# Platonism, Mereology and Whitehead's Process Ontology

Dwayne Richard Schulz
Bachelor of Arts, Economic History (Honours)
Bachelor of Laws

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Faculty of Arts,
School of Philosophical, Historical & International Studies.

**Abstract**

This thesis critically examines the process philosophy of Alfred North Whitehead focusing on the relationships between his platonism[1] and his mereology[2]. Particular attention is given to Whitehead's notion of the 'extensive continuum'; an ideal mereotopological[3] structure said to ground the geometrical nature and measurability of physical events. The thesis argues that Whitehead's platonism is inconsistent with his process philosophy and results in confused and mistaken ideas about inter alia the nature of spacetime and general relativity. The thesis makes the case however that Whitehead's thought does contain the seeds of a one-category ontology of *relational events* producing and binding themselves into processes, whose similarity and contingency is grounded in their own genetic nature rather than some super-sensible realm of forms beyond sense experience.

[1] By 'platonism' is meant the notion that the properties of objects instantiate, partake or mimic ideal or transcendental Forms. For example that the red in yesterday's sunset and the red of today's sunrise instantiate an ideal form Red that in itself is nowhere and no-when.

[2] By 'mereology' is meant a formal theory of part-whole relations in which there are axioms and definitions modelling the relation 'part of'. The term was invented by Leśniewski in 1927 and is derived from the Greek word meros (μέρος) for part. In modern times mereologies are usually formulated in the language quantificational logic.

[3] By 'mereotopological' is meant a formal theory of part-whole relations that also includes topological axioms and definitions modelling relations like 'connected to' and 'interior of'.

Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma at any university or equivalent institution and that, to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

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> The explanatory purpose of philosophy ... is to explain the emergence of the more abstract things from the more concrete things. It is a complete mistake to ask how concrete particular fact can be built up out of universals. The answer is 'in no way'.
>
> *Process & Reality* p.20.

> … the actualities constituting the process of the world are conceived as exemplifying the ingression (or 'participation') of other things which constitute the potentialities … The things which are temporal arise by their participation in the things which are eternal.
>
> *Process & Reality* pp.39 - 40.

## Introduction

Whitehead is known mostly for the process philosophy outlined in his 1929 magnum opus *Process and Reality*. But the ontology which Whitehead promotes is not a pure process ontology because he also counts amongst the furniture of the world abstract platonic forms which he called “eternal objects”. Eternal objects have two main functions in Whitehead’s philosophy. Firstly, they are responsible for everything that is general in the world; the apparent sameness of things. They ensure identity over time and space so that the laws of nature which apply over there are *the* very ones that apply here or that *the* colour blue I am seeing now is the very same colour blue I saw yesterday. Shapes and colours, geometry and gravity, persons and pyramids are all manifestations, or as Whitehead would say "ingressions"[4], of eternal objects.

The second function of eternal objects is to ground the apparent openness of events. According to Whitehead the passage of events is not a deterministic process with past events churning out future ones like a machine, but is a selection from amongst a set of real possibilities. Eternal objects are just these possibilities and must be accorded a degree of reality even if never instantiated in fact. They are “pure potentialities” that may

4 Whitehead uses the word “ingression” in place of words like “instantiation”, “realisation” or “manifestation”. He defines it in *Process & Reality* as referring to “the particular mode in which the potentiality of an eternal object is realized in a particular actual entity, contributing to the definiteness of that actual entity. p.23.

or may not come to be, leaving room in the world for free will and choice. Without them there would just be a "static monistic universe".[5]

Whitehead's platonic forms also figure centrally in his account of the geometry of space-time which he says is underpinned by an abstract realm of ideal "regions" called the "extensive continuum". The structure of this ideal continuum is described by a set of mereotopological axioms given in his theory of "extensive connection" in Part IV of *Process and Reality,* a set from which he purported to logically construct the fundamental geometric entities; points, lines, planes and higher dimensional figures, essential to physics and science as we know them. The extensive continuum, like Plato's 'Receptacle' is thus a kind of substructure grounding the geometrical properties of events in spacetime and enabling their objective measurement and comparison.

However, Whitehead's mereology had originally been formulated in the period between 1914 and 1920 in support of his view of space-time as an abstraction from or a relation between events as the ontological ultimates. In this period Whitehead argued that the orthodox theory of space as an aggregate of points, led to apparently unacceptable paradoxes and contradictions. For example, no matter how dense a collection of points is, it could never have the properties required for force and motion to be continuously transmitted from one place to another - ordinary phenomena like the transmission of stress across a surface would be impossible because isolated points can never touch. The concept of instantaneous velocity might be useful in calculus, but the idea of an object being accelerated zero metres in zero seconds was for Whitehead metaphysically incomprehensible. Points were therefore fictitious, albeit useful, products of the imagination which philosophy was obliged to uncover as abstractions from the true ontological ultimates.

In support of this nominalistic program Whitehead developed the method of extensive abstraction by which he purported to show that space-time points and other geometric elements could be logically constructed from infinite sets of mereologically ordered events. In this period Whitehead could be said to applying a well-worn method he and his collaborator Bertrand Russell had used many times in *Principia Mathematica* to logically construct abstract entities like numbers. Indeed Whitehead's original inquiries about the mereological construction of points had been in connection with his research for an

[5] *Process & Reality*, p.46.

intended fourth volume of *Principia Mathematica* on geometry. In this period Russell outlined a philosophy of logical empiricism in books like *The Problems of Philosophy* (1912) and *Our Knowledge of the External World* (1914) which seemed to overlap with Whitehead's own attitude at this time, a philosophy in which abstract entities were constructible from sense data. The virtue of logical construction over supposition Russell argued was like that of honest toil over theft, and in a famous remark he summarised their common outlook with the slogan, "that whenever possible, logical constructions are to be substituted for inferred entities." [6]

However, after 1920 the Platonistic tendencies in Whitehead's thinking became stronger. By the time of *Process and Reality* in 1929 the ideal substructure of the extensive continuum is said to underpin the whole dynamic play of events. On the one hand, most of the book outlines Whitehead's idea of nature as a "creative advance" in which "the many become one and are increased by one", i.e. an ontology in which events feed into and are transformed by other events in an ever-evolving dialectic between the momentum of the past and the urge towards the new. On the other hand however, Whitehead thinks of extension and the extensive continuum as an eternal, unchanging realm of forms governing the very geometry space-time. But it has been said that the chapter on extensive continuum in Part IV intrudes into *Process & Reality* like a "foreign body" with a certain "conceptual isolation" from the rest of the treatise[7] whose main notions like "extensive connection", "extensive part", and "region" are not even mentioned in *The Categoreal Scheme* set out in *Chapter II* which is meant to outline the architectonic of the whole work.

The prominence Whitehead gave to Part IV reflected an attitude that "extension is really the spatialisation of extension; and the extensiveness of time is really the temporalization of extension".[8] In other words Whitehead thought of the extensive continuum as somehow preceding actual events. And yet, in later works Whitehead could also say the seemingly opposite; "that the extension of space is the ghost of transition".[9] There is thus a deep ambiguity permeating Whitehead's scheme as to whether 'extension' or 'process' is primary, an ambivalence summed up in a statement he made in *Modes of Thought*

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6 Russell, R. *The Relation of Sense Data to Physics*, p.164.

7 Ringel, C. "Whitehead's Theory of Extension", p.3.

8 *Process & Reality,* p.289.

9 *Modes of Thought*, p.96.

published some years later in 1938, that “extension is derivative from process **and** is required by it”. [10] Both parts of that conjunction however cannot be true. Either extension is derivative from process and is not required by it, or extension is required in which case it cannot be derivative.

Whitehead's platonism therefore results in formulations that are often laboured if not downright obscurantist, an observation which applies to many other areas within his corpus, a proposition I aim to demonstrate throughout this thesis. There is nonetheless buried within *Process & Reality* the incipient idea of truly one-category ontology in which processes alone can account for the world as we know it without the categorical staples of Western philosophy since Plato and Aristotle, *Form* and *Substance*. This one-category process ontology is the philosophical gem shrouded within the obscurities of Whitehead’s platonistic system which this thesis also aims to examine. The thesis is broken down into five chapters.

**Chapter 1** gives a detailed account of the development and final presentation of Whitehead’s mature process ontology focusing on his platonism and notion of the extensive continuum.

**Chapter 2** critically examines the theological and philosophical justifications which Whitehead gave for his platonistic conception of eternal objects and the extensive continuum. It looks at the interpretational difficulties Whitehead’s platonism has created and how they can be resolved by developing that strand in his thinking which prioritises the concrete, particular and relational nature of events and processes. I argue that Whitehead had a notion of relational tropes that can be developed into a robust alternative to his eternal objects and a notion of probability that makes his “pure potentialities” superfluous.

**Chapter 3** looks at the deleterious effects of platonism on Whitehead’s alternative theory of general relativity, and puts the case that certain lines of thought in Whitehead suggest a process-oriented account of space-time more compatible with Einstein’s theory.

**Chapter 4** examines the historical antecedents to Whitehead’s platonism looking at its roots in Plato’s *Timaeus*, Leibniz, German *Naturphilosophie* and the philosophy of mathematics propounded by the mathematician Hermann Grassmann.

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10 *An Enquiry*, 1925 edition, in Note II, commenting on limitations of his earlier conception of extension, p.202.

**Chapter 5** critically engages with the details of Whitehead's mereotopology and his method of extensive abstraction highlighting some flaws in his reasoning and tracing some of them to his platonism about space.

# Chapter 1. The Evolution of Whitehead's Philosophy 1914 - 1935

## 1.1 Whitehead's Early Empiricistic Outlook

In his 1914 book *Our Knowledge of the External World* book Bertrand Russell attributes his definition of points and instants to Whitehead, saying his ideas were to form the basis of their projected fourth volume of *Principia Mathematica* on geometry. Russell went on to say that he owed to Russell not only his definition of points, but "the whole conception of the world of physics as a *construction* rather than an *inference".*[1] The planned fourth volume never appeared. Russell had first presented Whitehead's ideas on the mereological construction of points in March that year during the Lowell Lectures in Boston, drawing, much to Whitehead's consternation[2], on drafts of a paper which Whitehead had written called *The Relational Theory of Space.*[3] Whitehead eventually delivered that paper to the First Congress of Mathematical Philosophy in Paris on 8 April 1914. A French translation was later published in 1916 in the *Revue de Metaphysique et de Morale*, an English translation of which was published in 1978 by Patrick Hurley.[4]

The 1914 paper expresses the germs of a causal type theory of space that "we must not consider these physical bodies as existing first in space, then acting on one another, directly or indirectly. They are in space because they act on one another, and space is none other than the expression of certain properties of their interaction."[5]

> Geometry, as a mathematical theory, customarily takes for its point of departure all or part of the fundamental spatial entities: points, straight or curved lines, surfaces, and volumes. It takes them as simple, primitive ideas; that is, in abstract language, as

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1 Russell, B. "Preface", Our Knowledge of the External World, p.8. Whitehead's 1906 essay, Mathematical Concepts of the Material World was perhaps the beginning of his avowedly "Leibnizian" or relationist view of space in which "points are classes of objective reals, [that] disintegrate from instant to instant". He recalled later in 1920 that at that time he had already come to the conclusion that the ideas of point and relational space were inconsistent. Whitehead, A.N. "Einstein's theory: An alternative suggestion. The Times Educational Supplement, 12 February, 83, 1920. Referred to in Grattan-Guinness, p.434.

2 In his autobiography Russell attributes this incident as well as their growing differences on the war as having "put an end to our collaboration". In a letter he wrote in 1917 Whitehead expressed his dissatisfaction at the premature release of his ideas because it would "queer the pitch for the final exposition". p.101.

3 In the application document to the Royal Society Whitehead projected the fourth volume would contain chapters on projective, descriptive and metrical geometry and one on "the Constructions of space". See Gratten-Guiness, *The Search for Mathematical Roots,* p.414.

4 For an account of these events see Hurley, P. "Russell, Poincare and Whitehead's 'Relational Theory of Space'. Professor Hurley made an English translation of Whitehead's paper in 1978 which was published on microfiche in the Archives for Philosophical Research in 1978. Janet Fitzgerald published her own translation in her book *Whitehead's Early Philosophy of Space and Time* the following year. I would like to thank Professor Patrick Hurley for providing me with a copy of his translation and notes.

5 "The Relational Theory of Space", p.41.

> "variables" which are not logically functions of simpler variables. But if the relational theory of space is adopted, whether for the apparent world or for the physical world, this cannot be the first stage of geometrical research. For the relational theory of space it is essential that points, for example, be complex entities, logical functions of those relations between objects which constitute space. For, if a point is a simple thing, incapable of being logically defined by means of relations between objects, then points are, in fact, absolute positions. … Thus, the primary occupation of geometricians investigating the foundations of their science is to define points as a function of relations between objects.[6]

Whitehead continued slowly working out the details of his new theory in several papers over the next few years and in a paper read to a meeting of the British Association in 1915 entitled, *Space, Time & Relativity* he argued that the task of constructing points was "an unwritten chapter of mathematics, in much the same state as was the theory of parallels in the eighteenth century"[7].

> ...there are certain relations between events which we express by saying that they are relations between the temporal durations of these events, that is, between the temporal extensions of the events. [The durations of two events A and B may one precede the other, or may partially overlap, or may one contain the other, giving in all six possibilities.] The properties of the extension of an event in time are largely analogous to the extension of an object in space. Spatial extensions are expressed by relations between objects, temporal extensions by relations between events. The point in time is a set of relations between temporal extensions. It needs very little reflection to convince us that a point in time is no direct deliverance of experience. We live in durations, and not in points. But what community, beyond the mere name, is there between extension in time and extension in space ? In view of the intimate connection between time and space revealed by the modern theory of relativity, this question has taken on a new importance. I have not thought out an answer to this question.[8]

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6 "The Relational Theory of Space" p.40.

7 *Space, Time & Relativity* (1915), p.197

8 *Space, Time & Relativity* (1915), pp.197-198.

By 1919 Whitehead had thought out an answer, and in his book *An Enquiry Concerning the Principles of Natural Knowledge,* he came to the conclusion that the concepts of space, time and material objects were all abstractions from events. The logical procedure he had first canvassed in the 1914 paper for constructing points he now dubbed "the method of extensive abstraction" and he applied it to events rather than material objects to derive points in space-time. Whitehead purported to demonstrate how "the exactly determined concepts on which the whole fabric of science rests"[9] are based on the principle of "convergence to simplicity with diminution of extent"[10] and thus how space and time in the "new world of thought" opened up by Lamor, Lorentz, Einstein and Minkowski "are the first outcome of the simplest generalizations of experience..." [11]. The concepts of space and time were said to rest upon the more fundamental metaphysical fact of extension between events.

> An event $x$ may 'extend over' an event $y$, i.e. in other words $y$ may be a part of $x$. The concepts of space and time in the main, though not entirely, arise from the empirically determined properties of this relation of extension. It is evident from the universal and uniform application of the spatio-temporal concepts that they must arise from the utilization of the simplest characteristics without which no datum of knowledge would be recognised as an event belonging to the order of nature. Extension is a relation of this type. It is a property so simple that we hardly recognise it as such - it of course is so. Thus the event which is the passage of the car is a part of the whole life of the street. Also the passage of the wheel is part of the event which is the passage of the car....every element of space or of time (as conceived in science) is an abstract entity formed out of this relation (in association at certain stages with the relation of cogredience) by means of a determinate logical procedure (the method of extensive abstraction).[12]

Whitehead's method of extensive abstraction in the above works evinces a kind of hard-nosed empiricist attitude; as a means of de-constructing ideal entities and illustrating them as abstractions from our experience of finite series of concrete phenomena. Indeed the idea of 'logical construction' was part and parcel of his and Russell's program in *Principia*

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[9] *An Enquiry*, p.96.

[10] *An Enquiry*. p.96

[11] *An Enquiry*. pp.v - vi.

[12] *An Enquiry* pp.74-75.

*Mathematica* to reduce mathematics to logic or of demonstrating that mathematical entities like numbers were really shorthand for more complex formulations in logic and set theory involving for example isomorphic relations between sets. Russell had been applying this technique since about 1900, applying it to Peano's axioms for arithmetic discovering definitions for *0*, *number*, *successor*, and "the", which Peano had treated as primitive. He had also used his theory of definite descriptions in his famous paper "On Denoting" in the pages of *Mind* to deconstruct fictional objects like "the present king of France" or "the round square" which others like Meinong had argued were in some sense really 'subsisting' things.

In a similar vein Whitehead saw geometrical elements in this period as ideals which are only ever approximated to in practice. For example, in a paper for the *Proceedings of the Aristotelian Society* called "Time, Space and Material: Are They, and if so in What Sense, the Ultimate Data of Science"*?* also published in 1919, Whitehead described the set of events out of which points and instants were constructed as "routes of approximation" towards "non-existent ideals".

> A moment of a time-system is a route of approximation to the *non-existent ideal* of a duration without temporal extension. This route is composed of an infinite series of durations, extending over each other, the earlier in the series over the later, and so that there is no duration which they all cover. Such a series defines an instant of time, and will be here called a moment. All observation which endeavours to gain accuracy by instantaneousness is comprised within a duration as far down a momental series as possible, and is dated at that moment. Thus "nature at a moment" is the *ideal simplicity* of natural relations to which we approximate as we proceed along that momental series.[13]

Importantly, Whitehead did not just argue that all experience is of extended durations but of durations stitched together into continuous streams. This was the beginning of his *process* ontology, an ontology of events necessarily linked together into clusters and networks as opposed to mere collections. At first Whitehead tries to define this interlinking in mereological terms offering us the concept of 'extension in the making' as that process by which old events become **part of** the new.

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[13] *Time, Space, Material*, p.47. Emphases mine.

> Events never change. Nature develops, in the sense that an event $e$ becomes part of an event $e'$ which includes (extends over) $e$ and also extends into the futurity beyond $e$. Thus in a sense the event $e$ does change, namely in its relations to the events which were not and which become actual in the creative advance of nature....Thus we say that events pass but do not change. The passage of an event is its passing into some other event which is not it. An event in passing becomes part of larger events; and thus the passage of events is extension in the making.[14]

However, the conformance of say one stage of an embryo to a later one is not a simple matter of the first becoming part of the second but of one exercising causal influence over the other and so determining its character or form to some degree. It might also be construed as a case of one being a continuous topological transformation of the other. In either case, Whitehead was alluding to a relationship between events that was to become central to his later metaphysics, one which cannot be defined in mereological terms alone. To jump ahead slightly to the topic of the next chapter, by 1927 Whitehead was talking about this 'conformation' relation between events as a primary element of experience. In his book *Symbolism: Its Meaning and Effect,* he argued that experience is never of isolated phenomena or "simple occurrences" (bundles of "sense impressions" juxtaposed in simple serial order but of "causal efficacy" or "...the derivation of state from state, with the later state exhibiting conformity to the antecedent".

> My point is that this conformation of present fact to immediate past is more prominent both in apparent behaviour and in consciousness, when the organism is low grade. A flower turns to the light with much greater certainty than does a human being, and a stone conforms to the conditions set by its external environment with much greater certainty than does a flower. A dog anticipates the conformation of the immediate future to his present activity with the same certainty as a human being. When it comes to calculations and remote inferences, the dog fails. But the dog never acts as though the immediate future were irrelevant to the present. ”[15]

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[14] *An Enquiry*, p.62.

[15] *Symbolism*, p.41.

## 1.2 The Nature of Process in Whitehead's Later Works

### 1.2.1 Actual Occasions

By 1925 Whitehead's notion of conformation between events, or of events as determinants in the synthesis of others, had assumed greater importance. In the second edition notes to *An Enquiry* he even criticises his earlier view that extension was primary.

> The book is dominated by the idea that the relation of extension has a unique preeminence and that everything can be gotten out of it. During the development of the theme it gradually became evident that this is not the case, and cogredience had to be introduced. But the true doctrine that 'process' is the fundamental idea was not in my mind with sufficient emphasis. *Extension is derivative from process*, [my emphasis] and is required by it.[16]

In another publication of that year, *Science and the Modern World*, the relation between events which he had referred to in earlier works as 'conformation' and 'passage' was spoken about as "prehension". Prehension is first defined in a discussion of Francis Bacon's notion[17] that "all bodies whatsoever, though they have no sense,... have perception; for when one body is applied to another, there is a kind of election to embrace that which is agreeable, and to expel that which is ingrate"[18] a property which Bacon saw as exhibited for example by a weatherglass indicating the approach of a storm or a loadstone pointing towards magnetic north. A prehension is the means by which one feature or aspect of a thing is received or 'experienced' by another as well as the expression of that other in it. Whitehead defines prehension as "non-cognitive apprehension", and as the means by which there is a "gathering of things into ... unity"[19]. 'Prehension' is a term of art which Whitehead invented drawing on the Latin root 'Prehendere' meaning to catch/capture, seize/grasp, or apprehend/comprehend.[20] It is the root of words like "prehensile".

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16 *An Enquiry*, 1925 edition, Note II, p.202. By "cogredience" Whitehead means one event being ingredient in another as a spatio-temporal part of it. He seems to be alluding here to the problem that mereological axioms alone are insufficient for defining what we usually take to be continuity. More detailed discussion of the technicalities is left to Chapter 5.

17 In Bacon's *Natural History.* Bacon goes on to say "...and sometimes this perception ... is far more subtile than sense, so that sense is but a dull thing in comparison of it. We see a weatherglass will find the least difference of the weather in heat or cold when we find it not...[and] is sometimes at a great distance as when a loadstone draweth iron...[and are thereby] ... principal means for natural divination; for that which appeareth in these perceptions early in the great effects cometh after" quoted in *Science & the Modern World*, pp.41-42.

18 *Science and the Modern World*, p.69, full quote on p.41.

19 *Science and the Modern World*, p.69.

20 Helmut Maaßen p.1.

In the mature theory of *Process & Reality* prehension becomes the primary relation between ultimate evental units he calls "actual occasions"[21] or "actual entities". Each occasion via prehension, "mirrors within itself the *modes* of its predecessors, as memories which are fused into its own content"[22]. Actual occasions are unrepeatable particulars and similar to classical atoms in being ultimate but extended parcels of actuality. They are "the final real things"[23] or "res vera". But unlike atoms, actual occasions do not meander alone in the void, their relations to others are internal to their very being. An occasion stands to past occasions as a final commodity stands to the earlier stages of its manufacture; "the process itself is the constitution of the actual entity".[24]

### 1.2.2 Whitehead's Categoreal Scheme

At this point, a methodological issue needs to be mentioned that is important for interpreting what Whitehead means by 'actual occasions' and the relation of prehension. Whitehead's category of 'actual entity' is only one among several in his "categoreal" scheme, the most important of which is Creativity which he calls 'the Category of the Ultimate', or the "universal of universals". The next most important categories are grouped together in a class called the "Categories of Existence" which include 'actual entities', 'prehensions', 'nexus', 'subjective forms', 'eternal object', 'propositions', 'multiplicities' and 'contrasts'. I shall explore some of the other categories later but it need only be noted that "actual entities and eternal objects stand out with a certain extreme finality. The other types of existence have a certain intermediate character".[25]

As James Bradley [26] has argued, we need to keep in mind here Russell's comment after reading *Process and Reality*, that Whitehead, "had always had a leaning toward Kant",[27] as well as Whitehead's own express endorsement of Kant's transcendental methodology. In *Process & Reality*, Whitehead argues that he is attempting to reveal the conditions that make subjective experience of any kind, i.e. *experience as such*, possible. Thus in talking

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21 It is interesting to note that the word 'actual' derives from late Latin word like 'āctuālis', 'āctu', and 'āctus', forms of the infinitive meaning, 'to act', the root of other agentive type words like 'action', 'acting' etc.

22 *Science and the Modern World*, p.72.

23 *Process & Reality*, p.18

24 *Process & Reality*, p.219.

25 *Process & Reality*, p.22.

26 Bradley, J. "'The Critique of Pure Feeling': Bradley, Whitehead and the Anglo-Saxon Tradition", "Transcendentalism and Speculative Realism in Whitehead", and "The Speculative Generalization of the Function: A Key to Whitehead".

27 Russell, B. *Portraits from Memory. and Other Essays*. pp.102.

about actual occasions, Whitehead is not primarily making an empirical claim about microscopical entities at the base of the world, but mounting a transcendental[28] argument for certain universal categories of *experience.* [29] Although he often says his philosophy is a reversion to pre-Kantian modes of thought, Whitehead is definitely a Kantian in respect of methodology because he is attempting to reveal the universal conditions of subjectivity or experience as-such. A key phrase which opens up the door to Whitehead's thinking on this point is his statement that he "aspires to construct a critique of pure feeling, in the philosophical position in which Kant put his *Critique of* Pure *Reason"* and that Kant's *Transcendental Aesthetic*, was a distorted fragment of what should have been his main topic. [30]

Recall that in the *Critique of Pure Reason* Kant called the "Transcendental Aesthetic", "the science of all the principles of sensibility apriori"[31] by which he purported to prove that our experience of external objects depended upon what he called *pure intuitions*, viz those of 'space' and of 'time'. These were not things 'out there' experienced by the senses, but necessary pre-conditions of sensibility per se. The senses attributed incidental qualities to objects such as 'impenetrability', 'hardness', 'colour' and 'movement', whilst other faculties like the understanding attributed qualities such as 'substance', 'force' and 'divisibility'. But all of these qualities depended on more fundamental qualities like shape, size and location, which in turn implied the even more basic principles of 'space' and 'time'. The *pure intuitions* were therefore said to transcend all other categories, to be required by them, and making sensibility as such possible. Therefore, when Whitehead says he "aspires to construct a critique of pure feeling in the philosophical position in which Kant put his *Critique of Pure Reason*, he is saying that he too wishes to reveal the universal and necessary forms of experience and sensibility. His 'atomism' is therefore not not about the existence of certain natural kinds like quarks or electrons. His theory is not an exercise in physics to determine *what* there is, but an exercise in metaphysics to determine *how* experience must be. Although Whitehead does not pretend to deduce in a strict logical sense his own transcendental categories, they are he says, "generic notions inevitably

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[28] I use "transcendental" here in the Kantian sense of an argument which infers from the manifest features of experience certain universal forms which make it possible.

29 To reinforce the point, Bradley reminds us that when one of Whitehead's students once asked him how big an occasion was, Whitehead, with a smile, placed his thumb and his forefinger about an inch apart and replied "Oh, about so big". Bradley, J. p.171.

30 *Process & Reality*, p113.

31 *Critique of Pure Reason*, p.42.

presupposed in our reflective experience - presupposed but rarely expressed"[32]. His categorial scheme seeks to “frame a coherent, logical, necessary system of general ideas in terms of which every element of our experience can be interpreted", as "... bearing in itself its own warrant of universality…”[33].

So, an ‘actual occasion’ for him is an instance of a universal category of experience appropriate to many kinds of thing at many scales of existence. The examples he gives of them accordingly range widely from "electronic and protonic actual entities” to “yet more ultimate actual entities which can be dimly discerned in the quanta of energy"[34], from “God”[35] to "the most trivial puff of existence in far-off empty space",[36] from "macroscopic res verae through which the solidarity of the universe is effected” to "microscopic occasions whereby the world resolves itself into multiplicities”,[37] from "moments in the life-histories of enduring objects with conscious knowledge” [38] to the word 'United' in the utterance 'United States’.[39] Whitehead’s theory is therefore not really a species of physical atomism but what we might call a phenomenological or *experiential* atomism. He is not asserting the existence of some smallest unit of space-time or energy but that experience comes in discrete chunks to a finite degree of resolution. This seems an eminently reasonable assertion because as mortal creatures with finite senses and powers of observation we cannot experience anything with infinite precision but always in units that are dictated to us by our biology, technology etc., whether they be bits, pixels, words, light quanta or quarks. Such a view dovetails with Whitehead’s admiration for William James’ *radical empiricism* which was based on the simple observation that things like infinitesimals are inexperienceable and that real experience grows by definite "buds or drops of perception”.[40]

This kind of phenomenological atomism goes back at least to Whitehead's middle period texts as seen in the following passage from his 1920 work *The Concept of Nature*.

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32 *Process & Reality*, p.18.

33 *Process & Reality*, pp.3-4.

34 *Process & Reality*, p.91

35 *Process & Reality, p.*18

36 *Process & Reality, p.*18

37 *Process & Reality, p.107*

38 *Process & Reality, p.177.*

39 "Object and Subject", p.137.

40 *Process & Reality,* p.68.

> ...it would seem that every material entity is not really one entity. It is an essential multiplicity of entities. There seems to be no stopping this dissection of matter into multiplicities short of finding each ultimate entity occupying one individual point. This essential multiplicity of material entities is certainly not what is meant by science, nor does it correspond to anything disclosed in sense-awareness. It is absolutely necessary that at a certain stage in this dissociation of matter a halt should be called, and that the material entities thus obtained should be treated as units. The stage of arrest may be arbitrary or may be set by the characteristics of nature; but all reasoning in science ultimately drops its space-analysis and poses to itself the problem, 'Here is one material entity, what is happening to it as a unit entity? Yet this material entity is still retaining its extension, and as thus extended is a mere multiplicity. Thus there is an essential atomic property in nature which is independent of the dissociation of extension.[41]

His 1922 work *The Principle of Relativity* expresses a similar attitude regarding all perception and knowledge in a world where everything is inter-related.

> Observe that the practical atomicity of the physical and apparent characters is essential for the intelligibility of the apparent world to a finite mind with only partial perception. Without atomicity we could not isolate our problems; every statement would require a detailed expression of all the facts of nature. It has always been a reproach to those philosophers who emphasize the systematic relatedness of reality that they make truth impossible for us by requiring a knowledge of all as a condition for a knowledge of any.[42]

For Whitehead an atomic occasion is not further analysable into finer patterns or occurrences with equal measure of definiteness, which is to say that it is meaningless or at least impractical to talk about things happening in any definite way over shorter time-spans. For Whitehead the equably flowing time of physics, the ordered, comparative

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41 *The Concept of Nature,* pp.22-23.

42 *The Principle of Relativity*, p.73.

measure of relations *between* definite events does not apply to occasions in themselves.[43] One can of course imagine events to be divided without end[44] - the point however is that nature reveals itself to us through finite chunks which to all intents and purposes are indivisible.[45]

### 1.2.3 Prehension

While Whitehead's is an inherently relational philosophy it should not be mistaken for a holism like Hegel's in which everything is internally related to or dependent on everything else.[46] Firstly, prehension is not symmetrical between prehending and prehended occasions but is asymmetrical, or one-way, which is to say that if *x* is prehended by *y* then *y* is not prehended by *x*. The world is a series of interlinked directional processes in which the most recent events depend on those preceding them but not vice versa.

The other important quality of prehension is transitivity. Each occasion not only feels and incorporates qualities of past occasions but makes them available as objective "data" to those which succeed it. "It belongs to the nature of a being that it is a potential for every becoming"[47]. If occasion *x* is prehended by occasion *y* which is in turn prehended by occasion *z*, then occasion *z* will in some way indirectly sense or prehend occasion *x*. The transmissive nature of prehensions is clarified by Whitehead's frequent comparison of them to vectors.

> … in the creative advance, the nexus proper to the antecedent actual world is not destroyed. It is reproduced and added to by the new bonds of feeling with the novel actualities which transcend it and include it. But these bonds have always their vector character. Accordingly the ultimate physical entities for physical science are

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43 The issue of whether the duration of an occasion in and of itself can be analysed into smaller temporal durations has tied many a commentator into metaphysical knots. Van Haefton for example argues that the atomicity of occasions has something to do with the subjective aim or final cause of occasions which entails they are extended but indivisible. Chappel, Sipfle, Bradford Wallack and Nobo all take different views on whether an occasion unfolds over physical time. But Whitehead stresses that apart from definite events, i.e. events with reasonably certain boundaries distinguishing them from others, there can be no meaningful knowledge of anything. Thus in *An Enquiry* he asserts that despite the difficulty and arbitrariness of assigning definite boundaries to some events, the assumption of definiteness is absolutely essential to science and knowledge generally. He writes; "... it is a basal assumption essential for ratiocination relating to perceptual experience, that there are definite entities which are events. … It is the claim which is implicit in every advance towards exact observation, namely that there is something definite to be known" *An Enquiry*, p.74. It should be pointed out that later in this 1919 text Whitehead stresses that he is not propounding a theory of atomic events. I think he means that he is not propounding a physical atomism but a phenomenological one, a doctrine I think which he preserves in *Process & Reality.*

44 This notion of theoretical or hypothetical divisibility of occasions is discussed extensively in the sections dealing with "genetic" and "co-ordinate" division of the actual occasions which analyse the potential divisibility of occasions into temporal and spatial parts. These 'parts' are however described variously as "hypothetical", "quasi-", "might be", "indefinite" and "potential" subdivisions which is to say they exist ideally, at the level of the extensive continuum.

45 *Science and the Modern World*, p.103-104.

46 Some commentators like Nobo argue that in Whitehead's universe the cosmos is a kind of holograph in which everything is prehended by everything else. I disagree with this interpretation and set out more reasons in Appendix 2.

47 *Process & Reality*, p.22.

> always vectors indicating transference. In the world there is nothing static. But there is reproduction; and hence the permanence which is the result of order, and the cause of it. And yet there is always change; for time is cumulative as well as reproductive, and the cumulation of the many is not their reproduction as many.[48]

One important property about prehension that should also be mentioned is that it is not a relation defining a total order on all the occasions in the universe, which is to say that it is not the case that of any two occasions x and y, either x prehends y or y prehends x. Occasions do not prehend everything in their 'past', there are some occasions they don't prehend (what Whitehead called the "negative prehension" of "zero relevance"[49]) and some they prehend more intensely than others. For many if not all occasions there comes a point when the degree of their prehension by new occasions fades into utter insignificance and the universe (with the exception of God) forgets they ever existed.

### 1.2.4 Creativity

But the above passage brings out another dual aspect of occasions. On the one hand "there is reproduction" but on the other "there is always change". There is "the cumulation of the many" but not "their reproduction as many". What Whitehead is saying is that actual occasions are not reducible to the sum of their prehensions but active interpretations of the "data" given to them from the past - or as he sometimes put it, both subjects and "superjects", entities which arise from but *super*sede their objectively given conditions. Each occasion is a novel synthesis of the many influences to which it has been subject, a new particular arising from the fusion and integration of its own prehensions. This is expressed in his oft-quoted statement below.

> The ultimate metaphysical principle is the advance from disjunction to conjunction, creating a novel entity other than the entities given in disjunction. The novel entity is at once the togetherness of the many which it finds and it is also one among the disjunctive many which it leaves ... The many become one, and are increased by one. In their natures, entities are disjunctively many in process of passage into conjunctive unity.[50]

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48 *Process & Reality*, p.238.

49 *Process & Reality*, p.148.

50 *Process & Reality*, p.21. There is strong circumstantial evidence that Whitehead was responsible for introducing the word "creativity into the English language. Although there is one use of it in 1875, the word was not listed in any dictionary until 1961. The intellectual historian Paul Oskar Kristeller in his 1983 paper "Creativity and Tradition", *Journal of the History of Ideas*, 44:1, (1983), p.105, concluded that it did not gain wide currency until Whitehead's 1927 work *Religion in the Making*. The word did not make it into the OED until 1989. See Stephen Myer's paper in the journal *Configurations*. Other neologisms, such as "prehension" and "concrescence," have much older attestations, even going back to the 1640s. see Lewis S. Ford, in "Whiteheads Intellectual Adventure".

As discussed above, Whitehead makes the notion of creativity the pinnacle of his whole system, it is the 'The Category of the Ultimate'. He says that in all philosophies, "there is an ultimate which is actual in virtue of its accidents. It is only then capable of characterisation through its accidental embodiments, and apart from these accidents is devoid of actuality. In the philosophy of organism this ultimate is termed 'creativity'." [51] In earlier works of Whitehead's this creative impulse was spoken about as "extension in the making"; that process by which new events are added to the pre-existing stock of past events. In *Process & Reality* however, more emphasis is placed on the generation of novelty in what he called "the creative advance"; the coming into being of 'entities diverse from any entity in the 'many' which they unify'.[52] Thus while prehension both preserves and transmits forms from the past into the present, occasions are more than just mechanical reproductions, they are novel becomings in their own right adding something entirely new to the world.

### 1.2.5 Potentiality

Whitehead's notion of creativity is closely bound up with his notion of potentiality. He says that each occasion retains "the "message of *alternatives* which [have been] avoided"[53], which is to say that 'red might have been green, or what is loved might have been coldly esteemed'. Whitehead argues that the creative advance is a process of selecting and discarding possibilities.

> The notion of potentiality is fundamental for the understanding of existence, as soon as the notion of process is admitted. If the universe be interpreted in terms of static actuality, then potentiality vanishes. Everything is just what it is. Succession is mere appearance, rising from the limitation of perception. But if we start with process as fundamental, then the actualities of the present are deriving their characters from the process, and are bestowing their characters upon the future. Immediacy is the realisation of the potentialities of the past, and is the storehouse of the potentialities of the future. Hope and fear, joy and disillusion, obtain their meaning from the potentialities essential in the nature of things. We are following a trail in hope, or are fleeing from the pursuit in fear. The potentialities in immediate fact constitute the driving force of process.[54]

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51 *Process & Reality*, p.7.

[52] *Process & Reality*, p.21.

53 *Process & Reality*, p.149.

[54] *Modes of Thought*, p.99.

Whitehead's 'storehouse of potentialities' therefore entails a kind of modal realism that is central to his idea of process. He calls this realisation of potentialities, “concrescence”. Concrescence is the transition from contingency to necessity, from the mere possibility of a thing to that thing as a necessary fact in the constitution of subsequent entities. These "pure potentialities" can be repeatedly instantiated in different combinations other than the ones in which they have actually appeared, such that we can imagine for example the process that was the life of Plato instantiating a property other than that of wisdom, or the essence of Plato being instantiated at different times or worlds. Thus pure potentialities are not just abstractions but real and necessary factors in the constitution of actual entities.

> ... as soon as we abstract, so as to separate the notions of serial forms and of individual facts involved, we necessarily introduce the notion of potentiality: ... We say in effect, such and such facts are consistent with such and such serial forms. We are considering possibilities for individuals and possibilities for series. The mere immediate exemplification is only one aspect of our experience.[55]

### 1.2.6 Theory of Forms or “Eternal Objects”

Whitehead's pure potentialities are essentially the same as Plato's forms. They are abstract things whose hallmark is their generality or repeatability; the fact they can be shared by many occasions and transferred from one to another. In *Science and the Modern World* Whitehead christens them with the name "eternal object". They are vital to his conception of process because they tie the present to the past. Each event "mirrors within itself the *modes* of its predecessors, as *memories* which are fused into its own content".[56] An eternal object is said to transcend actuality as a "mere potential for the specific determination of fact"[57].

As early as 1919 Whitehead had spoken of the abstract objects which are "ingredient" in a stream of events as making up the “character” of that stream, and as making possible the comparison and identification of permanences ‘within’ it. “We are comparing objects in events whenever we can say, ‘there it is again’. Objects are the elements in nature which can be again"[58], they are "*recognita* amid events".[59] In these earlier works however

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55 *Modes of Thought*, p.99.

56 *Science and the Modern World*, p.72. [emphases mine].

57 *Process & Reality*, p.22.

58 *The Concept of Nature*, p.144

59 *An Enquiry*, p.82.

Whitehead had more of an empiricist attitude about such purported objects. In his 1920 work *The Concept of Nature* for example, he says that the idea of a self-same object enduring over time is a "pure illusion" abstracted from the flux, that the more permanent a thing is the more we are abstracting away from change and pragmatic context. He uses Cleopatra's Needle on the Charing Cross Embankment as an example.

> The static timeless element in the relation of Cleopatra's Needle to the Embankment is a pure illusion generated by the fact that for purposes of daily intercourse its emphasis is needless. What it comes to is this: Amidst the structure of events which form the medium within which the daily life of Londoners is passed we know how to identify a certain stream of events which maintain permanence of character, namely the character of being the situations of Cleopatra's Needle. Day by day and hour by hour we can find a certain chunk in the transitory life of nature and of that chunk we say, "There is Cleopatra's Needle". If we define the Needle in a sufficiently abstract manner we can say that it never changes. But a physicist who looks on that part of the life of nature as a dance of electrons, will tell you that daily it has lost some molecules and gained others, and even the plain man can see that it gets dirtier and is occasionally washed. Thus the question of change in the Needle is a mere matter of definition. The more abstract your definition, the more permanent the Needle. But whether your Needle change or be permanent, all you mean by stating that it is situated on the Charing Cross Embankment, is that amid the structure of events you know of a certain continuous limited stream of events, such that any chunk of that stream, during any hour, or any day, or any second, has the character of being the situation of Cleopatra's Needle.[60]

In scientific practice too, the degree to which one accorded permanence and substantiality to natural phenomena varied with pragmatic purpose. An electron for example can be thought of as either a distributed or a localised phenomenon depending on context. Although an electron is in some ways "an extremely small sphere in a ... test-tube"[61], it is also, according to Faraday's field theory, "an electric charge [that is] is everywhere".[62]

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60 *The Concept of* Nature, pp.166-167.

61 *The Concept of Nature*, p.147.

62 *The Concept of* Nature, p.146.

There is not just a single isolatable thing but the ingression of an electronic object into a "society of electronic occasions" [63] as well.

> The electron is not merely where its charge is. The charge is the quantitative character of certain events due to the ingression of the electron into nature. The electron is its whole field of force. Namely the electron is the systematic way in which all events are modified as the expression of its ingression.[64]

Thus the idea of an apparently simple entity like an electron is the result of abstracting away from a whole field or "situation" of events. But abstractness does not necessarily signify nothing. "It merely means that its existence is only one factor of a more concrete element of nature. So an electron is abstract because you cannot wipe out the whole structure of events and yet retain the electron in existence...the molecule is really in the event in the same sense as the grin is really on the cat's face".[65] Whitehead's reference here to Lewis Carroll's Cheshire Cat is telling, he is saying that properties like permanency and locality are to some extent artifacts of the imagination.

By the time of *Science and the Modern World* published in 1925 however, Whitehead began thinking of permanencies as less like the 'grin on the Cheshire Cat' and more like the super-sensible *Ideas* of Plato. To signify this shift he christens abstracta "eternal objects" and begins asserting they are above the vagaries of time and change; "a colour is eternal. It haunts time like a spirit. It comes and it goes. But where it comes it is the same colour. It neither survives nor does it live"[66]. Borrowing a line from Wordsworth, Whitehead compares their transcendental character to "The light that never was, on sea or land"[67]. In *Process and Reality* eternal objects are defined as "pure potentials for the specific determination of fact"[68], as "potentials for the process of becoming"[69] which "involve in their own nature indecision" because "they do not disclose in what entities this potentiality of ingression is realised"[70]. Whitehead however wants to retain the idea that

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63 *Process & Reality,* p.91.

64 *The Concept of Nature*, p.159.

65 *The Concept of Nature*. p.171.

66 *Science & the Modern World* p.87.

67 Science & the Modern World p.87.

68 *Process & Reality*, p.22.

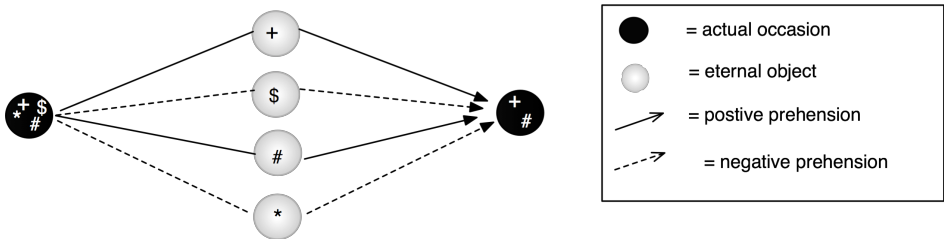
69 *Process & Reality*, p.29.

70 *Process & Reality*, p.29.

eternal objects are grounded in actuality in some way because they must by definition be potentially actualisable in it. In complete abstraction from actuality eternal objects are said to be "mere undifferentiated non-entities".[71]

Whitehead's theory of eternal objects and their function in the process of concrescence is illustrated in the following diagram.

**Diagram 1. Concrescence**




In the above diagram the half shaded circles represent eternal objects, dashed arrows; paths never taken or (negative prehensions), fully shaded circles; actualised occasions, and solid arrows; paths chosen (positive prehensions). Forms are 'transmitted' from occasion to occasion by the vector-like relation of prehension.

### 1.2.7 Whitehead's Onto-theology

In a chapter of *Science and the Modern World* entitled "Abstraction" Whitehead argues that the totality of eternal objects constitutes a 'realm' because they are all internally related to each other, so that the "analytical character" of this realm is its "primary metaphysical truth". [72] Each eternal object has its position in a "general systematic complex of mutual relatedness".[73] By 'analytic mutual relatedness' Whitehead explains that eternal objects are organised into objectively given structures which he calls "abstractive hierarchies". Abstractive hierarchies are the subject of formal sciences like logic, mathematics and geometry. An abstractive hierarchy is composed at its base of un-analysable first order forms which combine under certain relations into more complex second order objects, which are themselves combined under higher relations into third order objects, and so on. For a second order object Whitehead gives the example of three definite colours (A, B and C) in the trivalent geometric relation R holding between any three colourable faces of a regular tetrahedron. The complex abstract object R(A,B,C) is

71 *Process & Reality*, p.257.

72 *Science and the Modern World,* p.163.

73 *Science and the Modern World,* p.161.

then the three colourable faces of any tetrahedron coloured A, B and C. Each actual event was said to instantiate an “associated abstractive hierarchy”.

In themselves however eternal objects and abstractive hierarchies are all equally valid there being no reason why golden mountains, square circles and green algorithms couldn’t in principle be realised in actuality. Some additional factor is required so that genuine order and “aesthetic achievement” can find their way into an actual world that would otherwise be chaos. The entity which plays this role for Whitehead is God, whose “consequent” function is to select from the welter of infinite potentialities those most suitable for ingression into the course of events according to overarching ideals like harmony and beauty. God’s role in evaluating combinations of eternal forms for realization is summed up in the following passage.

> ... the actualities constituting the process of the world are conceived as exemplifying the ingression (or 'participation') of other things which constitute the potentialities of definiteness for any actual existence. The things which are temporal arise by their participation in the things which are eternal. The two sets are mediated by a thing which combines the actuality of what is temporal with the timelessness of what is potential. This final entity is the divine element in the world, by which the barren inefficient disjunction of abstract potentialities obtains primordially the efficient conjunction of ideal realization. This ideal realization of potentialities in a primordial actual entity constitutes the metaphysical stability whereby the actual process exemplifies general principles of metaphysics, and attains the ends proper to specific types of emergent order. By reason of the actuality of this primordial valuation of pure potentials, each eternal object has a definite, effective relevance to each concrescent process. Apart from such orderings, there would be a complete disjunction of eternal objects unrealized in the temporal world. Novelty would be meaningless, and inconceivable.[74]

As he makes clear in the above passage, the “divine element” or the Godhead itself is a hybrid of the two realms “combining the actuality of what is temporal with the timelessness of what is potential” and is both concrete occasion and pure potentiality, a "non-temporal actual entity”. Thus eternal objects are said to “subsist outside the realm of actual events

74 Process & Reality, p.40.

in God's "primordial nature [constituting] the Platonic world of ideas".[75] Whitehead describes why eternal objects require somewhere outside actuality to "subsist", as follows:

> Everything must be somewhere; and here 'somewhere' means 'some actual entity.' Accordingly the general potentiality of the universe must be somewhere; since it retains its proximate relevance to actual entities for which it is unrealized. This 'proximate relevance' reappears in subsequent concrescence as final causation regulative of the emergence of novelty. This 'somewhere' is the non-temporal actual entity. Thus 'proximate relevance' means 'relevance as in the primordial mind of God'. … The notion of '*subsistence*' is merely the notion of how eternal objects can be components of the primordial nature of God. […] But eternal objects, as in God's primordial nature, constitute the Platonic world of ideas.[76]

However, Whitehead does not think the realm of eternal objects has ontological primacy over the realm of actuality. He criticised neoplatonism for example as a "puerile metaphysics"[77] for treating actual entities as mere emanations from the transcendental. For him the "actuality of the universe is merely derivative from its solidarity in each actual entity"[78] not something independent of them. In contrast to traditional idealisms, Whitehead stresses the *interdependency* and ontological *equivalence* of the transcendental and actual realms. "The key to metaphysics is this doctrine of mutual immanence, each side lending to the other a factor necessary for its reality".[79]

At the end of the day however Whitehead still posits a transcendent world of forms as necessary to explain the "permanence", and "potentiality" of actual events. "We inevitably presuppose this realm of forms in abstraction from passage, loss and gain" in "all our thoughts of what has happened and can happen" and that an eternal object as humble as the multiplication table "essentially [qualifies] the course of history, whenever it is relevant."[80]

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75 *Process & Reality*, p.46.

76 *Process & Reality,* p.46.

77 *Adventures of Ideas*, p.130.

78 *Process & Reality*, p.200.

79 *Essays in Science and Philosophy* quoted in McHenry, L. *Whitehead and Bradley: A Comparative Analysis*. p.159.

80 *Modes of Thought,* p.68.

### 1.2.8 The Ideal Continuum

Whitehead's distinction between potentiality and actuality plays an important role in his account of the difference between the continuous and the discrete. In earlier works Whitehead could be seen as adopting the view that even though events did not contain points they were still infinitely divisible. In *The Concept of Nature* for example he says that "every event contains other events as parts of itself" [81] and that accordingly there are "no minimum durations ... no atomic structure"[82]. However by the time of *Process & Reality*, Whitehead had come to the decidedly opposite point of view that actuality is atomic, an idea summed up in his somewhat cryptic statement that "Continuity concerns what is potential; whereas actuality is incurably atomic."[83] When he says that 'continuity concerns what is potential' he is saying that that which is infinitely divisible is ideal; i.e. eternal objects. Whitehead goes on to say in fact that underneath the world of discrete finitely divisible events there is an ideal infinitely divisible space called "the extensive continuum". But unlike Newton's space or the 19th Century ether, the extensive continuum is not as "actual as anything else"[84]. It does not "minimize the factor of potentiality"[85] but is a "scheme of real potentiality which is exemplified in the mutual prehension of all actual entities".[86]

> All actual entities are related according to the determinations of this continuum; and all possible actual entities in the future must exemplify these determinations in their relations with the already actual world. The reality of the future is bound up with the reality of this continuum. It is the reality of what is potential, in its character of a real component of what is actual.[87]

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81 *The Concept of Nature*, p.76.

82 ibid.p.59. In modern parlance spaces with this property have come to be known as "gunk". David Lewis coined the term "gunk" in his book *Parts of Classes*. The idea of a gunky space has a pedigree possibly reaching as far back as Aristotle (see Roeper, P. "The Aristotelian Continuum. A Formal Characterization" and Newstead, A.G.J. "Aristotle and Modern Mathematical Theories of the Continuum" which makes the case that Aristotle may, like Whitehead, have thought that mathematical objects were derived by a process of abstraction from our experience of extended entities (in his case material bodies).

83 *Process & Reality*, p.61. The idea that true continua exist only potentially was of course originally set out in Aristotle's *Physics* and *Metaphysics*. Leibniz's view also turns on the difference between the actual and the potential. It is strange that Whitehead did not refer to either Aristotle's or Leibniz's works on this issue. Russell sums Leibniz up as follows: "what is actual may have an infinite number of parts, these parts are not indeterminate or arbitrary, but perfectly definite (G. ii. 379). Only space and time are continuous in Leibniz's sense, and these are purely ideal. In actuals, he says, the simple is prior to the aggregate; in ideals, the whole is prior to the part". (G. Ii. 379).Russell, B. *A Critical Exposition of the Philosophy of Leibniz* p.59.

84 *Process & Reality*, p.70

85 *Process & Reality*, p.70

86 *Process & Reality*, p.76

87 *Process & Reality,* p.66.

Whitehead argues that the extensive continuum is a necessary fact of existence as we know it, uniting disparate experiences and events across the whole universe under a single framework making comparison and measurement on the basis of homogeneous and permanent standards possible.

> …It is by reason of this disclosure of ultimate system that an intellectual comprehension of the physical universe is possible. There is a systematic framework permeating all relevant fact. By reference to this framework the variant, various, vagrant, evanescent details of the abundant world can have their mutual relations exhibited by their correlation to the common terms of a universal system. … Apart from these relations as facts in nature, such science is meaningless, a tale told by an idiot and credited by fools. For example, the conjecture by an eminent astronomer, based on measurements of photographic plates, that the period of the revolution of our galaxy of stars is about three hundred million years can only derive its meaning from the systematic geometrical relations which permeate the epoch. But he would have required the same reference to system, if he had made an analogous statement about the period of revolution of a child's top.[88]

Thus Whitehead's extensive continuum is an an infinitely divisible space housing all the various potentialities capable of instantiation in the realm of actual events, an idea inspired perhaps by Plato's notion of the 'Receptacle' in the *Timaeus*. It contains "contrary potentialities" rather than "definite atomic actualities" [89] as well as those potentialities which occasions must possess to be instantiated. It is "the generic morphology of the internal relations which bind the actual occasions in a nexus..."[90]. These 'generic morphological' relations that Whitehead is talking about are more abstract even than geometrical or mathematical relations and concern only the most fundamental of mereological and topological properties.

> The extensive continuum is one relational complex in which all potential objectifications find their niche. It underlies the whole world, past, present and future. Considered in its full generality ...the properties of this continuum are very few and do not include the relationships of metrical geometry. An extensive continuum is a

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[88] *Process & Reality*, p.327.

[89] *Process &Reality* p.67

[90] *Process & Reality*, p.288

> complex of entities united by the various allied relationships of whole to part and of overlapping so as to possess common parts, and of contact and of other relationships derived from these primary relationships. The notion of continuum involves both the property of indefinite divisibility and the property of unbounded extension. There are always entities beyond entities, because non-entity is no boundary. This extensive continuum expresses the solidarity of all possible standpoints throughout the whole process of the world. It is not a fact prior to the world; it is the first determination of order - that is of real potentiality arising out of the general character of the world.[91]

The axioms and definitions regulating the extensive continuum are discussed in Part IV of Chapter II of *Process & Reality* in which he proposes that the fundamental entities are "regions", "united by the various allied relationships of whole to part and of overlapping so as to possess common parts, and of contact…". Whitehead then purports to derive the fundamental geometrical relations and entities like points, straight lines and flat surfaces from these more generic mereo-topological axioms. Whitehead summarises the ideal nature of these mereo-topological and geometrical elements when he writes:

> The most fundamental elements in this scheme are those eternal objects in terms of which the general principles of coordinate division itself are expressed. These eternal objects express the theory of extension in its most general aspect. In this theory the notion of the atomicity of actual entities, each with its concrescent privacy, *has been entirely eliminated* (my emphasis). We are left with the theory of extensive connection, of whole and part, of points, lines, and surfaces, and of straightness and flatness.[92]

Thus the extensive continuum is indifferent as regards the dimensionality, size and shape of actual occasions stipulating only that they they obey some high level mereo-topological rules about wholes and parts and in this particular cosmic epoch some less general geometrical axioms concerning points, lines, planes and 'flat' loci.

### 1.2.9 The Dis-continuum of Actual Events

Thus it is the ideal platonic space which is a true, infinitely divisible continuum whereas the realm of actual events is atomic or discrete. However, the realm of actual events is also a

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[91] *Process & Reality*, p.66.

[92] *Process & Reality*, p.292.

"becoming of continuity" such that "actual occasions ... constitute a continuously extensive world". What Whitehead means by this is that events themselves compose by their prehensions of each other a kind of interconnected manifold.

> Thus the ultimate metaphysical truth is atomism. The creatures are atomic. In the present cosmic epoch there is a creation of continuity... But atomism does not exclude complexity and *universal relativity*. Each atom is a system of all things.[93]

The idea of an evental network or manifold laid out in *Process and Reality* built on a train of thought stretching back to his earlier works. In *An Enquiry Concerning the Principles of Natural Knowledge* in 1919 for example he argued against the material ether upholding instead the relational complex of events as the basis of all continuity in nature.

> The material called ether is merely the outcome of a metaphysical craving. The continuity of nature is the continuity of events and the doctrine of transmission should be construed as the doctrine of the co-extensiveness of events with space and time and their reciprocal interaction ...Something is going on everywhere and always"[94]

An ether of events implies that there is no such thing as empty or uneventful space. Actual space like any other actual thing is a field of events. Although Whitehead disagreed with the absolutist idea of space as an enduring substance-like stuff, his position in *Process & Reality* ought not to be confused with that of pure relationists like Leibniz of the *Monadology* who urged that space as perceived by the senses was an illusory phenomenon altogether. In Whitehead's mature theory so-called 'empty' space is itself nothing but a nexus of occasions of the "lowest"[95] kind. Thus for Whitehead in addition to ideal space there is concrete space which is a manifold of actual occasions, an “ether of events”.

> Wherever and whenever some thing is going on, there is an event. Furthermore, wherever and whenever in themselves presuppose an event, for space and time in themselves are abstractions from events. It is therefore a consequence of this doctrine that something is always going on everywhere, even in so-called empty space. This conclusion is in accord with modern physical science which presupposes

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93 *Process & Reality*, p.35.

94 *An Enquiry*, p.25.

95 *Process & Reality,* p.177.

> the play of an electromagnetic field throughout space and time. This doctrine of science has been thrown into the materialistic form of an all-pervading ether. But the ether is evidently a mere idle concept in the phraseology which Bacon applied to the doctrine of final causes, it is a barren virgin. Nothing is deduced from it; and the ether merely subserves the purpose of satisfying the demands of the materialistic theory. The important concept is that of the shifting facts of the fields of force. This is the concept of an ether of events which should be substituted for that of a material ether.[96]

### 1.3 Conclusion

In conclusion, according to Whitehead, the world is a creative process involving the constant generation of new event particulars on the basis, on the one hand, of earlier event particulars, and on the other, of eternal universals. Thus Whitehead bifurcates the world into two realms, one actual and the other ideal, one a stream of fleeting and singular events, the other a matrix of eternal, repeatable forms. The implication of the latter in the former ensures that there is not just flux but continuity as well, the transmission and reproduction of identities, patterns and qualities some of which must be instantiated in the stream of discrete events, and others which can (but needn't) be.

Whitehead foreshadowed the integrationist program of *Process & Reality* in his 1920 work *The Concept of Nature* in which he quotes Schelling and his colleague from Cambridge, the neoplatonist William Inge. The quote from Schelling says that nature "constructs itself", the one from Inge that common experience is underpinned by the reality of a "spiritual world". Thus the title of Whitehead's magnum opus is most apt. It is not *Process **in** Reality*, but *Process **and** Reality,* the conjunctive serving to counterpose the world of process to the deeper reality below. '*Process and Eternity*' would have served equally well. This transcendental realm has the structure of an extensive continuum constituting a "systematic framework permeating all relevant fact". This extensive continuum is characterised by relations of "whole and part, of points, lines, and surfaces, and of straightness and flatness". It grounds the measurability, comparability and unity of the visible universe and is a pre-requisite for scientific understanding of anything at all whether the revolutions of a galaxy or the spinning of a child's top.

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96 *The Concept of Nature*, p.78.

## Chapter 2. Critique of Whitehead's Platonism

Although Whitehead dismissed the neoplatonic idea of emanation he could be accused of propounding a metaphysics equally as "puerile" for positing a realm of eternal forms beyond physical sensibility. He seems guilty of committing the "error of mistaking the abstract for the concrete "[1] a habit of thought he famously coined as "the fallacy of misplaced concreteness".

> … the chief error in philosophy is overstatement. The aim at generalization is sound, but the estimate of success is exaggerated…. This fallacy consists in neglecting the degree of abstraction involved when an actual entity is considered merely so far as it exemplifies certain categories of thought. There are aspects of actualities which are simply ignored so long as we restrict thought to these categories. Thus the success of a philosophy is to be measured by its comparative avoidance of this fallacy, when thought is restricted within its categories.[2]

Whitehead says that the two chief functions of eternal objects as recognita and potentia are to explain, (1) the *identity* of events through space and time, i.e. their similarity and permanence, and (2) the *contingency* of events, i.e. the fact that some things might happen but don't and that other things do happen but mightn't have. What I would like to demonstrate in this chapter is that within the corpus of Whitehead's work are alternative explanations for these two apparent properties of process.

### 2.1 Identity and Analogy in Whitehead

The virtue of universals according to David Armstrong is that they are absolutely self-identical in their many guises or instances whilst the individuals in which they are implicated are only *similar* in respect of that universal. This car and this fire engine *resemble* each other in that they are both red but the universal 'red', and the instances of it in the car and in the fire engine are one and the same thing, they are *identical*. Universals are said to account for permanence on account of their identity over time, that they are exactly repeatable on different occasions. A kilogram of potatoes yesterday and a kilogram of onions today are both instances of 'a kilogram', and photosynthesis in cyanobacteria a billion years ago and photosynthesis in plants today are instances of 'photosynthesis' per se. The difficulty however is that actual events in experience are never absolutely identical and one never witnesses exact repetition which is only ever

1 *Science and the Modern World*, p.52.

2 *Process & Reality*, pp. 7-8.

approximated as an ideal like the oscillation of temperatures around a mean. Biology tells us there is a world of difference between photosynthesis in cyanobacteria and photosynthesis in plants and that the general term hides a wealth of details about different chemical pathways and reactants. Even much more rigidly defined phenomena in physics are fuzzy once a certain scale of resolution is reached. As Whitehead himself recognises the measurement of 'an' electron at one point in time and its measurement afterwards are two different but closely related events manifesting themselves in different, quite particular ways. Identities are only ever approached as imaginary ideals. In *Modes of Thought* for example, he says that whether we identify someone in childhood as the very same person in adulthood, depends on pragmatic context - in some situations it may be more prudent to stress their non-identity.

> Nothing in realized matter-of-fact retains complete identity with its antecedent self. This self-identity in the sphere of realized fact is only partial. It holds for certain purposes. It dominates in certain kinds of process. But in other sorts of process, the differences are important and the self-identity is an interesting fable. For the purpose of inheriting real-estate, the identity of the man of thirty years of age with the former baby of ten months is dominant. For the purposes of navigating a yacht, the differences between the man and the child are essential; the identity then sinks into a metaphysical irrelevancy.[3]

Whitehead goes on to point out that even though there is no such thing as absolute identity between different events there are likenesses or analogies. “Analogies survive amid diversity. The procedure of rationalism is the discussion of analogy. The limitation of rationalism is the inescapable diversity”.[4] But Whitehead falls back onto the notion of platonic universals to 'explain' these analogies, arguing in effect that Socrates the child and Socrates the man express a single eternal object of *Socrates* simpliciter.

> The whole understanding of the world consists in the analysis of process in terms of identities and diversities of the individuals involved. The peculiarities of the individuals are reflected in the peculiarities of the common process which is their interconnection. … But this possibility of abstraction, whereby individuals and the *forms* of process constituting their existence can be considered separately, brings out a fundamental intuition which lies at the basis of all thought. This intuition consists in

3 *Modes of Thought*. pp.94-95.

4 *Modes of Thought.* p.98.

> the essential passage, from the experience of individual fact to the conception of *character*. Thence we proceed to the concept of the *stability of character* amid the succession of facts. Thence we proceed to the concept of the *partial identity* of successive facts in a given route of succession. Thence we proceed to the *potentiality* of the facts for maintaining such partial identity amid such succession.… we necessarily introduce the notion of potentiality. … The notion of potentiality is fundamental for the understanding of existence as soon as the notion of process is admitted. If the universe be interpreted as static actuality then potentiality vanishes. Everything is just what it is.[5]

But would it not be simpler and less extravagant to say that analogy and similarity are just primitive facts about actual events, that instances approximate each other rather than some super-sensible eternal object? On such an account Socrates or even the universe as a whole is simply a nexus of interconnected occasions each stage of which resembles and is a causal determinant of its immediate successor. Analogous events might resemble each other but this would come down to the fact of physical mimesis and genealogical history, processes no more mysterious than the ‘reproduction’ of parents by their progeny, the 'copying' of text from one manuscript to another, or the ‘repetition’ of phonetic forms in two languages due to their evolutionary roots in a common ancestor.

In fact from very early on in the reception of Whitehead's philosophy those like John Dewey argued that abstract recognita were not needed to account for the similarity of events which could be explained along what he called “genetic-functional” lines.[6] As an alternative to the "ingression" of eternal objects from outside actuality, Dewey proposed instead the “egression” of incipient processes over time, the reinforcement of tendencies into routine patterns and stable regularities.

> There exist in nature indeterminate situations. Because of their indeterminate nature, the subsequent process is hesitant and tentative. The activity that is 'provoked' is incipient. If it becomes habitual, it is finally determinately egressive as a routine of nature, and it harmonizes the aggressively conflicting elements to which is due the indeterminacy of the original natural situation. When this routine-established mode of processive activity is observed it becomes the subject-matter of a natural law.[7]

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5 *Modes of Thought*. pp.98 - 99. Italics all mine.

6 See for example Dewey and Everett Hall.

7 Dewey, p.176.

Dewey's "genetic" theory of egression suggests the image of process as truly organic; as a chain of reproductive activities whose patterns of behaviour can evolve into seemingly permanent *forms* which are nonetheless prone to error, disturbance and breakdown, and of events as merely imperfect simulacra of each other. Whitehead seemed to be pulled in this direction at times and one can find throughout his writings an account not dissimilar to Dewey's. One aspect of this account involves what would now be called 'relational tropes'.

### 2.2 Relational Tropes in *Process & Reality*

Trope theorists argue that the *instances* of a given property like the particular shade of red here and now exist and that universals like 'redness' don't. The argument I wish to develop here is that *relational* tropes play a crucial role in Whitehead's system.[8] In most discussions tropes are thought of as monadic and entity-like such as the unique kind of wisdom had by Socrates or the particular love that Napoleon felt for Josephine. But tropes can also be thought of as both evanescent and relational. Their event-like character comes out when we think of the unique experiences we have of perceptual sensa like *that* particular shade of bluish green radiating from *that* eucalyptus forest *that* day in the mountains or the flash of hatred Catherine felt for Ann Boleyn on a particular morning in 1525. Tropes as things that happen; coming into then out of existence. Literature and poetry abound with evental tropes and are perhaps their stock in trade. Borges for example focuses on them in his famous story *Funes the Memorious* about a prodigious mnemonist; who, incapable of abstraction, suffered the world in all its overwhelming, fleeting and microscopic detail; "the shapes of the clouds in the south at dawn on the 30th of April of 1882" and "the lines in the spray which an oar raised in the Rio Negro on the eve of the battle of the Quebracho".

Evidence for Whitehead's friendliness towards the idea of relational tropes comes from the fact that the other philosopher to whom he most often recurred on the issue of universals was John Locke with his theory of "simple ideas". Locke defines "simple ideas" at the beginning of his *Essay Concerning Human Understanding,* as the direct ideas we have of particular individuals. Locke gives as an example the idea a child has of its wet nurse or mother. Such ideas are "are well framed in their minds; and, like pictures of them there,

[8] Tropes have an ancient pedigree and were apparently touched on by Aristotle but were investigated early last Century by Husserl who called them 'moments'. Tropes were in modern times revived by D.C. Williams and have more recently been championed by people like Peter Simons, Keith Campbell and Cynthia McDonald. See McDonald's article, "Tropes and Other Things" and D.W. Mertz for an historical overview. Also known as "abstract particulars", or "unit attributes. D.W.Mertz says the semantics of a sentence like "John's love for Mary is stronger than his love for Anne" are evidence of the acknowledgement of such entities in ordinary language. Mertz maintains that the logic of the above sentence can only be faithfully captured by a kind of second order logic which quantifies over property and relational instances. John's love for Mary is one thing (Love1) and his love for Anne is another (Love2) and each is a term in the binary asymmetrical relation 'stronger than'. Thus Love1(John, Mary) ∧ Love2 (John, Ann) ∧ Stronger than (Love1, Love2).

represent *only those individuals*" [my italics]. Other examples of simple ideas include those formed when encountering particular individuals in experience such as 'a leaf, a crow, a sheep, a grain of sand'. Locke argues that general sortals are arrived at by abstracting from simple ideas the details of their particularity in time and space.

> But it is beyond the power of human capacity to frame and retain distinct ideas of all the particular things we meet with: every bird and beast men saw, every tree and plant that affected the senses could not find a place in the most capacious understanding ...knowledge; which though founded in particular things enlarges itself by general views; to which things reduced into sorts under general names, are properly subservient....ideas become general by separating from them the circumstances of time and place and any other ideas that may determine them to this or that particular existence... [9]

Whitehead concurs with Locke that "there are not first the qualities and then the conjectural particular things but conversely"[10] and that the concept of a universal is abstracted "from these primary data by a process of comparison and analysis"[11].

In Whitehead's system however it is not monadic tropes which are central but relational ones which in *Process & Reality* he classifies as "contrasts". Contrasts appear for the first time in his first philosophical text, *An Enquiry Concerning the Principles of Natural Knowledge*, published in 1919 in which he argues that "what is known is not barely the things but the relations of things, and *not the relations in the abstract but specifically those things as related*".[12] In *Process & Reality*, he goes on to say that we have a bad habit of treating all relations as universals, a habit which ignores the "complex actual facts" forgetting that universals are abstractions from "contrasts...infected with the individual particularities of each of their relata"[13] and says that abstract relations stand to contrasts as "genera do to species"[14].

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9 Locke, J. *Essay Concerning Human Understanding*, quoted in *Process & Reality,* pp.52.

10 *Process & Reality*, p.53.

11 *Process & Reality,* p.152.

12 *An Enquiry,* p.13.[emphasis mine]

13 *Process & Reality*, p.228. To be clear, "contrasts" can be between different universals like that between red and blue say as well as between particular individuals or particular tropes.

14 *Process & Reality*, p.229.

> What are ordinarily termed 'relations' are abstractions from contrasts. A relation can be found in many contrasts; and when it is so found; it is said to relate the things contrasted.... For example, the complex nexus of ancient imperial Rome to European history, is not wholly expressible in universals. It is not merely the contrast of a sort of city, imperial, Roman, ancient, with a sort of history of a sort of continent, sea-indented, river-diversified, with alpine divisions, begirt by larger continental masses and oceanic wastes, civilized, barbarized, christianized, commercialized, industrialized. The nexus in question does involve such a complex contrast of universals. But it involves more. For it is the nexus of *that* Rome with *that* Europe. *We cannot be conscious of this nexus purely by the aid of conceptual feelings. This nexus is implicit, below consciousness, in our physical feelings*. (my emphasis)[15]

It is worth belabouring the point somewhat as Whitehead's implicit recognition of relational tropes is one that seems to have been almost entirely overlooked in the literature despite popping up quite regularly.[16] In his 1933 publication *Adventures of Ideas* for example he says that in speaking about relations as universals we abstract away the "actual connectedness" of the things connected, taking the relationship between Boston and Philadelphia as a case in point.

> It is generally held that relations are universals so that A can have the same relation to B as C has to D. For example, 'loving', 'believing', 'between' and 'greater than' are relations. ... But ... a relation cannot signify the actual connectedness of the actual individual things which constitute the actual course of history. For example, New York lies between Boston and Philadelphia. But the connectedness of the three towns is a *real particular fact*[17] on the earth's surface involving a particular part of the eastern seaboard of the United States. It is not the universal 'between'. It is a complex actual fact which, among other things exemplifies the abstract universal 'betweenness'.[18]

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15 *Process & Reality*, p.229.

16 John Lango's 2003 article, "Relational Particulars and Whitehead's Metaphysics" is the most notable exception. In it Lango argues for the ontological importance of relational particulars which he says cannot be reduced to monadic property particulars (contra Keith Campbell). He also argues that each prehension is a relational particular, a proposal I am completely in agreement with. Joseph. E. Early in his article "Process Structural Realism, Instance Ontology and Societal Order" also notices similarities between Mertz's instance realism and Whitehead's philosophy but does not develop this insight fully.

17 Italics mine.

18 *Adventures of Ideas*, p.230.

The above examples of the betweeness of New York relative to Boston and Philadelphia, could be mistakenly interpreted as static relations. However, relations like 'bigger than', 'heavier than', and 'loves' are from a process perspective abstractions from processes. I am *bigger* than my son now but he is growing and one day will be bigger than me, Romeo *loves* Juliet but won't love her or anything else after he commits suicide. The statement that I am bigger than my son focuses attention on a small passing phase in a complex series of events composing my getting older and his getting taller. Even relational tropes are events.

### 2.3 Prehensions as Relational Events

Relational tropes come quite naturally to any process metaphysics in which the hallmark of events is their relatedness to other events. One of the primary functions of eternal objects in Whitehead's system is in "introducing the multiplicity of actual entities"[19] from the past to those in the making. These eternal objects represent one occasion to another in the act of prehension such that it is "objectified" by that other from its own unique and particular 'viewpoint'. In his 1925 publication *Science and the Modern World* Whitehead noted that this perspectival approach[20] had obvious connotations with Leibniz's theory of monads.

> The things which are grasped into a realised unity, here and now, are not the castle, the cloud and the planet simply in themselves; but they are the castle, the cloud, and the planet from the standpoint, in space and time, of the prehensive unification. In other words, it is the perspective of the castle over there from the standpoint of the unification here. It is, therefore, aspects of the castle, the cloud and the planet which are grasped into unity here. You will remember that the idea of perspectives is quite familiar in philosophy. It was introduced by Leibniz, in the notion of his monads mirroring perspectives of the universe. I am using the same notion, only I am toning down his monads into the unified events in space and time [i.e., into actual occasions].[21]

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19 *Process & Reality,* p.59.

20 Elizabeth Kraus in her Metaphysics of Experience, pp.152-153, makes the point that Whitehead's studies in the area of projective geometry (which arose in the context of Renaissance studies in painting and cartography on how to project three-dimensional objects onto two dimensional surfaces), informed much of his later philosophy especially his theory of extension. Stephen Ross 's book Perspective in Whitehead's Metaphysics argues that the idea of perspective is central to understanding the whole of Whitehead's metaphysics.

21 *Science & the Modern World*, p.70.

Thus an actual occasion does not exist simpliciter with a bundle of monadic properties ‘out there’ but with a bunch of relations to other occasions both in the past and in the future. It has its own perspective on the things before it and is in the perspective of things that come after. The sun for example is not just a thing enduring in the void of space but that thing as prehended by the events composing the planets of our solar system, right down to weather events and human perceptual events here on earth. *How* the sun prehends and is prehended, the particular *ways* in which it experiences and is experienced are essential to its identity. Whitehead called this perspectival aspect of prehension ‘objectification’.

> In other words, each actuality is prehended by means of some element of its own definiteness. This is the doctrine of the 'objectification' of actual entities. Thus the primary stage in the concrescence of an actual entity is the way in which the antecedent universe enters into the constitution of the entity in question, so as to constitute the basis of its nascent individuality.[22]

> Thus objectification is an operation of mutually adjusted abstraction, or elimination, whereby the many occasions of the actual world become one complex datum. This fact of the elimination by reason of synthesis is sometimes termed the perspective of the actual world from the standpoint of that concrescence.[23]

Whitehead also speaks about objectification as the process whereby an actual occasion as fleeting subject passes over into an 'objectified' datum for later occasions. Since the objective phase in some sense lives beyond or supersedes the phase of "subjective immediacy" it is described by Whitehead as a "superject". "An actual entity is to be conceived both as a subject presiding over its own immediacy of becoming, and as a superject which is the atomic creature exercising its function of objective immortality."[24] Thus objectification in Whitehead's eyes is both the abstraction or reduction of an occasion to "an element of its own definiteness" as well as the 'transmission', 'repetition', 'reproduction' and 'reception' of this 'element' by others in the future.

### 2.3.1 Three Models of Objectification

Many commentators have therefore tended to think of objectification as a process whereby the monadic universals of a given occasion are literally repeated or reproduced in its prehending occasions. However this reproductional model raises many difficulties, for if

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22 *Process & Reality*, p.152.

23 *Process & Reality,* p.210.

24 *Process & Reality*, p.45.

properties are truly re-enacted then whenever x prehends property Q in y then x must itself possess Q. In other words we have to conclude that if I see the green tree then I am green too! This is odd to say the least. Commentators like Bradford Wallack have however taken the bull by the horns accepting its consequences. He describes this literal re-enaction of forms in the subject entity as a "flow of feeling" or an act of "sympathy" between occasions. He gives the example of one man experiencing another's anger which results in the first man feeling angry himself and quotes the following paragraph from *Process & Reality* in support.

> A simple physical feeling enjoys a characteristic which has been variously described as 're-enaction', 'reproduction' and 'conformation'. This characteristic can be more accurately explained in terms of the eternal objects involved. There are eternal objects determinant of the definiteness of the objective datum which is the 'cause' and eternal objects determinant of the definiteness of the subjective form belonging to the 'effect'. When there is re-enaction there is one eternal object with two-way functioning, namely, as partial determinant of the objective datum, and as partial determinant of the subjective form. In this two-way role, the eternal object is functioning relationally between the initial data on the one hand and the concrescent subject on the other.[25]

After arguing that literal 're-enaction' should be accepted as Whitehead's own approach, Wallack acquiesces to the bizarre consequences that follow.

> We may have difficulty in understanding how a colour for example, or a smell, are subjective forms. It is easy to understand them as forms of objective data: we perceive a ruby as red, we perceive a rose as fragrant. But we do not ordinarily imagine that we feel red or fragrant about those things. Yet Whitehead proposes that we do.[26]

One way around this difficulty might be to think of objectification as a *representational* process whereby a monadic property in the prehending occasion is represented by a different property in the datum. However this model is not very appealing either. At first instance it appears intuitively plausible. If I see a green tree for example, then the 'green' of the tree is represented by something like a definite percept in my mind or a definite

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25 Process & Reality, p.238. Whitehead himself uses the 'angry man' example in several places for example in the chapter "Objects & Subjects" in *Adventures of Ideas*. p.183.

26 Wallack, p.111.

photoreceptor reaction in my eye. If iron senses oxygenated water it represents its 'valency' by changing state and giving up electrons. Perhaps that is just what prehension is, a kind of correlation between states. But what entitles us to say that the second quality really is causally connected to the first? The representational model fails to account for *why* one is correlated to the other. There is no causal mechanism or link that does the correlating so we lapse back into a Humean theory of causality as nothing but a relatively frequent or constant conjunction of similar pairs of events.

The representational model is in fact a reversion to a model of perception which Whitehead criticised in Descartes, Locke, Hume and Kant due ultimately Whitehead says to their inability to think outside the subject-predicate logic of substances and qualities. According to Whitehead "a subject-predicate proposition is considered as expressing a high abstraction."[27] Taking statements like "this stone as grey" as a starting-point he says is "delusive"[28]. This is exactly what Descartes does in the Meditation II for example, when he says that in seeing a crowd of men in the street he only truly sees an image of "hats and coats which may cover automatic machines". Descartes then goes on to assert that the image in his head is correlated to real men merely by the "faculty of judgement". Whitehead harshly criticises Descartes because the "faculty of judgement" is no guarantee that the men in the street are real. Descartes leaves out of the picture not only the real men but the real causal connection between those men and the viewer's visual image and experience of them. It is a quick step from there to Hume's skepticism.

> In this passage it is assumed that Descartes - the Ego in question - is a particular, characterized only by universals. Thus his impressions - to use Hume's word - are characterizations by universals. Thus there is no perception of a particular actual entity. He arrives at the belief in the actual entity by 'the faculty of judgment'". But on this theory he has absolutely no analogy upon which to found any such inference with the faintest shred of probability. Hume, accepting Descartes' account of perception (in this passage), which also belongs to Locke in some sections of his Essay; easily draws the sceptical conclusion.[29]

I contend that the only way to avoid the problems associated with the reproductional and the representational models is to abandon talk of monadic properties and eternal objects

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27 *Process & Reality,* p.138.

28 *Process & Reality*, p.160.

29 *Process & Reality*, p.49.

altogether and to think about objectification in purely *event-relational* terms instead. On the event-relational model, objectification is just that *process* whereby there is a direct presentation of one entity to another via a *dyadic asymmetric relational event* between prehended and prehending occasions. Prehensions are like vectors. In the passage cited by Wallack above, Whitehead states the key elements of the relational model when he writes, “there is one eternal object with two-way functioning” and that “the eternal object is functioning relationally”. Instead of thinking about some monadic quality Q in event x being prehended by event y, we should think of Q itself as a binary asymmetrical relational event joining x and y together in a single process. For Icarus to feel the sun's warmth is for Icarus to be *warmed* by the sun and for someone to smell the fragrance of a rose is for them to be ‘fragranted’ by the rose. We avoid the absurd consequence that in smelling a rose’s fragrance we become fragrant ourselves (reproduction) and the equally absurd notion that when we see a red ruby, the red we see is in our mind but not the ruby (representation). The relational event model of objectification overcomes these difficulties by positing the existence of objective, causally efficacious *events* connecting prehended and prehending entities.[30]

The upshot of this is that even Whitehead's prehensions are events whose function is to introduce dative occasions to experient ones. This suggestion was I think first made in 1956 by R.M. Martin, one of the first logicians interested in mereology and Whitehead's event ontology, when he said in connection with his formal model of Whitehead's ontology that prehensions ought to be "handled as special kinds of actual entities"[31]. On this interpretation, for an occasion x to prehend y is for x to prehend y by means of, or through the prism of some prehensional event z. Instead of portraying prehensions as arrows and occasions as dots, on this view it would be more accurate to depict both prehensions and occasions as arrows. Event particulars are relational creature carrying many relata in their domain to many relata in their co-domain, mapping each of the former to all of the latter . Prehensions on this account are degenerate cases having only one event in their domain and one event in their co-domain as pictured in the diagram below.

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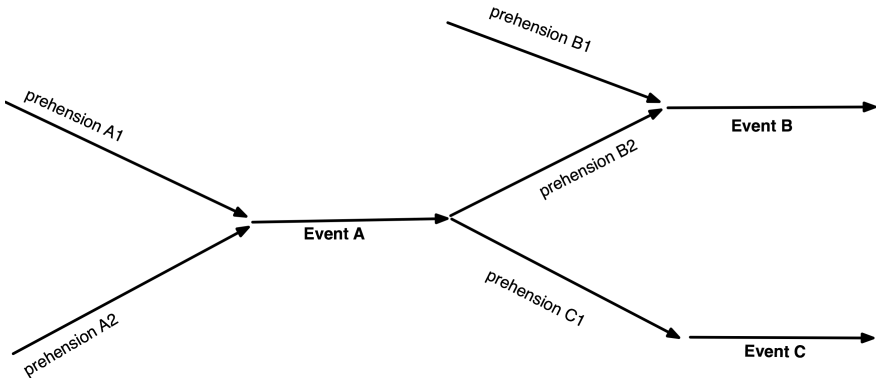
[30] In saying that an occasion cannot be separated even from those relations it has to succeeding occasions, I am not suggesting that such a relation is necessary at the time of its occurrence, only that it is a necessary component of its identity post-factum, i.e. retrospectively. At the time of an event x’s occurrence prospective events like y are merely contingent possibilities. After y occurs however then y is necessary to x because y is simply part of x's factual matrix. Prior to Henry VIII marrying Ann Boleyn, her relationship to him as a wife was a mere possibility. After the fact though, her relationship to him as wife became a necessary property of Henry VIII’s identity, who he actually was and is now. The Henry VIII that we know and refer to in this world was the one that married Ann Boleyn; a fact now and forever.

[31] R.M. Martin, p.5.

**Diagram 2. Prehensions as Relational Events or Arrows**




In the above diagram, A has two relata in its domain, (its prehensions A1and A2) and two relata in its co-domain (B2 and C1) the latter two being the means by which A is prehended by B and C respectively. We can say that prehensional event C1 is “C's perspective on A” or that it “introduces A to C”.

The above image of the world as relational events knitted together by other relational events is more in accord with Whitehead's fundamental principle that the unity of the world is derivative from the solidarity of actual occasions and that “actual entities are the only reasons”.[32]

### 2.3.2 Eternal Objects and the Issue of Individuation.

Unfortunately, the event-relational nature of objectification is obscured by Whitehead's use of words like “transference”, “reproduction” and “transmission” to describe it, all of which when taken literally trade on the idea of a monadic universal being repeated in the prehending event. Such talk has led to a fog of confusion descending on later commentary about Whitehead’s eternal objects and the process of objectification. Garland for example quite rightly denies that objectification is about occasions feeling abstract universals but interprets Whitehead as saying that occasions must therefore be “felt as whole individuals”. [33] It is not really clear how this solves the matter and is confronted by

[32] *Process &Reality*, p.37.

[33] Garland., "Whitehead’s Metaphysics: A Critical Examination", p.185. To be fair Garland rightly emphasises the vector character of feelings and the importance of particulars in Whitehead's thinking but he does not formulate expressly the idea that an individual feels via relational particulars which is to say that prehensions are concrete entities. The first to have done so I believe was John Lango in his 2002.

the problem that Whitehead says repeatedly that occasions are only experienced via certain of their *aspects*.[34].

> ...objectification relegates into irrelevance, or into subordinate relevance, the full constitution of the objectified entity. Some real component in the objectified entity assumes the role of being how that particular entity is a datum in the experience of the subject[35].

Anderson Weekes in a more recent paper[36] tackles the ancient question of individuation from the perspective of Whitehead's theory of objectification. He reads Whitehead as saying that simple eternal objects subsist transcendentally but complex eternal objects[37] come into existence when instantiated by some event on completion as a *superject.* Combinations of eternal objects are then said to enable events to be individuated descriptively because such combinations are always unique and infinite. On the other hand during its subjective phase of "absolute enjoyment", i.e. when the event is actually happening, it is said to be individuated "token reflexively". In addition the superject and subject phases are said to somehow overlap such that any given event can always be individuated both descriptively and token reflexively.

Weekes' interpretation seems to me obscure things even further raising more questions than it answers. Why should a complex of universals not be repeatable when each of its components is? What reason prevents two occasions having an infinity of eternal objects completely in common? Why is token-reflexivity unique to a subject but not a superject? And how can a completed phase overlap with something which wholly precedes it?

The source of Weekes' difficulties comes back to the fundamental problem of how eternal objects whose very essence is their repeatability could individuate unrepeatable event particulars which have all their eternal objects in common. If one entertains the notion that eternal objects are anything more than imaginary it is hard to escape the accusation levelled by those like Edward Pols that if an occasion's prehensions are nothing but

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34 Richard Rorty makes a similar mistake in his paper "The Subjectivist Principle and the Linguistic Turn" in which he says that Whitehead's actual entities become repeatables when they are prehended by later occasions. Although entities can be repeatedly prehended this does not mean they themselves are repeatables. An entity's effects reverberate so to speak down the generations but they themselves are quite distinct from their effects. True repeatables, to borrow David Armstrong's words,"are strictly identical in their different instantiations" Armstrong, D.M. *A World of States of Affairs*, p.27. The universal 'red' is the very same 'red' that is in the red rose or in the red wine. Whitehead identifies the exact same principle in *Science and the Modern World* as "The Translucency of Realisation" which he defines in the following way; "any eternal object is just itself in whatever mode of realisation is involved. There can be no distortion of the individual essence without thereby producing a different eternal object. *Science & The Modern World, p.171*.

35 *Process & Reality*, p.62.

36 Weekes, Anderson., "Abstraction and Individuation in Whitehead and Wiehl".

37 What Whitehead in *Science and the Modern World* calls its "associated abstractive hierarchy". See here my discussion at 1.2.7

“eternal objects that are together in a certain way” then occasions are composed of universals after all and thus indistinguishable in principle from others with identical components.[38] However, if we accept that eternal objects have no more than a faux existence the problem disappears - all of an occasion’s relations are unique and sufficient to individuate it at least theoretically if not in practice.

### 2.3.3 Demonstrative Reference and Whitehead’s “Real Essences”

Whitehead’s dualistic system accepts both concrete and abstract relations, both eternal objects and trope-like contrasts. This split is reflected in his notion that an occasion has two sets of relations to the rest of the universe; one set composed of relational particulars and the other of relational universals, the first he called its "real essence” the second its "abstract essence”.

> … the “organic doctrine” demands a real essence in the sense of a complete analysis of the relations, and inter-relations of the actual entities which are formative of the actual entity in question, and an “abstract essence” in which the specified actual entities are replaced by the notions of unspecified entities in such a combination; this is the notion of an unspecified actual entity. Thus, the real essence involves real objectifications of specified actual entities; the abstract essence is a complex eternal object. There is nothing self-contradictory in the thought of many actual entities with the same abstract essence; but there can only be one actual entity with the same real essence. For the real essence indicates “where” the entity is, that is to say, its status in the real world; the abstract essence omits the particularity of the status.

For Whitehead, the existence of an occasion’s real essence grounded demonstrative reference which he thought of as a universal feature of language. “Explicitly in the verbal sentence, or implicitly in the understanding of the subject entertaining it, every expression of a proposition includes demonstrative elements”.[39] Descriptions using abstract predicates are more prone to referential failure because two things with the same abstract qualities are indistinguishable linguistically. They might however be differentiable in sense

38 Pols, Edward. *Whitehead’s Metaphysics*, pp.158 - 195.

39 *Process & Reality*, p.43.

awareness because they possess exclusive concrete relations and qualities.[40] For instance if someone at a crowded party says, “The man with the tie is handsome” it might be impossible to discern who they are talking about because there are many things which are men and many wearing ties. In such situations demonstrative gestures such as the speaker pointing a finger combined with an indexical like, “That man there” resolve the ambiguity via the mechanism of some relational particular (like the referent’s locational relation to the speaker). Whitehead's recognition of the centrality of demonstrative reference goes back to at least his middle period in texts like *The Concept of Nature* where there can be found an extended discussion of the function of demonstration in calling attention to a referent's relations to a wider field or complex of objects.[41] Descriptive reference by contrast abstracts away from such relations delivering for thought “distinct individualities” or isolated “entities” whose advantage of communicability comes at the cost of a "loss in content".

> … the termini for sense-awareness are factors in the fact of nature, *primarily relata* [my emphasis] and only secondarily discriminated as distinct individualities. No characteristic of nature which is immediately posited for knowledge by sense-awareness can be explained. It is impenetrable by thought, in the sense that its peculiar essential character which enters into experience by sense-awareness is for thought merely the guardian of its individuality as a bare entity. Thus for thought red is merely a definite[42] entity, though for awareness ‘red’ has the content of its individuality. The transition from the red of awareness to the red of thought is accompanied by a definite loss of content, namely by the transition from the factor ‘red’ to the entity red. This loss in the transition to thought is compensated by the fact that thought is communicable whereas sense-awareness is incommunicable.[43]

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40 Richard Rorty misconstrues Whitehead on this point accusing him of believing that “the only sort of statement which can describe a non repeatable entity is a token-reflexive statement, and that the only entities which non-token-reflexive discourse can describe are repeatable entities”. I think Rorty overstates the case and is contradicted by the rather nuanced discussion of demonstratives and descriptions in The Concept of Nature. referred to above I think Whitehead would concur that an abstract description may very well be sufficient to identify a particular, in the same way as the term, “The French Emperor who lost the Battle of Waterloo” uniquely describes Napoleon. I think Whitehead should only be taken as asserting that abstract terms are not always sufficient to identify concrete particulars making demonstrative gestures sometimes necessary. Rorty, R. “The Subjectivist Principle and the Linguistic Turn”. p.140.

41 *Process & Reality*, p.9.

42 I think Whitehead’s meaning would be clarified here if he had used “defined” rather than “definite”.

43 *The Concept of Nature*, pp.12-13.

## 2.4 A Probablistic Account of the Contingency of Events

This now brings me to the other main function of eternal objects in Whitehead's system which is to ground what Whitehead called the potentiality of events, the apparent fact that every moment presents us with a plethora of alternatives for the future; of events which mightn't happen but will and of events which could happen but won't. On such an account, an event like the tossing of a coin, somehow enlivens abstract possibilia like 'the coin coming up heads', and 'the coin coming up tails', plus some kind of a 'decision' to actualise one or the other. In a truly platonic vein Whitehead expressed this idea of foregone alternatives by saying that "every occasion is a synthesis of being and not-being".[44]

I hope I have now established that for Whitehead *analogy* between concrete events is a vital moment in his philosophy, albeit one obscured by his attachment to eternal objects. Whitehead's emphasis on analogy however is also the basis for a non-platonic account he gives of the contingency of events. This account is set out in sections V - VII of Chapter IX of *Process & Reality* which contains his thoughts on probability. Those sections aim to give an "ultimate ground" to the veracity of judgments based on induction. He says that his position does not "differ fundamentally from those of Mr. Keynes as set out towards the conclusion of his Chapter XXI" in his *A Treatise on Probability*[45]. In that chapter Keynes argues says that the idea of "perfect analogy" has its place but says that states of the universe "identical in every particular, may never recur, and, even if identical states were to recur, we should not know it."[46] Instead, he says, science starts from a practical supposition about the "atomic character of natural law" working with "legal atoms"[47] which are assumed to exercise discrete and invariable effects on the totality whose changes "are solely due to a separate portion of the preceding state".[48] In the next chapter entitled "Induction and Analogy" Keynes goes on to argue that reasoning by imperfect analogy is only justified if we are entitled to assume "the amount of variety in the universe is limited in such a way that at least some objects are not so complex that their qualities fall into an

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[44] *Science & the Modern World*, p. 163.

[45] *Process & Reality,* p.206. Whitehead had been one of Keynes's PhD assessors along with W.E. Johnson in 1908 at Cambridge. It is not widely known that Whitehead's organicist philosophy influenced Keynes. The latter attended Whitehead's mathematics lectures in 1904 and both were members of the Apostles along with Russell where topics like "internal relations" were often discussed. See Winslow 1986.

[46] Keynes. p.286.

[47] I think in using the word "legal" here Keynes is alluding to the conventional and historically conditioned nature of these 'atoms'

[48] Keynes, p.287.

*infinite* number of independent groups"[49]. This is the "inductive hypothesis" which says that there must be a finite *apriori* probability for a certain conclusion before one is entitled to say that its probability is increased by the existence of additional evidence in its favour. Keynes, thus held that statements of probability are subjective weightings of a person's confidence about some event given (a) evidence in its favour but also (b) prior experience of analogous events. "To this extent the popular opinion that Induction depends upon experience for its validity is justified and does not involve a circular argument".[50] Thus although Keynes says of probability that it "may be called subjective"[51] and that it is "concerned with the degree of belief which it is rational to entertain in given conditions",[52] those beliefs are grounded in prior experience of analogous events.

Keynes' viewpoint is pretty much identical to Whitehead's who writes, "It is evident that the ultimate 'ground' to which all probability judgments must refer can be nothing else than the actual world as objectified in judging subjects. A judging subject is always passing judgment upon its own data."[53] Whitehead then observes that probability judgements about a given event presuppose the judging subject's limited sampling of a finite number of occasions within that vague but delimited region of the cosmos composing its "actual world". This environment has certain features which make probablistic reasoning a valid exercise in rational thought.

> … inductive reasoning gains its validity by reason of a suppressed premise. This tacit presupposition is that the particular future which is the logical subject of the judgment, inductively justified, shall include actualities **which have close analogy to some contemporary subject** enjoying assigned experience; for example**, an analogy to the judging subject in question, or to some sort of actuality** presupposed as in the actual world which is the logical subject of the inductive judgment. It is also presumed that this future is derived from the present by a continuity of inheritance in which this condition is maintained.[54]

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[49] Keynes, p.297.

[50] Keynes, p.300.

[51] Keynes, p.3.

[52] Keynes, p.3.

[53] Process & Reality, p.203.

[54] *Process & Reality*, p.204.

Whitehead is saying that we are entitled to make inductive judgments because the present is “a continuity of inheritance” from the past that therefore resembles it to some degree. But counter-posed to this tendency is a countervailing one for occasions to creatively supersede their 'data'. Induction might work *sometimes* because the present is analogous to the past but fails at others for the very same reason - the need for induction arises because prehension yields at most analogy rather than identity; imperfect reproduction rather than exact repetition. If, as Whitehead says, “the basis of all probability and induction is the fact of *analogy* between an environment presupposed and an environment directly experienced” [55] then by the same token “judging subjects” are prohibited from predicting with absolute certainly given x whether y or z will be the case. Talk about “pure potentialities” then is really disguised talk about a subject’s very real but limited ability to predict the future based on its sampling of prior analogous events.[56] To adapt a line from the *Timaeus* which Whitehead was fond of quoting himself, “one cannot look for anything more than a likely story”.

### 2.5 Conclusion

In Whitehead's scheme there are virtually two theories of the similarity and contingency of events; a transcendental one based on the ingression of eternal objects, the other an immanent one based on prehensional relations between events. In the immanent account event particulars alone do the real work of relating one thing to another and of stitching the universe together. Universals, and even tropes (if thought about statically) are derivative abstractions. This idea is expressed forcefully in his dictum that “no one crosses the same river twice...No thinker thinks twice; and…no subject experiences twice”[57] and is the foundation of a truly one-category ontology hidden below the platonistic superstructure of his metaphysical system. In this ontology analogies alone “survive amid diversity” and “the limitation of rationalism is the *inescapable diversity*”[58].

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[55] *Process & Reality*, p.207.

[56] As if to atone for his non-platonic account of probability in sections V - VII of Chapter IX, Whitehead gives in section VIII “an alternative statistical ground” based on eternal objects. It is however a page and half of rather shallow and unconvincing discussion about “the prehension by every creature of the graduated order of appetitions constituting the primordial nature of God”. Process & Reality, p.207.

[57] *Process & Reality*, p.29.

[58] *Modes of Thought*. p.98.

# Chapter 3. Whitehead's Platonistic Theory of Relativity

## 3.1 Whitehead's Stand against Subjectivism

In June 1921 Einstein visited London at the invitation of Lord Haldane, a famous politician and Hegelian philosopher who had been a very close friend of Whitehead's since their days in the Aristotelian Society during 1915 to meet. At a dinner party at Haldane's on the 10th and again on the morning of the 11th, Whitehead and Einstein had long private discussions about general relativity. On the second occasion the esteemed scientist had to retire to bed apparently exhausted by Whitehead's perplexing metaphysical objections.[1]

The source of Whitehead's disagreement with Einstein lay in his belief that physical theory and science generally require space to have a uniform metrical structure so that different observers could reach identical conclusions about the lengths and durations of different bodies and events. This uniform structure is not an enduring substance as in Newton but an *ideal* structure which informs but is separate from the realm of contingent events and without which measurement and comparison would be impossible. Whitehead thought that Einstein conflated the two realms making measurement dependent on the contingent situations of observers and thus prone to the charge of subjectivism.

Whitehead's stand against what he thought were subjectivist tendencies in modern thought can be seen as early as 1899 in his support for Russell against Poincare's conventionalism about the geometry of space.[2] Also, Whitehead tellingly begins each of his works on relativity with observations about the relationship between internal consciousness and external reality, discussing topics like perception, meaning and significance. Each of these discussions is concerned to develop a realist doctrine which asserts that we can and regularly do make reliable inferences and claims about an objectively given reality beyond the realm of immediate sensation.

The first chapter of *An Enquiry* for example mounts a refutation of Berkeley's skepticism, emphasising that we never experience relata as isolated phenomena but as always 'significant' of other entities and a wider world beyond them. Whitehead refers to Berkley's literary character Alciphron who professes a naive belief in the distant planets despite skeptical arguments from his interlocutor Euphranor who maintains that we never see

1 Desmet, R. "Whitehead and the British Reception of Einstein's Relativity: An Addendum to Victor Lowe's Whitehead Biography", p.24.

2 Whitehead wrote to Russell in December in response to a request for advice regarding his dispute with Poincare over whether the choice between different geometries was empirically under-determined and therefore a matter of convention, and whether distance could be defined apart from that choice. On the distance issue, Whitehead encourages Russell saying that "Space is the true arena on which to fight the battle with nominalism". See Desmet, R. "A Refutation of Russell's Stereotype", p.157.

things in themselves but always obscured as if through a "crimson cloud". Whitehead says; "What he [Alciphron] directly knows is his relation to some other elements in the universe - namely I, Alciphron am located in my percipient event 'here and now' and the immediately perceived appearance of the planet is characteristic of another event 'there and now'."[3] Whitehead declared himself to be in complete agreement with Dr. Johnson whose retort to Berkley was to stomp his foot into the ground, thus realising "the otherness of the paving stone"[4].

Likewise, in *The Concept of Nature*, he argues against what he calls "the bifurcation of nature", defending the idea that in sense perception "nature is disclosed as a complex of entities whose mutual relations are expressible in thought without reference to mind" and that philosophers become 'incoherent' when they "drag in the mind and talk of entities in the mind or out of the mind". *The Principle of Relativity* published in 1922 is motivated by the same train of thought, "our experience requires and exhibits a basis of uniformity, [which] exhibits itself as the uniformity of spatio-temporal relations".[5] Without "some knowledge of a systematically related structure of space-time we are dependent upon the contingent relations of bodies which we have not examined and cannot prejudge".[6] Without such a concept of externality, "...the distance of the star Sirius would be a phrase without meaning", [7] and judgements about equality of distance or speed would be without objective foundation.

Whitehead thus runs together the ideas of an objective and uniform reality with that of an ideal, linear and homogenous space space behind the world of actual events, a thesis recapitulated in his chapter on measurement in Part IV of *Process & Reality* which argues that any act of measurement relies on *intuitions* of congruence, permanence and straightness. For example, a physical standard of length like a particular metal rod stored by the Bureau of Standards in Paris is prone all kinds of microscopic distortions. Other more accurate standards (like the distanced travelled by a beam of light over a given period of time) suffer a similar problem once a certain degree of precision is reached. Any physical standard is in fact an “approximation to straightness”[8] which embodies and relies

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3 *An Enquiry*, p.13.

4 *An Enquiry*, p.13.

5 *The Principle of Relativity*, p.a3.

6 *The Principle of Relativity*, p.59.

7 *The Principle of Relativity*, p.5.

8 *Process & Reality*, p.328.

on our *intuition* that they remain congruent with what they measure through time and space. Congruency is more than physical coincidence between measuring tools and measured objects (such as when we place a tape measure and a length of wood side by side), but a fact concerning their movements and displacements within a systematic geometrical scheme. Ultimately he says all measurement relies on intuitions concerning the existence of straight lines and the congruency of segments thereof, which in the simplest Euclidian case, comes down to the congruence between straight line segments composing the opposite sides of parallelograms.

> … it has been shown by Cayley and von Staudt that the congruence of segments and the numerical measures of the distances involved are definable. The simplest case is that of Euclidean Geometry. In that case the basic fact is that the opposite sides of parallelograms are equal. A further complication is required to define congruence between segments which are not parallel. But it would serve no purpose to enter into the detailed solutions of this mathematical problem. But the illustration afforded by the particular case of the congruence of the opposite sides of parallelograms enables the general principle underlying the notion of congruence to be explained. *Two segments are congruent when there is a certain analogy between their functions in a systematic pattern of straight lines, which includes both of them*. The definition of this analogy is the definition of congruence in terms of non-metrical geometry. It is possible to discover diverse analogies which give definitions of congruence which are inconsistent with each other. That definition which enters importantly into the internal constitutions of the dominating social entities is the important definition for the cosmic epoch in question.[9]

In the general case, the straight lines Whitehead is referring to, are not ‘straight’ in the normal sense but in the sense of conforming to the abstract axioms defining one of the three main homogenous geometrical systems of constant curvature, the Euclidian, the elliptical and the hyperbolic. Definitions for the three man systems can be formulated in terms of Playfair's axiom of parallelism which says that for a given point P and a line L outside P in the same plane, Euclidian geometry stipulates that *all but one* straight line through P intersects L, elliptical geometry asserts that *all* lines through P intersect L, while hyperbolic geometry asserts that *some* but not all lines straight lines through P intersect L.

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[9] *Process & Reality*, p.331.

So Whitehead is just saying that a non-metrical system of such ‘straight lines’ is necessary as an apriori starting point for any kind of physical theory.

Whitehead says that this ideal linear structure cannot be experienced by the senses directly but is perceived through a special mode of perception called 'presentational immediacy'. Perception in the mode of presentational immediacy displays;

> …the real extensiveness of the contemporary world. It involves the contemporary actualities but only objectifies them as conditioned by extensive relations. It displays a *system* pervading the world, a world including and *transcending* the experient. It is a vivid display of systematic real potentiality, inclusive of the experient and *reaching beyond it*. In so far as straight lines can only be defined in terms of measurements, requiring particular actual occasions for their performance, the theory of geometry *lacks the requisite disengagement from particular physical fact*. The requisite geometrical forms can then only be introduced after examination of the particular actual occasions required for measurement. But the theory of projection explained above, requires that the definition of a complete straight line be *logically prior* to the particular actualities in the extensive environment.[10]

By focusing on the enduring, regular and systematic aspects of experience, perception in the mode of presentational immediacy is able to penetrate through phenomenal actuality to the ideal linear relations of the extensive continuum underpinning it.

> Depth of experience is gained by concentrating emphasis on the systematic structural systems in the environment, and discarding individual variations. Every element of systematic structure is emphasized, every individual aberration is pushed into the background. The variety sought is the variety of structures, and never the variety of individuals. … In every possible way, the more advanced organisms simplify their experience so as to emphasize those nexus with some element of tightness of systematic structure.[11]

Whitehead was here reading in a platonistic way Arthur Cayley’s claim that “projective geometry is all geometry”, a slogan that had been taken up and put into effect by Klein’s Erlangen Program. Cayley and Klein it will be recalled thought of projective geometry as

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[10] *Process & Reality*, p.324. [all emphases mine]. When Whitehead talks about ‘projectors’ he means linear relations or lines as presented to an experient occasion in its immediate experience as opposed to the linear relations existing objectively throughout the extensive continuum.

[11] *Process & Reality*, p.319.

the most general kind of geometry without concepts of distance, parallelism, or angularity and based on simple relations of incidence between lines in a point and points in a line. Other geometries like affine geometry (involving non-metrical non-intersecting parallel lines), and hyperbolic, elliptical and Euclidian geometries could be built up with additional axioms and metrics.[12] The geometries dealt with by Klein are all 'homogenous' or of constant curvature unlike the continuous or differentiable manifolds of Riemann which were of variable curvature. In 1908 Klein's programme took a big step forward when his younger colleague at Göttingen, Hermann Minkowski, recast Einstein's special theory of relativity in the language of a 4-dimensional homogenous geometry using projective concepts.

But if Whitehead's platonism means that the ideal of straightness is not something constructed on the basis of our experience of falling bodies, rays of light and plumb bobs but is "logically prior" to them and an intuition about the elements of an axiomatic system involving at minimum the congruence of line segments composing parallelograms, then nobody before the invention of axiomatic geometries in the 19th Century or for that matter no-one untrained in pure mathematics could be said to have ever understood such an everyday notion as that of a straight line. Whitehead's platonism about a linear structure of constant curvature "logically prior" to actual events thus biased him against the idea that the geometry of spacetime might be heterogenous as I show next.

### 3.2 Whitehead against Heterogeneous Space-time

The three main species of geometry, the Euclidian, hyperbolic and elliptical are thus all systems of 'straight lines' which is why Whitehead could concede that the curvature of space-time on the cosmic scale might be negative, zero or positive. What Whitehead could not abide however was the notion of a variably curved space-time contingent on the distribution of matter. For Einstein of course there are no gravitational fields 'in' space but instead the curvature of space-time itself explaining why the trajectories of massless photons are bent in the vicinity of large bodies like the sun. Whitehead's position had been rehearsed at Cambridge even before the invention of special relativity in 1905, when the young Bertrand Russell said in his 1897 book *An Essay on the Foundations of Geometry* that whilst space might conform to a non-Euclidian geometry of constant

12 A'Campo, N., Papadopoulos, A. "On Klein's So-called Non-Euclidian geometry". Most information I relate here on the technicalities and history of projective geometry and Klein's Erlangen program are taken from this paper.

curvature, a space of variable curvature, as suggested by those like William Kingdon Clifford[13], was “rather preposterous”.[14].

Whitehead was thus obliged to formulate an alternative theory of gravity based on the idea that "time is the stratification of nature … the old fashioned belief in the fundamental character of simultaneity" [15] and the idea of an independent gravitational field pervading space-time and acting at a distance similar to the way Newton’s classical gravitational force. Whitehead set out his alternative theory in his 1922 book *The Principle of Relativity*. The ontological foundation of his theory trades on the recurrent and ubiquitous ingression of certain abstract structures or "adjectival characters" through the events of nature. Generally speaking a field, including the gravitational field, for a given particle **m** at a point **P** in the four-dimensional hyper volume of space-time is,

> … a limited region stretching from **P** into **P’**s futurity … qualified by an adjective dependent upon **m** and **P** only … a three dimensional boundary region between **P’**s co-present and **P**’s kinematic future. … **P**’s causal future. … the adjectival character of the field of **m** at **P** consists in the correlated physical characters of the different event-particles of the field. The whole conception is practically the familiar one of tubes of force … [except that] … As we pass along a tube radiating from [the spatial point **P**] we keep to the same tube by allowing for the lapse of time required by the velocity c.[16]

Whitehead disagrees with Einstein for whom simultaneity is defined operationally as the coincidence of returning light signals in favour of the view that simultaneity is an apriori presupposition. Beginning with the notion of an "event particle", or an event "with all its dimensions ideally restricted" he defines linear "historical routes" of such particles. These are similar to Minkowski’s “world lines”. Whitehead seems to have in mind sequences of event particles belonging to the history of particular things like an electron, a rock, planet or a person but this isn’t necessarily the case. Such a linear series could simply be a tract through empty space. A "moment" or a "duration" through a given event particle P he defines as "a system of event particles representing all nature instantaneously

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13 Clifford was another Cambridge genius who anticipated the idea of general relativity. He was a staunch atheist and vociferous opponent of any beliefs ungrounded by evidence. He was also an early follower of Grassmann’s theory of extension from which he constructed his own geometrical algebras now known as Clifford algebras. Tragically he died aged only 34 in 1879.

14 This is actually a quote from an earlier 1893 paper but his attitude had not changed in the intervening 4 years. I take the quote from Sylvia Nickerson’s and Nicholas Griffin’s paper “Russell, Clifford, Whitehead and Differential Geometry”. p.29.

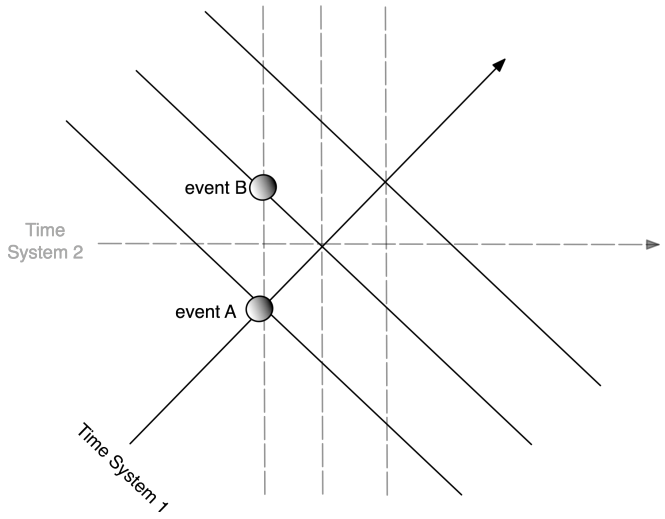
15 *The Principle of Relativity*, p.67.

16 *The Principle of Relativity*, p.36.

contemporaneous with P", *it is a temporal slice of some minimal durational extent through the entirety of nature*. P's duration must be said to exist for P even though P cannot in principle detect or sense other events within it. A contemporaneous duration must however be pre-supposed if we are to have any sense of an objectively given space within which events can be framed. Series of such temporal slices or moments arranged in parallel make up a "time system"[17]. There are different time systems and multiple intersecting moments for any given particle in contrast to the classical Newtonian theory which postulates but one time system and one "moment" incorporating P. It is therefore possible that "at some inconceivable distance from us there are events co-present with us now and also co-present with the birth of Queen Victoria..."[18].

There is no one privileged time system, each of them being but one aspect on the overarching "creative advance of nature", one amongst many perspectives on the "perpetual transition into novelty"[19]. Whitehead illustrates his notion of intersecting time systems with a diagram which I have reproduced below slightly modified to emphasise the difference between the two different time systems.

**Diagram 3. Intersecting Time Systems**
(Adapted from *Principle of Relativity 1922)*




In the above diagram, Time System 1 is represented by the series of solid diagonal lines for a given historic route (represented by the diagonal arrow pointing north east). The series of dashed vertical parallel lines represent Time System 2 for another historic route (represented by the horizontal arrow pointing due east). In Time System 1, event A precedes event B but in Time System 2, A and B are contemporaneous.

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[17] *The Principle of Relativity*, p.69. Emphases mine.

[18] *The Concept of Nature*, p.178.

[19] *The Concept of Nature,* p.178.

To the extent that relations between an event particle's historical route, and the historical routes of other events remain constant and parallel to it, we can abstract away from the concrete particularity of those relations and talk about a single enduring volume, gaining in the process a "facile representation of permanent space"[20] with a quasi-Euclidian geometry. It was on this metaphysical foundation that in 1922 Whitehead constructed an alternative mathematical machinery to Einstein's, predicting many of the same results but with some important exceptions. Whitehead had already summarised this position in an article for the *Times Education Supplement* in 1920:

> Now the spatial and temporal relations of event-particles to each other are expressed by the existence in space (in whatever sense that term is used) of points, straight lines, and planes. The qualitative properties and relations of these spatial elements furnish the set of conditions which are a necessary prerequisite of measurement. For it must be remembered that measurement is essentially the comparison of operations which are performed under the same set [of] assigned conditions. If there is no possibility of assigned conditions applicable to different circumstances, there can be no measurement. We cannot, therefore, begin to measure in space until we have determined a non-metrical geometry and have utilized it to assign the conditions of congruence agreeing with our sensible experiences…. For this reason I doubt the possibility of measurement in space which is heterogeneous as to its properties in different parts. I do not understand how the fixed conditions for measurement are to be obtained. In other words, I do not see how there can be definite rules of congruence applicable under all circumstances. *This objection does not touch the possibility of physical spaces of any uniform type, non-Euclidean or Euclidean* [my italics]. But Einstein's interpretation of his procedure postulates measurement in heterogeneous physical space, and I am very sceptical as to whether any real meaning can be attached to such a concept[21]

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[20] *The Principle of Relativity*, p.71.

[21] Whitehead, A.N., "Einstein's Theory; An Alternative Suggestion", The Times Educational Supplement, 12 Feb. 1920, p. 83; reprinted in Whitehead, Essays in Science and Philosophy(New York: Philosophical Library, 1948), pp. 241–8 (at 246–7). Taken from Sylvia Nickerson's and Nicholas Griffin's paper "Russell, Clifford, Whitehead and Differential Geometry". p.36.

Einstein himself had explored a similar approach with his 'Entwurf' theory prior to his realisation late in 1915 that the principle of general covariance[22] was implied by the principle of equivalence between accelerating reference frames and an homogenous gravitational field.[23] In other words if gravitational force ≃ acceleration, and an object in free-fall ≃ an object floating in space, then there is nothing for it but to say that the geometry of space-time must be curved in the neighbourhood of large centres of mass like the earth. In Minkowskian geometry, an observer who maintains a constant distance from some nearby centre of gravity (like Galileo atop the leaning tower of Pisa) has a straight world-line and objects falling towards the centre (like his two lead weights) are accelerating downwards with curved world-lines, but the principle of equivalence in General Relativity entails that it is Galileo who is accelerating with a curved-world line and that it is the two lead balls moving along straight world-lines or geodesics towards the centre. As Tim Maudlin writes in his book *Philosophy of Physics, Space and Time*;

> How does General Relativity explain gravitational phenomena in terms of the geometry of space-time? Once we accept that the trajectories of particles in free-fall are straight lines, it is clear that space-time cannot be Minkowskian.

Commentators sympathetic to Whitehead like Ronny Desmet and Gary Herstein have highlighted how Whitehead's theory was an attempt to account for gravity in a way that maintained a strict separation between geometry and physics in contrast to Einstein who they accuse of conflating the two. They mean that Whitehead sought to uphold uniform geometrical space as an apriori pre-requisite of physical science and the very measurability and comparability of events. They talk about a so-called “measurement problem” in general relativity, Desmet arguing for example that any purported difference in angle between the actual and expected paths of a light ray when it passes close to the sun requires “bringing the two angles into one space of comparison”[24]. He says that to reject this uniform space of comparison introduces an unnecessary bifurcation between human experience and the world of physical theory.

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[22] General covariance is described on Wikipedia as “the invariance of the *form* of physical laws under arbitrary differentiable coordinate transformations. The essential idea is that coordinates do not exist *a priori* in nature, but are only artifices used in describing nature, and hence should play no role in the formulation of fundamental physical laws…. Einstein recognised that the general principle of relativity should also apply to accelerated relative motions, and he used the newly developed tool of tensor calculus to extend the special theory's global Lorentz covariance (applying only to inertial frames) to the more general local Lorentz covariance (which applies to all frames), eventually producing his general theory of relativity. The local reduction of the general metric tensor to the Minkowski metric corresponds to free-falling (geodesic) motion, in this theory, thus encompassing the phenomenon of gravitation.”

[23] Norton, J. “General Covariance and the foundations of General Relativity”. p.803.

[24] Desmet, “Gary Herstein, Whitehead and the Measurement Problem”, p.247.

If I understand them correctly Desmet and Herstein are asserting that variably curved spaces are (1) unmeasurable and (2) at least suspect for contradicting 'common sense' or 'intuition'. The first proposition overstates the case for there are many kinds of variably curved spaces some of which are relatively homogenous and measurable, others which are massively heterogeneous and unmeasurable, and everything in between. For example the surface of a sphere is a space of constant curvature but a sphere with a slight bump on it is not. Both however are potentially measurable spaces (depending on things like whether they are finitely or infinitely divisible, whether composed of points, whether differentiable etc). Whilst one might concede that if the universe were not relatively homogenous at least in places then measurability might be unfeasibly difficult but this does not exclude the possibility of there being regions of variable curvature in the proximity of planets, stars and galaxies scattered amongst regions that are mostly flat as our universe actually appears to be. In addition, Einstein's theory tells us exactly how to translate measurements and co-ordinates between accelerating and inertial reference frames. In other words it tells us exactly how to measure variably curved spaces so it is not really clear what the so-called "measurement problem" actually is. This leaves objection (2) the argument from common sense. Firstly, common sense is neither universal, fixed nor necessarily veridical - it held once that the earth was flat and that the sun circled the earth. Furthermore, the abstract geometrical space which scientists and modern school children are familiar with today is probably an historical artefact and in many tribal, non-literate cultures lack words and concepts for abstract space and time having very different notions to the quantified homogenous, geometrical notions of space and time that we do. For example, some Aboriginal languages like Guugu Yimithirr in north Queensland lack individual-centred notions of 'left', 'right', 'front' and 'back' employing instead cardinal directions aligned to the east-west trajectory of the sun, [25] while Amazonian tribes like the Amondawa[26] and Piraha[27] are said to lack any sense of time as an all-encompassing background within which events are located. Also to the common sense of most scientifically informed people even today, the idea of constantly curved spacetime is no less 'weird' than the idea of a spacetime of variable curvature. Finally, Desmet and Herstein's proposition relies on the anthropocentric idea that the world must as a matter of principle conform to human intuition such that the test of a theory is its degree of alignment

[25] Haviland, J. "Guugu Yimithirr Cardinal Directions".

[26] Sinha, C., Da Silva, V. Zinken, J. Sampaio, W. "When time is not space: The social and linguistic construction of time intervals and temporal event relations in an Amazonian culture".

[27] Everettt, D. "Cultural Constraints on Grammar and Cognition in Piraha: Another Look at the Design Features of Human Language".

with it. Even accepting there is such a thing as universal human intuition, without this anthropic principle there is no reason why the manifold of actual events and the historical routes of occasions within it should not conform to a geometry of variable curvature. Whether it does or not is entirely an empirical question. On that count Whitehead's theory, whilst it might have been a respectable alternative to Einstein's at least amongst a minority of physicists until the 1950s,[28] has been disconfirmed by the mass of experimental evidence accumulated since. Russell could opine in his 1927 book *The Analysis of Matter* that the choice between the orthodox view espoused by people like Eddington and Whitehead's was purely conventional, "a mere question of convenience"[29] underdetermined by the known facts and experimental evidence. This was because Whitehead's theory yielded the same predictions regarding light bending, gravitational redshift, the precession of the perihelion of Mercury, and the Shapiro time-delay effect. However Whitehead's theory diverged from Einstein's implying at least five other effects[30] which could not be answered at that time but have been since. The first evidence came in 1971 with experimental data about the anisotropy of the locally measured gravitational constant.[31] Whitehead's argument that gravity wasn't the curvature of space-time itself but was like a magnetic field propagating through a flat four-dimensional Minkowski manifold is thus not only contradicted by swathes of observational data but virtually irrelevant to the study of gravitational phenomena whether Big Bang inflation, black holes or gravitational lensing.

Ultimately, the root of Whitehead's unease with general relativity was his commitment to the platonistic notion of a "hypothetical substructure of the universe, uniform under all the diverse phenomena".[32] Once we reject the existence of such an ideal substructure there

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28 Clifford, G. and Will, C. "On the Multiple Deaths of Whitehead's Theory of Gravity" referring to Bonnor, W.B. "Instrumentalism and Relativity", *British Journal for the Philosophy of Science*, 1958, vol.8., pp.291-294.

29 Russell, B. The Analysis of Matter, p.77. I take here much from Ronny Desmet's article "How did Whitehead become Einstein's Antagonist? On Poincare and Whitehead".

30 Clifford, G. and Will, C. "On the Multiple Deaths of Whitehead's Theory of Gravity". p.1.

31 Gibbons, G. and Clifford Will's paper "On the Multiple Deaths of Whitehead's Theory of Gravity" summarises five modern experiments which can be used to test Whitehead's theory, each of which they say disconfirms it. In some cases Whitehead's predictions are off by a factor of a million. Einstein's theory by contrast is said to pass all five tests "with flying colours". Bain, J. "Whitehead's Theory of Gravity" also gives a good summary of the differences between Whitehead's and Einstein's theories and explains how the former is now seen as having been decisively refuted. Sylvia Nickerson and Nicholas Griffin in their paper "Russell, Clifford, Whitehead and Differential Geometry", also review the debate and evidence against Whitehead. Whitehead himself predicted two things at variance with GR, the first was interference between gravitational and electromagnetic effects and the other secular acceleration of the centre of mass of two bodies ("the two body problem"). The second prediction was the one Clifford and Will refer to as disconfirmed in 1971 by accurate measurements of the gravitational effects of the tides. The alleged measurement problem in Einstein is also the subject of Gary Herstein's book *Whitehead and the Measurement Problem of Cosmology*. Unfortunately I have not been able to obtain a copy of Herstein's book for this thesis.

32 Whitehead, A. *Essays in Science and Philosophy*, p. 285. Quoted in Desmet, R. PhD. Chapter 6 The Minkowskian Background of Whitehead's Theory of Gravitation p.258, originally published as Whitehead's entry on "Mathematics" for the eleventh edition of the Encyclopaedia Britannica

is no compelling metaphysical reason why we should rule out Einstein's theory of a variably curved spacetime, the only test of which is whether it accords with the facts as it evidently does.

### 3.3 Whitehead's Prehensional Theory of Space

As I explained above, by 1925 Whitehead had arrived at the formulation that "extension is derivative from process, *and* is required by it" expressing a deep ambiguity in his system about the ontological significance of extension *vis a vis* process. The formula does however point to the idea of space as *derivative* from prehensive relations between occasions,[33] as an ideal measure abstracted from the concrete structure of inter-prehending events or "the ghost of transition".[34] Once we abandon the idea that events can only be measured against some ideal substructure and accept instead that they are only measurable relative to other events, then it might be expected that standards of measure will vary with changes in things like the heterogeneous distribution of matter in the cosmos. Developing a truly process-oriented approach to gravity consistent with Einstein's general theory would require jettisoning Whitehead's platonistic conception of the extensive continuum. [35] The glimmerings of such a theory can be seen in the reformulated notion of 'duration' in *Process& Reality* which abandons the idea of "all nature instantaneously" [36] as a primitive concept. In its stead duration is defined for some event x as that *set of occasions which are prehensively independent from* x. Contemporaneity is defined in terms of prehensive independence in the following way. Given any pair of occasions x and y, either x prehends y, or y prehends x or neither prehends the other. Occasions which are prehended by x are in its prehensive 'past', those which prehend x are in its prehensive 'future', and those which are independent from x, are in its prehensive 'present' composing its "duration". The set of occasions which are x's contemporaries or within its duration I will refer to here as $D^x$. Although x is prehensively independent from other occasions within $D^x$, any other pair of occasions y and z within $D^x$ may share successor occasions which prehend them both, or ancestor occasions which they both prehend, or may be contemporaneous with each other. In

33 In the *Monadology*, Leibniz treats extension as a purely phenomenal mode of perception between monads. Whitehead does not go this far, but his position is similar in some sense to Leibniz's because for him extension expresses causal independence between actual occasions.

34 *Modes of Thought*, p.96.

35 Llewellyn I think was the first to make this suggestion in his 1973 paper. Llewellyn, R. "Whitehead and Newton on Space and Time Structure". *Process Studies*, 3 (4):239-258 (1973). Footnote 4, p.255.

36 *The Principle of Relativity*, p.69.

other words the relation of contemporaneity is reflexive and symmetric but non-transitive.[37] Prehension defines what is known in mathematical order theory as a 'non-strict weak order' [38] on the set of all occasions which allows for example that in $D^x$, y might prehend z, but so long as neither prehends nor are prehended by x, they qualify as contemporaries of x and members of $D^x$. This kind of order has some very interesting properties. Firstly, although the relation of prehensive independence defines for each occasion a single duration, it is possible for an occasion to be a member of multiple non-identical durations. For example x may be a member of y's duration, $D^y$ , and a member of z's duration $D^z$ , so that $D^y$ and $D^z$ overlap at x, but $D^y$ may contain members that $D^x$ lacks or vice versa.[39] Thus y may be a contemporary of x and in its duration $D^x$, but contemporary with an event z which x is not. Each occasion x has but one unique duration but can be a member of many non-identical but overlapping durations.[40]

On the Newtonian theory of time contemporaneity is transitive and simultaneity absolute, so that all the occasions within any given duration are simultaneous with each other, and in the sense that for each individual member of a given duration there is only one duration that they belong to, (i.e., if $y \in D^x$ and $z \in D^x$ then $D^y = D^z$ and if $x \in D^y$ and $x \in D^z$ then $D^y = D^z$). Relativistic systems on the other hand are those in which simultaneity is not transitive; ones in which each particular occasion defines its own 'time system' relative to a given 'historical route' but can be located within the durations of many other occasions. In Whitehead's words, "There is a prevalent misconception that becoming involves the notion of a unique seriality for its advance into novelty. This is the classic notion of time which philosophy took over from common sense. Mankind made an unfortunate generalization

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37 I acknowledge here John Lango's very insightful article, "Whitehead's Category of Nexus of Actual Entities" which discusses at length how prehension can be understood on the basis of the logic of relations and I have taken much here from it.

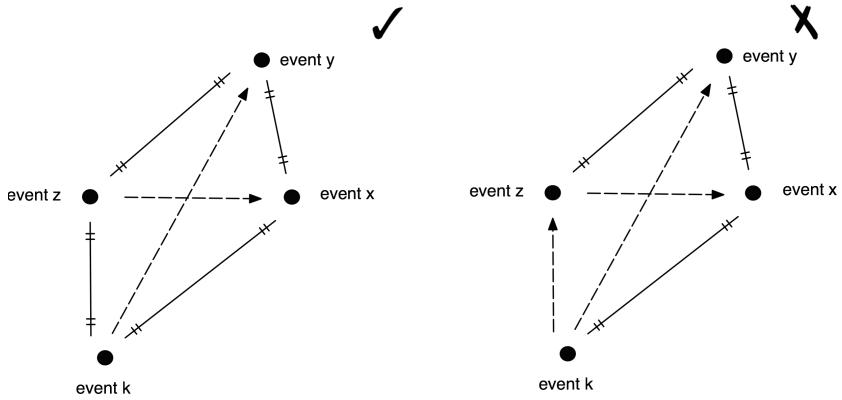
38 A weak ordering on a set A is a binary asymmetric, transitive and irreflexive relation R that is not a total order on A, (because there are at least two members of A, x and y which are 'indifferent' i.e.$\exists xy$: $\neg(Rxy) \wedge \neg(Ryx)$ is true. Defining the indifference relation thus; $Ixy \Leftrightarrow df \neg(Rxy) \wedge \neg(Ryx)$, a strict weak ordering is a weak ordering in which the indifference relation I is transitive such that for any x,y, z of A; $(Ixy \wedge Iyz) \rightarrow Ixz$. In the classical Newtonian model of time the temporal relation 'later than' is a strict weak ordering on the set of events because if event x is simultaneous with y and y is simultaneous with z, then x is simultaneous with z; i.e. the indifference relation "simultaneous with" is transitive. In a non-strict weak ordering the indifference relation is non-transitive.

39 Suppose that for some occasion x, its duration Dx contains another occasion y. By definition, neither x nor y prehend the other, i.e. $\neg(Pxy) \wedge \neg(Pyx)$ and so by definition x must also be in y's duration Dy. Therefore, $x \in Dy \rightarrow y \in Dx$. However, it will be the case in relativistic systems that Dy has other elements; a, b, c... $x \notin Dx$.

40 An article by Villard Allan White, "Whitehead, Special Relativity, and Simultaneity" says the change in position between Whitehead's earlier works and *Process & Reality* was due to Whitehead's newfound understanding of the "subjective" nature of concrescence as opposed to the objective relations of extension emphasised in the earlier works. If by this he means that Whitehead's later emphasis on topological transformations or relations of prehension, experience or 'feeling' between events came assumed greater prominence in his metaphysics then I am in complete agreement but I think it is a misreading to think of this transformation as being subjectivist in the sense of being a tilt towards idealism, anti-realism or solipsism in any way.

from its experience of enduring objects".[41] The spatio-temporal structure of events based on their prehensional relations is illustrated in the diagrams below.

**Diagram 4. Spacetime as the prehensional structure of events**




The above figure illustrate space-time relations on the basis of the asymmetric, irreflexive and transitive relation of prehension. A dashed line from *x* to *y* indicates that y prehends x. A barred line between x and y indicates that x neither prehends nor is prehended by y. The figure on the left illustrates the non-transitivity of contemporaneity. Event x is contemporaneous with y which is itself contemporaneous with z. Event x prehends z nonetheless. This can also be expressed by saying that durations $D^x$ and $D^z$ overlap at y but are non-identical. The figure on the right is an invalid representation because x prehends z which prehends k which means (given prehension is transitive) that x prehends k, but there is a barred line between k and x indicating mutual independence which cannot be true if x prehends k.

If we accept that occasions and historical routes of occasions are prehended differently by different occasions both qualitatively and quantitatively and that measurement is just the counting and comparison of such qualitative and quantitative relations then there seems to be no reason in principle why the spatio-temporal measure of an event must be homogeneous for all other prehending/observer events or why the geometry of the whole prehensional structure oughtn't to be variably curved for instance. The prehensional and spatio-temporal structure of the evental manifold could in theory conform to many geometries and the one scientists select should be determined exclusively by empirical data rather than by proximity to some ideal "substructure" thought to make measurement conveniently uniform for human observers.

### 3.4 Conclusion

Whitehead's prehensional account of the relativity of simultaneity is close in spirit to other causal and discrete theories of relativity like that of A.A. Robb outlined in his 1914 book *A Theory of Space and Time*, David Malament's and Rafael Sorkin's causal set theory, and

41 *Process & Reality*, p.35.

cellular automata models proposed by people like Stephen Wolfram and Edward Fredkin. Causal set theory asserts that much of relativity is deducible from axioms for partially ordered sets of discrete events ordered by an asymmetric, irreflexive and transitive relation interpreted as representing causal connection. In the words of Rafael Sorkin,"… this structure alone suffices to reproduce (to a high degree of approximation) everything that we mean by the geometry of spacetime".[42] One of the virtues of causet theories is discreteness; the number of events between any other two is finite. This eliminates many of the problems of infinities and renormalisation in physics and has the virtue of making measurement of properties like distance and volume trivial. Another boon of causet theories has been their representation of dynamics as a "growth process" in which new events are added to the stock of past events bit by bit, and whose order relative to their ancestors is unique but whose order relative to non-ancestors is arbitrary. Sorkin says in that this property of causets corresponds exactly to the principle of general covariance in standard relativistic physical theory [43] but does not require giving up the idea of 'becoming' such that the past and the future are "all laid out in one timeless tableau. … the 'Now' might be restored to physics without paying the price of a return to the absolute simultaneity of pre-relativistic days"[44]. Although the theory is still under development, it has already been used to make accurate predictions about fluctuations in the value of the cosmological constant and is being intensively studied by physicists like Lee Smolin as a basis for quantum gravity.[45]

The causal theory of space-time, of which Whitehead's prehensional model was an early precursor, has thus subsequently blossomed into a promising research program. Unfortunately Whitehead's work in this area was hampered by his attachment to a platonistic theory of the extensive continuum which he perceived as imposing a necessarily uniform or flat Minkowski geometry on the manifold of actual events, an idea which turns out it seems to have been a scientific dead-end.

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42 http://www.einstein-online.info/spotlights/causal_sets . Sorkin quotes Riemann on the virtues of discreteness, The question of the validity of the presuppositions of geometry in the infinitely small hangs together with the question of the inner ground of the metric relationships of space. In connection with the latter question... the above remark applies, that for a discrete manifold, the principle of its metric relationships is already contained in the concept of the manifold itself, whereas for a continuous manifold, it must come from somewhere else. Therefore, either the reality which underlies physical space must form a discrete manifold or else the basis of its metric relationships should be sought for outside it[…]."

43 This is what I have called above the "non-transitivity of contemporaneity" in Whitehead's prehensional model of space-time. It entails that from a common ancestor event, the one causal/ prehensional structure can be seen to grow in different ways such that the temporal order of two non-causally connected events in one growth path (co-ordinate system) is the reverse of that in the other, and yet laws of causality and causal connection are invariant.

44 Sorkin, R. "Geometry from Order: Causal Sets".

45 For overviews from a philosophical perspective see Callender and Wulrith . Other good overviews can be found in Sorkin, Dribus and Markopoulou.

# Chapter 4. Historical Roots of Whitehead's Process Platonism

## 4.1 Plato

There is surprisingly little emphasis today on the fact that when Whitehead says he is a platonist he ought to be taken literally or that Plato's idealist doctrines are at the core of his process philosophy,[46] in spite of his now famous statement that European philosophy "consists of a series of footnotes to Plato."[47] The first to highlight the strong connection between these two thinkers was the Plato scholar A.E. Taylor in his review of Whitehead's book *The Concept of Nature* in the pages of *Mind* in 1921 in which he described Whitehead's contribution to *Naturphilosophie* as "the finest contribution yet made by any man". Whitehead leant heavily on Taylor's works about Plato in *Process & Reality* and *Adventures of Ideas* and it seems that Taylor's own view may have influenced Whitehead shift towards a more openly platonic philosophy around 1925. In the preface to his famous 1926 book, *Plato: The Man and his Work,* Taylor reiterates his opinion that there was "an almost absolute equivalence of *Timaeus*' analysis with that of Whitehead in his *Principles of Natural Knowledge*"[48]. Others who have focused heavily on the Plato connection include Dorothy Emmet, who, in her 1932 book, claimed that Whitehead's cosmology was really just a "modern form of Platonism"[49] and Jorge Nobo who gave a so-called "revisionist"[50] (what I would call neoplatonist) reading of the extensive continuum in his 1986 text *Whitehead's Metaphysics of Extension and Solidarity*.[51]

I will limit myself to some remarks on the connection between three key ideas which had a self-acknowledged influence on Whitehead; Plato's theory of forms, Plato's idea of a field of abstract potentiality called 'the Receptacle', and Plato's own organicist cosmology of the world as analogous to a living being.

I have already discussed the influence of Plato's theory of forms so I shall only reiterate here that although Whitehead rejects what he calls the "absoluteness" of Plato's forms, i.e.

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46 John Lango gives some background on Plato's influence in his 2008 entry for the *Yearbook of Whiteheadian Process Thought*. Also a recent 2015 article by Eleonora Mingarelli in the journal *Process Studies* gives a detailed account of the role of Plato's Receptacle in Whitehead.

47 *Process & Reality*, p.39.

48 See Betegh, "The Timaeus of A.N. Whitehead and A.E. Taylor".

49 Emmet, D. *Whitehead's Philosophy of Organism*. p.241. See her chapters V and VIII more generally.

50 Theodore Vitalli's review of Nobo's book for example.

51 Nobo stresses the extensive continuum as the unified, indivisible ground and *source* of actual events underemphasising the indeterminacy, creativity and asymmetry of process in Whitehead. See my discussion in Appendix 3.

their ontological independence from the realm of actual events, he does say they are nonetheless irreducible and essential components of reality and that the two realms "require each other".[52] Eternal objects are said to be "essentially referent beyond themselves",[53] because their very essence is to be realisable in actuality. We could flip this around and just as easily say that in Whitehead's view the realm of actual occasions is also essentially referent beyond itself, to the realm of pure potentialities. Although eternal objects and Plato's forms are different they do share important features such as being eternal, non-actual and explanatory of the identity and contingency of events.

Regarding the idea of a *Receptacle* (*eidola*/ ὑποδοχή) as outlined in the *Timaeus,* Plato describes it in the following ways; as the "nurse of all becoming" (49a),"it never alters its characteristics, and continues to receive all things" (50c,) "it is by nature a matrix for everything" (50c), "the things which pass into it and out of it are images of the eternal beings" (50c), "that which all over itself is going to receive properly and repeatedly all the likenesses of all the intelligible and eternal things must in its nature itself be devoid of all character" (51a), "a seat for everything that comes to be … apprehended without the senses by a sort of spurious reasoning (52b). [54]

The Receptacle is therefore the most abstract and general of structures shared by all particular forms. Cornford says that Plato requires the Receptacle because there must be something, not totally unreal, that is the home of them all but which is not entirely separate from them[55]. There is little question that Whitehead's notion of the extensive continuum in *Process & Reality* as the most general realm or structure grounding specific geometrical forms was inspired by Plato's Receptacle even though there is no express acknowledgement as such there. Whitehead does however refer to the *Timaeus* several times in *Process & Reality* and one can find in later works express discussions about the connection. The lengthiest passage is from his book *Adventures of Ideas* published in 1933. In the relevant passage, Whitehead says that the Receptacle is close to the modern idea of a space-time but one devoid of particular physical laws.

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52 *Modes of Thought*, p.70.

53 *Modes of Thought*, p.69.

54 Plato, *Timaeus and Critias*. Translated by Lee, D and Johansen, T.K.

55 Cornford, F. *Plato's Cosmology*

> The notions of Harmony and of Mathematical Relations are only special exemplifications of a yet more general philosophical concept, namely, that of the general interconnectedness of things, which transforms the manifoldness of the many into the unity of the one. We speak in the singular of *The* Universe, of Nature, of …. , which can be translated as Process. There is the one all embracing fact which is the advancing history of the one Universe. This community of the world which is the matrix for all begetting, and whose essence is process with retention of connectedness, - this community is what Plato terms The Receptacle (ὑποδοχή) in our effort to define this meaning we must remember that Plato says that it is an obscure and difficult concept, and that in its own essence, the Receptacle is devoid of all forms. It is thus certainly not the ordinary geometrical space with its mathematical relations. Plato calls his Receptacle, "the foster mother of all becoming". He evidently conceived it as a necessary notion without which our analysis of Nature is defective. It is dangerous to neglect Plato's intuitions. He carefully varies his phrases in referring to it and implies that what he says is to be taken in its most abstract sense. The Receptacle imposes a common relationship on all that happens, but does not impose what that relationship shall be. It seems to be a somewhat more subtle notion than Aristotle's 'matter' which, of course, is not the 'matter' of Galileo and Newton. Plato's Receptacle may be conceived as the necessary community within which the course of history is set, in abstraction from all the particular historical facts. I have directed attention to Plato's doctrine of the Receptacle because, at the present moment, physical science is nearer to it than at any period since Plato's death. The space-time of modern mathematical physics conceived in abstraction from the particular mathematical formulae which applies to the happenings in it, is almost exactly Plato's Receptacle. It is to be noted that mathematical physicists are extremely uncertain as to what these formulae are exactly, nor do they believe that any such formulae can be derived from the mere notion of space-time. Thus, as Plato declares, space-time in itself is bare of all forms.[56]

On the question of Plato's own organicist cosmology in the *Timaeus*, it is partly thanks to Whitehead that this aspect of his writings now receives more attention. Prior to that modern readers primarily knew Plato for his theory of forms and his political and ethical theories so that the *Timaeus* was seen as something of an aberration. In the middle ages

56 *Adventures of Ideas*, pp.149 - 150.

however Cicero's translation was the only text available in the West[57] so Plato was associated primarily with his organicism. That changed in the Renaissance with the translations of many new manuscripts by people like Cosimo de' Medici, Marsilio Ficino, Bruni and Alberti[58]. Whitehead's debt to the organicist cosmology of the *Timaeus* is acknowledged in Chapter III of *Process & Reality*, "The Order of Nature", in which he compares the cosmology of Newton's *Scholium*[59] unfavourably to that of the *Timaeus*, saying that the *Scholium* contained no hint,

> of that aspect of self-production, of generation, of φύσις[60], of natura naturans[61], which is so prominent in nature. For the *Scholium*, nature is merely, and completely, there, externally designed and obedient. The full sweep of the modern doctrine of evolution would have confused the Newton of the *Scholium* but would have enlightened the Plato of the *Timaeus*.[62]

The *Timaeus* has the Demiurge taming the primeval chaos of the world by fashioning it "with an eye on the eternal"[63] in his own image as something with intelligence, soul and overarching order. The whole world is thus an organism whose parts function harmoniously together, as microcosms of the entire macrocosm suffused with all-encompassing World Soul. The word "soul" signified for Plato the principle of self-movement.[64] The idea of nature 'constructing itself' in accordance with higher principles of order and aesthetic achievement is seen in Whitehead's notion that occasions have a "subjective aim" which like a final cause lures or encourages them to develop in a way that harmonises with certain cosmic values like Beauty and intensity of feeling. Dorothy Emmet and Taylor argue in this connection that the ingression of an eternal object is analogous to the regulation of real developmental processes as described by Plato in *Philebus,* which are thought of as approximating or moving towards "a certain right

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[57] Russell, B. *A History of Western Philosophy*, p.157. See also Kantorowicz, E. "Plato in the Middle Ages", *The Philosophical Review*, Vol. 51, No. 3 (May, 1942), pp. 312-323.

[58] Arthur Herman's *The Cave and the Light* gives a good overview of this whole period and indeed the battle between Platonism and Aristotelianism in Western thought.

[59] The *Scholium* was an essay appended to the second edition of Newton's *Philosophiæ Naturalis Principia Mathematica* in 1726.

[60] φύσις (*fýsi*) meaning "nature".

[61] "natura naturans" is the term employed by Spinoza literally meaning "nature naturing" or "nature doing what nature does" but indicating nature as an active, self-producing entity or process.

[62] *Process & Reality*, p.93.

[63] *Timaeus*, 29a.

[64] My summary here is taken the *Internet Encyclopaedia of Philosophy* entry: "Plato: Organicism".

proportion" or "law of structure" such that "When they have 'become' they will embody it perfectly".[65] It is this sense of *natura naturans* or nature constructing itself which *Naturphilosophes* like Goethe, Schelling and Hegel took from Plato and from which Whitehead's own organic philosophy is directly descended.

### 4.2 Hermann Grassmann's Extension Theory

Whitehead's first book, *A Treatise on Universal Algebra* published in 1898 was mostly an exposition of the ideas of Hermann Grassmann but also with discussion of Boole and Schroeder and all the then contemporary logical calculi. Whitehead's *Universal Algebra* did enjoy a brief moment in the sun for a few years after publication[66] but was then virtually forgotten except by a handful of Whiteheadian philosophers. Grassmann was one of the originators of the concept of a vector[67] and his 1844 book, the *Ausdehnungslehere* (Extension Theory) virtually invented linear algebra which was contained a more general calculus distinguished by its use of product operators to construct higher order vector spaces. The main products are generated by the use of a binary anti-commutative operator called the "exterior product", (symbolised by a wedge "^").

Grassmann arrived at the idea of vectors musing on a straight line segment ABC and the formula AB + BC = AC (Diagram 5). His discovery was that the formula also holds even when C is in between A and B so long as AB and BA are thought of as having the same absolute magnitude but opposite orientations such that AB is identical to the negative of BA, and like positive and negative numbers, sum to zero, i.e. (AB) + (BA) = 0 or (AB) + -(AB) = 0 i.e., (AB) = -(BA) (Diagram 6).

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[65] Emmet, D. *Whitehead's Philosophy of Organism*, p.226.

[66] Russell reported to Moore in 1900 that at the Mathematical Congress in Paris he had found that Whitehead had "a great reputation" amongst "the Italians", viz Peano and his followers, and that they had all "read and much admired his book and were delighted to meet him". Grattan-Guinness, p.432. Peano went on to write his own book on Grassmann *Calcolo geometrico secondo l'Ausdehnungslehre di H. Grassmann preceduto dalle operazioni della logica deductive* whose English translation is; *Geometrical Calculus, According to the Ausdehnungslehre of H. Grassmann, Preceded by the Operations of Deductive Logic*, translated by Kannenberg, L.C. for the first time 2000.

67 The term "vector" was taken by Hamilton from the Latin for vehicle or transport but it was one Grassmann objected to. His preferred word was "strecke" or "stretches" connoting not just a sense of displacement but the idea of a thing extending itself out through or over a series of others.

**Diagram 5 Addition of line segments**

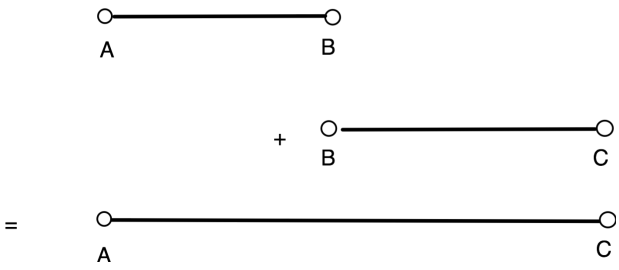
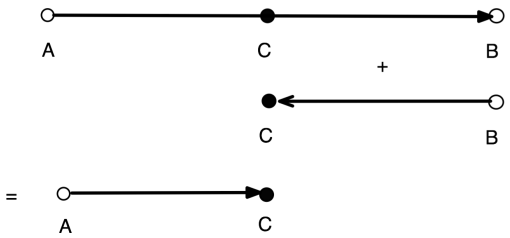


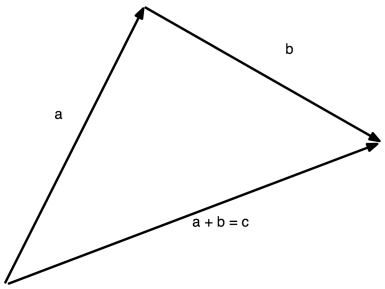

**Diagram 6a. Addition of Directed Line segments**

The rule AB + BC = AC still holds when C is in between A and B (on condition that BC = - CB.)




Grassmann generalised this idea of a ‘quantity with a direction’ to his notion of free vectors (those not bound to points in the line) showing that they could also be added in a consistent way according to the normal laws of addition. Again interpreting the calculus geometrically, the sum of vectors with magnitudes *a* and *b* could be said to vary with the difference in angle between them from a maximum of a + b (when the two vectors had the same direction or $180^{o}$) and a minimum of a - b (when the two vectors were oppositely directed or at $0^{o}$) with the magnitude and direction of their sum c like a centre of gravity depending on the contributions of the respective summands 'weights' (magnitudes and directions). The vector sum can be modelled geometrically as the base of a triangle formed by appending the tail of b, to the tip of a, such that c’s tail is the same as a’s and its tip the same as b’s, as in the diagram below.

**Diagram 6b. Vector Addition**



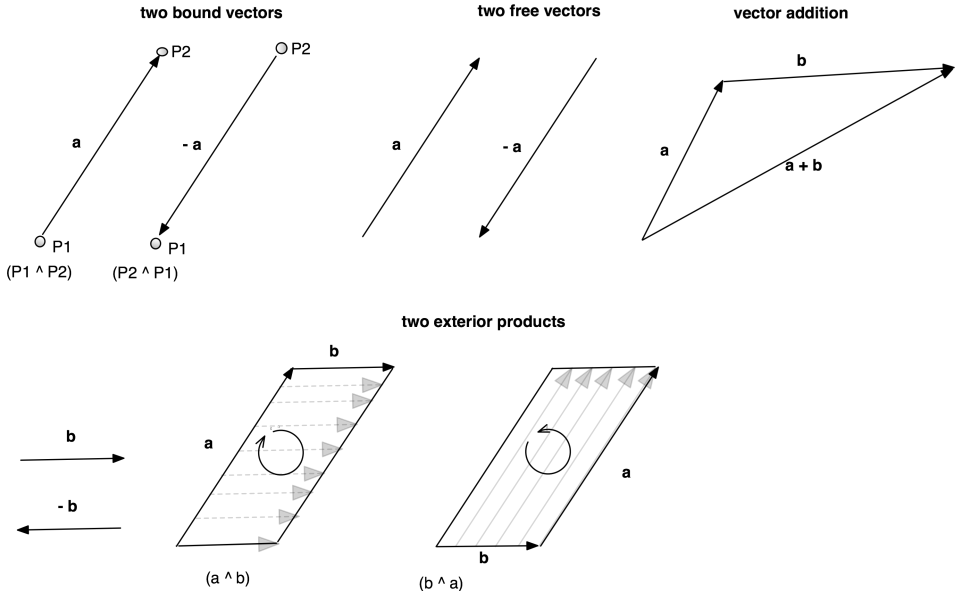

In the above, imagine **b** pivoting around the point where **a** and **b** meet, whilst tip of **c** stays tethered to the tip of **b**, such that it stretches and shrinks as the tip of **b** moves around. If **a** and **b** have the same direction so does **c** and its magnitude is just the magnitude of **a** + the magnitude of **b**. If **a** and **b** are oppositely directed then the magnitude of **c** is just the difference in magnitude of the summand vectors. If the directions of **a** and **b** are neither the same nor exactly opposite then the magnitude and direction of their sum is like a centre of gravity determined by the respective 'weights' of the summand vectors (i.e. their respective magnitudes and directions).

Returning now to Grassmann's exterior product, it is defined as generated by component vectors. For example, *a* and *b* are said to generate a "bi-vector" (*a* ^ *b*) which can be interpreted as an oriented plane (oppositely oriented to the bi-vector (*b* ^ *a*). Bi-vector generation can be again modelled geometrically by imagining the tails of two vector arrows being brought together, then while holding one constant, varying the position of the second by moving its tail along the length of the first, sweeping out a higher grade 'area' in the process (see Diagram 7). The exterior product of three vectors is a "tri-vector" or a directed volume. The exterior product can be used to compound vectors ad infinitum to generate directed geometric quantities of any dimension or "grade".

Vectors, bi-vectors, tri-vectors, and so on can be treated as distinct quantities that can be added together and multiplied by scalars such that some number x multiplied by a unit vector '*a'* gives a vector in the same direction but of 'magnitude' x, a scalar y applied to a bi-vector gives another with the same orientation but with a magnitude 'area' y times as large, and any element multiplied by -1 gives another with the same magnitude/area etc but with a reversed orientation. The outer product is therefore 'anti-commutative' defined as that property such that any two oppositely signed bi-vectors with the same components but in reverse order are identical; i.e. that (a ^ b) = - (b ^ a) and -(a ^ b) = (b ^ a).[68]

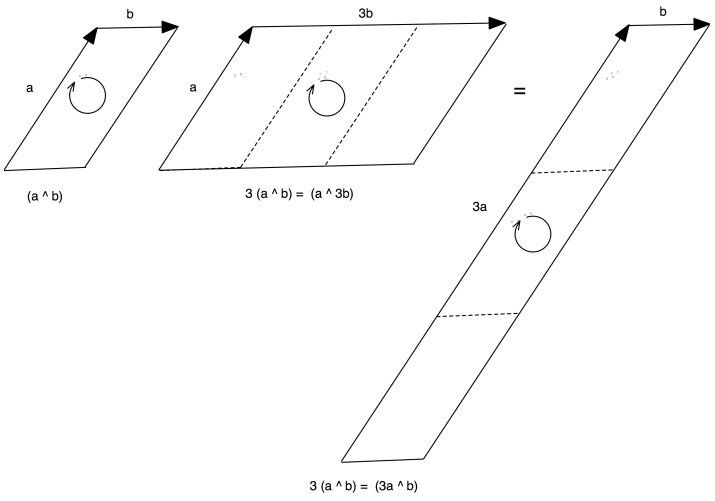
68 Anti-commutativity also entails that double negation cancels. Thus (a ^ b) = -(b ^a) = - - (a ^ b).

**Diagram 7. Addition and Exterior Product of Vectors**




Treating bivectors as single quantities that can be scaled up linearly however entails 'partial distributivity' of scalars over the exterior product or that scalars factor out, i.e. to multiply **x** and (a ^ b) together is to apply that factor to *one* vector but not the other in the product (a ^ b), so that **x(**a ^ b) = (**x**a ^ b) or (a ^ **x**b). Similarly, a parallelogram with area of 2km$^2$ having sides 1km and 2km can be scaled up by a factor of 3 to 6km$^2$ by multiplying either side of the parallelogram (but not both) by 3. This is illustrated in the following figures.

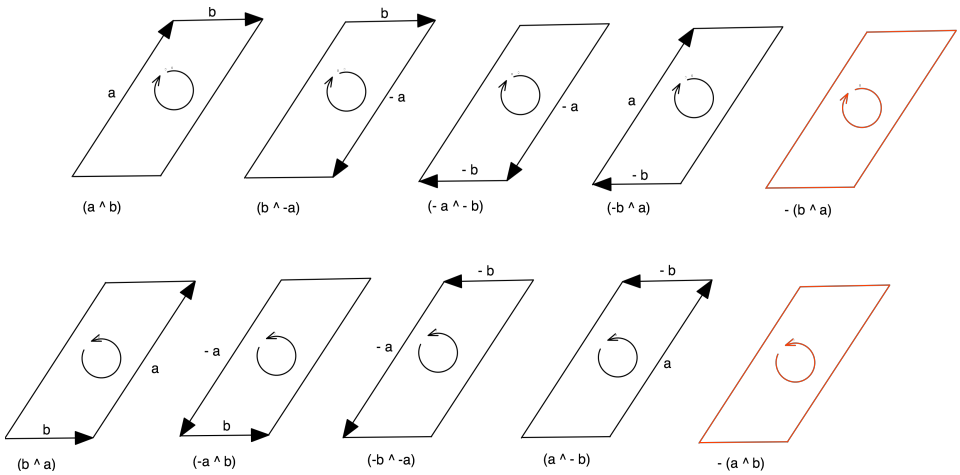
**Diagram 8. Scalar Multiplication of Exterior Products**




'Partial distributivity' applies equally to negative scalars so that -1(a ^ b) = (-a ^ b) or (a ^ -b) so that in combination with anti-commutativity it is easy to show that

(a ^ b) = (-b ^ a) or (b ^ -a)[69]. This latter property is known as *anti-symmetry*. Anti-symmetry says in short, the effect of switching the order of components in a product, can be reversed by changing the sign of either *one* (but only one) of the components. A geometric illustration of this fact can be seen in the figures in **diagram 9** below. The top row of products are oriented anti-clockwise, the lower ones clockwise. In the bottom row, each product has been created by reversing the order of elements in the product directly above it. Note how a reversal of orientation can also be produced by simply changing the sign of one (and only one) element in the pair or of the whole product (figures in red).

**Diagram 9. Anti-symmetry of the Exterior Product**




Importantly the product of a vector with itself, (its 'square') or with any scalar multiple of it is the 0 vector; i.e., for any x, (a ^ **x**a = 0)[70]. A geometrical 'explanation' for this is that one cannot construct or 'sweep out' a parallelogram from any two vectors with the same or opposite directions, the product of such vectors being the degenerate case where the 'area' or extent of the product magnitude reduces to zero. The same law holds for bi-vectors and higher grade products so that (a ^ b) ^ (a ^ b) = 0 too.

Anti-commutativity however only applies to products which have an even number of components. The components of oddly graded products like tri-vectors commute which is to say if a, b and c are all vectors then (a ^ b) ^ c = c ^ (a ^ b). Both sides of this equation

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69 Anti-commutativity says (a ^ b) = -(b ^ a). Applying non-distributivity we get (a ^ b) = (-b ^ a) or (b ^ -a).

70 To be accurate the product of an entity with itself is the zero product or multi-vector not the number 0. Another way to think about why the product of a vector with itself is zero is that anti-commutativity tells us that a^b = -b^a, so a^a = -a^a = -(a^a). If we think of (a^a) and -(a^a) as akin to numbers, the only 'number' that is equal to its negative is zero, so (a^a) = 0.

Some presentations start with this law then prove anti-commutativity using the fact that the exterior product distributes over addition thus:
(a + b) ^ (a + b) = 0
= (a ^ a) + (a ^ b) + (b ^ a) + (b ^ b)
= (a ^ b) + (b ^ a) = 0
therefore, 0 - (b ^ a) = (a ^ b) i.e. - (b ^ a) = (a ^ b).

obviously have the same absolute magnitude but they also have the same orientation (+ or -). Taking subscripts to indicate grade this is captured by the formula: $(a_m \wedge b_n) = -1^{mn} (b_n \wedge a_m)$.[71] Thus if $a_1$ is the vector a and $b_2$ is the bivector (c ^ d) then (a ^ b) = a ^ (c ^ d) = $(-1^{1 \times 2 = 2} = 1)$ $(b_2 \wedge a_1) = (c \wedge d) \wedge a$, or $a \wedge (c \wedge d) = (c \wedge d) \wedge a = (b \wedge a)$.

The most important extension of Grassmann's work was carried out by William Kingdon Clifford at Cambridge in the 1870s just prior to Whitehead's arrival. He developed a new system based on another product that Grassmann invented now known as the geometric product.[72] Clifford's systems are what are known today as 'Clifford' or 'Geometric Algebras'. Grassmann's extension theory has proven incredibly advanced for its time. Not only are vectors and vector addition now standard fare in physics, but his calculus and its derivative branches like Clifford algebra even today are revealing unexpected applications in many areas from quantum physics and gravity through to computer vision. The well known mathematician F.W. Lawvere has even argued that Grassmann's calculus anticipated many important concepts in category theory.[73] A Clifford algebra was promoted by Wolfgang Pauli and Dirac in the 1930s as a suitable formalism for the non-commutative quantum mechanics of interacting fermions and Pauli's exclusion principle. However Grassmann's calculus and Whitehead's contribution were relatively neglected by the broader mathematics community and it was not until the 1960s, when an ex-philosopher turned mathematician, David Hestenes, re-discovered them, that the tide began to turn. Hestenes came to Grassmann and Clifford algebras via Pauli and Dirac, and largely thanks to his efforts, the last few decades have seen a significant resurgence of interest in Grassmann and Clifford algebras together with a revival of Whitehead's reputation in this field. Hestenes has commented that Whitehead's work is probably still "the best and most complete exposition on the *Ausdehnungslehre* in English" and that he was "one step away from a mathematical system that truly deserves to be regarded as a

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71 If m . n is odd then m.n.-1 = -1 but if m.n is even m.n.-1 = +1.

72 The geometric product is the sum of a wedge product '^' and the 'dot' product '•', i.e. ab = (a^ b) + (a • b). The dot product is a scalar. If θ is the angle between vectors a and b then (a • b) is the absolute magnitude of a multiplied by the absolute magnitude of b multiplied by cos θ, i.e. (a • b) = |a| |b| cos θ. Alternatively, in a 2D co-ordinate space with co-ordinate axes x and y, if vectors A and B have co-ordinates (Ax, Ay) and (Bx, By) respectively, then the dot product A • B is (Ax.Bx) + (Ay.By). If vectors A and B are orthogonal then their dot product is 0. If the vectors are parallel then their geometric product is also 0. If they are neither orthogonal nor parallel then their geometic product will be a combination with a scalar as its first component and a bivector as its second component. Therefore one can show that the geometric product of the unit vector with itself or a2 = 1, and that the geometric product of two orthonormal unit vectors reduces to their exterior product or ab = a^b. This fact combined with the fact that the geometric product anti-commutes and is associative leads to the important fact that the geometric product of the unit bi-vector with itself is -1, i.e. (a^b)(a^b) = -1, because (a^b) (a^b) = abab = a(ba)b = a(-ab)b = (a-a)(bb) = (-aa)(bb) = -(aa)(bb) = -(1)(1) = -1. Thus we could say that geometric sqrt -1 = the unit bivector (a^b) similar to the unit imaginary number ***i***. Also in the 2D plane the effect of pre or post 'multiplying' the unit orthonormal base vectors a and b by the unit bivector (a^b) is to rotate them 90o clockwise or anti-clockwise. The geometric product as the sum of a bi-vector and a scalar seems odd but they are like complex numbers which are similarly compounds of two different kinds of numbers: imaginaries and reals. See Stephen Gull, Anthony Lasenby and Chris Doran's paper, "Imaginary Numbers are not Real — the Geometric Algebra of Spacetime" for more detailed explanation.

73 Lawvere, F.W. "Grassmann's Dialectics and Category Theory".

Universal Geometric Algebra"[74]. Interestingly, one of the applications of geometric algebras has been in "gauge theories" of General Relativity, which, like Whitehead's theory, speak of a gravitational field superimposed on a flat Minkowski space-time. Work on gauge theories has been mainly carried out by the astrophysics group at Cavendish Laboratory, Cambridge, headed by Anthony Lasenby and Chris Doran. In one of their papers they say that their theory is "constructed in a flat background spacetime and employs gauge fields to ensure that all relations between physical quantities are independent of the positions and orientations of the matter fields. In this manner all properties of the background spacetime are removed from physics, and what remains are a set of 'intrinsic' relations between physical fields".[75] The connection between such theories and Whitehead's own approach to general relativity is suggestive although Lasenby has stated that their theory has quite a different motivation and structure.[76]

I have of course only touched very superficially on the main features of Grassmann and Clifford algebras but one can glimpse how their virtue resides in their power to encode directional phenomena including rotations and oscillations. The philosophical interest of geometric calculii also lies in whether, as Peter Simons puts it, they afford us a "deeper insight into the nature of space and spacetime, or extension, as Grassmann and Whitehead would have said".[77] He speculates that this comes down to their natural ability to represent periodic, phase or wave-like behaviour which traditional tools such as complex numbers and quaternions can obscure.[78] At the root of geometric calculi however and the machinery they possess for representing directional and cyclic phenomena, is the simple notion of a combinable vector which expresses how physical changes are linked together as processes in simple and more complex ways into products with properties like

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[74] Quoted in Dawson, A. "Whitehead's Universal Algebra", in Weber, M. & Desmond, W. (eds), *Handbook of Whiteheadian Process Thought [Vol.2].*

[75] Lasenby, A., Doran, C., and Gull, S. "Gravity, Gauge Theories and Geometrical Algebra", arXiv:gr-qc/0405033v1.

[76] Professor Lasenby has stated that "I don't think there's too much relation with Gauge Theory Gravity. The latter works in a flat spacetime background, but as the name suggests, then finds the gauge fields that arise from local gauging of the symmetries of spacetime translation and rotation, whereas Whitehead's theory attempts to look at retarded effects from a collection of point masses, which I would therefore place closer to theories attempting a 'direct action' version of gravity, such as the ideas Hoyle & Narlikar …". Private email.

[77] Simons, P. "Vectors and Beyond". p.392.

[78] The imaginary number *i* is postulated to be the square root of -1. Applying standard algebra and commutativity of multiplication: ($\boldsymbol{i^1 = i}$), ($\boldsymbol{i^2}$ = -1, $\boldsymbol{i^3 = i i^2 = -i}$), and ($\boldsymbol{i^4 = ( -i.i)}$ = (-1. i)(i) = -1($i^2$) = -1(-1) = 1), ($\boldsymbol{i^5 = i^4. i = 1 (i) = i}$). Notice that the sequence comes back to where it started with $\boldsymbol{i^5 = i^1}$. Repeatedly multiplying *i* by itself generates the periodic series; i, -1, -i, 1, i, -1, -i, 1 …. similar to how -1 multiplied by itself results in the periodic series 1, -1, 1, -1… We can interpret multiplication by ***i*** geometrically as an anti-clockwise 1/4 rotation about the origin of a co-ordinate system where -***i*** and ***i*** are the lower and upper ends of the vertical axis, and -1 and 1 the left and right ends of the horizontal axis which gives us a special 2D space called the Argand plane. Complex numbers are compounds of imaginary and real numbers of the form (a +b***i***) where a and b are real numbers with the first component interpretable geometrically as the horizontal real co-ordinate and the seconds as the vertical imaginary co-ordinate in the Argand plane. Complex numbers can be added and multiplied in the usual way to encode partial rotations, translations etc.

non-commutativity and anti-symmetry that mimic many dynamical processes in nature. The affinity of vectors and events is why no doubt Whitehead compares occasions to vectors some 35 times in *Process & Reality*. The vector character of the world bears out he says the wisdom of Heraclitus.

> Mathematical physics translates the saying of Heraclitus, 'All things flow,' into its own language. It then becomes, All things are vectors. Mathematical physics also accepts the atomistic doctrine of Democritus. It translates it into the phrase, All flow of energy obeys 'quantum' conditions.

But there is another interesting historical connection between vectors and notions of space in the platonism of the German *Naturphilosophie* movement of the early 19th Century which created a pool of significant philosophical and cultural ideas from which Grassmann drew many of his insights.

### 4.3 The Naturphilosophie of Schelling and Schleiermacher

> Among the Greek authors he was chiefly attracted by Plato, who stood so near Schleiermacher's direction …".
>
> Robert Grassmann on his brother Hermann[79]

The philosophical background to Grassmann's calculus was the *Naturphilosphie* and dialectics of figures like Schelling and Schleiermacher and even further back Leibniz. Grassmann had attended Schleiermacher's lectures in Berlin between 1827 and 1830 while studying theology, and had made an intense study of his *Dialektik* with his brother Robert in 1840.[80] He had also been directly influenced by Schelling's *Naturphilosophie* through his father Justus who was also a published mathematician and crystallographer. Crystallography held a particular fascination for the naturphilosophes at that time who saw it as a model for self-structuring processes ("Selbst-Gestaltung") in general and as an

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[79] in from Robert Grassmann's biographical outline of Hermann Grassmann. Quoted in Petsche, H. "Ernst Abbe's reception of Grassmann in the light of Grassmann's reception of Schleiermacher", p.170.

80 Grassmann's father Justus Grassmann was also a philosophically inclined mathematician. A pioneer in crystallography, he was heavily influenced by Leibniz and Schelling's Naturphilosophie. Most importantly, Justus's 1824 book Raumlehre, (Space Theory) criticised Euclid for abstracting away from the idea of "direction" in favour of un-directional or uni-directional lengths. see Radu, M. "Justus Grassmann's Contributions to the Foundations of Mathematics: Mathematical and Philosophical Aspects" p.15. (see Lewis, A.C. "Grassmann's Ausdehnungslehre and Schleiermacher's Dialektik", p.109). Banks also speculates that Grassmann may also have been familiar with the works of the Leibnizian psychologist Herbart whose monadological metaphysics of point-like wesen or instantaneous forces existing together in combinatorial manifolds. Herbert did have a direct influence on Riemann's idea of a manifold.

analogy for biological self-organisation in particular. Leading mineralogists like Christian Weiss, who had a direct influence on Justus Grassmann, believed the key to understanding crystal formation lay in understanding the principles governing the "internal" side of nature which was invisible, non-localised and holistic compared to the "external" side which was divided into opposites. Crystal formation and even extension itself were expressions of the former domain in the latter.[81] The most important figure for Grassmann was however Schleiermacher whose *Dialektik,* heavily influenced by Leibniz and Schelling, claimed to be a technique for conceptual construction based on tensions between contrasting ideas. Grassmann's first 1844 version of the *Ausdehnungslehre*, (sometimes referred to as the "A1") which influenced Whitehead the most was shot through with Schleiermacherian vocabulary and dialectical philosophy, [82] a major reason why the book was virtually ignored in Grassmann's own lifetime.[83] Grassmann assumes, like Schleiermacher, that mathematical concepts are quite separate from our psychological intuitions of physical space and quantity.[84] Pure mathematics is instead a science of *pure thought forms*, a *formenlehre* [85] which generates new concepts from other concepts rather than from facts or from intuition.

1. The highest division of all the sciences is into the real and the formal, where the first represent in thought the existent as standing independently over against thought, and have their truth in the correspondence of thought with that existent. The formal sciences, on the other hand, have as their object that which is posited through thought itself and have their truth in the correspondence of the reasoning processes among themselves . . . pure mathematics is the theory of forms.[86]

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[81] For a more detailed article on Weiss and his influence see Heuser, M. "The Significance of Naturphilosophie for Justus and Hermann Grassmann".

[82] Lewis, A.C. "Grassmann's Ausdehnungslehre and Schleiermacher's Dialektik", p.112-114 and also Manfred, F., "Metaphysical Foundations", *A Cambridge Companion to Friedrich Schleiermacher*, Mariner, J. ((ed). 2005). One of Grassmann's biographers says that Schleiermacher introduced Grassmann to the "treasure chest of pre-Hegelian dialectics drawing inspiration from Plato, Spinoza, Kant, Schelling, and Romantic philosophy of nature" - (Petsche., H.J. *Hermann Grassmann Biography.* p.XVI). Grassmann however derided Hegelian dialectics as suffering from "unclearness and arbitrariness which negates all their results", (from the Introduction to the A1. quoted in Lewis. A.C., "The unity of logic, pedagogy and foundations in Grassmann's mathematical work", p.16).

[83] Others like Cassirer speculated that the book's implicit opposition to the Kantian idea of space as an apriori intuition retarded its acceptance in the German speaking world at that time.

[84] Grassmann's brother Robert later drew the express conclusion that Kant erred in assuming space is intuited a priori in his 1890 *Introduction to the Theory of Knowledge or Philosophy* ("Einleitung in die Wissenslehre oder Philosophie"). "As long as the child has not learned to move its eyes and hands, it knows nothing of space; it only experiences space by its movements. And human beings also have no aprioristic knowledge that space consists of three extensions; rather, they can assume any given number of extensions; but external space has only three extensions, and this, again, man knows only from experience." Petsche, H.J. *Hermann Grassmann Biography*. pp.234 - 235.

[85] Interestingly, Grassmann's brother Robert, wrote a book on logic called the *Formenlehre* (1872), which Lewis says was "very much an extension of the general theory of forms as given in the A1 in that it is concerned with specifying and developing the various operations which today are associated with set theory, and Boolean and other algebras". Lewis, A.C. *Grassmann's Ausdehnungslehre and Schleiermacher's Dialektik*, p.123.

[86] Lewis, A.C. "Grassmann ' s *Ausdehnungslehre* and Schleiermacher' s Dialektik", p.122.

For Grassmann pure mathematics is a dialectical and rational activity of uniting, opposing, equating, conjoining, separating and contrasting pure concepts like 'continuous' and ‘discrete' or 'equal' and ‘different'. His brother Robert who was a key figure in his own mathematical and philosophical development went on to write a book called *Formenlehre* in 1872. I shall not delve into the details of the Grassmann brothers' dialectical method here but emphasise its deeply rationalistic and platonistic overtones with its focus on "Denken Formen" and the elevation of pure over applied reasoning. The historian Oliver Dargold gives the flavour of the Grassmann brothers' approach when he says they;

> ... believed that to be rigorous, mathematics had to be pure, devoid of any recourse to experience or intuition. As arithmetic was closest to this ideal, Hermann designed his theory of extension by analogy with it. He and Robert then reworked arithmetic to make it perfectly pure. This implied a constructive, generative conception of numbers. Robert further provided a general theory of the composition of abstract quantities that embraced all branches of logic and mathematics. In this *Formenlehre* as in Hermann's arithmetic, the basic constructive tool was mathematical induction.[87]

Grassmann’s focus on relation, order and structure also helped open up a new line of thinking in Germany that was influential in the rise of “pure mathematics”. Ernst Cassirer in his famous book *Substance and Function* attributed Grassmann with having discovered an “allgemeinen Formwissenschaft” or a “general science of form” and as having helped initiate a shift in mathematics which put the concepts of ‘order’, 'form' and 'function above those of 'substance' and 'thing'.[88] Importantly, it also inspired mathematicians like Ernst Schroeder, Dedekind, Cantor, Peano and Klein who used Grassmannian arguments about the nature of number and extension in areas like measure theory against more empirically minded thinkers like Du Bois-Reymond, Fechner, Helmholtz, and Poincare.[89]

But Grassmann’s platonistic constructivism was something he inherited from the entire tradition of German *Naturphilosophie*, much of which was engaged in what Iain Hamilton Grant has called a *Platonistic physics*, drawing inspiration from the *Timaeus.* One of the

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[87] Darrigol, O. "Number and measure: Hermann von Helmholtz at the crossroads of mathematics, physics, and psychology", p.525.

[88] For the reception of Grassmann in the Germany outside mathematics and the idea of a general science of form and its influence in German intellectual life see Ziche, P. “New forms of science and new sciences of form: On the non-mathematical reception of Grassmann’s work”.

[89] See Olivier Darrigol's paper "Number and measure: Hermann von Helmholtz at the crossroads of mathematics, physics, and psychology".

key differences of *Naturphilosophie* in the writings of people like Schelling, Hegel and Schleiermacher compared to earlier Platonic traditions was that the ingression of the *Ideas* in becoming was not a harmonious and static process, but one involving tension and conflict between competing principles and tendencies. In the case of Hegel these conflicting tendencies assume dramatic proportions animating the 'sturm und drang' of history and the clash of civilisations. In others like Schleiermacher the polarities were more subdued, limited to the realm of ethics, statecraft and religion. In others like Schelling they drive the unfolding logic of auto-genesis found in nature whether in the growth of crystals or the evolution of stars. Even space, was believed to be the resultant of counterposed physical and metaphysical tendencies. Schelling for example saw space as "the form of separation of finite things from the infinite", and "pure extension as "the form of the being-for-itself (understood as a perfect separation from the Absolute) of things".[90] Christian Weiss, the crystallographer referred to earlier spoke about extension in terms of the unfolding of the interior unextended side of nature into a continuous multiplicity.[91] The English translator of Schelling's *First Outline of a System of the Philosophy of Nature,* Keith Peterson, calls Schelling's method "Logogenesis" because it;

> … begins with a genuine opposition of factors; either something is opposed to thought itself or there are two factors contesting in thought. … a dialectic ensues that necessitates a third synthetic moment, and this new whole can itself be treated as one factor or product at the next level or stage of development. We obtain the image of nested spheres of activity or "products." A product consisting of two simple "factors" can itself become one of two factors constituting another product or sphere of activity, and so what is a mere factor for one stage of development could itself be a product from the perspective of an earlier stage.[92]

Schelling's generation of "products" from the "opposition of factors" has an obvious and familiar echo in the writings of Grassmann on the exterior product. When Whitehead first learned about Grassmann from his Cambridge teacher Homersham Cox in the 1880s then, he was not only gaining insight into a powerful mathematical system but opening the

[90] Dezi, A. p.120.

[91] Heuser, M. p.54.

[92] Peterson, K. p.xxviii.

door to a rich philosophical tradition of constructivist and platonistic physics which was to colour his own idea of an ether of events and the actualisation of pure potentialities.[93]

### 4.4 Leibniz

> If someone were to reduce Plato to a system, he would render a great service to mankind, and it would then be clear that my own view approaches his somewhat.
>
> Leibniz, Letter to Nicolas Remond (1715)[94]

Another interesting fact about Grassmann's algebra is the express historical link he and others made between it and *analysis situs,* Leibniz's idea for a co-ordinate free, "distinctly geometrical or linear" calculus expressing "*situation*[95] directly as algebra expresses magnitude directly".[96] Leibniz's motivation for this project was ultimately tied to his theory that space and time (and thus extension and motion) were illusions as *unreal* as the reflections in a mirror. In his ontology the "really real" (ta ontos onta)[97] were monads or "substantial forms" that were akin to both Platonic *Ideas* and Aristotelean substances; eternal, without parts, indestructible, immaterial and utterly independent, each "a complete being which suffices of itself to determine by virtue of its own nature all that must happen to it"[98]. They were constituted only by their "perceptions" of one another so Leibniz compared them to "living mirrors". The essence of apparent motion was due to variations in the intensity to which monads perceived each other. Like a hall of mirrors, a change in the perceptions of one implied a change in the perceptions of every other. This was the ultimate reason why the apparent motion of bodies could only ever be experienced as change of relative position with respect to systems of other bodies.

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[93] Riche, J. p.237.

[94] Quoted in Turnbull, R. "Aseity and Dependence in Leibniz's Metaphysics".p.95.

[95] A situation is particular configurations of bodies or geometrical entities; how they are spatially situated with respect to one another.

96 The idea of an analysis situs by which by which one could "treat mechanics almost like geometry" and even "test the qualities of materials" was first expressed by Leibniz in a letter to Huygens in 1679. Huygens' response was skeptical. Leibniz continued working away at the idea for the next 40 years however. His essay *Characteristica Geometrica* in 1677 summarised in the letter to Huygens was not published in his lifetime. 170 years later, at Möbius' suggestion, Grassmann entered an essay competition in 1846 announced during Leibniz's bicentenary by the Furstlich Jablonowski'schen Gesellschaft in Leipzig for a dissertation on Leibniz's C*haracteristica Geometrica* which by then had only been partially in print for 13 years. Grassmann won the competition but was the only entrant. Möbius was the presiding judge. The idea that there is a direct lineage from analysis situs to Grassmann's calculus or to modern projective geometry has been challenged by the foremost historian of analysis situs, Vincenzo De Risi in his book *Geometry and Monadology: Leibniz's Analysis Situs and Philosophy of Space*. See also Valerie Debuiche's "Perspective in Leibniz's invention of *Characteristica Geometrica*: The problem of Desargues' influence".

[97] From a letter to a young scholar 1707. Quoted in Stuart Brown "Leibniz and Berkeley: Platonic Metaphysics and 'The Mechanical Philosophy", p.246.

[98] *Correspondence with Arnauld. (Discourse on Metaphysics)* p.153. quoted in Iltis, Carolyn. p.35.

> In fact, each substance is a kind of force of acting, i.e. an endeavour to change itself with respect to all the others according to certain laws of its own nature. Whence any substance whatever expresses the whole universe, according to its own point of view. And in the phenomena of motions this fact is especially apparent, for there every single body must be supposed to have a motion in common with any other, as if they were in the same ship, as well as its own motion, reciprocal to its bulk; how this could be so could not be imagined if motions were absolute and each body did not express all others. ("Motion is not Something Absolute")[99]

There are two ideas in the above passage relevant to my discussion here. One is Leibniz's idea that each substance is "a kind of force of acting" and the other is that motions of bodies are always relative. Both of these ideas have echoes in Whitehead.[100] I shall defer discussion of the first concept focusing for now on the latter.

### 4.4.1 Relativity of Motion

The above passage shows why Leibniz thought the idea of the vacuum as a concrete particular was illusory. There was no movement of bodies through a stuff-like space but just changes in monadic perceptions; even if an empty place existed, it "would only mark *the possibility* of that which is missing in relation with the actual one".[101] Space as conceived by Newton and Descartes was actually an obscured way of thinking about how monads do and *might* perceive each other, a distorted image partially reflecting but also obscuring a deeper truth about existing and possible relations between real substances. In his famous correspondence with Clarke Leibniz argues that whatever space is, it must be conceived as containing possible situations and orders of material bodies;

> I don't say that space is an order or situation, but an order of situations, or an order according to which situations are disposed, and that abstract space is that order of situations when they are conceived as being *possible* (5th Letter, §104, L 713-14).[102]

We see here also that Leibniz posits the existence not only a space of actual situations which Leibniz compares elsewhere to "a net which continuously receives a different form

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99 Leibniz, 1686 quoted in Arthur, p.16.

[100] Comparisons between Leibniz's monads and Whitehead's occasions have been made by many before such as Charles Hartshorne and Nicholas Rescher. The lines of influence on Whitehead run through Russell's book *A Critical Exposition of the Philosophy of Leibniz* published in 1900 and Louis Couturat's *La Logique de Leibniz* published in 1903. Russell's and Whitehead's colleague at Cambridge James Ward who had been taught by Lotze was also influential. See Pierfrancesco Basile's books and articles.

[101] De Risi, p.562.

[102] Arthur, p.35.

and thus changes"[103], but also a real but *abstract space* of all possible situations. Modern Leibniz scholars like Arthur and De Risi have demonstrated that what characterises Leibniz's mature theory then is not just the idea that the apparent motion of a body can only be experienced as a change in relation to some system of bodies hypothesised to be at rest (the "equivalence of hypotheses"), but also the idea that apparent motion trades on the existence of an *abstract space* of all possible situations[104]. In his earlier works Leibniz referred to this abstract space as the "Immensum" something that is "indifferent to different ways of being dissected"[105] that is "one, indivisible, immutable". For Leibniz, the abstract space of possibilities was the reason it was a genuine continuum, i.e. indivisible, because lacking in truly determinate parts, or having parts that are only *potentially* distinguishable. Material extensa were by contrast divisible into an actual infinity of other creatures.

> In fact space, just as time is an order, and precisely (in the case of space) the order of co-existence, which does not only include actual things but also the possible ones. Thus it is something indefinite, as is any continuum, the parts of which, are not actual but can be arbitrarily taken, like for instance, the parts of a unity, or fractions. … For while space is an ideal continuum, Mass is discrete, that is, an actual multiplicity, or a being through aggregation of infinite unities. In actuals, simples are anterior to aggregates in ideals the whole is prior to the parts. Neglect of this consideration, has engendered the famous labyrinth of the continuum.[106]

The parallels with Whitehead's dual notions of the extensive continuum and the ether of actual events are obvious. There are however very important differences, the main one being that Leibniz is a more consistent platonist; the vacuum and material bodies are illusory images of the really real, whereas for Whitehead, so-called empty space and material bodies are both aggregates of occasions, the former being "low-grade".[107] For both however, there is no enduring or substantive space through which bodies move and relative to whose points their movements can be plotted.

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[103] Arthur, p.18.

[104] (all possible states of perception in monads).

105 Leibniz in a letter of 1676. in Arthur, p.18.

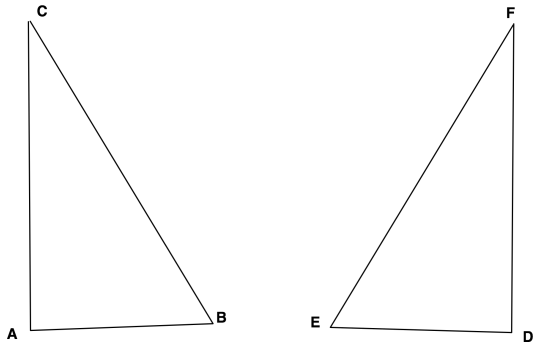
[106] Letter to De Bosses, 31 July 1709. From De Risi, p.568 - 569.

[107] Whitehead also says the extensive continuum *is* finitely divided by actual occasions in some instantiated "cosmic epoch" but In itself is infinitely divis*ible*, which is to say there is no end to the number of ways it could be potentially divided by occasions in other possible epochs or worlds. Leibniz on the other hand stresses that ideal space in itself is not something divisible or extended. He would say I think however that we can nonetheless construct a mental image of ideal space as geometrical, extended, with points and lines etc.

This is why Leibniz's plan to develop a coordinate-free analysis situs is an important adjunct to his metaphysics. For without an absolute space, the measurement and comparison of bodies and systems of bodies (situations) can only be accomplished by directly mapping them onto each other, i.e. by measuring them with respect to their congruency or non-congruency. A figure was said to have the same situation or be congruent with another when it could be "placed" on it in such a way that its properties remained invariant. One example often used to illustrate identical situation or congruence is that of two oppositely oriented right triangles as illustrated below. Leibniz writes in a letter to Huygens in 1679,

> ABC ≅ DEF means that the triangles ABC and DEF are congruent with respect to the order of their points, that they can occupy exactly the same place, and that one can be applied or placed on the other without changing anything in the two figures except their place. So if one places D upon A, E upon B, and F upon C, the two triangles, which are assumed equal and similar, obviously coincide.[108] …assuming in this that the first three points are connected by rigid lines (whether straight or curved does not matter) and the other three likewise.

**Diagram 10. Triplets of Points with equivalent situations or two 'congruent' triangles**




Now it has to be said that Leibniz's work on his *Characteristica Geometrica* was never brought to fruition and was in many ways quite inadequate. Solomon for example observes that in the above diagram the two figures are only congruent under a group of linear transformations which preserve straightness, length and angle, and which include either "reflection" in the plane or "rotation" in three-space, concepts Leibniz seems to allude to when speaking of the points being "connected by *rigid* lines", but which he was unable to formulate explicitly in the way found in modern treatments. What can be said

108 From Graham Solomon's PhD thesis, *Leibniz's Analysis Situs in Mathematical Context*, 1990, p.84.

however is that Leibniz was striving for an abstract calculus expressing the intrinsic relationship of a given entity relative to another without recourse to an extrinsic or absolute background. De Risi says the situational concept is necessarily relational, "there is no such thing as the absolute situation of a unique object, but only the relative situation of a set of objects". [109]

*Analysis situs* is then an adjunct to Leibniz's program of demonstrating substantive space as a phenomenal expression or abstraction from actual monads in the space of all possible situations. Its purpose was to replace the method of measuring and locating bodies and systems of bodies against a fixed frame of reference, with an alternative and relativistic way of measuring and locating bodies and situations with respect to each other.

### 4.4.2 Force or *vis viva*

The other defining property of monads is what Leibniz called their "appetition", an inner drive or endeavour compelling them to more perfect states of perception and manifesting God's own divine energy.[110] A monad "… is a being that is capable of action".[111] Endeavour was expressed in the phenomenal world by what Leibniz called "force" or "*vis viva*" (living energy) closely related to what we now call "kinetic energy". He argued against Descartes and his followers that the quantity of force $mv^2$ [112], rather than the quantity of absolute motion or $m|v|$, was preserved in mechanical interactions and unchanging for the universe as a whole. Leibniz argument was that motion was phenomenal and relative whereas force was "something more real" there being "sufficient grounds for attributing it to one body rather than to another"[113]. The absoluteness of *vis viva* compared to the relativity of motion was brought out in his correspondence with people like Arnauld.

> In order to say that something is moving we will require not only that it change its position with respect to other things but also that there be within itself a cause of change, a force, an action".[114]

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109 De Risi, Geometry and Monadology: Leibniz's Analysis Situs and Philosophy of Space, p.133. It is also worth noting that in the Characteristica Geometrica Leibniz's definition of a straight line by the formula 1B2B + 2B3B = 1B3B is a formula for the addition of line segments which was exactly how Grassmann began developing his notion of a vector.

110 Which is why he also called them "souls" or "entelechies".

111 Leibniz, *Principles of Nature and Grace Based on Reason,* 1714.

112 Leibniz discovered $mv^2$ from Huygens and experiments correlating the velocity of weights dropped into soft clay with the depth of displacement.

113 Leibniz, "Essay de dynamique 1692, quoted in Iltis, C. p.33.

114 *Critical Thoughts on Descartes* 1692, quoted in Robert, J.T. "Leibniz on Force and Absolute Motion", p.557.

> We are right in attributing motion to a boat rather than to the whole sea, even though abstractly speaking we could maintain another description of their motion, since motion abstracted from its cause is always a relative thing.[115]

> Movement in itself, separated from force is something merely relative and its subject cannot be determined; force however being something real and absolute, … we must not be surprised if nature preserves the same quantity of force but not the same quantity of motion.[116]

There are obviously difficulties with Leibniz's argument that $mv^2$ is absolute if motion and thus velocity are merely relative.[117] Leibniz was however trying to articulate the general idea of *energy* as something real, absolute and conserved in all physical interactions, a concept he did not have the resources to fully develop at that time. He is however the acknowledged pioneer of the concept of kinetic energy which found its complete expression some 200 years later in the work of scientists like Sadi Carnot and Lord Kelvin when they formulated the modern laws of thermodynamics.[118] Leibniz's embryonic notion of energy was why for him, a material body was not a totally illusory phenomenon like space, but a "well-founded" one, meaning that bodies were correlated with real and organically structured centres of force[119] or "sprinkled with souls and entelechies"[120] all the way to infinity.

> When I say that there is no part of matter that does not contain monads, I illustrate this with the example of the human body or that of some other animal, any of whose solid and fluid parts contain in themselves in turn other animals and plants. And this, I think, must be said again of any part of these living things, and so on to infinity. . .

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[115] *Letter to Arnauld* 14 July 1686, quoted in Robert, J.T. "Leibniz on Force and Absolute Motion", p.557.

[116] *Letter to Arnauld* 14 January 1688. quoted in Robert, J.T. "Leibniz on Force and Absolute Motion", p.558.

[117] See Robert, J.T. "Leibniz on Force and Absolute Motion", pp.553 - 573.

[118] The science historian Jane Coopersmith for example says, "The equivalence of this 'active force' to our concept of energy is striking, especially when we realize that Leibniz's concept of force is not Newton's but is given by the concept $mv^2$ … apart from the factor of $^1/_2$ this is identical to our modern expression of kinetic energy". Coopersmith, J. *Energy, the Subtle Concept: The Discovery of Feynman's Blocks from Leibniz to Einstein*. p.26.

[119] Bodies are not literally aggregates of monads. Each monad is a self-sufficient substance - they do not separate at one moment then come together the next like a school of fish. 'Aggregation' is virtual indicating greater intensity of perception between certain groups of monads. See Rutherford, D. "Metaphysics; the Late Period", pp.144 - 146, and Nachtomy, O. *Possibility, Agency and Individuality in Leibniz's Metaphysics*, 199 - 212.

[120] Letter to De Volder 24 March/3 April 1699. Quoted in Phemister, P. *Leibniz and the Natural World, Activity, Passivity and Corporeal Substance*. p.60.

> I shall use an analogy. Imagine a circle; in it draw three other circles which are the same size and as large as possible, and in any new circle and in the space between circles again draw the three largest circles of the same size which are possible. Imagine proceeding to infinity in this way: it does not follow that there is an infinitely small circle, or that there is a center having its own circle, in which (contrary to the hypothesis) no other is inscribed.[121]

Leibniz therefore arrives at view of pointless extensa similar to Whitehead's view of pointless events in his middle period texts, as well as a view of Newtonian space as a kind of distorted image of actual and possible situations between more fundamental entities and forces.

### 4.5 Conclusion

Whitehead's system then stands in a tradition of platonic constructivism which runs through Leibniz, Schelling, Schleiermacher and the Grassmanns. That tradition is not concerned to deny the reality of change in the vein of Parmenides, as much as it is concerned to affirm the active role of the transcendental in change and development throughout nature, as the passage from what Whitehead called the realm of "pure potentiality" to that of actuality. The naturphilosophes' understanding that reality did not just copy the original forms but was the resultant of a dynamic and sometimes conflictual interaction between them, was the philosophical background to Grassmann's theory of extension and Whitehead's own ether of events. In the modern platonic tradition following Leibniz, space, time and extension then are not substances in their own right, but constructed from relations between more basic relata. These relations however are seen through platonic lenses, as instantiations of abstract but real *Ideas* in-*form*-ing actuality. Whitehead's theory of extension reproduces this platonistic dichotomy. On the one hand extension is the product of prehensions between events, but on the other these prehensional relations are also ingressions of abstract geometric structures ultimately grounded in an ideal and mereotopological continuum. The axioms of this mereotopological realm are set out in Part IV of *Process & Reality,* and it is to them I now turn in more detail.

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[121] Letter to Des Bosses 11–17 March, 1706; G 305–306. Quoted in Nachtomy, p.74.

## Chapter 5. Whitehead's Platonistic Mereotopology

### 5.1 The 1914 Paper

Whitehead's "Theory of Extension" in Chapter II of Part IV of *Process & Reality* had a long period of gestation starting with his 1914 paper "A Relational Theory of Space" delivered to the First Congress of Mathematical Philosophy in Paris. Patrick Hurley, the translator of that paper describes it as neo-Kantian because it aims at "interconnecting different levels of experience, as opposed to connecting the content of experience to some extra-mental world"[1]. Like Poincare in his *Science and Hypothesis,* Whitehead draws strong distinctions between geometrical, perceptual and physical space and is motivated by a concern to demonstrate how our fragmented perceptual space is the basis from which the refined logical objects of geometry are derived. In *Our Knowledge of the External World* Russell shares Whitehead's concerns arguing that a method like Whitehead's was necessary because points could never be validly inferred from sense data, obliging philosophy to logically construct points from "some complex assemblage of immediately given objects, which will have the geometrical properties required".[2]

The 1914 paper is however just a very rough and informal sketch of some philosophical ideas about the ontological status of points with some technical suggestions for how they might be constructed. Its stated objective is to sketch out a relational theory of space in which points and other geometrical entities like lines and surfaces are not primitive but "complex entities, logical functions of those relations between objects which constitute space"[3]. Note that at this point Whitehead still understands space to be a relation between material objects rather than events. Whitehead distinguishes between three different senses of whole and part. (1) A sense of whole and part pertaining to mathematical classes corresponding to the 'all' and 'some' of logic. (2) A sense in which the whole is "different by its very nature" to the parts composing it - for example a pudding and the sugar contained in it as an ingredient, and (3) the "homogenous" sense of whole and part pertaining to spatial objects which are of the same kind as their parts. For example, "the head is part of the body of a horse, the province is part of the territory of a nation, and the meter is part of the kilometre". He finishes this first part of the paper with a discussion

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[1] Hurley, P. "Russell, Poincaré, and Whitehead's 'Relational Theory of Space". p.18.

[2] Russell, B. *Our Knowledge of the External World*. p.113-114.

[3] "The Relational Theory of Space", p.40.

about why the construal of space in terms of mathematical classes leads one to treating it as composed of points. This justifies the need he says for a different conception of spatial wholes even though the two notions of class and whole are quite similar.

Whitehead then defines a mereological relation called "σ inclusion" (which Whitehead symbolises with the symbol “Eσ” but which I will represent here with “>”) in terms of a more primitive relation R. R is said to be any relation within the set σ (the domain of all "direct relations” between physical objects and representing "some mode of physical action" [4]). Inclusion holds between entities a and b on the following condition: if “*x* is any entity having the relation R to *b*, then *x* also always has the relation R to *a*".

In other words:

$a>b \Longrightarrow_{df} \forall x\ (xRb \rightarrow xRa)$.

He then goes on to say "When these conditions are fulfilled, we say that *a* includes *b* with respect to σ , or: *b* is a σ-part of *a*. For example, to see the head of a dog is to see the dog; to touch the head of a dog is to touch the dog, and to pet the head of a dog is to pet the dog, etc."[5]. It is interesting that in this early text Whitehead defined mereological inclusion in terms of a more general causal type relation, as a ‘direct mode of physical action’. In *An Enquiry* and *The Concept of Nature* Whitehead treated ‘inclusion’ as primitive but switched back to treating it as derivative in *Process & Reality*, defining it not in terms of a causal relation but in terms of ‘connection’.[6] Moving now back to the 1914 paper, after defining inclusion in terms of a more primitive relation of “physical action”, Whitehead next defines the relation T as that anti-symmetric, transitive and extensional[7] relation which is a species of the Eσ inclusion relation.

Whitehead’s exposition becomes somewhat opaque at this point introducing distinctions between material T-points and spatial T-points, and material objects of T-space defined as collections (classes) of material T-points in relation to which the concepts of whole and part and the logical concepts of all and some, and that of T-inclusion “blend together”.

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4 "The Relational Theory of Space", p.45.

5 "The Relational Theory of Space", p.46.

6 The definitions for inclusion found in the 1914 paper and *Process & Reality* are very similar, connection being defined in *Process & Reality* as follows: “Region A is said to 'include' region B when every region connected with B is also connected with A” which might be rendered in logical notation as something like: $a>b \Longrightarrow_{df} \forall x\ (xCa \rightarrow xCb)$.

7 In the logical sense such that any other relation binding the exact same set of relata as T is equal to T.

Then Whitehead announces that the aim of the earlier definitions is to render precise the general concept of "an object progressively cut into smaller and smaller parts until its dimensions have disappeared and nothing remains but a point."[8] He equates the idea of a point with that of a limit but is careful to note that the notion of a mathematical limit applies only to sets of numbers and that an analogous but different notion is required for sets of geometrical entities, observing that the idea of infinite series of σ-objects arranged according to the inclusion relation is not by itself the idea of a "conceptual limit". [9]

He then defines "T serial classes", as those classes with σ objects as members, none of which is a member of the null class and such that for any two members x and y either Txy or Tyx, and "Geometric T series" as those T-serial classes such that for every x in the class there is unique object p such that every other object in the series (i) xTp, and (ii) there is no object y such that pTy. The entity p is described as “T-indivisible”. T-indivisible objects correspond to points, lines and planes and I shall refer to them here as “minima”. Whitehead gives as examples of geometric series a class of cylinders each nested inside the other whose unique common minima is their axis. Another example of a geometric T-series is a class of concentric spheres whose common geometrical minima is their centre point.

Whitehead’s exposition here is in many ways opaque, and the parts which are clear, if I have correctly understood them, erroneous. Firstly, the concepts of a “T-series” and a “Geometric T-Series” are unnecessary for defining points whose chief property according to the paper is their indivisibility. Whitehead stated objective is to construct points as *logical functions* or as “*conceptual limits*” based on relations between material objects, but the problem is that points seem to be on an ontological par with the things from which they are said to be derived. One could construct a higher dimensional figure by adding points together as easily as one could derive points by analysing material objects. Whitehead’s points in the 1914 paper are neither more nor less primitive than the wholes of which they are part and thus not really “conceptual” limits of a different ontological order to the wholes of which they are part.

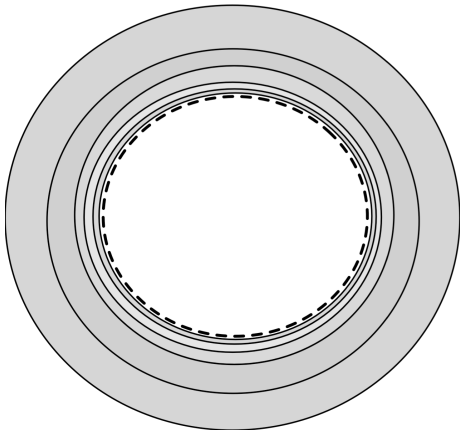
Another problem with the paper is that Whitehead fails to spell out the different conditions required for T-series to converge on the different kinds of minima. A cone for example can be partitioned in such a way that its last member is the disk composing it's base, but it can

8 "The Relational Theory of Space", p.52.

9 "The Relational Theory of Space", p.52.

also be partitioned such that the last member is the point at its apex. Different partitions will generate different kinds of minima of which there are infinitely many besides points, straight lines and flat surfaces; for example enclosed 3-dimensional figures, and open or closed curves. A disk with a hole in it for example can be partitioned into a set of concentric rings the least member of which is a 'hollow' circle.

**Diagram 11. Set of Concentric Rings Convergent on a Circle**



### 5.2 Middle Period Texts

Some of the above problems are addressed in the two later works of 1919 and 1920; *An Enquiry Concerning the Principles of Natural Knowledge* and *The Concept of Nature*. The most important difference is that points are no longer defined as the smallest members of mereologically ordered series but as entities of a different ontological order; as the *conceptual limits* towards which infinite series of diminishing events are imagined to approach. Whitehead's formulates a new mereological theory with six axioms and definitions that form the background to the new method. The listed axioms are not all independent, some being implied by others and the manner of exposition is rather informal, a fact heavily criticised by Lesniewski who was apparently appalled at its lack of logical rigour.[10]

I describe and render each axiom into standard first order logical notation drawing mainly on Simon's 1991 paper adopting the now common convention of using "<" to indicate the binary 'part of' operator. The domain of discourse is inhabited by events only and I use the variables are "x", "y", "z" and "a", "b" and "c".

[10] In his "Whitehead's Theory of Events", Lesniewski says that Tarski introduced him to Whitehead's theory in 1926. He says that he had "particular difficulty in extracting a general schema for precise formulation" from Whitehead's works such that he could not even attempt to resolve an issue Tarski had raised about the validity of Whitehead's deduction of certain theorems. Simons speaks about Lesniewski "taking offence" at Whitehead's lack of rigour in *Principia* which probably coloured his attitude to his later works which he had not intended to be strictly axiomatic. Simons, P. "Whitehead and Mereology", p.10.

**Proper Parthood**

Whitehead firstly defines the 'part of' relation to be one where the whole is "distinct from" or a "proper part" of the whole. By "distinct" and "proper" he meant that if x is part of y, then y has a part z that is not part of x.

(1) $(x < y) \rightarrow \exists z: (z < y) \wedge \neg(z < x)$

**Unbounded Above and Below**

The next axiom is that "every event extends over other events and is itself part of other events."

(2) $\forall x \exists yz: (x < y) \wedge (z < x)$

**Overlap**

The third axiom says that if all the parts of one event x are parts of a second y, and y is "distinct" from x, then x is part of y.

(3) $[(\forall x (x < y) \rightarrow (x<z)] \wedge [\exists a: (a<z) \wedge \neg(a < y)] \rightarrow (y < z)$

**Transitivity**

The fourth axiom says that the parthood relation is transitive.

(4) $(x < y) \wedge (y < z) \rightarrow (x < z)$

**Density**

The fifth axiom says that if an event is part of another, there is a third event which is 'between' both, i.e., something that is part of the first but includes the second.

(5) $\forall xy (x < y) \rightarrow \exists z (z < y) \wedge \neg(z < x)$

**Upward Directed**

The sixth axiom says that for any two events there is a third which includes both.

(6) $\forall xy \exists z: (x < z) \wedge (y < z)$

Whitehead said that (2),(4) and (6) imply something like the continuous ether. This claim turns out to be incorrect for reasons I'll explain later.

Whitehead then defines the relations of Intersection, Separation and Dissection.

**Intersection**

Two events x and y intersect "I" if there is a third thing z which is a part of them both or if one is part of the other. Simons refers to this by the more conventional term "overlap".

(7) $Ixy \Longrightarrow_{df} \exists z\,[(z<x) \wedge (z<y)] \vee [(x < y) \vee (y< x)]$

**Separation**

Two events are separated "S" if they do not intersect.

(8) $Sxy \Longrightarrow_{df} \neg Ixy$

**Dissection**

Whitehead speaks of a dissection of an event as being a "non-overlapping exhaustive analysis of an event into a set of parts". He has in mind a set of non-overlapping evental parts which sum to the event in question. Whitehead however does not employ the concept of 'sum' but gets the same result in a roundabout way by first defining "a separated set such that the set of intersectors of its members is identical to the set of intersectors of the event". Thus each dissection set dissects one and only one event, but an event may have many dissections.[11] 'Dissection' does not play an essential role in the system as a whole and even though it features in Whitehead's definition of 'junction' (discussed below) there are alternative and simpler renderings that do not rely on it. In any case a second order predicate "D" for "Dissection" of a set X could be defined with respect to some event y as follows.

(9) $DX$ *w.r.t.* $y \Longrightarrow_{df} \forall ab[\,\{(a \in X \wedge b \in X) \rightarrow (a < y) \wedge (b<y) \wedge (Sab)\} \wedge \forall z\,(Izy \leftrightarrow \exists c\,(c \in X) \wedge Izc)]$

11 The number of dissections there are for each event will obviously depend on whether events are characterised as infinitely or only finitely divisible. Whitehead thinks at this point that events are infinitely divisible but non-punctiform and says therefore that there are infinitely many dissections of any given event.

**Junction**

Two events x and y have junction when there is a third event z which intersects them both and for which a dissection set exists all of whose members are parts of either event.

(10a) $Jxy \Longrightarrow_{df} \exists z\, \exists Z$: DZ w.r.t. z $\wedge$ ( [$\forall a\, (a<z) \rightarrow (a<x) \vee (a<y)$]

As discussed above Whitehead's use of 'dissection' to define Junction is redundant. Commentators like Varzi and Palter have translated Junction as that simple mereological relation obtaining when an event z intersects x and y but has no parts separate from them. So leaving the dissection set out of definition 10a above we get:

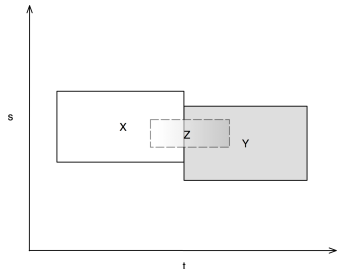
(10b) $Jxy \Longrightarrow_{df} \exists z\, [\forall a\, (a<z) \rightarrow (a<x) \vee (a<y)]$

Whitehead was after a purely mereological means of defining continuous events. He wrote,

> The concept of continuity of nature arises entirely from this relation of the junction between two events. Two joined events are continuous one with the other. Intersecting events are necessarily joined; but the notion of junction is wider than that of intersection, for it is possible for two separated events to be joined.[12]

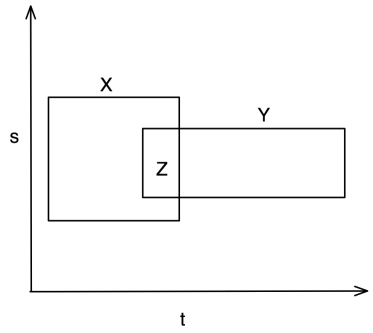
Whitehead had in mind that it was possible for two separated events, i.e. events without common parts, to be joined or contiguous, as occurs when one event immediately follows another, like the second half of a race following immediately after the first. The idea of continuity he seemed to have in mind is the one that obtains when two events are not separated by a third but touch directly. He is thinking about situations like those illustrated in Diagrams 12 and 13 below in which there is an event Z which intersects X as well as Y but has no parts which are not either part of X, or part of Y.

**Diagram 12. Junction as Shared Inclusion**



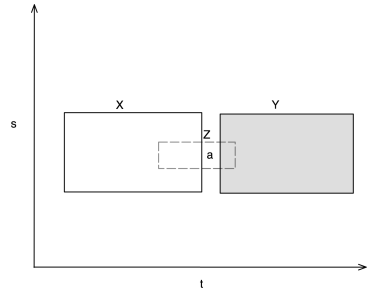

12 *An Enquiry*, p.102.

**Diagram 13. Junction as Overlap**



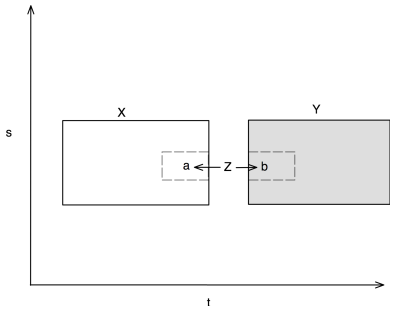

Whitehead's definition of junction seems to work well enough to exclude situations like that illustrated in the diagram below - even though event Z has some parts in X and some in Y, it also has a part that is part of neither (viz. 'a').

**Diagram 14. An Example of Where Junction Works**



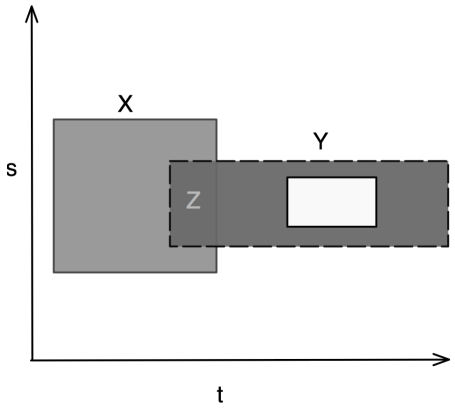

Junction is not however adequate as a definition of continuity because there are situations in which two events have junction but are not continuous. This is portrayed in the diagram below. In this situation, event Z is composed exclusively of a and b, i.e., it does not have any parts which are not in either X or Y, and yet X and Y are clearly separate events.

**Diagram 15. An Example of Where Junction Fails**



Roberto Casati and Achille Varzi have pointed out that junction may be a necessary property of continuity between events but it is not a sufficient one. This is probably why Whitehead abandoned a purely mereological system for one based on the notion of 'contact' or 'connection' in *Process & Reality (*even though he never expressly stated as such citing other reasons for this change). [13] Another problem raised by Palter is that junction permits evental holes, a situation illustrated in Diagram 16 below[14] . Palter calls the *Enquiry* definition "too weak" [15]

**Diagram 16. Holes**




and goes on to argue that the problem was overcome with a new definition of junction introduced by *The Concept of Nature* published the following year. The new definition says that x and y have junction if there is an event z that has x and y as parts and no parts that are not either part of x or part of y.

(11) $\mathcal{J}\, xy \Longrightarrow_{df} \exists z\, [(x<z) \wedge (y<z)] \wedge [\forall a\, (a<z) \rightarrow (a<x) \vee (a<y)]$

Whitehead himself however tended to think of both definitions as equivalent.[16] In any case Palter is mistaken about the superiority of the *Concept of Nature* definition because it excludes neither holes nor discontinuous events. Observe that X and Y in both Diagrams 15 and 16 have junction in the *Enquiry* sense because there is an event Z that has no parts not in either X or Y, but also junction in the *Concept of Nature* sense because the

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13 After criticisms from Laguna of his mereology, Whitehead changed his approach. I shall explore this issue in more depth later on.

14 Palter, R. p.46.

15 Palter, p.47.

16 See discussion in the *Concept of Nature*, p.77 where Whitehead says "If either of these alternative definitions is adopted as the definition of junction, the other definition appears as an axiom respecting the character of junction as we know it in nature".

event which is the sum of X and Y obviously has X and Y as parts and no other parts that are not either in X or in Y or in both, and yet the event in Diagram 15 is clearly discontinuous and the event in Diagram 16 has a hole in it.

Mereological axioms alone it would seem then are not powerful enough to define continuity or to exclude holes both of which are topological concepts. It is not just that holes are, as Roberto Casati and Achille Varzi have noted, "slippery, elusive entities",[17] but that behind Whitehead's definitions of continuity is a circular assumption that "all events are continuous or connected, that is, do not consist of two or more non-joined parts. It is only thus that it could appear plausible to him that purely mereological concepts could be used to define topological concepts such as being connected."[18]

Whitehead's mereologically based definitions are also too weak to capture topological notions like that of a boundary. In this context Whitehead uses 'junction' to define what he called 'adjointness' and 'injointness' by which he intended to capture the situation of two events being joined at a common boundary. But both definitions suffer from the same problem afflicting Whitehead's definition of continuity. I give attention to them here however because the idea of injoint and non-injoint inclusion is decisive for his definitions of extensive abstraction and the construction of points and lines discussed in the next two sections.

'Adjointness' is had by two events which share a common boundary and neither is included in the other whereas injointness is had when two events which share a common boundary but one is included in the other. Whitehead's mereological definitions say that adjointness occurs when two events have junction but are separate (no parts in common) and that injointness occurs when two events have junction but one has all its parts in common with the other.

**Adjointness**

(12) $\mathbf{Ad}\ xy \Longrightarrow_{df} Jxy \wedge Sxy$

**Injointness**

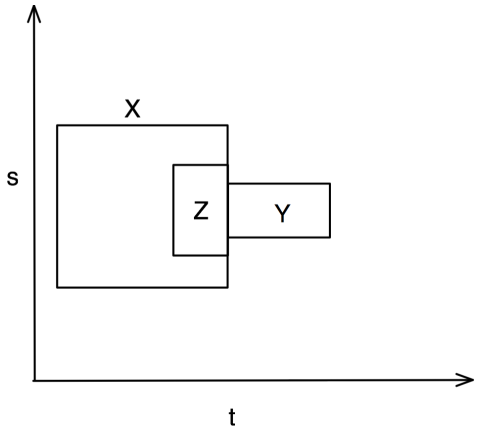
(13) $\mathbf{In}\ xy \Longrightarrow_{df} x<y \wedge \exists z\ Szx \wedge Ayz$

17 Casati, R and Varzi, A. *Holes and other Superficialities*, p.1.

18 Simons, P. "Whitehead's Mereology", p.11.

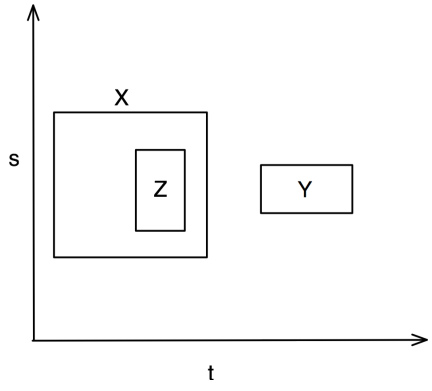
The above two definitions were meant to cover situations like those captured in the following diagram where X is adjoined to Y and injoined to Z.

**Diagram 17. Adjointness and Injointness**




Whitehead's definitions of adjointness and injointness obviously fail because his definitions of junction cannot exclude discontinuous events. This is seen in the next below where X and Y have junction (in the *Concept of Nature* sense because there is an event that extends over them both and nothing else, viz their sum 'X+Y') and separation, but are clearly not adjoined at a common boundary. Similarly, X and Z in the diagram below are by definition injoined (because Z is part of X, and Z is technically speaking adjoined to Y which is separate from X), but clearly do not share a common boundary.

**Diagram 18. Why Adjointness and Injointness also Fail**



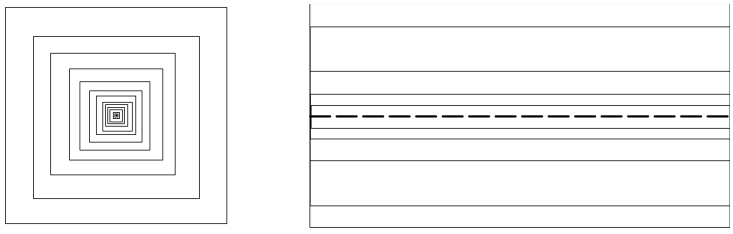
### 5.3 Extensive Abstraction in the Middle Period Texts

Whitehead also uses mereological concepts in these middle period texts to sketch out a program for defining geometrical entities like points, lines and planes through the method of extensive abstraction building on concepts first developed in the 1914 paper. Extensive abstraction as described in the middle period texts does not seem at first to hinge on Whitehead's flawed definitions of continuity and boundedness but as has been shown they do. These texts also refrain from defining points as merely being the 'smallest' or last members of infinite series of mereologically ordered objects but they do as I will show below. In these works he begins by defining an "abstractive class" as a set of events, such that of any two of its members, one extends over the other and such that there is no event which is extended over by every other in the set. In other words an abstractive set has no smallest member and no two members extending over each other. In modern notation we might render an abstractive set X in the following way:

(14) $\mathrm{Abst}X := \{x|\ \forall y \in X\ (x<y \vee y<x \wedge [x<y \rightarrow \neg(y<x)]\ )\ \wedge \neg(\exists z \in X: \forall k \in X, z<k)\}$

In a note to his discussion Whitehead introduces the concept of a limit that is *different* from the abstractive set of diminishing events, speaking about the 'lengths' of each event in an abstractive class tending to zero as the number of elements in the series tends to infinity. Whitehead then equates abstract entities like points and lines with *limits* of these geometric series. "Evidently the set of squares converges to a point, and the set of rectangles to a straight line. Similarly abstractive classes with three dimensional volumes are said to converge on areas"[19]. One issue of importance here to remember is that all abstractive sets have as members four-dimensional evental regions and that lower dimensional entities are all ideals abstracted from them. The limits of abstractive sets are thus things like points, lines, surfaces and three-dimensional volumes. The two-dimensional diagrams should not be interpreted literally but but as analogs for four-dimensional hyper-volumes.

**Diagram 19. Extensive Abstraction Convergence to a Limit Point and Line**

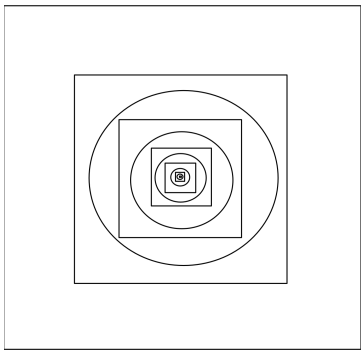


19 *An Enquiry*, p.106.

In the above diagram the infinite set of nested squares converge onto a limit point whereas the set of rectangles with diminishing heights but constant length converge onto a line.

Whitehead's definition of a geometrical limit differs from the definition adopted in his 1914 paper because it is not the least member of an abstractive series but a limit outside the series. In fact Whitehead says these limits are themselves *special kinds of sets*. These special sets are defined using a relation between abstractive sets he called "covering". An abstractive set A is said to cover another B when each of A's members extends over at least one member of B. Abstractive sets are said to be "K-equal" if they cover each other. As an example, take an abstractive set of nested squares and another set of nested disks fitting somewhere deeper inside it.[20]

**Diagram 20. A Set of Squares K-Equal with a Set of Disks**



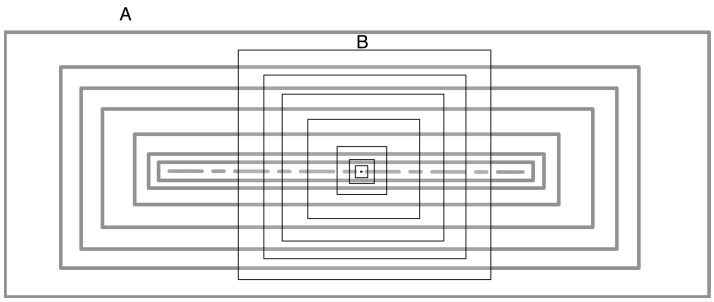
In the next step of Whitehead's construction of points, he defines what he calls a "prime" abstractive set" in respect to "a formative condition σ". It is not exactly clear what Whitehead has in mind when talking about "a formative condition σ" but probably just meant that quality or rule governing how the set is constructed. The indexing of abstractive sets to a particular σ condition recognises the issue I pointed out with the 1914 paper, that different sets converge on different kinds of limits depending on how they are constructed. Whitehead however does not specify the particular conditions determining different kinds of limits other than those for points. This lacuna therefore leaves unsolved the problem of circularity, of how to construct geometrical objects like lines and surfaces from abstractive sets that are not themselves defined by geometrical predicates like 'rectangularity'. Putting this issue to the side for the moment, Whitehead's definition of a prime abstractive set in the *Enquiry* is defined as one that is covered by every other abstractive set satisfying σ. The definition of a prime changes in the *Concept of Nature*

20 Note how the outer disk in Diagram 22 is part of the outermost square as well as of the second outermost square. The set of squares and the set of disks are however K-equal because each square contains at least one disk as part and each disk contains at least one square as part.

where a σ prime is characterised as one that only covers an abstractive set satisfying σ if it is also covered by that set. Like a prime number which is only divisible by numbers equal to it, a prime abstractive set only covers those sets to which it is K-equal - there is no set satisfying σ that it covers that does not cover it.[21] "In other words you cannot get any abstractive set satisfying the condition σ which exhibits intrinsic character more simple than that of a σ-prime."[22]

The concept of a prime set seems designed solve the problem of abstractive sets converging on non-point minima like enclosed surfaces or volumes. The reason is that if an abstractive set A composed of say cubic segments does converge on something like a smaller inner cube $\boldsymbol{a}$ say, then one could construct another abstractive set $\boldsymbol{a'}$ from regions inside $\boldsymbol{a}$. The abstractive set $\boldsymbol{a'}$ would be covered by A but would not cover A, and so A wouldn't be prime. Likewise if A is a set of rectangles convergent on a segment of its midline $\boldsymbol{m}$, another abstractive set B composed of square segments from which at least one is included in every rectangle of A could be constructed such that A would cover B but would not be covered by it. A would therefore be non-prime, as in the diagram below.

**Diagram 21. A non-Prime Abstractive Set of Rectangles**




In the final step of Whitehead's construction of points he defines an "abstractive element" as "the whole group of abstractive sets which are K-equal to any one of themselves. Thus all abstractive sets belonging to the same abstractive element are K-equal and converge to the same intrinsic character. Thus an abstractive element is the group of routes of approximation to a definite intrinsic character of ideal simplicity to be found as a limit

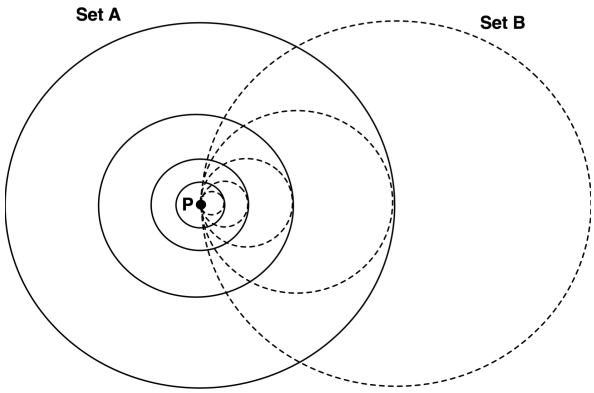
[21] We must remember here the definition of covering which is that A covers B if each member of A contains a member of B. If A and B cover each other it is not necessary for every member of A to be covered by a member of B, only that each member of B cover a member of A and that each member of A cover a member of B. Either set A or set B might have an outer first member not covered by a member of the other set. In other words mutual covering does not require A and B to be unbounded above.

22 *The Concept of Natur*e, p.87-88.

among natural facts"[23]. An "element" in respect to some given abstractive set A therefore contains all the abstractive sets that are K-equal to A. By identifying points with sets of all the K-equal prime abstractive sets and not just this or that prime set, Whitehead was probably trying to ensure that every constructed 'point' is unique. An element or elemental set with respect to σ is meant to be exhaustive - any prime abstractive set satisfying σ is included in it. Elemental sets were also the means by which Whitehead purported to demonstrate geometrical units as mental abstractions twice removed from actual events, as *sets* of 'indivisible' prime abstractive *sets,* i.e. sets of sets of actual events. Whitehead's characterisation of geometrical limits as ideal things therefore depends7 on the notion that sets of things are qualitatively different from the things they group together, (i.e. $x \neq \{x\} \neq \{\{x\}\}$). In this regard I also note that Whitehead's definition of a point is therefore not purely mereological but relies on set theoretical notions to achieve its aims.

There were however real problems with Whitehead's method at this stage of its development. Robert Palter points out one serious error in Whitehead's method and offers what I think is a false solution. He points out that the property of mutual covering or K-equality between two abstractive sets converging on the same point is not universal. Take the diagram below. The solid disks of Set A each contain a dashed disk from Set B but not vice versa.[24] In other words A covers B but B does not cover A, so A cannot be prime. The general lesson is that unless a formative condition σ can be found that excludes sets like B we cannot define prime sets.

**Diagram 22. Asymmetrical Covering**



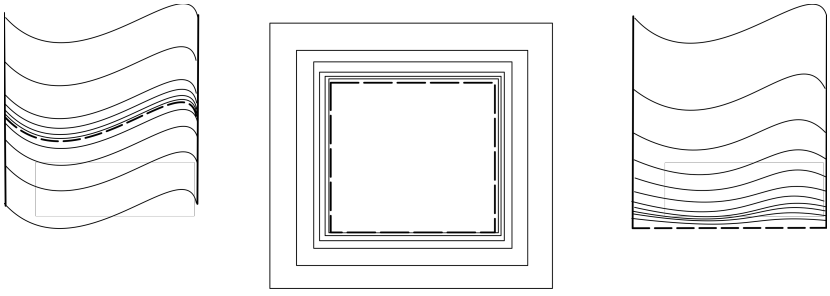

23 *The Concept of Nature*, p.84.

24 I have adapted this example from Palter, p.63.

Whitehead would have believed at the time that he could give a formula σ for excluding abstractive sets like B, by simply excluding sets with injoined members. However, as was shown above at the end of section 5.2, Whitehead's definition of injointness was inadequate. Palter says such sets can be excluded with a formative condition σ = "covering all the abstractive classes and elements constituting some given punct [an "instantaneous straight line]".[25] I cannot see how Palter's proposed σ condition excludes abstractive sets with injoined members. In addition he put the cart before the horse because straight lines are not defined. Palter's solution is therefore no solution at all. One alternative solution might be to just say that sets like B are prime, but then we have the inverse problem of not being able to give a formative condition σ that excludes sets with non-injoined members like A.

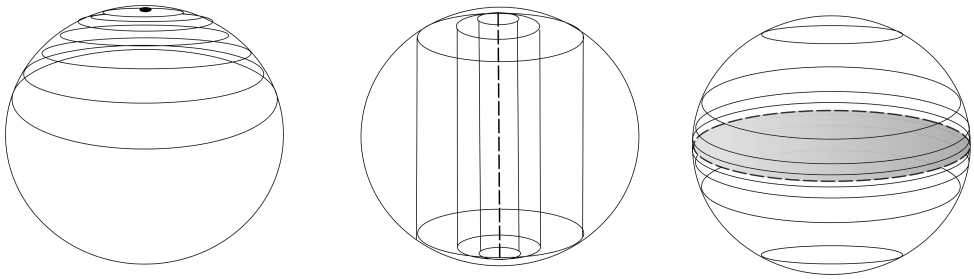
Another problem is that Whitehead says that abstractive sets composed of higher dimensional events in the shape of say hyper-cubes or spheres generate lower dimensional elements like points, lines and planes, an assertion he illustrates with diagrams of for example a set of concentric squares convergent on a point or a set of concentric rectangles of equal length convergent on a straight line. But even assuming Whitehead's mereological definition of points is correct, Whitehead does not construct higher level limits like lines or planes mereologically. This brings us back to the problem of circularity which I put to one side earlier. The problem is that Whitehead's middle period texts lack an explanation of how geometrical objects other than points can be defined purely mereologically. For example, whether abstractive sets converge on full bodied figures, surfaces, open curves, closed curves, straight lines or points would seem to depend on things like the *rate* of diminution in the boundary *lengths* and *shapes* of the parts as one descends down the set. Examples of how geometric properties of abstractive sets determine their different geometric limits is seen in the two diagrams below.

**Diagram 23. Convergence onto a curve, a square surface and straight line**



[25] Palter, p.63 and p.61 for the definition of a "punct".

**Diagram 24. Sections of a Sphere Convergent on a point, a line and a disc**



Whitehead's own illustrations show limits whose geometrical characters seem to be determined by the geometric properties of the series they are generated from. Although Whitehead warns his readers to exercise "caution" as to the "possibly misleading character" of spatial diagrams [26] he relies nonetheless on the geometric properties of the illustrated figures to make his argument.

One possible way of interpreting this lacuna is to say that extensive abstraction at this stage was still under development and that Whitehead thought of his work as but a *step* towards a more rigorous proof that geometrical forms were constructible mereologically. A more plausible answer to my mind is that thinking he had successfully defined points, it was only necessary in his mind to define lines and higher dimensional objects by classifying sets of points according to their properties in the elliptical, hyperbolic and Euclidian cases as he had in his early mathematical works like *The Axioms of Projective Geometry* and *The Axioms of Descriptive Geometry*. As Whitehead says in the first book, "Geometry is the science of cross classification. The fundamental class K, is the class of points; the selected set of subclasses of K is the class of (straight) lines. This set of subclasses is to be such that any two points lie on one and only one line, and that any line possesses at least three points. These properties of straight lines represent the properties which are common to all branches which usage terms Geometrical…".[27]

In any case, Whitehead's mereological definition of points is inadequate because it is incapable of excluding or including only abstractive sets with injoined members which is necessary for the definition of a prime set. In conclusion, the mereological systems developed in Whitehead's works between 1914 and 1920 did not deliver the geometrical results he was after. The question then remains of whether the system of Part IV of

26 *An Enquiry*, p.105.

27 *The Axioms of Projective Geometry*, p.5.

*Process & Reality* fared any better by adding topological concepts to Whitehead's mereology, an issue I discuss in the next section.

### 5.4 The System of 'Extensive Connection' in *Process and Reality*

By 1927 Whitehead had arrived at the position that continuity or connection should be treated as a primitive relation rather than one definable mereologically. He had been nudged in this direction by the first serious commentator on his mereological writings, Theodore de Laguna, who set out his own mereotopological formalism in three articles in the *Journal of Philosophy* in 1922. In his "Point, Line and Surface as Sets of Solids"[28] de Laguna set out from a position in which "neither 'containing' nor 'extending-over' are assumed as primitive, but both are defined in terms of the relation 'can connect'". Like Whitehead he also defined points, lines and surfaces using abstractive sets but thought of as sets of nested solids rather than events.

The theory of Extensive Connection in Part IV of *Process & Reality* essentially makes the case that the structure of space is one of regions *connected* to other regions rather than one of events *including* other events. It is then ultimately a topological rather than a mereological system because the 'part-of' relation is defined in terms of connection. On a more philosophical note, Simons argues that Whitehead in *Process & Reality* understands occasions to "enjoy", or to be the "contents" of spatial regions. Although Whitehead does use this terminology it is a little misleading. In *Process & Reality*, spatial occasions like any other occasion do not endure and so cannot experience changes of state such as being occupied one minute then being unoccupied the next. The things which endure are eternal objects and complexes of eternal objects. Regions in Whitehead's scheme are therefore ideal objects which occasions are said to instantiate rather than occupy in the sense we speak about a material object occupying a portion of substantial space. The geometrical shapes within these regions are likewise ideal objects. This is brought out in Whitehead's discussion of "flat loci" in Chapter III of Part IV which is about the definition a straight line.

> … a muddle arises between 'forms' and concrete physical things. Geometry starts with the purpose of investigating certain *forms* of physical things. But in its initial definitions of the 'point' and the 'line,' it seems immediately to postulate certain ultimate physical things of a very peculiar character. Plato himself appears to have

28 de Laguna, T. "Point, Line and Surface as Sets of Solids", p. 451.

> had some suspicion of this confusion when he "objected to recognizing points as a separate class of things at all." He ought to have gone further, and have made the same objection to all the geometrical entities, namely, points, lines, and surfaces. He wanted 'forms,' and he obtained new physical entities.
>
> According to the previous chapter, "extension' should be construed in terms of 'extensive connection'; that is to say, extension is a *form* of relationship between the actualities of a nexus. A point is a nexus of actual entities with a certain 'form'; and so is a 'segment'. Thus geometry is the investigation of the *morphology* of nexus.[29]

The technical aspects of Whitehead's theory of extension have been examined in detail by quite a few authors[30] so I do not intend to list in great detail each axiom in logical notation, except to the extent necessary to clarify and illustrate Whitehead's main achievements and what I see as the surrounding philosophical issues. The topological part of Whitehead's mereotopology in Part IV was introduced to remedy some of the defects in his earlier, purely mereological systems. The new system made the idea of 'direct connection' a primitive concept in terms of which 'mediate connection', 'inclusion' and external connection' (which he called 'adjointness' in the earlier works) could be defined. Whitehead uses the terms 'connection' or 'extensive connection' to mean that relation obtaining when two regions directly touch or overlap. Simons uses the symbol "$\odot$" to represent this relation, the symbol "$\boxdot$" for 'mediate connection' and "$\leq$" for the 'part-of' relation. I shall quote verbatim some of the relevant definitions then give Simons' first order logic translations.

**Definition 1**. Two regions are mediately connected when they are both connected with a third region.

Def. $\boxdot$ $x \boxdot y := \exists z\,(x \odot z \wedge z \odot y)$

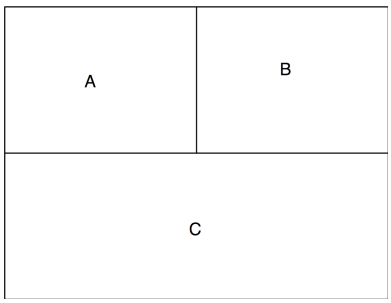
Observe how this definition allows directly connected regions like A and B in the diagram below to also be mediately connected via a region like C.

---

[29] *Process & Reality*, p.302. Emphases mine.

[30] Robert Palter in his 1960 book *Whitehead's Philosophy of Science*, Bowman Clarke with his 1981 and 1985 papers "A Calculus of Individuals Based on Connection" and "Individuals and Points", Peter Simons in his 1991 paper "Whitehead und die Mereologie", Gerla and Tortora in their 1992 article "La relazione di connessione in A. N. Whitehead", and most recently Sebastian Richard in 2011 with his "Whitehead's Mereotopology and the Project of Formal Ontology".

**Diagram 25. 'Direct connection' and 'mediate connection' aren't mutually exclusive**




**Definition 2**. Region A is said to 'include' region B when every region connected with B is also connected with A. As an alternative nomenclature, region B will be said to be 'part of' region A.

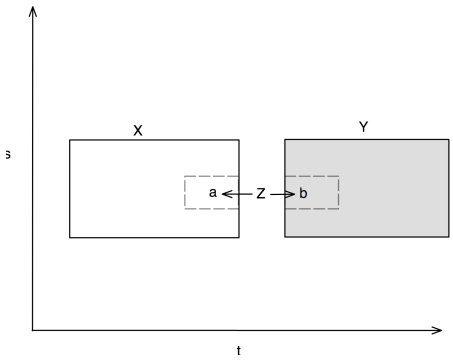
Def ≤ $x \leq y := \forall z (z \odot x \rightarrow z \odot y)$

**Defintion 7**. Two regions are externally connected ['adjoined'] when (i) they are connected and (ii) they do not overlap.

Def $><$ $x><y := (x \odot y \wedge \neg \exists z\ z \leq x \wedge z \leq y)$

The above definitions enabled Whitehead to avoid the problems which had plagued his earlier definitions of relations like 'junction', 'adjointness', and 'injointness'. We can test this assertion by revisiting Diagram 17 which represents two clearly separate events which Whitehead's earlier definitions of junction weren't able to exclude.

**Diagram 26. Junction and Connection.**




Whitehead's new definitions seem at first pass to also fail the test. Interpreting the diagram to now represent regions rather than events, X and Y might be thought to satisfy

the definition of being ‘directly connected’ in the Part IV sense for the same reason they have ‘junction’ in the middle period sense; region Z is composed of a and b which are entirely inside X and Y so that X and Y are unfortunately ‘directly connected’ in the Part IV sense. To rule out such situations we need an axiom that says that ‘any two regions are mediately connected. This is stated in the second arm of a conjunction in Whitehead’s “Assummption 2”.

**Assumption 2** No region is connected with all the other regions and any two regions are mediately connected.

Simons renders this as:

U⊡ $\forall x \forall y\ (x \boxdot y)$

The above axiom would solve the problem by ruling out the existence of blank areas in between regions a and b (or for that matter X and Y) positing instead the existence of intermediate regions joining them together. By requiring any two regions to be mediately connected Axiom 1 also entails that all pairs of directly connected regions are also mediately connected.

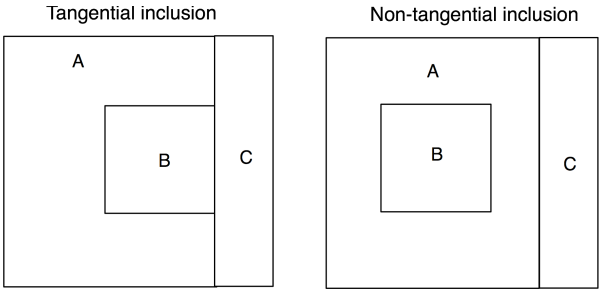
One of the main advantages of Whitehead’s topological definitions and axioms was that it enabled him to define tangential inclusion so as to be able to exclude abstractive sets convergent on their own boundaries. Recall the problem of asymmetrical covering illustrated in Diagram 22 brought about by the existence of abstractive sets with ‘injoined’ or ‘tangentially included’ members. In the system of extensive connection Whitehead was able to define tangential inclusion as external connection (defined as direct connection between two regions which have no part in common).

**Definition 8.** A region B is tangentially included in region A when (i) B is included in A and (ii) there are regions which are externally connected with both A and B.

This allows Whitehead to distinguish between tangential and non-tangential (or injoined and non-injoined) inclusion as illustrated in the diagram below. In the picture on the left, region C is externally connected to A and B, and A tangentially includes (or injoins) B. In

the picture on the right C is externally connected to A only which includes B non-tangentially.

**Diagram 27. Tangential inclusion**




So Whitehead's new definitions allow him to limit the definition of abstractive sets to those ordered by non-tangential inclusion[31] only and so exclude tangential sets from the definition of a prime abstractive set defining a geometric point.

**A Definition of lines and straight lines**

The ideas of Part IV are an advance on his earlier ideas in books like *The Concept of Nature* in other respects too. Firstly, whereas in the earlier works Whitehead refrained from giving express definitions for limits other than points, in *Process & Reality* he does appear to offer purely mereotopological definitions for lines and straight lines as well as flat planes and volumes. He gives definitions for a segment in Section IV of Chapter II and then a straight line segment in Section II of Chapter III, the second of which I paraphrase for the sake of brevity.

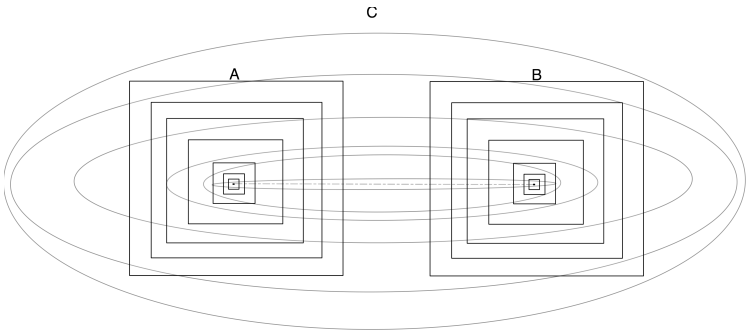
**Definition 18** A geometrical element is called a 'segment between two points P and Q when its members are prime in reference to the condition that the points P and Q are incident in it.

**Definition 3.** A straight line is a single geometrical element defined by an *ovate* prime abstractive set in reference to the condition of covering two points.

[31] Definition 10. A set of regions is called an 'abstractive set' when (i) any two member of the set are such that one of them includes the other non tangentially and (ii) there is no region included in every member of the set". *Process & Reality,* p.298.

To unpack these definitions we must first understand that Whitehead defines a given point in a way similar to that taken in the middle period texts, but this time as a set of all the *non-tangential* K-equal prime abstractive sets in respect of a given formative condition σ. The other important fact about his definition of straight line segments is that the relevant abstractive sets are restricted by a formative condition σ such as to only include those whose members are all "ovate" (what we would now call "convex"). The kind of convex, non-tangential abstractive set Whitehead was talking about in his definition of a straight line is illustrated in the figure below.

**Diagram 28. Whitehead's Definition of a Straight Line**




In the above diagram the two square abstractive sets A and B converge on points and are covered by the set of ellipses C which converges on the dashed line segment between the points defined by sets A and B. The formative condition σ for C requires it to cover A and B.

For any abstractive set convergent on a curved line, at least in Euclidian spaces, the inner regions would have to become non-convex as they approach the curved line segment, entailing the abstractive set is non-ovate after all. Whitehead thought of his definition as a major accomplishment because it demonstrated from first principles the uniqueness of a Euclidian straight line joining any two points, something of which he thought the modern definition[32] was incapable, having to import concepts that were ultimately traceable to the empirical notion of a "route of certain physical occurrences".[33]

Whitehead's use of ovate abstractive sets seems suspect at first sight because a convex figure is a usually defined geometrically by the inability to construct a straight line between

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[32] Euclid says that a straight line is a line "which lies evenly with the points on itself" or that it is a "breadthless length" while the modern definition according to Whitehead is "the shortest distance between two points". Whitehead says the first is obscure while the second imports the physical notion of distance.

[33] *Process & Reality*, p.302.

any two of its boundary points that doesn't intersect its interior. Whitehead acknowledges that convexity cannot be defined mereotopologically with such a standard first order definition. He says “we cannot define a single oval”[34] and opts instead to define convex regions with a second order definition that turns on an ovate region’s inclusion in the whole “*class* of ovals” defined by "enumerating all those peculiar [but mereotopological] properties possessed by individual members of the class, or by sub-sets of members of the class”[35] such as (i) only having as members regions which are such that if they intersect another member they do so in one and one only sub-region that is also a member, and (ii) having members which are such that if they are externally connected are “touching either in a 'complete locus'[36] of points or in a single point”.[37] Whitehead's method appears circular but is no less valid than listing as one of the defining features of even numbers say, the fact that adding two together results in a third even number, or defining friendly dogs by the unique relational fact that when any two meet they wag their tails. Similarly, Whitehead’s definition focuses on the many unique relationships between convex regions to define convexity. This seems odd because we usually define a property as a complex of simpler properties rather than by the properties of the set which includes all of the members with the property we are seeking to define. But that does not make his approach invalid. Whitehead does seem therefore to have succeeded in defining straight lines on the basis of mereotopological properties alone and any suspicion that he has smuggled geometrical concepts into the definiens is unwarranted.

Whitehead generalises his construction of straight lines to “flat loci” in two and three dimensions by simply varying the σ condition so that the prime ovate abstractive set has to cover more points. A two dimensional triangle for example is constructed from ovate abstractive sets required to cover any three non-collinear points and a three-dimensional tetrahedron by a set covering four non-coplanar points. A plane can then be constructed as that locus such that any three non-collinear points inside it form a triangle in the locus while stipulating that any finite number of points in the locus lie in some triangle in the

[34] *Process & Reality*, p.303.

[35] *Process & Reality*, p.303.

[36] A ‘single locus’ means here a single line segment or a bounded surface or volume.

[37] *Process & Reality*, p.304.

locus. Whitehead proposes a similar procedure for constructing three-dimensional “flat space” from tetrahedrons.[38]

### 5.5 Conclusion

Part IV of *Process and Reality* therefore caps off Whitehead's mereological investigations over the previous 14 years, proving how points, lines and 3-dimensional loci can all be logically constructed from 4-dimensional entities. As an exercise in axiomatic geometry, Whitehead's theory of extension enabled him to deduce points and Euclidian straight lines from more abstract, and what he thought of as more universal principles in a way that had eluded geometers since the time of Euclid and similar to how he and Russell in their logicist phase had constructed numbers from sets.

However, major questions remain about the method of extensive abstraction’s philosophical value and interpretation. One of the most important criticisms of Whitehead’s method was made in the 1950s by the eminent philosopher of science Adolf Grünbaum who attacked its “positivistic constructive” approach which he thought was “fatally ambiguous”. Grünbaum’s argument was that the method is “incapable of ascertaining meaningfully a difference in identity between the class of sensible volumes defining a given point and the class defining another point closely nearby, although the two points are separated from one another by a super-denumerable infinity of intervening points”, or that the method “...requires the notion of a non-denumerable infinity of perceivables" which is "patently "untenable”[39]. Grünbaum seems to construe Whitehead as saying that for any two points which are so close together that they are indistinguishable from one another in sense experience, there must exist two corresponding abstractive sets with outer members that are clearly distinguishable from one another in sense experience. To put some flesh on the bones of this argument, if two points 'a' and 'b' are located at some indiscernibly small distance apart, say $10^{-100}$ cm, then according to Grünbaum, Whitehead requires it to be clearly distinguishable in sense experience which of the two corresponding abstractive sets A and B is convergent on which point. Grünbaum's objection is that sense experience may very well be able to prehend the outermost members of A and B but it could never discern which points they converge onto, separated as they are by an undetectable $10^{-100}$ cm.

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[38] When Whitehead here talks about “flat space” he means that his definitions define Euclidian straight lines only. In an elliptical space for example there would be more than one prime abstractive set that covered the two abstractive sets defining any given pair of points, or in other words more than one 'straight' line connecting any two points.

39 Grünbaum, A. “Whitehead’s Method of Extensive Abstraction”, p.226.

Grünbaum's criticism however does not hit its mark because whatever its technical failings Whitehead's method is not meant to show how points are demonstrable in sense experience either directly or indirectly via their corresponding abstractive sets. It's objective is in fact the exact opposite - to show how points exist outside of sense experience altogether. In the middle period texts points are conceived of as conceptual objects which function as ideal limits approached by series of events but never actually attained; as fictive things suggested by the progressive whittling away in the mind's eye of 'bigger' and more basic logical objects to an infinite degree. Although there is a shift in *Process & Reality* towards a platonic interpretation with points seen as real things they do not *exist* in sense experience but *subsist* as ideal forms outside of it. In addition Whitehead's atomistic ontology in *Process & Reality* implies that the actual events experienced in sensibility are only finitely divisible. Infinitely divisible abstractive sets do not exist in actuality but in potentiality only. In either case Grünbaum's understanding of Whitehead's abstractive sets as things which must be experienceable as proxies for every point in a purported transfinite space is simply mistaken.

The shift in Whitehead's thinking between the two periods however does raise other other philosophical problems because of the change in the ontological status of the objects Whitehead abstracts from and to. In the earlier texts geometrical objects were mental fictions abstracted or constructed from our finite experience of concrete four-dimensional events but in *Process and Reality* geometrical objects are said to be abstracted from other ideal objects (objectively existing regions in the extensive continuum). The manifold of actual events in Whitehead's later system is discrete; an evolving ether of atomic events, and thus neither infinitely divisible nor composed of points. The spatial regions of the extensive continuum on the other hand are infinitely divisible because they are postulated to 'subsist' outside of the actual. But if geometrical entities like points, lines and planes are of the same ontological status as regions then what is the purpose of extensive abstraction when one could also have *postulated* their 'subsistence' in the same way? Whitehead's platonism therefore entails that extensive abstraction is redundant - one cannot logically construct *res vera*, things that are themselves ontological ultimates. If we take Whitehead's platonism seriously extensive abstraction becomes nothing but an exercise in geometry for its own sake lacking in philosophical purpose. From a truly process point of view however extensive abstraction serves its original function of demonstrating abstract entities as generalisations of tendencies encountered in finite experience quite admirably

in adherence with Russell's slogan "that whenever possible, logical constructions are to be substituted for inferred entities." [40]

40 Russell, R. *The Relation of Sense Data to Physics*, p.164.

## Summary

Whitehead's platonism and his platonistic theory of the extensive continuum lead him ultimately to profoundly confusing and mistaken doctrines on issues like objectification, general relativity and even the method of extensive abstraction itself. Whitehead justifies his eternal objects on the grounds that 'it is form which endures not substance' and that without 'pure potentialities' the world is just "a static monistic universe"[1]. But as Whitehead himself also acknowledged, apparently enduring objects whether the Pyramid of Cheops or the waveform of an electron are constantly evolving not just in substance but in *form* as well and in ways that are inherently creative and unpredictable. The more we treat either substances, forms or potentialities as enduring things the more we are abstracting away from the underlying reality of process. The individual atoms in my body, like planks in the Ship of Theseus, are always being replaced, but so too are the hormonal, chemical and neuronal *patterns* within my cells and tissues as I grow and mature. And the *possibilities* concerning my getting rich or contracting a virus likewise shift from moment to moment with contingent changes in circumstance and history. In other words forms and potentialities are abstractions from process as much as substances are. This is the implication of the incipient one-category ontology which Whitehead's philosophy seems to bound towards but ultimately shies away from. I have sketched in faint outline what a fully developed pure process ontology might look like. It is represented diagrammatically by a network of concatenated arrows each carrying many events in their domain to many events in their co-domain except for those special events called "prehensions" which carry single events to single events. In such an ontology events bind *themselves* together into processes that do not exhibit invariant forms but at most quasi-periodic, analogous and relatively self-sustaining patterns of behaviour that are just as equally prone to instability, crisis and breakdown. Such an ontology freed from Whitehead's eternal objects is more faithful to the morphologically dynamic, complex and evolutionary world going on around us in everything from ecological networks and climate systems to the history of the cosmos itself. It is also truer to Whitehead's own dictum that "… in separation from actual entities there is nothing, merely nonentity - 'The rest is silence'".[2]

[1] *Process & Reality*, p.46.

[2] *Process & Reality*, p.43.

## Appendix 1.Whitehead's Mistaken Use of Zeno's Dichotomy

Although Whitehead sometimes justified his "epochal" or atomistic theory of events on empiricist grounds referring to the limits of human perception and the need for science to work with units of matter and energy set by pragmatically determined thresholds, he also attempted to justify his atomic theory metaphysically, drawing on Zeno's Dichotomy argument. Importantly, he begins with the empirical observation of William James that "either your experience is of no content, of no change, or it is of a perceptible amount of content or change. Your acquaintance with reality grows literally by buds or drops of perception."[3] But Whitehead then proceeds to justify his atomism apriori with an argument against infinite divisibility along the following lines; if an event 'A' over a period of one second is infinitely divisible, then it is divisible into a first half 'a' required for before anything later can occur, which itself contains a first half 'b" which must occur before anything else does, but which itself contains a first half '*c'* and so on ad infinitum. Thus if *A* is infinitely divisible it cannot have an *earliest* first part, for any part deemed to be earliest would itself contain an earlier part. But says Whitehead, *A* must have an earliest first part that occurs after the instant *i* at which *A* begins because clearly at some point in time after *i* something happens to "effect a transition" into A. Whitehead argues that if an act of becoming is infinitely divisible then;

> ... there is nothing which becomes, so as to effect a transition into the second in question ...For at the beginning of the second of time there is no next instant at which something can become. ... every act of becoming must have an immediate successor, if we admit that something becomes. For otherwise we cannot point out what creature becomes as we enter upon the second in question. ... The conclusion is that in every act of becoming there is the becoming of something with temporal extension; but that the act itself is not extensive in the sense that it is divisible into earlier and later which correspond to the extensive divisibility of what has become."[4]

Whitehead's conclusion is that reality is atomic.

> Finally, the extensive continuity of the physical universe has usually been construed to mean that there is a continuity of becoming. But if we admit that 'something becomes', it is easy, by employing Zeno's method, to prove that there can be no

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[3] *Process & Reality*, p.68.

[4] *Process & Reality*, pp.68 - 69.

> continuity of becoming. There is a becoming of continuity, but no continuity of becoming. The actual occasions are the creatures which become, and they constitute a continuously extensive world. Thus the ultimate metaphysical truth is atomism. [5]

The nub of his argument is that "every act of becoming must have an immediate successor … For otherwise we cannot point out what creature becomes as we enter upon the second in question". It is fair enough to require each part of a continuous event to have an immediate successor, i.e. it seems reasonable to require that over each sub-interval, say each millisecond of *A*'s unfolding, *A* be occurring, that it cannot be the case for example that during the 30th millisecond *A* is not happening but during the 31st millisecond it is. In other words it is reasonable to assume that events are unbroken continuities without gaps. However, accepting this premise does not on its own imply there must be a unique and earliest first part of *A*. To get to that conclusion Whitehead argues that after the instant *i* at which *A* begins, there must must be some later event e that *immediately* follows *i* without any gaps in between. This argument has a hidden premise which is that A contains instantaneous parts like *i*. From here he goes on to say that no part of *A* which begins after the instant *i* can be immediately adjacent to *i.* If we accept that *i* exists then it is most certainly the case that that it cannot be immediately next to another event because as an instant, it is of zero durational extent. Just as there is no decimal number closest to another, so there can be no instantaneous event closest to another which raises the question of 'how to effect a transition' from *i* into A. But on Whitehead's own account, at least in his middle period texts, point instants *do not exist*. If we reject the existence of instants, the issue of non-contiguity between the so-called instant *i* of *A*'s beginning, and the later parts of *A,* does not arise. In other words Whitehead's first premise that events are gapless continuities might be true, but if we reject his second premise about the existence of instants we are not thereby forced to concede the existence of evental atoms.

Whitehead's Zeno-like argument for the epochal theory was therefore according to the standards of his own doctrines, unsound. What reason was there then for this lapse in position? I think the reason is that Whitehead's rejuvenated platonism in his later works led him into a confusion about whether there might be point-events in actuality after all. Recall that he concludes ideal points and instants 'subsist' as pure potentialities in the extensive continuum. Actual events are said to 'divide' the ideal continuum which is to say

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[5] *Process & Reality*, pp.35 - 36.

they 'occupy' or instantiate some extended region within it. These regions are analysable into an infinity of ideal parts convergent on ideal points which must subsist at an even higher level of abstraction. These higher level ideal points are correlated to locations inside of regions and at their boundaries. Thus each actual event in some sense has an ideal instantaneous beginning which is ideal and in principle unavailable to sense experience. But if points subsist as pure potentialities then there does not seem to be any reason against them being instantiable as point-events in actuality albeit ones that are in principle un-experienceable and acausal. Whether such point-events could be said to *compose* larger events encountered in human experience or whether they somehow exist alongside but independently of them is another question. In either case I think Whitehead's platonism leads to a confusion about the ontological status of points and to a false apriori deduction of the necessity for eventual atoms.

## Appendix 2. Nobo's Neo-Platonic Interpretation of the Extensive Continuum

I would like here to argue both for and against a position advocated by Jorge Nobo, in his book *Whitehead's Metaphysics of Extension and Solidarity,* which is that Whitehead holds a kind of neo-platonic theory of individual occasions as holographic images projected from an extensive continuum that is one, eternal and indivisible. I think Nobo is spot-on about Whitehead's platonism but he fails to recognise that Whitehead was inconsistent, wavering back and forth between it and a more naturalistic process ontology. Nobo's drive to find consistency where there is none leads him to say for example that the unity and indivisibility of the extensive continuum "grounds the mutual transcendence and the mutual immanence, of any two actual entities"[6] by which he means that a given occasion is literally *in every* other, regardless of whether the latter precede or succeed it in time and regardless of how spatially distant they might be.

> … a proper region mirrors, in addition to itself, all the proper regions that were already in existence relative to its own origination. In like manner, a proper region mirrors all the extension that, relative to its own origination, was as yet unactualized, as yet mere extension. A proper region, therefore, mirrors within itself the entire extensive universe - the universe, that is, such as it was relative to that region's origination.[7]

Nobo's idea of symmetrical prehension also precludes genuine creativity which he sees rather as a kind of divine energy emanating from the transcendental realm into individual actualities, "an inexhaustible metaphysical energy at the base of all existence and revealed to, as well as manifested in, all concrescent existents".[8] I think the textual evidence in *Process & Reality* against Nobo's idea of mutual immanence between actual occasions is compelling, but it has to be acknowledged that Whitehead's metaphysics is not entirely closed to the idea. As early as *The Concept of Nature* in 1920 for example, he wrote that systematic and lawful relations between events did not exclude apparently non-local phenomena using the transmission of electric charge as a possible example.

> The electron is not merely where its charge is. The charge is the quantitative character of certain events due to the ingression of the electron into nature. The electron is its whole field of force ... *to this conception of scientific objects, the rival*

[6] Nobo, p.205.

[7] Nobo, p.224.

[8] Nobo, p.175.

> *theories of action at a distance and action by transmission through a medium are both incomplete* [my emphasis]...the character of the stream of events which we are considering, bears marks of the existence of every other electron throughout the universe...The ether is the expression of this systematic modification of events throughout space and time. The best expression of the character of this modification is for physicists to find out. My theory...is ready to accept any outcome of physical research.[9]

It was on the basis of this kind of thinking that led Whitehead in later works to reject the very idea of self-sufficient spaces existing without reference to other spaces, denying what he calls "the fallacy of simple location". In *Science and the Modern World* for example he says that portions of space are "only entities as within the totality; you cannot extract them from their environment without destruction of their very essence...every volume mirrors in itself every other ...".[10] However, Whitehead's holism and any glimmering of the idea of reciprocal prehension in his work is strongly tempered by a healthy respect for the reality of causally efficacious particulars. So whilst the extensive continuum does not exclude on principle the possibility of mutual immanence, in relation to the plenum of actual events in this cosmic epoch, Whitehead always emphasises prehension as asymmetrical and compatible with one-way causality. "The passage of the cause into the effect is the cumulative character of time. The irreversibility of time depends on this fact".[11] That Whitehead understands prehension and immanence as circumscribed in this way is evidenced in numerous passages from *Process & Reality* which stress that the *intensity* to which an occasion is felt varies according to how it is processed by the mediating occasions through which it is felt. There are "inhibitions or additions, weakenings or intensifications due to the history of its production".[12] He describes human perception for example as a "complex amplifier [in which] the experiences of any part of the body are transmitted to one or more central occasions to be inherited with enhancements accruing upon the way".[13] Alternatively, the influence of some occasions can be progressively diminished by those later in the series such that they fade into insignificance. "For

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9 *The Concept of Nature,* pp.159 - 160.

10 *Science & the Modern World*, p.65.

11 *Process & Reality*, p.237.

12 *Process & Reality*, p.237.

13 *Process & Reality*, p.119.

example, in touch there is a reference to the stone in contact with the hand but the chain of occasions along the arm sinks into the background, almost into complete oblivion".[14] Whitehead's physicalistic understanding that prehension is of varying degrees of intensity and is not everywhere of equal force is brought out in the following passage:

> Generalising from the language of physics, the experience of M is an intensity arising out of specific sensa, directed from A, B, C. There is in fact a directed influx from A, B, C of *quantitative* feeling arising from specific forms of feeling. The experience has a vector character, a common *measure of intensity* [15], and specific forms conveying that intensity ... [and if] we remember that in physics 'vector' means definite transmission from elsewhere we see that this metaphysical description ... agrees absolutely with the general principles according to which the notions of modern physics are framed."[16]

That the intensity of prehension can fall away to zero means that each occasion only prehends a delimited 'universe' consisting of a specific number of other occasions, a limited sampling of the whole.[17] Each actual entity arises out if its own peculiar actual world"[18], "It must be remembered that the phrase "actual world" is like "yesterday" and "tomorrow", in that it alters its meaning according to standpoint.[19] Just as Descartes said, "this body is mine;" so he should have said, "this actual world is mine. My process of "being myself" is my origination from my possession of the world.[20]

In fact Whitehead expounds a "principle of intensive relevance" required to "rescue actual occasions from being undifferentiated repetitions of each other", which is fundamental to the meaning of concepts like 'alternative possibilities', 'more or less', 'important or negligible'. An occasion he says, "might have had more relevance, and it might have had less relevance, including the zero of relevance included in negative prehension".[21] Thus prehension is a localised, physical and this-worldly phenomenon - we do not have to worry

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14 *Process & Reality*, p.120.

15 emphases mine.

16 *Process & Reality*, p.116.

17 In relativistic terms an occasion's universe would be composed only of those events in its backwards light cone.

18 *Process & Reality*, p.284.

19 *Process & Reality*, p.65.

20 *Process & Reality*, p.81.

21 *Process & Reality*, p.148.

about how butterflies on earth today can prehend quarks on Alpha Centauri tomorrow. Generally speaking, prehension is at its most intense with occasions in the immediately preceding duration. Atoms, like persons and individuals are therefore, to a large extent (but not completely) *causa sui*; self-caused, or self-perpetuating patterns of process. In general, each occasion has the most intense feelings for those past occasions most proximate to itself, generally (but not necessarily) prehending those separated by intermediate occasions more weakly.

Finally I think Nobo overlooks an important aspect of Whitehead's philosophy which is its pluralism. Actual occasions are not conceived of as modes of abstract Being in-general. There is nothing like Aquinas' “ens commune”, Spinoza's “Substance”, or Hegel's “Absolute”. For Whitehead, in such "monistic schemes, the ultimate is illegitimately allowed a final, 'eminent' reality, beyond that ascribed to any of its accidents”[22] … “the point to be emphasized against them is "*the insistent particularity of things experienced*”[23]. For example, Bradley's argument that a wolf eating a lamb is really the Absolute qualified by the compound predicate "Wolf-eating-lamb" in such way that each particular is a mere *affectiones substantiae*, is he says, "a travesty of the evidence" because; "*That* wolf ate *that* lamb at *that* spot at *that* time: the wolf knew it, the lamb knew it; and the carrion birds knew it …”[24] This is just another way of saying that although Whitehead acknowledges the existence of a transcendental realm, he does not think it takes ontological precedence over actuality such that the latter is its mere expression. Rather the eternal realm concedes the particularity, concreteness and reality of *actual* occasions. As I have argued, Whitehead's thesis of ontological equivalence leads nonetheless to a host of problems which can only be resolved by excising eternal objects altogether. Nobo takes the alternative route of making Whitehead out to be more of a consistent platonist than he actually was, a route I believe is less faithful to the processual spirit at the heart of his philosophy.

22 *Process & Reality*, p.7.

23 *Process & Reality,* p.4.

24 *Process & Reality*, p.43.

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