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ABSTRACT

Let $\{Z_{jt} : j = 1, 2 \ldots k,\ t = 0\ \pm 1,\ \pm 2, \ldots\ldots\}$ be k independent stationary processes, with spectral density functions $S_{Zj}(w)$, $j = 1.2 \ldots k$. In many real world situations there is a need to compare two or more spectra. Tests to compare spectra already exist in the literature. In this paper we propose a test, based on Bartlett's modification of the likelihood ratio criterion, for comparing two or more spectra. Simulation studies show that for k=2 this test is comparable and in some cases better than existing test procedures. The performance of this test for k=3 is also assessed.

1. INTRODUCTION

In many applications there is a need to compare two or more stationary processes. In the frequency domain, one way of comparing these processes is to compare their spectra. Given a stationary process Z_t, the spectral density function is defined by

$$S_Z(w) = \frac{1}{\pi} \sum_{k=-\infty}^{\infty} \gamma(k) e^{-iwk} \tag{1.1}$$

where $\gamma(k)$ is the covariance function of Z_t and w, the frequency, is in the range $(0,\pi)$. Hence the spectrum is the Fourier transform of the covariance function. Since $\gamma(k)$ is an even function (1.1) is often written in the equivalent form

$$S_Z(w) = \frac{1}{\pi} \left[\gamma(0) + 2\sum_{k=1}^{\infty} \gamma(k)\cos(wk) \right] \tag{1.2}$$

Tests of hypotheses for the comparison of spectra have been proposed by various authors including Jones et al. (1972), De Souza and Thompson (1982), Shumway (1982), Swanepoel and Van Wyk (1986), Coates and Diggle (1986), Diggle and Fisher (1991). In this paper we propose a new test statistic based on the Bartlett's modification of the likelihood ratio criterion.

Given k independent stationary processes, the hypotheses to be tested are:

$$H_0: S_{Z_1}(w) = S_{Z_2}(w) = \dots\dots\dots\dots = S_{Z_k}(w) \qquad 0 < w < \pi$$

H_1: At least two spectra are significantly different from each other.

Using Monte Carlo simulations, we will show that for $k=2$, this test's performance, based on our proposed test statistic, in terms of size and power is comparable and in some cases better than other existing tests in the literature. Again using Monte Carlo simulations, it will also be shown that the test based on our test statistic performs reasonably well for $k = 3$.

In section 2 we summarise the basic theory for estimating $S_Z(w)$. In section 3 we briefly discuss some existing procedures for comparing spectra. In section 4 we present our test and in section 5 we discuss the results of Monte Carlo simulations and compare some of these results with results obtained by Coates and Diggle (1988) and Diggle and Fisher (1991). In section 6 we discuss some applications to real data.

2. ESTIMATORS AND ASSOCIATED DISTRIBUTIONS

Let $z_1, z_2, \dots\dots z_n$ be a stationary series and let λ_k be a data window such that it slopes down to zero on both sides. Then the spectral ordinates may be calculated for each frequency $w_p = \dfrac{2\pi p}{n}$, $p = 0, 1, \dots\dots n$, from the smoothed periodogram

$$\hat{S}_Z(w_p) = \frac{1}{\pi}\left(\lambda_0 c_0 + 2\sum_{k=1}^{M} \lambda_k c_k \cos(w_p k)\right) \tag{2.1}$$

where M is called the truncation point. It is known that applying an appropriate data window λ_k converts $\hat{S}_Z(w_p)$ in (2.1) into a consistent estimator of $S_Z(w_p)$. Jenkins (1963) has shown that $\dfrac{\nu \hat{S}_Z(w_p)}{S_Z(w_p)}$ approximately follows a χ^2 distribution with ν degrees of freedom, where ν is given by

$$\nu = \frac{2n}{\sum_{k=-M}^{M} \lambda_k^2} \tag{2.2}$$

In order to use (2.1) a suitable truncation point $M < n$ must be chosen.

Another consistent estimate as discussed in Swanepoel and Van Wyk (1986) can be obtained by averaging the spectral ordinates computed from m partitions each of length L For each partition the power spectra are computed using (2.1) with M replaced by n-1. Let $\hat{S}_{Z_j}(w_p)$ be the spectral ordinate at frequency w_p of the j th partition, j = 1,2,....m. Then the consistent estimator of $S_Z(w_p)$ for each

$$w_p = \frac{2\pi p}{L}, \qquad p = 0, 1, \ldots\ldots, \left[\frac{L}{2}\right],$$

where L is even, is given by

$$\hat{S}_Z(w_p) = m^{-1}\sum_{j=1}^{m}\hat{S}_{Z_j}(w_p) \tag{2.3}$$

Hence $\nu\frac{\hat{S}_Z(w_p)}{S_Z(w_p)}$ approximately follows a χ^2distribution with ν degrees of freedom where ν is calculated from (2.2) with M replaced by n-1.

If the observations come from a linear process then under fairly general conditions, it has been shown by several authors including Anderson (1971), Brillinger and Rosenblatt (1967), Hannan and Quinn (1970), Lomnicki and Zaremba (1957), that $\hat{S}_{Z_j}(w_p)$ follows asymptotically a normal distribution with

$$E\left[\hat{S}_Z(w_p)\right] \approx S_Z(w_p) \tag{2.4}$$

$$\mathrm{Var}\left[\hat{S}_Z(w_p)\right] \approx S_Z^2(w_p)\frac{\sum_{k=-(n-1)}^{n-1}\lambda_k^2}{n} \tag{2.5}$$

From (2.4 and (2.5) it can be seen that the asymptotic mean and variance are proportional to $S_Z(w_p)$ and $S_Z^2(w_p)$ respectively. Hence this suggests that a logarithmic transformation will, perhaps, stabilise the variance. It has been shown in Priestly (1981) that

$$E\left[\ln\hat{S}_Z(w_p)\right] \approx \ln S(w_p) \tag{2.6}$$

$$\mathrm{Var}\left[\ln\hat{S}_Z(w_p)\right] \approx \frac{\sum_{k=-(n-1)}^{n-1}\lambda_k^2}{N} \tag{2.7}$$

From (2.2) and (2,7) it follows that

$$\operatorname{Var}\left[\ln \hat{S}(w_p)\right] \approx 2/v \tag{2.8}$$

which is independent of w_p, $p = 0, 1, \ldots, n$.

3. SOME EXISTING TESTS FOR THE COMPARISON OF POWER SPECTRA

A conventional parametric method of testing for the equality of two spectra as discussed in Swanepoel and Van Wyk (1986) is as follows:

Let there be two stationary series $z_{11}, z_{12} \ldots\ldots z_{1n_1}$ and $z_{21}, z_{22}, \ldots\ldots z_{2n_2}$, whose generating processes have spectral densities $S_{Z_1}(w)$ and $S_{Z_2}(w)$ respectively. The test of hypotheses is

$$\begin{aligned} &H_0: \; S_{Z_1}(w) = S_{Z_2}(w) \qquad 0 \le w \le \pi \\ &\text{vs} \\ &H_1: \; S_{Z_1}(w) \ne S_{Z_2}(w) \end{aligned} \tag{3.1}$$

Let $\hat{S}_{Z_1}(w)$ and $\hat{S}_{Z_2}(w)$ be the estimators of $S_{Z_1}(w)$ and $S_{Z_2}(w)$ respectively as defined in (2.3). Since $\ln \hat{S}_{Z_i}(w)$ (i = 1,2) follows approximately a normal distribution with mean $\ln S_{Z_i}(w)$ and variance $2/v_i$, it follows that $\ln\left\{\frac{\hat{S}_{Z_1}(w)}{S_{Z_2}(w)}\right\}$ is approximately normally distributed with mean 0 and variance $2\left(\frac{1}{v_1}+\frac{1}{v_2}\right)$. Hence it follows that

$$X = \left\{\frac{2}{v_1}+\frac{2}{v_2}\right\}^{-1} \sum_{p=0}^{\left[\frac{L}{2}\right]} \left\{\ln \frac{\hat{S}_{Z_1}(w_p)}{\hat{S}_{Z_2}(w_p)}\right\}^2 \tag{3.2}$$

follows a χ^2 distribution with $\eta = \left[\frac{L}{2}\right]+1$ degrees of freedom.

In his discussion on discriminant analysis of time series, Shumway (1982) uses as a test statistic to test for pattern differences between spectra of two groups, the ratio of spectral estimators at each frequency in the range $(0, \pi)$, from the two groups. Under the null hypothesis of no difference, the test statistic follows an F-distribution with $2Ln_1$ and $2Ln_2$ degrees of freedom. n_1+1 and n_2+1 are the number of times series in each of the groups and L is the number of frequencies over which smoothing is introduced in order to obtain consistent estimators of the spectra.

To test (3.1), Swanepoel and Van Wyk (1986) have suggested three non parametric tests based on a Kolmogorov-Smirnov type statistic, a χ^2 statistic and a Kullback Leibler type statistic, respectively, for comparing two spectra. The bootstrap method is applied to obtain estimates of size and power. They have shown that estimates of size are consistent with the stated significance level and that the bootstrap procedure has higher power for all their test statistics than when the conventional procedure based on (3.2) is used.

To test (3.1), Coates and Diggle (1986) consider non parametric tests analogous to the maximum periodogram ordinate and cumulative periodogram tests for white noise, and a parametric test which is a likelihood ratio test based on a postulated linear model and a postulated quadratic model for the log spectral ratio. They show that the parametric approach gives a test that is at least as powerful and sometimes considerably more powerful than the non parametric tests. However the parametric test based on the quadratic model has higher power that that based on the linear model.

To test (3.1), Diggle and Fisher (1991) consider two nonparametric tests based on the Kolmogorov-Smirnov and Cramer-von Mises statistics, respectively. Estimates of size reveal that the test based on the Cramer-von Mises statistic is generally more conservative than that based on the Kolmogorov-Smirnov statistic. Estimates of power reveal that the test based on the Kolmogorov-Smirnov statistic has generally better power than the nonparametric and parametric tests considered by Coates and Dingle (1986).

4. "HOMOGENEITY OF VARIANCE TEST" FOR THE EQUALITY OF TWO OR MORE SPECTRA

In this section, we consider the Bartlett modification of the likelihood ratio test for homogeneity of variance and we obtain a test statistic based on smoothed averaged spectral estimators.

Let $\{z_{jt}, j = 1,2,....k., t = 1,2,....n\}$ be k independent stationary time series and let Z_1, $Z_2,...,Z_k$ be their respective generating processes and $S_{Z_1}(w), S_{Z_2}(w),,........,S_{Z_k}(w)$ be their corresponding spectra. We wish to test

$$H_0 : S_{Z_1}(w) = S_{Z_2}(w) = ...= S_{Z_k}(w) \qquad 0 \le w \le \pi$$

vs (4.1)

H_1 : At least two spectra are significantly different from each other.

Assume that each series has the same number of observations. Partition each series into an equal number of parts, say m, each with length L.

Let $\hat{S}_{Z_{ij}}(w_p)$ denote the estimate of the spectrum $S_{Z_{ij}}(w_p)$ of the i th partition of the j th process at frequency w_p, where i = 1,2,....m, j = 1,2,......k and $p = 1, 2, ..., \left[\frac{L}{2}\right]$. Then

$$\hat{S}_{Z_j}(w_p) = \frac{1}{m}\sum_{i=1}^{m}\hat{S}_{Z_{ij}}(w_p) \tag{4.2}$$

is a consistent estimator of $S_{Z_j}(w_p)$, j = 1, 2,.......k

It follows that under H_o

$$v\frac{\hat{S}_{Z_j}(w_p)}{S_{Z_j}(w_p)} \sim \chi^2_v \tag{4.3}$$

where v is defined as in. (2.2).

Let

$$S_j^2 = \nu_j \frac{\hat{S}_{Z_j}(w_p)}{S_{Z_j}(w_p)} \qquad j = 1,2,\ldots k \tag{4.4}$$

Then

$$S^2 = \frac{1}{k}\sum_{j=1}^{k} S_j^2 \quad \sim \quad \chi^2_{k\nu} \tag{4.5}$$

where $\nu_1 = \nu_2 = \ldots = \nu_k = \nu$.

Then by the likelihood ratio criterion and using Bartlett's result

$$Q^{(\nu)} = C^{-1}\left[k\nu \ln S^2 - \nu\sum_{j=1}^{k} \ln S_j^2\right] \tag{4.6}$$

is distributed approximately as chi square with (k-1) degrees of freedom, where

$$C = \frac{1+\left[\frac{k}{\nu} - \frac{1}{k\nu}\right]}{3(k-1)} \tag{4.7}$$

Hence

$$Q = \sum_{p=0}^{\left[\frac{L}{2}\right]} Q^{(\nu)} \quad \sim \quad \chi^2_{\eta} \tag{4.8}$$

where $\eta = \left(\left[L/2\right] + 1\right)(k-1)$.

Now

$$
\begin{aligned}
& k\nu \ln S^2 - \nu\sum_{j=1}^{k}\ln S_j^2 \\
= \quad & k\nu\ln\left\{\frac{1}{k}\sum_{j=1}^{k}\nu\frac{\hat{S}_{Z_j}\left(w_p\right)}{S_{Z_j}\left(w_p\right)}\right\} - \nu\sum_{j=1}^{k}\ln\left\{\nu\frac{\hat{S}_{Z_j}\left(w_p\right)}{S_{Z_j}\left(w_p\right)}\right\} \\
= \quad & \nu\left[\ln\left\{\left(\frac{1}{k}\right)^k\frac{\nu^k}{\left(S_Z\left(w_p\right)\right)^k}\left(\sum_{j=1}^{k}\hat{S}_{Z_j}\left(w_p\right)\right)^k\right\} - \ln\left\{\frac{\nu^k}{\left(S_Z\left(w_p\right)\right)^k}\prod_{j=1}^{k}\hat{S}_{Z_j}\left(w_p\right)\right\}\right] \\
= \quad & \nu\left[k\ln\left\{\frac{1}{k}\sum_{j=1}^{k}\hat{S}_{Z_j}\left(w_p\right)\right\} - \ln\left\{\prod_{j=1}^{k}\hat{S}_{Z_j}\left(w_p\right)\right\}\right] \\
= \quad & k\nu\ln\left\{\frac{1}{k}\sum_{j=1}^{k}\hat{S}_{Z_j}\left(w_p\right)\right\} - \nu\sum_{j=1}^{k}\ln\left\{\hat{S}_{Z_j}\left(w_p\right)\right\}
\end{aligned}
$$

Hence the test statistic in (4.8) becomes

$$
Q = \sum_{p=0}^{\left[\frac{L}{2}\right]} C^{-1}\left[\nu k\ln\left(\frac{1}{k}\sum_{j=1}^{k}\hat{S}_{Z_j}\left(w_p\right)\right) - \nu\sum_{j=1}^{k}\ln\left(\hat{S}_{Z_j}\left(w_p\right)\right)\right] \tag{4.9}
$$

where C is defined as in (4.7).

5. SIMULATION STUDY

5.1 Outline

For k=2, series of lengths n = 64, 256, 1024 were simulated from AR(1) processes

$$X_t = \phi X_{t-1} + a_t$$

and MA(1) processes

$$X_t = a_t - \theta a_{t-1}$$

where in each case a_t is a Gaussian white noise process. The series were partitioned into m = 2, 3, 4 parts for length 64, m = 4, 8, 16 parts for length 256 and m = 16, 20, 24 parts for length 1024. The series length n was adjusted as necessary to ensure that n/m became a whole number.

Distributional properties of the proposed test, based on Q defined by equation (4.9), were checked by obtaining estimates of the mean, variance and skewness. Estimates of size were obtained by applying the test to simultaneous pairs of AR(1) processes for

$\phi = 0, 0.1, 0.5, 0.9$

and MA(1) processes for

$\theta = 0.1, 0.5, 0.9$

This was done for the various values of n and m.

For the various values of m, estimates of power were obtained by applying the test to simultaneous of pairs of AR(1) processes,

for $n = 64$

$\phi = 0$ vs 0.2, 0.4, 0.6, 0.8
$\phi = 0.5$ vs 0.1, 0.3, 0.7, 0.9;

for $n = 256$

$\phi = 0$ vs 0.1, 0.2, 0.3, 0.4, 0.5, 0.6
$\phi = 0.5$ vs 0.1, 0.2, 0.3, 0.4, 0.6, 0.7, 0.8, 0.9;

for $n = 1024$,

$\phi = 0$ vs 0.05, 0,1 0.15,0.2 0.25
$\phi = 0.5$ vs 0.35, 0.4, 0.45, 0.55, 0.6, 0.65.

For comparison purposes estimates of power and size were also obtained for the test based on the statistic X defined in equation (3.2). Some of the results of the test based on Q were also compared to results, for the corresponding sample sizes, obtained by Coates and Diggle (1986) and Diggle and Fisher (1991).

Based on the estimates of size, a criterion was obtained to determine a range of suitable values of m for a particular value of n. Using this criterion, a value of m was selected for series of length 198 and 492 for $k = 2$, and for series of length 252 for $k = 3$. Estimates of mean, variance, skewness, size and power were obtained.

Each Monte Carlo test used 1000 randomizations. The choice of n and the AR and MA parameters were made for easy comparison with the results of Coates and Diggle (1986) and Diggle and Fisher (1991). In all cases a Parzen window was used for smoothing.

5.2 Distributional Properties
Theoretical means, variances and measures of skewness for the corresponding degrees of freedom are shown in table 1. Estimates of mean, variance and a measure of skewness for the various values of m and n for the test based on Q are shown in tables 2 (a) to (c). In most cases the means are slightly underestimated but reasonably close to the theoretical means. However the variances tend to be overestimated especially for very strong autoregressive dependence. Most of the estimated measures of skewness are slightly larger than the corresponding theoretical measures.

5.3 Size Estimates
Estimates of size for 10%, 5% and 1% significance levels together with their 95% confidence intervals for tests bases on Q and X are shown in tables 3 (a) to (c). The first line in each cell of these tables are the estimates of size and the second and third lines are the lower and upper 95% confidence interval estimates, respectively. For the test based on Q, for n=64, 256 and 1024 reasonably good estimates of size are achieved for m=2,3 partitions, m= 6,8 partitions and m=20,24 partitions respectively. However in all cases, size was overestimated as the AR(1) and MA(1) parameters tended to their upper limits. In all cases estimates of size for the test based on X are larger that those for the test based on Q since X is always larger than Q. The estimates of size based on Q compare favourably with those based of the two non-parametric tests shown in table 1 of Diggle and Fisher (1991). When prewhitening of the spectra was attempted to try and improve size estimates for series with strong autocorrelation dependence, it turned out in all cases that size was considerably underestimated. Hence no further attempts were made to improve the size estimates.

5.4 Power Estimates
Estimates of power for the test based on Q and on X , for the 10%, 5% and 1% significance levels for n=64, m=2,3, for n= 252, m=6,8 and for n=1024, m=20,24 are shown in tables 5 (a) to (f). In all cases the test based on X has slightly higher probability of rejecting H_0 when H_1 is true. This is due to the fact that X is always larger than Q and hence always has larger size associated with it than that the test based on Q. By comparison with results for corresponding parameter values and series lengths, in tables 2 and 3 in Diggle and Fisher (1991), the test based on Q has in most cases comparable power. In some cases the test based on Q, has slightly higher power and in some cases slightly lower power than the tests in Diggle and Fisher (1991). By comparison with the results in tables 2a, 2c, 3a and 3c, the test based on Q tends to generally perform much better than those non parametric tests and parametric tests in Coates and Diggle (1986). For example for n=64, AR(1) $\phi = 0.5$, AR(1) $\phi = 0.9$, 10% significance level, the power of the parametric test in Coates and Diggle (1986) for the linear and quadratic models are both 0.48, whereas the power of the test based on Q is 0.75 for 2 partitions and 0.83 for 3 partitions.

5.5 Criteria for Selecting suitable values of m

As mentioned in 5.3, for the values of n below, reasonably good estimates of size were obtained for the values of corresponding values of m . The corresponding degrees of freedom are obtained by [((n/2m)+1)(k-1)].

n	m		degrees of freedom	
64	2	3	17	11
256	6	8	22	17
1024	20	24	26	22

Based on the fact that there are between 4 and 6 degrees of freedom between the lower and upper limits for these values of n, the following criteria were developed and can be used as a guide to select suitable values of m.

$$10+\frac{5(n-64)}{(256-64)} \le \frac{n}{2m} \le 16+\frac{5(n-64)}{(256-64)}, \qquad 64 \le n \le 256$$

$$16+\frac{5(n-256)}{(1024-256)} \le \frac{n}{2m} \le 21+\frac{5(n-256)}{(1024-256)}, \qquad 256 \le n \le 1024$$

Since the square roots of the 64, 256, and 1024 are multiples of 8, these criteria can easily be extended for larger values of n if necessary. Also, since the degrees of freedom is a function of (k-1), these criteria can also be used as a guide to select m for values of k > 2.

For k = 2, the test based on Q and X were applied to series of lengths 198 and 492. Using the above criteria the range of suitable m values was found to be between 5 and 7 for n = 198, and between 11 and 14 for n = 492. Value of m=6 for n=198 and m=12 for n =492 were selected to obtain the estimates of the mean, variance, measure of skewness, size and power. These estimates are shown in tables 2(d), 2(e), 3(d), 3(e), 5(g) to 5(j). For the test based on Q, as before , in most cases the means were slightly underestimated but reasonably close to the theoretical means. However the variances tend to be overestimated especially for very strong autoregressive dependence. Most of the estimated measures of skewness were slightly larger than the corresponding theoretical measures. Reasonably good estimates of size were obtained were obtained for both Q and X except for when the AR(1) and MA(1) parameters tended to their upper limits. Reasonably good estimates of power were obtained.

For k = 3, the test based on Q was applied to series of length 252. Using the above criteria the range of suitable m values was found to be between 6 and 8. The number of partitions m= 6 was selected to obtain estimates of the mean, variance, measure of skewness size and power. These estimates are shown in tables 2(f), 4 and 6. In all cases the means were slightly overestimated but reasonably close to the theoretical means. However the variances tend to be overestimated and more so for very strong autoregressive dependence. Again most of the estimated measures of skewness were slightly larger than the corresponding theoretical measures. Reasonably good estimates of size were obtained except for when the AR(1) and MA(1) parameters tended to their upper limits. Reasonably good estimates of power were obtained.

6. APPLICATIONS

6.1 *Tree Ring Data*

In order to reconstruct climates from information from trees, one type of measurement that climatologists use are distances between the consecutive rings of trees. Figures 1, 2 and 3 show tree ring data for 3 separate sites about 10 km. apart at about the same altitude on Mount Egmont on the North Island of New Zealand. Each data set consists of standardised distances between rings, averaged over a number of trees in a particular site. Standardisation allows samples with large differences in growth rates to be combined and can be used to remove any undesired growth trends present. It is expected that there is no significant difference between growth as influenced by climate in the three different sites. Each series consisted of n=352 observations. After filtering the series, checks revealed no significant cross correlations between the series. The number of partitions to smooth the spectra i.e. the value of m, was chosen to be 8. This was done by using the selection criteria from section 5 as a guide. Figure 4 shows the smoothed spectra of the three sites and it is clear that they are quite similar. The test of hypotheses of no significant differences based on Q was applied and could not be rejected as can be seen from the results below.

Length of Each Time Series	352
Number of Partitions	8
Q Test Statistic	40.5355
Degrees of Freedom	46
p-value	0.6997

6.2 *Earthquake and Explosion Data*

It is clear from an examination of earthquake and nuclear explosion waveforms, that there are distinct differences in their patterns. It is therefore expected that there will be differences in their spectra as well. Figures 5 and 6 show the standardised waveforms of a nuclear explosion detonated in China in August 1995 and an earthquake which took place in the Solomon Islands in September 1995. The two events which were of similar strength were recorded at the same seismological station. Each series consists of 600 observations recorded over a 30 second interval. Even though the series are not quite stationary and no transformation could be found to make them stationary, the test for no significant difference in underlying spectra was nevertheless carried out. Using the criteria from section 5 as a guide, each series was partitioned into m=15 partitions and smoothed spectra were obtained. Figure 7 shows the smoothed spectra and it is clear that there are differences in the spectra. The test based on Q was applied to the spectra and was strongly rejected as can be seen from the results below.

Length of Each Time Series	600
Number of Partitions	15
Q Test Statistic	367.9952
Degrees of Freedom	21
p-value	0

7 CONCLUDING REMARKS

The results show that, even though the variation in Q is large for the various values of n and m, the distributional approximations to the chi-square distribution are reasonably adequate. The estimates of size and power of the test based of Q compare favourably with some existing tests and in some cases the test based on Q displayed superior power. Estimates of size tend to be smaller than the nominal size for a small number of partitions and larger than the nominal size for a large number of partitions.

Results for n=198 and 492 for k=2 and for n=252 for k=3 show that the criteria for the selection of m, i.e. the amount of smoothing, are a reasonably good guide for the choice of m. However it is clear that this idea needs more exploration before it can be firmly established what the optimum value of m should be.

We believe that the advantage the test based on Q has over existing tests in the literature is the ease with which it can be extended to the case for k >2. This is revealed in the results for n=252 for k=3. The results of the application of the test to real data in section 6 further strengthen our claim the Q is a reasonably good statistic for testing for significant differences between spectra.

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APPENDIX

TABLE 1
SOME PARAMETERS OF THE CHI-SQUARE DISTRIBUTION

Degrees of freedom	Mean	Variance	Median	Measure of Skewness
9	9	18	8.34	0.1556
11	11	22	10.34	0.1407
17	17	34	16.34	0.1132
21	21	42	20.34	0.1018
22	22	44	21.34	0.0995
26	26	52	25.34	0.0915
33	33	66	32.34	0.0812
44	44	88	43.34	0.0704

TABLE 2
ESTIMATES OF MEAN, VARIANCE AND A MEASURE OF SKEWNESS OF THE TEST STATISTIC Q

(a) n=64 k=2

Process	m	df	Mean	Variance	Skewness
AR(1)					
$\phi = 0$	2	17	14.8368	43.1865	0.1268
	3	11	10.0446	27.7782	0.1785
	4	9	9.1488	32.7989	0.2057
$\phi = 0.1$	2	17	15.0836	44.1320	0.1404
	3	11	10.1252	29.0984	0.2320
	4	9	9.2593	34.7796	0.2117
$\phi = 0.5$	2	17	14.7278	41.0457	0.1404
	3	11	10.0235	29.0237	0.1957
	4	9	9.3061	33.4233	0.1809
$\phi = 0.9$	2	17	15.6479	66.1720	0.1879
	3	11	12.2232	70.6033	0.2427
	4	9	9.3707	123.5592	0.3111
MA(1)					
$\theta = 0.1$	2	17	14.8842	44.4456	0.1576
	3	11	9.8477	27.0925	0.1783
	4	9	9.2421	33.8558	0.2580
$\theta = 0.5$	2	17	14.4105	37.8799	0.1433
	3	11	9.7336	27.4211	0.2305
	4	9	8.9426	29.4933	0.1696
$\theta = 0.9$	2	17	15.0960	43.9517	0.1848
	3	11	10.6215	38.9845	0.1988
	4	9	9.6598	37.5366	0.2266

(b) n=256 k=2

Process	m	df	Mean	Variance	Skewness
AR(1)					
$\phi = 0$	4	33	27.8413	73.3146	0.1001
	6	22	19.8779	58.4140	0.1277
	8	17	15.7344	49.5152	0.2206
$\phi = 0.1$	4	33	28.1639	69.8812	0.0921
	6	22	19.7541	56.7707	0.1714
	8	17	15.8571	48.0303	0.1535
$\phi = 0.5$	4	33	27.9340	72.8203	0.1226
	6	22	19.1332	52.9281	0.1627
	8	17	15.7881	46.0487	0.1763
$\phi = 0.9$	4	33	27.2947	73.6094	0.1492
	6	22	20.4012	73.3735	0.1858
	8	17	15.2492	77.8392	0.2156
MA(1)					
$\theta = 0.1$	4	33	27.9358	76.6844	0.1131
	6	22	19.6131	53.7048	0.1504
	8	17	15.5301	46.0897	0.1692
$\theta = 0.5$	4	33	27.5872	71.0168	0.1448
	6	22	19.2987	58.6445	0.1606
	8	17	15.4537	46.0844	0.1798
$\theta = 0.9$	4	33	28.8987	90.3192	0.1796
	6	22	20.4674	66.1798	0.1686
	8	17	15.2842	58.6626	0.1884

(c) n=1024 k=2

Process	m	df	Mean	Variance	Skewness
AR(1)					
$\phi = 0$	16	33	29.0872	80.6456	0.1306
	20	26	22.9297	58.0265	0.1394
	24	22	20.3582	61.3481	0.1432
$\phi = 0.1$	16	33	28.9167	81.3104	0.1665
	20	26	23.3438	66.7120	0.1250
	24	22	20.4902	59.5425	0.1344
$\phi = 0.5$	16	33	28.7496	79.3148	0.1122
	20	26	23.3963	68.9871	0.1371
	24	22	19.9210	52.5393	0.1473
$\phi = 0.9$	16	33	28.5661	90.2629	0.1476
	20	26	23.3746	83.9922	0.1748
	24	22	21.8382	103.8434	0.2017
MA(1)					
$\theta = 0.1$	16	33	29.0508	78.3870	0.1097
	20	26	23.5952	63.9143	0.1095
	24	22	20.1073	60.2599	0.1755
$\theta = 0.5$	16	33	28.8179	78.1336	0.0957
	20	26	23.5177	69.0848	0.1579
	24	22	20.2712	61.2867	0.1597
$\theta = 0.9$	16	33	29.8176	105.4583	0.1227
	20	26	24.7555	102.4055	0.1827
	24	22	22.2550	93.0493	0.1931

(d) n=198 k=2

Process	m	df	Mean	Variance	Skewness
AR(1)					
$\phi = 0$	6	17	15.3510	43.7239	0.1990
$\phi = 0.1$	6	17	15.2287	42.3803	0.1651
$\phi = 0.5$	6	17	14.8209	43.2554	0.1759
$\phi = 0.9$	6	17	16.4668	79.6309	0.2041
MA(1)					
$\theta = 0.1$	6	17	15.3501	41.1880	0.1638
$\theta = 0.5$	6	17	15.1381	41.3125	0.1492
$\theta = 0.9$	6	17	16.0646	51.0467	0.1710

(e) n=492 k =2

Process	m	df	Mean	Variance	Skewness
AR(1)					
$\phi = 0$	12	21	19.2862	57.4521	0.1790
$\phi = 0.1$	12	21	19.2904	52.3785	0.1716
$\phi = 0.5$	12	21	19.2143	55.1553	0.1128
$\phi = 0.9$	12	21	20.1103	97.8701	0.1934
MA(1)					
$\theta = 0.1$	12	21	19.4690	56.0665	0.1532
$\theta = 0.5$	12	21	19.6596	56.8382	0.1533
$\theta = 0.9$	12	21	19.8027	79.2542	0.1998

(f) n=252 k =3

Process	m	df	Mean	Variance	Skewness
AR(1)					
$\phi = 0$	6	33	39.2105	117.7417	0.1933
$\phi = 0.1$	6	33	39.6558	114.0331	0.0687
$\phi = 0.5$	6	33	38.8824	113.3108	0.0951
$\phi = 0.9$	6	33	38.7496	179.0240	0.1951
MA(1)					
$\theta = 0.1$	6	33	39.1330	102.4321	0.1038
$\theta = 0.5$	6	33	38.6913	100.0431	0.0939
$\theta = 0.9$	6	33	42.3929	141.2486	0.1249

TABLE 3
ESTIMATES OF SIZE WITH 95% CONFIDENCE INTERVALS FOR TEST STATISTICS Q AND X for k=2

(a) n=64

Process	m	df	Q			X		
			10%	5%	1%	10%	5%	1%
AR(1) $\phi = 0$	2	17	0.0740*	0.0490	0.0130	0.1330*	0.0920*	0.0420*
			0.0550	0.0352	0.0067	0.1140	0.0782	0.0357
			0.0930	0.0628	0.0193	0.1520	0.1058	0.0483
	3	11	0.0950	0.0500	0.0130	0.1310*	0.0860*	0.0360*
			0.0760	0.0362	0.0067	0.1120	0.0722	0.0297
			0.1140	0.0638	0.0193	0.1500	0.0998	0.0423
	4	9	0.1350*	0.0910*	0.0320*	0.1670*	0.1160*	0.0470*
			0.1160	0.0772	0.0257	0.1480	0.1022	0.0407
			0.1540	0.1048	0.0383	0.1860	0.1298	0.0533
$\phi = 0.1$	2	17	0.0840	0.0540	0.0160	0.1450*	0.1050*	0.0450*
			0.0650	0.0402	0.0097	0.1260	0.0912	0.0387
			0.1030	0.0678	0.0223	0.1640	0.1188	0.0513
	3	11	0.0940	0.0570	0.0180*	0.1470*	0.0780*	0.0320*
			0.0750	0.0432	0.0117	0.1280	0.0642	0.0257
			0.1130	0.0708	0.0243	0.1660	0.0918	0.0383
	4	9	0.1570*	0.1000*	0.0380*	0.1870*	0.1310*	0.0560*
			0.1380	0.0862	0.0317	0.1680	0.1170	0.0497
			0.1760	0.1138	0.0443	0.2050	0.1448	0.0623
$\phi = 0.5$	2	17	0.0710*	0.0380	0.0100	0.1310*	0.0830*	0.0300*
			0.0520	0.0242	0.0037	0.1120	0.0692	0.0237
			0.0900	0.0815	0.0163	0.1500	0.0968	0.0363
	3	11	0.0980	0.0550	0.0210*	0.1390*	0.0830*	0.0310*
			0.0790	0.0412	0.0147	0.1200	0.0692	0.0247
			0.1170	0.0688	0.0273	0.1579	0.0968	0.0373
	4	9	0.1420*	0.0860*	0.0290*	0.1760*	0.1150*	0.0430*
			0.1230	0.0722	0.0227	0.1570	0.1012	0.0367
			0.1610	0.0998	0.0353	0.1950	0.1288	0.0493
$\phi = 0.9$	2	17	0.1140	0.0710*	0.0230*	0.1910*	0.1370*	0.0610*
			0.0950	0.0572	0.0167	0.1720	0.1232	0.0547
			0.1130	0.0847	0.0293	0.2100	0.1508	0.0673
	3	11	0.1870*	0.1390*	0.0780*	0.2350*	0.1770*	0.0990*
			0.1680	0.1252	0.0717	0.2160	0.1632	0.0927
			0.2060	0.1528	0.0843	0.2540	0.1908	0.1053
	4	9	0.3070*	0.2380*	0.1520*	0.3340*	0.2640*	0.1720*
			0.2880	0.2242	0.1457	0.3150	0.2502	0.1657
			0.3260	0.2518	0.1583	0.3530	0.2778	0.1783

* Significant at the 5% level

Process	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
MA(1)								
$\theta = 0.1$	2	17	0.0840	0.0520	0.0110	0.1540*	0.1010*	0.0450*
			0.0650	0.0380	0.0047	0.1350	0.0872	0.0387
			0.1029	0.0658	0.0173	0.1729	0.1148	0.0513
	3	11	0.0860	0.0580	0.0090	0.1270*	0.0790*	0.0270*
			0.0670	0.0442	0.0027	0.1080	0.0652	0.0207
			0.1050	0.0718	0.0153	0.1460	0.0928	0.0333
	4	9	0.1490*	0.1040*	0.0390*	0.1910*	0.1310*	0.0520*
			0.1300	0.0902	0.0327	0.1720	0.1172	0.0457
			0.1680	0.1179	0.0453	0.2100	0.1448	0.0583
$\theta = 0.5$	2	17	0.0520*	0.0290*	0.0110	0.1190	0.0720*	0.0270*
			0.0330	0.0152	0.0047	0.1000	0.0583	0.0207
			0.0710	0.0428	0.0173	0.1380	0.0858	0.0333
	3	11	0.0780*	0.0510	0.0150	0.1220*	0.0740*	0.0260*
			0.0590	0.0372	0.0087	0.1030	0.0602	0.0197
			0.0970	0.0648	0.0213	0.1410	0.0878	0.0323
	4	9	0.1120*	0.0780*	0.0320*	0.1530*	0.1020*	0.0430
			0.1030	0.0642	0.0257	0.1340	0.0882	0.0367
			0.1410	0.0918	0.0383	0.1720	0.1158	0.0493
$\theta = 0.9$	2	17	0.0880*	0.0420*	0.0130	0.1590*	0.1070*	0.0330*
			0.0690	0.0282	0.0067	0.1400	0.0932	0.0267
			0.1070	0.0558	0.0193	0.1780	0.1208	0.0393
	3	11	0.1230*	0.0880*	0.0370*	0.1460*	0.1160*	0.0540*
			0.1040	0.0742	0.0307	0.1270	0.1022	0.0477
			0.1420	0.1028	0.0433	0.1650	0.1298	0.0603
	4	9	0.1670*	0.1070*	0.0520*	0.1980*	0.1350*	0.0690*
			0.1480	0.0932	0.0457	0.1790	0.1212	0.0627
			0.1860	0.1208	0.0583	0.2170	0.1488	0.0753

* Significant at the 5% level

(b) n=256

Process	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
AR(1)								
$\phi = 0$	4	33	0.0400*	0.0170*	0.0050	0.0730*	0.0340*	0.0120
			0.0210	0.0032	-0.0013	0.0540	0.0252	0.0057
			0.0590	0.0308	0.0113	0.0920	0.0528	0.0183
	6	22	0.0840	0.0540	0.0160	0.1080	0.0680*	0.0220*
			0.0650	0.0402	0.0097	0.0890	0.0542	0.0157
			0.1030	0.0678	0.0223	0.1270	0.0818	0.0283
	8	17	0.1140	0.0650*	0.0240*	0.1300*	0.0820*	0.0320*
			0.0950	0.0512	0.0177	0.1110	0.0682	0.0257
			0.1330	0.0788	0.0303	0.1490	0.0958	0.0383
$\phi = 0.1$	4	33	0.0310*	0.0200*	0.0050	0.0660*	0.0310*	0.0110
			0.0121	0.0062	-0.0013	0.0470	0.0172	0.0047
			0.0500	0.0338	0.0113	0.0850	0.0448	0.0173
	6	22	0.0880	0.0430	0.0110	0.1160	0.0660*	0.0170*
			0.0690	0.0292	0.0047	0.0970	0.0522	0.0107
			0.1070	0.0568	0.0173	0.1350	0.0798	0.0233
	8	17	0.0870	0.0480	0.0180*	0.1070	0.0570	0.0220*
			0.0680	0.0342	0.0117	0.0880	0.0432	0.0157
			0.1060	0.0618	0.0243	0.1260	0.0708	0.0283
$\phi = 0.5$	4	33	0.0540*	0.0310*	0.0080	0.0830	0.0570	0.0210*
			0.0350	0.0172	0.0017	0.0640	0.0432	0.0147
			0.0730	0.0448	0.0142	0.1020	0.0708	0.0273
	6	22	0.0720*	0.0330*	0.0120	0.0930	0.0550	0.0150
			0.0530	0.0192	0.0057	0.0740	0.0412	0.0087
			0.0910	0.0468	0.0183	0.1120	0.0688	0.0213
	8	17	0.0950	0.0570	0.0220*	0.1140	0.0690*	0.0270*
			0.0760	0.0430	0.0157	0.0950	0.0552	0.0207
			0.1140	0.0708	0.0283	0.1330	0.0828	0.0333
$\phi = 0.9$	4	33	0.0420*	0.0220*	0.0050	0.0660*	0.0410*	0.0120
			0.0230	0.0082	-0.0013	0.0470	0.0272	0.0057
			0.0610	0.0358	0.0113	0.0850	0.0548	0.0183
	6	22	0.1070	0.0690*	0.0260*	0.1320*	0.0850*	0.0370*
			0.0880	0.0552	0.0197	0.1130	0.0712	0.0307
			0.1260	0.0828	0.0323	0.1510	0.0988	0.0433
	8	17	0.1560*	0.1070*	0.0490*	0.1780*	0.1200*	0.0580*
			0.1370	0.0932	0.0427	0.1590	0.1062	0.0517
			0.1749	0.1208	0.0553	0.1970	0.1338	0.0643

* **Significant at the 5% level**

Process	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
MA(1)								
$\theta = 0.1$	4	33	0.0580*	0.0300*	0.0040	0.0870	0.0570	0.0140
			0.0390	0.0162	-0.0023	0.0680	0.0382	0.0077
			0.0770	0.0438	0.0103	0.1060	0.0658	0.0203
	6	22	0.0810	0.0400	0.0150	0.1070	0.0600	0.0210*
			0.0620	0.0262	0.0087	0.0880	0.0462	0.1047
			0.1000	0.0538	0.0213	0.1260	0.0738	0.0273
	8	17	0.1020	0.0570	0.0150	0.1200*	0.0730*	0.0200*
			0.0830	0.0432	0.0087	0.1010	0.0592	0.0137
			0.1210	0.0708	0.0213	0.1390	0.0868	0.0263
$\theta = 0.5$	4	33	0.0430*	0.0180*	0.0030	0.0800*	0.0420*	0.0130*
			0.0240	0.0042	-0.0033	0.0610	0.0282	0.0067
			0.0620	0.0318	0.0093	0.0990	0.0558	0.0193
	6	22	0.0790	0.0510	0.0130	0.1100	0.0640*	0.0250*
			0.0600	0.0372	0.0067	0.0910	0.0502	0.0187
			0.0980	0.0648	0.0193	0.1290	0.0778	0.0313
	8	17	0.0940	0.0600	0.0170*	0.1050	0.0700*	0.0200*
			0.0750	0.0462	0.0107	0.0860	0.0562	0.0137
			0.1130	0.0738	0.0233	0.1240	0.0838	0.0263
$\theta = 0.9$	4	33	0.0760	0.0420	0.0110	0.1170	0.0730*	0.0230*
			0.0570	0.0282	0.0047	0.0980	0.0592	0.0167
			0.0950	0.0558	0.0173	0.1360	0.0868	0.0292
	6	22	0.1110	0.0700*	0.0210*	0.1400*	0.0950*	0.0360*
			0.0920	0.0562	0.0147	0.1210	0.0812	0.0297
			0.1300	0.0838	0.0273	0.1590	0.1088	0.0423
	8	17	0.1350*	0.0870*	0.0350*	0.1510*	0.1020*	0.0470*
			0.1160	0.0732	0.0287	0.1320	0.0882	0.0407
			0.1540	0.1008	0.0413	0.1700	0.1159	0.0532

* Significant at the 5% level

(c) n=1024

Process	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
AR(1)								
$\phi = 0$	16	33	0.0620*	0.0290*	0.0090	0.0690*	0.0350*	0.0120
			0.0430	0.0152	0.0027	0.0560	0.0212	0.0057
			0.0810	0.0428	0.0153	0.0880	0.0488	0.0183
	20	26	0.0720*	0.0330	0.0060	0.0740*	0.0380*	0.0070
			0.0530	0.0192	-0.0003	0.0350	0.0242	0.0007
			0.0910	0.0468	0.0468	0.0930	0.0518	0.0133
	24	22	0.0870	0.0560	0.0210*	0.0920	0.0600	0.0210*
			0.0680	0.0422	0.0147	0.0730	0.0462	0.0147
			0.1060	0.0698	0.0273	0.1110	0.0738	0.0273
$\phi = 0.1$	16	33	0.0680*	0.0370	0.0110	0.0750*	0.0450*	0.0140
			0.0490	0.0232	0.0047	0.0560	0.0312	0.0077
			0.0870	0.0508	0.0173	0.0940	0.0558	0.0203
	20	26	0.0770*	0.0350*	0.0090	0.0910	0.0420	0.0100
			0.0580	0.0212	0.0027	0.0720	0.0282	0.0037
			0.0960	0.0488	0.0153	0.1100	0.0558	0.0163
	24	22	0.0980	0.0590	0.0150	0.1050	0.0640*	0.0180*
			0.0790	0.0452	0.0087	0.0860	0.0502	0.0117
			0.1170	0.0728	0.0213	0.1240	0.0778	0.0243
$\phi = 0.5$	16	33	0.0610*	0.0340*	0.0110*	0.0640*	0.0380	0.0120
			0.0420	0.0202	0.0047	0.0450	0.0242	0.0058
			0.0800	0.0478	0.0073	0.0830	0.0518	0.0183
	20	26	0.0820	0.0490	0.0160	0.0890	0.0550	0.0200*
			0.0630	0.0353	0.0097	0.0700	0.0412	0.0138
			0.1010	0.0628	0.0223	0.1080	0.0688	0.0263
	24	22	0.0800	0.0370	0.0070	0.0860	0.0480	0.0080
			0.0610	0.0232	0.0007	0.0670	0.0342	0.0017
			0.0990	0.0508	0.0133	0.1049	0.0618	0.0143
$\phi = 0.9$	16	33	0.0600*	0.0390	0.0090	0.0670*	0.0460	0.0130
			0.0410	0.0252	0.0027	0.0480	0.0322	0.0067
			0.0790	0.0528	0.0153	0.0860	0.0600	0.0193
	20	26	0.0960	0.0600	0.0290*	0.1030	0.0660*	0.0310*
			0.0770	0.0462	0.0227	0.0840	0.0522	0.0247
			0.1150	0.0738	0.0353	0.1220	0.0798	0.0373
	24	22	0.1600*	0.1060*	0.0520*	0.1700*	0.1140*	0.0560*
			0.1410	0.0922	0.0457	0.1510	0.1002	0.0498
			0.1790	0.1110	0.0583	0.1890	0.1278	0.0623

* Significant at the 5% level

Process	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
MA(1)								
$\theta = 0.1$	16	33	0.0520*	0.0330*	0.0110	0.0630*	0.0370	0.0130
			0.0330	0.0192	0.0047	0.0440	0.0232	0.0067
			0.0710	0.0468	0.0173	0.0820	0.0508	0.0193
	20	26	0.0720*	0.0370	0.0110	0.0800*	0.0450	0.0150
			0.0530	0.0232	0.0047	0.0610	0.0312	0.0087
			0.0910	0.0508	0.0129	0.0990	0.0588	0.0213
	24	22	0.0950	0.0530	0.0190*	0.1040	0.0570	0.0190*
			0.0760	0.0392	0.0127	0.0850	0.0432	0.0127
			0.1140	0.0668	0.0253	0.1230	0.0708	0.0253
$\theta = 0.5$	16	33	0.0540*	0.0260*	0.0060	0.0640*	0.0340*	0.0100
			0.0360	0.0122	-0.0003	0.0452	0.0202	0.0037
			0.0730	0.0398	0.0123	0.0830	0.0478	0.0163
	20	26	0.0910	0.0470	0.0130	0.1040	0.0600	0.0130
			0.0720	0.0332	0.0067	0.0850	0.0462	0.0067
			0.1099	0.0608	0.0193	0.1230	0.0738	0.0193
	24	22	0.1090	0.0660*	0.0180*	0.1150	0.0680*	0.0190*
			0.0900	0.0522	0.0117	0.0960	0.0542	0.0127
			0.1280	0.0798	0.0243	0.1340	0.0818	0.0253
$\theta = 0.9$	16	33	0.0870	0.0550	0.0210*	0.0970	0.0630	0.0240*
			0.0680	0.0412	0.0147	0.0780	0.0492	0.0177
			0.1060	0.0688	0.0273	0.1160	0.0768	0.0303
	20	26	0.1180	0.0820*	0.0430*	0.1260*	0.0870*	0.0480*
			0.0990	0.0682	0.0367	0.1070	0.0732	0.0447
			0.1370	0.0960	0.0493	0.1450	0.1008	0.0543
	24	22	0.1640*	0.1210*	0.0490*	0.1720*	0.1260*	0.0540*
			0.1450	0.1072	0.0427	0.1530	0.1122	0.0477
			0.1830	0.1347	0.0553	0.1910	0.1397	0.0603

* Significant at the 5% level

(d) n=198

Process	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
AR(1)								
$\phi = 0$	6	17	0.0910	0.0520	0.0170*	0.1140	0.0700*	0.0250*
			0.0720	0.0382	0.0107	0.0950	0.0562	0.0187
			0.1100	0.0658	0.0233	0.1330	0.0838	0.0313
$\phi = 0.1$	6	17	0.0810	0.0450	0.0170*	0.1090	0.0620	0.0230*
			0.0620	0.0312	0.0107	0.0900	0.0482	0.0167
			0.1000	0.0588	0.0233	0.1280	0.0758	0.0293
$\phi = 0.5$	6	17	0.0860	0.0420*	0.0160	0.1050	0.0610	0.0230*
			0.0670	0.0282	0.0097	0.0860	0.0472	0.0167
			0.1050	0.0558	0.0223	0.1240	0.0748	0.0293
$\phi = 0.9$	6	17	0.1240*	0.0900*	0.0410*	0.1440*	0.1060*	0.0530*
			0.1050	0.0762	0.0347	0.1250	0.0922	0.0467
			0.1430	0.1038	0.0473	0.1630	0.1198	0.0593
MA(1)								
$\theta = 0.1$	6	17	0.0810	0.0410	0.0100	0.0960	0.0520	0.0140
			0.0620	0.0272	0.0037	0.0770	0.0382	0.0077
			0.1000	0.0548	0.0163	0.1150	0.0658	0.0203
$\theta = 0.5$	6	17	0.0820	0.0530	0.0110	0.0950	0.0720*	0.0160
			0.0630	0.0392	0.0047	0.0760	0.0582	0.0097
			0.1010	0.0668	0.0173	0.1140	0.0858	0.0223
$\theta = 0.9$	6	17	0.1200*	0.0670*	0.0250*	0.1390*	0.0940*	0.0300*
			0.1010	0.0532	0.0187	0.1200	0.0802	0.0237
			0.1390	0.0808	0.0313	0.1580	0.1078	0.0363

* Significant at the 5% level

(e) n=492

Process	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
AR(1)								
$\phi = 0$	12	21	0.1070	0.0600	0.0150	0.1200*	0.0660*	0.0170*
			0.0880	0.0462	0.0087	0.1010	0.0522	0.0107
			0.1260	0.0738	0.0213	0.1390	0.0798	0.0233
$\phi = 0.1$	12	21	0.0930	0.0570	0.0160	0.0970	0.0640*	0.0180*
			0.0740	0.0432	0.0097	0.0780	0.0502	0.0117
			0.1120	0.0708	0.0223	0.0116	0.0778	0.0243
$\phi = 0.5$	12	21	0.0930	0.0520	0.0140	0.1050	0.0620	0.0160
			0.0740	0.0382	0.0077	0.0860	0.0482	0.0097
			0.1120	0.0658	0.0203	0.1240	0.0758	0.0223
$\phi = 0.9$	12	21	0.1360*	0.0910*	0.0480*	0.1480*	0.1010*	0.0510*
			0.1170	0.0772	0.0417	0.1290	0.0872	0.0447
			0.1550	0.1048	0.0543	0.1670	0.1148	0.0573
MA(1)								
$\theta = 0.1$	12	21	0.0930	0.0560	0.0180*	0.1010	0.0630	0.0230*
			0.0740	0.0422	0.0117	0.0820	0.0492	0.0167
			0.1120	0.0698	0.0243	0.1200	0.0768	0.0293
$\theta = 0.5$	12	21	0.1070	0.0610	0.0210*	0.1210	0.0720*	0.0230*
			0.0880	0.0472	0.0147	0.1020	0.0582	0.0167
			0.1260	0.0748	0.0273	0.1400	0.0858	0.0293
$\theta = 0.9$	12	21	0.1240*	0.0880*	0.0370*	0.1330*	0.0930*	0.0440*
			0.1050	0.0742	0.0307	0.1140	0.0792	0.0377
			0.1430	0.1018	0.0433	0.1520	0.1068	0.0503

* Significant at the 5% level

TABLE 4
ESTIMATES OF SIZE WITH 95% CONFIDENCE INTERVALS OF THE TEST STATISTICS Q FOR k=3 and n=252

Process	m	df	10%	5%	1%
AR(1)					
$\phi = 0$	6	44	0.0830	0.0550	0.0150
			0.0640	0.0412	0.0087
			0.1020	0.0688	0.0213
$\phi = 0.1$	6	44	0.1000	0.0520	0.0130
			0.0810	0.0382	0.0067
			0.1190	0.0658	0.0193
$\phi = 0.5$	6	44	0.0810	0.0470	0.0190*
			0.0620	0.0332	0.0127
			0.1000	0.0608	0.0253
$\phi = 0.9$	6	44	0.1600*	0.1040*	0.0560*
			0.1410	0.0902	0.0497
			0.1790	0.1178	0.0623
MA(1)					
$\theta = 0.1$	6	44	0.0890	0.0440	0.0090
			0.0700	0.0302	0.0027
			0.1080	0.0579	0.0153
$\theta = 0.5$	6	44	0.0760*	0.0320*	0.0070
			0.0570	0.0182	0.0007
			0.0950	0.0458	0.0133
$\theta = 0.9$	6	44	0.1600*	0.1030*	0.0360*
			0.1410	0.0892	0.0297
			0.1790	0.1168	0.0423

* Significant at the 5% level

TABLE 5
ESTIMATES OF POWER OF THE TEST STATISTICS Q AND X FOR
FOR k=2

(a)AR(1) $\phi = 0$ vs $\phi > 0$ n=64

ϕ	m	df	Q			X		
			10%	5%	1%	10%	5%	1%
0.0	2	17	0.0740	0.0490	0.0130	0.1330	0.0920	0.0420
0.2			0.1470	0.0930	0.0310	0.2340	0.1620	0.0830
0.4			0.3880	0.3030	0.1530	0.5230	0.4210	0.2830
0.6			0.7700	0.6870	0.5280	0.8650	0.7960	0.6590
0.8			0.9780	0.9650	0.9400	0.9910	0.9820	0.9610
0.0	3	11	0.0950	0.0500	0.0130	0.1310	0.0860	0.0360
0.2			0.1970	0.1320	0.0550	0.2540	0.1850	0.0870
0.4			0.5080	0.4180	0.2460	0.5690	0.4900	0.3280
0.6			0.8730	0.8100	0.6640	0.9030	0.8560	0.7420
0.8			0.9910	0.9870	0.9670	0.9940	0.9880	0.9820
0.0	4	9	0.1350	0.0910	0.0320	0.1670	0.1160	0.0470
0.2			0.2440	0.1810	0.0920	0.2850	0.2100	0.1130
0.4			0.6000	0.5230	0.3370	0.6400	0.5560	0.3940
0.6			0.9080	0.8810	0.7530	0.9250	0.8990	0.8010
0.8			0.9930	0.9890	0.9770	0.9930	0.9940	0.9850

(b)AR(1) $\phi = 0.5$ vs $\phi \neq 0.5$ n=64

ϕ	m	df	Q			X		
			10%	5%	1%	10%	5%	1%
0.1	2	17	0.4100	0.3080	0.1540	0.5210	0.4420	0.2860
0.3			0.1630	0.1060	0.0390	0.2680	0.1900	0.0950
0.5			0.0710	0.0380	0.0100	0.1310	0.0830	0.0300
0.7			0.1930	0.1180	0.0470	0.2790	0.2280	0.1660
0.9			0.7470	0. 6710	0.4700	0.8230	0.7760	0.6590
0.1	3	11	0.5120	0.4150	0.2670	0.5930	0.4920	0.3540
0.3			0.2270	0.1610	0.0720	0.2780	0.2120	0.1180
0.5			0.0980	0.0550	0.0210	0.1390	0.0830	0.0310
0.7			0.2930	0.2140	0.0970	0.3630	0.2820	0.1560
0.9			0.8300	0.7720	0.6420	0.8670	0.8240	0.7250
0.1	4	9	0.6130	0.5170	0.3440	0.6590	0.5640	0.4140
0.3			0.2900	0.2160	0.1100	0.3330	0.2570	0.1490
0.5			0.1420	0.0860	0.0290	0.1760	0.1150	0.0430
0.7			0.3400	0.2680	0.1680	0.3800	0.3080	0.1970
0.9			0.8810	0.8440	0.7680	0.8980	0.8600	0.8070

(c) AR(1) $\phi = 0$ vs $\phi > 0$ n=256

ϕ	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
0.0	4	33	0.0400	0.0170	0.0050	0.0730	0.0390	0.0120
0.1			0.0890	0.0430	0.0090	0.1260	0.0860	0.0220
0.2			0.2760	0.1850	0.0940	0.3550	0.2750	0.1440
0.3			0.5770	0.4760	0.2960	0.6570	0.5650	0.4000
0.4			0.8750	0.8310	0.6830	0.9060	0.8700	0.7660
0.5			0.9820	0.9730	0.9390	0.9880	0.9810	0.9610
0.6			0.9990	0.9980	0.9980	0.9990	0.9990	0.9980
0.0	6	22	0.0840	0.0540	0.0160	0.1080	0.0680	0.0220
0.1			0.1420	0.0940	0.0300	0.1840	0.1170	0.0470
0.2			0.3990	0.2940	0.1480	0.4470	0.3500	0.1950
0.3			0.7300	0.6470	0.4520	0.7720	0.6890	0.5070
0.4			0.9480	0.9230	0.8350	0.9580	0.9340	0.8720
0.5			0.9960	0.9920	0.9780	0.9970	0.9950	0.9870
0.6			1.0000	1.0000	0.9970	1.0000	1.0000	0.9980
0.0	8	17	0.1140	0.0650	0.0240	0.1300	0.0820	0.0320
0.1			0.1880	0.1120	0.0300	0.2180	0.1400	0.0480
0.2			0.4780	0.3700	0.2320	0.5150	0.4050	0.2580
0.3			0.7920	0.7330	0.5590	0.8180	0.7600	0.5930
0.4			0.9620	0.9450	0.8730	0.9650	0.9500	0.8960
0.5			0.9980	0.9950	0.9850	0.9980	0.9960	0.9880
0.6			1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

(d) AR(1) $\phi = 0.5$ vs $\phi \neq 0.5$ n=256

ϕ	m	df	Q			X		
			10%	5%	1%	10%	5%	1%
0.1	4	33	0.9090	0.8610	0.7210	0.9450	0.9040	0.8000
0.2			0.6290	0.5370	0.3490	0.7090	0.6210	0.4530
0.3			0.3110	0.2180	0.1100	0.3820	0.3070	0.1660
0.4			0.0950	0.0570	0.0210	0.1410	0.0930	0.0400
0.5			0.0540	0.0310	0.0080	0.0830	0.0570	0.0210
0.6			0.1240	0.0790	0.0260	0.1820	0.1170	0.0600
0.7			0.4080	0.3150	0.1520	0.5030	0.4030	0.2400
0.8			0.1872	0.8150	0.6690	0.9120	0.8690	0.7710
0.9			0.9980	0.9960	0.9930	0.9980	0.9980	0.9950
0.1	6	22	0.9630	0.9430	0.8590	0.9740	0.9530	0.8930
0.2			0.7820	0.7010	0.5550	0.8160	0.7460	0.5960
0.3			0.4440	0.3450	0.1960	0.4870	0.3980	0.2420
0.4			0.1520	0.0900	0.0380	0.1860	0.1140	0.0490
0.5			0.0720	0.0330	0.0120	0.0930	0.0550	0.0150
0.6			0.1670	0.1130	0.0480	0.2070	0.1390	0.1630
0.7			0.5560	0.4530	0.2910	0.6180	0.5160	0.3580
0.8			0.9490	0.9270	0.8310	0.9610	0.9420	0.8740
0.9			1.0000	1.0000	0.9980	1.0000	1.0000	1.0000
0.1	8	17	0.9870	0.9650	0.9300	0.9870	0.9700	0.9370
0.2			0.8500	0.7910	0.6370	0.8620	0.8210	0.6760
0.3			0.5100	0.4270	0.2740	0.5440	0.4580	0.3050
0.4			0.2000	0.1420	0.0600	0.2290	0.1660	0.0670
0.5			0.0950	0.0570	0.0220	0.1140	0.0690	0.0270
0.6			0.2480	0.1830	0.0910	0.2690	0.2140	0.1100
0.7			0.6680	0.5770	0.4050	0.6910	0.6040	0.4410
0.8			0.9630	0.9430	0.8800	0.9680	0.9550	0.9110
0.9			1.0000	1.0000	0.9980	1.0000	1.0000	0.9980

(e) AR(1) $\phi = 0$ vs $\phi > 0$ n=1024

ϕ	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
0.00	16	33	0.0620	0.0290	0.0090	0.0690	0.0350	0.0120
0.05			0.1240	0.0860	0.0290	0.1300	0.0980	0.0400
0.10			0.3110	0.2180	0.1060	0.3340	0.2350	0.1200
0.15			0.6400	0.5440	0.3530	0.6610	0.5660	0.3790
0.20			0.9070	0.8620	0.7300	0.9130	0.8750	0.7490
0.25			0.9860	0.9710	0.9330	0.9870	0.9750	0.9450
0.00	20	26	0.0720	0.0330	0.0060	0.0740	0.0380	0.0070
0.05			0.1330	0.0840	0.0230	0.1450	0.0940	0.0280
0.10			0.3630	0.2730	0.1440	0.3790	0.2840	0.1560
0.15			0.7190	0.6080	0.4290	0.7340	0.6250	0.4460
0.20			0.9470	0.9120	0.7930	0.9500	0.9190	0.8070
0.25			0.9920	0.9860	0.9670	0.9930	0.9870	0.9710
0.00	24	22	0.0870	0.0560	0.0210	0.0920	0.0600	0.0210
0.05			0.1680	0.1090	0.0370	0.1710	0.1160	0.0410
0.10			0.4410	0.3550	0.1990	0.4530	0.3660	0.2100
0.15			0.7440	0.6700	0.4980	0.7510	0.6790	0.5090
0.20			0.9420	0.9060	0.8310	0.9470	0.9120	0.8360
0.25			0.9980	0.9920	0.9800	0.9980	0.9940	0.9810

(f) AR(1) $\phi = 0.5$ vs $\phi \neq 0.5$ n=1024

ϕ	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
0.35	16	33	0.7810	0.6790	0.4960	0.7920	0.7070	0.5210
0.40			0.3800	0.2910	0.1500	0.4000	0.3150	0.1630
0.45			0.1190	0.0800	0.0280	0.1350	0.0900	0.0340
0.50			0.0540	0.0260	0.0060	0.0640	0.0340	0.0100
0.55			0.1390	0.0860	0.0240	0.1520	0.0940	0.0270
0.60			0.4260	0.3260	0.1840	0.4430	0.3570	0.1990
0.65			0.8200	0.7580	0.5870	0.8330	0.7750	0.6150
0.35	20	26	0.8020	0.7280	0.5380	0.8150	0.7410	0.5640
0.40			0.4520	0.3460	0.1880	0.4720	0.3610	0.2040
0.45			0.1500	0.0940	0.0370	0.1590	0.1010	0.0400
0.50			0.0910	0.0470	0.0130	0.1040	0.0600	0.0130
0.55			0.1420	0.0850	0.0.50	0.1510	0.0940	0.0370
0.60			0.4150	0.3310	0.1900	0.4270	0.3470	0.2030
0.65			0.7980	0.7320	0.5570	0.8090	0.7390	0.5740
0.35	24	22	0.8340	0.7670	0.6320	0.8450	0.7810	0.6470
0.40			0.5140	0.4210	0.2610	0.5270	0.4360	0.2710
0.45			0.2190	0.1500	0.0600	0.2290	0.1580	0.0670
0.50			0.0800	0.0370	0.0070	0.0860	0.0480	0.0080
0.55			0.2280	0.1510	0.0670	0.2400	0.1630	0.0760
0.60			0.5660	0.4850	0.3160	0.5810	0.4970	0.3270
0.65			0.9120	0.8770	0.7720	0.9140	0.8830	0.7820

(g) AR(1) $\phi = 0$ vs $\phi > 0$ n=198

ϕ	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
0.0	6	17	0.0910	0.0520	0.0170	0.1140	0.0700	0.0250
0.1			0.1340	0.0830	0.0280	0.1630	0.0990	0.0410
0.2			0.3200	0.2300	0.1140	0.3680	0.2660	0.1480
0.3			0.6740	0.5660	0.3670	0.7050	0.6110	0.4270
0.4			0.8950	0.8610	0.7420	0.9100	0.8810	0.7780
0.5			0.9870	0.9750	0.9390	0.9900	0.9800	0.9550
0.6			0.9980	0.9970	0.9930	0.9980	0.9980	0.9960

(h) AR(1) $\phi = 0.5$ vs $\phi \neq 0.5$ n=198

ϕ	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
0.1	6	17	0.9340	0.9660	0.7860	0.9440	0.9170	0.8230
0.2			0.7240	0.6500	0.4800	0.7620	0.6890	0.5350
0.3			0.4020	0.3130	0.1740	0.4390	0.3510	0.2070
0.4			0.1640	0.0980	0.0380	0.2000	0.1260	0.0530
0.5			0.0860	0.0420	0.0160	0.1050	0.0610	0.0230
0.6			0.1810	0.1340	0.0460	0.2170	0.1560	0.0720
0.7			0.5260	0.4270	0.2560	0.5680	0.4800	0.3200
0.8			0.9040	0.8550	0.7690	0.9200	0.8870	0.8010
0.9			0.9980	0.9950	0.9860	0.9980	0.9980	0.9900

(i) AR(1) $\phi = 0$ vs $\phi > 0$ n=492

ϕ	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
0.0	12	21	0.1070	0.0600	0.0150	0.1200	0.0660	0.0170
0.1			0.2290	0.1630	0.0720	0.2420	0.1760	0.0840
0.2			0.7010	0.6110	0.4320	0.7170	0.6410	0.4460
0.3			0.9640	0.9500	0.9000	0.9690	0.6540	0.9110
0.4			1.0000	1.0000	0.9980	1.0000	1.0000	1.0000

(j) AR(1) $\phi = 0.5$ vs $\phi \neq 0.5$ n=492

ϕ	m	df	Q 10%	Q 5%	Q 1%	X 10%	X 5%	X 1%
0.1	12	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.2			0.9820	0.9860	0.9310	0.9860	0.9730	0.9380
0.3			0.7560	0.6700	0.5140	0.7760	0.6950	0.5430
0.4			0.2580	0.1980	0.0910	0.2770	0.2110	0.1040
0.5			0.0930	0.0520	0.0140	0.1050	0.0620	0.0160
0.6			0.2580	0.1950	0.0930	0.3090	0.2140	0.1030
0.7			0.8710	0.8220	0.6800	0.8870	0.8350	0.7090
0.8			0.9970	0.9950	0.9930	0.9970	0.9950	0.9940
0.9			1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE 6
ESTIMATES OF POWER OF THE TEST STATISTIC Q
FOR n=252 k=3

(a)AR(1) $\phi = 0$ vs $\phi > 0$

ϕ	m	df	10%	5%	1%
0.0	6	44	0.0830	0.0550	0.0150
0.1			0.1620	0.1080	0.0420
0.2			0.4060	0.3030	0.1560
0.3			0.7850	0.7050	0.5370
0.4			0.9560	0.9290	0.8630
0.5			0.9990	0.9990	0.9990

(b)AR(1) $\phi = 0.5$ vs $\phi \neq 0.5$

ϕ	m	df	10%	5%	1%
0.1	6	44	0.9720	0.9490	0.8800
0.2			0.8160	0.7390	0.5700
0.3			0.4330	0.3340	0.1850
0.4			0.1500	0.0980	0.0370
0.5			0.0810	0.0470	0.0190
0.6			0.1970	0.1330	0.0500
0.7			0.6140	0.5200	0.3410
0.8			0.9620	0.9410	0.9980
0.9			1.0000	0.9980	0.9980

FIGURE 1

(a)

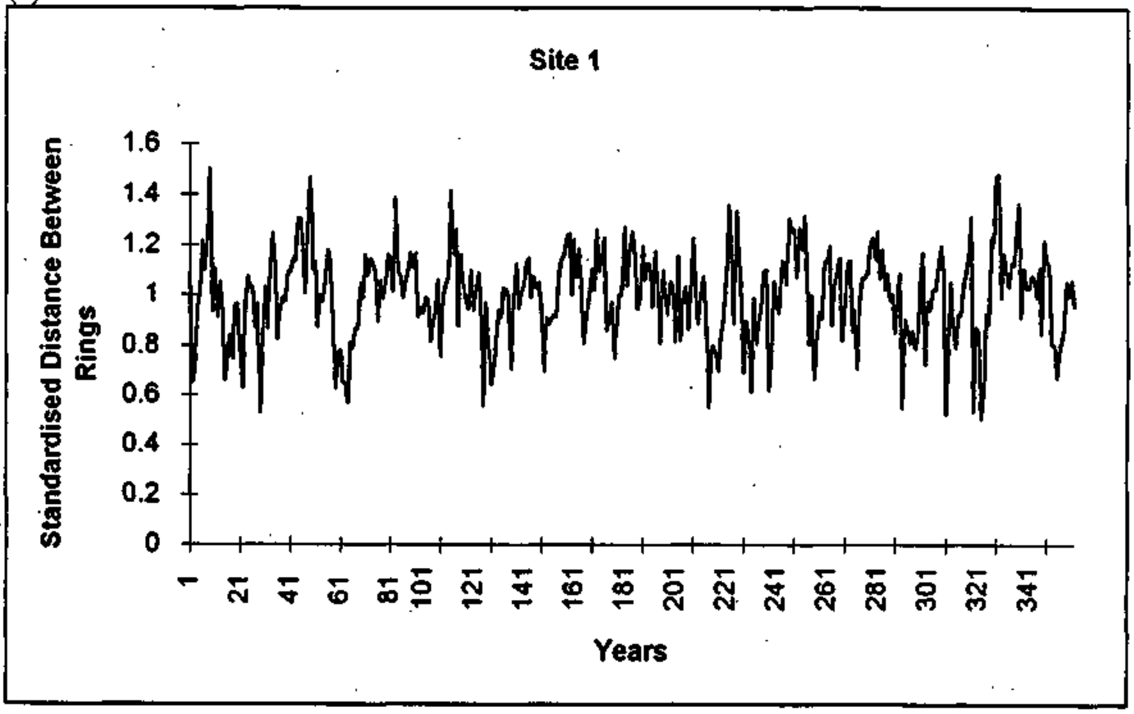


FIGURE 2

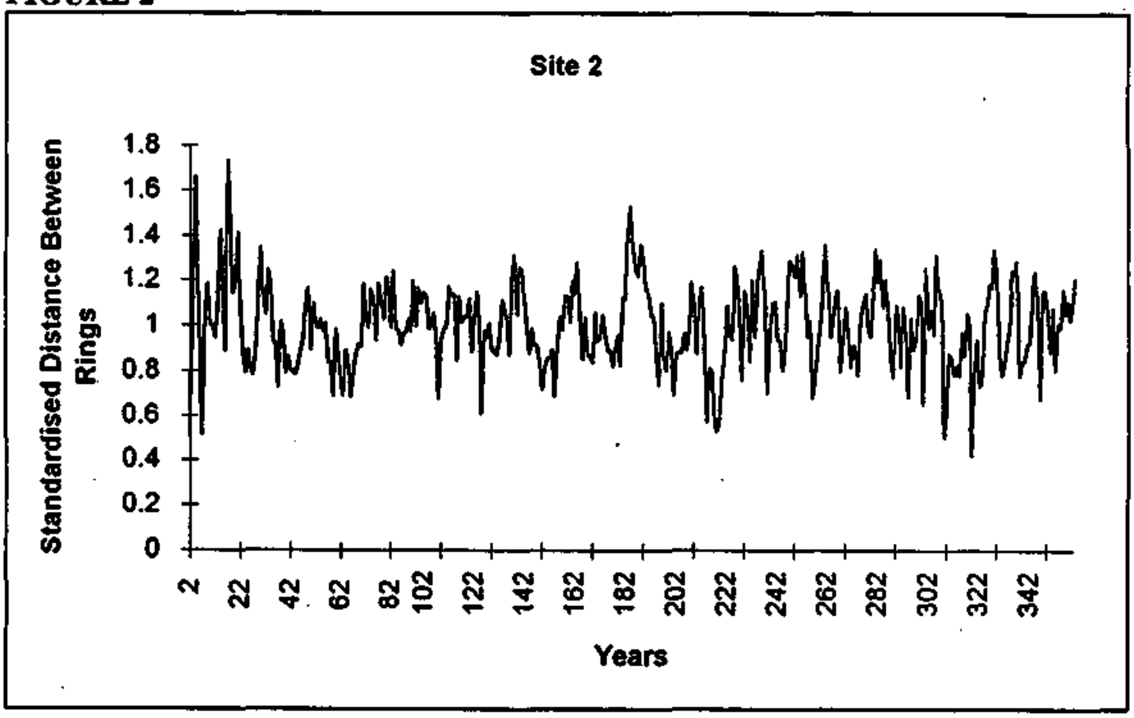


FIGURE 3

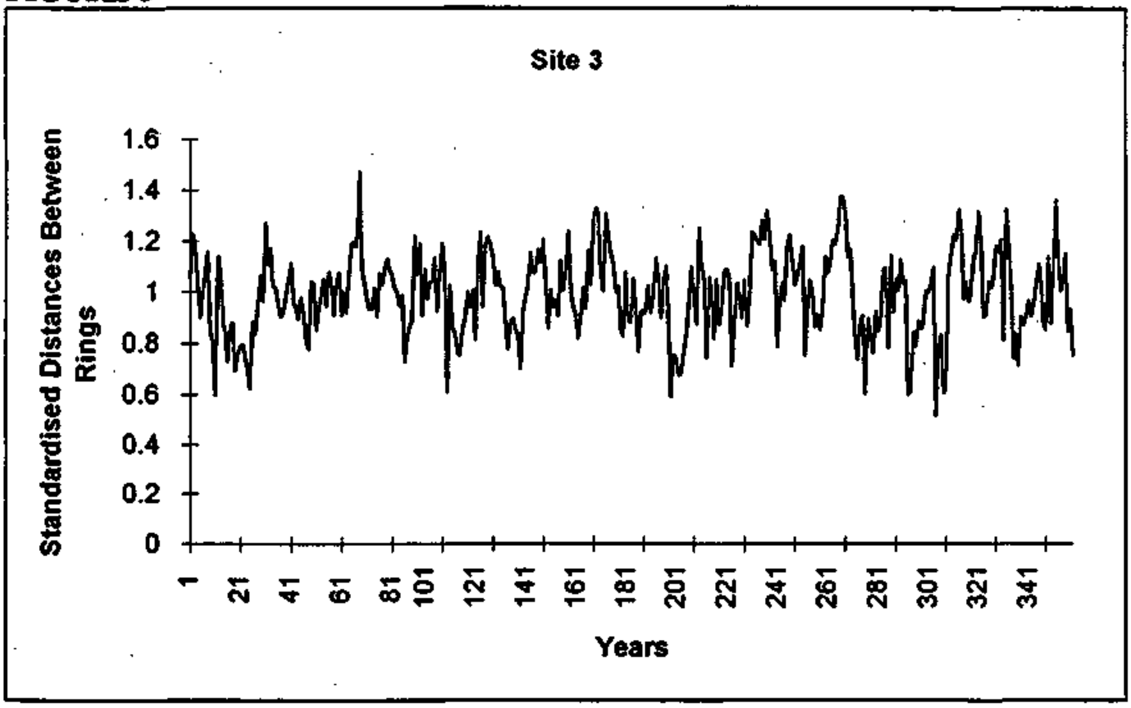


FIGURE 4

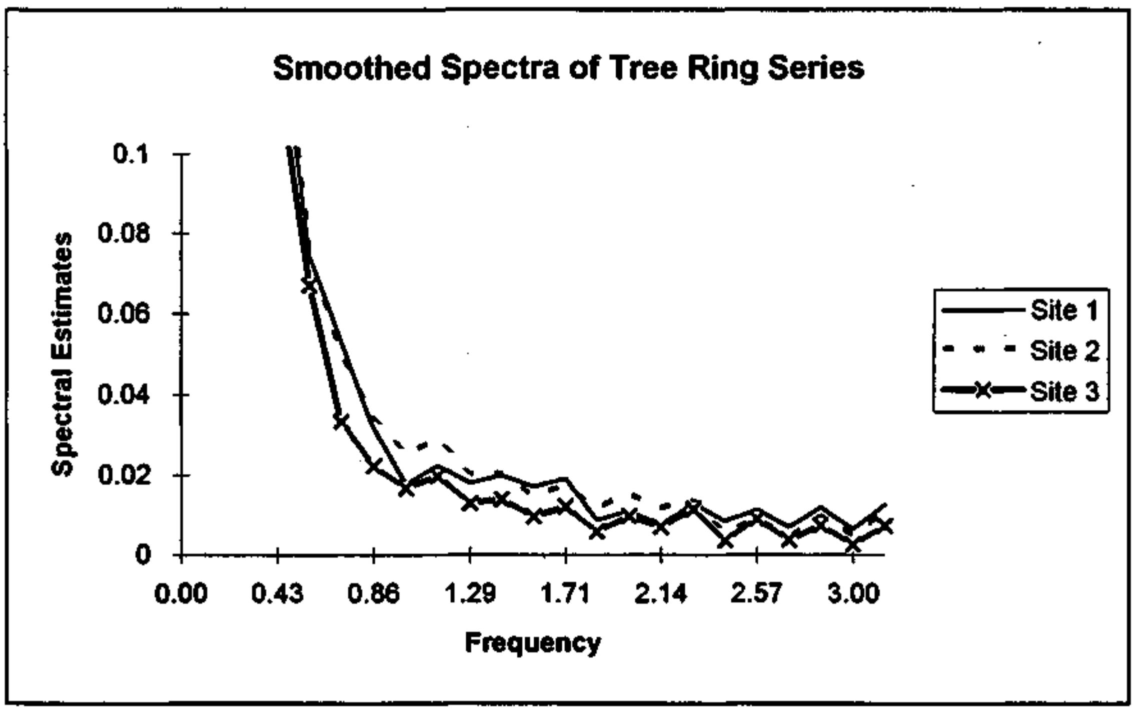


FIGURE 5

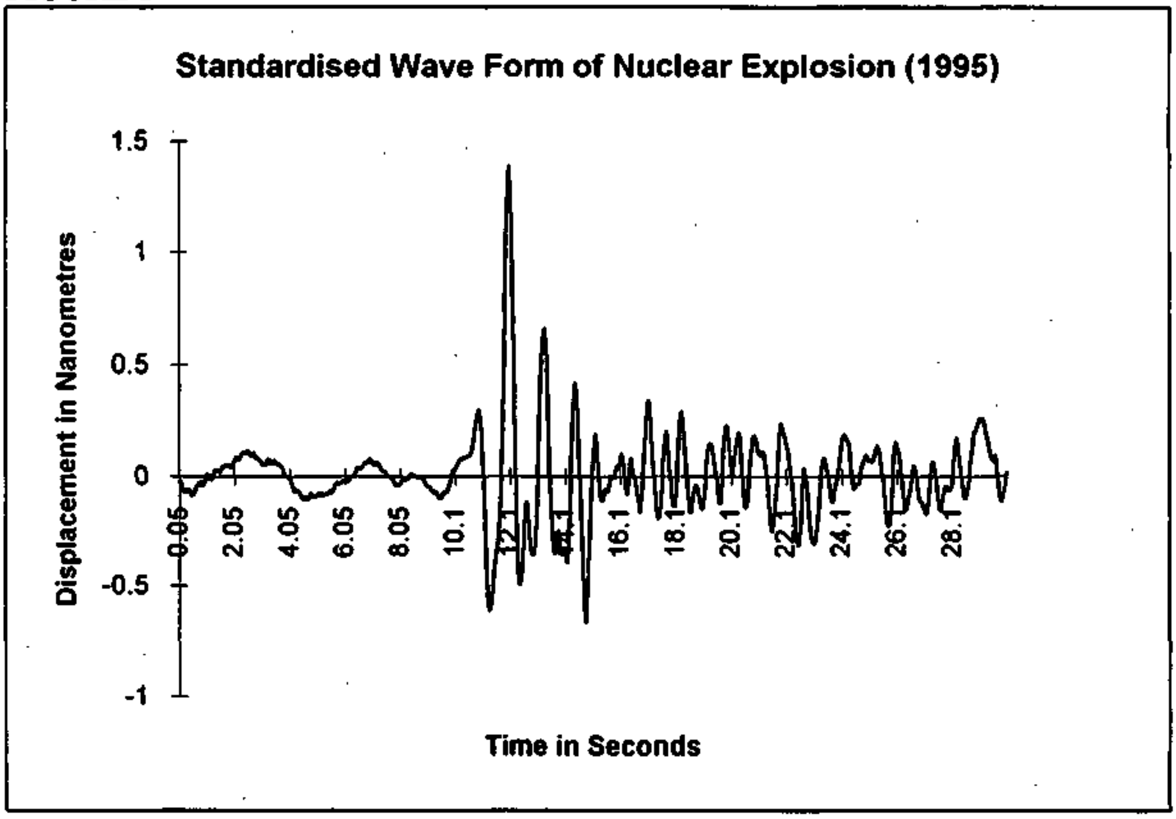


FIGURE 6

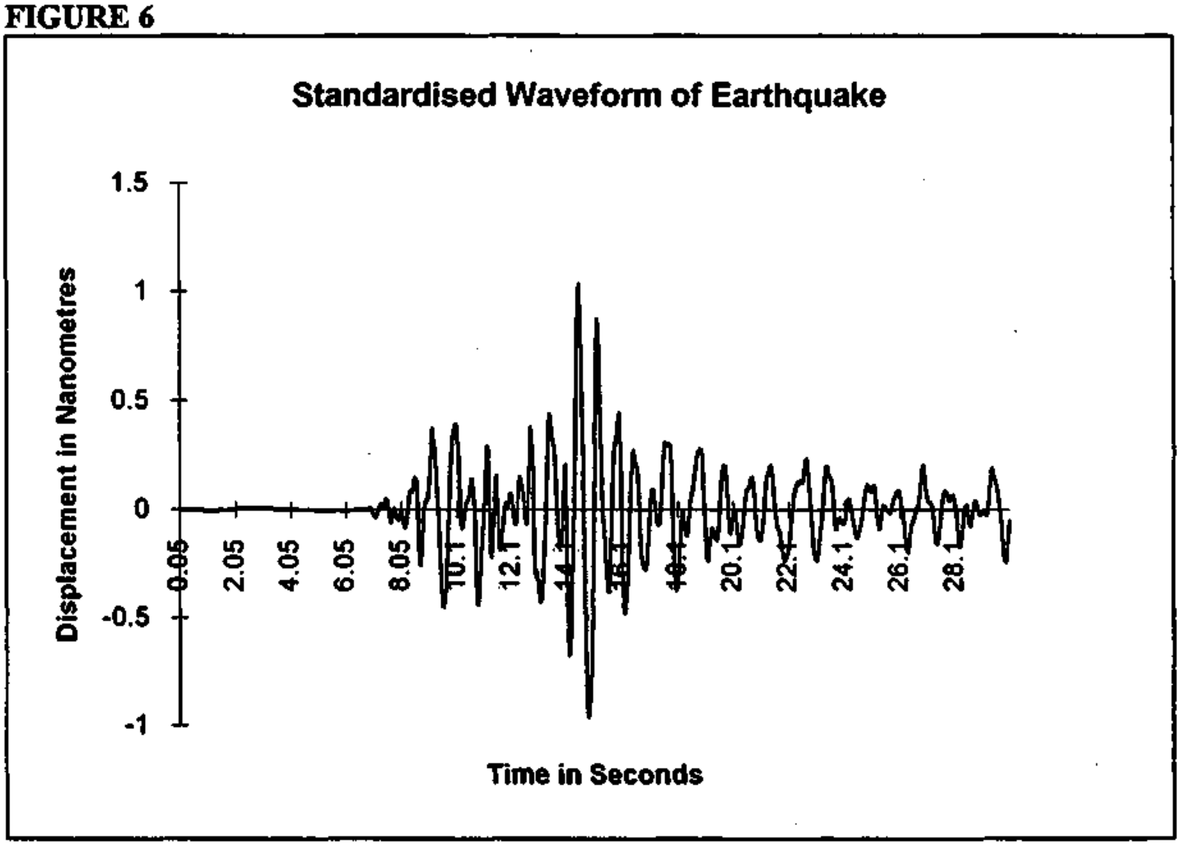
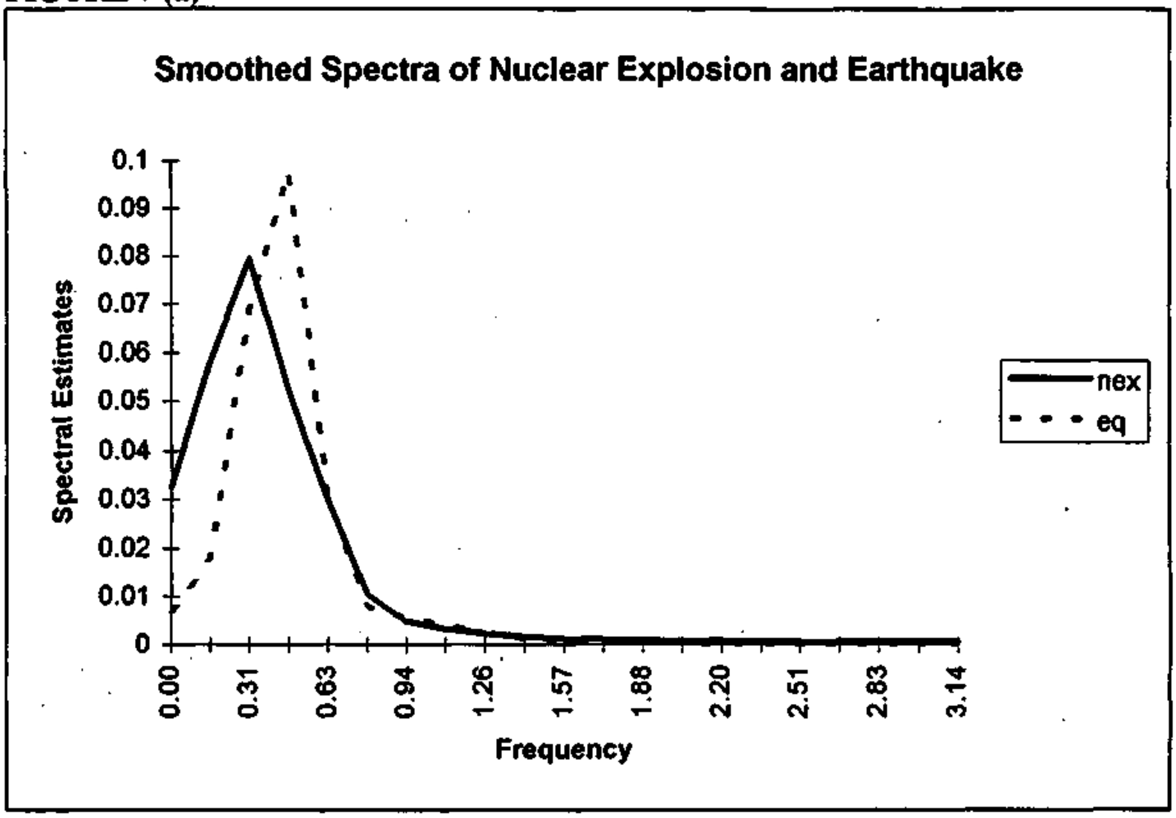
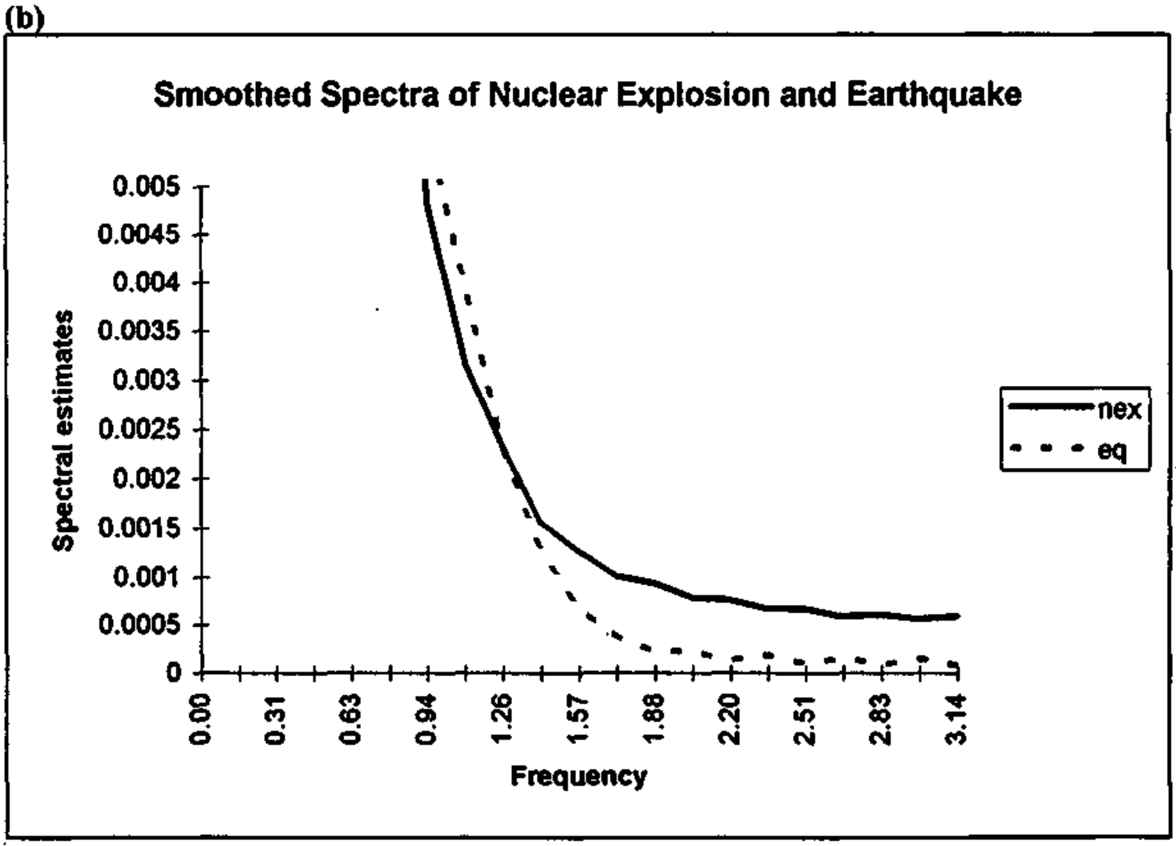


FIGURE 7 (a)



Smoothed Spectra of Nuclear Explosion and Earthquake
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Spectral Estimates
0.00
0.31
0.63
0.94
1.26
1.57
1.88
2.20
2.51
2.83
3.14
Frequency
nex
eq

(b)



Smoothed Spectra of Nuclear Explosion and Earthquake
0.005
0.0045
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
Spectral estimates
0.00
0.31
0.63
0.94
1.26
1.57
1.88
2.20
2.51
2.83
3.14
Frequency
nex
eq