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Abstract

In a world with costly transfers, some agents with high transaction costs may not find it feasible to trade. Hence, their consumption will co-vary with their endowment, leading to imperfect risk-sharing. In this paper, we augment the canonical risk-sharing model to incorporate frictions in the form of transaction costs. In this augmented model, given a particular network structure, risk sharing will happen within networks and not the whole universe of agents. We show that transaction costs and the implied network structure of trade have important implications for the tests of risk sharing. Using this model, we derive a structural risk-sharing test that uses consumption and production data alongside the trade network structure. We implement our method using data from the global trade of three major staple food commodities. Comparing our estimates with the benchmark of frictionless trade, we find some evidence of transaction costs impeding risk-sharing in these commodities.

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1 Introduction

Tests of perfect risk sharing are pervasive in economics. The classical model of risk sharing considers a social planner costlessly allocating a pool of aggregate endowment across risk-averse economic agents (Mace, 1991; Cochrane, 1991; Townsend, 1994, 1995). The model predicts that under optimality, the social planner allocates a fixed proportion of aggregate endowment to each agent. Therefore, the consumption of an agent varies only with the aggregate pool of endowments and is entirely independent of its own endowment (Mace, 1991; Cochrane, 1991; Townsend, 1994; Obstfeld, 1994; Bardhan and Udry, 1999). This prediction of the complete risk-sharing model has been tested several times and at different levels across panels of countries, states, regions, villages, and households. Although the justification across studies for testing the complete risk-sharing hypothesis may vary, what remains common is the canonical complete markets benchmark dictated by the planner's solution.

Even with a large body of literature testing the risk-sharing hypothesis, few (if any) have found evidence of perfect risk sharing (Canova and Ravn, 1996; Lewis, 1996). This has led economists to propose alternative explanations for why the data shows a positive correlation between income and consumption and deviates from the complete market benchmark. One strand of literature explains this observed correlation based on information frictions leading to moral hazard and limited commitment (Ligon, 1998; Ligon et al., 2002; Genicot and Ray, 2003; Laczó, 2015; Ábrahám and Laczó, 2018; Attanasio and Krutikova, 2020; Bold and Broer, 2021; Ambrus et al., 2022). Another small strand of literature has focused on frictions in the form of transaction costs to explain the apparent lack of risk sharing (Obstfeld and Rogoff, 2001; Schulhofer-Wohl, 2011; Fitzgerald, 2012; Jack and Suri, 2014; Clance et al., 2019; Bradford et al., 2022).

In this paper, we study the role of transaction costs in risk sharing. Two observations related to the canonical risk-sharing model are important for this paper. One, state contingent trade flows or informal transfers across agents are implicitly assumed in the model as a medium of insurance (Townsend, 1987, 1994). Two, the markets for risk may either be formal or informal but risk is shared via trade, therefore, transaction costs are important (Obstfeld and Rogoff, 2001; Fitzgerald, 2012; Jack and Suri, 2014). Our motivation comes from the idea that under transaction costs, risk sharing may not be perfect because agents with high transaction costs may not find it feasible to trade, hence their consumption may co-vary with their endowments.

If trade between agents is costly then do the theoretical and empirical implications of the risk sharing model change? This is the main question we pose in this paper. To understand the implications of costly trade, we introduce transaction costs in the simple model of risk sharing across N agents. We set up a theoretical structure where individual endowments are heterogeneous and agents are risk-averse. Therefore, trade between agents is welfare improving. We allow for costly resource transfer between individuals. We model these costs as iceberg-type trade costs popular in the trade literature. These costs are proportional to the volume of transfer and are different for different trade pairs. This problem naturally leads to studying the network structure of trade across agents. In a simple frictionless world, network structure is exogenous and every agent can form a trade link with all other agents costlessly (Jack and Suri, 2014). But with transaction costs, forming a trade link is costly and the trade network of an agent is dependent on model parameters including the bilateral trade costs. In this augmented model, given a particular network structure, risk sharing will happen within networks and not the whole universe of agents. As will be seen later, transaction costs and the implied network structure of trade have important implications for the tests of risk sharing.

Using simulations from the model, we first establish some basic results. We show that a reduction in transaction costs leads to: (i) greater linkages between agents and more connectedness in a network, (ii) reduced consumption inequality across countries, (iii) lower correlation between domestic consumption and endowment, and (iv) reduced consumption variation and higher global utility. Similar results are reported by Jack and Suri (2014) in a three agent risk-sharing model with fixed transaction costs. We show that these results hold in our generalized framework with iceberg costs and with an arbitrary number of players. Next, we derive a structural test of risk sharing which explicitly takes the network structure of trade into account while testing for risk sharing.

Our theoretical setup is general enough to fit into different contexts and can be used to study risk sharing with transaction costs among households in a village or across countries. However, structural estimation of the model requires information on trade linkages along with data on aggregate consumption and production. We implement our test on data from the global trade of rice, wheat, and maize since a long time series on aggregate consumption, production and trade network is easily available for these commodities. The variation in the locus of trade across the three commodities also allows us to test the robustness of our method to different network structures.

There are three main contributions of this paper. The first is in extending the standard model of risk sharing to include frictions in the form of transaction costs. In a complementary paper by Bradford et al. (2022), the authors develop a conceptual model of risk sharing with trade costs. We extend the ideas in Bradford et al. (2022) to a more general setting. Schulhofer-Wohl (2011) allows for the possibility of costly transfers in the standard social planner's allocation problem. He shows that income and consumption show greater correlation in households that are less risk averse or more costly to insure. In Schulhofer-Wohl's formulation, however, transaction costs do not vary at the trading pair level. We relax this assumption in our model. Jack and Suri (2014) develop a risk-sharing model with three individuals and fixed transaction cost. A unique feature of their model is that risk-sharing networks form endogenously based on transaction costs between agents. They show that such costs limit the size of the risk-sharing network and the extent to which state contingent transfers smooth consumption. We generalize their model to N agents with transaction costs proportional to the transfer.

Our second contribution is methodological. We develop a novel structural test of risk sharing which explicitly incorporates the network structure of trade in estimation. With costly transfers, network structure is endogenous and depends on transaction costs. We take this into account while estimating the extent of feasible risk sharing in a world with costly transfers. In a similar spirit, Laczó (2015) structurally estimates a risk-sharing model, but with frictions that arise in the form of limited commitment. She finds it a better fit for data from rural households in India. Fitzgerald (2012) embeds a gravity model of trade in intermediate goods in a standard DSGE model to propose a gravity based test of risk sharing. Fitzgerald's approach induces dependency across countries which is distinct from our case of perfect substitution; this difference has consequences on network formation and the role of trade costs in risk sharing.[1]

This paper also contributes to the literature studying specification issues in risk sharing tests. A recent strand of literature has established the omission of heterogeneous risk preferences as an important source of bias in the standard tests of risk sharing (Schulhofer-Wohl, 2011; Mazzocco and Saini, 2012; Asdrubali et al., 2019). Likewise, studies have suggested that risk sharing may happen within networks which may or may not span the entire universe of agents (Fafchamps and Lund, 2003; De Weerdt and Dercon, 2006; Bramoullé and Kranton, 2007; Attanasio et al., 2012; Attanasio and Krutikova, 2020). The implication is that aggregate shocks should be network specific (Ambrus et al., 2014, 2022; Bradford et al., 2022). Adding to this literature, we discuss how ignoring transactions costs and the implied network structure can lead to biased estimates of risk sharing. Incorporating networks into estimation also introduces a complication in the form of network dependence in the error structure. We allow errors in the risk sharing regression to exhibit such dependence and use results from Kojevnikov et al. (2021) to propose a network heteroskedasticity and autocorrelation robust variance estimator for conducting valid inference on the risk sharing parameters. The standard errors robust to general network dependence are presented along with the results.

The rest of the paper is laid out as follows. Section 2 briefly discusses the relevant literature with a focus on the role of networks in risk sharing. Section 3 presents the augmented model of risk sharing with transaction costs. Section 4 presents simulation based results from the model. Section 5 presents the structural risk-sharing test specification and discusses its connection with the conventional tests. Section 6 presents the empirical application estimates of risk sharing from the benchmark case of no trade frictions and the one with trade costs. Finally, section 7 concludes.

[1]Fitzgerald's test relies on bilateral import data and the estimation of gravity equations thereby explicitly accounting for the network structure of trade in risk sharing. However, the gravity based test of risk sharing does not directly relate to the standard tests of risk sharing which regress consumption on income. In our formulation, the risk-sharing test resembles the standard panel data regression of consumption on income and, like Fitzgerald, the null of no transactions costs is nested within the test specification. We also show how, under assumptions on the underlying network structure, fixed effects can be used to control for the influence of transaction costs on the test of risk sharing.

2 Related Literature

Earlier contributions to the literature studied risk sharing among households. Townsend (1994, 1995), Rosenzweig and Stark (1989) and Udry (1994) tested for perfect risk sharing among households within villages in developing countries. Likewise, Altug and Miller (1990), Mace (1991) and Cochrane (1991) studied risk sharing within households in a developed country context. At an aggregate level, Obstfeld (1994), Canova and Ravn (1996), Lewis (1996) tested for perfect risk sharing across countries. Irrespective of the level of aggregation, whether its households or countries, these papers have rejected the hypothesis of perfect risk sharing.

One criticism of the conventional test is that it ignores important sources of heterogeneity which can lead to false rejection of the null of full insurance. Schulhofer-Wohl (2011) shows that heterogeneity in risk preferences leads to heterogeneity in the responsiveness of consumption to aggregate shocks. Omission of this heterogeneity will bias the test against the null of full insurance (Schulhofer-Wohl, 2011; Mazzocco and Saini, 2012).

A related strand of literature studies how risk sharing happens among agents. In the context of rural households in developing countries, studies have shown that households share risk within close networks of family, kinship, and friends (Fafchamps, 1992; Fafchamps and Lund, 2003; De Weerdt and Dercon, 2006; Fafchamps and Gubert, 2007a,b; Kinnan and Townsend, 2012). Fafchamps and Lund (2003) use data on gifts and transfers from rural Filipino households to show that risk sharing through informal transfer happens primarily via networks of relatives and friends. This, they indicate, is consistent with models of incomplete insurance where enforcement of mutual insurance contracts is costly (Ligon, 1998; Ligon et al., 2002). Since formal institutions are weak in developing countries, the costs of enforcing a mutual insurance contract may be very high (Fafchamps and Gubert, 2007b). In such a scenario, households may rely on kinship networks and friends for mutual insurance (Fafchamps and Gubert, 2007a,b).

Though risk sharing may happen within existing networks with pre-existing ties, the motivations for sharing risk may itself influence the structure of mutual trade networks. Attanasio et al. (2012) show that under risk-sharing objectives and frictions in the form of no enforcement mechanisms, network formation happens among close friends and relatives. Likewise, Fafchamps and Gubert (2007a) show that geographic proximity is another important determinant of mutual insurance links. Ambrus et al. (2014) show that trade networks can develop in the form of risk-sharing islands where within-island risks are shared completely but across-island risk sharing is incomplete with network size being determined by the realization of shocks. The empirical implication of their result is that an agent's consumption will co-move with other members of that sub-network implying that the relevant aggregate shock will be network specific. Bramoullé et al. (2014) have a similar geometric network model as Ambrus et al. (2014) and focus on the simplest ways to characterize the entire network structure. Jack and Suri (2014) show that declining transaction costs improve the extent of risk sharing by expanding the number of participants within a trade network.

While the role of networks in risk sharing has been well documented, few studies have tested risk sharing at the network level. De Weerdt and Dercon (2006) test risk sharing at the network level using data from Tanzania. A network level test is possible since they have data on the complete insurance network of individuals within a village. Attanasio and Krutikova (2020) use the data from these Tanzanian households and their kinship networks to show that even withing networks, risk sharing in incomplete and the degree of risk sharing is dependent on the quality of information the households have about each other in the network.

In the absence of data on trade and insurance networks, papers that test for risk sharing within communities or villages have assumed that link formation is frictionless within the entire community or village (Townsend, 1994; Ravallion and Chaudhuri, 1997; Mazzocco and Saini, 2012; Munshi and Rosenzweig, 2016). These assumptions are easily violated with a single overlap between two different and otherwise disconnected sub-networks and would imply that these two networks function as one risk-sharing unit (Fafchamps and Lund, 2003; De Weerdt and Dercon, 2006). In other words, a household residing in a village having a kinship link with a neighboring village will make the two villages act as one risk-sharing network. In such a scenario, econometric specifications designed to test risk sharing within a village or sub-community within the village would be misspecified. A similar point is made by Ambrus et al. (2022) who study the implications of network-based localized information constraints on standard risk-sharing regressions. This paper builds on such insights. We first show that with trade frictions and costly link formation, the risk-sharing network does not span the entire universe of agents. Hence, given that the network is known to the econometrician, the test of feasible risk sharing should be done at the network level.

Most of the papers discussed here are in the context of risk sharing within households and focus on social connections in the form of kinship, family, and friends as mutual insurance networks. Our theoretical and empirical framework, however, considers risk sharing and trade networks among countries. Another point of departure from this literature is that we consider all-inclusive proportional transaction costs in contrast to the limited commitment friction setting in models of imperfect risk sharing among households. Nevertheless, as will be seen in the next section, many of the observations and findings from this literature are reflected in the theoretical and empirical implications of our model.

3 Risk-Sharing Model with Trade Network Transaction Costs

The standard risk-sharing model assumes a simple endowment based world where individual agents' endowments are state dependent and a social planner costlessly allocates resources among them. In the following section, we modify the standard model to accommodate bilateral transactions costs with trade flows among agents. We study the optimality conditions of the social planner's problem to illustrate the connection between aggregate consumption per agent and trading activity. This then necessitates re-framing the model in terms of formal networks and we use this setup to precisely characterize how all agents within a trade network are indirectly linked to each other. We then illustrate the empirically relevant aspects of the model.

3.1 Model

Consider N agents (countries, villages, households, etc.), each indexed with i. Each agent has a commonly known endowment y_i of a divisible homogeneous good with which they can trade;

let y_{ij}^E be exported endowment (from agent i to agent j) and y_i^D be domestic endowment (part of endowment not exported and consumed instead). Thus,

$$y_i = y_i^D + \sum_{j \neq i}^{N} y_{ij}^E \tag{1}$$

The global endowment is then $Y \equiv \sum_i y_i = \sum_{i=1}^N y_i^D + \sum_{i=1}^N \sum_{j \neq i}^N y_{ij}^E$. Likewise, we define total consumption for an agent as:

$$c_i = c_i^D + c_i^I \tag{2}$$

where c_i^D is the consumption from domestic endowment and c_i^I is the consumption from imports. Although important, we abstract away from the possibility of storage and intertemporal trade as a means to smooth consumption as we want to focus on frictions only in the form of transaction costs. Since there is no storage in this model:

$$c_i^D = y_i^D \tag{3}$$

Given this structure, we suppose that imports are costly. Therefore, one unit of export from agent j ends up being less than one unit for importing agent i. The fraction with which the exports are converted into imports is allowed to vary by each trading pair. This way of characterizing transaction costs is relatively crude but has foundations in the trade literature and is sufficient for our purposes. Suppose that for each unit of exports from agent j to agent i, only a fraction $\delta_{ji} \in [0, 1]$ is converted into imported consumption:

$$c_i^I = \sum_{j \neq i}^{N} y_{ji}^E \delta_{ji} \tag{4}$$

The term $1 - \delta_{ji}$ captures the loss from shipping costs (in terms of the good), "lost at sea" cargo, and wasteful rents extracted by some middleman; all of these prevent the full exported amount from being consumed by importers.[2]

The social planner's program (in a given year) is to maximize a utilitarian social welfare function, $\sum_{i=1}^{N} \alpha_i u_i(c_i)$ where α_i's are Pareto weights and $\sum_{i=1}^{N} \alpha_i = 1$, by choosing the endowment allocation per agent $(y_i^D, y_{ij}^E \ \ \forall j \neq i)_{\forall i}$ with constraints for agent i governed by the endowments: $c_i = y_i^D + \sum_{j\neq i}^{N} y_{ji}^E \delta_{ji}$, $y_i = y_i^D + \sum_{j\neq i}^{N} y_{ij}^E$, and $y_i^D, y_{ij}^E \geq 0$. To simplify the program, we can express it just in terms of exports and total endowments:[3]

$$\max_{y_{ji}^E \ \forall i, j\neq i} \sum_{i=1}^{N} \alpha_i u_i \left(y_i - \sum_{j\neq i}^{N} y_{ij}^E + \sum_{j\neq i}^{N} y_{ji}^E \delta_{ji} \right) \ \ s.t. \ \ y_{ji}^E \geq 0 \ \ \forall i, j \neq i \tag{5}$$

This setup generalizes the standard risk-sharing model. The system of first order conditions (FOC) is as follows, with Lagrangian multipliers, λ, for the non-negativity constraints:

$$\begin{cases} \alpha_j \frac{\partial u_j}{\partial c_j} \delta_{ij} - \alpha_i \frac{\partial u_i}{\partial c_i} - \lambda_{ij} = 0 \\ \lambda_{ij} y_{ij} = 0, \lambda_{ij} \geq 0, y_{ij} \geq 0 \end{cases} \quad \forall i, j \neq i \tag{6}$$

Since our focus is on an empirical setting with a known trade network, we assume knowledge of the set of trading partners for deriving the empirical expressions; we know which $y_{ij} = 0$.[4] Then, for a known set of trading partners and CRRA utility $u(c_i) = \frac{c_i^{1-\gamma}}{1-\gamma}$, we can derive a closed-form solution. The FOC system in equation (6) yields the linear system of equations (7).[5] An important parameter in this system that captures how trading partners are linked is the direct trade weight $\Delta_{ij} \equiv \left(\frac{\delta_{ij}\alpha_j}{\alpha_i}\right)^{-1/\gamma}$.

$$\Delta_{ij} \left(y_j - \sum_{k\neq j}^{N} y_{jk}^E + \sum_{k\neq j}^{N} y_{kj}^E \delta_{kj} \right) = \left(y_i - \sum_{k\neq i}^{N} y_{ik}^E + \sum_{k\neq i}^{N} y_{ki}^E \delta_{ki} \right) \ \ \forall i \text{ exporting to } j \tag{7}$$

[2]In this model, global consumption can be less than global endowment: $C \equiv \sum_i c_i = \sum_i (c_i^D + c_i^I) = \sum_{i=1}^{N} \left(y_i^D + \sum_{j\neq i}^{N} y_{ji}^E \delta_{ji} \right) \leq Y$.

[3]Note that we do not have the ceiling inequality constraint on exports; when you receive imports you can in fact export more than your own endowment.

[4]For our simulations, we evaluate all possible trading network shapes, required to solve the the full system in (6).

[5]Note that if there are heterogeneous preferences ($\gamma_i \neq \gamma_j$), then the FOC cannot be linearly transformed.

The system of equations in (7) can be solved for optimal exports, which then allows us to calculate optimal consumption c_i^*. In Appendix A, we show that the optimal consumption is linear in endowments, meaning $c_i^* = \sum_k y_k \omega_k^i$, where the weight $\omega_k^i \geq 0$ is an agent-k-specific function of parameters $(\Delta_{ij}, \delta_{ij})_{\forall i, j \neq i}$. Using this and equation (7) rewritten as $\Delta_{ij} c_j = c_i$, we can then express agent j's optimal consumption in terms of i's solution:

$$c_j^* = \frac{1}{\Delta_{ij}} \cdot \left(\sum_k y_k \omega_k^i \right) \text{ if } i \text{ exports to } j \tag{8}$$

Thus if two agents trade, their consumption share a common element that is scaled by the direct trade weight. Now consider an agent k that shares a trading partner with i (say k also exports to j) but i and k do not trade. In this case, there is no FOC directly pairing i and k, but they are indirectly linked via an intermediary (agent j). Define the indirect trade weight $\Delta_{i \to k}$ for two agents that are not directly linked: $\Delta_{i \to k} c_k^* = c_i^*$, where $\Delta_{i \to k} = \Delta_{ik}$ if the two are directly linked.[6] To finish characterizing the model and derive a general formula for $\Delta_{i \to k}$, we utilize network theory.

3.2 Trade Networks

The extensive part of our trade model, meaning whether two agents have any trade relationship, can be thought of as a directed graph or network. The graph represents each agent as a node (or vertex), and if two agents trade, they are connected to each other with an edge (or link). Since trade can be asymmetric, the graph is directed. If there is an undirected path connecting all agents in our directed graph, then our network is weakly connected and trade is "global". Otherwise there are "trading islands", called components: connected induced-subgraphs that are disjoint from other components. We will refer to each (weak) component of a disconnected graph as the "trade network", and describe the network within the subgraph. Thus, the universe of agents is the graph and each "trade network" is a subgraph, with autarkic countries being isolated nodes within the graph which are not connected to any subgraph. Define the trade network by the ordered pair

[6] In the example, i is linked to k via $c_i = c_k (\frac{\delta_{ij} \alpha_k}{\delta_{kj} \alpha_i})^{-1/\gamma}$, and $\Delta_{i \to k} = \Delta_{ij} / \Delta_{kj}$. In general $\delta_{ik} \neq \frac{\delta_{ij}}{\delta_{kj}}$.

$\mathcal{T} = (\mathcal{N}, \mathcal{S})$, where $\mathcal{N} = \{1, 2, \ldots, N\}$ is the set of nodes and $\mathcal{S}$ is the shape or adjacency matrix of directed links.

To link the previous section with the current discussion, first fix an exporter $\bar{i} \in \mathcal{N}$ within $\mathcal{T}$, called the index agent. Then, using equation (8), we can express $c_j^* \ \forall j \in \mathcal{N}$ in terms of $c_{\bar{i}}^*$ with a common term within network $\mathcal{T}$: $\mathcal{A}_{\mathcal{T}}^* \equiv c_{\bar{i}}^* = \sum_{k \in \mathcal{N}} y_k \omega_k^{\bar{i}}$, as shown in equation (9). We call $\mathcal{A}_{\mathcal{T}}^*$ the aggregate risk function, a concept originating from Bradford et al. (2022), which we generalize.

$$c_j^* = \frac{\mathcal{A}_{\mathcal{T}}^*}{\Delta_{\bar{i} \to j}} \quad \text{if } \bar{i}, j \in \mathcal{N} \tag{9}$$

Thus a network member's consumption is just the ratio of a network-specific function of endowments and parameters and an agent-specific function of parameters.[7] Thus, endowment shock based consumption risk is shared at the network level (rather than agent level), with heterogeneity in consumption being driven solely by exogenous parameters such as preferences, Pareto weights, and, trade costs. Furthermore, this risk is shared by everyone throughout the entire network, not just with one's direct trading partners.

Our main empirical specification is based on equation (9), which requires calculating the indirect trade weight; we derive a formula for the shortest-path $\Delta_{\bar{i} \to j}$ utilizing the observed network shape $\mathcal{S}$. Let $\mathcal{P}_{ik}$ be the collection of links for the geodesic (shortest-path) undirected path from i to k, meaning the minimum link-distance from node i to node k from the matrix $\tilde{\mathcal{S}} = \mathcal{S} + \mathcal{S}'$; in our case, this symmetric matrix is exactly the undirected adjacency matrix.[8] Thus $\mathcal{P}_{ik}$ is a sequence of elements of $\tilde{\mathcal{S}}$ along the undirected path.[9] For example, to get from i to l, suppose the smallest distance using $\tilde{\mathcal{S}}$ goes through links (i, j), (j, k), and (k, l). The indirect trade weight $\Delta_{i \to k}$, shown in equation (10), is a function of all of the trade costs δ along such a path. How these intermediary trade costs affect the end result is based on whether the links are export links ($\mathcal{S}(i, j) = 1$) or

[7] Note that it is superfluous to say "agent-network-specific" because if an agent is connected to two separate "networks", then in fact they are all one network.

[8] No symmetric trade is an outcome of our model and so there is no overlap between $\mathcal{S}$ and $\mathcal{S}'$; thus we do not need to divide such entries by 2, which would be required in the general case.

[9] A directed path only follows arrows in their specific direction; for our purposes, the indirect links that "connect" two non-trading network-partners are undirected via the first order conditions. The direction determines the δ order.

import links ($\mathcal{S}'(i,j)=1$).

$$\Delta_{i\to k} = \prod_{(r,s)\in\mathcal{P}_{ik}} \frac{(\Delta_{rs})^{\mathcal{S}(r,s)}}{(\Delta_{sr})^{\mathcal{S}'(r,s)}} = \left(\frac{\alpha_k}{\alpha_i}\prod_{(r,s)\in\mathcal{P}_{ik}} \frac{(\delta_{rs})^{\mathcal{S}(r,s)}}{(\delta_{sr})^{\mathcal{S}'(r,s)}}\right)^{-1/\gamma} \tag{10}$$

The exponent terms are elements of the adjacency matrices and are either 1 or 0. The first [second] intermediaries (in each pair) who export [import] are in the numerator and the opposite intermediaries are in the denominator. See proof of Proposition 2 in Appendix A. Note that $\mathcal{P}_{ik}$ are in general not unique; when calculating them, we order agents by their closeness to the index such that the path chosen is the geodesic path that passes through the most central agents.

3.3 Illustrative Examples

We illustrate the optimal consumption solution in our model through some examples. Consider first the case of a world with just three agents. Consider a particular trading pattern between the three agents: agent 1 exports to agents 2 and 3, and agents 2 and 3 do not trade with each other. Thus $y^E_{12} > 0$, $y^E_{13} > 0$, and $y^E_{32} = y^E_{23} = 0$, but also $y^E_{21} = y^E_{31} = 0$; a reverse trade flow between agents is suboptimal, meaning $y^E_{ij} y^E_{ji} = 0$ (see Appendix A). Then, we can solve the restricted system of equations from (7).

One interesting feature of the planner's solution is that it demands marginal utilities to be proportional among agents. In the three agent case, we can write the FOC's for agents 2 and 3 as $\Delta_{12}c_2 = c_1$ and $\Delta_{13}c_3 = c_1$. These show that the planner's solution will make agent 1's marginal utility to be proportional to 2 and 3. So in that sense, 1 is directly linked with 2 and 3. It also is straightforward to see that the marginal utility of 2 will also be proportional to 3, since, $c_2 = \dfrac{\Delta_{13}}{\Delta_{12}} c_3$. Although 2 and 3 do not trade with each other directly, they are indirectly linked with each other via agent 1. The implication of these direct and indirect linkages on risk sharing can be seen by solving for the optimal consumption of each agent.

$$c_1^* = \Delta_{13}\Delta_{12}\frac{\delta_{12}\delta_{13}y_1 + \delta_{13}y_2 + \delta_{12}y_3}{\delta_{12}\Delta_{12} + \delta_{13}\Delta_{13} + \delta_{12}\delta_{13}\Delta_{12}\Delta_{13}} \tag{11}$$

$$c_2^* = \Delta_{13}\frac{\delta_{12}\delta_{13}y_1 + \delta_{13}y_2 + \delta_{12}y_3}{\delta_{12}\Delta_{12} + \delta_{13}\Delta_{13} + \delta_{12}\delta_{13}\Delta_{12}\Delta_{13}} \tag{12}$$

$$c_3^* = \Delta_{12}\frac{\delta_{12}\delta_{13}y_1 + \delta_{13}y_2 + \delta_{12}y_3}{\delta_{12}\Delta_{12} + \delta_{13}\Delta_{13} + \delta_{12}\delta_{13}\Delta_{12}\Delta_{13}} \tag{13}$$

In this simple example, the optimal consumption of an agent is dependent only on a weighted average of their endowments or the aggregate risk. This is true despite the fact that agents 2 and 3 do not trade directly with each other. Agents 2 and 3 are linked with each other via their link with agent 1. Hence, they are part of the global network and share the same aggregate risk.

To see how a change in the trading pattern influences optimal consumption, we consider another example. Assume now that agent 1 exports to only agent 2 and agent 3 is autarkic. In that case, maintaining our assumption of no reverse trade flows, we have $y_{12}^E > 0$, $y_{21}^E = 0$ and $y_{31}^E = y_{13}^E = y_{32}^E = y_{23}^E = 0$. Given these assumptions, the optimal consumption for each agent is:

$$c_1^* = \Delta_{12}\frac{\delta_{12}y_1 + y_2}{\delta_{12}\Delta_{12} + 1}, \quad c_2^* = \frac{\delta_{12}y_1 + y_2}{\delta_{12}\Delta_{12} + 1}, \quad c_3^* = y_3 \tag{14}$$

So even with the changed network structure, we get the same result that optimal consumption is dependent only on the aggregate risk. But the aggregate risk function is specific to the trading network which now comprises of agents 1 and 2. This can be seen by comparing the aggregate risk expression for agent 1 in the first and the second example. Since agents 1 and 2 are both part of one network they share the same aggregate risk function. Agent 3 is the only member of its own network and hence its aggregate risk function is its own endowment.[10]

Two things are worth noting from the comparisons of these two examples. One, although linked agents share the same aggregate risk, the exact expression for aggregate risk will vary with the network structure. Second, all agents within a network will share the same aggregate risk but unconnected trade networks will have different expressions for aggregate risk.

[10] See Appendix A.4 for the derivation of optimal consumption solutions of these two examples.

4 Simulations

How do trade flows and the overall network change as trade costs and endowments shocks change? First, we consider the effects of a decrease in transaction costs, meaning an increase in δ. The model of Jack and Suri (2014) is a simplified version of our model with lump-sum transaction costs; they find that a cost reduction results in better consumption smoothing across shocks, increased number of transactions, and increased number of network participants. We show that this result holds in our generalized framework with iceberg costs and with a larger number of players.

In the example shown in Figure 1, the endowments are $[42, 81, 55]$ and all countries have identical trade costs and Pareto weights. With high trade costs, there is no trade. With a slightly lower cost, agent 2 exports to 1. With a further decrease in trade cost, agent 2 increases its amount to 1, and eventually exports to agent 3. In a larger example of five countries with endowments $[100, 50, 40, 90, 72]$, we see the trade networks growing and shifting as more countries start to trade as costs continually decrease. In terms of consumption smoothing, Figure 2 shows that as trade costs decrease, the social planner is able to smooth the endowment shocks such that consumption equalizes across countries. Increasing trade decreases inequality, but also decreases total consumption for any strictly positive trade cost level.

It is also interesting to see how adding new trading partners changes global risk sharing differentially. Consider 5 countries with endowments, $[100, 20, 20, 100, 22]$, all with equal Pareto weights and equal (small) trade costs. In this case, trade is efficient from the high endowment to low endowment countries. Consider five different trading scenarios displayed in Figure 3. First we have full autarky, then bilateral trade between 1 and 2, then additional isolated trade between 4-5, followed by letting 1 trade with 3 as well, and finally, connecting all five countries by having 4 trade with 3. How do these different scenarios affect welfare?

In Figure 4 we show how the previous trade scenarios affect the standard deviation of consumption [on the left] and global utility [on the right]. Not surprisingly, increasing trade reduces consumption inequality and promotes global utility. This is even the case when going from scenario 4 to scenario 5: note that in this scenario, all countries are still trading in scenario 4 but in scenario 5, the two distinct trading islands are linked by agent 3. Even in this case, risk sharing is improved. Thus a single linkage between two disparate networks will lead to risk sharing across all countries involved. Note however that the effect is smaller than when adding a completely autarkic agent to a network: the slope on utility increasing from scenario 4 to 5 is smaller than the rest. Again this is not surprising as we are simply allowing for smoother risk sharing.

Next, we consider the number of trades and players as the trade costs decrease; this is shown in Figure 5. Both the number of players in the network and the number of total trading increases. Finally, we consider the correlation between consumption and production, the main measure of risk sharing. As trade costs decrease, this correlation decreases, as shown in Figure 6.

5 Structural Risk-Sharing Test

Recall that our goal is to run a risk-sharing test controlling for trade costs and network structure. We can use our model to derive such a test. We discuss the main risk-sharing regression, the bias associated with ignoring trade costs, extensions, and network dependent inference.

5.1 Risk-Sharing Test Equation from Model

Consider the optimal consumption for a given time period t as defined in equation (9). Assuming that the observed consumption c_{jt} is measured with exponential multiplicative (zero mean) error compared to the model consumption: $c_{jt} = \exp(\varepsilon_{jt})c^*_{jt}$ and taking logs, equation (9) can be expressed as:

$$\ln(c_{jt}) = -\ln(\Delta_{jt}) + \ln(\mathcal{A}^*_{\mathcal{T}_t t}) + \varepsilon_{jt} \tag{15}$$

The first component $\Delta_{jt} \equiv \Delta_{\bar{i}_t \to j,t}$, is an agent-year specific parameter (relative to an index agent which changes by network and year, denoted with $\bar{i}_t$) and is a function of trade costs, Pareto weights, and preferences. In particular, it contains information about agent j and the index agent $\bar{i}$. If j does not trade directly with $\bar{i}$, then Δ_{jt} also contains information on all agents that indirectly link j and $\bar{i}$ based on the network shape $\mathcal{S}_t$. The indirect trade weight is indexed by j and t as it will be different for each agent based on its position in the network and will also change with time since we allow networks to change overtime.

The second component $\mathcal{A}^*_{\mathcal{T}_t t}$ is the aggregate risk function, which is the common element to consumption allocation of all agents in a given network, and thus varies at the network-year level. Network $\mathcal{T}$ is indexed by time, $\mathcal{T}_t$, to emphasize the fact that networks can change every period; both membership $\mathcal{N}_t$ and shape $\mathcal{S}_t$ affect consumption each year.

The empirical implication of the model is that, conditional on indirect trade weight and the aggregate risk function, consumption should be independent of own endowment. To arrive at a test, we follow the literature and add endowment as a covariate in equation (15).

$$\ln(c_{jt}) = -\ln(\Delta_{jt}) + \ln(\mathcal{A}^*_{\mathcal{T}_t t}) + \eta \cdot \ln(y_{jt}) + \varepsilon_{jt} \tag{16}$$

Equation (16) defines a misspecification test on our model. Under perfect risk sharing, $\eta = 0$. If $\eta \neq 0$, consumption is correlated with production and therefore the complete risk sharing hypothesis is rejected. Under that scenario, $(1-\eta)$ gives us a measure of the degree of risk sharing achieved in the network (Asdrubali et al., 1996; Crucini, 1999; Crucini and Hess, 1999; Asdrubali et al., 2020; Bradford et al., 2022). It is important to reiterate that Δ_{jt} and $\mathcal{A}^*_{\mathcal{T}_t t}$ are functions of parameters and hence will be estimated simultaneously with η.[11] We also add a constant to equation (16) to better justify the zero-mean assumption for ε_{jt}.

5.2 Ignoring Trade Costs and Bias

To see how equation (16) compares with the benchmark of no transactions costs, we derive the test assuming $\delta_{ijt} = 1$. The canonical risk-sharing regression is equivalent to the model with $\delta_{ijt} = 1$ for all trading pairs. This results in the following regression:

[11] Recall that y_{it} is contained in $\mathcal{A}^*_{\mathcal{T}_t t}$ and so separate identification of η relies on nonlinearity of $\ln(\mathcal{A}^*_{\mathcal{T}_t t})$ in $\ln(y_{jt})$; this is satisfied as $\mathcal{A}^*_{\mathcal{T}_t t}$ is linear in network participant endowments (see Appendix A). This restriction is implicit in other risk-sharing tests as well when controlling for the aggregate shock.

$$\ln(c_{jt}) = (1/\gamma)\ln(\alpha_j) + T_t + \eta \cdot \ln(y_{jt}) + \tilde{\varepsilon}_{jt} \tag{17}$$

Equation (17) is the standard test of risk sharing. Since there are no trade costs, the trading network is global and aggregate risk, T_t, is only indexed by time. Also, each Δ_{jt} just collapses to the Pareto weights. The standard tests, given in (17), are generally estimated using a two-way fixed effects approach where time fixed effects control for aggregate risks and unit fixed effects control for the Pareto weights. The two-way fixed effects approach is not generally appropriate for our case as we allow networks to change every year in an unrestricted manner. This implies that both Δ_{jt} and $\mathcal{A}^*_{\mathcal{T}_t t}$, which depend on the network shape and the position of the agent in the network, will vary across individuals and over time. Aggregate risk and indirect trade weight can only be subsumed into a network-year and a unit fixed effect when the networks are observed and are stable over time.[12]

If in reality we have $\delta_{ijt} < 1$, then the estimate of η from regression (17) is biased. To see how trade costs can influence the estimate of risk sharing, we decompose the trade weight term using the formula from equation (10) as $-\ln(\Delta_{jt}) = (1/\gamma)\ln(\alpha_j/\alpha_{\bar{i}_t}) + (1/\gamma)\ln(F(\boldsymbol{\delta}_{\mathcal{N}_t}|\mathcal{S}_t))$. The F term is a network specific function of trade costs for exporters and importers along country j's geodesic path to the index country in the network: $F(\boldsymbol{\delta}_{\mathcal{N}_t}|\mathcal{S}_t) = \prod_{(r,s)\in\mathcal{P}_{\bar{i}_t,j}} (\delta_{rst})^{\mathcal{S}_t(r,s)}/(\delta_{srt})^{\mathcal{S}'_t(r,s)}$. This captures the country specific trade cost network effects which limit risk sharing. The trade costs of each country along the path between a country and the central trading country (the index) is an intensive measure of how costly trade is for a country, which affects their consumption level.

The extent of the bias is based on the covariance $\text{Cov}(\log(y_{it}), \ln(F(\boldsymbol{\delta}_{\mathcal{N}_t}|\mathcal{S}_t)))$, which is likely non-zero for two reasons. First, the (optimal) network shape is endogenously determined by the social planner and hence is a function of the endowment distribution. The social planner wants to re-distribute endowment but takes into account the efficiency per transaction; very inefficient linkages are less likely to be optimal. In this case, inefficient countries are more likely to be left

[12]If every agent is linked directly or indirectly to every other agent (meaning no isolated networks) then global trading is one network and the aggregate risk function varies only at the yearly level: $\mathcal{A}^*_{\mathcal{T}_t t} = \mathcal{A}^*_t$. In this case, it can be subsumed by a year fixed effect.

out of the network; if a low endowment country is surrounded by inefficient countries, then their trade efficiency is likely low and they may be left out.

Second, the exogenous characteristics that affect trade costs may correlate with endowments. Low endowment countries may have high barriers to trade, which exacerbates their problem as they can benefit the most from sharing risk, but are least able. Countries with efficient trading cost network effects (those whose linkages are mostly exporters with low costs) may have higher endowments; in this case the covariance is positive and there is upward bias in η.

Given that the network shape is an equilibrium function of the endowments, and F is a highly nonlinear function of the shape and trade costs, the bias does not have a straightforward expression. We illustrate the bias with a simulated example.[13] For the countries in each network per year, we regress the consumption on production, initially alone, and then controlling for the aggregate risk function and trade costs. The results are presented in Table 1. The coefficient on production is akin to the risk sharing test; adding in the aggregate risk reduces the coefficient and significance by a small amount, but controlling for trade costs causes the production coefficient to be small and noisy. Thus, if the model captures the data generating process, controlling for trade costs can drastically change the risk sharing prediction.

5.3 Heterogeneous Preferences

As demonstrated in Schulhofer-Wohl (2011), it is more realistic to have agent specific preferences γ_i instead of a constant γ. In this case, the FOC still has an exponential proportional form $c_i^{\gamma_i} = c_j^{\gamma_j}(\alpha_j \delta_{ij}/\alpha_i)$. Just as before, we can, without loss, define $\mathcal{A}^*_{\mathcal{T}}$ to be agent $\bar{i}$'s consumption. Then by the FOC, $c_j = \tilde{\mathcal{A}}^{\gamma_{\bar{i}}/\gamma_j}/(\tilde{\Delta}_{\bar{i}\to j})^{-1/\gamma_j}$, where $\tilde{\Delta} \equiv \Delta^{-\gamma}$. Then the risk-sharing regression has scaled model terms per country:

$$\ln(c_{jt}) = \frac{\ln(\tilde{\Delta}_{jt})}{\gamma_j} + \frac{\gamma_{\bar{i}_t} \ln(\mathcal{A}_{\mathcal{T}_t t})}{\gamma_j} + \eta \cdot \ln(y_{jt}) + \varepsilon_{jt} \tag{18}$$

[13] We simulate the model for a 4 country case with homogeneous Pareto weights and fluctuating trade costs (random uniform draws averaged around 0.6) across 50 time periods.

Note that $\gamma_{\bar{i}_t}$ can change every year depending on the network structure; a previous member chosen for the index could be absent in the network the following year, and thus the identity of $\bar{i}_t = \bar{i}(\mathcal{N}_t)$ changes. Note the presence of the index country's γ term as a coefficient on $\mathcal{A}_{\mathcal{T}_t t}$. This is a network-specific term and thus even if one omits trading costs from the model, the correct model specification with heterogeneous preferences uses trade network information.

5.4 Estimation

The estimation steps are as follows. First we calculate the network shape $\mathcal{S}_t$ for each year using the observed set of bilateral trading partners. We set the network to be undirected and calculate $\mathcal{S}_t$ for each isolated network.[14] For each isolated network, we define the index based on the highest betweenness centrality measure for all countries in the network. We then calculate the geodesic (shortest) path between all countries in the network and the index. We generate the path of countries that links each country to the index. This allows us to calculate the indirect trade weight for each country j as Δ_{jt}, with the added step of noting which country in each link along the path is an exporter or importer.

To estimate equation (16) we use non-linear least squares with Logistic trade costs, $\delta_{ijt} = \exp(X_{ijt}\beta)/(1 + \exp(X_{ijt}\beta))$. To directly calculate $\mathcal{A}^*_{\mathcal{T}_t t}$, one can use the index country's consumption or solve the model for each ω_{it} (see Section 3.2). If the data allows, one can also capture $\mathcal{A}^*_{\mathcal{T}_t t}$ with a network-year indicator, and we use this approach in our application as we have a global network structure every year.[15] We set the Pareto weights α_{it} to the population of each country, scaled to sum to 1. For a given parameter vector: $(\gamma, \beta, \eta, T_t\ \forall t)$, we calculate Δ_{jt} for all countries and years, with the exact formula shown below. Then we plug Δ_{jt} into equation (16), calculate $\sum_{jt} \varepsilon^2_{jt}$, and iterate using modified Newton's method.

It is illustrative to expand $\ln(\Delta_{jt})$, as shown in equation (19). To simplify the notation, let

[14] We utilize the *nwcommands* package in Stata (Grund, 2015), which allows the user to format data into a network and calculate various network properties.

[15] In Appendix B.1 we show how to incorporate non-trading countries.

$S^1_{jt} = \{r_m | \mathcal{S}_t(r_{m1}, r_{m2}) = 1\}$ and $S^2_{jt} = \{s_n | \mathcal{S}'_t(s_{n1}, s_{n2}) = 1\}$.

$$\ln(\Delta_{jt}) = (-1/\gamma)\ln(\alpha_{jt}/\alpha_{\bar{i}_t t}) + (-1/\gamma)\sum_{S^1_{jt}} \ln(\delta_{(r_{m1},r_{m2})t}) - (-1/\gamma)\sum_{S^2_{jt}} \ln(\delta_{(s_{n2},s_{n1})t}) \tag{19}$$

Next, we can expand δ based on its functional form, as shown in equation (20).[16]

$$\begin{aligned} \ln(\Delta_{jt}) = (-1/\gamma)\ln(\alpha_{jt}/\alpha_{\bar{i}_t t}) + (-\beta/\gamma)\left(\sum_{S^1_{jt}} X_{(r_{m1},r_{m2})t} - \sum_{S^2_{jt}} X_{(s_{n2},s_{n1}),t}\right) \\ + (-1/\gamma)\left(\sum_{S^2_{jt}} \ln(1+\exp(\beta X_{(s_{n2},s_{n1}),t})) - \sum_{S^1_{jt}} \ln(1+\exp(\beta X_{(r_{m1},r_{m2}),t}))\right) \end{aligned} \tag{20}$$

5.5 Network Dependent Inference

Given the network structure of trade, we allow the errors (ε_{it}) in the risk sharing equation to exhibit general network dependence. This affects inference since the usual heteroskedasticity and autocorrelation robust (HAC) standard errors, which assume independence between cross sectional units, will be incorrect for the parameters of the regression equation in (16). To account for network dependence, we use the results in Kojevnikov et al. (2021) (KMS thereafter) to arrive at a consistent network-HAC variance estimator. The latter allows covariances between any two nodes to decay relatively fast as a function of the network-distance, $D_N(i,j) = d \in \mathbb{R}$, which is defined on $\mathcal{T}_N$ to be the length of the shortest-path between nodes, $i, j \in \mathcal{N}_N$.[17]

With network dependent data, the limit theorems needed to obtain asymptotic results including the variance-covariance matrix only hold if the network is not too dense. In particular, we need sufficient conditions which restrict not only the strength of network dependence between nodes but also the number of neighbors at a given distance, d, which ultimately depends on the network shape. To limit the strength of dependence, we assume that $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, \ldots, \varepsilon_{iT})' \in \mathbb{R}^T$ is condition-

[16] Note that this expression contains a summation of bilateral trade costs differences. Thus for X_{ijt} that do not vary across bilateral pairs, such as a time trend or constant, differential number of exporter/importer links identifies them. For a country that directly trades with the index, they only have a single trade cost in their expression: $\log(\delta_{\bar{i}jt})$.

[17] The indexing by N is to emphasize the fact that asymptotic theory follows due to the size of the network getting large, i.e. $N \to \infty$.

ally ψ-dependent given $\mathcal{C}_N$ with dependence coefficients given by $\lambda_N = \{\lambda_{N,d}\}_{d\geq 0}$, $\lambda_{N,0} = 1$. The notion of ψ-dependence is the primary way in which KMS measure the strength of covariance between two sets of nodes that are at least d-distance apart. The sequence of dependence coefficients λ_N provide an upper bound on such covariance terms where this sequence approaches zero as d increases.

The exact conditions needed to argue that the Law of Large Numbers and the Central Limit Theorem apply and consistent estimation of the variance is possible are given in Appendix B.2. These conditions have been adapted from KMS to fit the current setup.

Then given the regression equation in (39) of the appendix, a consistent estimator for the network HAC variance is then given by $\hat{\mathbf{Q}}_T\hat{\mathbf{\Omega}}_T\hat{\mathbf{Q}}_T$ where the middle (sandwich term) is constructed to be a weighted average of the covariances of node i with its neighbors, j, who are exactly d-*distance* away denoted by the set $\mathcal{N}_N^{\partial}(i,d)$. Formally, $\mathcal{N}_N^{\partial}(i,d) = \{j \in \mathcal{N}_N : D_N(i,j) = d\}$.

$$\hat{\mathbf{\Omega}}_T = \frac{1}{NT^2}\sum_{d\geq 0}\mathcal{K}_N(d)\left[\sum_{i\in\mathcal{N}_N}\sum_{j\in\mathcal{N}_N^{\partial}(i,d)}\mathbf{h}_i^{(1)}(\hat{\boldsymbol{\theta}})\hat{\boldsymbol{\varepsilon}}_i\hat{\boldsymbol{\varepsilon}}_j'\mathbf{h}_j^{(1)}(\hat{\boldsymbol{\theta}})'\right] \tag{21}$$

where $\mathbf{h}_i^{(1)}(\hat{\boldsymbol{\theta}})$ is the jacobian of $\mathbf{h}_i(\hat{\boldsymbol{\theta}})$ (see equation (39) in the appendix) and $\mathcal{K}_N(d) = \mathcal{K}(d/b_N)$ is the kernel used to weight the sample covariances as a function of the network distance. In particular, we use the Bartlett kernel i.e. $\mathcal{K}(x) = (1-|x|)\cdot\mathbb{1}\{|x| < 1\}$. The term b_N is the bandwidth parameter which depends on N and is used to truncate how many such covariance terms appear in the double sum. In our case, we use $b_N = \lfloor (N)^{1/3}\rfloor$ where $\lfloor\cdot\rfloor$ is the floor function.

Another thing to note is that while KMS do not impose any restriction on $\mathbf{\Omega}_T$, we assume that the off-diagonal elements of $\mathbf{\Omega}_T$ are zero. These terms correspond to $\mathbb{E}(\varepsilon_{it}\varepsilon_{jt'}|\mathbf{w}_{it},\mathbf{w}_{jt'})$, which are the error covariances at lags and leads. In other words, we assume that there is no dependence between node i and its network neighbors across different time points. Therefore, we only allow network dependence between contemporaneous errors, $\mathbb{E}(\varepsilon_{it}\varepsilon_{jt}|\mathbf{w}_{it},\mathbf{w}_{jt})$, and assume that such cross-sectional dependence becomes substantially weaker as one considers increasingly distant neighbors in the network.

6 Empirical Application

6.1 Data

We implement our risk-sharing test on data from global markets of rice, wheat, and maize. Data on consumption, production, exports, imports, and other aggregates at the country level are extracted from the Food and Agriculture Organization's (FAO) 'Food Balance Sheets' database (FAOSTAT, 2014). The FAO food balance sheets provide country-level time series of production, domestic supply, food consumption, stock variations, and trade of major agricultural commodities from 1960 to 2013. This enables us to construct large unbalanced panels.[18]

FAO also has data on trading pair-level trade flows for countries. This data is available from 1986 to 2013.[19] The dataset lists origin and destination country pairs with export and import amounts to the destination from the origin country. Our main usage of the trade-flow data is just the binary information on whether two countries have a trading relationship in a given year and in what direction. We do use the actual quantities to trim the dataset, as discussed in Section 6.3. To measure the trade flow, we use the export amount when measuring exports from origin to destination (with some exceptions).[20]

For estimation, we combine the FAO trade-flow data with data from the Centre d'Etudes Prospectives et d'Informations (CEPII) gravity dataset. The CEPII contains a set of variables that are generally used by researchers to understand the determinants of trade flows between countries

[18]These panels are unbalanced because of missing data for some years for some countries.

[19]For some year and country pairs, we observe exports not matching imports on a non-trivial level. The FAO reports this can occur for a variety of reasons. One explanation is "exported quantities could be destroyed or lost en route due to accidents, weather conditions". Note that our definition of trade-cost includes "lost at sea" as explaining part of the gap between imported consumption and exported endowment amount in the model. We do not believe that the trade cost can simply be identified by comparing the gap between the FAO reported export/import amounts because our definition also includes other possible reasons.

[20]The dataset is not balanced and thus we must use the import amount when measuring exports from destination country to origin country in certain cases. To be specific: there are cases where country A exports to country B but country A is only listed as a destination country with an "import" amount that country B declares from A (which was the export from A to B). Thus for these cases, we generate the missing row for an A to B export amount using the import reported to be received by B from A.

(Conte et al., 2020).[21] We use some of the variables in CEPII dataset to model the per unit bilateral trade costs between countries. To capture trade frictions in terms of tariffs and other indirect trade barriers, we also use data on Nominal Rates of Assistance from the Distortions to Agricultural Incentives database (Anderson and Signe, 2013). Finally, trade networks may also be influenced by relative productivity and comparative advantage. Data on indicators of a particular crop's productivity based on agro-climatic conditions and natural endowments is available from the FAO's Global Agro-Ecological Zones database (Fischer et al., 2021). This dataset provides country-level simulated potential yields from agronomic models. We use these simulated potential yields to calculate relative productivity for country pairs.

6.2 Trade Networks

Figure 7 shows the network plots for the three commodities. The nodes denote countries and the edges denote the trade linkages. Since these linkages would vary over time depending on the state of the world and the production realizations, we show networks for three time periods. As can be seen from Figure 7, the network structure varies both across commodities and across time. Our estimation routine would explicitly take the network structure of trade into account while testing for risk sharing.

Another feature of networks is the number of connections or the *degree* each node has in the network. The degree distribution of a network gives us some information about the type of network. In our case, the networks of all three commodities have a left-skewed degree distribution. This implies that most nodes have a small number of connections, but a few nodes have a very high number of connections. This means that trade in these commodities is dominated by one or a few countries. Figure 9 plots the average number of trade partners per country for each year in our dataset. Across all three commodities, we observe a rising number of trade partners over time. This indicates that trade linkages have possibly grown on account of greater connectivity and

[21] The CEPII provides trade pair level data on population weighted distance, time difference, whether the pairs were common colonies, common language, religious proximity, etc. It also provides data on macroeconomic variables like national income, population, total trade flows, and membership of regional trade agreements and the WTO.

lowering of freight and transaction costs.

6.3 Structural Test Estimates

We estimate equation (16) for 1986-2013. Our main sample includes countries with a population of at least 10 million and we drop small trading amounts (we consider an expanded sample for robustness).[22] Also, we ignore the smaller amount in any reciprocal trade of the same commodity in the sense that we do not consider it as a link in that commodity's trade network; this is an equilibrium outcome of the model and fits the data in most cases. Our standard errors are derived using the network HAC estimator given in (21) which allows for general network dependence in the errors of the risk sharing equation as discussed in Section 5.5. The results are displayed in Table 2 for the commodities maize, wheat, and rice respectively. All specifications include year fixed effects to control for aggregate shocks. This is feasible as we have a global network each year. This implies that the aggregate shock will be common to all (trading) countries and be distinct each year. As discussed, this is generally not the case, but happens to be true in this particular sample.

The baseline specifications in Table 2 show the results from the risk-sharing test without accounting for transaction costs. We observe that own production is significant in the baseline regression in all three commodities, implying a rejection of the perfect risk-sharing hypothesis.[23] The wheat market is closest to achieving perfect risk sharing followed by rice and maize. In particular, for maize, the baseline production coefficient η is 0.303 with a standard error of 0.133. For wheat, η is 0.180 (0.042) and for rice, η is 0.241 (0.053).

In the next set of specifications, we introduce constant transaction costs in estimation. The "constant trade cost" varies across each country-network pair as even if all countries have identical costs, the indirect Δ paths connecting them to the index country are different; some are longer/shorter, some have more exporters/importers, and these affect Δ even with constant δ. The

[22] We use separate cutoffs per commodity as the mean trading levels vary by an order of magnitude: we cut off trade levels of 800 for maize, 20,000 for wheat, and 500 for rice (each is below 10% of the mean). We tried various cutoffs based on rules like % of total imports, and the relative results (within commodities across models) were not sensitive.

[23] Relative results do not change if we include non-trading countries, only the coefficient on own production gets shifted down by running regression (36) instead.

pure effects of the network shape are captured here, via the relative number of exporters to importers along the path between a country and the index. Overall, we observe a minor decline in the estimated coefficient on production in this specification. For maize, the trade-cost-constant specification yields an estimate on η of 0.301 (0.133), a trivial change to the baseline. For wheat, η is now 0.173 (0.046). Relative to the baseline, the coefficient decreases by 3.7% and the standard error (SE) increases by 9.1%. The fit, as measured by the correlation between the outcome and predicted value, increases from 0.748 to 0.751. For rice, η is now 0.235 (0.052). The coefficient decreases by 2.4% and the SE decreases by 2.5%. The fit goes from 0.764 to 0.766.

In the third column for each commodity, we introduce transaction cost covariates inside δ, which adds link specific heterogeneity in trade costs. We consider log distance between the two countries and various economic/cultural similarity indices for the country pairs, which are standard variables in trade gravity models. Specifically, we include common language, time zone difference, GATT/WTO membership by destination and origin, relative (historical) productivity by country-pair, and rate of assistance from the World Bank "Distortion from Agricultural Incentives" Database.[24] We again observe a decline in η for all three commodities.

For maize, the trade-cost-controls specification yields an estimate on η of 0.273 (0.105). This is a 9.9% decline from the baseline and a 20.5% decrease in its standard error. The fit of the model increases from 0.715 to 0.745. For wheat, this specification yields a coefficient of 0.170 (0.047). The coefficient decreases by 5.7%, the SE increases by 12.1%, and the new fit is 0.755. Finally, for rice, the estimate is 0.218 (0.042); this is a 9.6% decrease in the production coefficient and a 21.4% decrease in the standard error. The new fit is 0.790. Overall, the variables included are not individually significant. This is in part due to how they are included: differentially aggregated per trading-pair along the country's path to the index. However, the downstream effects on consumption via the network structure with link specific trade cost variation does affect the production coefficient across various specifications.

[24] Some covariates are highly correlated for certain commodity networks and the problematic variables are dropped.

6.4 Heterogeneous Effects

Table 3 reports estimates from the heterogeneous risk preference model in equation (18) for baseline and trade cost specifications for all three commodities.[25] These regressions include country fixed effects in all specifications. We compare the baseline to the trade-cost-constant specification as the trade cost controls were not well estimated separately from the country-level slopes. Overall the results are noisier for production, consistent with Schulhofer-Wohl (2011).

For maize, the baseline model yields a production coefficient η of 0.255 (0.137), with the standard error in parentheses. The trade cost specification yields η equal to 0.1711 (0.08845). This is a 33.1% decrease in η and a 35.3% decrease in its standard error.

For wheat, the baseline η is 0.11481 (0.038932), and trade cost yields 0.11695 (0.066531). This is a 1.86% increase in η and a 71.9% increase in its standard error. Thus, while the coefficient increases a trivial amount, the predictive power of production decreases significantly. Finally, for rice, baseline η is 0.21078 (0.21541), and trade cost yields 0.18163 (0.20765). This is a 13.8% decrease in the coefficient and a 3.60% decrease in its standard error.

The trade-cost-constant specification under heterogeneous preferences yields larger changes to risk-sharing than in the homogeneous case. This is intuitive as the effects of the network shape are likely dependent on a country's location in the network, which the homogeneous specification averages out.

6.5 Network Distance from Center Effects

To see how centrality of the network affects risk sharing, we consider specifications in which we only include countries that are within a certain distance (network-distance) from the index. When only including agents within a 1-link radius from the index, the risk sharing parameter should be smaller than the whole sample. As one increases the number of links included (meaning going away from the center of the network center), risk-sharing would naturally go down; the less connected countries are not as integrated into the trading network. The effects of controlling for

[25]For countries without variation over time, we normalize their γ to 1.

trade costs should also matter at all levels of the radius. We test these hypotheses for all three commodities.

For maize in Table 4, at $\leq$ 3-links, the coefficient decreases by 9.9% from baseline, at $\leq$ 2-links it decreases by 11%, and at 1-link it decreases by 23%. In addition, at 1-link distance the statistical significance decreases. This indicates that near the center of the network, there is stronger evidence of risk-sharing in Maize after controlling for trade costs. For wheat in Table 5, $\leq$ 3-links yields a -5.7% change, $\leq$ 2-links has -6.6%, and 1-link has -10.4%. For rice in Table 6, $\leq$ 3-links yields a -9.6% change, $\leq$ 2-links has a similar decrease, and 1-link has a -5.6% change.

6.6 Alternative Specifications

As an alternative sample, we consider a 1 million population cutoff and keep all nonzero trade levels. Results are displayed in Table 7 and are weaker (smaller or no difference between baseline and trade costs) and there is less aggregate risk sharing; this is intuitive as many more countries who are less connected (and trade significantly less) are included in this sample.

For functional form robustness, first we consider a probit instead of logit for the trade costs, δ. Second, we consider a more substantial functional form change that still captures the indirect trade effects. We specify the log indirect trade weight as a single logit of the linear difference in covariates across export/import pairs. Across all commodities, we find similar risk-sharing estimates and some small changes to statistical significance of the controls.

7 Concluding Remarks

In this paper, we study the role of transaction costs in risk sharing. The standard model of risk sharing implicitly assumes costless trade across agents and is modeled at the level of consumption. We explicitly model trade flows between agents with iceberg-type transaction costs which vary with each trading pair and are proportional to the volume of trade flows. In this augmented model, link formation is costly and hence trade networks play an important role in risk sharing. This seemingly simple modification adds a non-trivial estimation hurdle. Although the model gives us an empirically testable specification that resembles the standard risk-sharing tests, the existence of the trade cost parameters requires a network approach. We estimate the model using consumption, production, and trade network data from the global food markets of rice, wheat, and maize. To conduct valid inference, we also adjust the standard errors for general network dependence.

Simulations from the model show that declining transaction costs lead to expansion of the network and greater connectedness within the network. This in turn leads to reduced consumption volatility and greater social welfare. We show that the omission of transactions costs and the implied network structure can bias the estimates of risk sharing.

Although the augmented risk-sharing model presented in this paper is general enough to fit into different contexts, estimation requires information on mutual insurance and trade networks. Given the availability of such information, the same model can be used to test for risk sharing under different contexts and settings. Extending the framework to incorporate storage, savings, or intertemporal trade are promising avenues for future research.

References

Ábrahám, Á. and Laczó, S. Efficient risk sharing with limited commitment and storage. *The Review of Economic Studies*, 85(3):1389–1424, 2018.

Altug, S. and Miller, R. A. Household choices in equilibrium. *Econometrica*, pages 543–570, 1990.

Ambrus, A., Mobius, M., and Szeidl, A. Consumption risk sharing in social networks. *American Economic Review*, 104(1):149–82, 2014.

Ambrus, A., Gao, W., and Milán, P. Informal risk sharing with local information. *The Review of Economic Studies*, 2022. ISSN 0034-6527. doi: 10.1093/restud/rdab091. URL Link.

Anderson, K. and Signe, N. Updated national and global estimates of distortions to agricultural incentives, 1955 to 2011. *The World Bank*, 2013. Retrieved from, worldbank.org.

Asdrubali, P., Sørensen, B. E., and Yosha, O. Channels of interstate risk sharing: United states 1963–1990. *The Quarterly Journal of Economics*, 111(4):1081–1110, 1996.

Asdrubali, P., Tedeschi, S., and Ventura, L. Heterogeneity in risk aversion and risk sharing regressions. *Journal of Applied Econometrics*, 34(5):827–835, 2019.

Asdrubali, P., Tedeschi, S., and Ventura, L. Household risk-sharing channels. *Quantitative Economics*, 11(3):1109–1142, 2020.

Attanasio, O. and Krutikova, S. Consumption insurance in networks with asymmetric information: Evidence from tanzania. *Journal of the European Economic Association*, 18(4):1589–1618, 2020.

Attanasio, O., Barr, A., Cardenas, J. C., Genicot, G., and Meghir, C. Risk pooling, risk preferences, and social networks. *American Economic Journal: Applied Economics*, 4(2):134–67, 2012.

Bardhan, P. and Udry, C. *Development Microeconomics*. OUP Oxford, 1999.

Bold, T. and Broer, T. Risk sharing in village economies revisited. *Journal of the European Economic Association*, 19(6):3207–3248, 2021.

Bradford, S. C., Negi, D. S., and Ramaswami, B. International risk sharing for food staples. *Journal of Development Economics*, 2022. Forthcoming: Retrieved from link.

Bramoullé, Y. and Kranton, R. Risk sharing across communities. *American Economic Review*, 97 (2):70–74, 2007.

Bramoullé, Y., Kranton, R., and D'amours, M. Strategic interaction and networks. *American Economic Review*, 104(3):898–930, 2014.

Canova, F. and Ravn, M. O. International consumption risk sharing. *International Economic Review*, pages 573–601, 1996.

Clance, M., Ma, W., and Naraidoo, R. International consumption risk sharing and trade transaction costs. *University of Pretoria Department of Economics Working Paper Series*, 2019.

Cochrane, J. H. A simple test of consumption insurance. *Journal of Political Economy*, 99(5): 957–976, 1991.

Conte, M., Cotterlaz, P., and Mayer, T. The CEPII Gravity Database. *CEPII*, 2020.

Crucini, M. J. On international and national dimensions of risk sharing. *Review of Economics and Statistics*, 81(1):73–84, 1999.

Crucini, M. J. and Hess, G. D. International and intranational risk sharing. *CESifo Working Paper Series*, 1999.

De Weerdt, J. and Dercon, S. Risk-sharing networks and insurance against illness. *Journal of Development Economics*, 81(2):337–356, 2006.

Fafchamps, M. Solidarity networks in preindustrial societies: Rational peasants with a moral economy. *Economic Development and Cultural Change*, 41(1):147–174, 1992.

Fafchamps, M. and Gubert, F. The formation of risk sharing networks. *Journal of Development Economics*, 83(2):326–350, 2007a.

Fafchamps, M. and Gubert, F. Risk sharing and network formation. *American Economic Review*, 97(2):75–79, 2007b.

Fafchamps, M. and Lund, S. Risk-sharing networks in rural philippines. *Journal of Development Economics*, 71(2):261–287, 2003.

FAOSTAT. Food Balance Sheets, 2014. Retrieved from www.fao.org/faostat.

Fischer, G., Nachtergaele, F., Van Velthuizen, H., Chiozza, F., Franceschini, G., Henry, M., Muchoney, D., and Tramberend, S. *Global Agro-Ecological Zones v4–Model documentation*. Food & Agriculture Org., 2021. Retrieved from, fao.org.

Fitzgerald, D. Trade costs, asset market frictions, and risk sharing. *American Economic Review*, 102(6):2700–2733, 2012.

Genicot, G. and Ray, D. Group formation in risk-sharing arrangements. *The Review of Economic Studies*, 70(1):87–113, 2003.

Grund, T. U. Network analysis using Stata, 2015. Retrieved from nwcommands.org.

Jack, W. and Suri, T. Risk sharing and transactions costs: Evidence from kenya's mobile money revolution. *American Economic Review*, 104(1):183–223, 2014.

Kinnan, C. and Townsend, R. Kinship and financial networks, formal financial access, and risk reduction. *American Economic Review*, 102(3):289–93, 2012.

Kojevnikov, D., Marmer, V., and Song, K. Limit theorems for network dependent random variables. *Journal of Econometrics*, 222(2):882–908, 2021.

Laczó, S. Risk sharing with limited commitment and preference heterogeneity: Structural estimation and testing. *Journal of the European Economic Association*, 13(2):265–292, 2015.

Lewis, K. K. What can explain the apparent lack of international consumption risk sharing? *Journal of Political Economy*, 104(2):267–297, 1996.

Ligon, E. Risk sharing and information in village economies. *The Review of Economic Studies*, 65 (4):847–864, 1998.

Ligon, E., Thomas, J. P., and Worrall, T. Informal insurance arrangements with limited commitment: Theory and evidence from village economies. *The Review of Economic Studies*, 69(1): 209–244, 2002.

Mace, B. J. Full insurance in the presence of aggregate uncertainty. *Journal of Political Economy*, 99(5):928–956, 1991.

Mazzocco, M. and Saini, S. Testing efficient risk sharing with heterogeneous risk preferences. *American Economic Review*, 102(1):428–68, 2012.

Munshi, K. and Rosenzweig, M. Networks and misallocation: Insurance, migration, and the rural-urban wage gap. *American Economic Review*, 106(1):46–98, 2016.

Obstfeld, M. Are industrial-country consumption risks globally diversified? *Capital mobility: The impact on consumption, investment and growth*, pages 13–44, 1994.

Obstfeld, M. and Rogoff, K. The six major puzzles in international macroeconomics: Is there a common cause? *NBER Macroeconomics Annual*, 15:390–403, 2001.

Ravallion, M. and Chaudhuri, S. Risk and insurance in village india: Comment. *Econometrica: Journal of the Econometric Society*, pages 171–184, 1997.

Rosenzweig, M. R. and Stark, O. Consumption smoothing, migration, and marriage: Evidence from rural india. *Journal of Political Economy*, 97(4):905–926, 1989.

Schulhofer-Wohl, S. Heterogeneity and tests of risk sharing. *Journal of Political Economy*, 119 (5):925–958, 2011.

Townsend, R. M. Arrow-debreu programs as microfoundations of macroeconomics. In *Advances in Economic Theory: Fifth World Congress*, volume 2, pages 379–428, Cambridge, MA, 1987. Cambridge University Press.

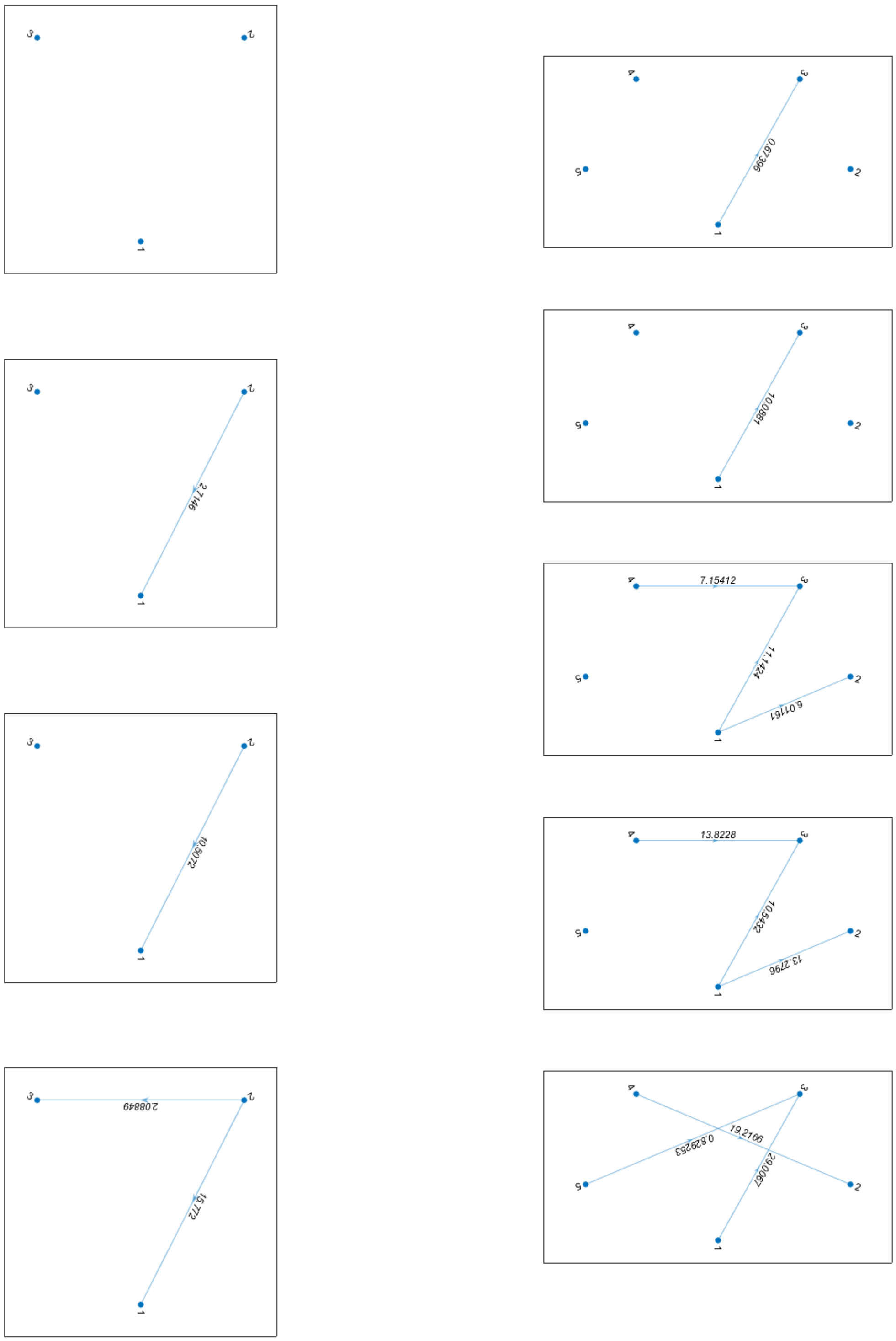
Townsend, R. M. Risk and insurance in village india. *Econometrica*, pages 539–591, 1994.

Townsend, R. M. Consumption insurance: An evaluation of risk-bearing systems in low-income economies. *Journal of Economic perspectives*, 9(3):83–102, 1995.

Udry, C. Risk and insurance in a rural credit market: An empirical investigation in northern nigeria. *The Review of Economic Studies*, 61(3):495–526, 1994.

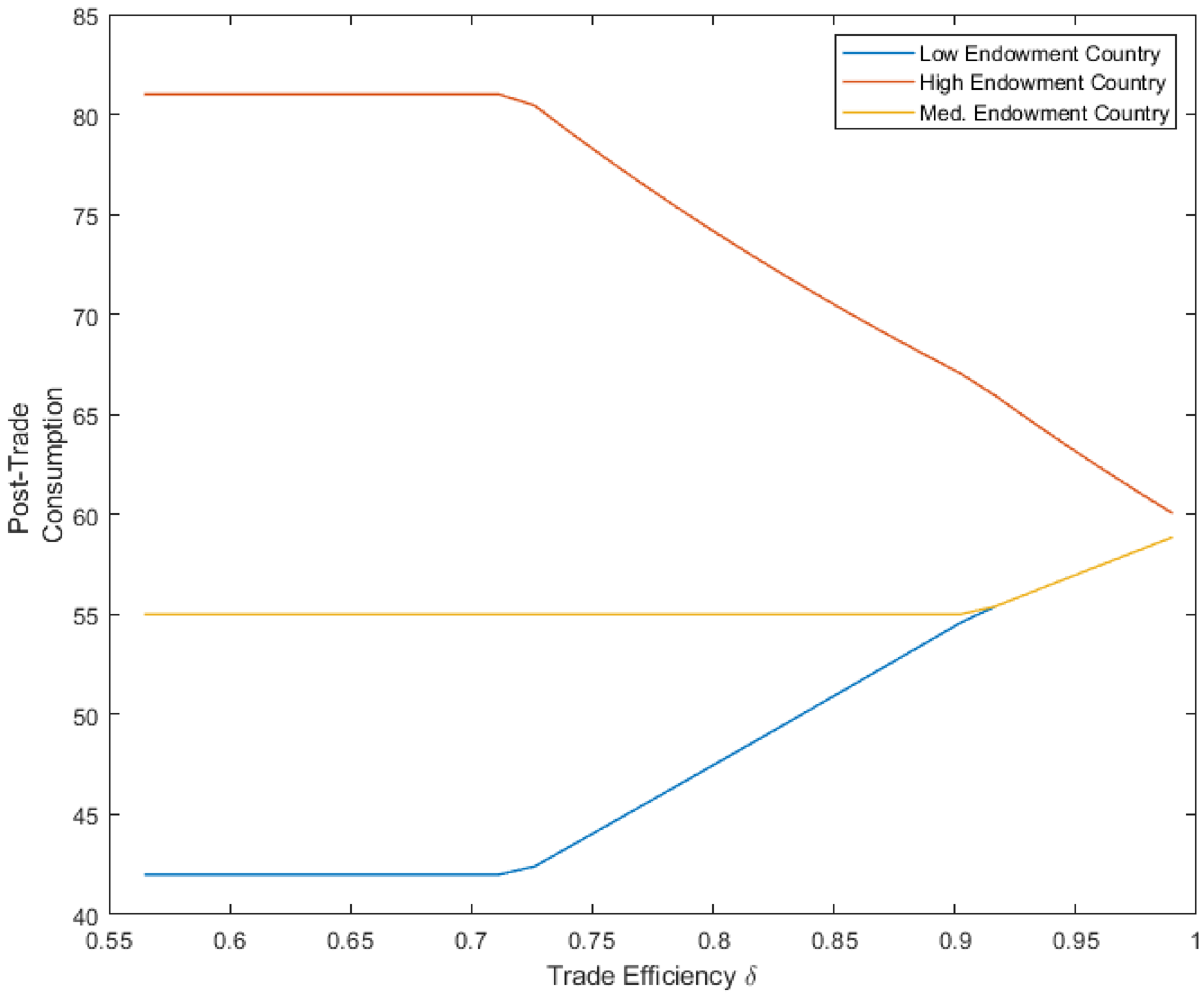
Figures

Figure 1: **Effect of Increasing δ on Trade Network**



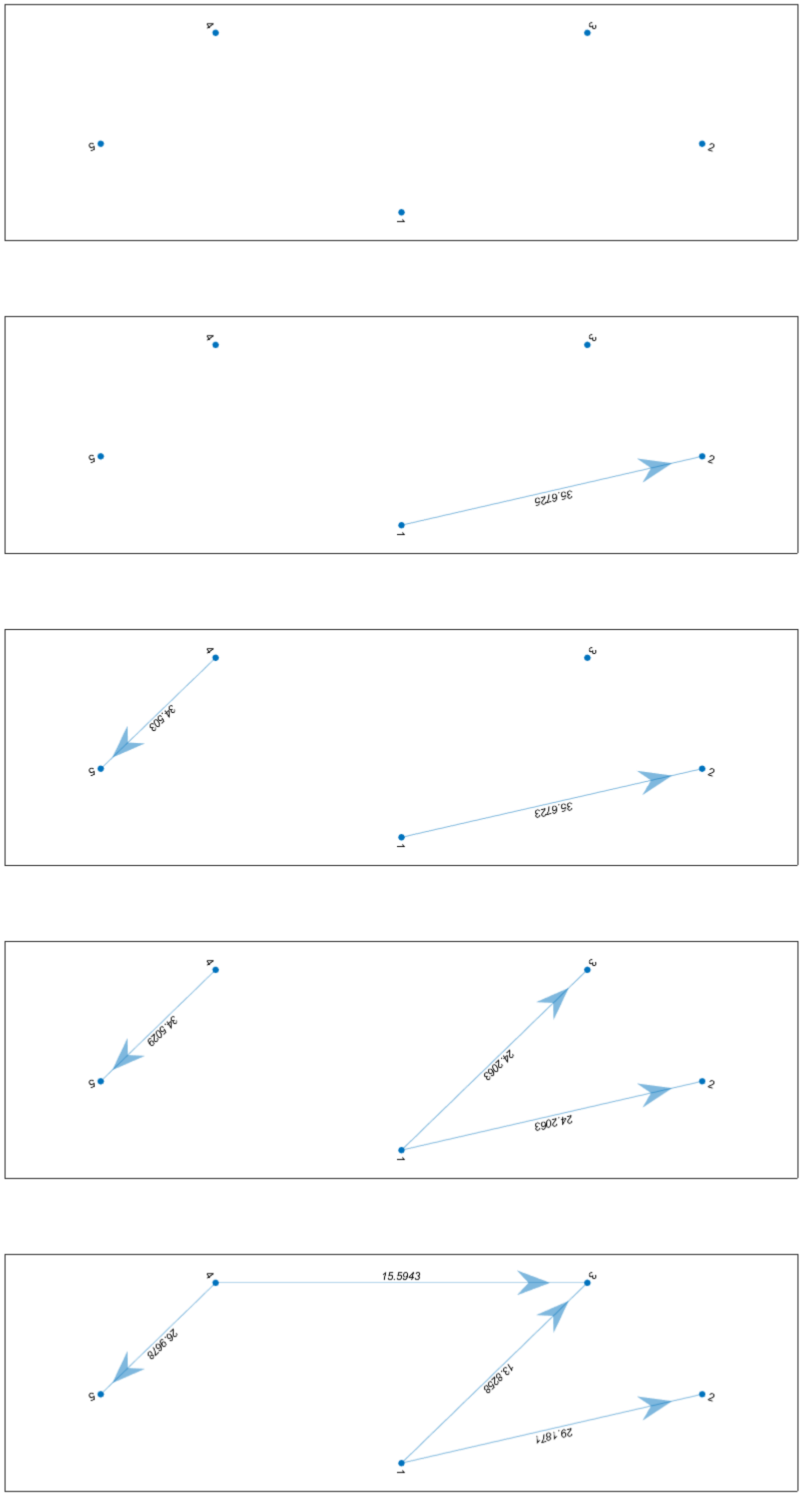
Note: The figure plots the optimal network shapes as δ increase or trade costs decrease. The left panel is a 3 country example and the right panel is a 5 country example.

Figure 2: **Effect of Increasing δ on Consumption**



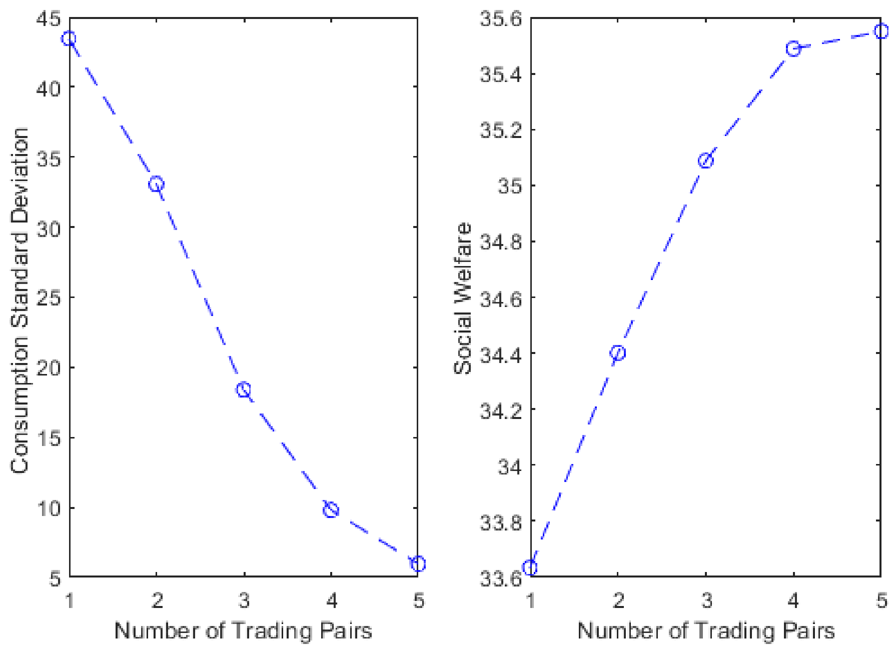
Note: The figure plots the optimal consumption (from social planner's problem) in a 3 country example as δ increase of trade costs decrease.

Figure 3: **Adding Trade Partners**



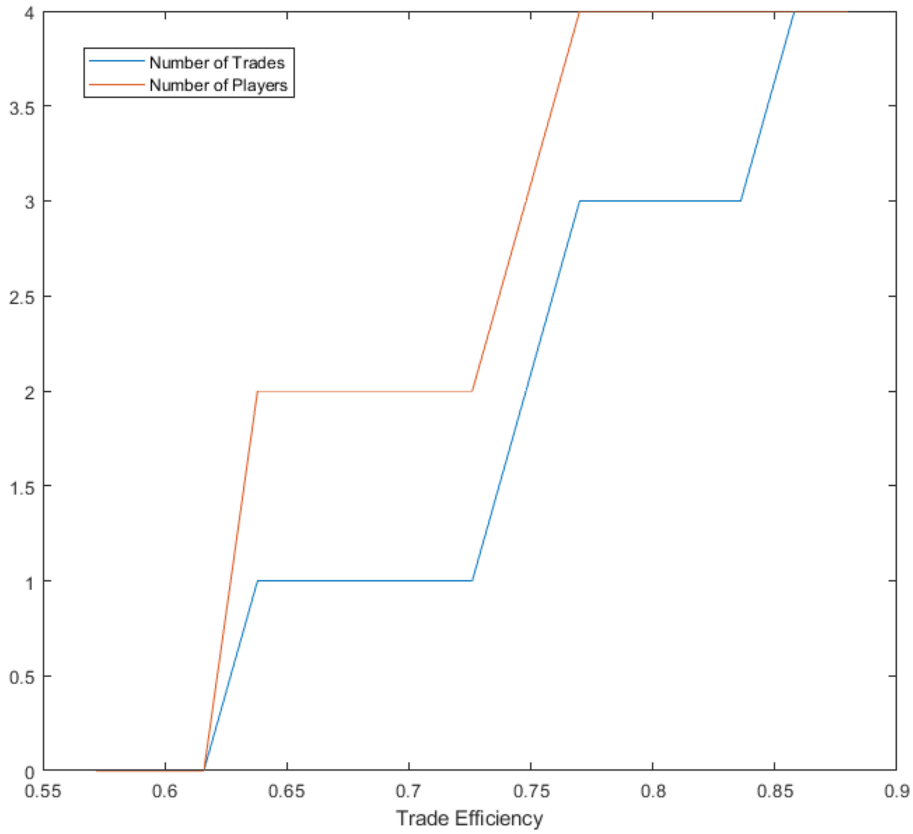
Note: The figure plots the network shapes of a 5 country example under different trading scenarios.

Figure 4: **Efficiency Effects of Adding Partners**
Consumption Variance and Global Utility



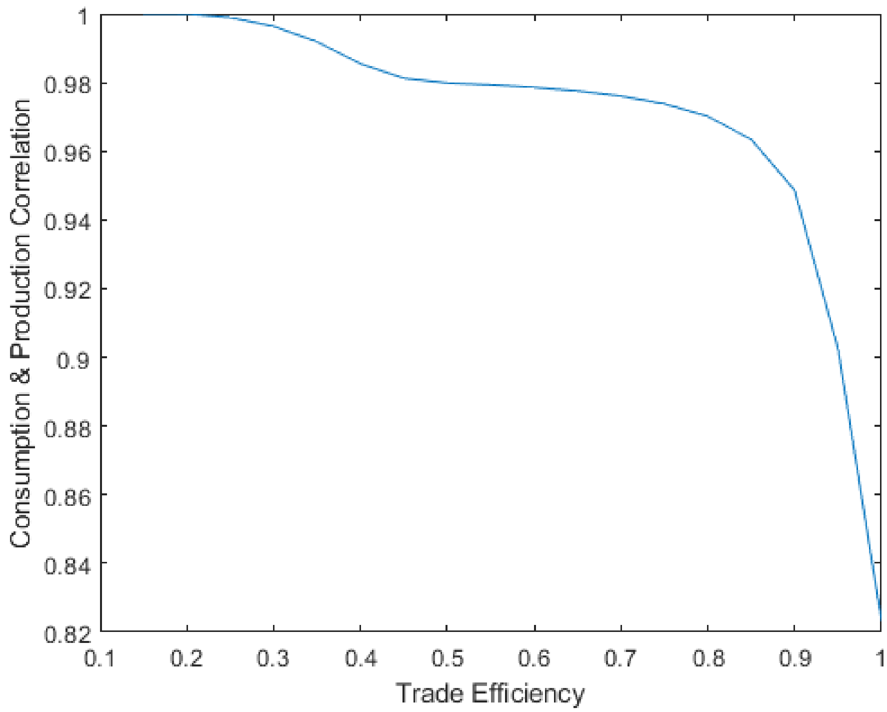
Note: The figure plots the consumption standard deviation and social welfare of a 5 country example when increasing the number of trading pairs.

Figure 5: **Trade Efficiency and the Amount of Trade Activity**



Note: The figure plots the number of trading links and number of members in the network as trade costs decrease.

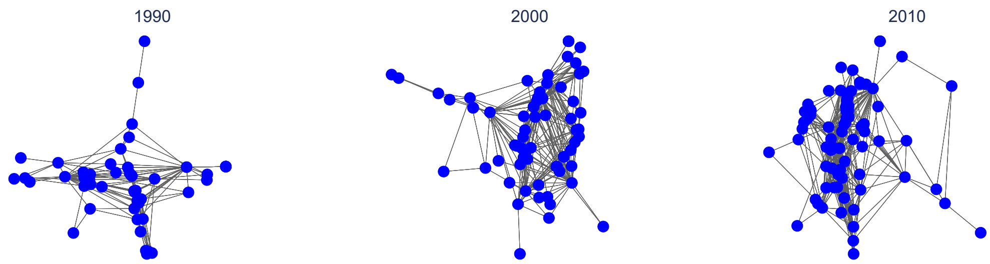
Figure 6: **Trade Efficiency and Risk Sharing**



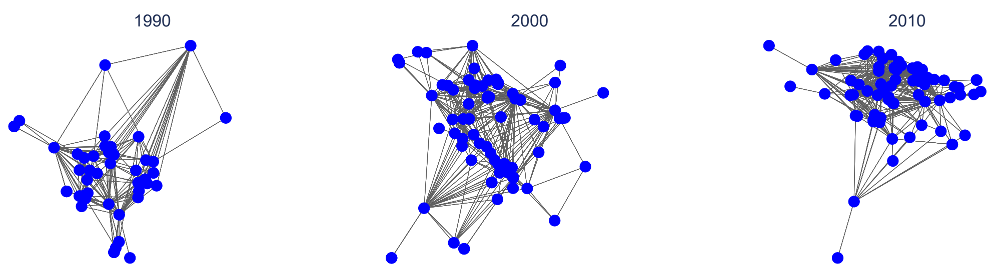
Note: The figure plots the correlation between own consumption and production as trade costs decrease.

Figure 7: **Trade Networks for the Three Commodities**

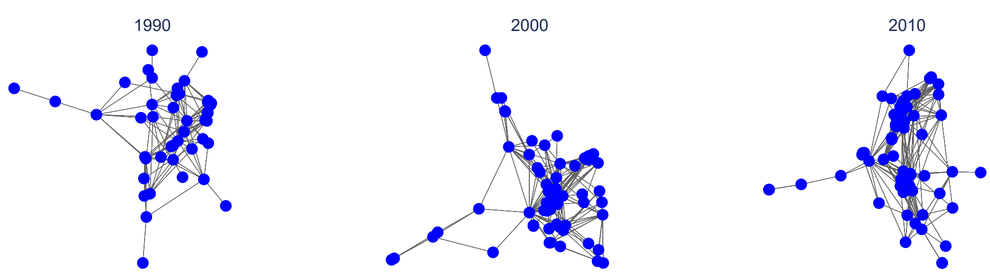
(a) Rice



(b) Wheat

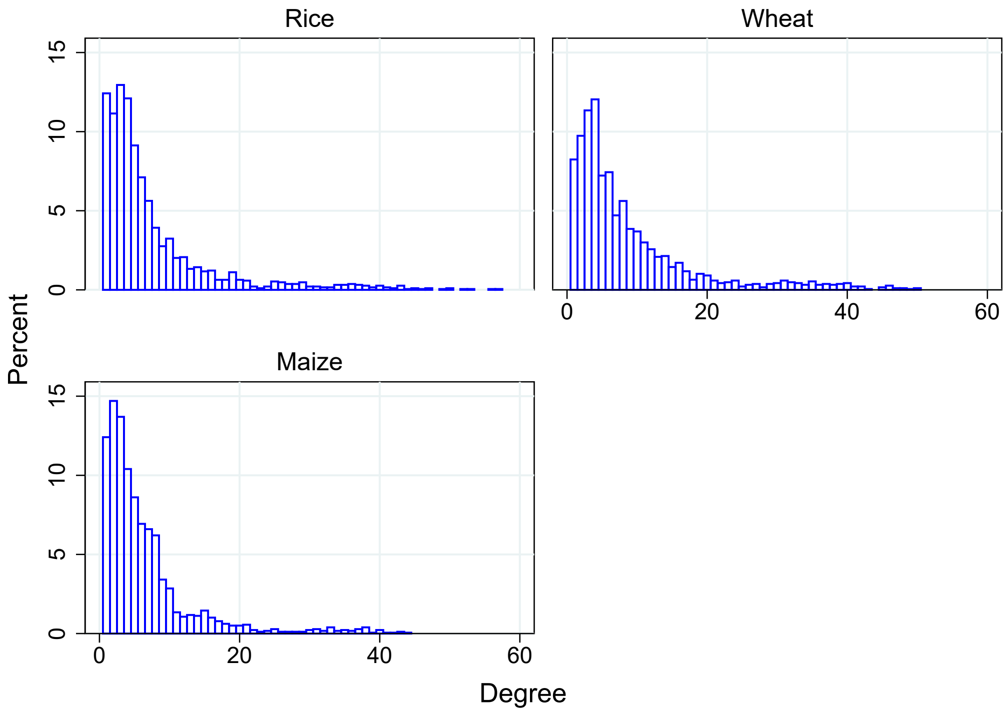


(c) Maize



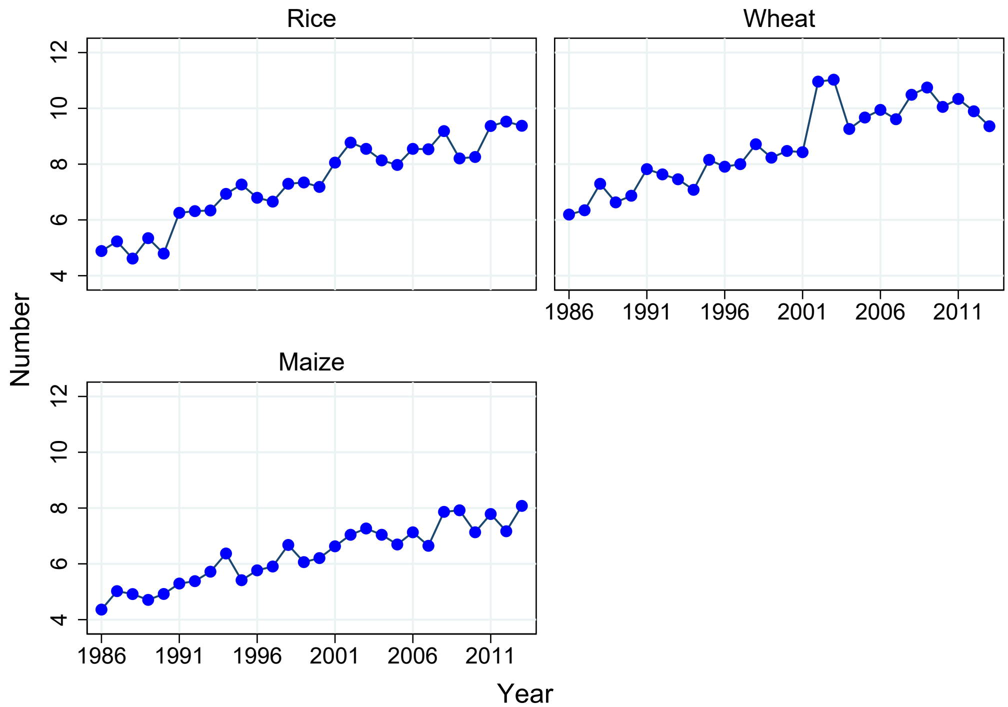
Note: The figure presents the network plots for rice, wheat, and maize for three slices of time. The blue dots are nodes or trading countries and the lines or edges denote trade links between countries.

Figure 8: **Degree Distribution of Nodes in Networks**



Note: The figure plots the histogram of number of trade links per country for rice, wheat, and maize.

Figure 9: **Average Number of Trade Partners: 1986-2013**



Note: The figure plots trends in the average number of trade partners per country for rice, wheat, and maize.

Tables

Table 1: **Simulated Risk-Sharing Regression and Bias**

Dependent Variable: log consumption	(1)	(2)	(3)
Log production	0.3913***	0.3377**	0.0647
	(0.0157)	(0.0457)	(0.0325)
Control for $\mathcal{A}$	No	Yes	Yes
Control for Δ	No	No	Yes
Observations	128	128	128
R^2	0.546	0.673	0.925

Note: Table present the estimates of risk sharing regressions from simulated data. The term $\mathcal{A}$ refers to the aggregate risk function. The term Δ refers to the indirect trade weight, which captures trade costs. Network dependent standard errors in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 2: **Structural Risk-Sharing Test Estimates**

Commodity:	Maize			Wheat			Rice		
Dependent Variable: log consumption	Baseline	Trade Cost Constant	Trade Cost Controls	Baseline	Trade Cost Constant	Trade Cost Controls	Baseline	Trade Cost Constant	Trade Cost Controls
Log production	0.303*	0.301*	0.273**	0.180***	0.173***	0.170***	0.241***	0.235***	0.218***
	(0.133)	(0.133)	(0.105)	(0.042)	(0.046)	(0.047)	(0.053)	(0.052)	(0.042)
Preference parameter	1.288***	1.296***	1.244***	1.252***	1.247***	1.245***	1.121***	1.148***	1.121***
	(0.33)	(0.33)	(0.29)	(0.14)	(0.134)	(0.126)	(0.138)	(0.148)	(0.132)
Constant	10.755***	10.820***	11.066***	12.677***	12.999***	13.157***	13.771***	13.969***	14.272***
	(1.931)	(1.951)	(1.567)	(0.502)	(0.685)	(0.74)	(0.924)	(0.908)	(0.808)
Trade-cost constant		15.039	21.497		9.849	41.312		12.945	23.426
		(8.398)	(297.184)		(8.116)	(40.952)		(7.748)	(78.239)
Common language			58.24						3.035
			(87.51)						(8.957)
Log weighted distance			-3.736			-4.999			-3.66
			(38.448)			(5.435)			(10.102)
Country i GATT member			-10.754			6.121			-15.965
			(16.295)			(8.021)			(15.396)
Country j GATT member			66.167			3.105			37.28
			(44.034)			(5.829)			(20.932)
Time difference between i and j			-0.584			0.047			0.029
			(7.757)			(1.007)			(1.862)
Log relative productivity			-4.768			-0.795			-0.637
			(5.877)			(1.398)			(1.646)
Nominal rate of assistance			-2.235			-0.186			0.472
			(1.296)			(0.849)			(0.532)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1752	1752	1752	1722	1722	1722	1860	1860	1860

Notes: The preference parameter is a coefficient both on the Pareto weights log ratio and the trade cost. Common language is a dummy variable coded as 1 if a language is spoken by at least 9% of the population in both countries. Population weighted distance between the two countries in kilometers. Time difference in number of hours difference between country i and j. GATT membership is a dummy variable coded as 1 if the country is GATT/WTO member. Relative (historical) productivity is calculate for country-pair and is extracted from the GAEZ database. Nominal rates of assistance (scaled by 10) are from the World Bank, Distortion from Agricultural Incentives Database. Network dependent standard errors in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 3: **Heterogeneous Effects**

Commodity:	Maize		Wheat		Rice	
Dependent Variable: log consumption	Baseline	Trade Cost Constant	Baseline	Trade Cost Constant	Baseline	Trade Cost Constant
Log production	0.255	0.171	0.115**	0.117	0.211	0.182
	(0.137)	(0.088)	(0.039)	(0.067)	(0.215)	(0.208)
Constant	11.548***	12.509***	13.343***	13.493***	13.899***	13.415***
	(0.089)	(0.056)	(0.650)	(3.022)	(0.361)	(0.576)
Trade-cost constant		19.987		19.989***		19.957***
		(19.822)		(1.917)		(0.535)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country γ-FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1752	1752	1722	1722	1860	1860

Notes: Country $\gamma-$FE refers to country specific slopes for preferences. Network dependent standard errors in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 4: **Maize Distance from Center Effects**

	1-link		$\leq$ 2-links		$\leq$ 3-links	
Dependent Variable: log consumption	Baseline	Trade Cost	Baseline	Trade Cost	Baseline	Trade Cost
Log production	0.215*	0.165*	0.294*	0.262*	0.303*	0.273**
	(0.106)	(0.066)	(0.133)	(0.103)	(0.133)	(0.105)
Preference parameter	1.241***	1.126***	1.295***	1.251***	1.288***	1.244***
	(0.272)	(0.206)	(0.328)	(0.287)	(0.330)	(0.290)
Constant	12.487***	13.000***	10.889***	11.224***	10.755***	11.066***
	(1.573)	(1.064)	(1.923)	(1.521)	(1.931)	(1.567)
Trade-cost constant		-9.339		21.497		21.497
		(327.038)		(280.349)		(297.184)
Common language		58.240		58.240		58.240
		(62.365)		(78.147)		(87.510)
Log weighted distance		-3.917		-3.917		-3.736
		(40.005)		(36.295)		(38.448)
Country i GATT member		-10.754		-10.754		-10.754
		(36.392)		(16.134)		(16.295)
Country j GATT member		66.589		66.589		66.167
		(41.909)		(40.323)		(44.034)
Time difference between i and j		1.914		-0.597		-0.584
		(6.287)		(7.317)		(7.757)
Log relative productivity		-3.962		-4.854		-4.768
		(5.323)		(5.368)		(5.877)
Nominal rate of assistance		-2.235		-2.235		-2.235
		(2.086)		(1.183)		(1.296)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1002	1002	1722	1722	1752	1752

Notes: The preference parameter is a coefficient both on the Pareto weights log ratio and the trade cost. Common language is a dummy variable coded as 1 if a language is spoken by at least 9% of the population in both countries. Population weighted distance between the two countries in kilometers. Time difference in number of hours difference between country i and j. GATT membership is a dummy variable coded as 1 if the country is GATT/WTO member. Relative (historical) productivity is calculate for country-pair and is extracted from the GAEZ database. Nominal rates of assistance (scaled by 10) are from the World Bank, Distortion from Agricultural Incentives Database. Network dependent standard errors in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 5: **Wheat Distance from Center Effects**

	1-link		$\leq$ 2-links		$\leq$ 3-links	
Dependent Variable: log consumption	Baseline	Trade Cost	Baseline	Trade Cost	Baseline	Trade Cost
Log production	0.126***	0.113***	0.178***	0.166***	0.180***	0.170***
	(0.017)	(0.018)	(0.040)	(0.044)	(0.042)	(0.047)
Preference parameter	1.318***	1.246***	1.258***	1.242***	1.252***	1.245***
	(0.134)	(0.123)	(0.139)	(0.124)	(0.140)	(0.126)
Constant	13.440***	14.195***	12.720***	13.248***	12.677***	13.157***
	(0.226)	(0.311)	(0.480)	(0.705)	(0.502)	(0.740)
Trade-cost constant		39.724		41.312		41.312
		(53.927)		(39.931)		(40.952)
Log weighted distance		-5.554		-5.411		-4.999
		(6.185)		(5.132)		(5.435)
Country i GATT member		-7.272		5.193		6.121
		(7.240)		(7.091)		(8.021)
Country i GATT member		14.894**		6.694		3.105
		(5.645)		(6.121)		(5.829)
Time difference between i and j		0.047		0.047		0.047
		(0.756)		(0.961)		(1.007)
Log relative productivity		-1.331*		-0.849		-0.795
		(0.542)		(1.358)		(1.398)
Nominal rate of assistance		0.328		0.328		-0.186
		(0.385)		(1.307)		(0.849)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	995	995	1714	1714	1722	1722

Notes: The preference parameter is a coefficient both on the Pareto weights log ratio and the trade cost. Common language is a dummy variable coded as 1 if a language is spoken by at least 9% of the population in both countries. Population weighted distance between the two countries in kilometers. Time difference in number of hours difference between country i and j. GATT membership is a dummy variable coded as 1 if the country is GATT/WTO member. Relative (historical) productivity is calculate for country-pair and is extracted from the GAEZ database. Nominal rates of assistance (scaled by 10) are from the World Bank, Distortion from Agricultural Incentives Database. Network dependent standard errors in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 6: **Rice Distance from Center Effects**

	1-link		≤ 2-links		≤ 3-links	
Dependent Variable: log consumption	Baseline	Trade Cost	Baseline	Trade Cost	Baseline	Trade Cost
Log production	0.193***	0.182***	0.241***	0.217***	0.241***	0.218***
	(0.039)	(0.035)	(0.053)	(0.042)	(0.053)	(0.042)
Preference parameter	1.059***	1.065***	1.121***	1.122***	1.121***	1.121***
	(0.121)	(0.124)	(0.139)	(0.132)	(0.138)	(0.132)
Constant	14.362***	14.703***	13.771***	14.278***	13.771***	14.272***
	(0.764)	(0.768)	(0.925)	(0.810)	(0.924)	(0.808)
Trade-cost constant		23.426		23.426		23.426
		(194.162)		(77.281)		(78.239)
Common language		-4.792		2.930		3.035
		(12.871)		(8.706)		(8.957)
Log weighted distance		-4.000		-3.678		-3.660
		(25.037)		(9.983)		(10.102)
Country i GATT member		-12.384		-15.997		-15.965
		(39.554)		(15.403)		(15.396)
Country j GATT member		38.185		37.010		37.280
		(23.191)		(20.806)		(20.932)
Time difference between i and j		0.539		0.064		0.029
		(4.803)		(1.847)		(1.862)
Log relative productivity		-0.637		-0.637		-0.637
		(3.223)		(1.631)		(1.646)
Nominal rate of assistance		-0.527		0.460		0.472
		(0.989)		(0.529)		(0.532)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1141	1141	1856	1856	1860	1860

Notes: The preference parameter is a coefficient both on the Pareto weights log ratio and the trade cost. Common language is a dummy variable coded as 1 if a language is spoken by at least 9% of the population in both countries. Population weighted distance between the two countries in kilometers. Time difference in number of hours difference between country i and j. GATT membership is a dummy variable coded as 1 if the country is GATT/WTO member. Relative (historical) productivity is calculate for country-pair and is extracted from the GAEZ database. Nominal rates of assistance (scaled by 10) are from the World Bank, Distortion from Agricultural Incentives Database. Network dependent standard errors in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 7: **Sample Size Robustness**

Commodity:	Maize		Wheat		Rice	
Dependent Variable: log consumption	Baseline	Trade Cost	Baseline	Trade Cost	Baseline	Trade Cost
Log production	0.449***	0.442***	0.298***	0.300***	0.300***	0.299***
	(0.082)	(0.082)	(0.051)	(0.052)	(0.047)	(0.046)
Preference	1.757***	1.775***	1.400***	1.427***	1.427***	1.471***
	(0.514)	(0.531)	(0.209)	(0.217)	(0.241)	(0.255)
Constant	8.527***	8.881***	12.553***	12.671***	12.445***	12.404***
	(1.233)	(1.266)	(0.830)	(0.830)	(0.938)	(0.929)
Trade-cost constant		25.816		26.386		25.441
		(62.268)		(55.941)		(79.638)
Common language		-4.404				8.862
		(12.425)				(17.949)
Log weighted distance		-5.873		-4.149		-5.588
		(9.067)		(8.280)		(10.707)
Country i GATT member		17.382		29.344		71.175
		(14.706)		(16.758)		(90.864)
Country j GATT member		13.732		-2.737		-3.175
		(15.638)		(16.671)		(15.758)
Time difference between i and j		0.945		-0.049		2.898
		(2.166)		(1.562)		(2.813)
Log relative productivity		-0.788		0.395		0.982
		(3.543)		(3.277)		(1.999)
Nominal rate of assistance		2.020		1.933		0.075
		(1.250)		(1.346)		(0.897)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3583	3583	3572	3572	3723	3723

Notes: The preference parameter is a coefficient both on the Pareto weights log ratio and the trade cost. Common language is a dummy variable coded as 1 if a language is spoken by at least 9% of the population in both countries. Population weighted distance between the two countries in kilometers. Time difference in number of hours difference between country i and j. GATT membership is a dummy variable coded as 1 if the country is GATT/WTO member. Relative (historical) productivity is calculate for country-pair and is extracted from the GAEZ database. Nominal rates of assistance (scaled by 10) are from the World Bank, Distortion from Agricultural Incentives Database. Network dependent standard errors in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Appendix

A Model Details

A.1 Symmetric Trade Result

Symmetric trading is not optimal. This allows us to ignore cases of two countries exporting to each other.

Lemma 1. *Given a linear trade cost of the form in equation (4) and with nonzero fractional bilateral trade parameter $\delta_{ij} \in (0,1]$ $\forall i \forall j \neq i$ (assuming both not equal to 1), any nonzero bilateral trade between i and j is one directional, meaning $y_{ij}y_{ji} = 0, y_{ij} \geq 0, y_{ji} \geq 0$.*

Proof. Suppose both i and j trade with each other, meaning $y_{ij} > 0$ and $y_{ji} > 0$. Then, based on the FOC system in equation (6), the two conditions would need to hold simultaneously:

$$\frac{\partial U}{\partial c_i}\delta_{ji} = \frac{\partial U}{\partial c_j} \cap \frac{\partial U}{\partial c_j}\delta_{ij} = \frac{\partial U}{\partial c_i}$$

These can only hold if $\delta_{ji} = 1/\delta_{ij}$. However that can only happen if both are equal to 1 or if one is greater than 1 (either case violating the assumptions). Thus one of these first order conditions does not hold with equality, meaning at least one export is zero. □

A.2 Optimal Consumption Result

Proposition 1. *Optimal c_i^* is a linear combination of endowments.*

Proof. Given a known set of exporters, the system of equations that define the optimal export choices can be written as the following:

$$\Delta_{ij}\left(y_j - \sum_{k\neq j}^{N} y_{jk}^E + \sum_{k\neq j}^{N} y_{kj}^E\delta_{kj}\right) = \left(y_i - \sum_{k\neq i}^{N} y_{ik}^E + \sum_{k\neq i}^{N} y_{ki}^E\delta_{ki}\right) \quad \forall \text{ exporter } i$$

Implicit in this equation is that $y_{ij} > 0$ and thus $y_{ji} = 0$ (by Lemma 1 in Appendix A).

This can be rearranged into an $\mathbf{Ax} = \mathbf{b}$ matrix format, with $\mathbf{x} = [y_{ij}^E, ...]$. $\mathbf{b} = [y_i - \Delta_{ij}y_j, ...]$. The formula for $\mathbf{A}$ is convoluted and involves the products of trade costs, relative Pareto weights, and the preference parameter. Then the solution by inversion, due to being square (and assuming full rank), is $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$. This is simply a linear combination of the vector $\mathbf{b}$, which in this case is just a linear combination of endowments $y_i \forall i$. Thus the export choices are linear in endowments. Since consumption is also just a linear combination of exports and endowments, optimal consumption will be a linear combination of endowments. □

Specifically, the endowments are y_i and y_j from the same trade network. Optimal consumption has the following pattern, where $\Omega \equiv [\Delta_{ij}, \delta_{ij}\ \forall i \forall j \neq i]$, and f, g^{ik}, h^{ki} are all functions $c_i^* = (y_i \cdot f(\Omega) - \sum_{k\neq i}^{N} g^{ik}(y_k \forall k, \Omega) + \sum_{k\neq i}^{N} h^{ki}(y_k \forall k, \Omega)\delta_{ki})/f(\Omega)$. Both agent pair specific functions (g^{ik} and h^{ki}) are simply weighted sums of all endowments. The endowments of any agent connected via the trade network has non-zero weight. Any agent in a separate network has zero weight.

A.3 Indirect Trade Weight Result

Proposition 2. *The (geodesic path) indirect trade weight from index i to ending partner k is:*

$$\Delta_{i\to k} = \left(\frac{\alpha_k}{\alpha_i} \prod_{(r,s)\in\mathcal{P}_{ik}} \frac{\delta_{rs}^{\mathcal{S}(r,s)}}{\delta_{sr}^{\mathcal{S}'(r,s)}}\right)^{-1/\gamma} \tag{22}$$

Proof. Consider the undirected geodesic path from i to k, which exists if $i, k \in \mathcal{T}$, but may not be unique. There exists a path $i, k_1, ..., k_{n-1}, k_n = k$ with n steps. The first FOC comparing i to k_1 is either $c_i = \Delta_{i,k_1} c_{k_1}$ or $c_{k_1} = \Delta_{k_1,i} c_i$ depending on which is the importer or exporter; either $\mathcal{S}'(i,k_1) = 1$ or $\mathcal{S}(i,k_1) = 1$, which due to Lemma 1 are mutually exclusive. The second is either $c_{k_1} = \Delta_{k_1,k_2} c_{k_2}$ or $c_{k_2} = \Delta_{k_2,k_1} c_{k_1}$. The combined chain is then:

$$c_i = \Delta_{i,k_1}^{\mathcal{S}(i,k_1)} \cdot \Delta_{k_1,i}^{-\mathcal{S}'(i,k_1)} \cdot \Delta_{k_1,k_2}^{\mathcal{S}(k_1,k_2)} \cdot \Delta_{k_2,k_1}^{-\mathcal{S}'(k_1,k_2)} c_{k_2}$$

If $n = 2$ then the expression above is the final answer. Now suppose it holds for $n = m$, meaning the path from i to k with m steps yields: $c_i = \prod_{(r,s)\in\mathcal{P}_{ik_m}} \Delta_{rs}^{\mathcal{S}(r,s)} \Delta_{sr}^{-\mathcal{S}'(r,s)} c_{k_m}$.

Now suppose we actually need one more link to reach k from i, meaning $n = m + 1$. If m exports to $m + 1 = k$, then $c_m = \Delta_{m,k} c_k$ and if k exports to m then $c_k = \Delta_{k,m} c_m$. Then we can compare c_i to c_k through c_{k_m}:

$$c_i = \Delta_{i\to k_m} c_{k_m} = \Delta_{i\to k_m} \cdot (\Delta_{m,k}^{\mathcal{S}(m,k)} \Delta_{k,m}^{-\mathcal{S}'(m,k)} c_k)$$

Thus we've shown $\Delta_{i\to k} = \prod_{(r,s)\in\mathcal{P}_{ik}} \Delta_{rs}^{\mathcal{S}(r,s)} \Delta_{sr}^{-\mathcal{S}'(r,s)}$. Substituting out $\Delta_{ik} \equiv (\delta_{ik}\alpha_k/\alpha_i)^{-1/\gamma}$ immediately yields the result. □

Note that the geodesic path is not in general the "lowest cost" path; geodesic paths only consider the number of links and there may be a "longer" path that has a lower indirect cost.[26] Also, in the case of heterogeneous γ, the ratios of γ_i along the path change the formula in scaling each intermediary direct Δ.

A.4 Optimal Consumption Solution with Examples

Consider a case of three agents. The setup is same as the general model in the paper. The social planner's problem is to maximize the social welfare function

$$\begin{aligned} U = {} & \alpha_1 u_1(y_1 - y_{12}^E - y_{13}^E + \delta_{21} y_{21}^E + \delta_{31} y_{31}^E) \\ & + \alpha_2 u_2(y_2 - y_{21}^E - y_{23}^E + \delta_{12} y_{12}^E + \delta_{32} y_{32}^E) \\ & + \alpha_3 u_3(y_3 - y_{31}^E - y_{32}^E + \delta_{13} y_{13}^E + \delta_{23} y_{23}^E) \end{aligned} \tag{23}$$

[26] Finding the lowest-cost path from all paths is computationally restrictive; restricting the set of paths based on some fixed length above the geodesic length and comparing the trade weights from each path is feasible.

subject to the non-negativity constraint, i.e., $y^E_{ji} \geq 0$ for all i and $j \neq i$. With a CRRA utility function, the first order conditions can be expressed as a system of equations:

$$\Delta_{12}(y_2 - y^E_{21} - y^E_{23} + \delta_{12}y^E_{12} + \delta_{32}y^E_{32}) = (y_1 - y^E_{12} - y^E_{13} + \delta_{21}y^E_{21} + \delta_{31}y^E_{31}) + \lambda_{12} \tag{24}$$

$$\Delta_{13}(y_3 - y^E_{31} - y^E_{32} + \delta_{13}y^E_{13} + \delta_{23}y^E_{23}) = (y_1 - y^E_{12} - y^E_{13} + \delta_{21}y^E_{21} + \delta_{31}y^E_{31}) + \lambda_{13} \tag{25}$$

$$\Delta_{21}(y_1 - y^E_{12} - y^E_{13} + \delta_{21}y^E_{21} + \delta_{31}y^E_{31}) = (y_2 - y^E_{21} - y^E_{23} + \delta_{12}y^E_{12} + \delta_{32}y^E_{32}) + \lambda_{21} \tag{26}$$

$$\Delta_{23}(y_3 - y^E_{31} - y^E_{32} + \delta_{13}y^E_{13} + \delta_{23}y^E_{23}) = (y_2 - y^E_{21} - y^E_{23} + \delta_{12}y^E_{12} + \delta_{32}y^E_{32}) + \lambda_{23} \tag{27}$$

$$\Delta_{31}(y_1 - y^E_{12} - y^E_{13} + \delta_{21}y^E_{21} + \delta_{31}y^E_{31}) = (y_3 - y^E_{31} - y^E_{32} + \delta_{13}y^E_{13} + \delta_{23}y^E_{23}) + \lambda_{31} \tag{28}$$

$$\Delta_{32}(y_2 - y^E_{21} - y^E_{23} + \delta_{12}y^E_{12} + \delta_{32}y^E_{32}) = (y_3 - y^E_{31} - y^E_{32} + \delta_{13}y^E_{13} + \delta_{23}y^E_{23}) + \lambda_{32} \tag{29}$$

where Δ_{ij} is a function of Pareto weights, utility parameter and transaction costs. In the examples below, we solve this system with two trading network examples.

A.4.1 Example 1

Consider agent 1 exporting to agents 2 and 3. Given that 1 is the exporter, a reverse trade flow from either 2 or 3 to 1 is suboptimal (meaning $y^E_{ij}y^E_{ji} = 0$). So, $y^E_{12} > 0$, $y^E_{13} > 0$ but $y^E_{21} = y^E_{31} = 0$. Additionally, we assume that 2 and 3 do not trade with each other, therefore, $y^E_{32} = y^E_{23} = 0$. Given these assumptions, the system with equalities simplifies to the following:

$$\Delta_{12}(y_2 + \delta_{12}y^E_{12}) = y_1 - y^E_{12} - y^E_{13} \tag{30}$$

$$\Delta_{13}(y_y + \delta_{13}y^E_{13}) = y_1 - y^E_{12} - y^E_{13} \tag{31}$$

The solutions for y^E_{12} and y^E_{13} are:

$$y^E_{12} = \frac{-\Delta_{12}y_2 + \Delta_{13}y_3 + \Delta_{13}\delta_{13}y_1 - \Delta_{12}\delta_{13}\Delta_{13}y_2}{\Delta_{12}\delta_{12} + \Delta_{13}\delta_{13} + \Delta_{12}\delta_{12}\Delta_{13}\delta_{13}} \tag{32}$$

$$y^E_{13} = \frac{\Delta_{12}y_2 - \Delta_{13}y_3 + \Delta_{12}\delta_{12}y_1 - \Delta_{12}\Delta_{13}\delta_{12}y_3}{\Delta_{12}\delta_{12} + \Delta_{13}\delta_{13} + \Delta_{12}\delta_{12}\Delta_{13}\delta_{13}} \tag{33}$$

These expressions can be substituted in the consumption equation of the three agents to get the optimal consumption expressions reported in the paper.

A.4.2 Example 2

Now consider the case when agent 1 exports to only agent 2 and agent 3 does not trade. In that case, we have $y^E_{12} > 0$, $y^E_{21} = 0$ and $y^E_{31} = y^E_{13} = y^E_{32} = y^E_{23} = 0$. This simplifies the system to:

$$\Delta_{12}(y_2 + \delta_{12}y^E_{12}) = y_1 - y^E_{12} \tag{34}$$

We can solve this to get y^E_{12} :

$$y^E_{12} = \frac{y_1 - \Delta_{12}y_2}{\Delta_{12}\delta_{12} + 1} \tag{35}$$

B Estimation Details

B.1 Trading Participation Selection

The risk-sharing equation is derived from the FOC of the model; for agents who are not part of any network in a given year, their consumption is simply equal to their production: $\ln(c_{jt}) = \ln(y_{jt}) + \varepsilon_{jt}$, and so there is no "risk-sharing" parameter to test since they are unable to share risk in that period. Note however that the fact that the agent is not trading is endogenous to the model; the realization of shocks in a year combined with trade costs could lead to an agent not trading. Thus it does raise the question of selection bias present in the sample if we only estimate the risk-sharing regression for trading agents.

If one simply wants to capture how being in a network reduces dependence on own production, then conditioning on the network participants is sufficient. However if one is interested in understanding the extent of global risk sharing induced by variation in endowment shocks, then one should include excluded countries as their autarkic status in a given year reveals the extent of risk sharing.[27] If the selection is based on observables, then an augmented risk-sharing equation is sufficient to capture membership as follows (expressed in terms of a single network):

$$\ln(c_{it}) = \ln(c^*_{it}) \cdot \mathbb{1}[i_t \in \mathcal{T}_t] + \ln(y_{it}) \cdot \mathbb{1}[i_t \notin \mathcal{T}_t] + \eta \ln(y_{it}) + \varepsilon_{it} \tag{36}$$

B.2 Asymptotic Derivations

The risk sharing test controlling for aggregate risk function and trade costs is given as:

$$\ln(c_{jt}) = \ln(\mathcal{A}^*_{\mathcal{T}_t t}) + \eta \cdot \ln(y_{jt}) - \ln(\Delta^*_{jt}) + \varepsilon_{jt}, \tag{37}$$

for each $j \in \mathcal{N}_N = \{1, \ldots, N\}$ and $t = 1, 2, \ldots, T$. Let $\mathbf{x}_{it} = (x_{it1}, x_{it2})$, $\mathbf{z}_{it} = \left(\{z^j_{it}\}_1^{|I_{it}|}, \{z^k_{it}\}_1^{|E_{it}|}\right)$ where $|I_{it}|$ and $|E_{it}|$ denote the cardinality of the importer and exporter sets, respectively of country i at time t. Let $\mathbf{f}_t = (f1_t, \ldots, fT_t)$ denote the vector of binary time indicators. Let $\mathbf{w}_{it} = (\mathbf{d}_t, \mathbf{x}_{it}, \mathbf{z}_{it})$ and $\boldsymbol{\theta} = (\boldsymbol{\alpha}, \boldsymbol{\beta})'$. Here $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4)'$ and $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_T)'$. Then, one may rewrite the risk sharing regression as:

$$\begin{aligned} y_{it} &= \mathbf{f}_t\boldsymbol{\alpha} + \beta_1 x_{it1} + \beta_2 x_{it2} + \beta_2 \left[\sum_{j \in I_{it}} g(z^j_{it1}, z^j_{it2}; \beta_3, \beta_4) - \sum_{l \in E_{it}} g(z^l_{it1}, z^l_{it2}; \beta_3, \beta_4)\right] + \varepsilon_{it} \\ &\equiv h(\mathbf{w}_{it}, \boldsymbol{\theta}) + \varepsilon_{it} \end{aligned} \tag{38}$$

where $g(\cdot;\cdot) = \log\left(\frac{\exp(\cdot)}{1+\exp(\cdot)}\right)$. The summations over j and l are adding over countries that indirectly connect i with the index country $\bar{i}$ in the importer and exporter sets, respectively. Hence, $\{(y_{it}, \mathbf{w}_{it});\ i \in \mathcal{N}_N; t = 1, 2, \ldots, T\}$ represent the observed sample from the population. Then,

[27] This would require a different approach: regress the probability of a network forming of a certain type as a function of endowment shocks. This is a selection equation that the post-selection risk-sharing regression ignores.

stacking equation (38) across time periods, we can write the regression more compactly as[28]

$$\mathbf{y}_i = \mathbf{h}_i(\boldsymbol{\theta}) + \boldsymbol{\varepsilon}_i,\ i \in \mathcal{N}_N \tag{39}$$

where $\mathbf{y}_i = (y_{i1}, \ldots, y_{iT})'$, $\mathbf{h}_i(\boldsymbol{\theta}) = (h_{i1}(\boldsymbol{\theta}), \ldots, h_{iT}(\boldsymbol{\theta}))'$, and $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, \ldots, \varepsilon_{iT})'$.

Assumption 1. $\mathbb{E}(\varepsilon_{it}|\mathbf{w}_i) = \mathbb{E}(\varepsilon_{it}|\mathbf{w}_{it}) = 0$

Assumption 1 implies that strict exogeneity holds with respect to risk sharing regression equation. Given the estimating equation in 39, we minimize the following nonlinear least squares objective function,

$$\min_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} \frac{1}{NT} \sum_{i\in\mathcal{N}_N} \hat{\boldsymbol{\varepsilon}}_i^2 \equiv \min_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} \frac{1}{NT} \sum_{i\in\mathcal{N}_N} (\mathbf{y}_i - \mathbf{h}_i(\hat{\boldsymbol{\theta}}))^2 \tag{40}$$

with the first order conditions for $\hat{\boldsymbol{\theta}}$ given as

$$\frac{1}{NT} \sum_{i\in\mathcal{N}_N} \nabla_{\boldsymbol{\theta}} \mathbf{h}_i(\hat{\boldsymbol{\theta}})' \hat{\boldsymbol{\varepsilon}}_i = \frac{1}{NT} \sum_{i\in\mathcal{N}_N} \mathbf{h}_i^{(1)}(\hat{\boldsymbol{\theta}}) \hat{\boldsymbol{\varepsilon}}_i = \mathbf{0} \tag{41}$$

where

$$\mathbf{h}_i^{(1)}(\hat{\boldsymbol{\theta}})' = \nabla_{\boldsymbol{\theta}} \mathbf{h}_i(\hat{\boldsymbol{\theta}}) = \begin{pmatrix} \frac{\partial h_{i1}(\hat{\boldsymbol{\theta}})}{\partial \theta_1} & \cdots & \frac{\partial h_{i1}(\hat{\boldsymbol{\theta}})}{\partial \theta_M} \\ \frac{\partial h_{i2}(\hat{\boldsymbol{\theta}})}{\partial \theta_1} & \cdots & \frac{\partial h_{i2}(\hat{\boldsymbol{\theta}})}{\partial \theta_M} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_{iT}(\hat{\boldsymbol{\theta}})}{\partial \theta_1} & \cdots & \frac{\partial h_{iT}(\hat{\boldsymbol{\theta}})}{\partial \theta_M} \end{pmatrix}_{(T\times M)}$$

Then, by first-order taylor expansion of equation (41) around the true $\boldsymbol{\theta}$, we have the following influence function representation for $\hat{\boldsymbol{\theta}}$

$$\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) = -\mathbf{Q}_T^{-1} \left(\frac{1}{\sqrt{N}} \sum_{i\in\mathcal{N}_N} \mathbf{v}_i \right) + o_p(1) \tag{42}$$

where, $\mathbf{v}_i = \mathbf{h}_i^{(1)}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_i = \frac{1}{T}\sum_{t=1}^T \nabla_{\boldsymbol{\theta}} h_{it}(\boldsymbol{\theta})\varepsilon_{it}$ and $\mathbf{Q}_T = \operatorname*{plim}_{N\to\infty} \frac{1}{NT}\sum_{i\in\mathcal{N}_N} \nabla_{\boldsymbol{\theta}}\{\mathbf{h}_i^{(1)}(\tilde{\boldsymbol{\theta}})\tilde{\boldsymbol{\varepsilon}}_i\}$
$= -\frac{1}{T}\mathbb{E}[\mathbf{h}_i^{(1)}(\boldsymbol{\theta})\mathbf{h}_i^{(1)}(\boldsymbol{\theta})']$.

Now, in order to obtain the asymptotic distribution of $\hat{\boldsymbol{\theta}}$, we need to apply the LLN and CLT for network dependent data to the sequence, $N^{-1/2}\sum_{i\in\mathcal{N}_N} \mathbf{v}_i$, where $\mathbb{E}(\mathbf{v}_i|\mathbf{w}_i) = \mathbf{0}$ due to assumption 1a).[29] Provided that certain regularity conditions hold (mentioned below) which are informed by KMS, we will obtain the following asymptotic normality result.

$$\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \sim \mathrm{N}\left(\mathbf{0}, \mathbf{Q}_T^{-1}\boldsymbol{\Omega}_T\mathbf{Q}_T^{-1}\right) \tag{43}$$

[28] Note that we do not explicitly introduce a binary indicator s_{it} for missing data given that our application involves an unbalanced panel. We simply assume that the observed data is conditionally independent of the errors, ε_{it}.

[29] Such an aggregation helps to express the original panel problem solely in terms of the cross sectional dimension. This matters for the asymptotic argument in our setting where we assume T to be fixed and N to approach infinity.

with the network-HAC variance is given by

$$\mathbf{\Omega}_T = \text{Avar}(N^{-1/2}\sum_{i\in\mathcal{N}_N}\mathbf{v}_i) = \lim_{N\to\infty}\frac{1}{NT^2}\sum_{d\geq 0}\sum_{i\in\mathcal{N}_N}\sum_{j\in\mathcal{N}_N^{\partial}(i,d)}\mathbb{E}(\mathbf{v}_i\mathbf{v}_j'|\mathbf{w}_i,\mathbf{w}_j) \tag{44}$$

where $\mathcal{N}_N^{\partial}(i,d) = \{j\in\mathcal{N}_N : D_N(i,j) = d\}$ is the set of neighbors of node i who are *exactly* d-links away. In our context, these cross-correlations result from network dependence among the errors $\boldsymbol{\varepsilon}_i$'s. This is because

$$\mathbb{E}(\mathbf{v}_i\mathbf{v}_j'|\mathbf{w}_i,\mathbf{w}_j) = \mathbf{h}_i^{(1)}(\boldsymbol{\theta})\mathbb{E}(\boldsymbol{\varepsilon}_i\boldsymbol{\varepsilon}_j'|\mathbf{w}_i,\mathbf{w}_j)\mathbf{h}_j^{(1)}(\boldsymbol{\theta})'$$

In particular, we assume weak dependence which means that these covariances diminish as the network-distance between any two pairs of nodes, i and j, grows large. To limit the strength of dependence, we impose regularity conditions that assume the network data to be conditionally ψ-dependent given the σ-algebra, $\mathcal{C}_N$.

Note that we have assumed conditional ψ-dependence with respect to $\boldsymbol{\varepsilon}_i$. This is easily translated into conditional ψ-dependence of the linear transformation, $\mathbf{v}_i = \mathbf{h}_i^{(1)}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_i$ given $\mathcal{C}_N$ with dependence coefficients, λ_N, from using Lemma 2.1 of KMS. The other conditions required for the LLN and CLT include moments conditions for $\mathbf{v}_i$, as follows

Assumption 2 (Moment conditions)**.** *i) For some $\epsilon > 0$, $\sup_{N\geq 1}\max_{i\in\mathcal{N}_N}||\mathbf{v}_i||_{\mathcal{C}_N,1+\epsilon} < \infty$ a.s; ii) For some $p > 4$, $\sup_{N\geq 1}\max_{i\in\mathcal{N}_N}||\mathbf{v}_i||_{\mathcal{C}_N,p} < \infty$ a.s. where $||\mathbf{v}_i||_{\mathcal{C}_N,p} = \left(\mathbb{E}[|\mathbf{v}_i|^p|\mathcal{C}_N]^{1/p}\right)$.*

Assumption 3 (Denseness and strength of network dependence)**.** *i) $\frac{1}{N}\sum_{d\geq 0}\zeta_N^{\partial}(d)\lambda_{N,d}\to_{a.s.} 0$ where $\zeta_N^{\partial}(d) = N^{-1}\sum_{i\in\mathcal{N}_N}|\mathcal{N}_N^{\partial}(i,d)|$; ii) There exists a positive sequence, $m_N\to\infty$ such that for $k = 1,2$*

$$N^{1-k}\cdot\mathbf{\Omega}_N^{-k}\sum_{d\geq 0}a_N(d,m_N;k)\lambda_{N,d}^{1-\frac{2+k}{p}}\to_{a.s.} 0$$
$$N^{3/2}\cdot\mathbf{\Omega}_N^{-1/2}\lambda_{N,m_N}^{1-1/p}\to_{a.s.} 0$$

as $N\to\infty$, where $p > 4$ and $a_N(\cdot,\cdot;)$ is the measure of network denseness used in KMS.

where $\zeta_N^{\partial}(d)$ gives us the average neighborhood size and $a_N(\cdot,\cdot;)$ is a combination of average neighborhood size and average neighborhood shell size. These are being used as measures of network denseness. As KMS put it, this condition appears in the form of a tradeoff between network denseness and strength of dependence needed to ensure that covariance decays as a function of network-distance, d.

A consistent estimator for the network HAC variance can then be obtained from constructing a sample analogue of the expression in (43) where the middle term is taken to be a weighted average of the sample covariances of $\mathbf{h}_i^{(1)}(\hat{\boldsymbol{\theta}})\hat{\boldsymbol{\varepsilon}}_i$ and $\mathbf{h}_j^{(1)}(\hat{\boldsymbol{\theta}})\hat{\boldsymbol{\varepsilon}}_j$ as follows:

$$\hat{\mathbf{\Omega}}_T = \frac{1}{NT^2}\sum_{d\geq 0}\mathcal{K}_N(d)\left[\sum_{i\in\mathcal{N}_N}\sum_{j\in\mathcal{N}_N^{\partial}(i,d)}\mathbf{h}_i^{(1)}(\hat{\boldsymbol{\theta}})\hat{\boldsymbol{\varepsilon}}_i\hat{\boldsymbol{\varepsilon}}_j'\mathbf{h}_j^{(1)}(\hat{\boldsymbol{\theta}})'\right]$$

with $\mathcal{K}_N(d) = \mathcal{K}(d/b_N)$ as the weight given to sample covariances as a function of the distance.

Consistency of $\hat{\mathbf{\Omega}}_T$ can be established using the results in KMS if we assume the following conditions hold along with Assumption 2) ii).

Assumption 4 (Consistency of the HAC estimator)**.** *There exists a* $p > 4$ *such that i) lim*$_{N\to\infty} \sum_{d\geq 1} |\mathcal{K}_N(d) - 1| \zeta_N^{\partial}(d) \lambda_{N,d}^{1-2/p} = 0$ *a.s. and ii) lim*$_{N\to\infty} N^{-1} \sum_{d\geq 0} a_N(d, b_N; 2) \lambda_{N,d}^{1-4/p} = 0$ *a.s.*