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THE VOLATILITY OF THE SOCIALLY OPTIMAL LEVEL OF INVESTMENT

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ABSTRACT

In this paper an annual series for the socially optimal level of investment from 1960-61 to 1993-94 for Australia is derived from a vintage production function and compared to the actual series for investment over the same period. The vintage production function can be expected to yield a smoother socially optimal investment series than that derived from a non-vintage, malleable capital production function. Even so, the resulting series for socially optimal investment is much more volatile than the series for actual investment. Several alternative assumptions are tried in an attempt to further smooth the socially optimal investment series. These include smoothing the assumed values of the exogenous variables, modelling adjustment costs and delivery lags, and changing the form of the production function. While these approaches do succeed in smoothing the investment series, in order for the socially optimal investment series to be as smooth as the actual series the assumed values of the parameters must be quite unrealistic. In the conclusion we suggest that in future research on the socially optimal level of investment the role of liquidity constraints and of irreversible investment should be investigated.

Key Words: Investment, vintage, production function, adjustment costs, investment volatility

Journal of Economic Literature Classification: E13, E22, E23.

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1. Introduction.

In public debate and discussion of macroeconomic performance, the level of investment is subject to frequent scrutiny and comment. Often concerns are raised that the level of investment is too low. Higher levels of investment are urged in the expectation that they will, by raising labour productivity, raise living standards. In spite of these concerns, economists have devoted little attention to calculating the socially optimal level of investment spending. It is true that there is a considerable literature on modelling the investment behaviour of privately optimising firms in an attempt to explain the pattern of actual investment (see Chirinko, 1993, for a survey). However, the privately optimal level of investment for the individual firm may be different from the socially optimal level. Some reasons for this are discussed in the conclusion. Abel et.al. (1989) have developed a criterion for determining whether an economy is dynamically efficient. Their rule is a generalisation of the rule, in Diamond (1965), for dynamic efficiency in a steady state . However, while these rules maximise social welfare, and, hence, investment is socially optimal, they assume that the economy is in a steady state. An advantage of the approach in our paper is that it avoids steady state assumptions. For instance, interest rates and employment growth are allowed to vary over the planning horizon instead of setting these variables at constant long run levels. Hence, it is hoped that this flexibility makes the model more empirically useful.

In this paper we develop and apply to Australia a method for calculating the socially optimal level of investment. This exercise is part of a broader project in which we are attempting to calculate the socially optimal level of aggregate saving and its socially optimal disposition between investment expenditure and the accumulation of foreign assets. Our calculations yield a series for the socially optimal level of investment which has, compared with the series of the actual level of investment, a very high coefficient of variation.[1] This is notwithstanding the use of a vintage putty-clay production function to calculate our series. Such a production function will yield a

[1] The series for the socially optimal level of investment is calculated as a proportion of GDP. This avoids the problems of a non-stationary series. Bertola and Caballero (1994) adopt the same procedure to assess the volatility of investment.

smoother series than a production function with malleable capital.

It seems, at first sight at least, unreasonable to argue for a socially optimal investment series with such a high level of volatility. Because of this we investigate various ways in which the series can be smoothed. These include smoothing the exogenous variables, varying the elasticity of substitution between labour and capital and introducing adjustment costs of investment. Even with the choice of reasonable degrees of smoothing using these techniques we find that the series for the socially optimal level of investment spending is still very volatile compared with the series for the actual level of investment. This result is discussed in the conclusion.

2. A Neoclassical Vintage Model : The Base Case.

2.1 Why a vintage model ?

In neoclassical theory, the (non-vintage) production function is of the general form :

$$Y_j = f(K_j, L_j) \tag{1}$$

where K_j and L_j are the capital stock and employment levels at time j, respectively. The early neoclassical model of investment was developed by Jorgenson (1963). It assumes perfect competition and determines the optimal capital stock at the level of the firm by assuming either that output is exogenously given and the firm faces constant returns to scale, or that there are diminishing returns to scale and output is endogenous. One of these alternative assumptions is necessary for the capital stock to be determinate at the level of the firm, which establishes microfoundations for aggregate investment. However, if one is prepared to model aggregate investment assuming that aggregate employment is exogenous, then the problem of an indeterminate capital stock no longer exists. Instead, the optimal capital stock is determined by only one first order condition: that the marginal product of capital equals the (real) user cost of capital.

The problem is that if optimal investment were derived from this first order condition, it would exhibit massive volatility. A simple example gives the intuition for this result. Suppose that the ratio of investment to output is 0.2 and the ratio of the capital stock to output is 4.0, plausible ratios for Australia. A change in the user cost of capital which causes the desired capital stock to change by say 1% will cause investment to change by 20%, from 20% of output to 24% of output. The early neoclassical models tried to overcome this problem by defining investment as a distributed lag function of the optimal capital stock. The lag function is intended to capture the observation that investment projects take time to complete as a result of, for instance, unforseen delivery lags. The effect is to reduce the volatility of investment which would occur if the entire capital stock were to respond instantly to a change in the user cost of capital. Since the lag function is not derived explicitly from optimising behaviour, it introduces an *ad hoc* element into the investment model. In later models capital is assumed to be costly to adjust, which is captured by an adjustment cost function. Like delivery lags, the adjustment cost model also has the effect of smoothing the series for aggregate optimal investment.

A vintage production function, as used in this paper, will yield a smoother investment series than the non-vintage model, ceteris paribus.[2] This is because the vintage model requires a change in only the marginal vintage of capital, rather than the entire stock of capital, in response to a change in the user cost of capital. Aggregate investment in each year is defined to constitute a vintage of capital. Hence, in the simple numerical example given above, it would not be the aggregate capital stock that changes by 1%, but simply aggregate investment. The resulting series for investment will clearly exhibit less volatility than the non-vintage model. We can then apply modifications, such as adjustment costs, to the base case in order to further reduce investment volatility.

[2] The vintage model allows for a relaxation of the assumptions of homogeneous capital and disembodied technical progress, implicit in the non-vintage model. Instead, technical progress is embodied in successive vintages of capital, which are increasingly efficient.

2.2 Determining the socially optimal level of investment.

Using the form of the vintage production function derived in Appendix A, in this section a series for the socially optimal level of investment for Australia for the period 1960-61 to 1993-94 is derived from the intertemporal optimising problem outlined in Appendix B (see also Guest and McDonald, 1995). The model is characterised by a representative agent who maximises utility and borrows or lends in a perfect world capital market. To generate a base case it is assumed that $\sigma = 1$, so the production function is Cobb-Douglas. Given a Cobb-Douglas production function, the resulting equation for socially optimal investment is (see Appendix B)

$$I_j = \left\{ \frac{\alpha A_j (L_{j+1} - (1-\delta)L_j)^{1-\alpha}}{r_j + \delta + (m-\delta)\dfrac{\phi_j}{\Omega_j}} \right\}^{\frac{1}{1-\alpha}} \qquad (2)$$

where r_k is the rate of interest and the variables ϕ_k and Ω_k are functions of the future pattern of interest rates over the planning horizon and m is the proportion of debt to be repaid each year.

To calculate the level of socially optimal investment, measures of A_k, r_k, α, δ, m and the planning horizon, h, are needed. The calculation of the parameters α, δ, m and h are explained in Appendix C. From these calculations the values are 0.37 for α, 0.051 for δ, 0.15 for m and 130 years for h. These calculations are based partly on econometric estimation in this study and partly on estimates in the literature. The value for h is based on the principle that the value chosen be such that a longer planning horizon does not alter optimal outcomes by more than a specified degree of tolerance (see Appendix C).

The values for the efficiency parameter, A_k, are determined for the Cobb-Douglas case by:

$$A_k = \frac{Y_{k+1} - Y_k(1-\delta)}{I_k^{\alpha}[L_{k+1} - (1-\delta)L_k]^{1-\alpha}} \qquad k = 1,\ldots,h-1 \tag{3}$$

where Y_k, I_k and L_k are the observed values described above. The values for A_k are shown in Chart 1.

The calculation of the interest rate, r_k, is described in McDonald, Tacconi and Kaur (1991). It is a real world rate and should be interpreted as the rate at which Australia can trade current consumption for future consumption. Essentially, the calculation of r_k involves taking the 10 year government bond rates for the USA, UK and West Germany, then adjusting the rates for errors in forecasting inflation, and deflating the resulting series by the Australian consumer price index expressed in the currency of the respective country. A weighted average of the rates calculated by this procedure gives an interest rate for each year from 1960-61 to 1993-94, illustrated in Chart 2. It can be interpreted as the social opportunity cost of consumption. Over the period 1960-61 to 1993-94 the average value of the interest rate is 4.57%. This value is adopted for all future years beyond 1993-94.

3. Interpreting volatility in the socially optimal investment series.

The series for socially optimal investment from 1961 to 1994 is calculated using equation (2) with the values for the parameters and the exogenous variables described above. In Chart 3 this series is compared to the series for actual investment, where both series are expressed as a proportion of GDP. The socially optimal series is obviously much more volatile than the series for actual investment. We measure volatility as the degree of relative dispersion around the mean, given by the coefficient of variation, *V*.[3] The value of *V* for the actual and socially optimal investment series is 0.099 and 0.452,

[3] The coefficient of variation, V, is equal to the standard deviation of the series divided by its mean. Hence, this gives a measure of volatility which is independent of the mean.

respectively. Also, we measure the degree of persistence in the series by the first order serial correlation coefficient, ρ. The value of ρ for the actual series is 0.793 (with a t statistic of 6.18), while that for the socially optimal series is 0.367 (with a t statistic of 0.167). These facts indicate the much lower volatility and higher degree of persistence in the actual investment series compared to the socially optimal series. Also, there is a considerable difference between the levels of the actual and socially optimal investment series. The average actual I/Y ratio for the period is 0.249, while the average socially optimal I/Y ratio is 0.430. This issue is addressed in 3.2.1 below.

The purpose of this paper is to investigate ways of smoothing the socially optimal investment series illustrated in Chart 3. The effectiveness in smoothing the series is judged by how much the volatility is reduced. It is worth emphasising that our socially optimal series in Chart 3 is already a less volatile series than would have been generated using a non-vintage production function, as discussed in section 2.1.

3.1.1 The effect of smoothing the exogenous variables.

In this section we consider the smoothing effect on socially optimal investment of smoothing the series for the exogenous variables: r_j, A_j and employment growth. We then consider the smoothing effect on socially optimal investment of lowering the value of the elasticity of substitution, followed by an adjustment cost model and a model of delivery lags.

Consider three scenarios illustrated in charts 4, 5 and 6. In each scenario one of the three exogenous variables contributing to investment volatility is held constant from 1960-61 to 1993-94, while the other variables are allowed to vary as in the original socially optimal investment series in chart 3. In chart 4, employment growth is held constant at 1.141%.[4] In chart 5, the rate of interest is held constant at 4.57% - the

[4] The rate of employment growth is assumed to equal the rate of population growth. This implies that the labour force participation rate is constant throughout the time horizon. This prevents, even for very long time horizons, the labour force participation rate either approaching zero or exceeding one. Population is assumed to grow at 1.141%. This is the average annual growth rate from 1991 to 2011 for three series (A/B, C and D) of projected population growth rates in ABS Catalogue No.3204.0: Projections of the Population of Australia.

average of the interest rates for the period 1960-61 to 1993-94. In chart 6, technical progress, the rate of growth of the efficiency parameter, A_j, is held constant at 0.59%.[5] In this case the initial value, in 1961, for A_j is derived from econometric estimation of the production function, with parameters α, δ and the growth rate of A_j constrained to equal their chosen values (see Appendix C).

Table 1 gives the values for V and ρ of the socially optimal investment series for each of the three scenarios. Holding employment growth constant (Chart 4) has almost no effect on volatility. In this case, V is reduced from 0.452 to 0.446. The degree of persistence in the series is lower and, hence, further away from the degree of persistence shown in the actual series. The series with a constant interest rate (Chart 5) has a greater dampening effect on volatility, although the value of V is still four times that of the actual investment series. The series based on constant technical progress is in fact more volatile than the unsmoothed series (the base case). This is partly because the calculated value of A_j is a residual which varies inversely with the employment term in (2). Hence, from (2) the effect of A_j multiplied by the employment term is to reduce volatility in the investment series. Paradoxically, this smoothing effect of A_j on investment is lost when A_j itself is smoothed. However, the series based on constant technical progress has a first order serial correlation coefficient that is much higher than the other smoothed series considered so far and is closest to that for the actual series.

[5] The rate of 0.59% is derived from Dixon and McDonald (1992), as discussed in Appendix C.

TABLE 1

Scenario*	Coefficient of Variation, V.	1st order serial corr., ρ
actual investment series	0.099	0.793 (6.18)
base case i.e. unsmoothed series	0.452	0.445 (2.76)
constant employment growth	0.446	0.185 (1.05)
constant interest rate	0.346	0.060 (0.35)
constant technical progress	0.539	0.719 (5.82)
CES production fn.($\sigma = 0.5$) (constant technical progress)	0.480	0.608 (5.33)
Adjustment cost model (constant technical progress; adjustment costs = 20%)	0.290	0.639 (4.61)
Delivery lags model (constant technical progress; $\phi = 0.5$)	0.456	0.900 (12.16)
20% adjustment costs;** delivery lags(constant technical progress; ($\phi = 0.5$))	0.194	0.810 (7.65)
30% adjustment costs; delivery lags (constant technical progress; $\phi = 0.5$)	0.100	0.59 (4.17)

* The scenarios are discussed in the text.

** These assumptions are used in the McKibbin and Siegloff study.

However, despite the greater degree of volatility in the series based on a constant rate of technical progress, there are good reasons for using this basis. The measured rate of technical progress varies due to cyclical factors, in particular labour hoarding in economic downturns, as well as variations in "true" technical progress. Therefore, the unobservable "true" rate of technical progress may reasonably be assumed to vary less than the measured rate. Chart 1 shows that the calculated values of A_j, from which the measured rate of technical progress is derived, exhibit an implausible degree of volatility. On the other hand, arbitrarily smoothing employment and the interest rate is less justified. On these grounds, the simulations in the rest of this paper assume constant technical progress, but variable employment and interest rate as described above.

3.1.2 A mean-adjusted series with constant technical progress.

Now consider the mean level of socially optimal investment. The mean socially optimal level of investment for the base case is 18 percentage points of GDP higher than the average actual level of investment (Chart 3). Similarly, in the case where technical progress is assumed constant from 1960-61 to 1993-94 (Chart 6), the mean level of socially optimal investment is 19 percentage points above the mean actual level. Because we are concerned with the volatility and persistence of the investment series, rather than the mean, for all further simulations we assume that firms do not persistently "under-invest" (or "over-invest") by imposing the condition that the mean actual investment level is equal to the mean socially optimal level.[6] This is achieved by determining the value for the efficiency parameter, A_j, in the first year of the horizon (1960-61) such that this condition holds. In subsequent years, constant technical progress at the rate $a = 0.59\%$ is assumed. The resulting mean-adjusted series for socially optimal investment is compared to the series for actual investment in Chart 7.

The assumptions for mean-adjustment and constant technical progress are maintained for all further simulations. It is clear from Chart 7 and Table 1 that the

[6] The possibility of persistent "under-investment" is discussed in the conclusion.

socially optimal investment series with constant technical progress and mean-adjustment still exhibits much greater volatility than the actual series. (Note that mean-adjustment does not affect V or ρ.)

3.3 Sensitivity to α, the output elasticity of investment.

From equation (2), socially optimal investment is a function of α, the output elasticity of investment. In this section we conduct simulations to determine the sensitivity of the volatility in the socially optimal investment series to variations in the value of α from its chosen value of 0.37. The results are illustrated in Chart 8. Reducing the value of α below 0.75, which is an unrealistically high value, has a negligible impact on V. For instance, the implausibly low value of α of 0.05 yields $V = 0.509$, a reduction of only 0.03 from the case where $\alpha = 0.37$. We conclude that reductions in α have a negligible effect in bringing the degree of volatility of socially optimal investment closer to the degree of volatility of actual investment.

3.4 The CES production function.

It might be thought that a lower value of the elasticity of substitution, σ, would generate a less volatile series for socially optimal investment. This is because if capital is less substitutable for labour then the choice of the capital-labour ratio will be less responsive to changes in the rate of interest. In pursuing this possibility simulations with a CES production function were used. The CES form of the production function is (see Appendix A)

$$Y_j = (1-\delta)Y_{j-1} + A_{j-1}\left[\alpha I_{j-1}^{-\theta} + (1-\alpha)(L_j-(1-\delta)L_{j-1})^{-\theta}\right]^{-1/\theta} \qquad (4)$$

where $\sigma = 1/(1+\theta)$. In the case where $\sigma < 1$, the first order condition (B.6) yields the socially optimal level of investment as

$$I_j = \left[\frac{\left(\frac{\alpha A_j}{r_j+\delta}\right)^{-\frac{\theta}{1+\theta}} - \alpha}{1-\alpha} \right]^{\frac{1}{\theta}} (L_{j+1}-(1-\delta)L_j) \tag{5}$$

For purposes of comparison, in these simulations the values of all exogenous variables and parameter values, except for σ, are equal to the values in the Cobb-Douglas case with constant technical progress and adjusted mean. This isolates the impact of lowering the value of σ. Calculations reveal that lower values of σ have a limited impact on volatility and persistence in the series. For instance, Chart 9 compares the case where σ=0.5 with the Cobb-Douglas case where σ=1.0. The value of V falls from 0.539 for σ=1.0 to 0.480 for σ=0.5. The value of ρ falls from 0.656 to 0.608.

Chart 10 illustrates the sensitivity of V to the value of σ. Starting from the Cobb-Douglas case (σ=1.0), reductions in the value of σ have a small impact on the measure of volatility, V. The value of V falls from 0.539 for σ=1 to a value of 0.458 for σ=0.1, which is in any case an implausibly low value of σ. This is not a significant reduction in volatility and is still considerably greater than the value for the actual investment series, which is a value of V of 0.099.

To seek an explanation for why a lower value of σ has little impact on V, consider the elasticity of investment with respect to the interest rate, ϵ_r, calculated from (5):

$$\epsilon_r = \frac{\alpha-1}{1+\theta}\left[1 + \frac{\alpha}{\left(\frac{r_j+\delta}{\alpha A_j}\right)^{\frac{\theta}{1+\theta}} - \alpha}\right] \tag{6}$$

This elasticity is an important determinant of investment volatility.[7] Also note that

[7] The other determinants of investment volatility in this model are: (i) changes in employment, the impact of which on volatility is invariant with respect to θ and therefore σ (from (6)); (ii) changes in A_j, the impact of which on volatility varies with respect to σ in a similar way to the interest rate (but note that in this section of the paper A_j is smoothed to have a constant rate of growth); (iii) changes in the exogenous level of output in year 1 of each plan, Y_1, since it is the investment share of output, $(I/Y)_1$, that is being measured.

$\sigma = \frac{1}{1+\theta}$. From inspection of (6), the responsiveness of ϵ_r to a change in θ is not unit free. Rather, it is dependent on the units in which A_j is measured. It appears that for the units in which the Australian data used in this paper are measured, a change in the value of θ, and therefore σ, has little impact on ϵ_r , and therefore on V.

The conclusion from these simulations is that with a CES production function, reducing the elasticity of substitution does not significantly reduce the volatility in the socially optimal investment series.

3.5 An adjustment cost model.

An approach which has been used extensively in the literature is to model adjustment costs in the accumulation of capital. The model presented here is similar to that used in McKibbin and Siegloff (1988), who broadly follow the approach established by Lucas (1967) and Hayashi (1982). Assume one dollar of investment expenditure effectively yields less than one dollar of new capital, because real resources are used up by the disruptions to the existing production process caused by the installation of new capital goods and the retraining of workers. These costs are assumed to increase at an increasing rate, so that marginal adjustment costs are increasing, which ensures a solution for socially optimal investment. Also, adjustment costs are negatively affected by the size of the firm as measured by its capital stock in year k, K_k. Let J_k equal the effective accumulation of capital in year k after adjustment costs have been deducted. Then:

$$J_k = I_k\left[1 - 0.5\,\mu\left(\frac{I}{K}\right)_k\right] \tag{7}$$

where $0.5\,\mu\left(\frac{I^2}{K}\right)_k$ is the cost of installing J_k units of capital.

The Cobb-Douglas form of the production function is now:

$$Y_j = (1-\delta)^{j-1} Y_1 + \sum_{k=1}^{j-1} (1-\delta)^{j-k-1} A_k J_k^{\alpha} (L_{k+1} - (1-\delta) L_k)^{1-\alpha} \tag{8}$$

The necessary adjustment to the first order condition is set out in Appendix B. The value for μ is chosen to equal 3.89. This implies that one dollar of investment expenditure yields an average of eighty cents of capital net of adjustment costs. The size of the adjustment costs chosen here is approximately in the mid-range of the magnitudes used by McKibbin and Siegloff (1988). The resulting series for socially optimal investment is shown in chart 11. Table 1 shows that the introduction of adjustment costs reduces the volatility of the socially optimal investment series by a large amount and reduces the degree of persistence in the series by a small amount. However, the volatility of the socially optimal series is still approximately 3 times higher than the volatility of the actual investment series. The degree of persistence in the series is below the degree of persistence for the actual series.

A value of μ could be chosen such that the volatility of the socially optimal investment series is equal to the volatility of the actual series. However, simulations showed that this value is approximately 20, which implies nearly 100% adjustment costs - clearly a nonsensical proposition. Thus, the plausible value of μ, as chosen above, generates a socially optimal investment series with considerably more volatility than the actual series.

3.6 Modelling delivery lags.

The final approach is to include unanticipated delivery lags. These lags refer to the lag between the time the decision to invest is made and the time the expenditure on the investment is actually made. The existence of these lags implies that in any particular period some investment expenditure is determined by decisions made in earlier periods, based on information about interest rates and profitability in the earlier period.

Jorgenson (1963) models delivery lags in the following way. Jorgenson assumes that for an investment decision taken in year k, a proportion, υ_k, of the project is completed in year k, υ_{k+1} is completed in year k+1, υ_{k+2} in year k+2, and so on. Since these delivery lags are unanticipated they do not enter the optimising decision of the firm in the Jorgenson model. This is a weakness of the model, since firms would presumably begin to anticipate delivery lags, which would then become a constraint in the optimising process. Jorgenson then assumes that $\sum_{t=k}^{k+\infty} \nu_t = 1$ and he estimates the υ's in a distributed lag function for investment. To introduce delivery lags into our calculations of the socially optimal series of investment, assume that the υ's decline geometrically so that socially optimal investment in year j, I_j^*, is an exponentially weighted moving average of socially optimal investment decisions taken in years 1, 2,...,j. McKibbin and Siegloff (1988) use this lag distribution function. A Koyck transformation yields the following equation for socially optimal investment, I_j^* :

$$I_j^* = \phi I_j + (1-\phi) I_{j-1}^* \qquad (9)$$

We choose a value of the smoothing parameter, ϕ , equal to 0.5. This implies that following the decision to invest, half the investment expenditure is made in the first year, a quarter in the second year, and so on. This is approximately equivalent to the time lags estimated by McKibbin and Siegloff (1988). The resulting series is shown in Chart 12. The volatility, *V*, of this series is 0.456, still approximately 4 times higher than that for the actual investment series, and higher than the volatility of the series with adjustment costs. The degree of persistence in the series is increased considerably and is actually higher than for the actual series (see Table 1).

Analagously to the argument in the case of the adjustment cost model, a value could be chosen for ϕ which would generate a series with the same degree of volatility as the actual series. However, experimentation shows that this value is close to zero, implying close to infinite delivery lags, which is again implausible.

We then combine adjustment costs of 20% and delivery lags ($\phi = 0.5$) as in the series constructed by McKibbin and Siegloff (1988). Their series tracks actual investment quite well. However, an important departure from our approach is their assumption that some firms do not optimise freely but, rather, face liquidity constraints in their investment decisions. In fact, liquidity constraints dominate in their explanation of actual investment, in that they conclude that 90% of investment is explained by the liquidity constraint variable. Nevertheless, we adopt their twin assumptions regarding adjustment costs and delivery lags. The resulting series for socially optimal investment is shown in Chart 13. The value for *V* is 0.194. This is still twice the degree of volatility in the actual investment series.

The combination of delivery lags, adjustment costs and constant technical progress can yield a series with a degree of volatility equal to the volatility of the actual series. One such scenario is illustrated in Chart 13. This scenario involves 30% adjustment costs (an increase from 20% in Chart 12), constant technical progress and delivery lags where $\phi = 0.5$ (the value in Chart 12).

Conclusion.

This paper compares, for Australia, the volatility of the socially optimal series of investment with the volatility of the actual series. Various series of the socially optimal level of investment are constructed. These series are considerably more volatile than the actual investment series. In constructing the socially optimal series from a vintage production function several factors were included which smooth the series. These are : (i) a constant rate of employment growth; (ii) a constant interest rate; (iii) lowering the values of α, the elasticity of output with respect to capital, and σ, the elasticity of substitution; (iv) adjustment costs; and (v) delivery lags. Of these smoothing techniques, only adjustment costs is both defensible on theoretical grounds and yield a significantly lower level of volatility. For instance, smoothing employment is ineffective in smoothing the series and is *ad hoc*; smoothing the interest rate yields a smoother series but is *ad hoc*; lowering α and σ do not significantly smooth the series; and delivery lags are *ad hoc* because they are not part of the optimising problem but act simply as a statistical filter. Furthermore, adopting a value for the adjustment cost parameter based on the literature yields a series which is still three times as volatile as the actual investment series. By increasing the adjustment cost parameter to the unreasonably high level of 30% and introducing *ad hoc* delivery lags of one year the socially optimal investment series can be smoothed to give a degree of volatility equal to that for the actual investment series. This case is illustrated in Chart 14.

The main point to be drawn from the simulations is that the volatility of the socially optimal investment series is excessive, relative to the volatility of the actual series. Of course, the actual series will differ from the socially optimal series whenever there is a wedge between the profit maximising level of investment chosen by firms and the socially optimal level. However, some reasons for this wedge would not explain the difference in volatility. These reasons include:

(i) distortions due to tax rates which alter the relative prices of capital and labour. It is unlikely that tax rates themselves are sufficiently volatile to explain investment volatility.

(ii) Externalities drive a wedge between the social rate of discount and the private rate of discount. This will occur if individuals are myopic in that, for instance, they use a discount rate which is higher than the social discount rate. Another form of market failure results where some risk is diversifiable to the economy but has not been diversified by private agents - this will mean the private rate of discount is above the social rate, which would lower the actual level of investment relative to the socially optimal level. Again it is difficult to imagine why the size of the wedge between the private and social rate of discount should be volatile.

(iii) Imperfect competition - both monopoly and monopsony power - results in a level of investment below the socially optimal level. But it seems unlikely that the degree of imperfect competition is volatile.

On the other hand, there are some reasons for divergence between the actual and socially optimal investment series which could perhaps account for the lower volatility of the actual investment series, and which we have not modelled. These factors include:

(i) Liquidity constraints on investment, or credit rationing, occur as a result of information asymmetries between borrowers and lenders. De Meza and Webb (1987) show that there may be too little investment under credit rationing. However, Williamson (1986) and Keeton (1979) show that under other circumstances credit rationing is socially optimal. So whether a properly specified process determining the socially optimal level of investment would include liquidity constraints is a moot point. Nevertheless, there is evidence that the actual investment series is influenced by liquidity constraints. Recent empirical work at the Reserve Bank of Australia by Mills et.al. (1994) strongly suggests that financial variables, such as a firm's cash flow and liquid assets, are important determinants of the actual level of investment. McKibbin and Siegloff (1988) also suggest liquidity constraints as one possible interpretation for their model of investment. Binding liquidity constraints will dampen the response of investment to a fall in interest rates, thereby reducing the volatility of the actual investment series. It is interesting to note that the coefficient of variation of the series for the share of gross operating surplus in GDP for Australia for the period 1959-60 to 1993-94 is 0.09, which is very close to 0.10, the coefficient of variation of the actual investment series. Thus, in as far as current profitability, as a measure of the liquidity constraint, determines investment it has a smoothing effect relative to the more traditional neoclassical factors.

(ii) Our model of investment determination has not taken into account uncertainty and irreversibility of investment decisions. Dixit and Pindyck (1994) argue that since investment decisions are often irreversible and undertaken in the face of uncertainty, the opportunity to invest is akin to holding a call option. Hence, it can be optimal to wait longer before investing than would be the case with perfect foresight. Bertola and Caballero (1994) derive an aggregate series of privately optimal investment from a model of irreversible investment and idiosyncratic uncertainty at the level of the firm. Applying this model to U.S. data yields an aggregate investment series which has a degree of volatility close to the degree of volatility of the aggregate series of actual investment. To achieve this correspondence of volatility Bertola and Caballero have to assume a degree of idiosyncratic uncertainty which they consider high. However, as they point out, they do not include other smoothing processes such as the costs of adjustment of investment.

One conclusion for future work suggested by the calculations in this paper is that the technique developed here to calculate the socially optimal series of investment should be extended to include liquidity constraints and idiosyncratic uncertainty. This is because, as shown in this paper, reasonable incorporation of other smoothing mechanisms yields a series for the socially optimal level of investment which has a much higher volatility than the series for actual investment.

APPENDIX A

This appendix gives the derivation of the particular form adopted for the vintage production function.

The model assumes "putty-clay" technology with a production function of the general form:

$$Y_j = \sum_{k=j-T}^{j-1} A_k \, F_j(I_k(1-\delta)^{j-k-1};\, l_{kj}) \tag{A.1}$$

where T is the age of the oldest plant in use at time j; $l_{k,j}$ is the labour in period j working on capital installed in period k; A_k is the efficiency parameter which captures neutral technical progress; the productivity of capital declines at the rate δ per period; and the investment goods installed in period k, I_k, have a 1 period gestation. Hence, investment in period j-1 produces output in period j. The putty-clay case assumes that the labour requirement of each vintage of capital is exogenous to the firm between the time it is installed and the time it is scrapped. Usually this is embodied in a labour requirement for installed capital which is constant over time.

For the general CES case where σ can take any value, (A.1) becomes:

$$Y_j = \sum_{k=j-T}^{j-1} A_k\left[\alpha\left(I_k(1-\delta)^{j-k-1}\right)^{-\theta} + (1-\alpha)(l_{kj})^{-\theta}\right]^{-\frac{1}{\theta}} \tag{A.2}$$

Equation (A.2) is not a convenient form for empirical applications for the following reasons. Firstly, it contains a sequence of past values of investment. This sequence will be larger, the larger the value of T. Secondly, $l_{k,j}$ is not readily observable. We observe aggregate employment in a given period but not the labour employed on capital of a given vintage. This problem is solved rather easily in the putty-putty case, since, in that case, labour is allocated competitively such that the marginal product of labour is equated across all vintages. This means that $l_{k,j}$ can be expressed in terms of the (constant) marginal product. This in turn implies that the aggregate production can be expressed in terms of the observable variable, L_j, aggregate employment. In the putty-clay model, however, labour cannot be allocated such that the marginal product of labour is equated across all vintages. Rather, as shown below, labour working on new capital, $l_{j-1,j}$, is equal to the increase in the aggregate labour force, L_j - L_{j-1}, plus labour released by scrapping of economically obsolescent capital, δL_{j-1}. Hence, $l_{j-1,j}$ is defined in terms of observable variables.

The sequence of past values of investment can be eliminated by the following procedure. Assume that capital installed in period k has a labour/capital ratio of z_k

$$z_k = \frac{l_{k,j}}{I_{k,j}} \qquad \text{(A.3)}$$

$$\therefore \quad l_{k,j} = z_k I_{k,j}$$

where z_k is the labour required to operate one unit of capital of vintage k and $I_{k,j}$ is the amount of capital, which was installed in period k, working in period j.. It is assumed, following the putty-clay approach, that over the period of time in which the capital is used, z_k is constant. This means that changes in relative factor prices after period k have no effect on z_k.

With constant exponential depreciation of capital at rate δ,

$$I_{k,j} = I_k(1-\delta)^{j-k-1} \qquad \text{(A.4))}$$

Therefore, the labour working on any vintage j at time k is given by

$$l_{k,j} = z_k I_k (1-\delta)^{j-k-1} \qquad \text{(A.5)}$$

Using (A.5) to substitute for l_{kj} in (A.2):

$$Y_j = \sum_{k=j-T}^{j-1} A_k\left[\alpha\left(I_k(1-\delta)^{j-k-1}\right)^{-\theta} + (1-\alpha)\left(z_k I_k(1-\delta)^{j-k-1}\right)^{-\theta}\right]^{-\frac{1}{\theta}}$$
$$= \sum_{k=j-T}^{j-1} A_k(1-\delta)^{j-k-1}\left[\alpha I_k^{-\theta} + (1-\alpha)(z_k I_k)^{-\theta}\right]^{-\frac{1}{\theta}} \quad \text{(A.6)}$$

Hence, output is a distributed lag of the output produced from past vintages of capital. From here we adopt a two step procedure. The first step is to transform (A.1) to an autoregressive form. The second step is to express the unobservable variable, z_k, in terms of observable variables.

Expanding (A.6), for the case where T=3:

$$Y_j = A_{j-3}(1-\delta)^2\left[\alpha I_{j-3}^{-\theta} + (1-\alpha)(z_{j-3}I_{j-3})^{-\theta}\right]^{-\frac{1}{\theta}}$$
$$+ A_{j-2}(1-\delta)\left[\alpha I_{j-2}^{-\theta} + (1-\alpha)(z_{j-2}I_{j-2})^{-\theta}\right]^{-\frac{1}{\theta}} \quad \text{(A.7)}$$
$$+ A_{j-1}\left[\alpha I_{j-1}^{-\theta} + (1-\alpha)(z_{j-1}I_{j-1})^{-\theta}\right]^{-\frac{1}{\theta}}$$

Furthermore,

$$Y_{j-1} = \sum_{k=j-1-T}^{j-2} A_k(1-\delta)^{j-k-2}\left[\alpha I_k^{-\theta} + (1-\alpha)(z_k I_k)^{-\theta}\right]^{-\frac{1}{\theta}}$$

$$= A_{j-4}(1-\delta)^2\left[\alpha I_{j-4}^{-\theta} + (1-\alpha)(z_{j-4}I_{j-4})^{-\theta}\right]^{-\frac{1}{\theta}}$$
$$+ A_{j-3}(1-\delta)\left[\alpha I_{j-3}^{-\theta} + (1-\alpha)(z_{j-3}I_{j-3})^{-\theta}\right]^{-\frac{1}{\theta}} \quad \text{(A.8)}$$
$$+ A_{j-2}\left[\alpha I_{j-2}^{-\theta} + (1-\alpha)(z_{j-2}I_{j-2})^{-\theta}\right]^{-\frac{1}{\theta}}$$

Subtracting (A.9) from (A.7):

$$\therefore \quad (1-\delta)Y_{j-1} = A_{j-4}(1-\delta)^3\left[\alpha I_{j-4}^{-\theta} + (1-\alpha)(z_{j-4}I_{j-4})^{-\theta}\right]^{-\frac{1}{\theta}}$$

$$+ A_{j-3}(1-\delta)^2\left[\alpha I_{j-3}^{-\theta} + (1-\alpha)(z_{j-3}I_{j-3})^{-\theta}\right]^{-\frac{1}{\theta}} \qquad \text{(A.9)}$$

$$+ A_{j-2}(1-\delta)\left[\alpha I_{j-2}^{-\theta} + (1-\alpha)(z_{j-2}I_{j-2})^{-\theta}\right]^{-\frac{1}{\theta}}$$

$$Y_j - (1-\delta)Y_{j-1} = A_{j-1}\left[\alpha I_{j-1}^{-\theta} + (1-\alpha)(z_{j-1}I_{j-1})^{-\theta}\right]^{-\frac{1}{\theta}} \qquad \text{(A.10)}$$

$$- A_{j-4}(1-\delta)^3\left[\alpha I_{j-4}^{-\theta} + (1-\alpha)(z_{j-4}I_{j-4})^{-\theta}\right]^{-\frac{1}{\theta}}$$

Thus

$$Y_j = (1-\delta)Y_{j-1} + A_{j-1}\left[\alpha I_{j-1}^{-\theta} + (1-\alpha)(z_{j-1}I_{j-1})^{-\theta}\right]^{-\frac{1}{\theta}} \qquad \text{(A.11)}$$

$$- A_{j-4}(1-\delta)^3\left[\alpha I_{j-4}^{-\theta} + (1-\alpha)(z_{j-4}I_{j-4})^{-\theta}\right]^{-\frac{1}{\theta}}$$

And, in general, for any value of T:

$$Y_j = (1-\delta)Y_{j-1} + A_{j-1}\left[\alpha I_k^{-\theta} + (1-\alpha)(z_k I_k)^{-\theta}\right]^{-\frac{1}{\theta}} \qquad \text{(A.12)}$$

$$-A_{j-1-T}(1-\delta)^T\left[\alpha I_{j-1-T}^{-\theta} + (1-\alpha)(z_{j-1-T}I_{j-1-T})^{-\theta}\right]^{-\frac{1}{\theta}}$$

Casual observation suggests that T, the age of the oldest vintage of capital in use, is likely to be quite old. Hence, taking the limit as $T \to \infty$ and noting that $(1-\delta)^T \to 0$ as $T \to \infty$, (A.12) simplifies to

$$Y_j = (1-\delta)Y_{j-1} + A_{j-1}\left[\alpha I_{j-1}^{-\theta} + (1-\alpha)(z_{j-1}I_{j-1})^{-\theta}\right]^{-\frac{1}{\theta}} \qquad \text{(A.13)}$$

In equation (A.13) output in one period depends on the level of investment in the previous period and not on the levels of investment in any earlier periods.

Equation (A.13) still contains the unobservable variable, z. However, z can be expresses in terms of the observable variable, L, aggregate employment. Aggregate employment in period j is the sum of the labour employed on each vintage of capital in use in period j. Hence,

$$\begin{aligned} L_j &= \sum_{k=j-T}^{j-1} l_{k,j} \\ &= \sum_{k=j-T}^{j-1} z_k\, I_k(1-\delta)^{j-k-1} \end{aligned} \tag{A.14}$$

$$\textit{Similarly,}\quad L_{j-1} = \sum_{k=j-1-T}^{j-2} z_k\, I_k(1-\delta)^{j-k-2} \tag{A.15}$$

From (A.14) and (A.15), taking the case where T=3.

$$\begin{aligned} L_j - (1-\delta)L_{j-1} &= z_{j-3}I_{j-3}(1-\delta)^2 + z_{j-2}I_{j-2}(1-\delta) + z_{j-1}I_{j-1} \\ &\quad - z_{j-4}I_{j-4}(1-\delta)^3 + z_{j-3}I_{j-3}(1-\delta)^2 + z_{j-2}I_{j-2}(1-\delta) \\ &= z_{j-1}I_{j-1} - z_{j-4}I_{j-4}(1-\delta)^3 \end{aligned}$$

In general, for any value of T:

$$L_j - (1-\delta)L_{j-1} = z_{j-1}I_{j-1} - z_{j-T-1}I_{j-T-1}(1-\delta)^T \tag{A.16}$$

$$\begin{aligned} &\lim_{T\to\infty}\left[L_j - (1-\delta)L_{j-1}\right] = z_{j-1}I_{j-1} \\ &\therefore\quad \lim_{T\to\infty} z_{j-1} = \frac{\left[L_j - (1-\delta)L_{j-1}\right]}{I_{j-1}} \end{aligned} \tag{A.17}$$

Substituting (A.17) into (A.13), the functional form for a putty-clay CES production function using the aggregate variables Y, I and L is :

$$Y_j = (1-\delta)Y_{j-1} + A_{j-1}\left[\alpha I_{j-1}^{\ -\theta} + (1-\alpha)(L_j-(1-\delta)L_{j-1})^{-\theta}\right]^{-1/\theta} \tag{A.18}$$

Finally, an advantage of this functional form is that it enables the value of the efficiency parameter, A_k, to be derived for each year from $k=1,\ldots,j-1$. This is evident from (A.18), in which A_k can be determined given the actual observed values of Y_j, Y_{j-1}, I_{j-1}, L_j and L_{j-1}, and given a specific functional form and parameter values.

The Cobb-Douglas function:

In the special case where $\sigma = 1$, the Cobb-Douglas case:

$$Y_j = \sum_{k=j-T}^{j-1} A_k\left[I_k(1-\delta)^{j-k-1}\right]^{\alpha} l_{kj}^{\,1-\alpha} \qquad \text{(A.19)}$$

Following the same procedure as in the CES case, use (A.5) to substitute for l_{kj}:

$$Y_j = \sum_{k=j-T}^{j-1} A_k(1-\delta)^{j-k-1} I_k^{\alpha}(z_k I_k)^{1-\alpha} \qquad \text{(A.20)}$$

Expanding, for the case where T=3 :

$$\begin{aligned} Y_j &= A_{j-3}(1-\delta)^2 I_{j-3}^{\alpha}(z_{j-3}I_{j-3})^{1-\alpha} \qquad \text{(A.21)} \\ &\quad + A_{j-2}(1-\delta)I_{j-2}^{\alpha}(z_{j-2}I_{j-2})^{1-\alpha} + A_{j-1}I_{j-1}^{\alpha}(z_{j-1}I_{j-1})^{1-\alpha} \end{aligned}$$

And in general, for any value of T :

$$\begin{aligned} Y_j &= (1-\delta)Y_{j-1} + A_{j-1}I_{j-1}^{\alpha}(z_{j-1}I_{j-1})^{1-\alpha} \qquad \text{(A.25)} \\ &\quad - A_{j-1-T}I_{j-1-T}^{\alpha}(1-\delta)^T(z_{j-1-T}I_{j-1-T})^{1-\alpha} \end{aligned}$$

Taking the limit as T approaches infinity :

$$Y_j = (1-\delta)Y_{j-1} + A_{j-1}I_{j-1}^{\alpha}(z_{j-1}I_{j-1})^{1-\alpha} \qquad \text{(A.26)}$$

Furthermore

$$Y_{j-1} = \sum_{k=j-1-T}^{j-2} A_k(1-\delta)^{(j-2-k)\alpha} I_k^{\alpha} \, (z_k I_k)^{1-\alpha}$$

$$= A_{j-4}(1-\delta)^2 I_{j-4}^{\alpha}(z_{j-4}I_{j-4})^{1-\alpha} \qquad (A.22)$$

$$+ A_{j-3}(1-\delta)I_{j-3}^{\alpha}(z_{j-3}I_{j-3})^{1-\alpha} + A_{j-2}I_{j-2}^{\alpha} \, (z_{j-2}I_{j-2})^{1-\alpha}$$

$$\therefore \quad (1-\delta)Y_{j-1} = A_{j-4}(1-\delta)^3 I_{j-4}^{\alpha} \, (z_{j-4}I_{j-4})^{1-\alpha}$$

$$+ A_{j-3}(1-\delta)^2 I_{j-3}^{\alpha} \, (z_{j-3}I_{j-3})^{1-\alpha} + A_{j-2}(1-\delta)I_{j-2}^{\alpha} \, (z_{j-2}I_{j-2})^{1-\alpha}$$

(*A*.23)

Subtracting (*A*.23) *from* (*A*.21)

$$Y_j - (1-\delta)Y_{j-1} = A_{j-1}I_{j-1}^{\alpha}(z_{j-1}I_{j-1})^{1-\alpha} - A_{j-4}(1-\delta)^3 I_{j-4}^{\alpha}(z_{j-4}I_{j-4})^{1-\alpha}$$

$$\textit{Thus} \quad Y_j = (1-\delta)Y_{j-1} + A_{j-1}I_{j-1}^{\alpha} \, (z_{j-1}I_{j-1})^{1-\alpha}$$

$$- A_{j-4}(1-\delta)^3 I_{j-4}^{\alpha}(z_{j-4}I_{j-4})^{1-\alpha}$$

(*A*.24)

Substituting from (A.17):

$$Y_j = (1-\delta)Y_{j-1} + A_{j-1}I_{j-1}^{\alpha} \left(L_j-(1-\delta)L_{j-1}\right)^{1-\alpha} \qquad (A.27)$$

The approach in deriving (A.27) can be compared to the approach adopted by Dadkhah and Zahedi (1986), in estimating a non-vintage production function. The also use a Koyck transformation to derive an autoregressive equation in output. They assume a Leonteif production function and also assume that capital is the limiting factor, such that:

$$Y_j = aK_j + u_j \qquad (A.28)$$

where

Y_j = output produced during period j,

K_j = capital stock at the beginning of period j,

u_j = error term.

In their model, the production function is combined with the capital stock identity:

$$K_j = (1-\lambda)K_{j-1} + I_{j-1} \tag{A.29}$$

to get

$$Y_j = (1-\delta)Y_{j-1} + aI_{j-1} + v_j \tag{A.30}$$

where

I_{j-1} = gross investment during period j-1,

δ = the depreciation rate,

and $v_j = u_j - (1-\delta)u_{j-1}$.

Hence, the similarity between the approach of Dadkhah and Zahedi and our approach is the derivation of an autoregressive equation in output. The difference is that their equation, (A.30), is based on a non-vintage Leonteif production function, whereas our equation (A.27) is based on a vintage production function with variable factor proportions.

APPENDIX B

The optimisation problem, which determines socially optimal investment, is outlined here (see Guest and McDonald, 1995). A representative agent maximises a concave utility function:

$$U = U(C_1, C_2, .., C_h, W_h) \qquad \text{(B.1)}$$

where C_j is aggregate consumption in year j, h is the length of the planning horizon in years, W_h is the level of real wealth at the end of the planning horizon. Consumption is subject to an output constraint and an international borrowing constraint. The output constraint in period j is given by the production function:

$$Y_j = Y_1(1-\delta)^{j-1} + \sum_{k=1}^{j-1}(1-\delta)^{j-1-k} A_k\, F_k(I_k) \qquad j = 2, \ldots, h \qquad (B.2)$$

The international borrowing constraint is :

$$C_j = Y_j - I_j + B_j - (m+r_1)(1-m)^{j-1}D_0 - \sum_{k=1}^{j-1}(m+r_k)(1-m)^{j-k-1}B_k \qquad j = 1, \ldots, h \qquad (B.3)$$

where D_0 is the level of overseas debt inherited in year 1; m is the (constant) proportion of the debt to be repaid in each year; r is the interest rate; B_j is the level of overseas borrowing in year j. Terminal wealth is defined as

$$W_h = (1-\delta)^h K_0 + \sum_{k=1}^{h-1}(1-\delta)^{h-k} I_k - (1-m)^h D_0 - \sum_{k=1}^{h}(1-m)^{h-k} B_k \qquad \text{(B.4)}$$

where K_0 is the capital stock inherited in year 1.

The maximisation problem which defines intertemporal balance may be written

$$\max_{\substack{(C_1 \dots C_h)\\(I_1 \dots I_{h-1})\\(B_1 \dots B_h)\\(Y_2 \dots Y_h)\\(W_h)}} \Gamma = U(C_1, C_2, \dots, C_h, W_h)$$

$$+ \sum_{j=1}^{h} \lambda_j \left[Y_j - I_j - C_j + B_j - (m+r)(1-m)^{j-1} D_0 - \sum_{k=1}^{j-1} (m+r_k)((1-m)^{j-k-1} B_k \right]$$

$$+ \sum_{j=2}^{h} \psi_j \left[(1-\delta)^{j-1} Y_1 + \sum_{k=1}^{j-1} (1-\delta)^{j-k-1} F_k(I_k) - Y_j \right]$$

$$+ \phi \left[(1-\delta)^h K_0 + \sum_{k=1}^{h-1} (1-\delta)^{h-k} I_k - (1-m)^h D_0 - \sum_{k=1}^{h} (1-m)^{h-k} B_k - W_h \right] \qquad (B.5)$$

where λ_j, $j=1,\dots,h$, ψ_j, $j=2,\dots,h$, and ϕ are Lagrange multipliers.

The planning horizon, h, can be chosen to be any positive finite number. The planning horizon chosen for the simulations is 130 years (see Appendix C).

The first order condition for the determination of socially optimal investment is

$$\frac{\partial F_j}{\partial I_j} = r_j + \delta + (m-\delta)\frac{\Phi_j}{\Omega_j} \qquad \text{(B.6)}$$

where

$$\Phi_j = \sum_{i=1}^{h-j-1} \left\{ (r_{j+1} - r_j)(1-\delta)^{i-1} \left[(1-m)^{h-j-i-1} + \sum_{z=1}^{h-j-i-1} (1-m)^{z-1} \right. \right.$$

$$\left. \left. \prod_{x=z}^{h-j-i-1} (1+r_{h-x}) \right] \right\} \qquad j = 1,\ldots,h-1$$

$$\Omega_j = \sum_{v=j+1}^{h-1} \left\{ \left[\prod_{k=v}^{h-1} (1+r_k) + \sum_{l=v+1}^{h-1} (r_v - r_l)(1-m)^{h-l} \prod_{x=v+1}^{l-1} (1+r_x) \right] (1-\delta)^{v-j-1} \right\}$$

$$+ (1-\delta)^{k-j-1} \qquad j = 1,\ldots,h-1$$

As the interest rate is exogenous, the model is separable in that the socially optimal values for investment and output are determined from (B.5) and (19), which are independent of the utility function. From (B.5), assuming $\sigma = 1$ (the Cobb-Douglas functional form), socially optimal investment is given by

$$I_j = \left\{ \frac{\alpha A_j (L_{j+1} - (1-\delta)L_j)^{1-\alpha}}{r_j + \delta + (m-\delta)\dfrac{\Phi_j}{\Omega_j}} \right\}^{\frac{1}{1-\alpha}} \qquad \text{(B.7)}$$

__The adjustment cost model.__

The first order condition (B.6) above is different in the case of adjustment costs, as follows:

$$\frac{\partial F_j}{\partial I_j} = \alpha A_j \left(I_j - 0.5\mu \frac{I_j^2}{K_j} \right)^{\alpha-1} \left(1 - \mu \frac{I_j}{K_j} \right) [L_{j+1} - (1-\delta) L_j]^{1-\alpha} \qquad \text{(B.8)}$$

This yields the following non-linear equation in I_j to be solved for I_j :

$$\alpha A_j [L_{j+1} - (1-\delta)L_j]^{1-\alpha} \left(1 - \mu \frac{I_j}{K_j}\right) \left(I_j - 0.5\mu \frac{I_j^2}{K_j}\right)^{\alpha - 1} = r_j + \delta + (m - \delta) \frac{\Phi_j + \mu \frac{I_j}{K_j} (1-\delta)^{h-j}}{\Omega_j} \tag{B.9}$$

APPENDIX C

Determining parameter values.

Determining a value for α.

The parameter α is the elasticity of output with respect to capital. As a first step in determining a value of α, the vintage production function is estimated in Cobb-Douglas form. The vintage production function is fitted to the annual time series for output, investment and employment from 1960-61 to 1993-94. The output series is Gross Domestic Product at 1988-89 prices[8], the investment series is Gross Fixed Capital Expenditure (Private plus Public) at 1988-89 prices[9], and the employment series is aggregate weekly hours worked by all employed persons[10].

The estimating equation is:

$$\frac{Y_j-(1-\delta)Y_{j-1}}{L_j-(1-\delta)L_{j-1}} = A_0 e^{a(j-1)}\left(\frac{I_{j-1}}{(L_j-(1-\delta)L_{j-1})}\right)^{\alpha} \tag{C.1}$$

The value of δ is set exogenously at 0.051, based on ABS calculations (see below). It is the average reducing balance depreciation rate from 1966-67 to 1992-93 (see below for discussion). The estimated values of the other three parameters are (t statistics in parentheses):

α = 0.52 (5.32) ; A_0 = 2.84 (3.68) ; a = 0.007 (1.50)

The estimate of α seems too high, given data on factor shares and the estimates of α from overseas studies.[11] An upper bound for α can be inferred from the data on the

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[8] Source: ABS Catalogue No. 5206.0

[9] Source: ABS Catalogue No. 1313.0

[10] Source: ABS CAtalogue No. 6203.0

[11] There are no other Australian estimates of α using vintage production functions.

wages share of income, which averaged 0.5744 for Australia over the period 1966-1994.[12] Given some degree of imperfect competition, labour is paid less than its marginal product, which implies that the output elasticity of labour, $1-\alpha$, is greater than the wages share of income. On this basis a lower bound for $(1-\alpha)$ is 0.5744 implying an upper bound for α of $1-0.5744=0.4256$. This is well below the econometric estimate of 0.52. Therefore we set the value of α as follows. On the plausible assumption that market power rents are at least 5 percentage points of GDP, the value α is set at 0.37, which is approximately 5.3 percentage points below the upper bound of 0.4256.

This value is broadly consistent with the overseas literature using vintage production functions. These studies produced a very wide range of estimates of α. You (1976) uses annual data for U.S manufacturing industry and a Cobb-Douglas production function. He obtains an estimate of α of 0.55. Malcolmsen and Prior (1979) find an average estimate of 0.28 for α using data on U.K. manufacturing industry. McHugh and Lane (1983) adopt the same approach and data set as You (1976), with the same estimating equation, except that they utilise information across regions of the U.S. They estimate α between 0.15 and 0.32. Earlier work by Wickens (1970) using annual U.S. data on GDP, aggregate employment (man-hours worked) and capital stock for the period 1900-1960, suggestes a range of estimates of α based on assumptions about values of the initial capital stock, K_0. The average of these estimates is 0.26. Other early studies by Intriligator (1965) and Solow (1962) found an average estimate of α of 0.24.

Having replaced the estimated value of α, the estimated growth rate, a, of the efficiency parameter is also replaced by a value based on the empirical evidence for Australia. Evidence for the value of a, often referred to as the growth rate of total factor productivity in the Australian literature, can be found in several recent studies - for instance, Dixon and McDonald (1992) and Dowrick (1990). Also, cited in Dixon and McDonald, is an OECD study for Australia by Englander and Mittelstadt (1988) and an

[12] From ABS data service, DX.

ABS study by Aspden (1990).[13] The estimates from these four studies range from 0.005 to 0.024 and the average is approximately 0.0114. However, the Dixon and McDonald (1990) study is the appropriate source for our purposes since their data set and sample period most closely resemble ours. The main difference, which must be accounted for, is that they do not use total GDP but rather total output from those industries for which there is a satisfactory measure of output. They call this sector the M sector. The four industries for which there is not a satisfactory measure of output are called the N sector and consist of: construction; finance, property and business services; public administration and defence; and community services. To convert the Dixon and McDonald estimate of total factor productivity to an estimate consistent with the use of total GDP, which consists of the M+N sectors, we assumed that total factor productivity in the N sector is zero. In the N sector, by definition, output is equal to input and so productivity growth is zero. To calculate the necessary adjustment let a_M and a_{GDP} be the values of a for the M sector and GDP, respectively; and let s_N be the share of the N sector output in GDP. Then $a_{GDP}=a_M(1-s_N)$. The average value of s_N for Australia from 1967 to 1994 is

[13] These results are not directly comparable because they use an aggregate (i.e.non-vintage) production function. Nevertheless, it can be shown that under certain assumptions the vintage Cobb-Douglas production function reduces to the non-vintage version. This result can be shown quite easily using some results established earlier in this chapter. It was shown in section 2.2 that in the long run or steady state, in which the rate of interest, r, the growth rate, a, of the efficiency parameter, and the growth rate, l, of employment, are all held constant throughout the time horizon, then the growth rate, γ, of output, as j approaches ∞ is given by

$$\gamma = (1+a)^{\frac{1}{1-\alpha}}(1+l) - 1$$

where i is the growth rate of investment. This also holds for the non-vintage Cobb-Douglas production function in a steady state in which the capital stock grows at the same rate as output, shown as follows:

$$Y_j = A_j K_j^{\alpha} L_j^{1-\alpha}$$

$$\textit{Hence,} \quad \frac{Y_j}{Y_{j-1}} = \frac{A_j K_j^{\alpha} L_j^{1-\alpha}}{A_{j-1} K_{j-1}^{\alpha} L_{j-1}^{1-\alpha}}$$

$$\textit{Let} \quad 1+\gamma = \frac{Y_j}{Y_{j-1}} = \frac{K_j}{K_{j-1}}$$

$$\therefore \quad (1+\gamma) = (1+a)^{\frac{1}{1-\alpha}}(1+l)$$

0.3425.[14] The resulting value for a_{GDP} is 0.0059.

Other parameter values.

The parameter δ is the rate of depreciation. The ABS method of calculating depreciation is followed. Depreciation is measured as the change in the present discounted value (PDV) of the future service potential of the asset. The discount rate used is the internal rate of return of the asset. The ABS adopts the convention of a straight line (prime cost) depreciation function in order to calculate the depreciation charge each year. Using figures for end-year gross capital stock and consumption of fixed capital for both private enterprises and public authorities, the average straight line depreciation rate from 1966-67[15] to 1993-94 is 0.033. Using figures for end-year net capital stock[16] and consumption of fixed capital, the average reducing balance depreciation rate is 0.051. The model in this thesis assumes a reducing balance depreciation function. Accordingly, the value for depreciation adopted for all years is 0.051.

The parameter, m, is set at 0.15, implying that 90 percent of a loan would be repaid after 14 years. Although repayment plans for project finance differ widely, the assumption here is consistent with a typical project finance repayment plan.

Determining a planning horizon.

From equation (21), I_k is a function of Φ_k and Ω_k, which are functions of future rates of interest and therefore the length of the planning horizon, h, (see Appendix B). Hence, the question of the appropriate length of the planning horizon arises. The

[14] This is the maximum period for which data is available on the ABS data service, DX.

[15] This series for capital stock begins in 1966-67.

[16] Net capital stock is defined as gross capital stock less accumulated capital consumption - that is the written down value of the capital stock, which can be used to calculate the reducing balance depreciation rate.

choice of a planning horizon is discussed in Guest, 1994. The general principle we adopt is that the value for h should be chosen such that a longer horizon does not alter the optimal outcomes by more than a specified degree of tolerance.

This can be explained as follows. Assume a particular length for the planning horizon and calculate the socially optimal value of I_k. Then increase the planning horizon by one year thereby adding another observation to the series for the future rate of interest. This will yield a new value for the socially optimal value of I_k. If adding this year to the planning horizon changes the value of the socially optimal level of I_k by less than the specified degree of tolerance, then it was not important to add it. The degree of tolerance is set at 1 percentage point of GDP. If this procedure were to be repeated in calculating I_k for k=1960-61,..,1993-94 it may yield a different planning horizon for each calculation of I_k. To avoid this, we choose the planning horizon where k=1993-94 and apply this in the calculation of I_k for all k from 1960-61 to 1993-94. In summary, we choose the value of h such that an increase in h affects the value of optimal investment to GDP in 1993-94 by less than a degree of tolerance, chosen as 1 percentage point. Under this criterion, simulation exercises result in a value of h of 130 years, so h=130 was chosen.

APPENDIX D Values of Exogenous Variables

	A_1	L	I	Y	r
1960-61	43.67	150.07	29395	117163	4.04
1961-62	60.29	153.95	29984	120360	3.76
1962-63	59.70	158.79	32036	127771	2.82
1963-64	58.03	165.78	35373	136586	1.75
1964-65	31.23	172.63	39852	145219	1.84
1965-66	70.52	186.09	42134	148922	2.38
1966-67	52.63	188.70	42898	158770	2.83
1967-68	91.37	193.40	45237	165247	3.25
1968-69	64.96	195.20	49074	179489	3.90
1969-70	56.31	199.10	51653	188848	3.44
1970-71	55.40	207.80	54584	199074	0.89
1971-72	65.37	214.50	56363	207809	0.66
1972-73	51.15	215.30	57138	214875	3.17
1973-74	75.11	222.10	58847	221962	4.03
1974-75	107.15	219.20	56070	227379	4.56
1975-76	70.73	215.40	58970	237354	6.35
1976-77	59.12	216.08	59991	244607	6.56
1977-78	81.58	214.15	59334	246049	4.86
1978-79	65.13	220.07	63339	261707	4.08
1979-80	57.51	222.20	63626	268292	4.81
1980-81	85.02	229.16	69826	276102	7.73
1981-82	51.36	227.79	74398	284949	9.15
1982-83	85.56	220.43	66165	278534	8.32
1983-84	64.31	224.34	68018	293143	8.43
1984-85	65.48	236.69	74295	307268	7.78
1985-86	61.67	247.01	77474	321035	4.61
1986-87	71.65	250.23	76049	327284	3.15
1987-88	62.53	259.64	81200	342885	4.83
1988-89	80.24	268.87	89828	354515	6.71
1989-90	56.83	270.48	89553	366909	7.55
1990-91	85.43	268.45	80515	366448	6.67
1991-92	99.68	264.89	75659	371766	4.89
1992-93	77.06	262.36	76973	381603	2.19
1993-94	na	269.04	79352	394894	3.35

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Chart 1 The efficiency parameter, A_j

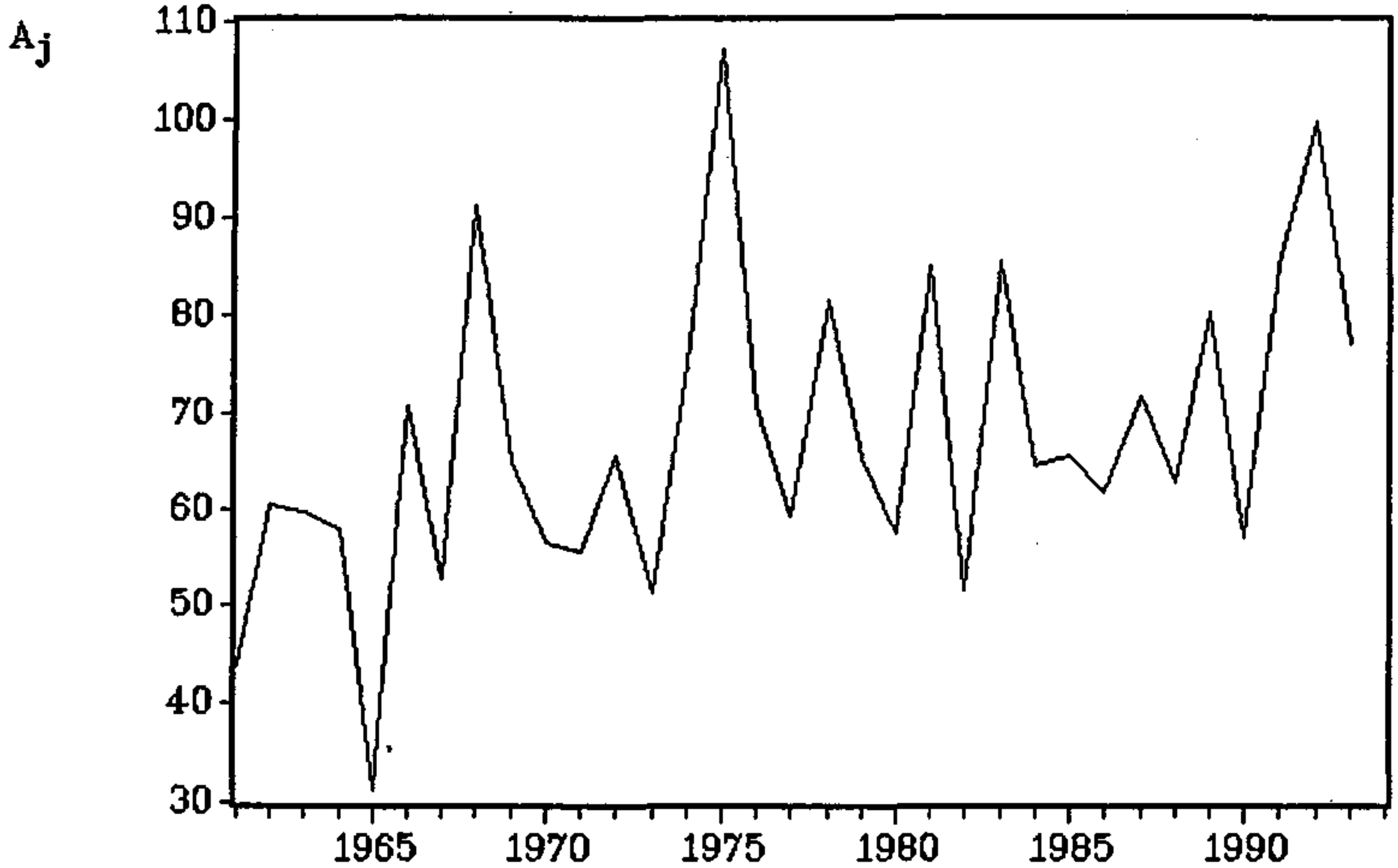

CHART 2 The interest rate, r_j.

r_j represents the social opportunity cost of consumption.

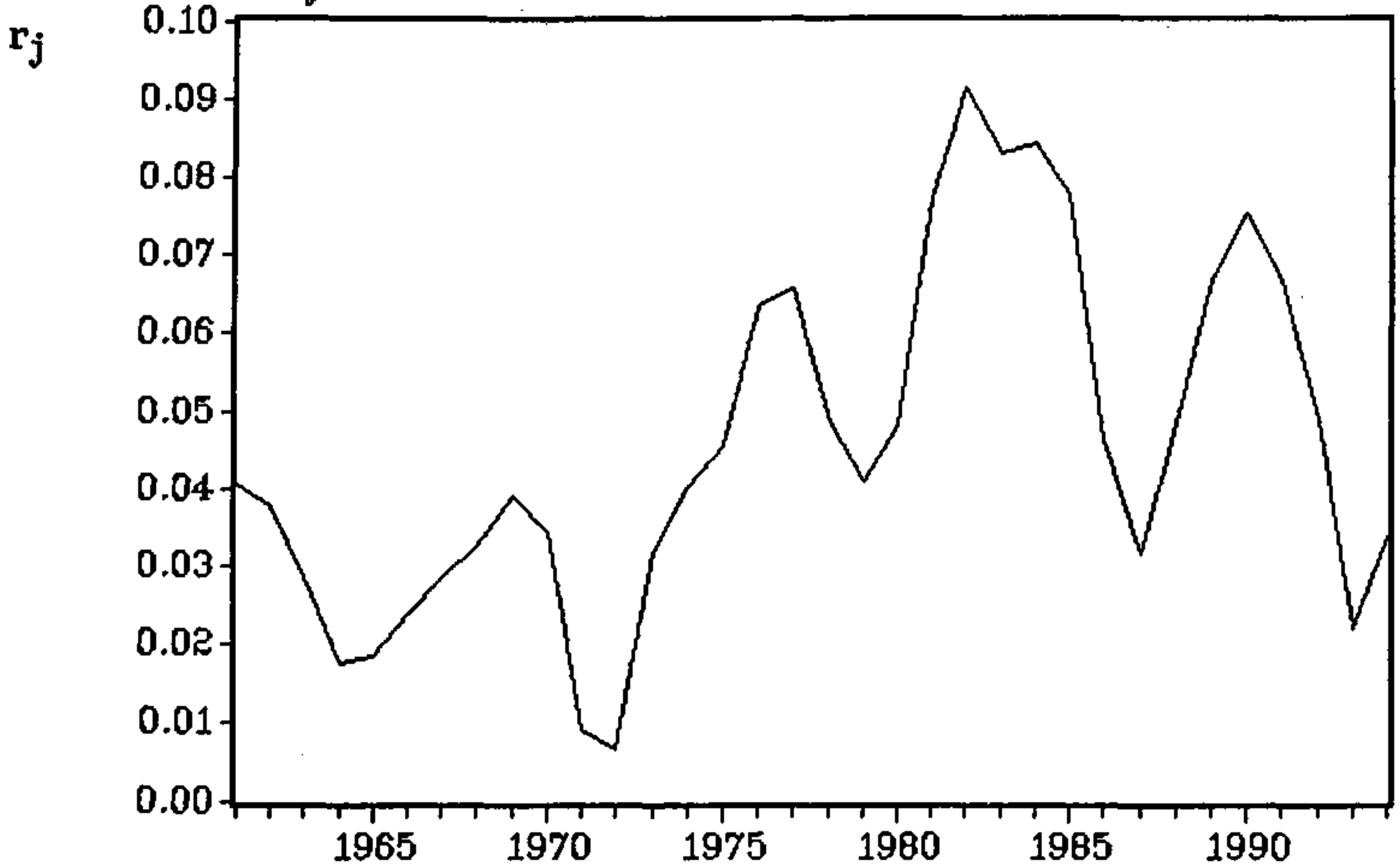

Chart 3 Optimal investment – the base case

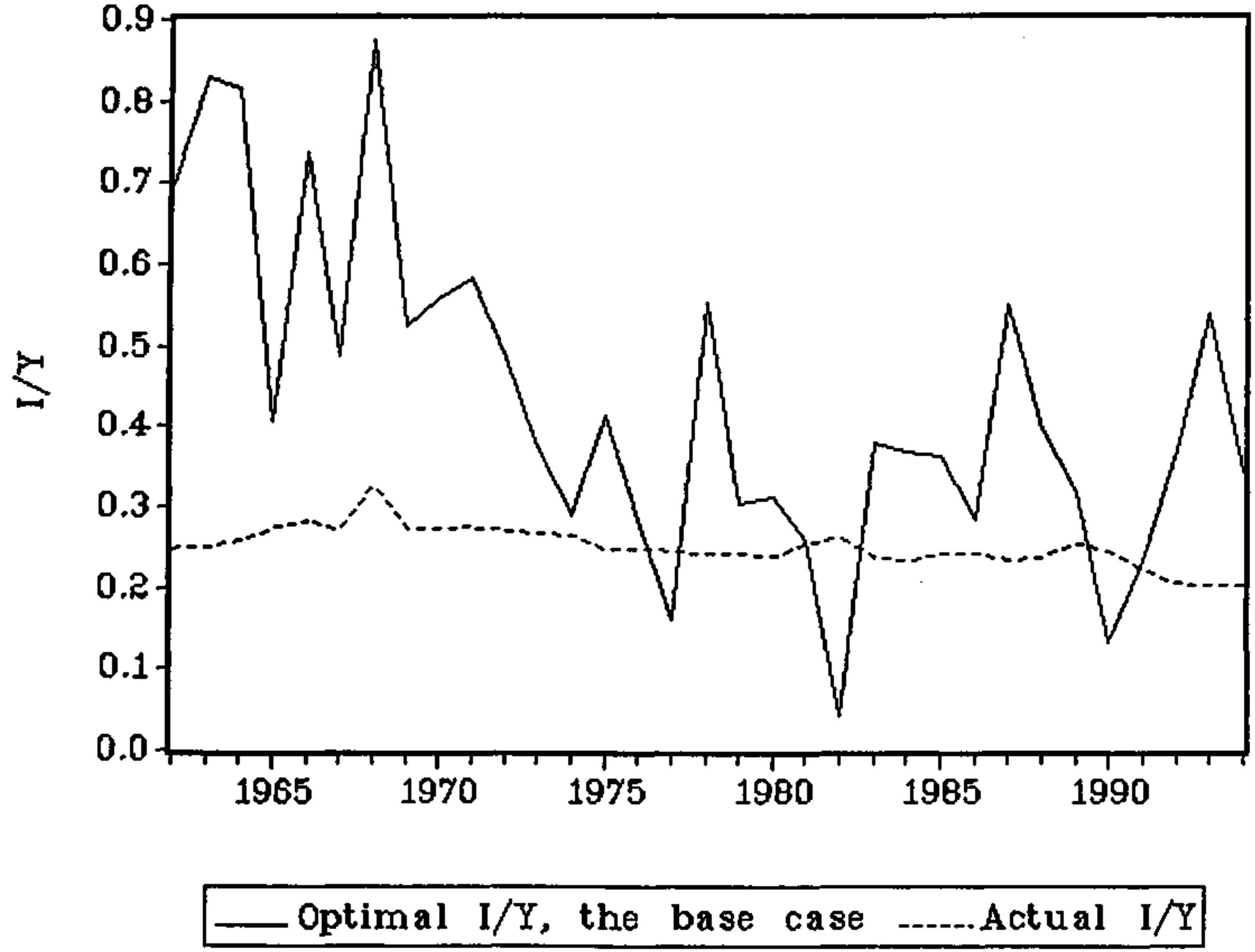

Chart 4 The effect of constant employment growth

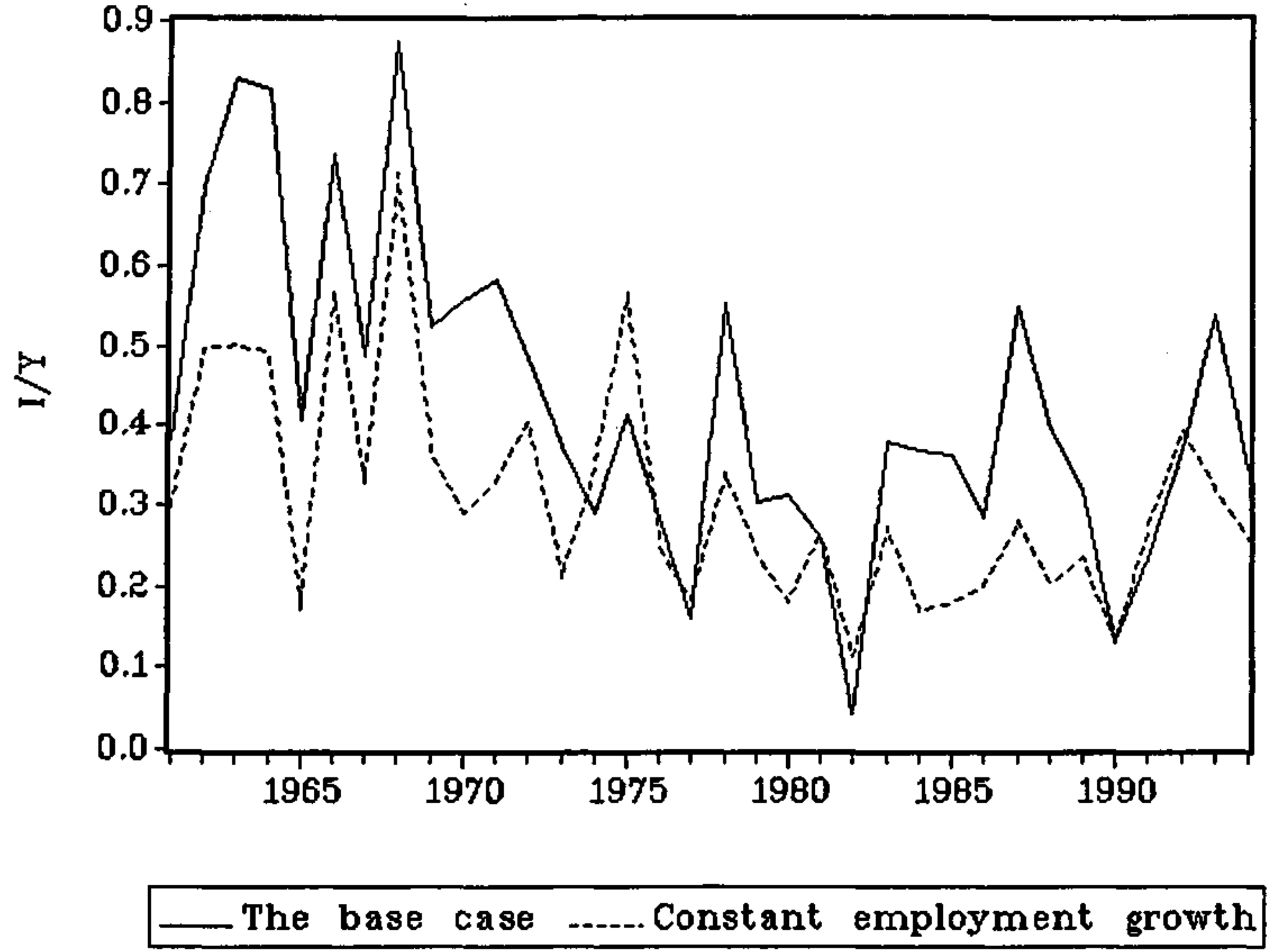

Chart 5 The effect of a constant interest rate

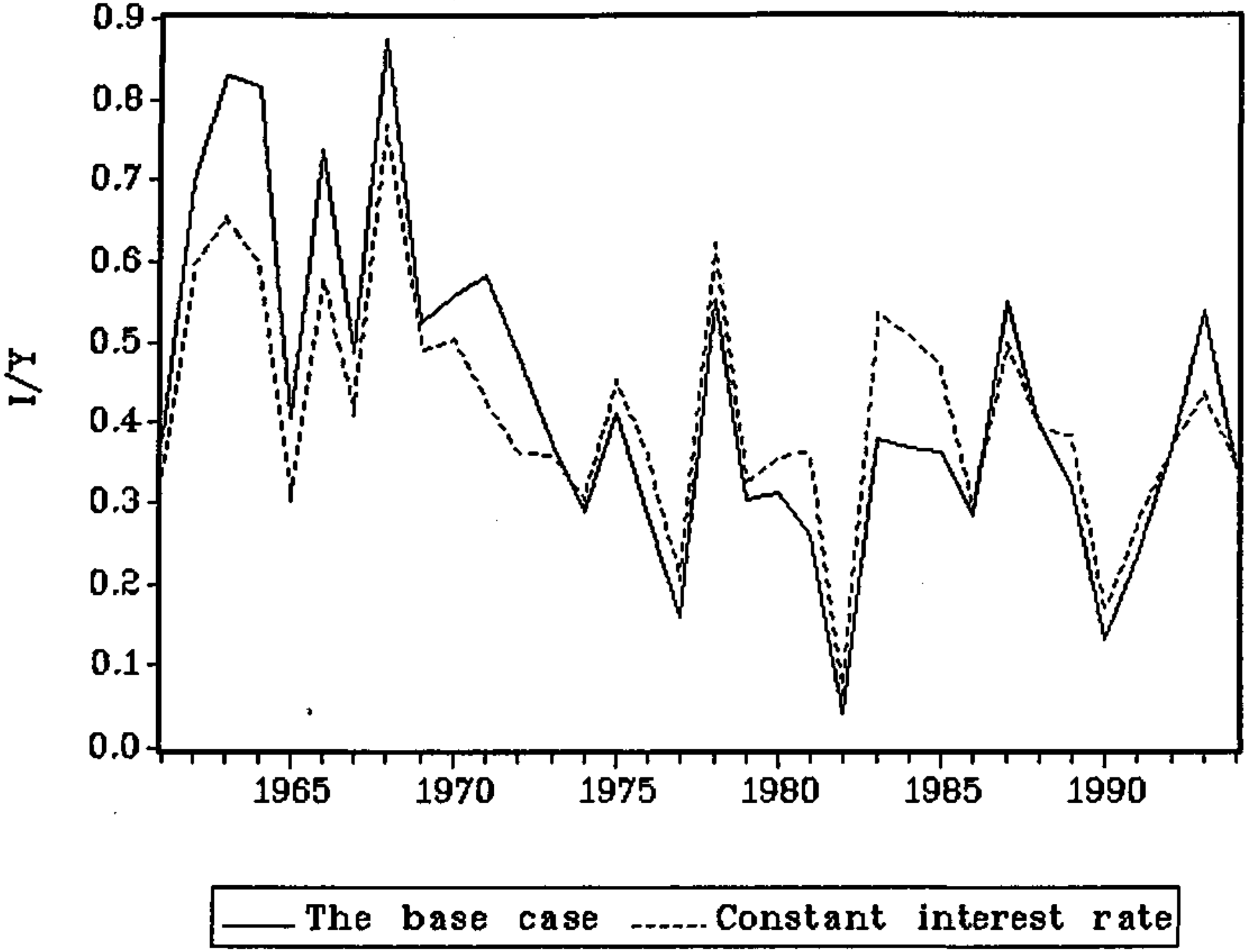

Chart 6 The effect of constant technical progress

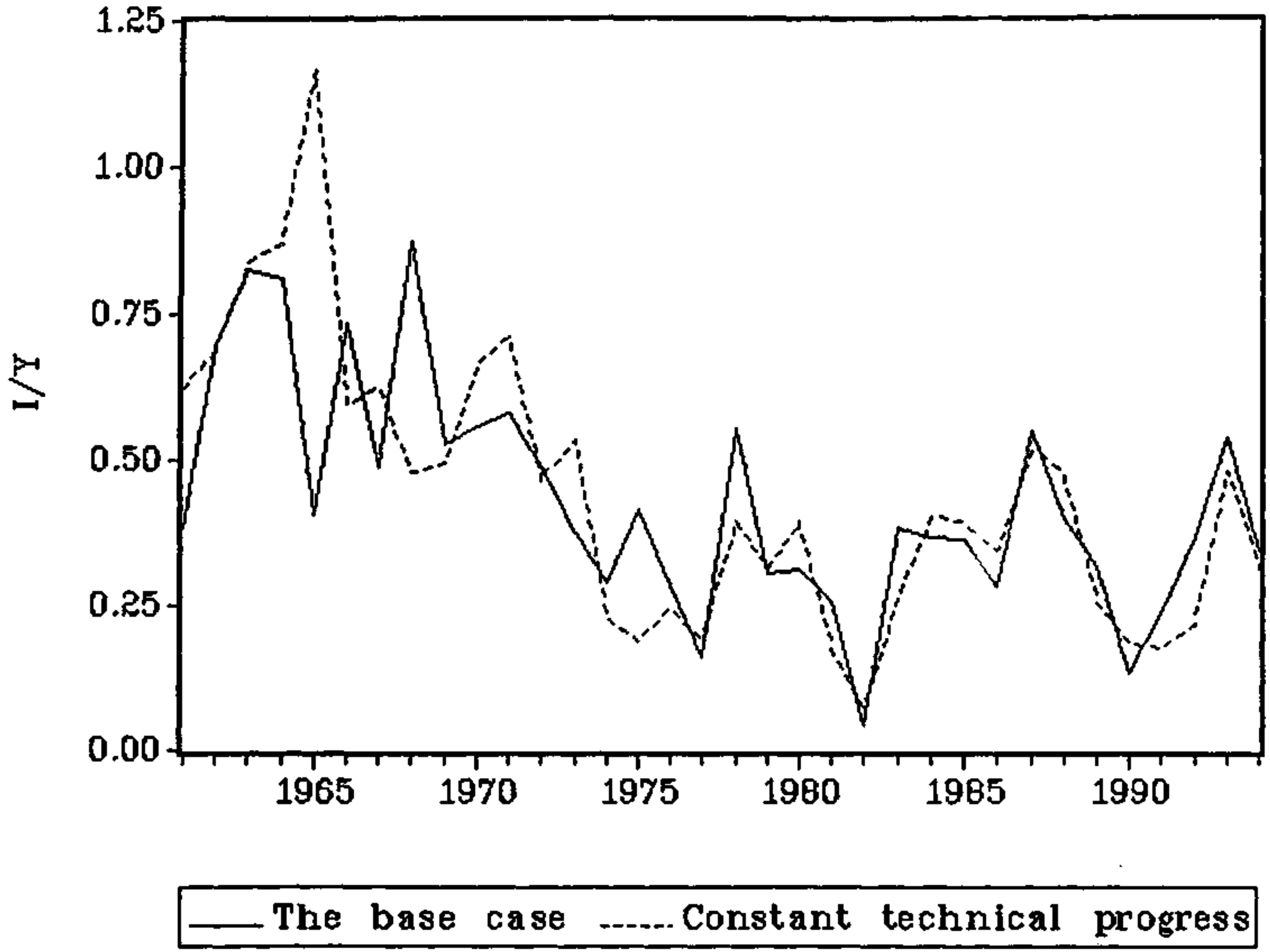

Chart 7 The effect of constant technical progress, mean adjusted.

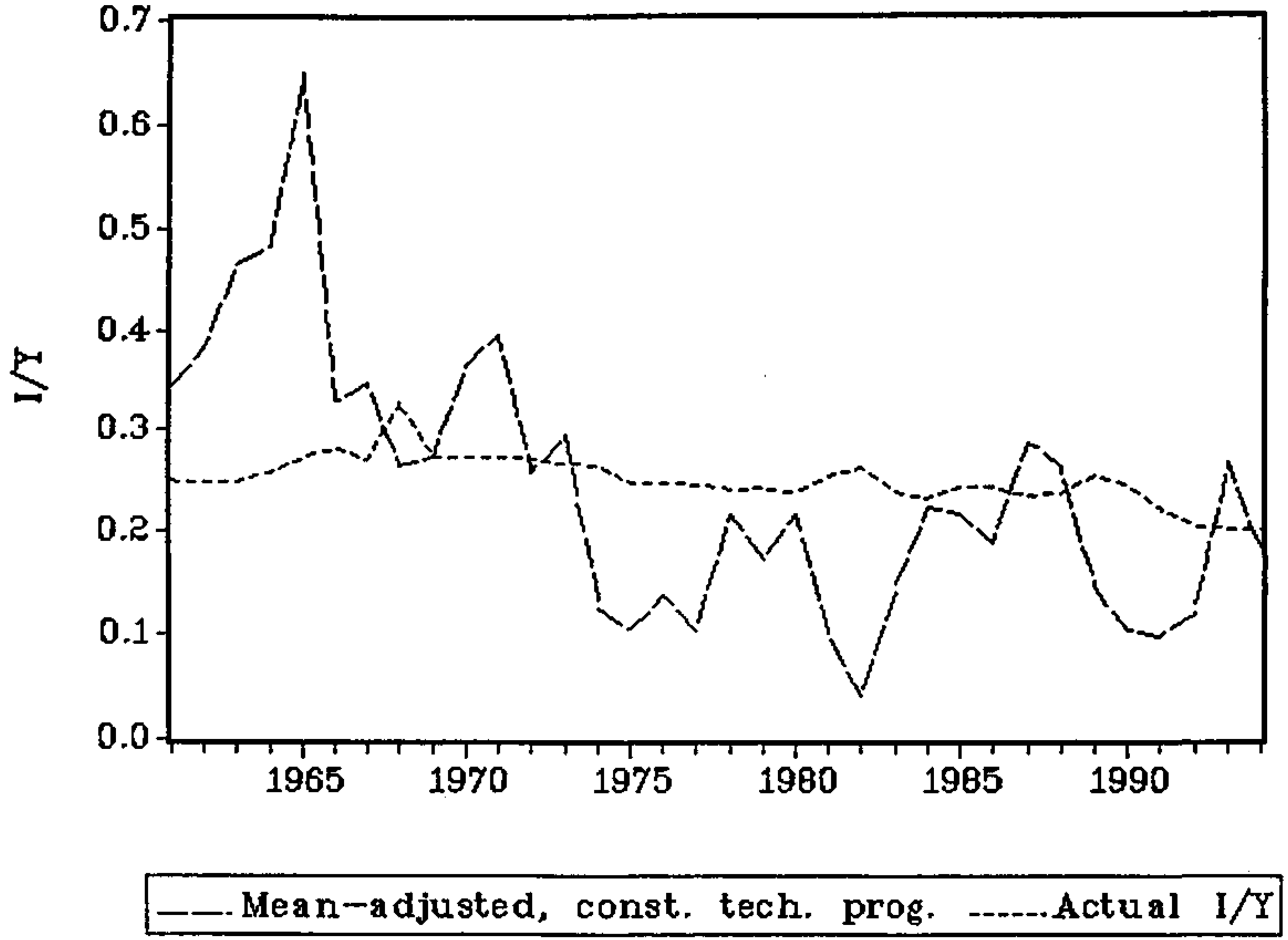

Chart 8 Sensitivity of volatility to values of the parameter α

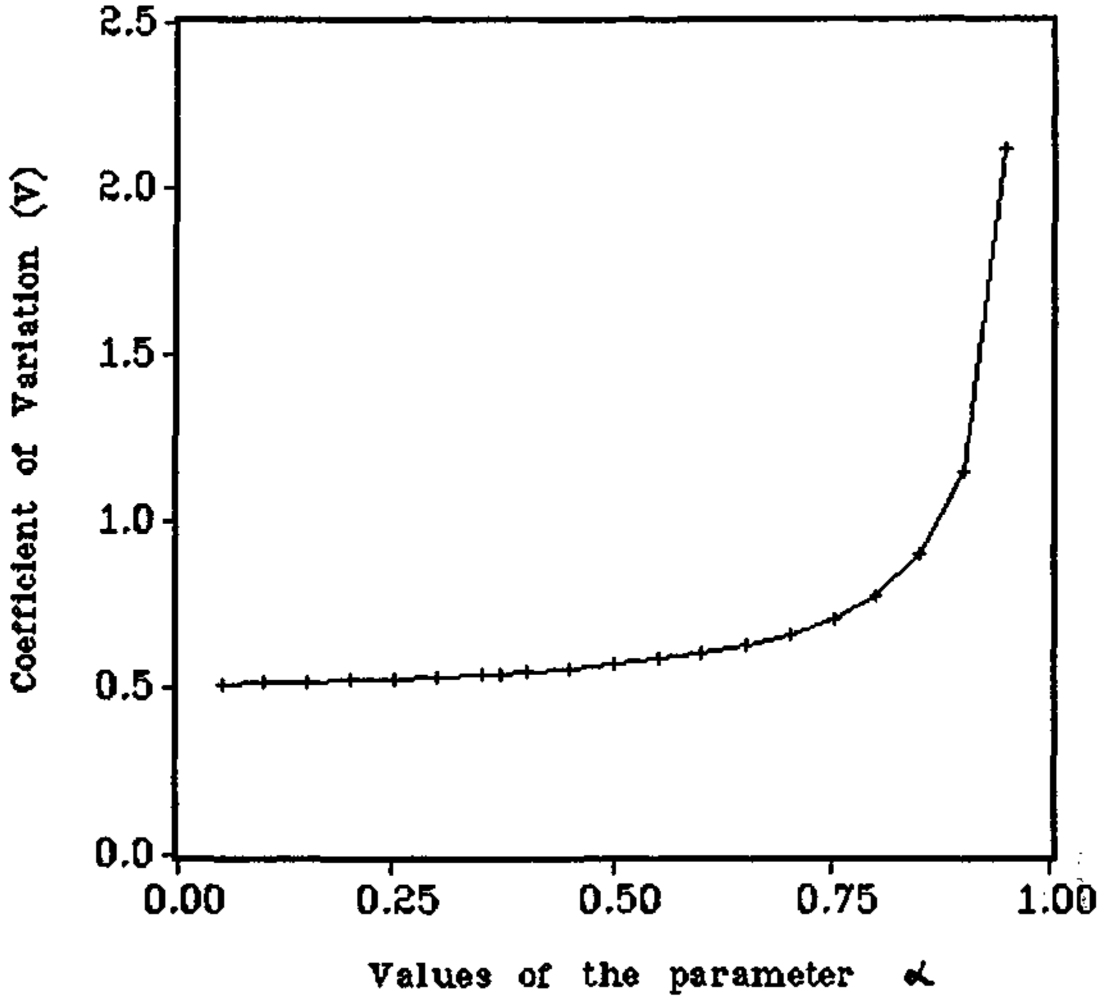

Chart 9 Comparison of Cobb–Douglas and CES production functions.

Both series are mean-adjusted and technical progress is constant.

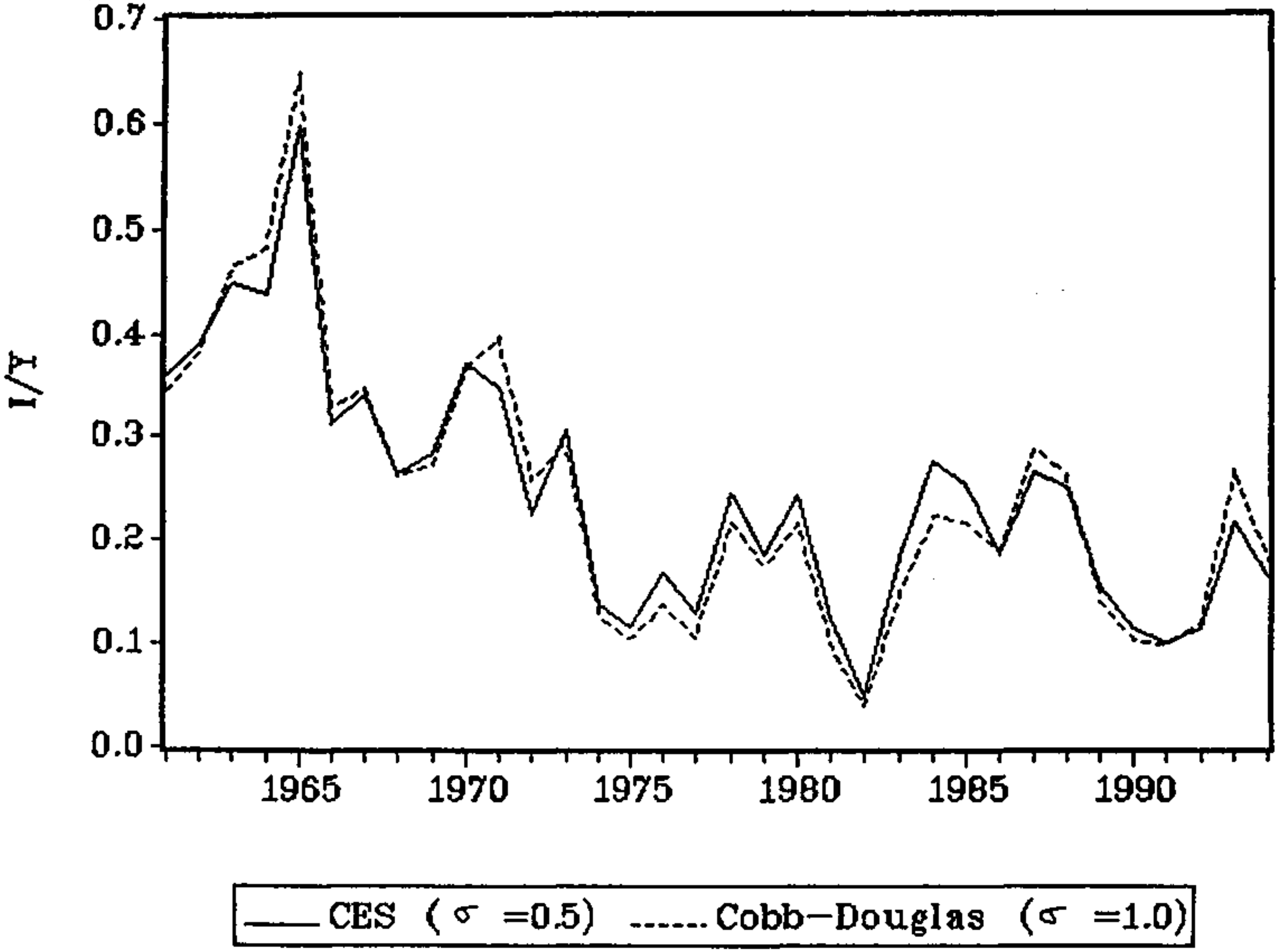

Chart 10 Sensitivity of volatility to the elasticity of subst. (σ)

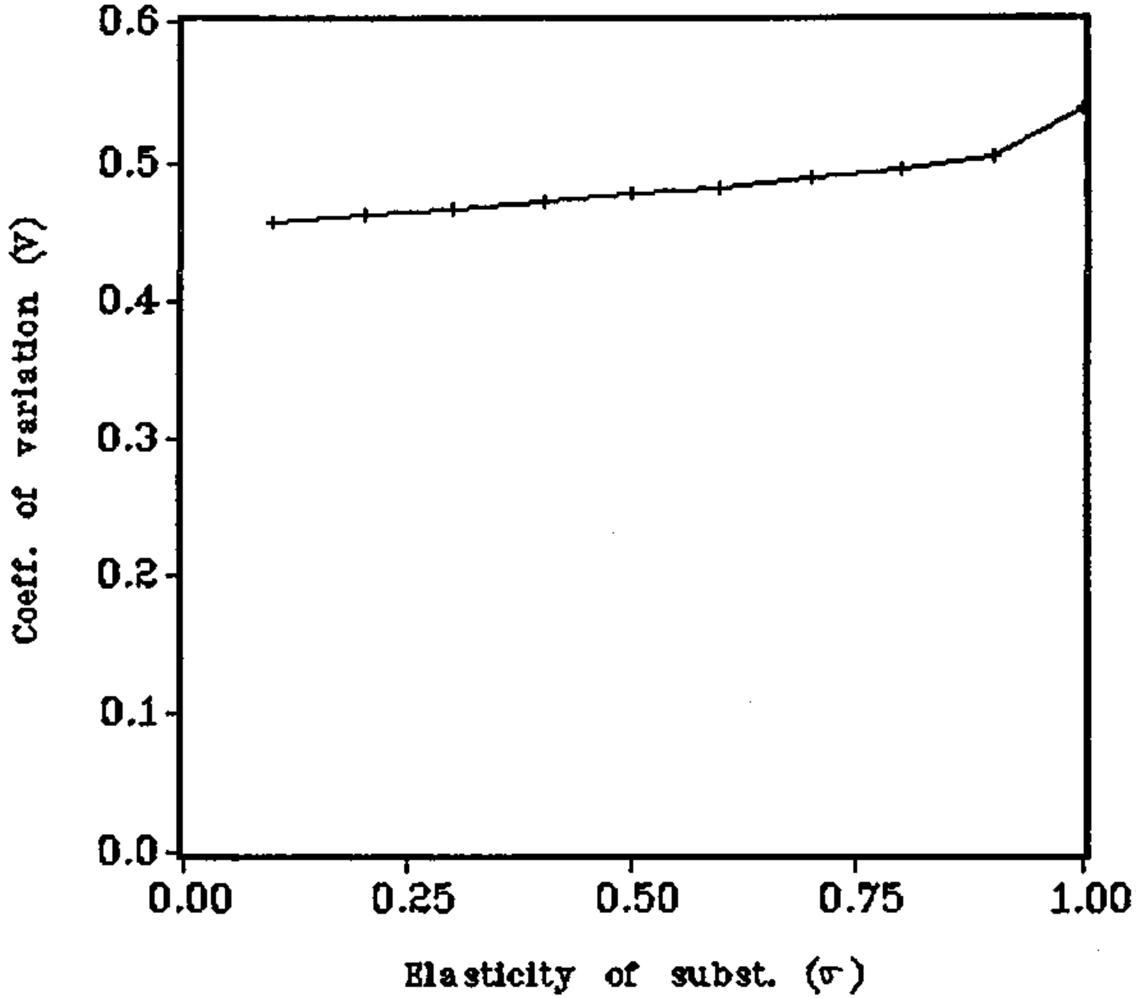

Chart 11 Adjustment cost model

Both series are mean-adjusted and technical progress is constant.

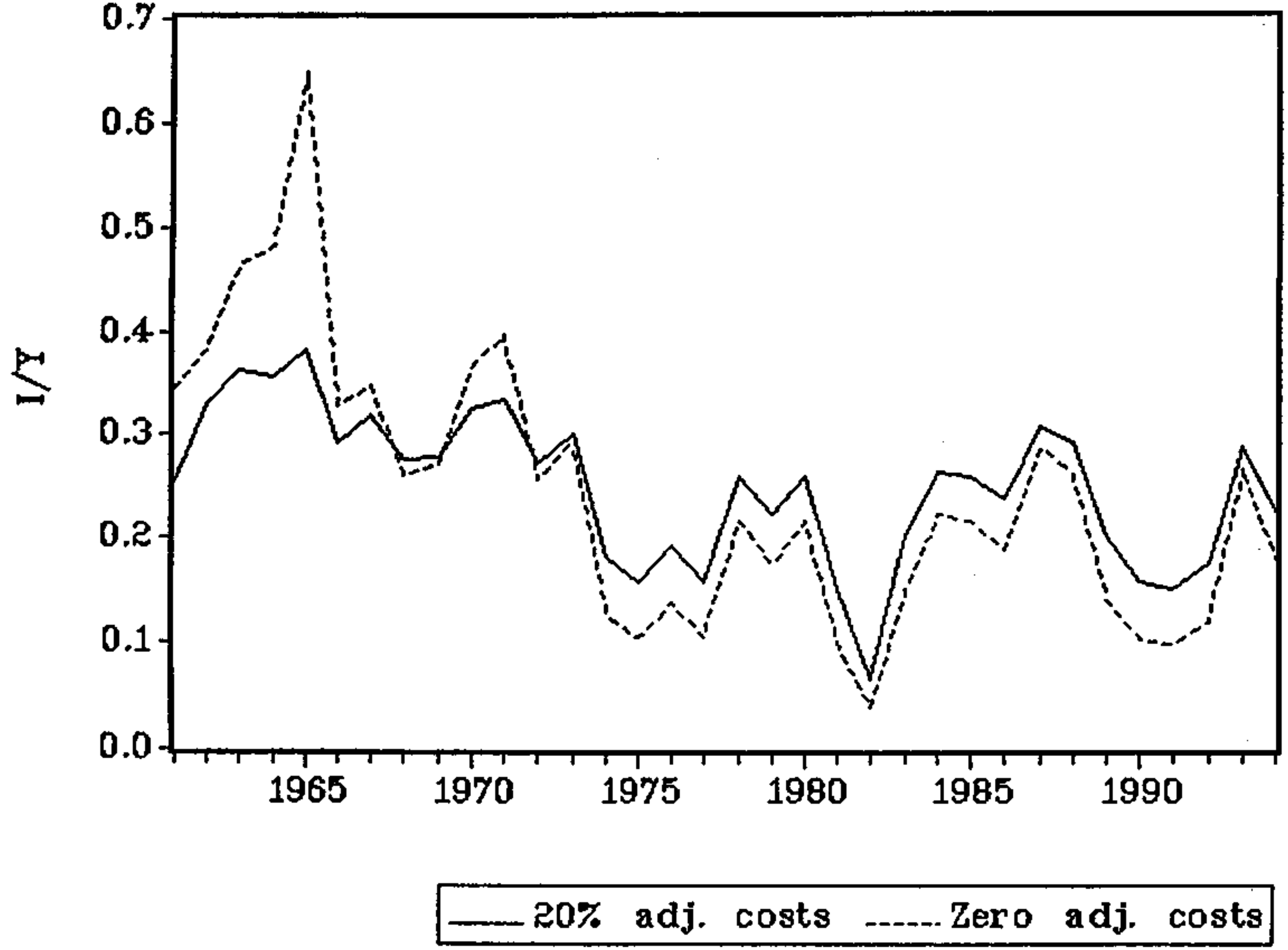

Chart 12 Delivery lags / exponential smoothing model

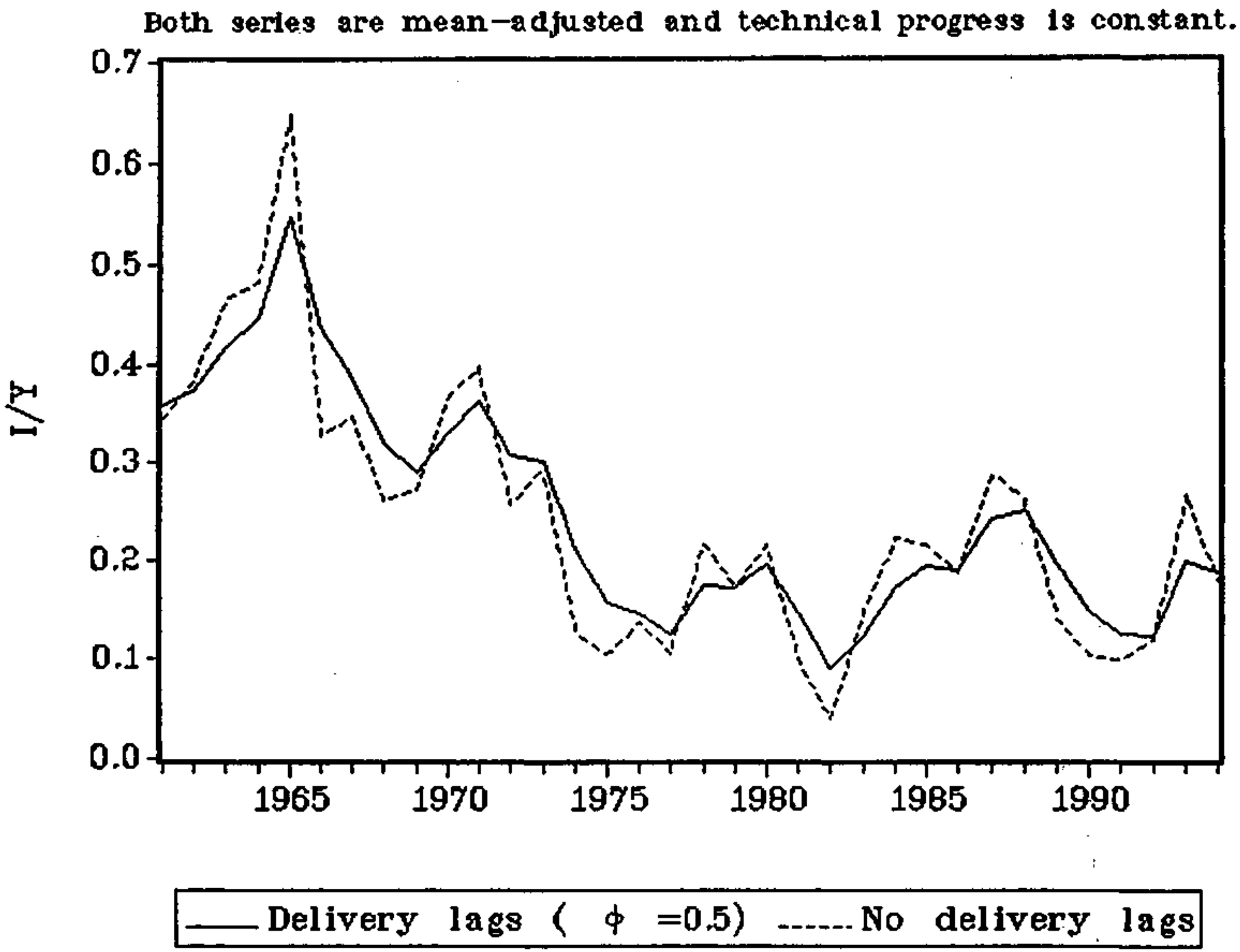

Chart 13 20 % adjustment costs; one year delivery lags (ϕ =0.5); constant technical progress.

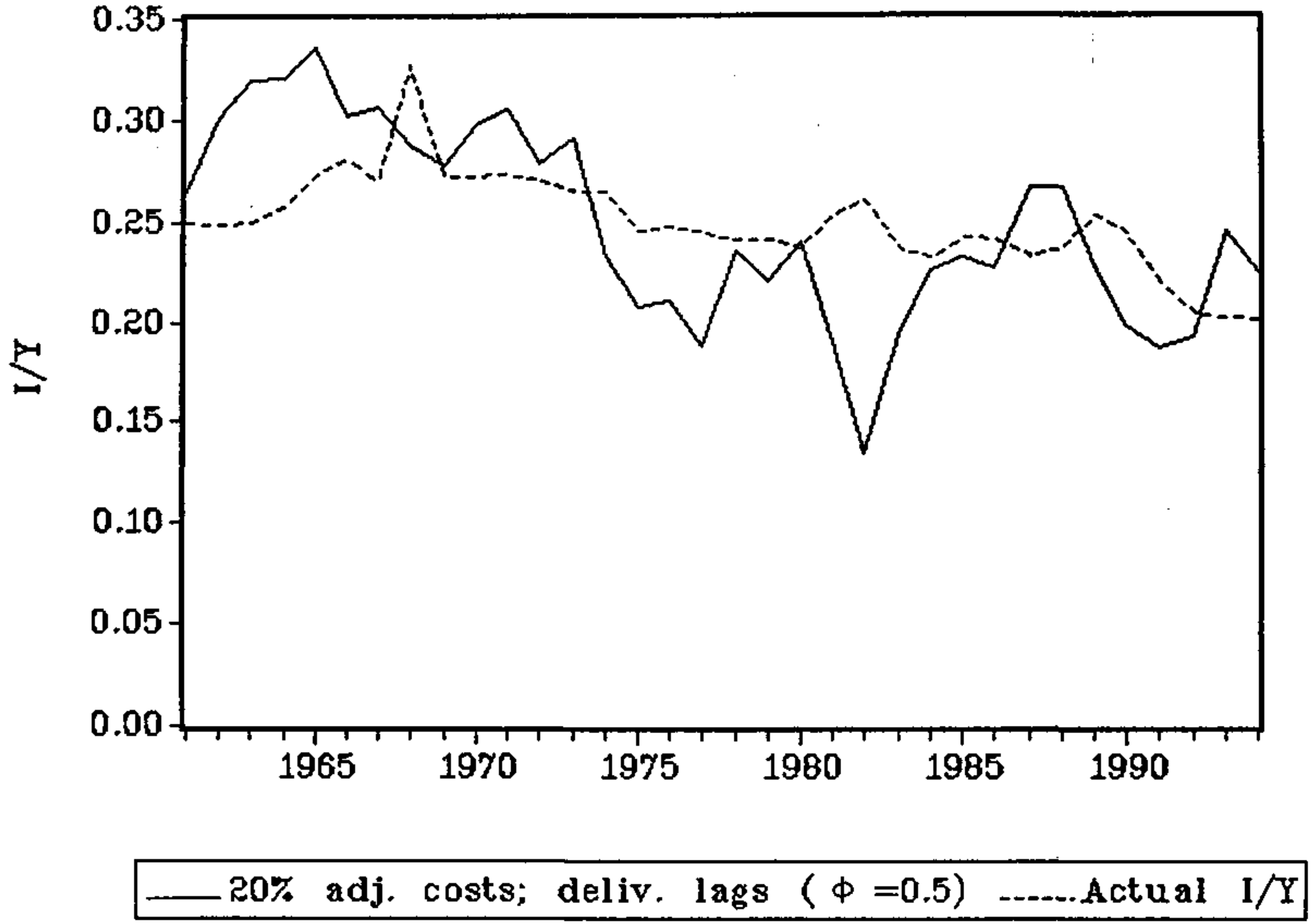

Chart 14 30% adjustment costs plus delivery lags (ϕ =0.5) plus constant technical progress.

Volatility for the optimal and actual series are equal.