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INCREASING RETURNS, SPECIALIZATION, AND THE THEORY OF THE ALLOCATION OF TIME

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Increasing Returns, Specialization, and the Theory of the Allocation of Time

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Abstract

This paper develops a general equilibrium model to analyze the interactions between the division of labor based on endogenous comparative advantage and the allocation of time. The model follows the assumption of consumption technology in Becker [1965] and the trichotomy assumption on the time allocation in Gronau [1977], and demonstrates a process in which the productivity of time, the time spent at each consumption activity, the time spent at all consumption activities, consumption variety, and percapita real income increase as transaction efficiency is improved. We find that labor supply and wage rate increase concurrently as the division of labor evolves. The labor supply will be rigid regardless of the level of the wage rate if the economy has reached complete division of labor. Our model also shows that increases in the preference for time-intensive consumption can increase the labor supply and the productivity if the degree of economies of complementarity is sufficiently significant. The labor supply will bend back as the preference for time-intensive consumption increases if the economy has exhausted the potential for further division of labor.

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1. Introduction

Instead of regarding the leisure as the residual of the work time, Becker [1965] viewed the time spent at leisure as an input combined with market goods to produce the final satisfaction.[1] This assumption implies a dichotomy of market production and home production. Gronau [1977] extended Becker's dichotomy to a trichotomy by distinguishing work at home -- the production of goods and services which have good or perfect substitutes in the market, for example, child caring, cooking, and clothes washing -- from the consumption activities, which have poor or no market substitutes.[2] Gronau's trichotomy of work in the market, work at home, and leisure suggests that strong interactions exist between the division of labor and the allocation of time. Considered by Smith [1776] as the primary source of economic growth, the division of labor will shift the production, as well as the time spent at production, from self-sufficient activities to market activities. In the words of Reynolds [1983], that is a "transfer of household activities to specialized commercial producers whose activities are more readily dectected." Locay [1990] vividly described this process: "*On sitting down for an everyday meal, a typical European-American family in the seventeenth-century New England would find that it had itself produced almost all the components of the meal... the food was prepared entirely at home... also that the food was home grown on land cleared by the family. The family grew the crops, raised the livestock, harvested and stored the products, and in general did all the processing necessary to prepare the food for consumption. And family production was not limited to food production. The house... was likely to have been bulit by the family, and it certainly was maintained by them. The chairs and table the family used may also have been*

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[1] Becker [1965, p.495]: "Households will be assumed to combine time and market goods to produce more basic commodities that directly enter their utility functions. One such commodity is the seeing of a play, which depends on the input of actors, script, theatre and the playgoer's time." Becker's concept of the leisure has significant advantage over the traditional concept when we consider the consumption variety as shown in the following sections. However, a model that follows the traditional approach to explore the interactions between the division of labor and the time allocation is provided at Appendix 1.

[2] Gronau [1977, p.1100]: "(Becker's approach) has been put to wide use... A fact that seemed to have been overlooked is that the theory does not really deal with household production in the common sense of the term. It does deal with... consumption technology, but has very little to say on home production... at least in the case of women, one should distinguish between work at home and leisure, but this distinction... disappeared in Becker's more general formulation."

home produced, as well as the clothes the family members wore to the meal... a present-day New England... modern family rarely grows its own food or builds its own home and furniture, and the clothes of its members are usually store bought. Even the food of the meal is usually considerably processed before the household purchases it, and the family often avoids any food processing by eating out."

Owing to increasing returns, ex ante identical individuals can greatly enhance their productivity by specialization through the division of labor, which is associated with a market network.[3] The extent of the market, as well as the level of specialization and division of labor, as Coase [1946, 1960] pointed out, is limited by transaction costs.[4] A household must allocate part of its time on self-sufficient activities due to transaction costs outweighing the benefit from further division of labor and specialization.[5] Households will purchase some previously self-provided goods from the market, therefore shift time out of these self-sufficient activities, and reallocate time among other production and consumption activities as the division of labor evolves. Increases in the level of specialization and division of labor will change the allocation of time by shrinking the number of self-sufficient activities.

The first purpose of this paper is to explore, based on an analysis framework developed by Borland, Ng, Shi, and Yang, systematically the welfare implications of the interactions between the allocation of time and the economic development related to specialization and division of labor. Two empirical findings are used to further motivate this paper. First, as Juster and Stafford [1991] reported, more time spent at market work, higher wages, and the rising share of total female work time in the labor market are strongly associated with meals out, the market substitute of the preparation of meals at home.[6] Second, observing the time allocation of women in the U.S. and in

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[3] We consider there is no exogenous but endogenous comparative advantage in the economy. See Yang [1994] for the difference between the two concepts.

[4] That is, in Smith's conjecture [1776, pp.31-32], the division of labor depends upon the extent of the market which is determined by transportation efficiency.

[5] Reynolds [1983, p.947]: "(for the era in which population and output are growing at about the same rate, the 'traditional' or 'conventional' economy) it is better to say that the economy is dominated at this stage by *household production*. Each family produces not only most of its own food, but most of its housing and clothing, plus a wide range of services -- education, healing, recreational activity, religious observance."

[6] Juster and Stafford [1991, p.492]: "Market work and higher wages are strongly associated with meals out... the substitution of goods for own time (meals out and prepared food at the grocery)... coincides with the rising share of total female work time in the labor market."

Norway as shown in Table 1, we can find that the total work time and the housework time declined as the market work time and the leisure uprose from 1965 to 1980s.[7] We will explain these two phenomenons in the following sections by focusing on increasing returns and specialization.

TABLE 1

Changes in Time Allocation of Female in U.S. and Norway, 1965—1980s[8]

(Hours per Week)

	U.S.			Norway		
Activity	1965	1981	Changes	1971	1980	Changes
(1) Total work	60.9	54.4	−6.5	54.6	50.6	−4.0
(2) Market work	18.9	23.9	+5.0	13.3	17.6	+4.3
(3) Housework	41.8	30.5	−11.3	41.3	33.0	−8.3
(4) Leisure	35.4	41.9	+6.5	39.2	45.2	+6.0
(5) Personal care	71.9	71.6	−0.3	74.2	72.1	−2.1
Total=(1)+(4)+(5)	168.0	168.0	0	168.0	168.0	0

note: (1) = (2) + (3)

**Data Resource: Juster and Stafford [1991, p. 477, Table 3] [9]

[7] In Table 1, we emphasize the changes in the time allocation of female since women provide substantially more housework than men. (According to the estimates of Morgan, Sirageldin, and Baerwaldt [1966], the output of wives at home constitutes close to 40% of measured GNP.) We suppose the increases in the extent of the market, which substitute home production with market goods, have much stronger impacts on the time allocation of women. This can be supported by the strong correlation between meals out and the rising share of total female work time in the labor market.

[8] It is obvious, from 1965 to 1980s, there were changes in the division of labor between men and women inside the household. For men, however, the time spent at housework increased only 2.3 hours per week in the U.S., and 1.4 hours per week in Norway, while for women the time spent at housework decreased 11.3 hours per week in the U.S., and 8.3 hours in Norway. In this paper, we will focus on the division of labor between households, which is organized by the market.

[9] In Norway, the increases in the leisure of female can be explained by the decreases in time spent at work and at personal care. However, the former can explain 66% of the changes in the leisure. For the U.S. female, the time spent at personal care had almost no changes between 1965 and 1981. The increases in the leisure can be completely explained by the decreases in the total work time.

The economic growth has involved not only the shifts of production from self-sufficient to market activities but also the increases in the variety of goods available.[10] Without increases in the productivity, the production and the consumption of a larger variety of goods both require more time input, assuming the time spent at leisure is the summation of time spent at individual consumption activity.[11] Will the leisure be traded off for a greater consumption variety as the economy develops? The second purpose of this paper is to simultaneously endogenize time allocation and consumption variety in a general equilibrium model, and to explain the concurrent increases in the consumption variety and leisure as transaction efficiency is improved.

In most time allocation theories the central issues is to explain how increases in the productivity will change the allocation of time. It is of interest, however, to consider how will changes in people's preference for the leisure affect the productivity? Traditional wisdom, for example, Weber [1930], seems to conclude that people's preference for more time-intensive consumption technology is negatively associated with the productivity. Buchanan [1994] agreed with this view and argued, since the labor supply limits the level of specialization and division of labor, the productivity must be hurt by the change in people's preference toward more time-intensive consumption.[12] The third purpose of this paper is to explore what the relationship between the preference for time-intensive consumption and the productivity is, and to show that this relationship is not necessarily negative due to the complicated interactions of three tradeoffs between economies of specialization and transaction costs, between benefit and cost of consumption variety, and between leisure and work.

This paper is arranged as follows. The model is specified in Section 2. The general equilibrium is solved in Section 3. The economic implications are derived from the comparative statics analysis in Section 4, and the conclusion is drawn in Section 5.

2. The Model

[10] See Locay [1990, p.966]

[11] There is no exogenous technical progress in production functions in our story.

[12] One basic feature of our model is the extent of the market, as well as the supply of labor, is limited by transaction costs.

Consider an economy with M ex ante identical consumer-producer households and m goods (or services, of course). As in Gronau [1977], we assume the goods can either be purchased in the market or self-provided. The good purchased in the market and the good self-provided are perfect substitutes. The self-provided amounts of good i are x_i, the amounts sold in the market are x_i^s, and the amount purchaseds in the market are x_i^d. The transaction cost coefficient for each unit of goods bought is $1-k$, and kx_i^d is the amounts a household obtains from purchasing. The total amounts consumed of good i are $x_i + kx_i^d$. Following Becker [1965] , we assume households combine consumption time which has no market substitute and goods to produce 'basic goods', which enter their utility functions directly. Denoting the basic good i as Z_i and

$$(1) \qquad Z_i = (x_i + kx_i^d)t_i^\beta$$

where t_i is the time spent at consumption combined with good i to produce basic good Z_i. The basic good can be viewed as a final product produced by a Cobb-Douglas production function, the 'consumption technology', with good x_i and consumption time t_i as inputs. β represents the degree of time intensity in producing this basic good. A higher β represents a stronger desire of the household for time-intensive consumption. Following Yang and Shi [1992], we assume that the more basic goods are consumed by a household, the greater is the utility cost of calculating optimum consumption, production and trade. Suppose that c is the cost coefficient for a household to manage one basic good, and cm is a proportion of utility that is lost due to management of m basic goods. Each household is assumed to have an identical CES utility function, given by

$$(2) \qquad u = (1-cm)V, \qquad V = \left(\sum_{i=1}^{m} Z_i^{-\rho}\right)^{-1/\rho}, \qquad c \in (0,1), \qquad \rho \in (0,-1)$$

where cm represents the management cost of consumption variety and V represents a preference for diverse consumption. V increases monotonically with the number of basic goods, m, which is a decision variable. The elasticity of substitution between input varieties is $1/(1+\rho)$. The degree of economies of complementarity between each two basic goods is represented by $-1/\rho$.

The system of production for each consumer-producer household is

$$(3) \qquad x_i + x_i^s = l_i - \alpha, \qquad \alpha \in (0,1)$$

$$\sum_{i=1}^{m}(l_i + t_i) = 1, \qquad l_i \in [0,1), \qquad t_i \in (0,1), \qquad i = 1,2,\ldots,m$$

where $x_i + x_i^s$ is the output level of good i; l_i, representing the household's level of specialization in producing good i, is the time used by a household at producing good i, The parameter α, representing the degree of economies of specialization, is the fixed learning or training costs.[13] This system of production functions and endowment constraints displays economies of specialization since the labor productivity increases with a household's level of specialization. Note that the economies of specialization based on the fixed learning costs is household specific, that is, increasing returns in this economy are localized. $\sum_{i=1}^{m} l_i$ is the total time a household spent at work, and $\sum_{i=1}^{m} t_i$ is the total time spent at consumption. Three tradeoffs have been formalized within the single model in this section: the tradeoff between time allocated on market and non-market production activities and on consumption activities, which is taken from Becker model [1965] and Gronau model [1977]; the tradeoff between economies of specialization and economies of complementarity, which is taken from Dixit-Stiglitz model [1977]; the tradeoff between economies of specialization and transaction costs, which is the basic feature of Yang-Shi model [1992]. These tradeoffs will be used to generate concurrent increases in the level of specialization, the amounts and variety of

[13] This specification of production function follows Becker [1981] and Rosen [1983], who assumed that the fixed learning or training cost, or investment, generates economies of specialization through increasing utilization rate of the investment in learning and training. According to Marshall [1890, pp.250-51], the fixed learning cost in acquiring knowledge and skill is caused by a diffcult learning process that needs intensive exertion of the central nerve system in coordinating local nerve centers, and reflex action becomes routes which do not need much exertion of the central nerve system as the learning process has reached a threshold level.

goods produced and consumed, the time spent at market production activities, and the time spent at consumption activities due to improvements in the transaction efficiency.[14]

The budget constraint (or trade balance) is given by

$$(4) \qquad \sum_{i=1}^{m}(p_i x_i^s) = \sum_{i=1}^{m}(p_i x_i^d)$$

where p_i is the price of good i .

As in Yang and Shi [1992], a Walrasian regime or a multilateral bargaining game is assumed, and the following lemma has been established by Yang and Shi [1992] and Wen [1994].

Lemma 1

According to the Kuhn-Tucker conditions, for a household's optimum decision, a household sells at most one good and does not buy and sell or self-provide the same good.

[14] Basically we construct our model following Gronau's model, which is an extension of Becker's model. Gronau's model [1977, p.1105] is summarized as follows:

(1) $Z = Z(X, L)$; (2) $X = X_M + X_H$; (3) $X_H = f(H)$;
(4) $X_M = WN + V$; (5) $L + H + N = T$

where Z is the amount of commodity, which is a combination of goods and services (X) and consumption time (L) -- in our model, this combination takes a Cobb-Douglas function form, but instead of viewing all goods and services as a whole as in Gronau, our model, following Becker's approach, makes distinction between different goods and services, that enables us to endogenize the consumption variety -- X_M denotes goods purchased from the market, X_H denotes goods produced at home with the production function f, N is the time spent at market work, H is the time spent at home production, T is the total time available, W is wage rate, and V is other resources of income. Between Gronau's model and ours, the basic distinctions are: First, we assume transaction costs exist while Gronau's model assumes perfect transaction efficiency in purchasing market goods. Second, we assume increasing returns in production as Gronau's model takes the assumption of decreasing returns. Third, what we construct here is a general equilibrium model, without the ex ante dichotomy of consumer and firm, instead of a partial equilibrium one.

Signifying the utility of a household selling good i by u_i and taking Lemma 1 into account, the decision problem for it is

$$(5) \qquad \text{Max:} \quad u_i = (1-cm)[(x_i t_i^{\beta})^{-\rho} + \sum_{r\in R}(kx_r^d t_r^{\beta})^{-\rho} + \sum_{j\in J}(x_j t_j^{\beta})^{-\rho}]^{-1/\rho}$$

$$\text{s.t.} \quad x_i + x_i^s = l_i - \alpha, \quad x_j = l_j - \alpha, \quad \forall j \in J \quad \text{(production function)}$$

$$l_i + \sum_{j\in J} l_j + t_i + \sum_{r\in R} t_r + \sum_{j\in J} t_j = 1 \quad \text{(endowment constraint)}$$

$$p_i x_i^s = \sum_{r\in R}(p_r x_r^d) \quad \text{(budget constraint)}$$

where R, consisting of $n-1$ elements, is the set of goods the household buys in the market; J, consisting of $m-n$ elements, is the set of goods the household self-provides at home. n is the number of goods traded by the household. Decision variables for this maximization problem are l_i, l_j, t_i, t_j, t_r, x_i, x_i^s, x_j, x_r^d, n, and m, where $n \in (1,m)$, and $m \in (1,\infty)$. The first-order conditions for the problem yield the optimum l_i, l_j, t_i, t_j, t_r, x_i, x_i^s, x_j, x_r^d, n, and m as functions of relative prices of all trade goods. The optimum x_i^s and x_r^d represent individual household's demand and supply functions. Inserting the optimum decisions into u_i produces an indirect utility function.

The $n-1$ utility equation conditions for $n-1$ types of households selling different goods determine $n-1$ relative prices of n traded goods, and the $n-1$ market clearing conditions determine $n-1$ relative numbers of individuals selling n traded goods

$$(6) \qquad p_i/p_r = 1, \qquad M_i/M_r = 1$$

where M_s is the number of households selling good s, $s = i,r$. The other market clearing condition is not independent of (6) due to Walras' law. Inserting the equilibrium relative prices into the first-order conditions for the maximization problem in (5) yields equilibrium values for all decision variables, given by

$$(7)\qquad \beta x_i = \beta x_j = t_i = t_j = \beta[1-\alpha(m-n+1)]/\{(1+\beta)[K(n-1)+m-n+1]\}$$
$$= -\rho\alpha\beta/[(1-K)(1+(1+\beta)\rho)], \quad \forall j \in J, \quad i = 1,\ldots,n$$
$$\beta x_r^d = t_r = \beta K[1-\alpha(m-n+1)]/\{(1+\beta)[K(n-1)+m-n+1]\}$$
$$= -\rho\alpha\beta K/\{(1-K)[1+(1+\beta)\rho]\}$$
$$x_i^s = (n-1)x_r^d, \qquad \forall r \in R, \qquad i = 1,\ldots,n$$

$$(8)\qquad l_i = \{-\rho\alpha[1+K(n-1)]/\{(1-K)[1+(1+\beta)\rho]\}\} + \alpha, \qquad i = 1,\ldots,n$$
$$l_j = \{-\rho\alpha/\{(1-K)[1+(1+\beta)\rho]\}\} + \alpha, \qquad \forall j \in J$$

$$(9)\qquad m = [\rho(1-K)/(\alpha K) + 1/c]/(1-\rho)$$
$$n = \{1-K[1+(1+\beta)\rho]\}m/(1-K)\} - [1+(1+\beta)\rho]/\alpha + 1$$
$$= 1 + [1/(1-\rho)]\{\rho/(K\alpha) + \{1-K[1+(1+\beta)\rho]\}/[c(1-K)]\}$$
$$-[1+(1+\beta)\rho]/\alpha$$

$$(10)\qquad u = \beta^{\beta}(1+\beta)^{-(1+\beta)}(1-cm)$$
$$[1-\alpha(m-n+1)]^{1+\beta}[K(n-1)+(m-n+1)]^{-[1+(1+\beta)\rho]/\rho}$$
$$= \beta^{\beta}(-\rho)^{2+\beta}(1-\rho)^{(1-\rho)/\rho}\alpha^{(1+\beta\rho)/\rho}c^{1/\rho}K^{-1}(1-K)^{-(1+\beta)}$$
$$[c(1-K)+\alpha K]^{1-1/\rho}[1+(1+\beta)\rho]^{-[1+(1+\beta)\rho]/\rho}$$

where $K = k^{-\rho/[1+(1+\beta)\rho]}$. Note that it is necessary for $-1/\rho > 1+\beta$ to keep the number of goods to be positive since $\partial u/\partial n < 0$ if $-1/\rho < 1+\beta$. This means the economies of complementarity must be sufficiently large. The second-order conditions for the interior maximum points of n and m are also satisfied if $-1/\rho > 1+\beta$. Due to the symmetry, m, n and other variables in (7) to (10) are the same for all households. Hence, we can use n to represent not only each household's number of traded goods but also to represent the number of traded goods and level of division of labor in the economy as a whole.

3. Comparative Statics Analysis and its Economic Implications

Given $-1/\rho > 1+\beta$, we can obtain the following comparative statics:

(11) $dn/dk > 0$, $dn/dc < 0$, $dn/d\alpha > 0$, $dm/dk > 0$,

$dm/dc < 0$, $dm/d\alpha > 0$, $d(m-n)/dk < 0$, $d[(m-n)/m]/dk < 0$

(12) $dx_i^s/dk > 0$, $dx_i^s/d\alpha > 0$, $dx_r^d/dk > 0$, $dx_r^d/d\alpha > 0$

$d(Mx_i^s/n)/dk = d[M(n-1)x_r^d/n]/dk > 0$

(13) $dt_s/dk > 0$, $dt_s/d\alpha > 0$, $s = i,j,r$

$dl_i/dk > 0$, $dl_i/d\alpha > 0$, $d(m-n)l_j/dk < 0$, $d(m-n)l_j/d\alpha < 0$

where n is the number of traded goods, that represents the level of division of labor; m is the number of all goods produced and consumed in the economy, that represents the degree of diversity of goods; and $m-n$ is the number of self-provided goods. $(m-n)/m$ represents the degree of self-sufficiency. Note that an interior solution of n is greater than 1 and no goods are traded at the corner solution with $n-1$. k is transaction efficiency coefficient; c is the variety management cost coefficient, and $1/c$ is the efficiency in managing a variety of goods; α is the degree of economies of specialization. The per household value of production sold in the market, which is equal to per household total demand for all traded goods, is x_i^s. Mx_i^s/n is therefore equal to the equilibrium aggregate demand and supply for one traded good. l_i, as previously defined, is the labor share spent at producing the traded good, representing the level of specialization for a household and the degree of commercialization; l_j is the labor share spent at producing non-traded good j, $\forall j \in J$; t_s is the time share spent at consuming good s, $s = i,j,r,r \in R$. Define L as the total time spent at producing goods, market and nonmarket, and H as the total time spent at consumption activities, which can be viewed as the total leisure time, we have

$$(14) \qquad H=\sum_{s=1}^{m} t_s=[-\rho/(1-\rho)]\{1+\alpha K/[c(1-K)]\}, \qquad L=1-H=l_i+\sum_{j\in J} l_j$$

and the comparative statics of H and L are

$$(15) \qquad dH/dk>0, \qquad dH/dc<0, \qquad dH/d\alpha>0$$

$$dL/dk<0, \qquad dL/dc>0, \qquad dL/d\alpha<0$$

The following proposition can thus be derived from (11) to (15)

Proposition 1

The number of traded goods and the level of division of labor increase, and the degree of self-sufficiency decreases as transaction efficiency is improved. In this process the time spent at producing the traded good, the level of specialization, the time spent at a single consumption activity, the time spent at all consumption activities, the number of goods, the extent of the market, labor productivity, and per household real income increase as transaction efficiency is improved. The time spent at producing total non-traded goods and at total production activities decrease as transaction efficiency is improved. The number of traded goods, the time spent at producing the traded good and at consumption activities increase, and the time spent at producing total non-traded goods and at total production activities decrease as the degree of economies of specialization increases. An increase in a household's management efficiency of consumption variety increases the number of goods, the number of traded goods, and the time spent at each and all consumption activities.

Our model predicts that the time spent at leisure and at work for the market increase concurrently, and the time spent at total work and at housework decrease concurrently, as the economy develops due to the division of labor. This predication can be verified by the changes in the time allocation of female in the U.S. and Norway, 1965-1980s, as shown in Table 1. Proposition 1 can also expalin the average time uses of Israeli married women (1968) in Table 2.

TABLE 2

Average Time Uses Of Israeli Married Women, by Education Status (1968)

(Hours per Day)

Activities	Years of Schooling			Changes (Hours)		
	(1) 0-8	(2) 9-12	(3) 13 +	(2)–(1)	(3)–(2)	(3)–(1)
a. Total work	8.15	7.55	7.32	–0.60	–0.23	–0.83
b. Work at home	6.94	5.84	4.75	–1.10	–1.09	–2.19
c. Market work	1.21	1.71	2.57	+0.50	+0.86	+1.36
d. Leisure	5.10	5.78	6.08	+0.68	+0.30	+0.98
e. Physiological needs	10.21	10.04	10.06	–0.17	+0.02	–0.15

Note: a = b + c

a + d + e = 24 (This may not always be satisfied because of missing data.)

** Data Resource: Gronau [1977, p.1103, Table 2]

Observing Table 2, we can find that the time spent at leisure and at work for the market increase concurrently, and the time spent at total work and at housework decrease concurrently, as years of schooling increase.[15] Using our model, there are two possible ways to interpret this result. First, more years of schooling represent higher fixed learning investments (costs), and higher degrees of economies of specialization. As Proposition 1 predicts, the time spent at leisure and at work for the market

[15] Note that the changes in time spent at physiological needs is relative unimportant, compared to other factors.

increase, and the time spent at total work and at housework decrease, as the degree of economies of specialization increases. Second, it is reasonable that a higher education level can improve the transaction efficiency (communication efficiency). By Proposition 1, the time spent at leisure and at work for the market increase, and the time spent at total work and at housework decrease, as transaction efficiency is improved.In the following subsections, we report some further contributions of this model which synthesizes the theory of specialization and division of labor and the theory of the allocation of time.

The Supply of Labor

Traditional theories usually concern the effects of changes in wage rate, income, or the relative cost (productivity) of time and good, on the time allocation. Our model emphasizes the effects of increases in the extent of the market, the level of specialization, and the level of division of labor, on the allocation of time. Improvements in transaction efficiency shift time out of non-market production activities, into market production activity, and increase the labor supply for the market. Consider the changes in the everyday life of the typical New England family. As the division of labor evolves, the economy grows due to specialization, and the New England family no more self-provides the house, the chairs, the table, and the food. Purchasing these goods from the market instead of self-providing implies that the New England family must supply more amounts of traded good to the market for obtaining the necessary purchasing power. In a retrospective view, we are treating the topic of labor supply according to Young's classic concept [1928]: Demand and supply are two sides of the division of labor.

By this approach, we can explain why the more time spent at market work, the higher wages, and the rising share of total female work time in the labor market are strongly associated with meals out, as Juster and Stafford [1991] has observed. Improvements in the transaction efficiency of dining service induce households to substitute meals at home with meals out. Some households, therefore, will specialize in providing meals, and other households now taking the dining service from the market can save their time for market production and for leisure. The increases in time spent at market work will increase the level of specialization, and thus increase the labor productivity, which implies higher wages. Since women provide the major part of housework, the substitution of meals out for preparing food at home will strongly coincide with the rising share of total female work time in the labor market.

Another application of our model is the effects of tax on the labor supply. Besides the negative effect of tax on the marginal wage rate and income, our model suggests that the negative correlation between tax and transaction efficiency should be noted, since that will impose negative impacts on the extent of the market, and thus on the supply of labor.[16] Our model also suggests that the high level of self-sufficiency is related to low transaction efficiency. It is not surprising, therefore, that Swedish men are the clear leaders in home improvements time, averaging over 4 hours per week in 1984, compared to 2.8 hours in the U.S. and less than 1 hour in Japan, considering the high tax rates in Sweden in 1984.[17]

According to (11), the number of traded goods n will eventually reach the number of all goods m since n increases more quickly than m as transaction efficiency is improved. As soon as n has reached m, the comparative statics of n and m in (11) are no longer relevant. Letting $m=n$, $l_j = x_j = t_j = 0$ and solving the general equilibrium for complete division of labor yields

(16) $$m = n = \{\rho(1-K)/K + [1+(1+\beta)\rho]/c\}/(1+\beta\rho), \quad dm/dk > 0, \quad dm/dc < 0$$

(17a) $$x_i = (1-\alpha)/\{(1+\beta)[1+K(m-1)]\}, \quad x_r^d = Kx_i, \quad t_i = \beta x_i, \quad t_r = \beta K x_i$$

(17b) $$l_i = (1+\alpha\beta)/(1+\beta), \quad dl_i/d\alpha > 0, \quad dl_i/d\beta < 0$$

where the labor supply is a constant if the economy has reached complete division of labor as transaction efficiency is improved. Although there is no labor market in our model, we can acquire the average wage rate by counting the value of goods a household receives from the market through a hour's work at producing the traded good. Denoting w as the avereage wage rate, then $w \approx k(n-1)x_r^d/l_i \approx k(l_i-\alpha)/l_i$ if

[16] The negative relationship between arbitrary tax extraction and transaction efficiency, as well as its consequence, has been examined by North [1981].

[17] See Juster and Stafford [1991, p. 497].

n is sufficiently large and $(n-1)x_r^d = x_i^s >> x_i$. The wage rate w is a monotone increasing function of l_i because of increasing returns. The relationship between the wage rate and labor supply is depicted in Figure 1. The wage rate and labor supply increase concurrently as the division of labor evolves. As soon as the economy has reached complete division of labor, the labor supply will be a constant, no matter how high the wage rate is. The supply of labor for the market will be rigid as the economy has exhausted the potential for further specialization and division of labor.

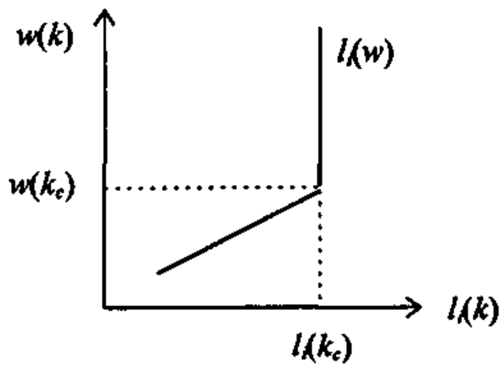


FIGURE 1

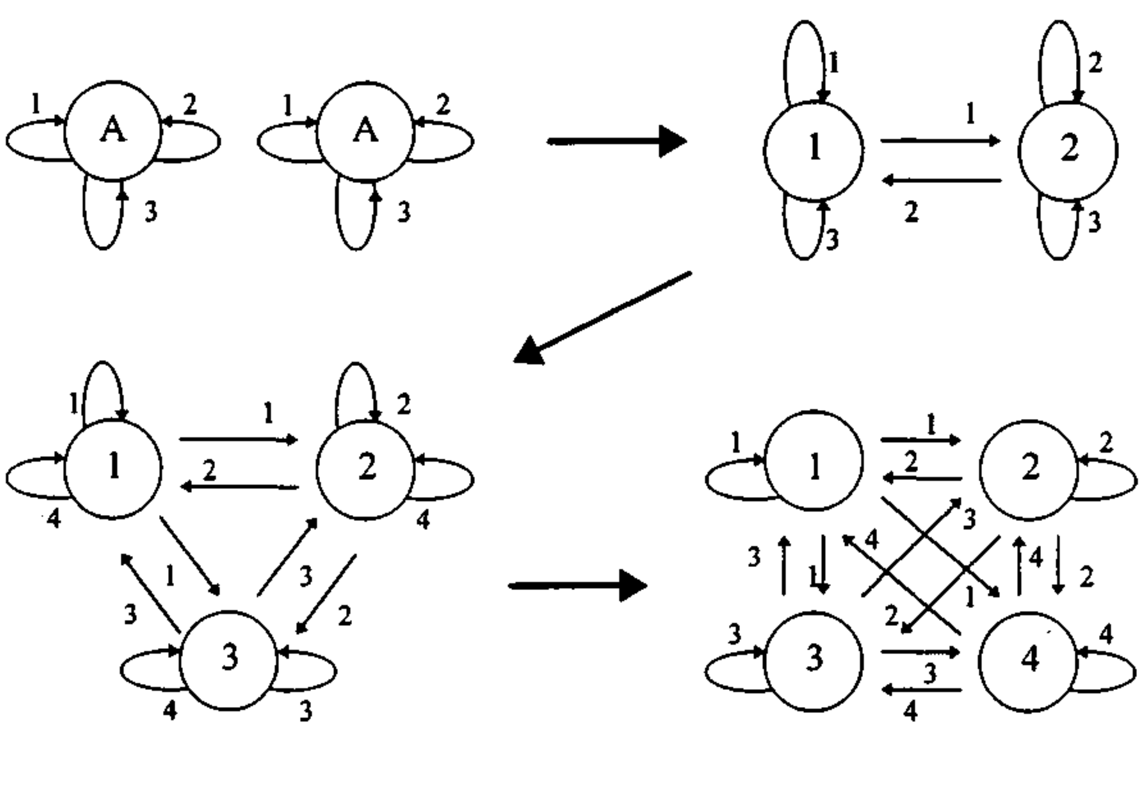
The Relationship between Labor Supply and Wage Rate

The Concurrent Increases in Leisure and Consumption Variety

Instead of resorting to a strong income effect dominating a weak substitution effect to explain the concurrent increases in leisure and productivity of time, our approach highlights the effect of division of labor on shrinking the number of self-sufficient activities. The productivity of time, and the time spent at each and at all consumption activities will then increase concurrently as the division of labor evolves due to improvements in transaction efficiency.

Our general equilibrium model, furthermore, predicts that increases in the level of specialization and division of labor will increases both leisure and consumption variety. An increase in the consumption variety will develop a related new demand for time spent at consuming more kinds of goods, and induce a decrease in total work time. On the other hand, producing the higher consumption variety requires an increase of time in at least one production activity. Since there is no exogenous technical progress in our story, only specialization and division of labor can provide the necessary high productivity and at the same time satisfy the related new time demand by shrinking the number of self-sufficient activities to exploit increasing returns. Figure 2 provides an intuitive illustration of how the evolution of division of labor may proceed. The lines in each panel of Figure 1 signify goods flows. The small arrows indicate direction of good flows. The numbers beside the lines signify goods involved. A circle with number i signifies a person selling good i.

(a) Autarky, n=1, m=3 (b) Division of Labor, n=2, m=3



(c) Division of Labor, n=3, m=4 (d) Division of Labor, n=4, m=4

FIGURE 2

The Evolution of Division of Labor

Panel (a) denotes autarky where each household self-provodes 3 goods, because of low transaction efficiency. As transaction efficiency is slightly improved, the economy evolves to the state depicted in panel (b) where each household sells one good, buys one good, trades two goods, and engages in two production activities. The level of specialization, the time spent at market work and at leisure increase since the number of production activities for each person now is reduced from 3 to 2. When transaction efficiency is further improved, the economy evolves to panel (c), where each household sells one good, buys two goods, trades three goods, and engages in two production activities. The consumption variety increases since now the number of goods is increased from 3 to 4. The economy evolves to panel (d), where each household sells one good, buys three goods, and engages in only one production activity, as transaction efficiency is further improved. The level of specialization, the time spent at market work and at leisure are higher than in panel (c) since the number of production activities is further reduced from 2 to 1. Compared to autarky, leisure and consumption variety both increase in panel (c) and (d).[18]

The Effect of the Preference for Time-Intensive Consumption on Productivity

Differentiating m, n, x_i, x_j, and x_r^d with respect to β, the degree of time-intensity in consumption, yields

(18) $$dm/d\beta = \{-\rho/[\alpha K^2(1-\rho)]\}(dK/d\beta) < 0$$

(19) $$dn/d\beta = \{-\rho/[\alpha c(1-\rho)(1-K)]\}\{c(1-k)\{1+(1/K)(\rho^2 \ln k)/[1+(1+\beta)\rho]^2\} +\alpha K\{1+[(1+\beta)/(1-K)]\{(\rho^2 \ln k)/[1+(1+\beta)\rho]^2\}\}$$

(20) $$dx_i/d\beta = dx_j/d\beta = \{(\alpha\rho^2)/\{(1-K)[1+(1+\beta)\rho]^2\}\} \{1-\{(\rho K \ln k)/\{(1-K)[1+(1+\beta)\rho]\}\}\}$$

[18] Our model predicts that the leisure and consumption variety will increase concurrently as the economy evolves from one structure with lower transaction efficiency to another structure with higher transaction efficiency, although our illustration shows a process in which the leisure and the consumption variety increase in turn from one panel to another. It can be assumed, however, the economy will evolve from autarky to panel (c) or (d).

(21) $dx_r^d/d\beta = \{(\alpha\rho^2 K)/\{(1-K)[1+(1+\beta)\rho]^2\}\}$
$\{1-\{(\rho\ln k)/\{(1-K)[1+(1+\beta)\rho]\}\}\}$

From (19) to (21), we can prove (see Appendix 2) that $dx_i/d\beta = dx_j/d\beta > 0$, $dx_r^d/d\beta > 0$, and $dn/d\beta > 0$ if ρ is sufficiently large, that is, if the degree of economies of complementarity between each two consumption activities, $-1/\rho$, is sufficiently large; $dx_i/d\beta = dx_j/d\beta < 0$, $dx_r^d/d\beta < 0$, and $dn/d\beta < 0$ if ρ is very small and satisfies $\rho > -1/(1+\beta)$.[19] Since $l_i = (n-1)x_r^d + x_i + \alpha$ and $l_j = x_j + \alpha$, we can establish that $dl_i/d\beta > 0$ and $dl_j/d\beta > 0$ if ρ is sufficiently large; $dl_i/d\beta < 0$ and $dl_j/d\beta < 0$ if ρ is very small and satisfies $\rho > -1/(1+\beta)$. Noting that, owing to increasing returns, the productivity increases as the labor input increases, we then can establish the following proposition

Proposition 2

The number of goods, the consumption variety, decreases as people's preference for time-intensive consumption increases. The number of traded goods, the level of division of labor, the level of specialization, and the productivity increase as people's preference for time-intensive consumption increases if the degree of economies of complementarity in consumption is sufficiently significant, and may decrease as people's preference for time-intensive consumption increases if the degree of economies of complementarity in consumption is not significant.

[19] Note that the conditions for $dx_i/d\beta = dx_j/d\beta < 0$, $dx_r^d/d\beta < 0$, and $dn/d\beta < 0$ may not exist, hence the latter part of proposition 2 may not exist, if the necessary upper threshold of ρ is too small that $\rho > -1/(1+\beta)$ is not satisfied. See Appendix 2.

Proposition 2 is proven in Appendix 2. It concludes that the increase in the preference for time-intensive consumption can be a source of productivity growth, instead of weakening productivity, if the preference for diverse consumption is strong enough. The economic intuition implied in this result is explained as follows. People have the incentive to withdraw time from some production activities as they put more value on the time spent at consumption,. This will decrease the consumption variety, as shown in (18). Assuming people strongly prefer diverse consumption, then what kind of production activities will they withdraw time from? The logical choice is shrinking the number of self-sufficient activities, by further specialization and division of labor, to satisfy the new demand for time since it has the least impact on the consumption variety by exploiting increasing returns. In other words, a strong desire for both time-intensive activities and variety of consumption puts a greater demand on productivity that can be achieved only from a higher level of specialization and division of labor. Hence an increase in β for a given large value of ρ will raise the level of division of labor and productivity.

There are several implications in Proposition 2. First, the coexistence of high consumption diversity, abundant leisure, and high productivity -- the basic features of developed economies -- indicates that high levels of specialization and division of labor exist in modern economies. Second, concurrent increases in preference for time-intensive consumption and preference for diverse consumption can be the spur for productivity growth. The commercial advertisements in the modern everyday life, therefore, play an important role in stimulating economic development by changing people's preference toward more time-intensive and more diverse consumption. Third, a tighter restriction on endowments can generate a higher level of specialization and division of labor. In our story, the increase in the preference for time-intensive consumption tightens, while the strong preference for diverse consumption further tightens, the endowment constraint of time. Specialization and division of labor can be the answer to resource shortage. Forth, the traditional wisdom, that assumes the preference for time-intensive consumption weakens the productivity, is a special case of our model. It may hold only if the degree of economies of complementarity is not significant.

Note that $d(m-n)/d\beta < 0$ if ρ is sufficiently large, this implies that the number of traded goods will eventually reach the number of all goods as the preference for time-intensive consumption increases if the degree of complementarity is sufficiently significant. Combining (17b) and (19)--(21) yields

$$(22)\qquad dl_t/d\beta = d[(n-1)x_r^d + x_t + \alpha]/d\beta > 0 \quad \text{for } m < n \quad \text{if } \rho \text{ is sufficiently large}$$
$$dl_t/d\beta < 0 \quad \text{for } m = n$$

The economic intuition of (22) is straightforward. An incraccess in the preference for time-intensive consumption must decrease the labor supply if the economy has reached complete division of labor since there are no self-sufficient activities to withdraw time from. It implies in (22) a backbending phenomenon in labor supply. The level of division of labor and the labor supply for the market will increase, if the economy does not reach complete division of labor, as the preference for time-intensive consumption increases, assuming the degree of economies of complementarity is sufficiently significant. The labor supply for the market will fall as the preference for time-intensive consumption increases if the economy has reached complete division of labor. Assuming ρ is sufficiently large, the relationship between labor supply and the preference for leisure is depicted in Figure 3, where $l_t(\beta_c) = (1+\alpha\beta)/(1+\beta)$ and the economy reaches complete division of labor if $\beta \geq \beta_c$. The labor supply curve will bend back as the economy has exhausted the potential for further division of labor.

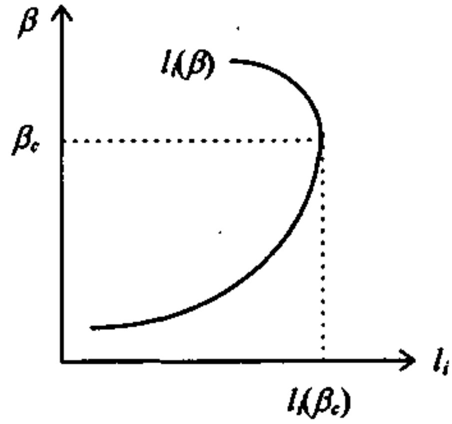


FIGURE 3

The Relationship between l_i and β

when ρ is Sufficiently Large

5. Conclusion

Following Becker's assumption [1965] of consumption combining time and goods to produce the final satisfication and Gronau's trichotomy assumption [1977] on the time allocation, the model in the current paper is featured with three tradeoffs: the tradeoff between economies of specialization and transaction costs, between economies of specialization and economies of complementarity, and between time allocated on production activities and on consumption activities. These features are used to explore the the interactions between the division of labor based on endogenous comparative advantage and the allocation of time. Our model demonstrates a process in which the productivity of time, the time spent at each consumption activity and at all consumption activities, consumption variety, and per capita real income increase as the level of division of labor increases due to improvements in the transaction efficiency. In this process, more and more non-market goods turn to be commodities, the degree of self-sufficiency and the time spent at self-sufficient production activities decrease concurrently, and people allocate more and more time on producing trade goods while they enjoy more leisure by exploiting increasing returns through specialization.

We find that labor supply and wage rate increase concurrently as the division of labor evolves. The labor supply will be rigid if the economy has exhausted its potential for further division of labor. Our model also shows that increases in people's preference for time-intensive consumption can increase the productivity if the degree of economies of complementarity is sufficiently significant. The labor supply will bend back as the preference for time-intensive consumption increases after the economy reaches complete division of labor.

Buchanan [1994] pointed out: "Imagine...what it would be like to try to produce everything on your own, with no economic interaction with others. How much could a person produce in total independence from any exchange nexus? A person's life would indeed be solitary, nasty, brutish, and short, to use Thomas Hobbes's description in a different context. We might want to add the word '*tiring,*' since the valued output that would be secured from the maximal inputs of labor would scarcely ensure survival." What are the welfare implications of division of labor? Besides the obvious increases in per capital real income, productivity, and consumption variety, the division of labor and the accompanied specialization free people from spending their time on low-productivity, self-sufficient only, non-market production activities. The reallocation of this freed time from the shrinked non-market production activities to consumption activities increases the utility directly and constitutes an important part of the welfare from division of labor.

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Appendix 1: A Model Following Traditional Leisure/Labor-Supply Analysis

Here we construct another model that specifies the relationship between specialization, the division of labor and time allocation following the traditional approach which assumes that total work effort enters negatively into the utility function.[20] An economy with M ex ante identical households and m goods or services is assumed, and the symbols used below have the same denotations as in Section 2. Each household is assumed to have an identical utility function, into which the total time spent at work entering negatively, given by a Cobb-Douglas function form

$$\text{(A.1)} \qquad \ln u = V + \beta \ln(1-L)$$

$$V = \sum_{i=1}^{m} \ln(x_i + kx_i^d), \qquad L = \sum_{i=1}^{m} l_i, \qquad \beta > 0$$

where β represents the degree of people's preference for leisure relative to any other consumer good. The endowment constraint is given by

$$\text{(A.2)} \qquad \sum_{i=1}^{m} l_i + H = 1, \qquad l_i \in (0,1), \qquad H \in (0,1), \qquad i = 1,\dots,m$$

The production function is given by (3), and the budget constraint by (4) as in Section 2. A Walrasian regime or a multilateral bargaining game is assumed. Following Lemma 1, we have the decision problem for a household selling good i

$$\text{(A.3)} \qquad \text{Max:} \quad \ln u_i = \ln x_i + \sum_{r\in R} \ln(kx_r^d) + \sum_{j\in J} \ln x_j + \beta \ln H$$

$$\text{s.t.} \quad x_i + x_i^s = l_i - \alpha, \quad x_j = l_j - \alpha, \quad \forall j \in J \quad \text{(production function)}$$

$$l_i + \sum_{j\in J} l_j + H = 1 \quad \text{(endowment constraint)}$$

$$p_i x_i^s = \sum_{r\in R} p_r x_r^d \quad \text{(budget constraint)}$$

[20] For example, see Henderson and Quandt [1985, pp.24-25].

The $n-1$ utility equation conditions for $n-1$ types of households selling different goods determine $n-1$ relative prices of n traded goods and the $n-1$ market clearing conditions determine $n-1$ relative numbers of households selling n traded goods. The equilibrium is

$$\text{(A.4)} \qquad p_i/p_r = 1, \qquad M_i/M_r = 1$$

Inserting the equilibrium relative prices into the first-order conditions for the maximization problem in (A.3) yields equilibrium values for all decision problems, given by

$$\text{(A.5)} \qquad x_i = x_j = x_r^d = [1-(1+m-n)\alpha]/(\beta+m) = \alpha/K,$$

$$x_i^s = [(n-1)+(n-1)(n-m-1)\alpha]/(\beta+m)$$

$$=(1/K)(1+1/K)\alpha m - 1/K + \beta\alpha/K^2$$

$$\text{(A.6)} \qquad l_i = [n+(n^2-mn-n+\beta+m)\alpha]/(\beta+m)$$

$$=(1/K)(1+1/K)\alpha m - 1/K + \beta\alpha/K^2 + (1+1/K)\alpha$$

$$l_j = [1+(n+\beta-1)\alpha]/(\beta+m) = (1+1/K)\alpha$$

$$H = \beta[1-(1+m-n)\alpha]/(\beta+m) = \beta\alpha/K$$

$$\text{(A.7)} \qquad n = (1+1/K)m + (1-1/A) + \beta/K$$

$$u = k^{n-1}\beta^\beta \{[1-(1+m-n)\alpha]/(\beta+m)\}^{\beta+m}$$

where $K = -\ln k$. Following (A.5) to (A.7), the comparative statics are given by

$$\text{(A.8)} \qquad du/dk > 0, \qquad dn/dk > 0, \qquad dn/d\beta > 0, \qquad dn/d\alpha > 0$$

$$\text{(A.9)} \qquad dx_i^s/dk > 0, \qquad dx_i^s/d\beta > 0, \qquad dx_i^s/d\alpha > 0, \qquad dx_r^d/dk > 0,$$

$$dx_r^d/d\beta > 0, \qquad dx_r^d/d\alpha > 0, \qquad d(Mx_i^s/n)/dk > 0, \qquad d(Mx_i^s/n)/d\beta > 0$$

$$\text{(A.10)} \qquad dH/dk > 0, \qquad dH/d\beta > 0, \qquad dH/d\alpha > 0$$

$$\text{(A.11)} \qquad dl_i/dk > 0, \qquad dl_i/d\beta > 0, \qquad dl_i/d\alpha > 0,$$

$$d(m-n)l_j/dk < 0, \qquad d(m-n)l_j/d\beta < 0$$

and we can obtain the following proposition from (A.8) -- (A.11)

Proposition A.1

(1) The number of traded goods and the level of division of labor increase, and the degree of self-sufficiency decreases as transaction efficiency is improved. In this process the level of specialization, labor productivity, the extent of the market, leisure, and per household real income increase as transaction efficiency is improved. The time of a household spent at producing the traded good increases and at producing total non-traded goods decreases as transaction efficiency or the degree of economies of specialization increases. The number of traded goods, the level of division of labor, and the extent of the market increase as the degree of economies of specialization increases.

(2) The number of traded goods, the level of division of labor, the level of specialization, leisure, and the productivity increase as people's preference for leisure increases.

Note that the second part of Proposition A.1 is a special case of Proposition 2 since Cobb-Douglas utility function is a special case of CES utility function as $\rho \to 0$, that means the degree of economies of complementarity is infinitely large.

The number of traded goods will eventually reach the number of all goods as transaction efficiency is improved since $d(m-n)/dk < 0$. Letting $m = n$, $x_j = l_j = 0$ and solving the general equilibrium for complete division of labor yields

(A.12) $$x_i = x_r^d = (1-\alpha)/(m+\beta), \qquad H = [\beta(1-\alpha)]/(m+\beta),$$
$$l_i = (m+\alpha\beta)/(m+\beta), \qquad dl_i/d\beta < 0, \qquad l_i + H = 1$$

Denoting w as the average wage rate, then $w \approx k(n-1)x_r^d/l_i \approx k(l_i - \alpha)/l_i$ if $(n-1)x_r^d = x_i^s >> x_i$. w is a monotone increasing function of k and l_i due to increasing returns. Combining (A.10) -- (A.12) yields Figure A.1 and Figure A.2, where the economy reaches complete division of labor if $k \geq k_c$ or $\beta \geq \beta_c$. Note that labor supply and leisure increase concurrently as the preference for leisure increases if $\beta < \beta_c$; labor supply decreases and leisure increases as the preference for leisure increases if $\beta \geq \beta_c$.

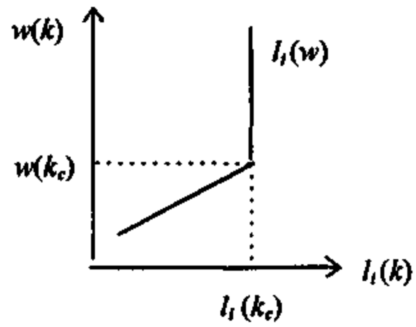

FIGURE A.1

The Relationship between Labor Supply and Wage Rate

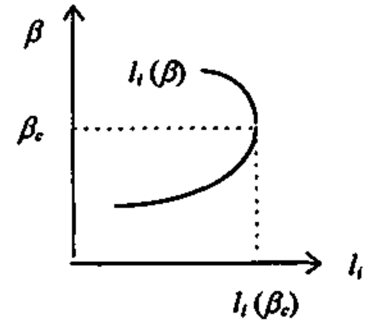

FIGURE A.2

The Relationship between Labor Supply and the Preference for Leisure

Appendix 2 : Proof of Proposition 2

We want to prove the following statement which establishes Proposition 2: $dn/d\beta > 0$, $dx_t/d\beta > 0$, $dx_j/d\beta > 0$, and $dx_r^d/d\beta > 0$ hold if ρ is sufficiently large; $dn/d\beta < 0$, $dx_t/d\beta = dx_j/d\beta < 0$, and $dx_r^d/d\beta < 0$ may hold if ρ is very small and satisfies $\rho > -1/(1+\beta)$. We shall prove that $dn/d\beta > 0$ holds if ρ is sufficiently large and $dn/d\beta < 0$ may hold if ρ is very small and satisfies $\rho > -1/(1+\beta)$, then the remaining part of the statement can be easily proved by the same approach. Denoting $F(k,\beta,\rho) = 1+(1+\beta)[\ln k/(1-K)]\{\rho/[1+(1+\beta)\rho]\}^2$ and $G(k,\beta,\rho) = 1+(\ln k/K)$ $\{\rho/[1+(1+\beta)\rho]\}^2$, we know from (19) that $dn/d\beta > 0$ if $F > 0$ and $G > 0$, and $dn/d\beta < 0$ if $F < 0$ and $G < 0$. It can be shown that $F > 0$ if and only if $\rho > f(\rho)$ and $G > 0$ if and only if $\rho > g(\rho)$, where $f(\rho) = -1/\{(1+\beta)+[-\ln k(1+\beta)/(1-K)]^{1/2}\}$ and $g(\rho) = -1/[(1+\beta)+(-\ln k/K)^{1/2}]$. We then establish the statement by proving below that $\rho > f(\rho)$ and $\rho > g(\rho)$ if ρ is sufficiently large, and $\rho < f(\rho)$ and $\rho < g(\rho)$ may exist if ρ is very small and satisfies $\rho > -1/(1+\beta)$.

Since $df/d\rho > 0$, $dg/d\rho < 0$, and $f, g \in (-1,0)$, the solutions for $\rho = f(\rho)$ and $\rho = g(\rho)$ must exist for $\rho \in (-1,0)$. Assuming $\rho_f = f(\rho_f)$ and $\rho_g = g(\rho_g)$, $\rho_f, \rho_g \in (-1,0)$, we have $\rho > f(\rho)$ for $\rho > \rho_f$ and $\rho > g(\rho)$ for $\rho > \rho_g$ since

$-1 < f(\rho \to -1) < f(\rho \to 0) < 0$ and $dg/d\rho < 0$. We know that $\rho > -1/(1+\beta)$ must be satisfied for the number of goods to be positive. Assuming $\rho_{\min} = -1/(1+\beta)$, we can then obtain Table A since $dn/d\beta > 0$ if $\rho > f(\rho)$ and $\rho > g(\rho)$, and $dn/d\beta < 0$ if $\rho < f(\rho)$ and $\rho < g(\rho)$. From Table A we can conclude that $dn/d\beta > 0$ if $\rho > \max(\rho_f, \rho_g)$ and $dn/d\beta < 0$ if $\min(\rho_f, \rho_g) > \rho > \rho_{\min}$. The statement in the beginning that verifies Proposition 2 can then be established.

TABLE A

The Sign of $dn/d\beta$

The Sign of $dn/d\beta$	$\rho_{\min} > \max(\rho_f, \rho_g)$	$\max(\rho_f, \rho_g) > \rho_{\min} > \min(\rho_f, \rho_g)$	$\min(\rho_f, \rho_g) > \rho_{\min}$
$\rho > \max(\rho_f, \rho_g)$	+	+	+
$\max(\rho_f, \rho_g) > \rho > \min(\rho_f, \rho_g)$	N.E.	?	?
$\min(\rho_f, \rho_g) > \rho$	N.E.	N.E.	–

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