The Information Content of the Term Structure of Interest

by

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February 1999

WORKING PAPER NO. 64

Not for quotation
Comments welcome.

ABSTRACT

This paper presents the results of an alternative test of the rational expectations theory of the term structure of interest rates. The validity of the expectations hypothesis of term structure has also been examined by other researchers. While there is more often rejection of the expectations hypothesis, no other theory (data-consistent with the entire yield curve) provides empirically adequate explanation of this phenomenon. The study considers postwar U.S. pure discount (zero coupon) bond yields with various maturities, starting from one month throughout 60 months. Based on the *ex − post* formation of rational expectations, we quantify the expectations error and test the level of truth of the expectations hypothesis, that is, the strength of the departure of the yield curve from the expectations theory. The results suggest that a significant amount of information available at no cost to market agents is not incorporated in forming people's expectations.

Keywords: expectations error, information set, moving average, rational expectations, term premium, term structure, stochastic error, yield.

JEL classification: C22, D84, E43, G14

1. INTRODUCTION

To explain the determinants of the shape of the term structure[1] of nominal interest rates, researchers have employed a number of theories. The most cited theories, however, are the market expectations hypotheses[2] (EH). A simple generalization of these theories demonstrate that the yield of the longer-term interest rate is determined by an average of current and future yields associated with the shorter-term interest rates plus a time-invariant term premium.

Since the introduction of the efficient market hypothesis in finance during the 1960's, in general the expectations theory of the term structure of interest rates accommodates *rational expectations* (Muth 1961) as a necessary assumption. Different econometric models have been developed for testing the rational expectations hypothesis (REH), see *inter alia* Shiller (1979, 1990), Shiller *et al.* (1983), Campbell and Shiller (1987, 1991), Fama (1984), Fama and Bliss (1987) Hamilton (1988), Froot (1989) and Hall *et al.* (1992). Two main questions have been considered by researchers: 1) whether the REH of the term structure of interest rates holds, and 2) how much information about the future yields is contained in the current spot rates.

The empirical studies in general are not supportive to the REH, particularly when the term structure of the U.S. interests rates is considered. For a summary of the results on the predictive power of the spread on discount yields for U.S. Treasury securities, refer to both Table 1 in Rudebusch (1995, p. 249) and Table 1-4 in Roberds and Whiteman (1996, pp. 26-27) and the references therein. Based on U.S. and UK data, Driffill *et al.* (1997) further provide evidence against the REH, while Cuthbertson (1996), Hurn *et al.* (1995) and Taylor (1992) have employed UK yields. On the contrary, when the yield curve from other than U.S. and UK countries is examined, there is less empirical evidence against the REH, see for example, among others, Engsted (1996), Engsted and Tanggaard (1995), Estella and Mishkin (1997), Gerlach and Smets (1997), Hardouvelis (1994), Boero and Torricelli (1997).

A recent study by Johnson (1997) examines the validity of the REH of the term structure of interest rates based on postwar U.S. data from the McCulloch (1990) pure discount bond yields. Johnson considers tests based on the $ex-post$ errors, expectations errors, built on the difference between the actual longer-term (n-period to maturity) yield rate and the equivalent one predicted by the REH from the shorter-term (one-period to maturity) yield rate. Although, the REH is rejected by other researchers, Johnson (1997) argues that a model, which he calls the "*noise model*", might provide some useful approximation to the relationship, at least within

[1] Such a differentiation, with respect to the length of time on which funds are either borrowed or lent, is known as the *term structure* of interest rates or the *yield curve*. In the literature, however, this concept more or less is loosely defined (Bliss 1996). It could include the discount function, the forward rate curve and zero-coupon yield curve (discount rate function). We refer to the "*term structure*" based on its zero-coupon version except where it is otherwise specified.

[2] There is great variety across different "*weak expectations hypothesis*" such as the *Local* Expectations Hypothesis (L-EH), Yield-to-Maturity Hypothesis (YTM-EH), Return-to-Maturity Hypothesis (RTM-EH), and the Unbiased Expectations Hypothesis (U-EH), as defined by Cox *et al.* (1981).

the one month and other short-term discount bonds up to 12 months to maturity.

Similarly to Johnson (1997), this paper quantifies the expectations error and tests the level of *falseness* of the expectations hypothesis, that is, the strength of the departure of the yield curve from the expectations theory. However, there are some important differences. Firstly, our study considers the extended McCulloch and Kwon (1993) dataset[3] of pure discount bonds. Next, we perform tests for the validity of the REH using zero coupon bond yields in a more general setting and on various maturities, starting from one month through to 60 months. Finally, we test the model by taking into account the serial correlation among the expectations errors under the REH. Johnson (1997, p. 1240) builds on previous research by Mankiw and Miron (1986), where one and two period to maturity interest rate series are under investigation. Of course, this is the only combination when the expectations error does not follow a moving average (MA) process. We stress that, if the time to maturity of the long-term bond, n, is more than two times the sampling interval, (eg. for interest rates sampled monthly m=1, $n > 2$) then the expectations error has a MA(n-2) structure.

In order to remove the MA(h-1) error structure, where the positive integer h gives the fraction between the forecast and the sampling interval respectively, the natural response is to apply generalized least squares (GLS) estimation. However, there are some pitfalls when one estimates a model with RE, see Flood and Garber (1980), Hansen and Hodrick (1980). To ensure consistent and asymptotically efficient estimation, we consider a forward filter (FF) as introduced by Hayashi and Sims (HS) (1983). Transforming both the dependent variable and the independent variables with the Hayashi-Sims forward filter (HSFF), we apply an instrumental variable (IV) procedure on the transformed regression equation using as instruments untransformed variables similar to those considered by Johnson (1997). Based on Hayashi-Sims' instrumental variable (HS-IV) estimation, our results indicate that a significant amount of information freely available to market agents is not incorporated in forming people's expectations. In contrast, if one uses OLS regressions, the information content in the expectations errors is significantly understated. Our results, in general, agree with the findings reported by many other researchers when the postwar US term structure is investigated, the REH is tested and consequently rejected.

This paper is organized as follows. Section 2 discusses theoretical foundations that are applied in the empirical work and Section 3 summarizes the data. The empirical analysis is carried out in Section 4, while a summary of the conclusions and implications is presented in Section 5.

[3]The end of the month data set used by Johnson, published in McCulloch (1990), coverts the time period up to February 1987. The new data set, McCulloch and Kwon (1993), matches precisely the previous one but with an added digit of precision and a few minor corrections for the time subperiod from August 1983 to February 1987.

2. METHODOLOGY

2.1 An Introduction to Empirical Studies of the Rational Expectations Hypothesis

A standard practice when investigating the term structure of interest rates is to assume rational expectations of the form $R_{t+1,m} = \mathrm{E}_t[R_{t+1,m}] + \varepsilon_{t+1}$. Note that: $R_{t+1,m}$ is the yield on a m-period zero-coupon bond; E_t denotes the mathematical expectation conditional on the available full public information set $\mathbb{I}_t$ up to time period t ($\mathbb{I}_t \subset \mathbb{I}_{t+1}$) and; ε_{t+1} – is a mean zero forecast error which is orthogonal to $\mathbb{I}_t$. Then, the REH of the term structure implies[4] that the n-period yield, $R_{t,n}$, apart from the time-invariant term premium, should equal the investors' expectations of the average level of current and future m-period yields, $R_{t,m}$, associated with the shorter-term interest rate. This relationship can be expressed as follows

$$R_{t,n} = L_k + \frac{1}{k} \sum_{j=0}^{k-1} \mathrm{E}_t(R_{t+mj,m}) \,, \tag{1}$$

where $k = \frac{n}{m}$ is a positive integer and L_k is a term premium which may vary with k, but is assumed to be constant through time. The presence of a constant term premium agrees with many of the twists of the REH. While REH imposes such an extreme assumption, "*no portion of the expectations hypothesis' failure can be attributed to time variation in term premiums*" (Bekaert *et al.*, 1997b, p. 8). Moreover, constant term premia are indicative of any possible source of deviation from the '*true*' term structure (Hurn *et al.*, 1992).

Following Campbell and Shiller (1991), equation (1) implies that a maturity specific multiple of the (long-short) term spread is able to predict the shorter-term, m-period to maturity, change in the longer, n-period to maturity, term interest rate yield,

$$R_{t+m,n-m} - R_{t,n} = \alpha_1 + \beta_1 \frac{m}{n-m} (R_{t,n} - R_{t,m}) + v_{t+m} \,, \tag{2}$$

where the error term in the regression, v_{t+m}, follows a moving average process of order $m-1$.

After subtracting $R_{t,m}$ from both sides of equation (1), we have the current spread between the long-term (n-period to maturity) rate and the short-term (m-period to maturity) rate equals the difference between the average expected future short-term rates and the current short-term rate plus a constant (risk premium) as follows

$$R_{t,n} - R_{t,m} = L_k + \frac{1}{k} \sum_{j=0}^{k-1} \mathrm{E}_t(R_{t+mj,m}) - R_{t,m} \,. \tag{3}$$

Alternatively, the REH can be tested[5] by regressing the *ex post* value of the short-

[4] There is more than one way to define a linearized version of the REH. Such techniques can be extended to the coupon-bearing bonds, see Shiller (1979) and Shiller *et al.* (1983).

[5] Another way to test the predictive power of the term structure of interest rates is by examining the forward

term changes, known as a realized difference $S^*_{n,m} \equiv \frac{1}{k}\sum_{j=0}^{k-1} R_{t+mj,m} - R_{t,m}$ on a constant and the current spread, $S_{n,m} \equiv R_{t,n} - R_{t,m}$, as given with the following regression equation

$$\frac{1}{k}\sum_{j=0}^{k-1} R_{t+mj,m} - R_{t,m} = \alpha_2 + \beta_2 (R_{t,n} - R_{t,m}) + w_{t+n-m} , \tag{4}$$

where the innovation term w_{t+n-m} is a moving average process of order $n-m-1$. The REH implies that the slope coefficient β_1 and β_2 should be unity in equation (2) and (4) equivalently. In other words, the current spread $S_{t,n}$ (maturity specific spread $\frac{m}{n-m}S_{t,n}$) should be a good predictor of the future m-period change in the longer term bond yield (of the change between the weighted average of future m-period rates and the short-term bond yield). Two recent papers by Rudebusch (1995) and Roberds and Whiteman (1996) review the REH with U.S. nominal interest rates data. According to the former study, one can find a clear $U-$*shaped* pattern of the predictive power of the spread at various maturities (Table 1., p. 249), while the latter study explains the "*predictability smile*" in the term structure for post-war U.S. nominal yields and even compares the movements in long rates to Mona Lisa's "*smirk*" (Table 1-4, pp. 26-27). Hence, there is some support for the predictive power and thus for the REH in both the very short- and the very long-end of the yield curve. On the contrary, the hypothesis fails in the middle range of the curve when the shorter-term rate in the spread is between two months and two years.

There is a plethora of evidence for biases in OLS regressions with variables from overlapping samples (Hansen and Hodrick (1980)) or when testing a rational expectations model with pre-determined, but not exogenous regressors, see Mankiw and Shapiro (1986). More recently, Bekaert *et al.* (1997 b & a) further explore the "peso problem" and the small sample bias distortions. Moreover, the researchers demonstrate the high persistence of short term yields and, via well-designed Monte Carlo studies, model the small sample distributions in the yield curve. They show how much stronger the rejection of the REH is, particularly in studies with U.S. data.

Note that the aim of the paper is not to perform tests for the unity of the slope coefficients[6] as given with equations (2) and (4), but to examine an alternative test to REH, more specifically the level of falseness of the REH, similar to Johnson (1997). We stress that, if the time to maturity of the long-term bond is more than two times the sampling interval, i.e. for interest rates sampled monthly, while $k = \frac{n}{m} > 2$, the innovation term in both regressions given with equation (2) and (4) follows moving average process.

yields implicitly as predictors of the future spot rates, see Fama (1984), Fama and Bliss (1987) and Stambaugh (1988). Alternatively, the REH can be approached via vector autoregression (VAR) modeling, see Campbell and Shiller (1987, 1991) or by the error correction (ECM) – cointegrating modeling, refer to Stock and Watson (1988), Hall et *al.* (1992), Shea (1992), Pagan et *al.* (1996) and, Anderson (1997).

[6] Empirical studies on the information content of the term structure of U.S. interest rates are summarized in Melino (1988), Shiller (1990), Rudebusch (1995), Campbell (1995) and, Roberds and Whiteman (1996) among others.

2.2 The Econometric Introduction to the Expectations Errors Model

When the expectations are formed rationally, the $ex-post$ $n-$period discount yield $R^*_{t,n}$, known as a perfect foresight, is a simple average of the current and future $m-$period yields

$$R^*_{t,n} = \frac{1}{k} \sum_{j=0}^{k-1} R_{t+mj,m} \,. \tag{5}$$

From equation (1), one can write the model for the yield of the n-period to maturity bond as $R_{t,n}$=$\mathrm{E}_t[R^*_{t,n}]$+L_k. The forecast error of this model, $R^*_{t,n}$-$R_{t,n}$+L_k, must be uncorrelated with the available full public information set $\mathbb{I}_t$ known at time t. Note that in practice researchers impose a lesser restrictive set of information Ω_t, subset of $\mathbb{I}_t$, which usually includes current and lagged interest rates. With $z_{t,n}$ we denote the *expectations* (specification or forecast) *error*, $R^*_{t,n} - R_{t,n}$. Of course, this random variable follows a MA(n-m-1) process. Similar to Johnson (1997), the constant term premium in (1) is supressed and after a substitution in equation (5) the expectations error implies

$$z_{t,n} \;=\; \frac{1}{k} \sum_{j=0}^{k-1} (R_{t+mj,m}) \;-\; \frac{1}{k} \sum_{j=0}^{k-1} \mathrm{E}_t(R_{t+mj,m}) \;=\; R^*_{t,n} \;-\; R_{t,n} \,. \tag{6}$$

Therefore, a testable assumption is that each expectations error ($z_{t,n} = R^*_{t,n} - R_{t,n}$) is orthogonal to Ω_t. Thus, a regression of $z_{t,n}$ on a constant and variables such that all are in Ω_t under the null of REH should produce an insignificant $F-$test on the coefficients. Also the coefficient of determination R^2 is expected to be insignificant if not close to zero. However, the residuals of such OLS are serially correlated due to the moving average error structure of the stochastic part of the expectations error.

To resolve the problem, the first natural response is to apply GLS estimation. However, there are some pitfalls when one estimates a model with RE, see Flood and Garber (1980), Hansen and Hodrick (1980). To ensure consistent and asymptotically efficient estimaton, we consider a forward filter to the linear model, as introduced by Hayashi and Sims (1983).

The idea behind Hayashi and Sims (HS) approach is as follows. Let V=$E[u_{t,n}u'_{t,n}]$, where $u_{t,n}$ is the MA(n-m-1) error term from the OLS regression of the expectations error, $z_{t,n}$, on a constant and some predetermined but not necessarily exogenous regressors $X_{t,n}$ ($t = 1, 2, 3, \ldots, T$ and $\frac{n}{m} > 2$), all in Ω_t. Choose W such that WVW'=$\sigma^2 I$, where I is $T\times T$ identity matrix. Then, after premultiplication both sides of the above regression (dependent and independent variables) with W, the new error term, $u^*_{t,n}$=$Wu_{t,n}$, will have a diagonal scalar covariance matrix. While GLS approach uses a lower triangular matrix, Hayashi and Sims suggest W to be upper triangular. Under the RE hypothesis, each element of $u^*_{t,n}$ is a linear combination of current and future $u_{t+s,n}$ ($s = 1, 2, 3, \ldots, q$) and therefore is orthogonal to current and past values of $X_{t,n}$ ($t = 1, 2, 3, \ldots, T$ and $\frac{n}{m} > 2$), i.e. orthogonal to Ω_t.

For example, when m=1 and n=3, the error term is MA(1) process, $u_{t,3}$=a_t+θa_{t-1}, where a_t is a white-noise process. The GLS is based on a backward transformation of the regression equation with $(1+\theta L)^{-1}$ while Hayashi and Sims propose a forward filter $(1+\theta L^{-1})^{-1}$ which transforms the MA disturbance[7] into its one-step ahead forecasting error. Hence, transforming both the dependent variable and independent variables, the serial correlation is removed from the innovation term from the right-hand side of the OLS regression. We apply IV regression using untransformed variables as instruments similar to those considered by Johnson.

The non-trivial part is to find a consistent estimator for the variance-covariance matrix. We consider two approximations to the upper triangular matrix W. The common feature in both procedures is that we model the serial correlations in the error term in the regression model based on both the full information embodied in the interest rate series and the known MA structure of the expectations error. For the first approximation, we assume that the short-term rate follows an AR(1) model, see the Appendix for the derivation of the variance-covariance matrix. To estimate the moving average structure of the innovation term of the latter, initially we regress the expectations error on a constant and predetermined regressors, $X_{t,n}$ ($t = 1, 2, 3, \ldots, T$ and $k=\frac{n}{m} > 2$), such that all are in Ω_t. For the innovation term of this regression one needs to estimate an invertible MA(q) process

$$u_{t,n} \;=\; \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_3\varepsilon_{t-3} + \cdots + \theta_q\varepsilon_{t-q}, \tag{7}$$

where ε_t is zero mean and uncorrelated across time white noise process and q=n-2. Fitting a MA(q) model is not an easy task for estimation. To avoid the computational hurdle, we use a simple, fast and robust estimator for fitting moving average models, see Galbraith and Zinde-Walsh (1994) for more details. To the innovation term initially, we fit a higher order AR(p), where $p > q$ and n is defined as above. The moving average parameters of this stochastic process [8] are estimated by simple transformations of the AR estimates, see Galbraith and Zinde-Walsh (p. 145, 1994), equation (3). The corresponding Hayashi-Sims forward transformation leads to an IV estimation which produces consistent and asymptotically efficient estimates.

To test the level of the information content of the term structure of interest rates, we first duplicate Johnson's results for our data set for any n such that $n > 2$. Then we run HS-IV regressions with the transformed expectations error as dependent variable on a constant and the transformed regressors but using untransformed instruments, namely, $z_{t-n+1-j,n}$, the lagged expectations errors or, $S_{t-j,n}$, the lagged spread between the n, $n > 2$, period to maturity yield as a long-term rate and the one month to maturity yield as a short-term rate. Note that there is no serial dependence in the OLS residuals when the time to maturity of the long-term rate is twice the equivalent time from the short-term rate. Before performing these tests to quantify the "*falseness*" of the REH by projecting the expectations error ($z_{t,n}$)

[7] If θ exceeds one in modulus, then the forward filter is $(1+\theta^{-1}L^{-1})^{-1}$. Of course, if θ=1 in modulus the autoregressive representation of $u_{t,n}$ does not exist (Hayashi and Sims (1983)) but a generalized method of moments estimation (Hansen (1982)) is consistent and efficient within this class of models.

[8] Special attention is taken to test for invertibility of these processes, i.e. if the roots of the pure autoregressive (moving average) polynomials are strictly greater than one in modulus. Whenever a non-invertable representation occurs, i.e. the root is greater than one we use the reciprocal value of the root.

on various elements of the information set Ω_t, as specified above, we summarize the data in the next section.

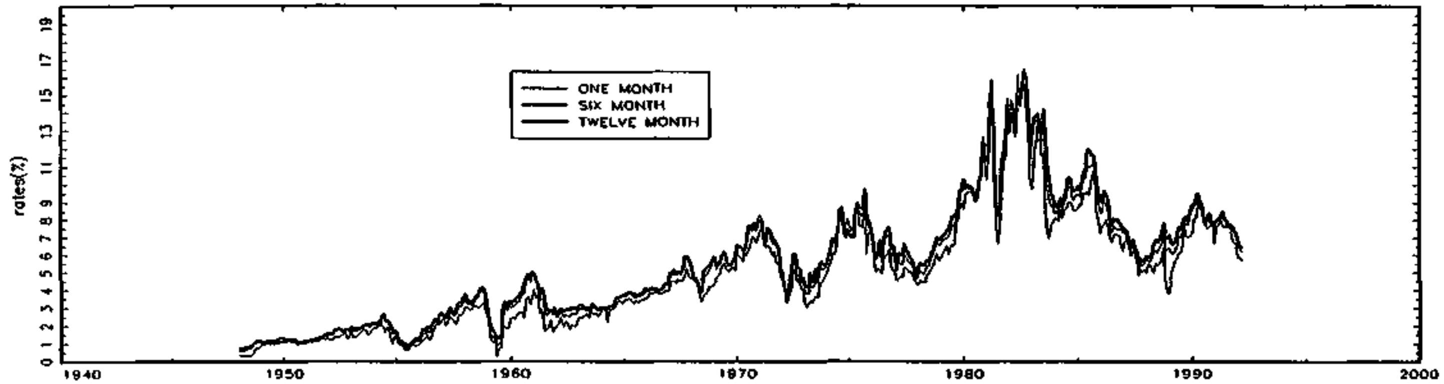
3. DATA

We consider zero coupon bonds, end of the month nominal interest rates based on postwar U.S. Government securities from the McCulloch and Kwon (1993) dataset of 531 observations starting from December, 1946 through February 1991. This is an updated version of McCulloch (1990). These pure discount bond yields are estimated by a spline interpolation technique, see McCulloch 1975. The data consists of theoretical zero coupon bonds, instantaneous forward yield and par bond yield curve. Moreover, the series are tax adjusted continuously compounded yields to a broad range of maturities.

In empirical studies, the postwar U.S. interest rates are usually examined for a time period which is post the Treasury Accord of 1951 (March 4, 1951) and it is further split into two sub-periods to account for the structural change in the Federal Reserve System which occurred in October 1979. Before October, 1979, the Fed's target was based on both the growth rate of money supply, $M1$, and the nominal level of short-term interest rates. On October 6, 1979 the Fed officially adopted M1 as its only intermediate target but kept some non-borrowed reserve as inter- and intra-week instruments to exercise monetary control. After October, 1982, there were some new shifts in the monetary targeting. According to Mishkin (1997), the Fed reversed to a "*policy of smoothing*" the term structure by targeting on borrowed reserves (discount loan borrorings). As a result, the post October 1982 interest rates are a little more "controled" and not so erratic in behaviour as they had been during the change 1979:10 to 1982:10, for more details on the Fed's role see Stigum (1990, pp. 19-29). Nevertheless, the short-term rates after October 1982 are more freely in fluctuation in comparisson with those before the first change in October 1979.

We divide our full sample period to four subperiods. The first two coincide with Johnson's study, namely the starting one – post the Treasury Accord, March 4, 1951 throughout the structural break in the Federal Reserve System, October 1979, 1951:12 to 1979:09, and the other is the full sample period used by Johnson, 1951:12 to 1987:02. We add two more subperiods: from the start, 1951:12 to 1982:09, and finally, our total sample period of 471 observations, based on 1951:12 to 1991:02. The following zero coupon bonds are chosen for our study: one-month, three-month, four-month and so on up to 18-months and 24- 36- 48- and, 60-months to maturity, 20 series in total.

U.S. ZERO-COUPON YIELD CURVE



TAX-ADJUSTED CONTINUOUS-COMPOUNDED INTEREST RATES, 1947:12 -1991:2

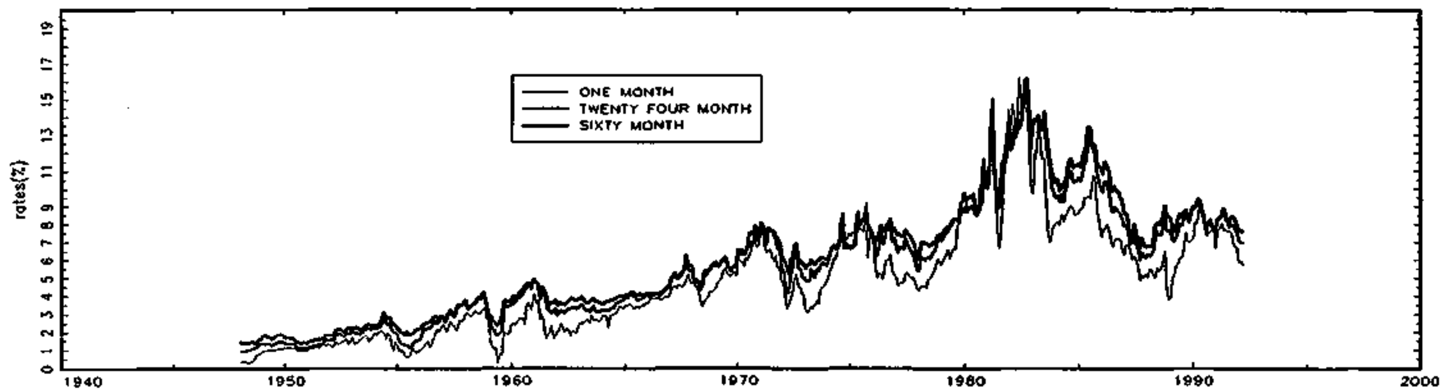


Table 1A: *Summary Statistics* of the *Expectations Error*

exp.error	Dec. 1951 - Feb. 1991 *MEAN*	*VAR*	*SKE*	*KURT*	Dec. 1951 - Sep. 1979 *MEAN*	*VAR*	*SKE*	*KURT*
$z_{t,3}$	-0.32	0.24	-2.43	14.95	-0.24	0.10	-0.83	2.67
$z_{t,4}$	-0.41	0.38	-2.64	15.24	-0.32	0.15	-0.82	1.74
$z_{t,5}$	-0.48	0.53	-2.41	13.39	-0.39	0.20	-0.87	1.57
$z_{t,6}$	-0.54	0.67	-2.10	11.11	-0.44	0.27	-0.96	1.49
$z_{t,7}$	-0.59	0.79	-1.69	8.24	-0.48	0.34	-0.99	1.50
$z_{t,8}$	-0.62	0.90	-1.35	6.07	-0.50	0.41	-0.99	1.37
$z_{t,9}$	-0.65	0.99	-1.09	4.53	-0.52	0.47	-0.97	1.19
$z_{t,10}$	-0.67	1.07	-0.93	3.72	-0.53	0.53	-0.94	0.99
$z_{t,11}$	-0.68	1.16	-0.83	3.30	-0.53	0.58	-0.88	0.70
$z_{t,12}$	-0.70	1.25	-0.78	3.09	-0.54	0.63	-0.82	0.41
$z_{t,13}$	-0.72	1.34	-0.76	2.92	-0.55	0.68	-0.75	0.17
$z_{t,14}$	-0.73	1.43	-0.74	2.74	-0.55	0.72	-0.68	-0.03
$z_{t,15}$	-0.75	1.52	-0.74	2.54	-0.56	0.76	-0.63	-0.17
$z_{t,16}$	-0.76	1.61	-0.74	2.32	-0.57	0.79	-0.59	-0.26
$z_{t,17}$	-0.77	1.70	-0.75	2.07	-0.57	0.83	-0.55	-0.32
$z_{t,18}$	-0.78	1.79	-0.75	1.83	-0.58	0.87	-0.53	-0.35
$z_{t,24}$	-0.82	2.29	-0.63	1.08	-0.62	1.02	-0.50	-0.35
$z_{t,36}$	-0.85	3.03	-0.56	0.74	-0.67	0.97	-0.69	-0.11
$z_{t,48}$	-0.85	3.48	-0.70	1.13	-0.62	0.71	-0.60	-0.38
$z_{t,60}$	-0.84	3.79	-0.84	1.26	-0.52	0.52	-0.37	-0.36

Table 1B: *Summary Statistics of the Expectations Error*

exp.error	Dec. 1951 - Feb. 1987 *MEAN*	*VAR*	*SKE*	*KURT*	Dec. 1951 - Sep. 1982 *MEAN*	*VAR*	*SKE*	*KURT*
$z_{t,3}$	-0.30	0.25	-2.50	15.38	-0.29	0.27	-2.60	15.67
$z_{t,4}$	-0.40	0.40	-2.67	15.27	-0.37	0.42	-2.83	16.02
$z_{t,5}$	-0.48	0.56	-2.41	13.07	-0.44	0.56	-2.58	14.71
$z_{t,6}$	-0.54	0.71	-2.08	10.69	-0.48	0.65	-2.04	11.78
$z_{t,7}$	-0.59	0.85	-1.67	7.83	-0.51	0.76	-1.62	8.91
$z_{t,8}$	-0.62	0.97	-1.33	5.71	-0.52	0.83	-1.20	6.55
$z_{t,9}$	-0.64	1.06	-1.07	4.21	-0.52	0.88	-0.82	4.59
$z_{t,10}$	-0.66	1.16	-0.92	3.42	-0.52	0.92	-0.58	3.65
$z_{t,11}$	-0.68	1.25	-0.83	3.01	-0.52	0.96	-0.40	3.17
$z_{t,12}$	-0.69	1.35	-0.77	2.78	-0.51	1.00	-0.32	3.10
$z_{t,13}$	-0.71	1.46	-0.76	2.61	-0.51	1.07	-0.33	3.11
$z_{t,14}$	-0.72	1.56	-0.75	2.44	-0.50	1.11	-0.32	3.23
$z_{t,15}$	-0.73	1.66	-0.75	2.26	-0.49	1.11	-0.15	2.67
$z_{t,16}$	-0.74	1.76	-0.76	2.05	-0.47	1.10	0.02	1.82
$z_{t,17}$	-0.75	1.85	-0.78	1.83	-0.46	1.11	0.08	1.24
$z_{t,18}$	-0.75	1.94	-0.79	1.64	-0.45	1.14	0.06	0.85
$z_{t,24}$	-0.76	2.42	-0.70	1.10	-0.41	1.43	0.09	0.27
$z_{t,36}$	-0.66	2.59	-0.41	0.84	-0.36	1.68	0.29	0.38
$z_{t,48}$	-0.55	2.63	-0.56	2.02	-0.34	1.44	0.80	1.34
$z_{t,60}$	-0.39	2.33	-0.77	3.82	-0.32	1.05	1.28	3.11

Notes: For the expectations error for any (sub)-period, a sample mean, a sample variance, a coefficient of skewness and a coefficient of excess kurtosis are computed and denoted with MEAN, VAR, SKE and KURT respectively.

4. EMPIRICAL ANALYSIS

For the selected time series and sample period, to avoid repetition, we depict only some time series. As can be seen from Figure 1, the yields on the zero coupon bonds are moving close together within an "*upwards*" trend until the early 1980's. During this time period, the most extreme high values were recorded. On the contrary, one can observe a reversing and hence declining "trend" for the last quarter of the sample time period. Note that the most striking feature of the last sub-period, starting soon after the structural change in the Federal Reserve System (October 1979), is the significant increase in the volatility level, refer to Figure 1.

Strong persistence is a common feature among these U.S. spot interest rates. Although there is no clear evidence of an exact unit root, the common practice is to assume that the interest rates are integrated of order one stochastic processes, while the spread between any two yields is integrated of order zero. Note that all the inferences we have made are based on stationary variables, see for more details Johnson (1997). Since many studies have reported summary statistics[9] for this data set, no need exists to repeat these. However, we depict the basic summary statistics of the expectations error in Table 1. The averages of the specification errors for all maturities and for all time periods are negative, which based on the model violates the zero term premium assumption or the pure REH. In fact, one can speculate that these premiums are not constant over the observed time, increasingly at the shorter term bonds but reaching a saturated level for some inter-mediate maturities, see Table 1A and Table 1B. Similarly, the variability among the expectations errors is minor for short-term maturities. It is a function of the observed time period also. Furthermore, the distributions of the expectations error indicate generally an asymmetric tail towards more negative values, i.e. skewness to the left. The only exception being when both relatively longer maturities are considered and for the time subperiod from 1951 to 1982. The only relatively flat distribution compared to the normal distribution is for the time period before the first structural change in the Federal Reserve System (October 1979), see the right panel of Table 1A.

In summary, for the sample period, we observe that the selected U.S. spot interest

[9] Among others see Engsted and Tanggaard (1994), Pagan *et al.* (1996), Campbell (1995), Campbell *et al.* (1997) and, Henry (1995).

rate yields experience considerable change, both in shape of their levels and direction and volatility structure.

We examine[10] the testable assumption that the expectations errors are orthogonal to Ω_t – the subset of all public available information. Under the null hypothesis that market participants build their expectations in accordance with the REH, a regression of $z_{t,n}$ on a constant and regressors – variables in Ω_t should not have explanatory power. In other words, such regressions should produce insignificant F–type Wald test on the coefficients, while the coefficient of determination,[11] R^2, is expected to be insignificant also if not close to zero. To be consistent with the previous study, we follow Johnson with respect to the selection of the regressors, i.e. we use as regressors the expectations error, $z_{t-n+1-j,k}$, and the spread, $S_{t-j,k}$, where the lag $j = 0, 1, 2, ..., 11$ is the same as suggested[12] by Johnson (1997).

Parameter estimates, Wald test statistics and coefficients of determination from both methodologies for the two main time periods are summarized in Table 2, 3, 4 and 5. The long-term bond is chosen with time to maturity of three and 12 months accordingly. This is because the level of noise, according to Johnson (1997, p. 1239) is "*small*" and although one can refute the REH, the model does provide some "*useful approximation to the relationship between interest rates at different maturities, at least for maturities of up to 12 months*". The estimated regressions are represented with an estimate for the constant and all independent variables, see the left column in any panel. The standard errors from the OLS are set in small parentheses. Robust covariance matrix estimation is used to construct heteroscedasticity and autocorrelation consistent (HAC) standard errors as suggested by West (1994), Newey and West (1994) and Den Haan and Levin (1996). The corrected standard errors are set in large parentheses, while the Wald test statistics are denoted with F^*, $F^\dagger$ and $F^\ddagger$ for West (1994), Newey and West (1994) and Den Haan and Levin (1996) respectively.

[10] GAUSS 3.2.17, a product of Aptech Systems, is used in our empirical work. We use some of the HAC procedures developed by Ka-fu Wong and Den Haan and Levin to compute a robust covariance matrix as suggested by the West (1994), Newey and West (1994) and Den Haan and Levin (1997).

[11] It is well known that such a goodness-of-fit measures are purely descriptive. In an initial version of the paper based on the GLS transformation we use Buse (1973) correction to the R^2 estimation. The current estimation of the coefficient of determination is based on the transformed regression.

[12] Of course, this is more or less an ad hoc procedure. We try different lags but with no significant change of both our findings and conclusions.

Table 2A: OLS and $HS-IV$ Regression of the Expectations Error on a Constant and Twelve Lagged Expectations Error. One and Three Months to Maturity.

OLS 1951 – 1991	HS – IV 1951 – 1991
Z_{LS} SE_{LS} SE^{*}_{W} $SE^{\dagger}_{NW}$ $SE^{\ddagger}_{HL}$	$Z_{AR(1)}$ $SE_{AR(1)}$ $Z_{AR(p)}$ $SE_{AR(p)}$
-0.23 (0.04) {0.06} {0.06} {0.07}	-0.20 (0.03) -0.19 (0.03)
0.15 (0.06) {0.08 } {0.06} {0.06}	0.48 (0.08) 0.56 (0.10)
-0.05 (0.07) {0.08 } {0.08} {0.10}	-0.42 (0.12) -0.56 (0.14)
0.03 (0.07) {0.08 } {0.02} {0.05}	0.30 (0.13) 0.45 (0.16)
0.02 (0.07) {0.09 } {0.06} {0.18}	-0.16 (0.13) -0.30 (0.16)
-0.04 (0.07) {0.07 } {0.06} {0.04}	0.07 (0.13) 0.17 (0.16)
-0.11 (0.07) {0.07 } {0.06} {0.06}	-0.17 (0.13) -0.25 (0.16)
0.18 (0.07) {0.06 } {0.06} {0.11}	0.24 (0.13) 0.30 (0.16)
0.06 (0.07) {0.07 } {0.06} {0.08}	0.06 (0.13) 0.02 (0.16)
0.05 (0.07) {0.06 } {0.05} {0.09}	-0.03 (0.13) -0.02 (0.15)
0.07 (0.07) {0.05 } {0.05} {0.07}	0.16 (0.12) 0.17 (0.14)
-0.11 (0.07) {0.10 } {0.12} {0.14}	-0.22 (0.11) -0.24 (0.12)
0.02 (0.06) {0.08 } {0.04} {0.10}	0.10 (0.07) 0.12 (0.08)
F=36.5 F^{*}=18.6 $F^{\dagger}$=123.6 $F^{\ddagger}$=83.4	$F_{AR(1)}$=79.2 $F_{AR(p)}$=89.4
R^2=0.076 R^2_{adj}=0.051	R^2=0.147 R^2=0.163
	R^2_{adj}=0.124 R^2_{adj}=0.141

Table 2B: OLS and $HS-IV$ Regression of the Expectations Error on a Constant and Twelve Lagged Yield's Spreads. One and Three Months to Maturity.

OLS 1951 – 1991	HS – IV 1951 – 1991
S_{LS} SE_{LS} SE^{*}_{W} $SE^{\dagger}_{NW}$ $SE^{\ddagger}_{HL}$	$S_{AR(1)}$ $SE_{AR(1)}$ $S_{AR(p)}$ $SE_{AR(p)}$
-0.03 (0.05) {0.07} {0.07} {0.11}	-0.02 (0.04) -0.02 (0.04)
-0.45 (0.08) {0.12} {0.13} {0.15}	-0.55 (0.12) -0.57 (0.14)
-0.06 (0.08) {0.10} {0.15} {0.18}	0.01 (0.14) 0.05 (0.17)
0.05 (0.08) {0.12} {0.07} {0.05}	0.03 (0.13) -0.00 (0.16)
-0.13 (0.08) {0.16} {0.18} {0.25}	-0.14 (0.13) -0.12 (0.16)
-0.07 (0.08) {0.09} {0.10} {0.19}	-0.06 (0.13) -0.07 (0.16)
-0.07 (0.08) {0.08} {0.08} {0.13}	-0.08 (0.13) -0.07 (0.16)
-0.20 (0.08) {0.13} {0.14} {0.12}	-0.19 (0.13) -0.20 (0.16)
0.09 (0.08) {0.10} {0.09} {0.08}	0.07 (0.13) 0.08 (0.16)
0.15 (0.08) {0.11} {0.07} {0.05}	0.16 (0.13) 0.15 (0.16)
0.03 (0.08) {0.11} {0.13} {0.17}	0.04 (0.13) 0.06 (0.16)
-0.13 (0.08) {0.08} {0.10} {0.08}	-0.17 (0.13) -0.20 (0.15)
0.10 (0.08) {0.09} {0.07} {0.10}	-0.04 (0.10) -0.02 (0.11)
F=84.2 F^{*}=48.3 $F^{\dagger}$=88.8 $F^{\ddagger}$=205.8	$F_{AR(1)}$=99.1 $F_{AR(p)}$=100.8
R^2=0.159 R^2_{adj}=0.136	R^2=0.178 R^2=0.180
	R^2_{adj}=0.155 R^2_{adj}=0.158

Notes: The fitted regression equations are represented with an estimate for the constant and all 12 independent variables defined as in the paper. These estimates, based on lags of the expectations error and the spread, from the OLS are denoted with Z_{LS} and S_{LS}. Accordingly, $Z_{AR(1)}$ and $Z_{AR(p)}$ are those estimates from the HS-IV procedure when AR(1) model for the short-term rate is assumed or the AR(p) approximation of order higher than the order of the MA process of the correlated innovation term from the OLS is considered. The standard errors from the OLS or HS-IV procedures are set in small parentheses. Robust covariance matrix estimation is used to construct a heteroscedasticity and autocorrelation consistent (HAC) standard errors as originally suggested by Andrews (1991). We consider three methods, namely the moving average HAC by West (1994), the non-parametric HAC by Newey and West (1994) and the parametric vector autoregression (VARHAC) by Den Haan and Levin (1996). The corrected standard errors are set in second parentheses and denoted with $*$, $\dagger$ and $\ddagger$ for the three HAC estimators respectevely. Similarly, F^{*}, $F^{\dagger}$ and $F^{\ddagger}$ denote the chi-squares asymptotically distributed Wald test statistics. These, equivalent to F type test statistics, have degree of freedom equals the number of regressors, while the constant is not counted. Critical values for the chi-squares distribution with 12 degree of freedom at 5%, 1% and 0.5% significance level are 21.03, 26.22 and 28.30 respectively.

Table 3A: *OLS* and *HS − IV* Regression of the Expectations Error on a Constant and Twelve Lagged Expectations Error. One and Three Months to Maturity.

OLS 1951 − 1979	HS − IV 1951 − 1979
Z_{LS} SE_{LS} SE^{*}_{W} $SE^{\dagger}_{NW}$ $SE^{\ddagger}_{HL}$	$Z_{AR(1)}$ $SE_{AR(1)}$ $Z_{AR(p)}$ $SE_{AR(p)}$
-0.12 (0.03) {0.04} {0.04} {0.05}	-0.11 (0.03) -0.11 (0.03)
0.12 (0.06) {0.10} {0.04} {0.07}	0.34 (0.09) 0.32 (0.08)
0.09 (0.07) {0.09} {0.06} {0.11}	-0.11 (0.11) -0.08 (0.11)
0.13 (0.07) {0.09} {0.04} {0.08}	0.25 (0.12) 0.22 (0.11)
0.05 (0.07) {0.07} {0.06} {0.08}	-0.04 (0.12) -0.01 (0.11)
-0.10 (0.07) {0.08} {0.07} {0.05}	-0.07 (0.12) -0.08 (0.11)
-0.05 (0.07) {0.08} {0.08} {0.07}	-0.08 (0.12) -0.07 (0.11)
0.07 (0.07) {0.08} {0.08} {0.07}	0.11 (0.12) 0.10 (0.11)
0.09 (0.07) {0.07} {0.05} {0.05}	0.06 (0.12) 0.07 (0.11)
-0.01 (0.07) {0.07} {0.04} {0.09}	-0.001 (0.12) -0.01 (0.11)
0.04 (0.07) {0.07} {0.09} {0.10}	0.02 (0.11) 0.02 (0.11)
0.12 (0.07) {0.09} {0.09} {0.10}	0.14 (0.11) 0.14 (0.10)
-0.04 (0.06) {0.09} {0.08} {0.09}	-0.07 (0.08) -0.07 (0.08)
F=38.5 F^{*}=22.0 $F^{\dagger}$=111.3 $F^{\ddagger}$=62.2	$F_{AR(1)}$=65.1 $F_{AR(p)}$=62.4
R^2=0.112 R^2_{adj}=0.077	R^2=0.172 R^2=0.166
	R^2_{adj}=0.140 R^2_{adj}=0.133

Table 3B: *OLS* and *HS − IV* Regression of the Expectations Error on a Constant and Twelve Lagged Yield's Spreads. One and Three Months to Maturity.

OLS 1951 − 1979	HS − IV 1951 − 1979
S_{LS} SE_{LS} SE^{*}_{W} $SE^{\dagger}_{NW}$ $SE^{\ddagger}_{HL}$	$S_{AR(1)}$ $SE_{AR(1)}$ $S_{AR(p)}$ $SE_{AR(p)}$
0.05 (0.04) {0.05} {0.04} {0.04}	0.05 (0.04) 0.05 (0.04)
-0.50 (0.09) {0.11} {0.07} {0.08}	-0.57 (0.14) -0.55 (0.12)
-0.13 (0.10) {0.10} {0.08} {0.09}	-0.08 (0.16) -0.10 (0.14)
-0.08 (0.10) {0.11} {0.09} {0.10}	-0.11 (0.15) -0.09 (0.13)
-0.25 (0.10) {0.11} {0.14} {0.14}	-0.22 (0.16) -0.24 (0.14)
-0.01 (0.10) {0.10} {0.09} {0.09}	-0.03 (0.16) -0.01 (0.14)
-0.05 (0.10) {0.10} {0.09} {0.09}	-0.03 (0.16) -0.04 (0.14)
-0.17 (0.10) {0.10} {0.08} {0.09}	-0.19 (0.16) -0.18 (0.14)
-0.03 (0.10) {0.10} {0.09} {0.11}	0.01 (0.16) -0.01 (0.13)
0.25 (0.10) {0.10} {0.07} {0.10}	0.20 (0.16) 0.22 (0.14)
0.15 (0.10) {0.10} {0.10} {0.11}	0.19 (0.16) 0.17 (0.14)
-0.19 (0.10) {0.10} {0.08} {0.10}	-0.23 (0.16) -0.21 (0.14)
-0.08 (0.10) {0.10} {0.10} {0.11}	-0.04 (0.12) -0.05 (0.11)
F^{*}=103.5 F^{*}=105.7 $F^{\dagger}$=145.6 $F^{\ddagger}$=111.8	$F_{AR(1)}$=135.0 $F_{AR(p)}$=131.3
R^2=0.251 R^2_{adj}=0.222	R^2=0.30 R^2=0.292
	R^2_{adj}=0.273 R^2_{adj}=0.264

Notes: The fitted regression equations are represented with an estimate for the constant and all 12 independent variables defined as in the paper. These estimates, based on lags of the expectations error and the spread, from the OLS are denoted with Z_{LS} and S_{LS}. Accordingly, $Z_{AR(1)}$ and $Z_{AR(p)}$ are those estimates from the HS-IV procedure when AR(1) model for the short-term rate is assumed or the AR(p) approximation of order higher than the order of the MA process of the correlated innovation term from the OLS is considered. The standard errors from the OLS or HS-IV procedures are set in small parentheses. Robust covariance matrix estimation is used to construct a heteroscedasticity and autocorrelation consistent (HAC) standard errors as originally suggested by Andrews (1991). We consider three methods, namely the moving average HAC by West (1994), the non-parametric HAC by Newey and West (1994) and the parametric vector autoregression (VARHAC) by Den Haan and Levin (1996). The corrected standard errors are set in second parentheses and denoted with *, † and ‡ for the three HAC estimators respectevely. Similarly, F^{*}, $F^{\dagger}$ and $F^{\ddagger}$ denote the chi-squares asymptotically distributed Wald test statistics. These, equivalent to F type test statistics, have degree of freedom equals the number of regressors, while the constant is not counted. Critical values for the chi-squares distribution with 12 degree of freedom at 5%, 1% and 0.5% significance level are 21.03, 26.22 and 28.30 respectively.

Table 4A: *OLS* and *HS − IV* Regression of the Expectations Error on a Constant and Twelve Lagged Expectations Error. One and Twelve Months to Maturity.

OLS 1951 − 1991	HS − IV 1951 − 1991
Z_{LS} SE_{LS} SE^{*}_{W} $SE^{\dagger}_{NW}$ $SE^{\ddagger}_{HL}$	$Z_{AR(1)}$ $SE_{AR(1)}$ $Z_{AR(p)}$ $SE_{AR(p)}$
-0.69 (0.07) {0.26} {0.25} {0.28}	-0.55 (0.07) -0.55 (0.08)
0.23 (0.11) {0.14} {0.11} {0.19}	2.31 (0.69) 2.19 (0.73)
-0.08 (0.16) {0.07} {0.11} {0.13}	-2.85 (1.26) -2.63 (1.30)
0.04 (0.17) {0.06} {0.09} {0.07}	0.91 (1.33) 0.55 (1.35)
0.001 (0.16) {0.06} {0.11} {0.06}	-0.22 (1.32) 0.16 (1.29)
-0.04 (0.16) {0.06} {0.10} {0.09}	0.07 (1.31) 0.01 (1.23)
-0.02 (0.16) {0.07} {0.06} {0.05}	-0.15 (1.31) -0.31 (1.24)
0.02 (0.16) {0.08} {0.06} {0.06}	0.32 (1.31) 0.44 (1.23)
0.06 (0.16) {0.06} {0.10} {0.08}	0.27 (1.31) 0.52 (1.24)
-0.07 (0.16) {0.07} {0.04} {0.06}	-0.67 (1.33) -1.10 (1.29)
-0.10 (0.17) {0.06} {0.07} {0.05}	0.51 (1.33) 0.47 (1.31)
-0.09 (0.16) {0.06} {0.09} {0.07}	-0.99 (1.16) -0.64 (1.17)
0.09 (0.11) {0.12} {0.11} {0.19}	0.71 (0.51) 0.56 (0.52)
F=19.8 F^{*}=18.7 $F^{\dagger}$=30.5 $F^{\ddagger}$=43.6	$F_{AR(1)}$= 71.9 $F_{AR(p)}$= 66.8
R^2= 0.044 R^2_{adj}=0.017	R^2= 0.143 R^2=0.134
	R^2_{adj}=0.118 R^2_{adj}=0.109

Table 4B: *OLS* and *HS − IV* Regression of the Expectations Error on a Constant and Twelve Lagged Yield's Spreads. One and Twelve Months to Maturity.

OLS 1951 − 1991	HS − IV 1951 − 1991
S_{LS} SE_{LS} SE^{*}_{W} $SE^{\dagger}_{NW}$ $SE^{\ddagger}_{HL}$	$S_{AR(1)}$ $SE_{AR(1)}$ $S_{AR(p)}$ $SE_{AR(p)}$
-0.2 (0.11) {0.29} {0.21} {0.30}	0.06 (0.12) 0.02 (0.12)
-0.63 (0.12) {0.15} {0.16} {0.16}	-2.51 (0.70) -2.36 (0.66)
-0.18 (0.13) {0.08} {0.05} {0.06}	1.07 (0.96) 1.00 (0.88)
0.03 (0.14) {0.07} {0.08} {0.10}	0.45 (0.94) 0.65 (0.89)
-0.09 (0.14) {0.07} {0.08} {0.14}	-0.44 (0.94) -0.61 (0.90)
0.03 (0.14) {0.07} {0.07} {0.07}	0.35 (0.95) 0.16 (0.90)
0.02 (0.14) {0.08} {0.06} {0.08}	0.06 (0.96) 0.19 (0.90)
-0.14 (0.14) {0.09} {0.16} {0.14}	-0.24 (0.96) -0.29 (0.90)
0.1 (0.14) {0.06} {0.07} {0.09}	0.06 (0.97) -0.14 (0.91)
0.22 (0.14) {0.06} {0.09} {0.05}	0.48 (0.97) 0.66 (0.91)
0.001 (0.14) {0.07} {0.11} {0.05}	-0.17 (0.96) -0.06 (0.88)
0.01 (0.14) {0.07} {0.07} {0.08}	0.10 (0.95) 0.02 (0.87)
-0.02 (0.12) {0.12} {0.16} {0.16}	-0.17 (0.53) -0.16 (0.50)
F= 88.6 F^{*}=39.1 $F^{\dagger}$=98.4 $F^{\ddagger}$=92.9	$F_{AR(1)}$= 207.5 $F_{AR(p)}$= 187.9
R^2= 0.169 R^2_{adj}=0.146	R^2= 0.318 R^2=0.296
	R^2_{adj}=0.3 R^2_{adj}=0.277

Notes: The fitted regression equations are represented with an estimate for the constant and all 12 independent variables defined as in the paper. These estimates, based on lags of the expectations error and the spread, from the OLS are denoted with Z_{LS} and S_{LS}. Accordingly, $Z_{AR(1)}$ and $Z_{AR(p)}$ are those estimates from the HS-IV procedure when AR(1) model for the short-term rate is assumed or the AR(p) approximation of order higher than the order of the MA process of the correlated innovation term from the OLS is considered. The standard errors from the OLS or HS-IV procedures are set in small parentheses. Robust covariance matrix estimation is used to construct a heteroscedasticity and autocorrelation consistent (HAC) standard errors as originally suggested by Andrews (1991). We consider three methods, namely the moving average HAC by West (1994), the non-parametric HAC by Newey and West (1994) and the parametric vector autoregression (VARHAC) by Den Haan and Levin (1996). The corrected standard errors are set in second parentheses and denoted with ∗, † and ‡ for the three HAC estimators respectevely. Similarly, F^{*}, $F^{\dagger}$ and $F^{\ddagger}$ denote the chi-squares asymptotically distributed Wald test statistics. These, equivalent to F type test statistics, have degree of freedom equals the number of regressors, while the constant is not counted. Critical values for the chi-squares distribution with 12 degree of freedom at 5%, 1% and 0.5% significance level are 21.03, 26.22 and 28.30 respectively.

Table 5A: *OLS* and *HS – IV* Regression of the Expectations Error on a Constant and Twelve Lagged Expectations Error. One and Twelve Months to Maturity.

OLS 1951 – 1979	HS – IV 1951 – 1979
Z_{LS} SE_{LS} SE^{*}_{W} $SE^{\dagger}_{NW}$ $SE^{\ddagger}_{HL}$	$Z_{AR(1)}$ $SE_{AR(1)}$ $Z_{AR(p)}$ $SE_{AR(p)}$
-0.6 (0.07) {0.20} {0.17} {0.27}	-0.56 (0.07) -0.57 (0.08)
0.14 (0.13) {0.16} {0.09} {0.14}	2.05 (0.81) 2.00 (0.93)
0.04 (0.19) {0.07} {0.08} {0.08}	-2.45 (1.42) -2.67 (1.64)
0.13 (0.19) {0.06} {0.10} {0.09}	0.68 (1.45) 0.91 (1.68)
-0.07 (0.19) {0.07} {0.08} {0.09}	-0.11 (1.45) -0.05 (1.67)
0.03 (0.19) {0.08} {0.10} {0.08}	-0.02 (1.45) 0.001 (1.66)
-0.10 (0.19) {0.07} {0.09} {0.08}	0.21 (1.45) 0.25 (1.65)
-0.01 (0.19) {0.06} {0.08} {0.07}	-0.03 (1.45) 0.02 (1.66)
-0.01 (0.19) {0.08} {0.07} {0.08}	0.10 (1.45) 0.01 (1.66)
-0.12 (0.19) {0.07} {0.04} {0.06}	-0.42 (1.46) -0.70 (1.68)
0.02 (0.19) {0.07} {0.05} {0.06}	-0.12 (1.46) -0.06 (1.67)
0.04 (0.19) {0.06} {0.05} {0.09}	0.81 (1.32) 1.06 (1.49)
-0.17 (0.13) {0.14} {0.10} {0.13}	-0.74 (0.62) -0.82 (0.69)
F= 31 F^{*}= 19.2 $F^{\dagger}$= 59 $F^{\ddagger}$= 18.6	$F_{AR(1)}$= 77.2 $F_{AR(p)}$= 65.7
R^2= 0.097 R^2_{adj}=0.060	R^2= 0.217 R^2=0.190
	R^2_{adj}=0.184 R^2_{adj}=0.157

Table 5B: *OLS* and *HS – IV* Regression of the Expectations Error on a Constant and Twelve Lagged Yield's Spreads. One and Twelve Months to Maturity.

OLS 1951 – 1979	HS – IV 1951 – 1979
S_{LS} SE_{LS} SE^{*}_{W} $SE^{\dagger}_{NW}$ $SE^{\ddagger}_{HL}$	$S_{AR(1)}$ $SE_{AR(1)}$ $S_{AR(p)}$ $SE_{AR(p)}$
-0.22 (0.10) {0.32} {0.33} {0.31}	-0.18 (0.12) -0.18 (0.13)
-0.60 (0.14) {0.17} {0.11} {0.13}	-1.14 (0.94) -1.19 (1.04)
-0.21 (0.16) {0.10} {0.06} {0.10}	0.02 (1.27) -0.03 (1.35)
-0.12 (0.16) {0.10} {0.05} {0.08}	-0.41 (1.25) -0.30 (1.33)
-0.11 (0.16) {0.07} {0.07} {0.08}	1.04 (1.26) 1.09 (1.34)
0.05 (0.17) {0.09} {0.09} {0.10}	-0.33 (1.28) -0.36 (1.36)
0.05 (0.17) {0.08} {0.08} {0.11}	-0.29 (1.28) -0.16 (1.35)
-0.02 (0.17) {0.09} {0.06} {0.08}	0.31 (1.28) 0.18 (1.34)
0.04 (0.17) {0.09} {0.09} {0.09}	-0.71 (1.28) -0.85 (1.34)
0.34 (0.17) {0.07} {0.08} {0.06}	0.81 (1.29) 0.75 (1.35)
0.14 (0.17) {0.08} {0.07} {0.08}	0.17 (1.29) 0.42 (1.35)
0.02 (0.17) {0.09} {0.13} {0.10}	-0.22 (1.27) -0.25 (1.33)
-0.06 (0.15) {0.18} {0.07} {0.11}	0.17 (0.77)
F= 72.7 F^{*}= 81.7 $F^{\dagger}$=405.9 $F^{\ddagger}$= 73.4	$F_{AR(1)}$= 66.3 $F_{AR(p)}$= 66.4
R^2= 0.196 R^2_{adj}=0.163	R^2= 0.193 R^2=0.190
	R^2_{adj}=0.161 R^2_{adj}=0.158

Notes: The fitted regression equations are represented with an estimate for the constant and all 12 independent variables defined as in the paper. These estimates, based on lags of the expectations error and the spread, from the OLS are denoted with Z_{LS} and S_{LS}. Accordingly, $Z_{AR(1)}$ and $Z_{AR(p)}$ are those estimates from the HS-IV procedure when AR(1) model for the short-term rate is assumed or the AR(p) approximation of order higher than the order of the MA process of the correlated innovation term from the OLS is considered. The standard errors from the OLS or HS-IV procedures are set in small parentheses. Robust covariance matrix estimation is used to construct a heteroscedasticity and autocorrelation consistent (HAC) standard errors as originally suggested by Andrews (1991). We consider three methods, namely the moving average HAC by West (1994), the non-parametric HAC by Newey and West (1994) and the parametric vector autoregression (VARHAC) by Den Haan and Levin (1996). The corrected standard errors are set in second parentheses and denoted with $*$, $\dagger$ and $\ddagger$ for the three HAC estimators respectevely. Similarly, F^{*}, $F^{\dagger}$ and $F^{\ddagger}$ denote the chi-squares asymptotically distributed Wald test statistics. These, equivalent to F type test statistics, have degree of freedom equals the number of regressors, while the constant is not counted. Critical values for the chi-squares distribution with 12 degree of freedom at 5%, 1% and 0.5% significance level are 21.03, 26.22 and 28.30 respectively.

To our full sample period from 1951:12 to 1991:02, we first perform the OLS procedure, without the FF for serial dependence, as Johnson (1997) did in his study. For example, when the expectations error, built on the combination between the one and the three months bonds to maturity, is regressed on a constant and its own lags, refer to the left panel of Table 2A, the coefficient of determination (R^2) is about 7.61% and even lower for the adjusted coefficient of determination (R^2_{adj}) yielding 5.10%. Such results leave the impression that we are not far away from what the theory predicts.

On the contrary, when we correct for serial correlations, using the HSFF procedure with the same sets of regressors, the coefficients for R^2 (R^2_{adj}) are twice as high, yielding 14.73% (12.42%), see the bottom right corner of Table 1A. Based on the HS-IV procedure, there is a dramatic increase in the unexplained variability between the *ex−post* three period to maturity yield and the perfect foresight predicted by the theory. Clearly, this bias is due to the moving average structure of the expectations error. We observe similar results when the long-term rate is other than three months to maturity bond, see Table 4 and 5.

When regressors in the OLS are lags of the spread, we observe R^2=15.91% and R^2_{adj}=13.64% respectively. The level of noise absorbed in the spread, based on one year historical information, is at least twice as high as the equivalent one transmitted into the expectations error. The regression, corrected with $HSFF$, produces only marginally higher R^2 (R^2_{adj}) value of 17.76% (15.54%) for the spread equation, see the bottom right corner of Table 2B.

Alternatively, we can check the significance of the coefficients and the joint significance of the coefficients using t and F-type tests respectively. It appears that the most significant are the constant and the first lagged variable and this is applicable to both type of regressions and for the two sets of regressors.

Next, the level of the amount of information available to agents at no cost, but not adequately incorporated in forming their expectations about future interest rates, can be assessed with a formal Wald test on the estimates. Under the null hypothesis of no significant coeficients, the equivalent F-type Wald test statistic is asymptotically distributed as χ^2 variate with a degree of freedom equal to the number of regressors, while the constant is not counted. The rejection of the null hypothesis is overwhelming, with p-values equaling zero[13] from below, i.e. $p < 0.001$. The performed HSFF regressions yield overall even more significant values, i.e. higher Wald test statistics. Hence, the rejection of the REH is even stronger once we correct for the MA structure. Tests performed when higher than three months to maturity bond[14] is the long term rate yield similar results. After correction for serial dependences, the level of noise left in the system is not only in conflict with the oldest theory of interest rates (REH), but it is not supportive of the claim that one can build a useful approximation to the term structure of interest based on it.

Furthermore, we report the coefficient of determination from all regressions based on the one- and k period ($k > 2$) period discount bond in Table 6. The four sample periods, denoted with a lower case letter, namely a for the subperiod 1951:12 to 1979:9, b for the subperiod 1951:12 to 1982:9, c for the subperiod 1951:12 to 1987:2, and d for the subperiod 1951:12 to 1991:2, are the second, third, fourth and fifth columns in Table 6 respectively. When lags of the spread are used as regressors, the amount of model noise is smaller than the equivalent, but based on the expectations error. During the most turbulent time period ending in October 1982, the two methods produced on average the lowest R^2. Moreover, the HSFF estimation agrees with the results from the OLS procedure, particularly for short-term maturities (9 months or less), see Table 5. The striking feature is that the OLS is biased downwards. Overall, the coefficients of determination are much lower than the equivalent ones based on the HSFF estimation. For example, OLS estimates R^2 of 11.16% and 25.14% for the sample ending in 1979 when lags of $z_{t,k}$ and $S_{t,k}$ are considered. The HSFF R^2's are 17.19% and 29.99% equivalently. Note that, when OLS is applied to the lagged expectations error of 18 months or more to maturity, the magnitude of the findings is even more striking, i.e. 10, 25, 28 or more times smaller R^2 than those estimated with the HS-IV method, see the last five rows of Table 6.

[13] The single exception being when both the regressors are lags of expectations error and the HAC standard errors, as suggested by West (1994), are considered.

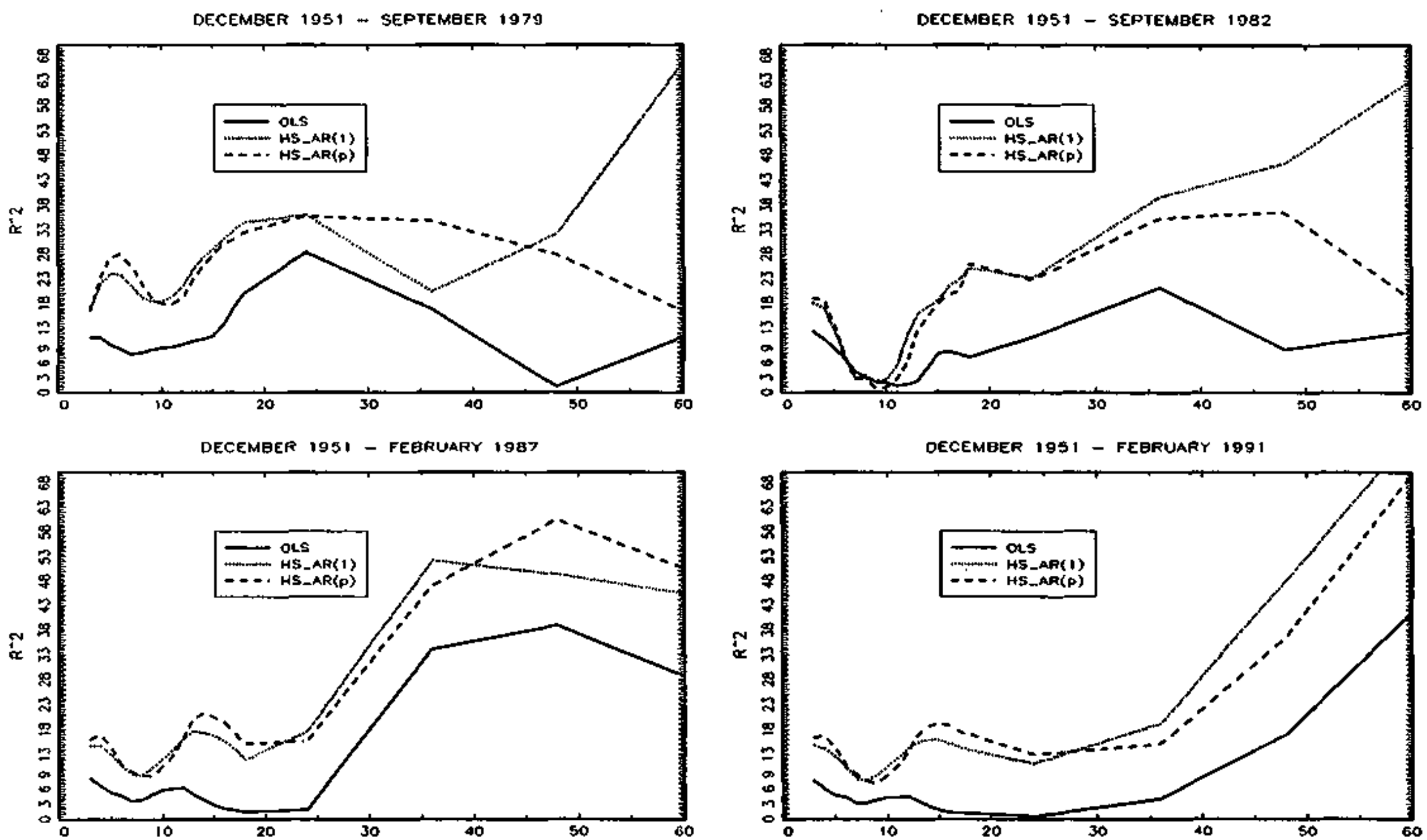
[14] The rest of the regression output, i.e. tests' statistics similar to these discussed in Table 4 and 5, leads to the same conclusions. Hence they are not reported here. All results can be provided on request.

Table 6: R^2 for OLS and HS-IV Regressions

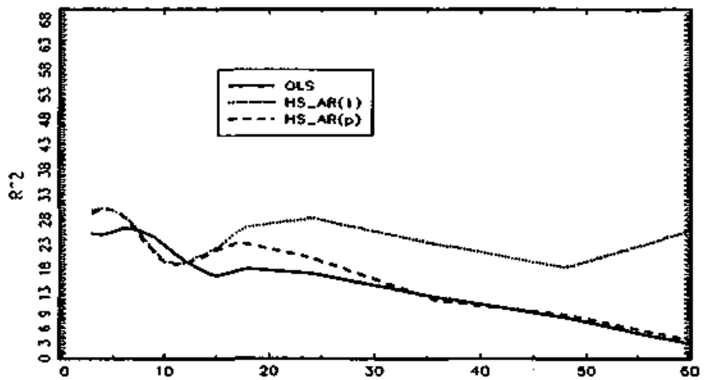
Regressors	OLS^a	HS-IV^a	OLS^b	HS-IV^b	OLS^c	HS-IV^c	OLS^d	HS-IV^d
$z_{t-n+1-j,3}$	11.16	17.19	12.29	18.04	8.20	14.71	7.61	14.73
$S_{t-j,3}$	25.14	29.99	19.30	20.58	17.12	18.76	15.91	17.76
$z_{t-n+1-j,4}$	11.24	22.22	10.84	16.92	6.56	14.65	5.81	14.12
$S_{t-j,4}$	24.99	30.38	17.06	18.71	15.44	19.33	13.80	17.64
$z_{t-n+1-j,5}$	9.72	24.02	9.07	12.31	5.24	12.88	4.59	12.47
$S_{t-j,5}$	25.56	29.88	15.63	17.01	16.17	22.38	14.57	20.58
$z_{t-n+1-j,6}$	8.85	23.83	6.93	7.17	4.71	10.74	4.23	10.18
$S_{t-j,6}$	26.37	28.82	14.47	17.52	17.34	25.48	15.57	23.42
$z_{t-n+1-j,7}$	7.76	21.49	4.26	2.72	3.56	9.05	3.05	8.10
$S_{t-j,7}$	26.22	26.78	15.18	18.84	17.99	27.55	16.20	25.12
$z_{t-n+1-j,8}$	8.06	19.27	3.02	3.45	3.81	8.72	3.03	7.56
$S_{t-j,8}$	25.46	24.12	17.84	19.32	18.37	28.66	16.76	26.13
$z_{t-n+1-j,9}$	8.71	18.41	2.06	2.14	5.01	10.32	3.73	8.83
$S_{t-j,9}$	24.44	21.56	18.41	18.77	18.78	29.83	17.07	26.93
$z_{t-n+1-j,10}$	8.92	18.26	1.94	2.56	5.72	12.07	4.22	10.60
$S_{t-j,10}$	22.89	19.55	16.73	16.30	18.86	31.35	17.00	27.84
$z_{t-n+1-j,11}$	9.12	19.42	1.32	5.38	5.95	13.84	4.27	12.19
$S_{t-j,11}$	21.12	19.14	14.00	13.84	18.84	32.98	16.94	28.91
$z_{t-n+1-j,12}$	9.73	21.66	1.57	11.70	6.16	16.30	4.44	14.26
$S_{t-j,12}$	19.55	19.29	11.60	10.44	18.84	34.47	16.88	30.02
$z_{t-n+1-j,13}$	10.28	24.89	2.21	15.82	4.82	17.57	3.44	15.39
$S_{t-j,13}$	18.35	20.31	10.97	8.89	19.05	36.00	16.91	31.05
$z_{t-n+1-j,14}$	10.73	27.20	5.08	17.44	3.72	17.28	2.45	15.88
$S_{t-j,14}$	17.29	21.26	9.68	8.59	19.15	37.18	16.75	31.74
$z_{t-n+1-j,15}$	11.34	29.04	7.88	18.56	2.75	16.75	1.66	15.76
$S_{t-j,15}$	16.63	22.36	6.54	8.82	18.99	37.89	16.43	32.15
$z_{t-n+1-j,16}$	13.71	30.99	8.27	21.33	1.99	15.62	1.22	15.09
$S_{t-j,16}$	17.16	24.11	5.23	11.72	18.64	37.95	15.99	32.15
$z_{t-n+1-j,17}$	17.19	32.90	7.77	22.76	1.67	14.04	1.14	14.36
$S_{t-j,17}$	17.62	25.59	4.25	14.69	18.35	37.89	15.61	32.33
$z_{t-n+1-j,18}$	20.06	34.41	7.10	25.10	1.42	11.86	1.08	13.77
$S_{t-j,18}$	18.08	26.64	4.23	17.30	18.07	37.94	15.19	32.43
$z_{t-n+1-j,24}$	28.34	35.97	11.20	23.19	1.73	17.62	0.43	11.07
$S_{t-j,24}$	17.06	28.41	9.39	25.46	14.26	35.39	12.27	33.97
$z_{t-n+1-j,36}$	16.96	20.42	21.16	39.56	33.99	52.17	3.93	19.17
$S_{t-j,36}$	12.27	23.01	21.60	33.37	11.21	29.92	12.05	41.71
$z_{t-n+1-j,48}$	1.12	32.00	8.73	46.45	38.82	49.25	16.96	48.14
$S_{t-j,48}$	8.16	18.30	21.88	29.10	18.04	19.15	10.66	42.49
$z_{t-n+1-j,60}$	30.11	66.62	7.33	62.95	28.72	45.42	41.33	77.90
$S_{t-j,60}$	2.82	25.77	19.37	22.97	23.93	16.31	9.25	36.69

Notes: The table shows the R^2 from the both regressions: OLS and HS-IV of the expectations error, $z_{t,k}$ on a constant and either variable in the first column. The four sample periods are denoted with lower case letter, namely a for the subperiod 1951:12 to 1979:9, b for the subperiod 1951:12 to 1982:9, c for the subperiod 1951:12 to 1987:2 and, d for the subperiod 1951:12 to 1991:2.

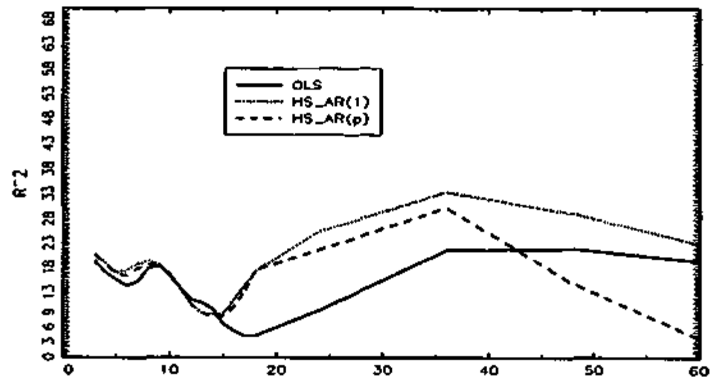
The working hypothesis that the expectations error is close to white noise when all publicly available information is incorporated in the market agents' expectations, as the theory suggests, is further investigated. The innovations term from the OLS of the expectations error on a constant and other instruments is initially estimated.


DECEMBER 1951 – SEPTEMBER 1979
DECEMBER 1951 – SEPTEMBER 1982
DECEMBER 1951 – FEBRUARY 1987
DECEMBER 1951 – FEBRUARY 1991
OLS
HS_AR(1)
HS_AR(p)
R^2
0 3 6 9 13 18 23 28 33 38 43 48 53 58 63 68
0 10 20 30 40 50 60

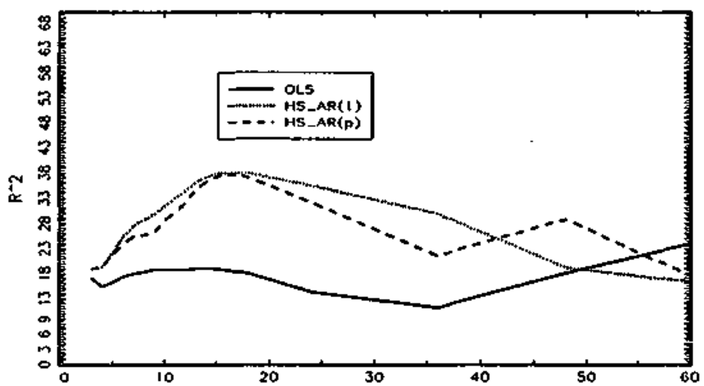
DECEMBER 1951 - SEPTEMBER 1979



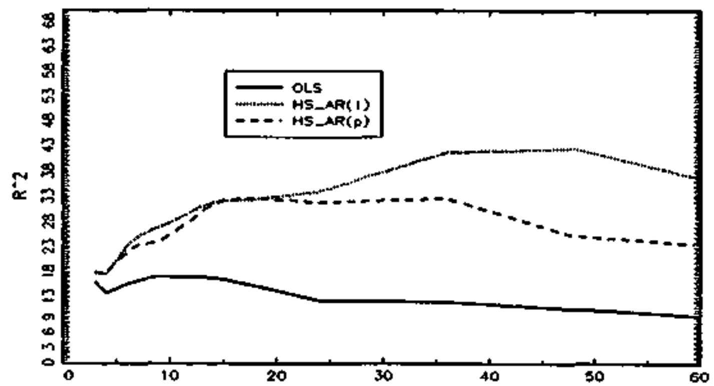
DECEMBER 1951 - SEPTEMBER 1982



DECEMBER 1951 - FEBRUARY 1987



DECEMBER 1951 - FEBRUARY 1991



Then a higher order[15] model AR(p) is fitted to to this error term. The approximated variance-covariance matrix is based on an invertible MA process as suggested by Galbraith and Zinde-Walsh (1994). Then, we repeat the HSFF procedure to remove the MA(q) structure of the expectations error. In Figure 2 and Figure 3, we depict the coefficients of determination from the OLS and HS-IV regresions (and for any subperiod considered as before), where we use both approxiamtions to the variance-covariance matrix, AR(1) process for the short term rate and AR(p) process for the innovations of the residuals of the original OLS equivalently. The results are not significantly different from those reported in Table 6. The common features are as follows. When lags of the expectations error are used as regressors, OLS produces relatively lower R^2 and thus supports Johnson's findings, at least for bonds with maturity up to 24 months. However, the HSFF regression yields much higher R^2 values. The dependency from the time to maturity is clearer here for both the full time period, December 1951 to February 1991, and the two sub-samples ending in September 1979 and in September 1982.

On the other hand, when lagged spreads are used as regressors, both OLS and HSFF's R^2 are relatively high compared with those from the expectations error regressions. There is also not much difference when the subperiod December 1951 to September 1979 is considered. Hence, one is not able to reject Johnson's hypothesis. There is not time dependency for any period or subperiod, i.e. R^2 values are relatively stable (constant). For the full time period and the other two subperiods (to September 1982 and February 1987) bonds with maturity up to 48 months are not so well approximated by the REH, see the much higher R^2 values from the HSFF procedure in Figure 3. Generally, the coefficients of determination when OLS is applied are generally below their correspondents from the HS-IV procedure.

To summarize, the quantity of unexplained regression for both OLS and HSFF procedures is significantly different most of the time when one takes into account and corrects for the MA structure in the expectations error. Of course, the conclusions reached by Johnson (1997, p. 1245) that the findings, based on OLS estimation procedure, "*contradicts a large body of work that reports apparently decisive rejections of the expectations theory*" are true but biased if one does not take into account the moving average error structure of the expectations error.

5. CONCLUSIONS

We present the results of an alternative test of the rational expectations theory of the term structure of interest rates. Applying different methodologies to test the informations content of the yield curve, the validity of the rational expectations hypothesis of term structure has been questioned by many researchers. There is

[15] We set the order of p=2k+4.

more often rejection of this theory. Nevertheless, the REH remains a candidate for the theory that is data-consistent with the entire yield curve. When the REH fails to survive, researchers argue that the exact consistency of the theory is a second order matter and the importance should be judged on how well the model explains (describes) the economy. Based on the *ex − post* formation of rational expectations, we quantify the expectations error and test the level of falseness of the expectations hypothesis, that is, the strength of the departure of the yield curve from the expectations theory. The results suggest that a significant amount of information at no cost available to market agents is not incorporated in forming people's expectations. One can argue that less attention has been paid to recognition of the *empirical adequacy* of this theory when rejection occurs as a consequence of a failure by the market participants to form their expectations in a *rational* manner, and when some important publicly available information is not accommodated in the model. In conclusion, a theory such as REH has been extensively tested by many researchers. Quite often, these examiners have been puzzled by their results. In any reconciliation of such, sometimes even paradoxical empirical findings have to be taken with care.

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7. APPENDIX

There are many ways one can write the time series process for the short-term rate.[16] Without loss of generality, we can assume that it consists of two components, a non-stochastic, $\tilde{R}_{t,m}$ a non-stochastic part, and $e_{(.)}$ a stochastic one. The shorter-term, m-period to maturity bond rate, is given as $R_{t,m} = \tilde{R}_{t,m} + e_{t,t+i}$, such that $e_{(t,t+i)}$ are *i.i.d.* with zero mean and constant variance, σ^2, where $i = m \times j$ $(j = 0, 1, 2, \ldots, (k-1))$. Then, using equations (1) and (6), one can write the expectations error as follows

$$z_{t,n} = c_{t,n} + \nu_{t,n}. \tag{8}$$

The first term in equation (7) is deterministic while the second term represents the stochastic part. Note that at time period t, the latter term takes the form of

$$\nu_n = \frac{1}{k}(e_t + e_{t+m} + e_{t+m\times 2} + \cdots + e_{t+m\times(k-1)}), \tag{9}$$

where $e_{t,t+i}$ are such random fluctuations in the short-term yield which occurred in the period $t+i$, but were not anticipated at period t.

We assume next that the zero coupon bond with one month to maturity (m=1) has AR(1) representation.[17] Then, the short term rate takes the form of

$$R_{t+j,1} = \mu + \phi R_{t-1+j,1} + e_{t+j}\ , \tag{10}$$

where the innovations, e_{t+j}, are *i.i.d.* $(0, \sigma_e^2)$ distributed. When the term to maturity of the longer zero coupon bond is three months, i.e. k=3, j takes values $0, 1, 2$ at time t, $t+1$ and $t = 2$ respectively. Then, ignoring the constant term premium in equation (1), the REH implies $R_{t,3} = \frac{1}{3}\ [R_{t,1} + \mathrm{E}_t(R_{t+1,1}) + \mathrm{E}_t(R_{t+2,1})\ ]$, while the perfect foresight, $R^*_{t,3}$, is given by the simple average of the current and future two one-period yield. The expectations error $z_{t,3} = R^*_{t,3} - R_{t,3} = \frac{1}{3}\ [(1+\phi)\ e_{t+1} + e_{t+2}]$ follows a MA(1) process. The variance-covariance matrix can be easily computed once we estimate ϕ from the above univariate AR(1) process. The variance of this stochastic process takes the form of $Var(z_{t,3}) = \frac{\sigma_e^2}{9}[1 + (1+\phi)^2]$, while the covariance of the expectations error is $Cov(z_{t,3}\ ,\ z_{t-1,3}) = \frac{\sigma_e^2}{9}\ (1+\phi)$.

In general, when k=n the expectations error $z_{t,n} = R^*_{t,n} - R_{t,n}$ is given as follows

$$z_{t,n} = \frac{1}{n}[(1+\phi+\phi^2+\cdots+\phi^{n-2})e_{t+1} + (1+\phi+\phi^2+\cdots+\phi^{n-3})e_{t+2} + \cdots \tag{11}$$
$$\cdots + (1+\phi+\phi^2)e_{t+n-3} + (1+\phi)e_{t+n-2} + e_{t+n-1}].$$

By changing variables, $\theta_1 = \frac{(1+\phi+\phi^2+\cdots+\phi^{n-2})}{n}$, $\theta_2 = \frac{(1+\phi+\phi^2+\cdots+\phi^{n-3})}{n}$ an so on $\theta_{n-2} = \frac{(1+\phi)}{n}$, and after some algebra one can simplify to the following variance-covariance matrix

$$Var(z_{t,n}) = \sigma_e^2[1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_{n-2}^2], \tag{12}$$
$$Cov(z_{t,n}, z_{t,n-1}) = \sigma_e^2[1\theta_1 + \theta_1\theta_2 + \theta_2\theta_3 + \cdots + \theta_{n-3}\theta_{n-2}],$$

and so on

$$Cov(z_{t,n}, z_{t,n-(n-3)}) = \sigma_e^2[1\theta_{n-3} + \theta_1\theta_{n-2}], \tag{13}$$
$$Cov(z_{t,n}, z_{t,n-(n-2)}) = \sigma_e^2[1\theta_{n-2}].$$

[16] For the time series, based on the short-term interest rate, common practice in the literature is to assume an AR(1) process or even random walk, $R_{t,m} = R_{t-1,m} + e_{t,t+i}$, process with zero mean and covariance stationary process with variance σ^2.

[17] In continuous time, a popular model for the instantaneous short-term interest rate is the Ornstein-Uhlenbeck stochastic process (Campbell et al. 1997), which is equivalent to the AR(1) model.