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Abstract

This paper studies linear and nonlinear autoregressive leading indicator models of business cycles in OECD countries. The models use the spread between short-term and long-term interest rates as leading indicators for GDP, and their success in capturing business cycles is gauged by the non-parametric procedures developed by Harding and Pagan (2001). Our preliminary findings indicate that bivariate nonlinear models of output and the interest rate spread can successfully capture the shape of the business cycle. In particular, they can capture the features of recession and the deviation of the actual path of the cycles from a triangular approximation to this path, both characteristics that other models of GDP fail to reproduce.

Keywords: Business Cycles, Leading Indicators, Nonlinear Models, Yield Spread.

JEL classification: C22, C23, E17, E37.

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1. Introduction

One of the most striking aspects of macroeconomic time series data is its cyclical behaviour. Series such as output, consumption and investment all undergo temporary fluctuations about trend, and the fluctations in different series often occur at roughly the same time. There are huge literatures that attempt to measure, explain and predict these so-called business cycles, and much of this work focusses on output, which is used as an indicator of the multivariate cycle. There is general consensus that movements in output are representative of the business cycle, but there are vigorous debates about how to define and measure cycles in output, how to model them, and how to predict features such as turning points and recessions. Detrending issues fuel many of these debates, but other important issues include the possible nonlinearity in business cycles, and which variables are most useful for predicting output.

Recent work by Harding and Pagan (2001) has addressed several fronts of these debates. These authors point out the gap between policy makers' focus on turning points in the levels of output and academic interest in modeling the moments of detrended data, and they advocate the use of a cycle dating algorithm that identifies the turning points in levels by analysing changes in levels. The algorithm requires no detrending of data to define the cycle. Further, once turning points have been established, it is straightforward to measure various cyclical properties of peak to trough and trough to peak states, such as duration and amplitude. Harding and Pagan (2001) go on to develop some nonparametric techniques that allow one to evaluate models of output, by comparing the simulated predicted cyclical properties of such models with the corresponding properties of the data. This model evaluation technique can be used on models of detrended output, (one simply integrates the simulated data back into levels at the evaluation stage), and thus it can be used to compare a set of alternative models of output growth.

This paper studies linear and nonlinear autoregressive leading indicator models of output growth in OECD countries. Our models use the spread between short-term and long-term interest rates as leading indicators for growth in GDP, and their success in capturing business cycles is gauged by the non-parametric procedures developed by Harding and Pagan (2001). Our primary aim is to develop forecasting models that can predict turning points and other salient features of business cycles, and we particularly interested in assessing the predictive ability of nonlinear specifications relative to linear specifications. Authors such as Teräsvirta and Anderson (1992), Clements and Krolzig (1998), Jansen and Oh (1999) have previously studied the forecasting ability of *univariate* nonlinear specifications of output, but univariate analysis is of limited practical relevance, given the large literatures on leading indicators and variables that Granger-cause output. Here, we ask whether nonlinear leading indicator models can out-predict (linear) vector autoregressions.

We build on the work in Anderson and Vahid (2001), who extended the (linear) class of autoregressive leading indicator (ARLI) models developed by Zellner and Hong (1989), Zellner et al (1991) and Zellner and Min (1999) to include nonlinear autoregressive specifications (called NARLI models). NARLI models allow for differences in behaviour over different phases of the business cycle, and they also allow for asymmetries in how the indicator leads output. In line with recent research undertaken by Stock and Watson (1989), Davis and Fagan (1997), Friedman and Kuttner (1998), Estrella and Mishkin (1998) and others, we use yield spreads as our leading indicators. The predictive power of the spread is well established, but apart from a few investigations conducted in logit/probit settings (see, eg Estrella and Mishkin (1998), Karutnaratne (1999) and Birchenhall et al (2000)), most research on this issue has stayed within the confines of conventional linear regression. Our nonlinear specifications are nevertheless consistent with Galbraith and Tkacz (2000), who test for and find asymmetries in the link between the yield spread and output in G7 countries. See also De Long and Summers (1988), Cover (1992), Karras (1996), Choi (1999) and Weiss (1999), who model asymmetries in the relationship between monetary policy and output.

By using Harding and Pagan's (2001) model evaluation technique, we assess the ability of our models to capture the characteristics of cycles, rather than ability to deliver an accurate point forecast of a mean. We feel that the former is more relevant when modeling business cycles, because in this context the mean is rarely of direct interest. Other useful evaluation methods involve the comparison of probability forecasts of cycle-related events such as turning points or recessions. See Neftçi (1982), Diebold and Rudebusch (1989), Fair (1993) and Anderson and

Vahid (2001) for examples. While Harding and Pagan's (2001) techniques involve a definition of the cycle, this cycle is a user defined observable function of the data at hand. Related concepts such as peaks, phases and recessions are also user defined. This is slightly different from "declared" cycles such as NBER cycles, but it allows one to compare *measurable* properties of the data with a models' simulated predictions. It is also useful when evaluating business cycle models of countries where analogues of NBER cycles are not available.

We find that bivariate nonlinear models of output and the interest rate spread can predict the characteristic features of each country's business cycle. They can capture the amplitude and duration of both peak to trough and trough to peak states, and they can capture the deviation of the actual path of the cycles from a triangular approximation to this path. Linear leading indicator models of GDP fail to reproduce this last property, as do univariate nonlinear models. Thus, the nonlinearity in a bivariate framework appears to be important. Like other researchers, we find that the spread outperforms other financial indicators.

The next section of this paper provides a brief introduction to Burns and Mitchell's (1946) graphical approach of observing the business cycle, and compares this to the cycles defined by Harding and Pagan's (2001) adaptation of the Bry and Boschin (1971) algorithm (BBQ). It also outlines some non-parametric measures of business cycle characteristics (BCC), including an "excess index" that captures business cycle shape. We then explain how to use these measures as a benchmark for model evaluation. Section 3 describes our data, and we develop and evaluate our linear and nonlinear bivariate autoregressive leading indicator models in Sections 4 and 5. The paper concludes in section 6 with a summary of our findings.

2. Definitions and Measurement of Business Cycles

Early research on business cycles developed a tradition of studying graphs of economic indicators and noting the times at which these series reached peaks or troughs. Burns and Mitchell (1946) at the NBER formalised this tradition, identifying "reference cycles" on the basis of the dates when different series simultaneously reached peaks and troughs. Although Koopmans (1947) argued against these classical techniques on the basis that a graphical approach lacked a statistical foundation, the NBER has continued to determine “reference cycles” for the United States, and it regularly publishes a set of turning points that reflect a consensus view of when graphs have turned. Both policy makers and academics view the NBER turning points as indicative of “the (US) business cycle”, despite the ad-hoc way in which these points are determined.

Koopmans (1947) advocated the parametric modeling of time series as a scientific alternative to the classical approach, and since then, much business cycle research has been based on modeling the data. However, the presence of trends in most macroeconomic time series complicates the interpretation of parametric models. One cannot conduct standard statistical inference on models of trending data, and the cross correlation and serial correlation properties of detrended series (often called cyclical components) are very sensitive to the type of detrending procedure used. Different detrending methods imply different cyclical components, which need not “turn” when the classical business cycle turns. It follows that models of detrended data provide only indirect information about classical cycles, and one needs to keep this in mind when interpreting business cycle models. See Harvey and Jaeger (1993) or Canova (1998) for further discussion on the modeling of detrended data.

Harding and Pagan (2001) provide a rigorous link between these two approaches. Since the main problem with the classical approach is the absence of formal definitions for peaks and troughs, they design an algorithm that can be applied to raw data to date these events. Their algorithm mimicks the classical procedures used to find troughs and peaks, so that it identifies turning points that match the classical “reference cycle” very closely. Once turning points have been established, one can then measure various properties of classical cycles. We discuss these properties in more detail below. The difficulties in interpreting parametric models of detrended data are resolved by studying their implied classical properties. To do this, one firstly uses the parametric model to obtain simulated series of detrended data, then integrates the detrended series to obtain levels series, passes the implied levels series through the cycle dating algorithm and finally compares the measured properties of the simulated levels series with measures based on the original data.

2.1. Harding and Pagan's BBQ and nonparametric measures of BCC.

The turning points in "reference" cycles are typically located by finding local maxima and minima, subject to the conditions that peaks and troughs alternate, and that periods of contraction and expansion are sufficiently long. Such patterns are relatively easy to locate on a graph, and a good cycle dating algorithm needs to be able to recognize these same patterns.

The Bry and Boschan (1971) algorithm used by the NBER is one such algorithm. Designed for use on monthly data, this algorithm identifies a local maximum (or minimum) when

$$y_t > (<)\, y_{t\pm k} \quad \text{for } k = 1, ..., K \quad \text{and } K = 5 \text{ months}, \tag{2.1}$$

provided that each phase of the cycle lasts at least six months and the whole cycle lasts at least fifteen months. Harding and Pagan (2001) modify this algorithm so that it can be used on quarterly data, and this modified algorithm is known as the BBQ. The BBQ algorithm identifies turning points when

$$y_t > (<)\, y_{t\pm k} \quad \text{for } k = 1, ..., K \quad \text{and } K = 2 \text{ quarters}, \tag{2.2}$$

provided that each phase of a cycle lasts at least two quarters and the whole cycle lasts at least five quarters. The advantage of using an algorithm rather than a graph to locate dates of turning points is that it provides objective rather than subjective output, and one gets the same dates each time the algorithm is used.

Having identified the turning points in a series, one can then measure and study various business cycle characteristics (BCC). Harding and Pagan (2001) focus on four characteristics, which include measures of the length and size of a cycle, as well as measures of the cycle's impact and its shape. The four BCC are listed in Table 1, and since researchers are often interested in differences between recessionary and expansionary phases of the business cycle, it is useful to calculate the measures separately, for peak-to-trough phases and trough-to-peak phases.

Figure 1 provides a diagram that illustrates how each characteristic can be measured in a peak to trough episode. During this recessionary phase, the economy moves along the curved path from point X to point Y. The base of the triangle XYZ given by the length of the line XZ shows the duration (D) of the phase, i.e. how long it takes (in quarters) for the phase to be completed. The height of the triangle XYZ given by the length of the line YZ shows the amplitude (A) of the recession, i.e. the total change in output as the economy moves from its peak at X to its trough at Y. We convert this into a percentage change in our analysis.

Table 1: Business Cycle Characteristics (BCC)

Measure	**Description**
Mean Duration (quarters)	How many quarters it takes on average to complete each phase of the cycle
Mean Amplitude (%)	Total percentage decrease or increase. in output per phase of the cycle on average
Cumulation (%)	Cumulative percentage loss or gain in output per phase of the cycle on average
Excess (%)	The average excess percentage value per quarter relative to the triangle approximation

The impact of the recession can be measured by estimating the cumulated loss in output as the economy moves from the peak to the trough. This is the area above the curve path. The area of the triangle XYZ, i.e. $C_{Tri} = \frac{1}{2}DA$ provides a crude measure that Harding and Pagan (2001) call the "triangle approximation" to cumulative losses, and we illustrate this in Figure 1(a). A better approximation of the cumulated losses in output is given by the sum of the areas of the T small rectangles r, for $t = 1, \ldots T$, where $T = D$, and each rectangle relates to a single quarter during the phase. This "rectangle approximation", given by $C_R = \sum_{t=1}^{T} r_t$, is illustrated in Figure 1(b). The approximation can be further improved if we subtact the small corner triangles from the rectangles to obtain the shaded area in Figure 1(c). Since $A = \sum_{t=1}^{T} \alpha_t$, where each α_t measures the change in amplitude during quarter t, the total area in these small triangles is given by $S = \frac{1}{2}\sum_{t=1}^{T} \alpha_t = \frac{1}{2}A$. The "bias corrected" measure of cumulated output loss is then given by

$$C_{BC} = C_R - \frac{1}{2}A, \tag{2.3}$$

and after converting this into a percentage, this is our measure of cumulated output loss.

The final business cycle characteristic of interest measures the difference between the "bias corrected" cumulated output loss and the triangle approximation to this loss. We refer to this as an "Excess" measure and calculate it using the convention that $-Ex = C_R - \frac{1}{2}A - \frac{1}{2}DA$. The relevant area is illustrated in Figure 1(d). Harding and Pagan (2001) used this measure to create an "excess index" given by

$$E = \frac{\frac{1}{2}DA + \frac{1}{2}A - C_R}{D}, \tag{2.4}$$

and since this index describes the shape of the actual business cycle relative to the triangle approximation, it provides a measure of the shape of the cycle. We express the excess measure as a percentage in the work that follows.

The above measures relate to a single recession, but one can summarise the business cycle characteristics of a given macroeconomic variable by calculating the means of each BCC for over all peak-to-trough and over all trough-to-peak phases. These eight summary statistics (calculated without any prior detrending of the series) provide a natural benchmark for the evaluation of a business cycle model, because a good model should imply the same BCCs as those that are present in the data. Parametric models are, of course, typically modelling detrended data. However, this doesn't prevent the simulation of detrended data and then the integration of the simulated series to obtain an analogue of the original data together with its BCC measures. A sufficient number of simulations based on a parametric model will allow one to estimate the empirical density functions for each of the eight characteristics of interest, and these densities can then be compared with the relevant characteristics in the original data. If an observed BCC lies in the upper or lower tails of the simulated density, then this provides evidence against the parametric model. In our applications, we undertake 10000 simulations, and reject a parametric model if an observed BCC falls in the lower 5% or upper 5% tail of the relevant empirical distribution.

3. DATA AND MODELS

Our data consists of quarterly time series of real output (gross domestic product), short term interest rates and long term interest rates for the United States, Canada, the United Kingdom and Australia. We provide detailed information on data sources, our samples, and precise descriptions of our raw series in Appendix 1, and we base our benchmark analyses of business cycle characteristics on the natural logarithms of real GDP. Our spread variables are calculated by taking the difference between the interest rates on the long-term bond and the short term bond, and the variables in our parametric models are output growth (calculated as 100×the differenced logarithms of real GDP) and the spread. We use the notation y_t to denote output growth (which we will call output) and s_t to denote the interest rate spread. Graphs of all variables are provided in Appendix 1.

Summary statistics of the business cycle characteristics for each of our log(GDP) series are provided in Table 2. Here, it is quite clear that each characteristic varies considerably from country to country, and that the characteristics of the peak-to-trough phase are quite different from those of the trough-to-peak phase.

Table 2: Benchmark Business Cycle Characteristics

	USA	Canada	UK*	Australia	Netherlands
Mean Duration					
PT	3.8	4.0	4.4	3.7	2.5
TP	20.4	16.0	25.5	29.5	14.4
Mean Amplitude					
PT	-2.1	-3.2	-3.2	-3.1	-2.4
TP	22.9	17.2	21.5	28.3	14.9
Cumulation					
PT	-4.2	-6.6	-9.6	-5.2	-3.1
TP	342	257	381	458	185
Excess					
PT	-0.1	0.3	-0.1	0.1	-0.0
TP	1.4	1.4	-.5	1.5	1.0

Note: The UK figures relate to a cycle with a minimum length of 4 quarters rather than 5. This makes our analysis comparable to Harding and Pagan (2001)

The literature on nonlinearities in business cycles has developed in response to observed differences between recessionary and expansionary phases, but its success in modelling these differences has been questioned. Harding and Pagan (2001) note that a random walk model with drift can produce asymmetries similar to those in Table 2, although this model fails to capture "excess" in the trough-to-peak phase. A Markov-switching model (see Hamilton (1989)) also fails according to this criterion, which leads Harding and Pagan to conclude that "there is little evidence that nonlinear effects are important to the nature of the business cycle". Harding and Pagan (2001) also experiment with putting serial correlation into their model and forming a vector autoregression of output and investment, but none of these models capture "excess" in the trough-to-peak phase.

Our nonlinear leading indicator models differ from the above in two ways. Firstly, we model the joint behaviour of a *financial* indicator and output (rather than the joint behaviour of real variables). This is consistent with the view that monetary policy affects the business cycle, and it is also consistent with evidence that monetary variables Granger cause output. The other important feature of our models is that our nonlinear specifications are *bivariate* rather than univariate. Most previous work on nonlinearities in business cycles has focussed nonlinear univariate specifications of output, thereby ignoring the multivariate nature of business cycles.

We develop our models, one country at a time, to make sure that we account for country specific characteristics. In each case, we estimate a random walk in output to provide a baseline univariate specification, and then a VAR in output and the spread to provide a baseline bivariate model. We use AIC to guide our lag-length choices for the VAR, but eliminate lagged variables if they are statistically insignificant and their removal does not lead to serially correlated residuals. We estimate these restricted VARs both equation by equation using OLS, and as a Seemingly Unrelated Regression, but there is never much difference between the two, and our simulations are based on the latter. We call our restricted VARs ARLI models, and we compare the simulated properties of these models with those of the random walk models and the raw data. This allows us to assess how the lag structure and the financial indicator in each ARLI model accounts for ability to capture business cycle characteristics.

We develop our nonlinear models by conducting specification tests on each equation in the ARLI model. These tests include Tsay (1989) tests for threshold behaviour, tests by Luukonnen et al (1988) and Teräsvirta (1994) for evidence of smooth transition autoregressive behaviour, and other tests for structural change.

Statistically significant evidence of nonlinearity is found for each country, but the implied nonlinear form is different in each case. We use the results of the nonlinearity tests to guide our specification for each equation, and as above, we remove statistically insignificant explanators from equations provided that their removal does not lead to serially correlated residuals. Further details are provided below, in our country by country analysis.

4. Empirical Analysis

4.1. The United States

Univariate analysis of y_t found that an AR(2) model was preferred to a random walk (in ln(real GDP)), but there was no evidence to support a univariate nonlinear specification for y_t. The relevant equations for the period 1961:1 to 2000:4 (160 observations) were

$$\widehat{y}_t = \underset{(0.10)}{0.53} + \underset{(0.08)}{0.26} y_{t-1} + \underset{(0.08)}{0.13} y_{t-2} \qquad \hat{\sigma}_{\text{MLE}} = .822, \quad \text{AIC}=2.483, \qquad \text{and}$$

$$\widehat{y}_t = \underset{(0.07)}{0.87} \qquad \hat{\sigma}_{\text{MLE}} = .870, \quad \text{AIC}=2.572.$$

Bivariate analysis suggested a longer lag structure, with AIC selecting a VAR(5) in output and the spread as a baseline model. However, after omitting the insignificant lags and testing for serial correlation present in the residuals of each equation, we obtained the ARLI models presented in Appendix 2, which use just three lags. There was little difference between the results obtained using equation by equation OLS estimation, and system estimation. Both models show that the spread Granger-causes output, and that feedback is also present.

Nonlinearity tests based on a VAR(3) found widespread evidence of nonlinearity in each equation, as shown in Table 3. Both TAR and STAR tests supported regime switching behaviour in output, with any lag of s_t driving changes in regime. The spread equation is clearly nonlinear, regardless of the hypothesized switching variable. Related tests (see Luukkonen et al (1988)), and tests for the presence of a lagged depth of recession variable (see Beaudry and Koop, 1993) also found evidence of regime behaviour.

We estimated TAR and STAR specifications for each equation, using s_{t-1} as a transition variable, then s_{t-2} and then s_{t-3}. From these experiments, an

LSTAR model for output with s_{t-2} as the transition variable fitted the data best. For the spread equation, an LSTAR model with s_{t-1} as the transition fitted the data better than models with other transition variables, but the difference between alternative specifications was slight. The two chosen equations are given in Appendix 2, and we call this model our Bi-NARLI model. Given that lagged spread variables were possible transition variables for both equations, we also considered the possibility of a common nonlinear factor (see Anderson and Vahid (1998)). Such a factor was supported, using s_{t-2} as a transition variable, and the corresponding Com-NARLI model is given in Appendix 2.

Table 3: P-values of Nonlinearity Tests on USA Data

Test	Transition Variable	Output Equation	Spread Equation	Common Nonlinearity
TAR test	y_{t-1}	0.5822	0.0000	0.6101
(Tsay, 1986)	y_{t-2}	0.3144	0.0041	0.4077
	y_{t-3}	0.2613	0.0121	0.8752
	s_{t-1}	0.0051	0.0001	0.0079
	s_{t-2}	0.0006	0.0032	0.0050
	s_{t-3}	0.0189	0.0000	0.0455
STAR test	y_{t-1}	0.6607	0.0151	0.9397
(Teräsvirta,	y_{t-2}	0.3525	0.0000	0.2973
1994)	y_{t-3}	0.4074	0.0019	0.6387
	s_{t-1}	0.0024	0.0005	0.0399
	s_{t-2}	0.0007	0.0377	0.3018
	s_{t-3}	0.0134	0.0000	0.0067

Note: The last column tests the null hypothesis that any nonlinearities in the individual equations are the same. See Anderson and Vahid (1998) for details.

Table 4 presents the summary results of Harding and Pagan's (2001) procedure applied to 10,000 simulations of 160 observations of the DGP implied by each model. The results for our linear models are similar to those in Harding and Pagan. The random walk model fails to capture most aspects of the peak-to-trough stage and it is also unable to predict the shape of the trough-to-peak episodes (expansions). The latter is true for all of our linear models. In contrast, the nonlinear models can capture all features. In particular, the observed "excess" for expansions is within the simulated confidence intervals, showing that these models provide plausable specifications of the true data generating process.

Table 4: Simulated Business Cycle Characteristics for the USA.

	Raw Data	RW + Drift	AR(2)	ARLI	Bi NARLI	Com-NARLI
Duration						
PT	3.8	2.4* (2.0,3.4)	3.1 (2.0,4.5)	3.3 (2.2,5.0)	2.9 (2.0,4.0)	3.1 (2.0,4.4)
TP	20.4	36.5 (13.0,80.0)	31.0 (15.0,64.5)	27.6 (14.3,49.3)	37.0 (17.3,73.0)	32.6 (16.0,66.5)
Amplitude						
PT	−2.1	−1.0* (−1.7,−0.5)	−1.4 (−2.3,−0.7)	−1.5 (−2.5,−0.8)	−1.7 (−3.1,−0.7)	−1.9 (−3.2,−0.8)
TP	22.9	34.2 (12.2,75.6)	30.9 (15.0,64.5)	27.0 (13.3,50.3)	35.5 (16.6,69.9)	34.4 (16.3,69.6)
Cumulation						
PT	−4.2	−1.4* (−2.7,−0.5)	−2.6 (−6.0,−0.7)	−3.2 (−7.4,−0.9)	−2.8 (−6.2,−0.7)	−3.4 (−7.5,−0.9)
TP	342	1025 (97,3451)	865 (156,2649)	655 (136,1841)	1107 (195,3254)	958 (188,2901)
Excess						
PT	-0.10	0.00 (−0.15,0.15)	0.00 (−0.16,0.16)	0.00 (−0.16,0.17)	0.02 (−0.21,0.27)	0.02 (−0.19,0.24)
TP	1.36	0.05 (−1.43,1.58)	−0.00* (−1.40,1.34)	0.04* (−1.12,1.28)	0.08 (−1.20,1.38)	0.05 (−1.23,1.36)

Note: The values in parentheses are bounds of 90% confidence intervals derived from the simulated distributions. The asterisks highlight those sample statistics whose 90% bounds do not contain the observed cycle characteristic.

4.2. Canada

Univariate analysis of y_t found that an AR(1) specification was preferred to a random walk (in ln real GDP), but there was no evidence to support a longer lag structure for y_t. The relevant equations for the sample of 156 observations were

$$\widehat{y}_t = \underset{(0.10)}{0.63} + \underset{(0.08)}{0.31} y_{t-1} \qquad \hat{\sigma}_{\text{MLE}} = .874, \quad \text{AIC}=2.584, \qquad \text{and}$$

$$\widehat{y}_t = \underset{(0.07)}{0.91} \qquad \hat{\sigma}_{\text{MLE}} = .913, \quad \text{AIC}=2.669,$$

and we found no statistically significant evidence of univariate nonlinear behaviour in y_t.

The AIC criterion selected a VAR(2) in output and the spread as a baseline bivariate model, and after omitting the insignificant lags and testing for serial correlation present in the residuals of each equation, we obtained the ARLI models presented in Appendix 3. As in the US case, there was little difference between the results obtained using equation by equation OLS estimation, and system estimation. Both models show that the spread Granger-causes output, and that weak feedback is also present.

Although there was no evidence of an omitted CDR variable in either equation, TAR and STAR tests (see Table 5) found strong evidence of nonlinearity. These tests supported regime switching behaviour in each equation, with either y_{t-1} or s_{t-1} driving changes in regime. In general, the evidence supporting y_{t-1} as a transition variable in individual equations was stronger, and rejected the null that the nonlinearity was common. On the other hand, the evidence that s_{t-1} was driving regime changes was slightly weaker, and more consistent with a common nonlinear factor hypothesis.

Table 5: P-values of Nonlinearity Tests on Canadian Data

Test	Transition Variable	Output Equation	Spread Equation	Common Nonlinearity
TAR test (Tsay, 1986)	y_{t-1}	0.0025	0.0100	0.0090
	y_{t-2}	0.1229	0.9406	0.7835
	s_{t-1}	0.0368	0.0079	0.1974
	s_{t-2}	0.0718	0.1192	0.5477
STAR test (Teräsvirta, 1994)	y_{t-1}	0.0020	0.0019	0.0383
	y_{t-2}	0.1897	0.4443	0.7835
	s_{t-1}	0.1147	0.0062	0.4617
	s_{t-2}	0.1718	0.0224	0.9547

Note: The last column tests the null hypothesis that any nonlinearities in the individual equations are the same. See Anderson and Vahid (1998) for details.

We estimated TAR and STAR specifications for each equation, using y_{t-1} as a transition variable, and then again using s_{t-1} as the transition. STAR models with y_{t-1} as the transition fitted the data a little better and are presented in Appendix 3. The transition functions for each model are quite different, with y_t showing more pronounced evidence of threshold behaviour than $s_{t.}$ Despite the evidence against using y_{t-1} as a common transition variable, we estimated a system in which regimes changed with this variable. The resulting Com-NARLI model is given in Appendix 3. This model is quite different from the single equation STAR models (in that it is almost a threshold model, and the lower regime contains more observations), and relative to its single equation counterparts, it does not fit the data as well. The residuals of the single equation STAR models showed no evidence of serial correlation and only weak evidence of ARCH. Not surprisingly, the residuals of the Com-NARLI system were badly behaved, providing evidence that the common factor restriction was not appropriate.

Table 6: Simulated Business Cycle Characteristics for Canada.

	Raw Data	RW + Drift	AR(1)	ARLI	Bi NARLI	Com-NARLI
Duration						
PT	4.0	2.4* (2.0,3.4)	2.8 (2.0,4.0)	2.9 (2.0,4.0)	3.7 (2.0,6.0)	3.3 (2.0,5.2)
TP	16.0	36.4 (12.6,82.0)	27.6 (12.5,56.0)	27.6 (12.1,58.0)	36.9 (10.0,89.0)	36.0 (6.0,99.0)
Amplitude						
PT	−3.2	−1.1* (−1.8,−0.5)	−1.4* (−2.2,−0.7)	−1.4* (−2.3,−0.7)	−2.5 (−5.8,−0.7)	−1.9 (−3.9,−0.6)
TP	17.2	35.9 (12.3,79.8)	28.8 (12.4,60.0)	28.5 (11.8,60.0)	39.0 (9.5,95.6)	43.4 (5.0,132.1)
Cumulation						
PT	−6.6	−1.4* (−2.9,−0.5)	−2.2* (−4.8,−0.8)	−2.5* (−5.5,−0.8)	−6.7 (−21.5,−0.7)	−3.4 (−11.0,−0.7)
TP	257	1061 (91,3604)	694 (106,2128)	691 (96,2177)	1192 (55,4378)	1520 (16,6862)
Excess						
PT	0.27	−0.00* (−0.16,0.16)	0.00* (−0.15,0.14)	0.00* (−0.16,0.16)	0.06 (−0.19,0.35)	0.00 (−0.22,0.25)
TP	1.35	0.04 (−1.54,1.68)	0.03 (−1.31,1.40)	0.02 (−1.44,1.44)	0.14 (−1.81,2.24)	0.23 (−2.38,3.18)

Note: The values in parentheses are bounds of 90% confidence intervals derived from the simulated distributions. The asterisks highlight those sample statistics whose 90% bounds do not contain the observed cycle characteristic.

Output was simulated from each of the univariate and bivariate models, so that we could assess how well each "captures" the business cycle. We used each estimated model to simulate 10,000 output series of 158 observations, and then used Harding and Pagan's procedures to analyse the classical business cycle features implied by each series. Table 6 presents the features that are present in the Canadian data, and then the summary results for each set of simulations.

The results for the linear models are very interesting, because although each model can capture the trough to peak characteristics of the cycle, they are unable to predict the characteristics of peak to trough episodes (recessions). This inability is present, regardless of whether or not the interest rate spread is included in the model. However, once we use a bivariate nonlinear specification, there is a clear improvement. The bi-NARLI model can capture all features of recessionary phases that the linear models failed to capture. Out of all the models studied in this exercise, the mean prediction made for each peak to trough characteristic is closest to the corresponding observed value, when the Bi-NARLI model is used. Conversely, if we are interested in predicting any of the observed trough to peak statistics, then the ARLI model performs best. Relative to the linear models, the Com-NARLI model also performs well, although it fails to capture the shape of peak to trough episodes, like the linear models.

4.3. Australia

Harding and Pagan (2001) report that a random walk with drift model performs reasonably well in capturing the shape of the business cycle in Australia. They show that there is no significant autocorrelation in the growth rate of the Australian GDP, and they establish that the only characteristic of the shape of the business cycle in Australia that a random walk with drift model cannot reproduce is the trough to peak excess. Our data set spans a slightly different sample, and we also find no significant autocorrelation in the growth rate of the Australian GDP. However, our simulations find that a random walk with drift model fails in producing the peak to trough amplitude as well as the trough to peak excess. Here, we ask if a bivariate model of GDP growth and the interest rate spread can produce the shape characteristics of the Australian business cycles.

In a linear bivariate model of GDP growth and interest rate spread, Grangercausality tests show no evidence that the spread Granger-causes GDP growth. In fact, it seems that GDP growth is unpredictable on the basis of the bivariate information set that includes lags of GDP growth and the spread. This supports the random walk with drift model. However, there is strong evidence of heteroskedasticity in the GDP growth equation, which is quite apparent from the graph of the GDP growth in Appendix 1.

The unconditional variance of an autoregressive process depends on the variance of its innovations as well as the autoregressive parameters. Likelihood comparisons of models with a structural break in the autoregressive parameters and models with a break in the innovation variance favoured the latter, and supported a break in the variance of innovations in the first quarter of 1984. Conditional on a break in the innovation variance in 1984:1, the preferred model for the GDP growth is,

$$\hat{y}_t = \underset{(0.08)}{0.83} + \underset{(0.04)}{0.08} s_{t-2}. \tag{4.1}$$

Under the maintained hypothesis of heteroskedasticity, there is no evidence of nonlinearity or serial correlation in the errors of this equation.

As in the case of other countries, the interest rate spread in Australia, plotted in Appendix 1, has more interesting dynamics than GDP growth. There is strong evidence of nonlinearity in the time series model of interest rate spread in Australia with two lags of GDP growth and the spread on the right hand side. The tests of linearity against an LSTAR alternative provide strong evidence for the first lag of the spread as the transition variable. The best fitting LSTAR model has a centrality parameter of -0.35 and a very large smoothing parameter, which suggests that a TAR model will be appropriate for the spread. The fitted TAR model to the spread is,

$$\hat{s}_t = \underset{(0.13)}{0.18} + \underset{(0.07)}{0.81} s_{t-1} + (s_{t-1} < -0.35) \times \left(-\underset{(0.14)}{0.28} s_{t-2} - \underset{(0.21)}{0.69} y_{t-2} \right). \tag{4.2}$$

Table 7 shows the calculated shape statistics for the actual Australian data and for the simulated data from three models. The first is a random walk with drift model. The second, is a random walk with drift model with a break in variance in 1984:1. We have simulated data from this model to assess if the variance break is sufficient for producing a model that can capture the shape characteristics of the Australian business cycle. The third model, is the bivariate nonlinear model explained above.

Table 7: Simulated Business Cycle Characteristics for Australia

	Aus Data	RW + drift	RW + drift +hetero	TAR
Mean Duration (qrts)				
PT	3.7	2.8 (2.0,4.0)	3.9 (2.3,6.3)	3.0 (2.0,4.8)
TP	29.5	21.0 (10.0,41.0)	10.7* (4.8,21.7)	20.3 (6.5,45.5)
Mean Amplitude (%)				
PT	−3.1	−1.7* (−2.6,−1.0)	−5.3 (−8.6,−2.8)	−2.4 (−4.1,−1.0)
TP	28.3	22.1 (10.4,42.4)	17.8 (8.3,34.2)	22.5 (7.9,48.0)
Cumulation (%)				
PT	−5.2	−2.8 (−5.6,−1.1)	−12.8 (−30.2,−3.5)	−4.3 (−10.3,−1.1)
TP	457	391 (68,1143)	151 (23.4,478)	406 (30,1338)
Excess (%)				
PT	0.19	−0.00 (−0.20,0.20)	−0.00 (−0.62,0.62)	−0.01 (−0.32,0.31)
TP	1.53	0.02* (−1.14,1.18)	0.06 (−1.46,1.65)	0.27 (−1.22,2.02)

Note: The values in parenthese are bounds of 90% confidence intervals derived from the simulated distributions. The asterisks highlight those sample statistics whose 90% bounds do not contain the observed cycle characteristics.

4.4. United Kingdom

Univariate analysis of y_t found that a random walk (in ln real GDP) model was preferred, but there was significant evidence of change in the variance of the growth rate. The standard deviation of the output growth in the first half of the sample is 1.28, and 0.70 in the second half.. There is also significant evidence of positive serial correlation in the growth rate in the latter half of the sample. Determination of the break point by maximizing the likelihood leads to a break point at 1991:1. The following equations show the dramatic change in the time series properties of the output growth before and after this date:

$$y_t = \underset{(0.12)}{0.64} - \underset{(0.09)}{0.05} y_{t-1} + \hat{\varepsilon}_t \qquad \hat{\sigma}_\varepsilon = 1.16 \quad \text{for 1960:1 to 1990:4,}$$

$$y_t = \underset{(0.08)}{0.22} + \underset{(0.11)}{0.67} y_{t-1} + \hat{\varepsilon}_t \qquad \hat{\sigma}_\varepsilon = 0.31 \quad \text{for 1991:1 to 2000:2.}$$

This change is so dramatic that it begs the question of how to proceed. If we accept that the structure of the growth process has changed in the last ten years, would there be any point in estimating a time series model for the period before the change? The period after the change is too short to consider developing a nonlinear autoregressive leading indicator model for it. It has particularly been a quiet period with not many pronounced cycles, which makes the evaluation of models on the basis of their "business cycle characteristics" inapprorpriate. Alternatively, one can assume that this evidence is a sample peculiarity and ignore it, or speculate that it is a manifestation of other kinds of nonlineariy. We pursue the latter.

The AIC criterions selected a VAR(3) in output and the spread as a baseline bivariate model, and after omitting the insignificant lags and testing for serial correlation present in the residuals of each equation, we obtained the ARLI models presented in Appendix 4. As in other cases, there was little difference between the results obtained using equation by equation OLS estimation, and system estimation. Both models show that the spread Granger-causes output, and that weak feedback is also present.

Although there was no evidence of an omitted CDR variable in either equation, RESET, TAR and STAR tests found strong evidence of nonlinearity in the output equation. The p-values for the latter three tests were 0.0150, 0.0015 and 0.0048, with y_{t-2} as the transition variable in the TAR and STAR tests. Only the STAR test with y_{t-2} as the transition variable found evidence of nonlinearity in the spread equation (the p-value was 0.0421), and a common nonlinearity test rejected the null hypothesis that the output and spead equations had a common nonlinear STAR factor.

We estimated STAR specifications for each equation, using y_{t-2} as a transition variable. The maximum likelihood estimate of the smoothing parameter in the spread equation was quite large, and the centre of its transition function was 1.94, which effectively meant that we had a threshold model with very few observations in one regime. This model is very similar to the linear model, with some outlying observations removed. Since the number of outlying observations only just exceeded the number of estimated parameters, we decided to simply use the linear model for the spread[1]. Thus, our Bi-NARLI model has a non-linear output equation and a linear spread equation. The estimated linear and nonlinear models for output and the spread are included in Appendix 4.

Table 8: Simulated Business Cycle Characteristics for the UK.

	Raw Data	RW + Drift	AR(1)+ Break	ARLI	Bi NARLI
Duration					
PT	4.4	3.0* (2.3,4.0)	3.2* (2.3,4.3)	3.4 (2.4,4.7)	3.9 (2.5,5.8)
TP	25.5	14.7* (8.6,24.5)	13.6* (7.6,23.0)	15.3 (8.9,25.5)	16.6 (9.1,29.0)
Amplitude					
PT	−3.2	−1.7* (−2.3,−1.1)	−1.9* (−2.8,−1.3)	−1.9* (−2.7,−1.2)	−2.3 (−3.8,−1.3)
TP	21.5	12.2* (7.3,20.2)	11.9* (6.9,19.9)	12.9 (7.3,21.6)	14.8 (7.4,27.0)
Cumulation					
PT	−9.6	−3.0* (−5.4,−1.4)	−3.7* (−7.0,−1.6)	−3.9* (−7.8,−1.6)	−6.4 (−15.9,−1.8)
TP	381	158 (45,394)	141* (37,356)	173 (47,435)	233 (51,626)
Excess					
PT	−0.14	−0.00* (−0.13,0.13)	0.00 (−0.17,0.15)	0.00 (−0.15,0.15)	0.01 (−0.17,0.20)
TP	−0.50	0.00 (−0.53,0.53)	0.03 (−0.55,0.63)	−0.00 (−0.61,0.61)	0.04 (−0.73,0.79)

Note: The values in parentheses are bounds of 90% confidence intervals derived from the simulated distributions. The asterisks highlight those sample statistics whose 90% bounds do not contain the observed cycle characteristic.

Table 8 presents the features that are present in the UK data, and then summary results for simulations based on each model. As in Harding and Pagan (2001), we reduce the minimum duration of a complete cycle to 4 quarters in the

[1] Readers may rest assured that this decision was made before the abilities of models in capturing business cycle characteristics were evaluated.

case of UK, because otherwise the algorithm does not choose 1974 as a recession. The results quite clearly favour the nonlinear autoregressive leading indicator model. None of the linear models can produce cycles that have the similar shape to the real data in recessions (peak to trough). Allowing for structural break in 1990:4 in a univariate time series model of output does not help at all. The biNARLI model is the only model that can capture all features of the business cycle that the linear models failed to capture[2].

4.5. Netherlands

The graph of output growth for the Netherlands suggests structural change in this series, and further analysis based on univariate models suggests a structural break midway between 1974, for both the random walk model and the AR(3) chosen by AIC. The relevant equations for the random walk models are

$$\begin{aligned}
\widehat{y}_t &= \underset{(0.11)}{0.73}, \quad \hat{\sigma}_{\text{MLE}} = 1.1994, \quad \text{for the 1966:1-1997:4 sample, and} \\
\widehat{y}_t &= \underset{(0.26)}{1.17}, \quad \hat{\sigma}_{\text{MLE}} = 1.4678 \quad \text{together with } \widehat{y}_t = \underset{(0.11)}{0.57}, \quad \hat{\sigma}_{\text{MLE}} = 1.0406
\end{aligned}$$

for the 1966:1-1974:2 and 1974:3-1997:3 subsamples. The third lag in the AR(3) model became insignificant after allowing for the structural break, and only one lag was needed for the latter subsample (see Appendix 5 for these equations), but apart from this evidence of structural change, univariate nonlinearity tests found no further evidence of nonlinear behaviour in y_t.

The AIC criterion selected a VAR(3) in output and the spread as a baseline bivariate model, and after omitting the insignificant lags and testing for serial correlation present in the residuals of each equation, we obtained the ARLI models presented in Appendix 5. As for the other countries, there was little difference between the results obtained using equation by equation OLS estimation, and system estimation. Both models indicated that the spread Granger-causes output, and that weak feedback was present.

[2] We also studied the model with the non-linear spread equation which we had ruled out in favour of the one with linear spread equation. The results are very close to the results of the BiNARLI model presented in the table.

Analysis on the output equation in the VAR(3) found no evidence of nonlinearity (apart from the structural change). Given that our experience with the UK case had shown that formal modeling of a structural change was unnecessary once we allowed for nonlinearity elsewhere in the system, we maintained the output equation from our ARLI model and turned to the modeling of nonlinearities in the spread equation. The rationale behind this was that if the spread equation were nonlinear, then the lagged spread variables in the (linear) output equation would feed nonlinearities into data generating process for y_t. Formal nonlinearity tests on the spread equation (see Table 9) provided strong support for regime switching behaviour, but did not clearly indicate an appropriate specification. Accordingly, we estimated both TAR and STAR specifications with y_{t-1}, y_{t-3}, s_{t-1}, s_{t-2} or s_{t-3} driving changes in regime, and found that a STAR model with s_{t-2} as the transition variable led to a fit that was much better than all other alternatives. Our resulting Bi-NARLI model (which is given in Appendix 5) consists of the ARLI output equation and a nonlinear STAR model of the spread.

Table 9: P-Values of Nonlinearity Tests on the Spread for Netherlands

Transition Variable	Luukonen et al Test (1988)	TAR test (Tsay, 1989)	STAR test (Teräsvirta, 1994)
y_{t-1}	0.0086	0.0069	0.0075
y_{t-2}	0.3908	0.2191	0.3030
y_{t-3}	0.0292	0.1409	0.0188
s_{t-1}	0.0069	0.0328	0.0767
s_{t-2}	0.0006	0.2406	0.0006
s_{t-3}	0.0018	0.0005	0.0034

Table 10 shows the ability of the various models to capture the features of the Netherlands' business cycle. Here, we see that all models describe the business cycle rather well, with the random walk with break model, the ARLI model and the Bi-NARLI model capturing all features of interest. The simulations show that the simple random walk model with drift is inadequate, but there is no clear winner out of the three models that perform well. It appears that the presence of the spread in the output equation helps to explain business cycle characteristics, although this is not necessary if one accounts for nonlinearities by incorporating a break in the output process.

Table 10: Simulated Business Cycle Characteristics for Netherlands.

	Raw Data	RW + drift	RW + drift+break	AR with break	ARLI	Bi NARLI
Duration						
PT	2.5	3.1 (2.2,4.5)	3.1 (2.2,4.4)	3.0 (2.0,4.3)	3.0 (2.0,4.3)	3.1 (2.0,4.5)
TP	14.4	16.7 (9.0,30.0)	15.6 (8.4,27.3)	17.5 (8.3,34.3)	21.9 (10.7,42.5)	21.8 (10.4,43.0)
Amplitude						
PT	−2.4	−2.0 (−2.8,−1.2)	−1.9 (−2.7,−1.1)	−1.6* (−2.3,−0.9)	−1.7 (−2.5,−0.9)	−1.7 (−2.7,−1.0)
TP	14.9	16.3 (8.9,29.0)	14.0 (7.2,25.0)	13.9 (6.5,27.1)	19.9 (9.9,38.0)	20.0 (9.4,39.0)
Cumulation						
PT	−3.1	−3.4 (−6.7,−1.4)	−3.3 (−6.6,−1.4)	−2.6 (−5.3,−1.0)	−2.9 (−3.1,−1.1)	−3.1 (−6.7,−1.1)
TP	185	229 (53,619)	188 (40,509)	207 (34,617)	357 (69,1021)	363 (64,1075)
Excess						
PT	-0.03	0.00 (−0.19,0.19)	0.00 (−0.18,0.17)	0.00 (−0.18,0.18)	0.00 (−0.20,0.19)	0.00 (−0.20,0.19)
TP	1.02	0.02* (−0.80,0.87)	0.20 (−0.58,1.12)	0.14 (−0.67,1.05)	0.04 (−1.01,1.08)	0.01 (−1.05,1.07)

Note: The values in parentheses are bounds of 90% confidence intervals derived from the simulated distributions. The asterisks highlight those sample statistics whose 90% bounds do not contain the observed cycle characteristic.

5. Conclusion and Directions for Further Research

In this paper we develop bivariate nonlinear models of output and the interest rate spreads for five OECD countries. Our primary aim is to develop models that can account for key business cycle characteristics, and we evaluate our models by assessing whether or not they imply the cyclical features that are present in observed data. We find that our models can capture the amplitude, depth and shape characteristics of cycles, and that they also account for differences between expansionary and recessionary phases.

Our results are consistent with other research that has established a clear link between yield spreads and output, but they differ from most previous work because our framework is nonlinear, rather than linear. We find that the nonlinearity in our specifications is important, because most linear models of output and the spread have difficulty in accounting for the characteristics of recessions. The linear models also fail to capture the shape of expansions. With respect to differences across countries, our findings support Galbraith and Tkacz (2000). These authors found that the evidence of asymmetries in the spread-output relationship was stronger in North American countries, than in Europe.We find that the nonlinearity in the output equations for non-North American countries is more subtle, arising from nonlinearity in just one of the two equations in the bivariate system.

Harding and Pagan (2001) find that univariate nonlinear representations of output do not capture business cycles, and this, together with the work reported here, suggests that the bivariate nature of our models is important. Given Harding and Pagan's (2001) conclusions that VAR models with investment and consumption do not capture business cycle features, we also believe that the use of the spread, rather than other possible explanators is also important. We have undertaken other research (in Anderson and Vahid (2001)) that shows that bivariate nonlinear models of output and money are inferior to those of output and the spread, and this suggests that the spread is capturing more than just the effects of monetary policy on output.

Recent work in the Federal Reserve System in the USA (see, eg Kozicki (1997)) has advocated the use of error correction terms from models of financial markets in models of real variables, and we think that this insight is important. The yield spread is, of course a valid error correction term in modeling the bond market, and as such, it summarizes many features of the bond sector. We believe that further research that employs international interest rate differentials, and error correction terms from stock markets and exchange rate markets may also lead to useful models of output, but we do not pursue this idea further here.

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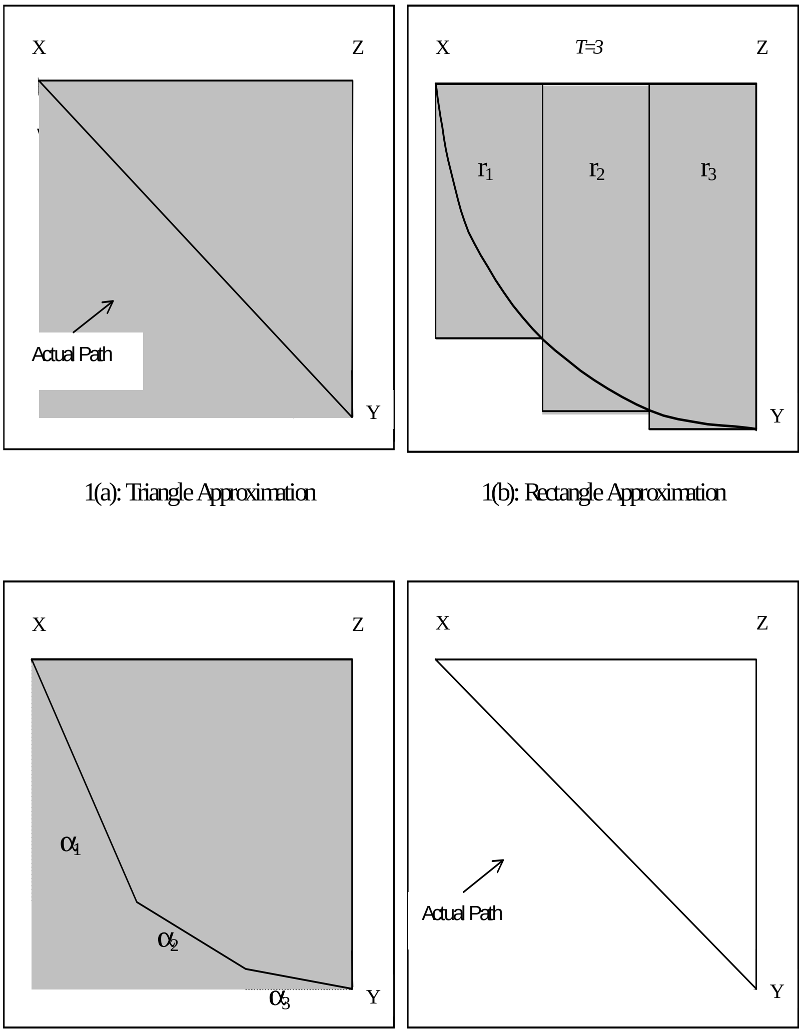
Figure 1: Calculation of Cumulative Loss and Excess



1(a): Triangle Approximation

1(b): Rectangle Approximation

1(c): Bias Corrected Approximation

1(d): Excess

APPENDIX 1: DATA

Precise descriptions of the raw series that we use in this analysis are given below. Unless otherwise stated, we have drawn all data from the OECD portion of the DX database. We use the logarithms of real GDP when we undertake our benchmark analysis, and our models are functions of output growth ($y_t = 100 \times \Delta \ln(GDP)$) and the interest rate spread (s_t = Long-term interest rate - Short-term interest rate). The effective samples used for analysis are shorter than the raw series because of lagged variables in the models.

USA (1960:1 to 2000:4)

Output: Real Gross Domestic Product: (Billions of Chained 1996 Dollars, seasonally adjusted at annual rates, from the U.S. Department of Commerce, Bureau of Economic Analysis).

Short-Term Interest Rates: 3-Month Treasury (Secondary) Bill Market Rates (Averages over business days expressed as a percentage, H15 Release from the Federal Reserve Board of Governors).

Long-Term Interest Rates: 10-Year Treasury Bond Constant Maturity Rates (Averages over business days expressed as a percentage, H15 Release from the Federal Reserve Board of Governors).

The effective sample for analysis consisted of 160 observations, dating from 1961:1 to 2000:4.

CANADA (1961:1 to 2000:3)

Output: Real Gross Domestic Product (seasonally adjusted in constant 1992 prices, series CAN.NAGVTT01.NCALSA).

Short-Term Interest Rates: Interest rates on 90 day deposit receipts. (expressed as a percentage pa, series CAN.IRT3DR01.ST).

Long-Term Interest Rates: Yields on long term government bonds (>10 Years). (expressed as a percentage pa, series CAN.IRLGV06.ST)..

The effective sample for analysis consisted of 156 observations, dating from 1961:4 to 2000:3.

AUSTRALIA (1969:3 to 2000:3)

Output: Real Gross Domestic Product, (seasonally adjusted in 1998/1999 prices, series AUS.NAGVTT01.NCALSA).

Short-Term Interest Rates: Yield on 3-Month Treasury Note (expressed as a percentage pa, series FIRMMTNIY3).

Long-Term Interest Rates: Yield on Long-Term Treasury Bonds (expressed as a percentage pa, series AUS.IRLTGV02.ST).

The effective sample for analysis consisted of 122 observations, dating from 1970:2 to 2000:3.

UNITED KINGDOM (1960:1 to 2000:2)

Output: Real Gross Domestic Product (seasonally adjusted in constant 1995 prices, series GBR.NAGVTT01.NCALSA).

Short-Term Interest Rates: 3 Month Treasury Bill Rates. (expressed as a percentage pa, series 11260C..ZF... from the IFS portion of the DX database).

Long-Term Interest Rates: Yields on 10 Year Government Bonds (expressed as a percentage pa, series GBR.IRLTGV02.ST)..

The effective sample for analysis consisted of 158 observations, dating from 1961:1 to 2000:2

NETHERLANDS (1965:1 to 1997:4)

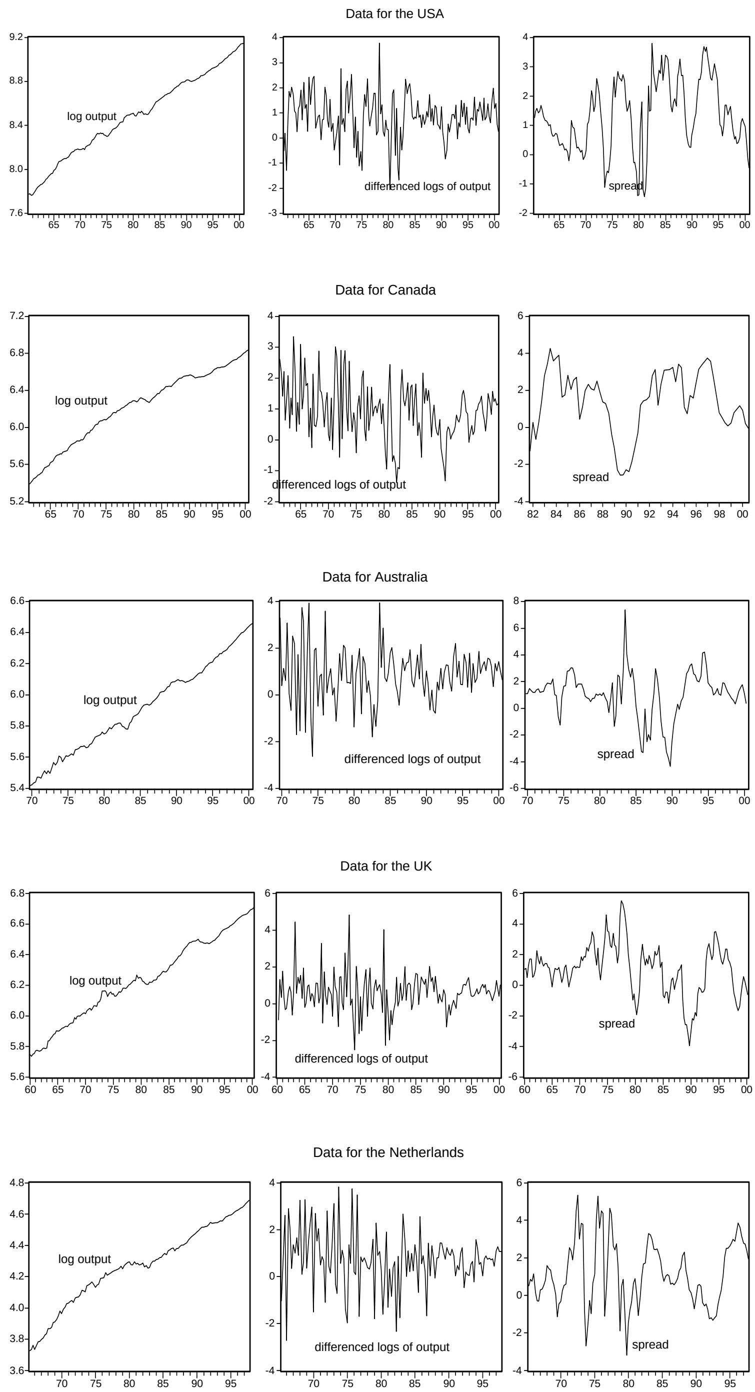
Output: Real Gross Domestic Product (seasonally adjusted volume index with 1995 base, series NLD.NAGVVO01.IXOBSA).

Short-Term Interest Rates: 3-Month Loans to Local Authorities (quarterly averages over monthly data, expressed as a percentage pa, series taken from the OECD portion of McGill University database).

Long-Term Interest Rates: 10 Year Government Bonds Yield. (expressed as a percentage pa, series 13861..ZF..from IFS portion of the DX data-base).

The effective sample for analysis consisted of 128 observations, dating from 1966:1 to 1997:4.

Figure 2: Data



APPENDIX 2: BIVARIATE MODELS OF OUTPUT AND SPREAD

(USA: 1961:1 - 2000:4)

(Standard errors are in brackets)

ARLI-OLS model of output and spread:

$$\hat{y}_t = \underset{(0.11)}{0.29} + \underset{(0.08)}{0.20} y_{t-1} + \underset{(0.08)}{0.14} y_{t-2} + \underset{(0.06)}{0.20} s_{t-2} \qquad \hat{\sigma}_{\text{MLE}} = 0.79$$

$$\hat{s}_t = \underset{(0.08)}{0.26} - \underset{(0.05)}{0.15} y_{t-2} + \underset{(0.08)}{1.06} s_{t-1} - \underset{(0.11)}{0.36} s_{t-2} + \underset{(0.08)}{0.19} s_{t-3} \qquad \hat{\sigma}_{\text{MLE}} = 0.55$$

ARLI-SYS model of output and spread:

$$\hat{y}_t = \underset{(0.11)}{0.29} + \underset{(0.08)}{0.19} y_{t-1} + \underset{(0.08)}{0.14} y_{t-2} + \underset{(0.06)}{0.20} s_{t-2} \qquad \hat{\sigma}_{\text{MLE}} = 0.79$$

$$\hat{s}_t = \underset{(0.08)}{0.26} - \underset{(0.05)}{0.15} y_{t-2} + \underset{(0.08)}{1.06} s_{t-1} - \underset{(0.11)}{0.35} s_{t-2} + \underset{(0.08)}{0.18} s_{t-3} \qquad \hat{\sigma}_{\text{MLE}} = 0.55$$

Bi-NARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= -\underset{(0.23)}{0.52} y_{t-1} + \underset{(0.24)}{0.49} y_{t-2} + \underset{(0.29)}{0.50} y_{t-3} - \underset{(0.26)}{0.66} s_{t-1} + \underset{(0.36)}{1.37} s_{t-2} + \\ &\quad f_{yt} \times (\underset{(0.16)}{0.81} + \underset{(0.24)}{0.71} y_{t-1} - \underset{(0.25)}{0.43} y_{t-2} - \underset{(0.29)}{0.57} y_{t-3} + \underset{(0.29)}{0.73} s_{t-1} - \underset{(0.37)}{1.40} s_{t-2}) \\ f_{yt} &= (1 + \exp\{-14 (s_{t-2} - 0.024)\})^{-1} \qquad \hat{\sigma}_{\text{MLE}} = 0.71 \end{aligned}$$

$$\begin{aligned} \hat{s}_t &= \underset{(0.09)}{0.45} - \underset{(0.08)}{0.24} y_{t-2} + \underset{(0.13)}{1.19} s_{t-1} - \underset{(0.14)}{0.56} s_{t-2} + \\ &\quad f_{st} \times (\underset{(0.25)}{0.21} y_{t-2} - \underset{(0.07)}{0.11} y_{t-3} - \underset{(0.13)}{0.47} s_{t-1} + \underset{(0.20)}{0.35} s_{t-2} + \underset{(0.09)}{0.34} s_{t-3}) \\ f_{st} &= (1 + \exp\{-6.79 (s_{t-1} - 1.24)\})^{-1} \qquad \hat{\sigma}_{\text{MLE}} = 0.49 \end{aligned}$$

Com-NARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.43)}{-1.45} - \underset{(0.24)}{0.39} y_{t-1} + \underset{(0.27)}{0.76} y_{t-2} + \underset{(0.18)}{0.29} s_{t-3} - \underset{(0.53)}{1.89} com_t, \qquad \hat{\sigma}_{\text{MLE}} = 0.73 \\ \hat{s}_t &= \underset{(0.29)}{1.25} + \underset{(0.13)}{0.23} y_{t-1} - \underset{(0.16)}{0.46} y_{t-2} + \underset{(0.07)}{1.03} s_{t-1} - \underset{(0.11)}{0.19} s_{t-2} + com_t, \qquad \hat{\sigma}_{\text{MLE}} = 0.52 \\ com_t &= (1 + \exp\{-1.19 (s_{t-2} + 0.55)\})^{-1} \left(\underset{(0.40)}{-1.27} - \underset{(0.13)}{0.32} y_{t-1} + \underset{(0.17)}{0.39} y_{t-2} + \underset{(0.08)}{0.19} s_{t-3} \right) \end{aligned}$$

APPENDIX 3: BIVARIATE MODELS OF OUTPUT AND SPREAD

(Canada: 1961:4 - 2000:3)

(Standard errors are in brackets)

ARLI-OLS model of output and spread:

$$\hat{y}_t = \underset{(0.10)}{0.55} + \underset{(0.08)}{0.19} y_{t-1} + \underset{(0.04)}{0.20} s_{t-2} \qquad \hat{\sigma}_{\text{MLE}} = 0.8175$$

$$\hat{s}_t = \underset{(0.09)}{0.23} - \underset{(0.07)}{0.11} y_{t-1} + \underset{(0.08)}{1.03} s_{t-1} - \underset{(0.08)}{0.16} s_{t-2} \qquad \hat{\sigma}_{\text{MLE}} = 0.7623$$

ARLI-SYS model of output and spread:

$$\hat{y}_t = \underset{(0.10)}{0.55} + \underset{(0.08)}{0.19} y_{t-1} + \underset{(0.06)}{0.20} s_{t-2} \qquad \hat{\sigma}_{\text{MLE}} = 0.8175$$

$$\hat{s}_t = \underset{(0.09)}{0.23} - \underset{(0.07)}{0.12} y_{t-1} + \underset{(0.08)}{1.02} s_{t-1} - \underset{(0.08)}{0.16} s_{t-2} \qquad \hat{\sigma}_{\text{MLE}} = 0.7623$$

Bi-NARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.16)}{0.37} + \underset{(0.20)}{0.64} y_{t-2} + \underset{(0.11)}{0.42} s_{t-1} + \underset{(0.04)}{0.13} s_{t-2} + \\ &\quad f_{yt} \times (\underset{(0.21)}{0.21} + \underset{(0.08)}{0.22} y_{t-1} - \underset{(0.20)}{0.64} y_{t-2} - \underset{(0.11)}{0.42} s_{t-2}) \\ f_{yt} &= (1 + \exp\{-41.69\,(y_{t-1} + 0.042)\})^{-1} \qquad \hat{\sigma}_{\text{MLE}} = 0.7593 \end{aligned}$$

$$\begin{aligned} \hat{s}_t &= \underset{(0.73)}{2.37} + \underset{(0.89)}{1.68} y_{t-1} - \underset{(0.38)}{0.61} y_{t-2} + \underset{(0.15)}{0.56} s_{t-2} + \\ &\quad f_{st} \times (\underset{(0.73)}{-2.37} - \underset{(0.89)}{1.68} y_{t-1} + \underset{(0.38)}{0.61} y_{t-2} + \underset{(0.09)}{1.14} s_{t-1} - \underset{(0.17)}{0.79} s_{t-2}) \\ f_{st} &= (1 + \exp\{-4.72\,(y_{t-1} + 0.32)\})^{-1} \qquad \hat{\sigma}_{\text{MLE}} = 0.7037 \end{aligned}$$

Com-NARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.11)}{0.66} + \underset{(0.10)}{0.37} y_{t-1} + \underset{(0.08)}{0.42} s_{t-2} + com_t, \qquad \hat{\sigma}_{\text{MLE}} = 0.7629 \\ \hat{s}_t &= \underset{(0.10)}{0.17} - \underset{(0.09)}{0.27} y_{t-1} + \underset{(0.08)}{1.02} s_{t-1} - \underset{(0.09)}{0.29} s_{t-2} - \underset{(0.25)}{0.67} com_t, \qquad \hat{\sigma}_{\text{MLE}} = 0.7367 \\ com_t &= (1 + \exp\{-211\,(y_{t-1} - 0.38)\})^{-1} \left(\underset{(0.10)}{-0.37} y_{t-1} + \underset{(0.06)}{0.12} y_{t-2} + \underset{(0.16)}{0.16} s_{t-1} - \underset{(0.08)}{0.42} s_{t-2} \right) \end{aligned}$$

APPENDIX 4: BIVARIATE MODELS OF OUTPUT AND THE SPREAD FOR THE UK (1960:1 - 2000:2)

ARLI-OLS model of output and spread:

$$\hat{y}_t = \underset{(0.10)}{0.38} + \underset{(0.08)}{0.16} y_{t-3} + \underset{(0.05)}{0.12} s_{t-2} \qquad \hat{\sigma} = 1.01$$

$$\hat{s}_t = \underset{(0.07)}{0.20} - \underset{(0.05)}{0.16} y_{t-3} + \underset{(0.07)}{1.15} s_{t-1} - \underset{(0.08)}{0.27} s_{t-2} \qquad \hat{\sigma} = 0.67$$

Bi-NARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.11)}{0.26} + \underset{(0.11)}{0.22} y_{t-2} + \underset{(0.47)}{1.10} s_{t-1} - \underset{(0.61)}{1.30} s_{t-2} + \underset{(0.36)}{0.75} s_{t-3} + \\ &\quad f_{yt} \times (\underset{(0.26)}{0.77} y_{t-1} - \underset{(0.15)}{0.39} y_{t-3} - \underset{(0.78)}{1.70} s_{t-1} + \underset{(1.03)}{2.11} s_{t-2} - \underset{(0.65)}{1.21} s_{t-3}) \\ f_{yt} &= (1 + \exp\{-1.44 (y_{t-2} - 0.38)\})^{-1} \qquad \hat{\sigma} = 0.94 \end{aligned}$$

$$\hat{s}_t = \underset{(0.07)}{0.20} - \underset{(0.05)}{0.16} y_{t-3} + \underset{(0.07)}{1.15} s_{t-1} - \underset{(0.08)}{0.27} s_{t-2} \qquad \hat{\sigma} = 0.67$$

APPENDIX 5: MODELS OF OUTPUT AND SPREAD

(Netherlands: 1966:2 - 1997:4)

(Standard errors are in brackets)

Reduced AR(3) model that incorporates structural change:

$$\hat{y}_t = \underset{(0.40)}{2.44} - \underset{(0.16)}{0.62} y_{t-1} - \underset{(0.16)}{0.42} y_{t-2} \quad \text{(for 1966:1-1974:2)} \qquad \hat{\sigma}_{\text{MLE}} = 1.4157$$

$$\hat{y}_t = \underset{(0.12)}{0.65} - \underset{(0.10)}{0.15} y_{t-1} \quad \text{(for 1974:3-1997:42)} \qquad \hat{\sigma}_{\text{MLE}} = 1.0572$$

ARLI-OLS model of output and spread:

$$\hat{y}_t = \underset{(0.15)}{0.60} - \underset{(0.08)}{0.23} y_{t-1} + \underset{(0.08)}{0.20} y_{t-3} + \underset{(0.06)}{0.14} s_{t-2} \qquad \hat{\sigma}_{\text{MLE}} = 1.1233$$

$$\hat{s}_t = \underset{(0.14)}{0.51} - \underset{(0.08)}{0.19} y_{t-2} - \underset{(0.07)}{0.18} y_{t-3} + \underset{(0.09)}{0.99} s_{t-1} - \underset{(0.12)}{0.41} s_{t-2} + \underset{(0.09)}{0.22} s_{t-3}, \quad \hat{\sigma}_{\text{MLE}} = 0.9628$$

ARLI-SYS model of output and spread:

$$\hat{y}_t = \underset{(0.14)}{0.59} - \underset{(0.08)}{0.23} y_{t-1} + \underset{(0.08)}{0.20} y_{t-3} + \underset{(0.06)}{0.14} s_{t-2} \qquad \hat{\sigma}_{\text{MLE}} = 1.1233$$

$$\hat{s}_t = \underset{(0.13)}{0.51} - \underset{(0.07)}{0.18} y_{t-2} - \underset{(0.07)}{0.19} y_{t-3} + \underset{(0.08)}{0.99} s_{t-1} - \underset{(0.12)}{0.41} s_{t-2} + \underset{(0.09)}{0.22} s_{t-3}, \quad \hat{\sigma}_{\text{MLE}} = 0.9628$$

Bi-NARLI model of output and spread:

$$\hat{y}_t = \underset{(0.15)}{0.60} - \underset{(0.08)}{0.23} y_{t-1} + \underset{(0.08)}{0.20} y_{t-3} + \underset{(0.06)}{0.14} s_{t-2} \qquad \hat{\sigma}_{\text{MLE}} = 1.1233$$

$$\hat{s}_t = \underset{(0.11)}{0.28} - \underset{(0.07)}{0.11} y_{t-2} + \underset{(0.13)}{1.16} s_{t-1} - \underset{(0.14)}{0.46} s_{t-2} + \underset{(0.08)}{0.20} s_{t-3}$$

$$f_{st} \times (+\underset{(0.21)}{1.01} y_{t-1} - \underset{(0.68)}{1.59} y_{t-2} - \underset{(0.28)}{0.82} y_{t-3})$$

$$f_{st} = (1 + \exp\{-16.18\,(s_{t-2} - 3.81)\})^{-1} \qquad \hat{\sigma}_{\text{MLE}} = 0.6107$$