and cancellations are only part of disruptions that may happen to airline operations, unfortunately. For comprehensive consideration of scenarios for a recourse model, other disruptions should be taken into account. One example is aircraft unavailability disruption. The operating and passenger inconvenience costs for aircraft unavailability are not directly available in literature such as *the Challenge*. One possible way to attack this difficulty is to understand the influence of aircraft unavailability on flight delays and cancellations so that the costs incurred can be estimated using known costs of flight delays and cancellations. The penalty term to be used in a recourse model can then be determined. This is a nontrivial task, unfortunately, as the propagated influence of aircraft unavailability on delays and cancellations is complicated. A disruption management tool, such as mixed integer models and algorithms proposed by Dickson et al. (2011), is thus essential for this purpose. The situation is even more complicated for global disruptions such as airport capacity reduction. The restriction on the number of departures and landings at a given period at an airport has profound influence on operations of crews, flights and aircraft. The determination of penalty terms for such type of scenarios calls for a sophisticated disruption management tool. A simulation model such as SimAir (Rosenberger et al. 2002), will also be particularly helpful because it can record each disruption and evaluate its influence on total costs.

8. Conclusion and Further Research

The field of airline planning under uncertainty has been increasingly active over the last decade. All the prototype tools and models proposed in the literature are still substantially different from the requirements from the airline companies (Clausen et al. 2010). Virtually all papers published address single resource systems (aircraft, crew or passenger recovery). The computational speed is still too long for real time operations. Nevertheless, airline planning under uncertainty is a problem rich in opportunity and high impact. Much research still to be done to close the gap between the research community and the real world airline operations.

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