

A note on “An Inframarginal Analysis of the Ricardian Model”

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Abstract

This is a note about Cheng et al’s paper, in which we consider a dual structure of division of labor and trade in the model of Cheng et al that is missed by the authors. While the inclusion of the dual structure will not change major results in the paper of Cheng, et al., it explores an interesting way to use a general equilibrium model to describe a dual structure with underemployment in a transitional period of economic development.

JEL: F11

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1. Introduction

This note considers a dual structure of division of labor and trade in the model of Cheng, Sachs, and Yang (2000) that is missed by the authors. While the inclusion of the dual structure will not change major results in their paper, it explores an interesting way to use a general equilibrium model to describe a dual structure with underemployment in a transitional period of economic development.

2. The model setting

The specifications of the current model are the same as in Cheng, et al. Consider a world consisting of country 1 and country 2. In country i , a continuum of consumer-producers has mass M_i ($i=1, 2$). The individuals within a country are assumed to be identical. Hence, two countries may be considered as being comprised of two groups of *ex ante* different individuals. As consumer-producers, the individuals consume two goods, x and y , and decide their own configurations of production and trade activities.

The utility function for individuals of group i or in country i is

$$(1) \quad U_i = (x_i + kx_i^d)^\beta (y_i + ky_i^d)^{1-\beta}$$

Where x_i, y_i are quantities of goods x and y self-provided, x_i^d, y_i^d are quantities of the goods bought from the market, and k is the trading efficiency coefficient. Assume that the production functions for a consumer-producer in country i are

$$(2) \quad x^p \equiv x_i + x_i^s = a_{ix} l_{ix} \text{ and } y^p \equiv y_i + y_i^s = a_{iy} l_{iy}$$

where x^p and y^p are respective output levels of the two goods produced by a person in country i and l_{ij} is a type i -individual's amount of labor allocated in producing good j or level of specialization of a type i -individual in producing good j . a_{ij} is a type i -individual's labor productivity in producing good j . The labor endowment constraint for a person in country i is $l_{ix} + l_{iy} =$

1. Let country 1 have a comparative advantage in producing good x , thus

$$(3) \quad \frac{a_{1x}}{a_{1y}} > \frac{a_{2x}}{a_{2y}}$$

3. Cheng et al's method to solve the model

In Cheng et al (2000), they consider four structures in the model, all these structures and configurations included in these structures are showed in Figure 1.

Configuration A_i (autarky) is shown in Figure 1. This configuration is defined by $x_i, y_i > 0, x_i^s = x_i^d = y_i^s = y_i^d = 0, i = 1, 2$. Structure A is that all individuals choose configuration A.

Configuration of partial specialization in the comparative advantage good, denoted by $(xy/y)_1$ and $(xy/x)_2$, is shown in Figure 1, Configuration $(xy/y)_1$ is relevant for individuals in country 1 and is defined by $x_1, y_1, x_1^s, y_1^d > 0, x_1^d = y_1^s = 0$; configuration $(xy/x)_2$ is relevant to individuals in country 2 and is defined by $x_2, y_2, x_2^d, y_2^s > 0, x_2^s = y_2^d = 0$.

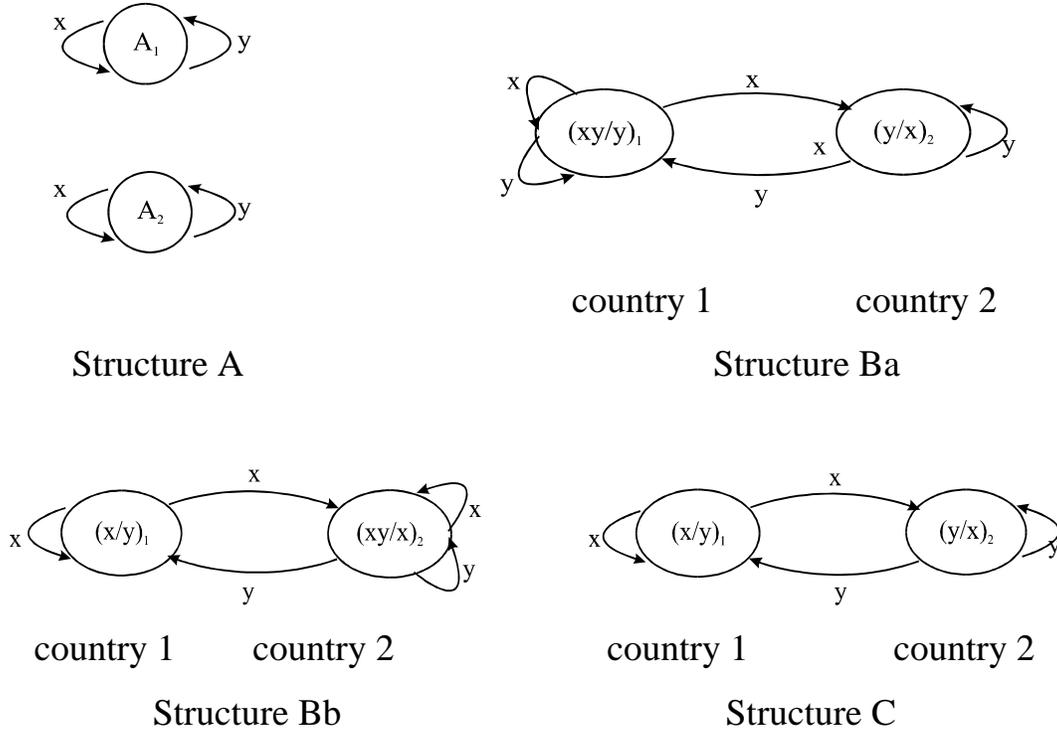


Figure 1: Configurations and Structures

Configuration of complete specialization in the comparative advantage good, denoted by $(x/y)_1$ and $(y/x)_2$ is shown in Figure 1. Configuration $(x/y)_1$ is defined by $x_1, x_1^s, y_1^d > 0, x_1^d = y_1 = y_1^s = 0$; configuration $(y/x)_2$ is defined by $y_2, y_2^s, x_2^d > 0, y_2^d = x_2 = x_2^s = 0$.

The general equilibrium of the world economy is defined as a resource allocation and a structure of trade network that satisfies (1) the requirement that each individual maximizes utility at a given set of prices with respect to configurations and quantities of production, trade, and consumption and (2) the set of prices clears the market. The individuals make their utility maximization decisions based on the infra-marginal analysis. There is a partial or corner equilibrium for a given structure and the general equilibrium is one of the four corner equilibria.

Table 1 gives all information about the four corner equilibria. Table 2 gives the general equilibrium and its inframarginal analysis.

Table1: Four Corner Equilibria in the Ricardo Model

Structures	Relative price (p_x/p_y)	Relevant Parameter Interval	Per Capita Real Income (Utility)	
			Country 1	Country 2
A	N.A.		$U_1(A) = (\beta a_{1x})^\beta$ $[(1-\beta)a_{1y}]^{1-\beta}$	$U_2(A) = (\beta a_{2x})^\beta$ $[(1-\beta)a_{2y}]^{1-\beta}$
Ba	a_{1y}/ka_{1x}	$k < k_1 \equiv$ $M_1 a_{1y}(1-\beta)/\beta M_2 a_{2y}$ < 1	$U_1(A)$	$U_2(A)$ $(k^2 a_{2y} a_{1x}/a_{2x} a_{1y})^\beta$
Bb	ka_{2y}/a_{2x}	$k < k_2 \equiv$ $M_2 a_{2x} \beta / (1-\beta) M_1 a_{1x}$ < 1	$U_1(A)$ $(k^2 a_{2y} a_{1x}/a_{2x} a_{1y})^{1-\beta}$	$U_2(A)$
C	$\beta M_2 a_{2y}$ $\div M_1 a_{1x}(1-\beta)$		$U_1(A) [\beta k M_2 a_{2y}$ $\div M_1 a_{1y}(1-\beta)]^{1-\beta}$	$U_2(A) [(1-\beta)k M_1 a_{1x} \div M_2 a_{2x} \beta]^\beta$

Table2: General Equilibrium and Its Inframarginal Comparative Statics of the Ricardo Model

Parameter		$k > k_0$			
Intervals	$k < k_0$	$M_1/M_2 >$		$M_1/M_2 <$	
		$(a_{2x}a_{2y}/a_{1x}a_{1y})^{0.5}\beta/(1-\beta),$		$(a_{2x}a_{2y}/a_{1x}a_{1y})^{0.5}\beta/(1-\beta),$	
		$k \in (k_0, k_1)$	$k \in (k_1, 1)$	$k \in (k_0, k_2)$	$k \in (k_2, 1)$
Equilibrium Structure	A	Ba	C	Bb	C

where $k_0 \equiv (a_{2x}a_{1y}/a_{1x}a_{2y})^{0.5}$, $k_1 \equiv (1-\beta)a_{1y}M_1/\beta a_{2y}M_2$, $k_2 \equiv \beta a_{2x}M_2/(1-\beta)a_{1x}M_1$.

Using table 2, Cheng et al derive the following proposition:

Proposition 1: The general equilibrium structure is determined by the two countries' relative productivity, relative preferences, relative population size and the level of trading efficiency. Given other parameters, improvements in trading efficiency can make the general equilibrium structure jump from autarky to partial division of labor and then to complete division of labor. For given transaction conditions, relative population size, and relative tastes for the two goods, the greater the degree of comparative advantage, the more likely that the equilibrium level of division of labor will be higher. For given transaction conditions, the more that the relative population size is in balance with relative tastes and relative productivity, the more likely it will be that the equilibrium level of division of labor is higher. As the equilibrium level of division of labor increases, the equilibrium aggregate productivity for society as a whole increases. In the process of moving to a

high level of division of labor, a country may receive more gains from trade even if its terms of trade deteriorate.

4. The Correct Solution to the General Equilibrium and Its Inframarginal Comparative Statics

Cheng, et al. omit two possible dual structures in the transitional period from autarky to the complete division of labor. One of them is structure PC, as shown in Fig 2(a), in which the population in country 1 is divided between configurations A and (x/y) and all individuals in country 2 choose configuration (y/x). There is a dual structure in country 1 in the sense that ex ante identical individuals have different ex post productivity and commercialized income. Those who choose autarky cannot find a job to work in the market (commercialized income is 0). Their labor productivity in producing x is lower than those who specialize in producing x and trade with foreigners. They appear to be in the unemployment. We can consider them to be underemployed in the sense that they cannot find a job in the market. Those x specialists have much higher commercialized income than their identical fellow citizens, though per capita real income (utility) is the same between two types of individuals. Another structure CP, as shown in Fig 2(b), is symmetric to PC.

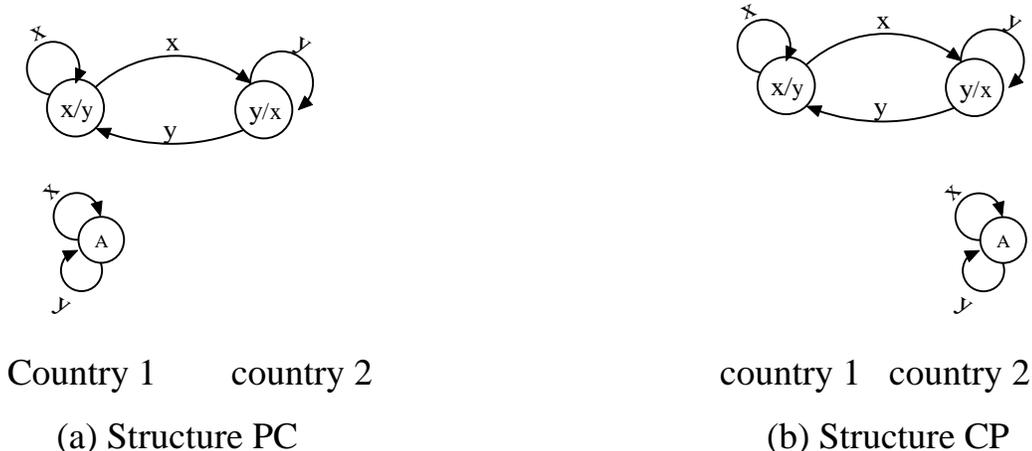


Figure 2: Dual Structures

Table 3 gives all information of the corner equilibria in structures PC and CP. Following inframarginal analysis proposed by Cheng, et al. and counting the two missed structures, we can solve the general equilibrium and its inframarginal comparative statics as shown in Table 4. First, we consider a given structure, for instance structure PC. We can compare values of indirect utility function for those individuals in country 1 who choose configuration (x/y) with their utility values for configurations (y/x), (xy/x), and A under the corner equilibrium relative price of x to y in structure PC, which is given in Table 3. The corner equilibrium in PC is a general equilibrium only if all individuals have no incentive to deviate from their chosen configurations under the corner equilibrium relative price. This condition requires that their utility value for (x/y) is not lower than that for configurations (y/x), (xy/x), and A under the corner equilibrium relative price in PC. This gives three semi-inequalities. Similarly, the condition under which individuals choosing A will not deviate from the chosen configuration under the corner equilibrium relative price in PC will give three more semi-inequalities with

only parameters. Finally, the condition under which individuals choosing (y/x) in country 2 will not deviate from the chosen configuration under the corner equilibrium relative price in PC will give three more semi-inequalities with only parameters. All the semi-inequalities can be used to identify a parameter subspace within which nobody has incentive to deviate from their chosen configurations in structure PC under the corner equilibrium relative price in this structure. In other words, within this parameter subspace the corner equilibrium in PC is a general equilibrium. We can conduct inframarginal analysis for each structure. All information of the inframarginal analysis is summarized in Table 4.

Table3: The Corner Equilibria in the Two Dual Structures

Structures	Relative price (p_x/p_y)	Numbers of individuals choosing various configurations	Per Capita Real Income (Utility)	
			Country 1	Country 2
PC	a_{1y}/ka_{1x}	$M_{1x} = \frac{\beta a_{2y}k}{(1-\beta)a_{1y}}M_2$ $M_{1A} = M_1 - \frac{\beta a_{2y}k}{(1-\beta)a_{1y}}M_2$ $M_{2y} = M_2$	$U_1(A)$	$U_2(A)$ $(k^2 a_{2y} a_{1x} / a_{2x} a_{1y})^\beta$
CP	ka_{2y}/a_{2x}	$M_{2y} = \frac{(1-\beta)a_{1x}k}{\beta a_{2x}}M_2$ $M_{2A} = M_2 - \frac{(1-\beta)a_{1x}k}{\beta a_{2x}}M_2$ $M_{1x} = M_1$	$U_1(A)$ $(k^2 a_{2y} a_{1x} / a_{2x} a_{1y})^{1-\beta}$	$U_2(A)$

Table 4: Inframarginal Comparative Statics of General Equilibrium

Parameter		$k > k_0$			
Intervals	$k < k_0$	$M_1/M_2 >$		$M_1/M_2 <$	
		$(a_{2x}a_{2y}/a_{1x}a_{1y})^{0.5}\beta/(1-\beta),$		$(a_{2x}a_{2y}/a_{1x}a_{1y})^{0.5}\beta/(1-\beta),$	
		$k \in (k_0, k_1)$	$k \in (k_1, 1)$	$k \in (k_0, k_2)$	$k \in (k_2, 1)$
Equilibrium Structure	A	PC, Ba	C	CP, Bb	C

where $k_0 \equiv (a_{2x}a_{1y}/a_{1x}a_{2y})^{0.5}$, $k_1 \equiv (1-\beta)a_{1y}M_1/\beta a_{2y}M_2$, $k_2 \equiv \beta a_{2x}M_2/(1-\beta)a_{1x}M_1$.

Note that structures PC and CP involve complete specialization in one country and coexistence of autarky and complete specialization in the other. The country with coexistence of autarky and complete specialization in this structure looks underdeveloped in the sense that it receives none of gains from trade. Also, ex ante identical individuals in the less developed country in this structure are divided between a professional occupation that trades with the foreign country and those who are self-sufficient and not involved in commercialized production. These self-sufficient individuals look like they are underemployed since they cannot find a job in the market. We call such structures as dual structures. The country with complete specialization obtains all of the gains from trade. The other country has a domestic dual structure between the commercialized sector and self-sufficient sector (autarky) which looks like underemployment.

Using table 4, we have following Proposition 2:

Proposition 2: As transaction efficiency increases from a very low to a very high level, the equilibrium level of domestic and international division of labor increases from complete autarky in both countries to the complete division of labor in both countries. In the transitional stage, a dual structure may occur. In this dual structure, the country with complete specialization obtains all of the gains from trade, while the other country has a domestic dual structure between the commercialized sector and self-sufficient sector (autarky) which looks like underemployment. The dual structures disappear as individuals in all countries are involved in international division of labor.

5. Conclusion

It is interesting to note that in the transitional period from autarky to the complete division of labor (structure C), multiple equilibria may occur. Cheng, et al. consider only structures Ba and Bb. They miss dual structures PC and CP and possible multiple equilibria. If we follow Yang and Zhang (1999) to assume that individuals in each country are divided between the two groups and individuals of one group have infinitesimally lower transaction efficiency coefficient than those of the other group, then the corner equilibria in structures Ba and Bb will not occur in equilibrium. The corner equilibrium in a dual structure will be the general equilibrium in the transitional stage from autarky to the complete division of labor. This provides an interesting way to describe the dual structure in economic development using the notion of general equilibrium. This general equilibrium approach to analyzing dual structures in economic development is not only more convincing than neoclassical models of dual structure based on ad hoc disequilibrium in labor market (Lewis, 1955 and Fei and Ranis,

1964), but it is also a nice way to formalize Lewis's original idea (Lewis, 1955) that dual the structure is between commercialized and autarkical sectors rather than between industrial and traditional agricultural sectors.

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