Department of Economics
Discussion Papers
ISSN 1441-5429

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No. 06/03

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Diversity of Specialization Patterns, Schur Convexity and Transference: A Note on the Axiomatic Measurement of the Division of Labor

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Summary. This short note makes a preliminary investigation into the axiomatic measurement of the division of labor. We first introduce a matrix that we refer to as labor specialization matrix (LSM) to describe specialization patterns of individuals, and then present some axioms for reasonable measures of the division of labor. Remarkable among the axioms are the Schur convexity and the transference condition, both being related to increasing returns to specialization. From the axioms are derived a class of measures, which are then exemplified.

Keywords: Diversity of specialization patterns; Labor specialization matrix; Measurement of division of labor; Schur-convexity; Transference.

JEL: C43.

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1. Introduction

Despite the fact that the division of labor constitutes *the* ultimate source for economic growth in the classical economic theory and in Adam Smith in particular, and that the recent two decades have witnessed an increasing revival of interest in putting specialization and the division of labor back to the core of economic theory (see, e.g., Stigler 1876, Rosen 1983, Yang and Ng 1998, Buchanan 1998, Sun, Yang and Zhou forthcoming), much more work remains to be done regarding formal analyses of the division of labor. It is well understood among the economics profession that the progressive division of labor often comes as expanding team work and/or enlarged trade network. But how to quantify/measure the division of labor has not yet been addressed to date. This paper aims at filling in this void by developing an axiomatic characterization of the measurement of division of labor.

To be precise, what indeed is the division of labor? "The division of labor may be defined as the division of a process or employment into parts, each of which is carried out by separate persons"(Groenewegen 1987, p. 901). In short, different persons allocate their labor among activities *differently*. If all individuals do exactly the same thing(s) --- recall the hypothetical persons in Smith's classical pin factory story who each take care of all the 18 stages in producing any single pin --- then we would expect no division of labor at all. The crucial feature of division of labor is therefore the heterogeneity or diversity of labor- allocation patterns among individuals. We'll introduce some matrix to describe the labor specialization patterns, based on which measures of the division of labor are to be developed.

It might be thought that measurement of the division of labor for an economy with multiple persons and multiple activities may be similar in spirit to that of the multiple dimensional inequality, a literature that has been well established (see, e.g., Kolm 1977, Maasoumi 1986, Foster and Sen 1997). Yet, as will be analyzed below in detail, what is underlying the division of labor is the productivity gains that essentially arises from increasing returns to specialization while the income inequality literature is largely concerned with the social welfare, explicitly or implicitly, in one form or another. That of course doesn't mean that the measure of the latter are completely useless for measuring the division of labor – it does help much --- but that some subtle notions that are particularly relevant to explaining increasing returns to labor specialization are needed. Also related is the measurement of diversity, an emerging but rapidly growing literature (e.g., Weitzman 1992, Nehring and Puppe 2002). But the most often discussed diversity measures appear to be inconsistent with some basic axioms imposed on the measurement of the division of labor, as analyzed below.

2.Analysis

2.1 Labor specialization matrix

Consider a production economy with n individuals. Each individual is endowed some labor (time), normalized as units, which can be allocated among any m activities, $m \geq 1$.[1] We assume $n \geq m$ to capture the idea that the division of labor is potentially limited by the population size, which is often loosely referred to as the extent of the market in the literature. Denote by l_{ij} individual i's labor input in activity j $(i \in N \equiv \{1,2,...,n\},\ j \in M \equiv \{1,2,...,m\})$, refereed to as the specialization level of individual i in activity j. Thus, the row vector $(l_{i1}, l_{i2}, \cdots, l_{im})(i \in N)$ not only describes the specialization level of individual i in each activity but also specifies his specialization *pattern* engaging in m activities as a whole. Intuitively, the division of labor simply refers to the fact that different individuals are assigned different tasks. In other words, specialization patterns are different between individuals. The more diverse the patterns, the higher the division of labor. Putting all individuals' specialization patterns together, we get a labor specialization matrix,

$$L = (l_{ij})_{n \times m}$$

[1] As is well known, Adam Smith forcefully argued long ago that difference in dexterity and expertise between people of different occupations tend to be the consequence rather than the cause of the division of labor. We may therefore presume that individuals' production functions in any activity are the same, though the production technology for activities may differ from each other. Of course that does not preclude the emergence of the division of labor among the ex ante identical agents, see, for example, Rosen (1983).

with $\sum_j l_{ij} \le 1, l_{ij} \ge 0,\ i \in N, j \in M$. For any $i \in N$, $(l_{i1}, l_{i2}, \cdots, l_{im})$ corresponding to a point in R_+^m.

Even casual observation would suggest that increasing variety of activities constitutes one crucial element in progress in the division of labor. That is, for an economy with a fixed population N, we expect to be able to derive some measure that applies to any $m \ge 1$, partially for possible comparison between two economies of roughly the same population size, or between two points of the time of a given economy wherein the population size remains almost unchanged during the said period of time. Furthermore, to quantify the division of labor, only cardinal measures are considered in what follows. Denote by LSM_m the set of all possible labor specialization matrices (LSM) of order m. Developing a cardinal measure of the division of labor for a production economy is equivalent to establishing a function, denoted as *D*, that maps from $\bigcup_{m=1}^{N} LSM_m$, denoted as LSM hereafter, into R^+ (the set of all non-negative real numbers).

2.2 Axioms

Should all individuals have the same specialization pattern, it seems fair to say that there is no division of labor among individuals at all, or, the division of labor is at the minimum. Formally, we introduce the axiom,

Minimum $\forall L \in LSM$, $D(L)=0$ if all the rows of L are the same.

Strictly speaking, this axiom simply serves to define the function value for a particular class of LSM, which may be seen as a kind of benchmark case, and does not qualitatively alter the characterization to be presented in the next subsection.

Another appealing axiom is

Continuity $\forall m$, $\forall L \in LSM_m$, $D(L)$ is continuous with any element of L.

As mentioned earlier, one fundamental feature of the division of labor lies in the diversity or "heterogeneity" of specialization patterns of individuals. Apparently one can construct very many classes of measures of diversity, and ecological and biological diversities in particular that are consistent with the Minimum axiom (see, e.g., Magurran

1988, Weitzman 1992, Solow et al 1993, Agosti et al 2000). To elaborate our theoretical discussion on measurement of the division of labor, we further introduce the following "monotonicity" axiom,

Schur-Convexity. $\forall\ L \in LSM$, $D(LB) \leq D(L)$ holds for any bi-stochastic matrix B of appropriate order.

Compared with *L,* the summation of the elements of any row of *LB* remains unchanged yet the distribution of the labor inputs among all activities for any individual $i(i \in N)$ becomes more "dispersed" or " averaged". What is of particular interest is that the labor allocations of *all* individuals are "averaged" in the same manner, characterized by bi-stochastic matrix B. The "averaged" specialization patterns as such naturally leads to a lower level of the division of labor, i.e. $D(LB) \leq D(L)$. Note the Schur-Convexity axiom holds for any $m \geq 1$.

Another axiom, which seems to us to be particularly relevant to measuring the division of labor, is what may be referred to as

Transference If $l_{ij_1} \geq \sum_{s \in \underline{n}} l_{sj_1} / n$ and $0 < l_{ij_2} \leq \sum_{s \in \underline{n}} l_{sj_2} / n$, then increasing l_{ij_1} by a small amount and decreasing l_{ij_2} by the same amount results in an increase in the measure *D(L).*

This axiom refers to a special case in re-allocating labor within the economy. Intuitively, if one agent becomes even more specialized in what he is already specialized in than the average of the economy at the cost of being less specialized in what he is less specialized than the average level, then the division of labor increases.

Parallel to the *minimum* axiom may come the following,

Maximum If $n = m, D(L) \leq D(\pi_n), \forall L \in M$, π_n is any $n \times n$ permutation matrix. That is, if everyone completely specializes in one activity that anyone else does not undertake, the heterogeneity of specialization patterns reaches the highest level.

However, the *Maximum* condition is *not* independent of the *Transference* because any $n \times n$ division matrix can be "transferred" to a permutation matrix by at most $n \times (n-1)$ steps of *Transference* operations. Also note that $D(\pi_n)$ may change with n.

The final axiom, which we refer to as simple symmetry, is about measuring the division of labor for economies of only two activities, i.e., m=2, in which each individual allocates exhaustively *all* her labor between the two activities. That is, $l_{i1}+l_{i2}=1, \forall i \in N$.

Simple Symmetry $D\begin{bmatrix} l_{11} \\ l_{21} \\ \vdots \\ l_{n1} \end{bmatrix} = D\begin{bmatrix} l_{12} \\ l_{22} \\ \vdots \\ l_{n2} \end{bmatrix}$ where $l_{i1}+l_{i2}=1, l_{i1}, l_{i2} \geq 0, \forall i \in N$

As analyzed in the above, the measure of the division of labor can be largely seen as quantifying the differentiation or heterogeneity of specialization levels among individuals. One can readily observe that the distribution of $\{l_{12}, \cdots, l_{n2}\}$ is simply the "mirror image" of that of $\{l_{11}, \cdots, l_{n1}\}$ due to $l_{i1}+l_{i2}=1, \forall i \in N$, and therefore the heterogeneity in each should be of the same magnitude.

In sum, we present in this subsection five axioms on measures of the division of labor: *Minimum, Continuity, Schur-Convexity, Transference and Simple Symmetry.*

2.3 Characterization

Naturally one may expect the notion of Schur-Convexity to closely relate to the convexity of functions in some way. We introduce,

Lemma 1. For any $L, L' \in M$, the following conditions are equivalent to one another: (1) there exists a $m \times m$ bi-stochastic matrix B, such that $L' = LB$;[2] (2) $f(L') \leq f(L)$ holds true for any Schur-convex function *f;* (3) $\sum_{j=1}^{m} d(l_{1j}, \cdots l_{nj}) \geq \sum_{j=1}^{m} d(l'_{1j}, \cdots l'_{nj})$ holds true for any convex function $d: R^n \to R$.

Proof. Consider the transposes of labor specialization matrices. The above lemma immediately follows from Theorem 3 in Kolm (1977) and Proposition 5.1 in Mosler (1994). **QED**

Note any bistochastic matrix can be expressed as a convex combination of some permutation matrices according to the Birkhoff Theorem. Thus, intuitively, for a given

[2] Algebraically, L is said to majorize L'. For sophisticated discussion on the notion of majorization and its extensive applications, see the special issue on majorization of *Linear Algebra and Its Application*

division matrix L and a stochastic matrix B, LB means that not only the labor inputs on different activities for any individual are averaged somehow, but the averaging of labor allocation for all individuals is done *in the same manner*. This interpretation provides a simple way to understand why any reasonable measures, such as those to be proposed below, that embody the notion of the "diversity" of specialization patterns among individuals should satisfy Schur-convexity. Kolm(1977, pp.5-6) details some interesting intuitive discussion regarding the inequality of consumption amongst individuals.

For economies wherein exist increasing returns to specialization in all activities, it appears more transparent why the concept of Schur-convexity does matter in measuring the division of labor. Consider a "symmetric Smithian economy" in which agents are intrinsically identical in all productive activities, namely, all the individuals' production function of any product j, $j \in M$, is the same, denoted as f_j. Let p_j be the price of product j, $j \in M$. Further define $d(l_{1j}, l_{2j}, \cdots, l_{nj}) \equiv p_j \sum_{i=1}^{n} f_j(l_{ij})$. Then $\sum_{j=1}^{m} d(l_{1j}, \cdots l_{nj})$ can be interpreted as the total monetary value (the wealth) of the produce of the economy, and thus Schur-convexity of measures, as demonstrated by Lemma, implies that the expansion in the network of division of labor increases the total wealth of the production economy. The concept of Schur-convexity is thus shown to be rooted in economies of division of labor based on increasing returns to specialization. In contrast, in the study of distribution inequality of multi-dimensional incomes, the notion of Schur-convexity also plays a key role but as the mathematical equivalent of the Pigou-Dalton principle (eg., Foster, Majumdar and Mitra 1990).

Lemma 2. Axioms Schur-convexity, Transference and Simple Symmetry imply that for any m, and any $L \in LSM_m$, $D(L)$ can be represented as $\sum_{j=1}^{m} g(l_{1j}, \cdots, l_{nj})$, where $g(x_1, \cdots, x_n)$ is a convex function and increases (decreases) with any $x_i \geq \bar{x} \equiv \frac{1}{n}\sum_{s=1}^{n} x_s$ $(x_i < \bar{x})$.

(1994), in particular Mosler (1994). A general discussion on the majorization relation between vectors can be found in Bhatia(1997, ch.2).

Proof By Lemma 1, Schur-convexirty implies that $D(L)$ can be represented by $\sum_{j=1}^{m} g(l_{1j},\cdots,l_{nj})$ for any convex function g. Without loss of generality, suppose $l_{1j^*} \geq \sum_{s\in N} l_{sj^*}/n$ for individual 1 and activity j^* (the case that $l_{1j^*} < \sum_{s\in N} l_{sj^*}/n$ could be addressed in the same way as follows). Consider a new labor specialization matrix of n-by-2, $\begin{bmatrix} l_{1j^*} & 1-l_{1j^*} \\ l_{2j^*} & 1-l_{2j^*} \\ \vdots & \vdots \\ l_{nj^*} & 1-l_{nj^*} \end{bmatrix}$. Apparently, $1-l_{1j^*} \leq \sum_{s\in N}(1-l_{sj^*})/n$. Let l_{1j^*} $(1-l_{1j^*})$ increase (decrease) by $\varepsilon > 0$ and the other elements remain unchanged. Transference requires that $g(l_{1j^*}+\varepsilon, l_{2j^*},\cdots,l_{nj^*})$ $+g(1-l_{1j^*}-\varepsilon, 1-l_{2j^*},\cdots,1-l_{nj^*}) > g(l_{1j^*},l_{2j^*},\cdots,l_{nj^*}) + g(1-l_{1j^*},1-l_{2j^*},\cdots,1-l_{nj^*})$. But by Simple Symmetry, $g(l_{1j^*},l_{2j^*},\cdots,l_{nj^*}) = g(1-l_{1j^*},1-l_{2j^*},\cdots,1-l_{nj^*})$, $g(l_{1j^*}+\varepsilon,l_{2j^*},\cdots,l_{nj^*})$ $= g(1-l_{1j^*}-\varepsilon,1-l_{2j^*},\cdots,1-l_{nj^*})$. Thus, $g(l_{1j^*}+\varepsilon,l_{2j^*},\cdots,l_{nj^*}) > g(l_{1j^*},l_{2j^*},\cdots,l_{nj^*})$. **QED**

Note Lemma 2 reveals that the simple symmetry imposed on the measure D(L), functions mapping from $LSM \equiv \bigcup_{m=1}^{N} LSM_m$ into R^+, applies to function g that maps vectors into R^+.

We now introduce our main result, which states that the measure of the division of labor based on a given labor specialization matrix is additively decomposable by the activities. In fact, the measure amounts to the sum of function values of each column of the labor specialization matrix.

Theorem. Axioms Minimum, Continuity, Schur-Convexity, Transference and Simple Symmetry imply that $D(L)$ can be represented as $\sum_{j=1}^{m} g(l_{1j},\cdots,l_{nj})$, where $g(x_1,\cdots,x_n)$ is a convex function satisfying $g(x,\cdots,x)=0$ for any $x \in [0,1]$ and increases (decreases) with any $x_i \geq \bar{x} \equiv \frac{1}{n}\sum_{s=1}^{n} x_s$ $(x_i < \bar{x})$.

Proof It follows from Lemmas 1 and 2 that D(L) is represented as $\sum_{j=1}^{m} g(l_{1j},\cdots,l_{nj})$ where function $g(x_1,\cdots,x_n)$ is convex and increases (decreases) in each $x_i \geq \bar{x} \equiv \frac{1}{n}\sum_{s=1}^{n} x_s$ $(x_i < \bar{x})$. Thus, it suffices to only verify that $\forall x \in [0,1], g(x,\cdots,x)=0$. To keep notations neat, let

$f(x) \equiv g(x,\cdots,x)$. By convexity of g, $\forall x \in [0,1]$, $f(x)+f(0) \geq 2f(x/2)$ and $f(1-x)+f(1) \geq 2f(1-x/2)$. But applying the Minimum axiom to economies of 2 activities results in $f(x)+f(1-x) + f(0)+f(1) = 2[f(x/2) + f(1-x/2)] = 0$. Therefore, $f(x)+f(0) = 2f(x/2)$. Similar analysis for any $\forall k \geq 3$ yields $(k-1)f(x/(k-1)) + f(0) = kf(x/k)$. Thus, for any $x_1, x_2 \in [0,1]$, $h(x_2,x_1) \equiv f(x_2)-f(x_1) = kf(x_2/k)-kf(x_1/k) = kh(x_2/k, x_1/k)$ for any positive integer k. Continuity of $g(...)$ implies continuity of $h(\cdot,\cdot)$. It is easy to verify (refer to proof of Lemma 4 in Sun and Ng 2000, p. 317) that for any real number $r>0$, $h(rx_2,rx_1) = rh(x_2,x_1)$ provided that $rx_1, rx_2 \in [0,1]$. Hence $f(x)-f(0) = h(x,0) = xh(1,0) = x[f(1)-f(0)]$, $\forall x \in [0,1]$, $\forall r \in R^+$ subject to $\forall rx \in [0,1]$. That is, $f(x) = x[f(1)-f(0)] + f(0)$. But Simple Symmetry implies, $f(x) = f(1-x)$. We have $x[f(1)-f(0)] = (1-x)[f(1)-f(0)]$, $\forall x \in [0,1]$. Hence, $f(1)-f(0)=0$, $f(x)=f(0)$, $\forall x \in [0,1]$. Noting $f(x)+f(1-x/2)=0$ $\forall x \in [0,1]$ for economies of two activities due to the Minimum axiom, we obtain $f(0)=f(1/2)=0$. Thus $g(x,\cdots,x) \equiv f(x) = 0$, $\forall x \in [0,1]$. **QED**

Note the Transference condition, simple though it seems, fundamentally differs from the notion of Schur-convexity in the sense that the latter plays an important role in both the study of inequality and the measure for division of labor, yet it is not this case for the former. We can illustrate this point by considering a simple case. Consider an economy with only two persons and two activities, for which the division matrix is supposed to be either $L_A = \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$ or $L_B = \begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix}$ Define $h_A(\varepsilon) \equiv D\left(\begin{bmatrix} 0.6+\varepsilon & 0.4-\varepsilon \\ 0.5 & 0.5 \end{bmatrix}\right)$ and $h_B(\varepsilon) \equiv D\left(\begin{bmatrix} 0.6+\varepsilon & 0.4-\varepsilon \\ 0.7 & 0.3 \end{bmatrix}\right)$ where D is a measure and ε is a sufficiently small positive real number. Thus $h_A(\varepsilon)$ and $h_B(\varepsilon)$ means that individual 1 transfers some labor effort from the second activity to the first activity. The transference of D requires that $h_A(\varepsilon)$ increase with ε while $h_B(\varepsilon)$ decrease with ε. On the other hand, we can also view the matrices L_A and L_B as something describing the distribution of two attributes of income among two persons in which the *ij*-element is the share of the *i-th* type of income distributed to individual j. Similar to $h_A(\varepsilon)$ and $h_B(\varepsilon)$, we can also define $s_A(\varepsilon) \equiv I\left(\begin{bmatrix} 0.6+\varepsilon & 0.4-\varepsilon \\ 0.5 & 0.5 \end{bmatrix}\right)$ and

$s_B(\varepsilon) \equiv I\left(\begin{bmatrix} 0.6+\varepsilon & 0.4-\varepsilon \\ 0.7 & 0.3 \end{bmatrix}\right)$, where I is a measure of the inequality of multi-dimensional incomes. One can easily imagine some popular measures in the study of income inequality such that both $s_A(\varepsilon)$ and $s_B(\varepsilon)$ increase with ε. In other words, the notion of *the transference* introduced in section 2 does not apply to the study of inequality of incomes, for which the so-called "transfer principle" is embodied by the Schur-convexity already (Maasoumi 1986).

This simple numerical example reveals that the concept of transference is much richer and more complex than it seems to be. In studying the measurement of the division of labor, the heterogeneity of specialization patterns fundamentally matters, which in turn means that the degree of diversity of specialization patterns should be counted in reckoning the impact of the labor re-allocation of individuals upon the extent of division of labor. In a more general sense, we actually face something like a "network effect" when we attempt to explore the complicated relationship between the individuals' decision in choosing a specialization pattern and the extent of the social division of labor as a whole. How to generalize the transference merits further exploration.

One may see any labor specialization matrix of *n-by-m* as *n* points from $\{(x_1,\ldots,x_m) \mid x_i \geq 0, \sum_{s=1}^{m} x_s \leq 1\}$ and measuring diversity of specialization patterns correspondingly as "dispersion" of the said points. The prevailing approach in measurements of diversity, represented in particular by Weitzman (1992), as recently characterized by Bossert, Pattanaik and Xu (2001), is based on the pair-wise distances (dissimilarities) of all points (elements). As such, it can be readily shown that even for economies of only three individuals and two activities, Weitzman's (1992) measure is not consistent with the Transference axiom. Sun and Ng (2000) provides an axiomatic characterization of the pair-wise distances. Their measure, however, is not compatible with the axioms presented in this paper either, for their "structural dissimilarity (difference)" index is essentially a linear one and hence inconsistent with (Schur-) convexity.

2.4 Examples

As analyzed in the preceding subsection, measuring the division of labor in a manner consistent with the axioms presented above is essentially no more than constructing a vector function, $g(x_1,\cdots,x_n)$, of some particular properties, as stated in the Theorems. We now introduce three further specific measures, which may be of practical use. But note that $g(x_1,\cdots,x_n)$ in Theorem cannot be strictly convex at all points. In fact, it is implied by Simple symmetry and Minimum that at any point $(x_1,\cdots,x_n)$ with $x_1=\cdots=x_n=x, x\in[0,1]$, $g(x,\cdots,x)+g(0,\cdots,0)=0$ and $2g(x/2,\cdots,x/2)=0$. But strict convexity requires $g(x,\cdots,x)+g(0,\cdots,0)>2g(x/2,\cdots,x/2)$ for any $x>0$. Thus, we can only expect to construct some convex function(s) $g(x_1,\cdots,x_n)$ that are not *strictly* convex at points $(x_1,\cdots,x_n)$ with $x_1=\cdots=x_n=x$, $\forall x\in[0,1]$.

The first measure is modified from Shannon's information entropy formula.[3] For any *n-by-m* $L\in LSM$,

$$D_1(L)\equiv\sum_j[\sum_i l_{ij}\ln(l_{ij})-\sum_s l_{sj}\ln(\frac{1}{n}\sum_s l_{sj})]$$

This measure has all the appealing properties of entropy-type indices. The other two measures are based on the Euclidean distances, but are somehow different from each other,

$$D_2(L)\equiv\sum_j\sum_{i\neq s}(l_{ij}-l_{sj})^2$$

$$D_3(L)\equiv\sum_j\sum_i(l_{ij}-\frac{1}{n}\sum_{s=1}^{n}l_{sj})^2$$

Note D_2 is based on the pair-wise distances while D_3 is based on the dispersion of specialization levels from the central location (the average on a given activity for the whole economy/team under consideration). Another measure based on "dispersion" may also come to mind, $D_4(L)\equiv\sum_j\sum_i(l_{ij}-\frac{1}{n-1}\sum_{\substack{s=1\\s\neq i}}^{n}l_{sj})^2$. Note $D_3(L)=\left(\frac{n-1}{n}\right)^2D_4(L)$. It is easy to show that all the four measures satisfy the function properties stated in Theorem. Which

[3] For an excellent analysis of the Shannon entropy measure of ecological diversity, see Pielou (1977) and Weitzman (1992).

measure is practically useful may largely depend on the economy in question and the date available. We would speculate that if $\sum_j l_{ij} \approx 1$ for all i, the Euclidean distance based measures maybe used, since, for example, $D_2(L) \equiv \sum_{i<j} d_{ij}^2$ where d_{ij}^2 is the Euclidean distance between two points $l_i = (l_{i1}, l_{i2}, \cdots, l_{im})$ and $l_j = (l_{j1}, l_{j2}, \cdots, l_{jm})$ on the simplex S_{m-1}. Otherwise, the entropy index seems an appropriate measure.

3. Conclusion

This note addresses the axiomatic measurement of the extent of division of labor. We present some axioms, from which a class f measures are derived. We also present some further specified measures. It goes without saying that this study is rather preliminary in that only interpersonal labor-specialization differences, or diversity of individual specializations, is considered. When other important issues, for instance, joint production and (dis)economies of scope are taken into account (e.g., Panzar and Willig 1981), the measurement could be significantly complicated. We leave this for future study.

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