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# Identification of Equilibrium Structures of Endogenous Specialisation: A Unified Approach Exemplified

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**Abstract**

A new method for identifying equilibria and conducting comparative statics of structures of the division of labour is proposed and illustrated by some models that are most typical of the literature on endogenous specialisation. The method starts with analysing individuals' decisions on specialisation under given price signals, and then solves for the equilibria and comparative statics in one step, resulting in a complete characterisation of subspaces of parameters in which different structures of specialisation occur in equilibrium. This approach proves highly powerful, particularly in dealing with models with substitutions between markets (for products and those for labour) and/or *ex ante* heterogeneous agents.

Notes

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## 1. A sketchy review of existing literature

In the rapidly growing literature of endogenous specialisation, of which the modern pioneering studies are Rosen (1978), Becker (1981) and Yang (1988) among others, a technically challenging problem is that many interesting models with intra-firm and inter-firm division of labour among *ex ante* heterogeneous agents, are not tractable by the existing tool kit, and therefore an obstacle to further progress needs to be overcome. This paper develops a new apparatus to fill the void.

We review some of the existing analytical tools before introducing the new method. To solve for the equilibrium of economies with *ex ante* identical agents (largely for addressing the Smithian specialisation, rather than the more discussed Ricardian specialisation based on *ex ante* comparative advantages among people/countries), Yang (1988) proposes what may be referred to as a two-step approach to identify the general equilibrium structure of the division of labour (as well as the associated equilibrium prices) among all possible structures. In the first step, for each structure, the "equilibrium" prices and utilities are calculated by equalizing utilities across all specialisation patterns in the said structure. Secondly, the very structure with the highest utility is selected as the general equilibrium structure of specialisation, in which no agent has an incentive to deviate from her specialisation pattern. Years later, in their ambitious monograph, Yang and Ng (1993) covers, and applies the two-step algorithm to many specific models. Sun, Yang and Yao (1999) further argue that the two-step approach applies to a much larger class of models. To be sure, the two-step approach has proved quite powerful in solving many endogenous specialisation models (for the latest survey, refer to Yang 2003). The assumption of *ex ante* identical agents, however, is a severe limitation to its application to many interesting economic issues (say, Yang's (2001) model on foreign direct investment between two countries, to merely mention one example). Moreover, the two-step approach may miss some equilibrium prices even for quite simple models of *ex ante* identical agents, as shown in Examples 1 and 3 in Section 3.

To deal with a model of *ex ante* two-type agents, Sun, Yang and Zhou (1999) elaborate on the two-step approach and develop what maybe termed as a "benchmark price" analytical method. Similar to the two-step approach, market-clearing prices are worked out for each structure in the first step. But differing from the two-step approach, individuals' utilities are then compared among constituent specialisation patterns (occupations) in this structure and occupations in other structures *under these market clearance prices* to identify the parameter subspace within which this structure occurs in general equilibrium (agents have no incentive to deviate from this structure).[1] The benchmark method has been employed successfully to analyse important phenomena such as globalisation and dual structures emerging from international trade (see Sachs, Yang and Zhang 2000). Yet, this approach does not apply to models with possible substitution between markets (for intermediate products and for labour, for instance), as in Li's (2001) multiple national enterprise model, for the simple reason that the markets for some products or services are absent in some structures and hence no price for them could be used to compare utilities across occupations occurring in different structures of the division of labour.

A much better known approach to equilibrium computation is of course the conventional simplex triangulation (see, for instance, Scarf and

[1] Strictly speaking, the subset of parameters identified as such is often not a (sub-)space. But we rather loosely refer to them as subspaces throughout this paper.

Hansen 1973; van der Laan and Talman 1987). But this approach, as an algorithm, requires well framed excess demand functions in order to start the fixed-point searching process in the price simplex. But in most models of endogenous specialisation as surveyed in Yang (2003), it is rather difficult to construct the excess demand across different structures of the division of labour with different goods or services actually traded in the markets. In addition, the algorithm, at best, converges to a (locally unique) equilibrium price vector from the starting point, while in models of endogenous specialisation, there may exist a continuum of equilibrium prices (refer to examples in Section 3). Most importantly, the simple triangulation algorithm, complicated and time-consuming as it is, does not convey the information of the structure of the division of labour, which is a major concern in this literature.

## 2.Brief introduction to a unified approach

The key idea of our unified approach could be summarised as the following: For any price vector $(p, w)$ where $p$ is the non-labour commodity price vector and $w$ the wage vector, the agent maximises utility $u(p, w)$ across specialisation patterns. The market clearing condition (and utility equalization condition among *ex ante* identical agents) *directly* leads to the general equilibrium structure and prices. As such, a complete characterisation of parameter subspaces, in which different structures of the division of labour occur in equilibrium, is obtained.

Put in a few more details, the approach proceeds as follows. For any given prices $(p, w)$ where the price vector $p = (p_1, ..., p_m)$ and wage vector $w = (w_1, ..., w_n)$, each *possible* kind of expert maximises utility for his/her potential occupation(s). For simplicity (without loss of generality), consider the situation in which there is only one type of expert who buys units of labour from the market, and agents of all the other occupations sell either some type of product or his/her own labour only. Due to the increasing returns to specialisation, agents may specialize in producing some products while trading for other products, provided that the transaction cost is not too high. For any person, the maximised utility under $(p,w)$ if she chooses to sell product $i$ is denoted as $u_i(p,w)$, $i=1,2,...,m$, while the indirect utility if she chooses to sell labour in labour market $j$ is denoted as $u_{Lj}(p,w)$, $j=1,2,...,n$, and the indirect utility if she buys labour is denoted as $u_0(p,w)$. The said agent derives the highest real income among $u_i(p,w)$, $i=1,2,...,m$; $u_{Lj}(p,w)$, $j=1,2,...,n$, and $u_0(p,w)$, under a given price signal $(p,w)$ by choosing the corresponding specialisation pattern. However, which pattern brings about the highest real income apparently depends on the relative prices. By means of partitioning the price space into several subspaces (in each of which some particular specialisation pattern(s) brings about the highest real income), and by the market clearance condition (and utility equalization among agents who are *ex ante* identical but may be *ex post* different in specialisation), we can identify the subspace of parameters, in which some particular structure of specialisation occurs in equilibrium. In short, we partition the price space first, based on which we then partition the parameter space to identify the condition under which the equilibrium specialisation structures and prices occur.

It should be emphasised that this approach identifies in one step, subspaces of parameters in which different structures of endogenous specialisation occur and in which all equilibrium prices are computed. In other words, this approach also applies to comparative statics and in fact constitutes a complete characterisation of the subspaces of parameters for different structures of labour specialisation. Note in some parameter subspaces, the equilibrium prices may be shadow prices and some occupations and markets may actually not occur. Indeed, it is often the case, as shown below by some typical models in the literature of endogenous specialisation.

Compared to existing methods solving for the equilibrium structure of the division of labour (for all the situations to which the benchmark price approach and/or the two-step approach apply), our method not only works but also simplifies the algebraic manipulation significantly in many cases. Some equilibrium prices (shadow prices) might be missed by the above two approaches, as shown by examples in the next section. Furthermore, our unified approach applies to a more general class of models than does the benchmark price one or the two-step one, particularly to models with substitution between markets and with *ex ante* heterogeneous agents.

## 3. Examples

We illustrate the new method by examining four examples, which are typical of the models in the literature of endogenous labour specialisation. Price taking behavior is assumed throughout. Example 1 is a basic model in which agents are *ex ante* identical and all the goods are consumption ones. Example 2 incorporates producer goods and a labour market, hence firms are allowed, but agents are still assumed to be *ex ante* identical. Example 3 considers an economy with consumption goods only but *ex ante* differences among agents are allowed. Example 4 allows both (*ex ante*) heterogeneous agents and producer goods, and the emergence of multinational firms from the expansion of trade, and division of labour is made endogenous.

*Example 1. Ex ante identical agents with only consumption goods*

Consider a simple economy with a continuum of *ex ante* identical agents and two consumption goods, X and Y. Each agent is endowed with one unit of labour. The production functions for X and Y are $f(l_X) = l_X - \alpha$ and $f(l_Y) = l_Y - \beta$ respectively, where $l_X(l_Y)$ is the labour input in the production of good X (Y). Labour is assumed the only input for production. Utility is a Cobb-Douglas function, $u(x, y) = xy$ and the transaction function is specified as $g(z) = kz, Z \in \{X, Y\}$, where $k \in (0,1)$ characterises the trading efficiency.

For any given price of X (in terms of Y), $p \equiv p_X / p_Y$, the agent's production-trade choice is made among (1) producing only good X, selling X and buying Y; (2) producing only good Y, selling Y and buying X; and (3) producing both but selling neither (autarky). For a relative price p, the indirect utility of agents choosing trading X [Y] for Y [X] is $u_X = \frac{(1-\alpha)^2}{4}kp$ [$u_Y = \frac{(1-\beta)^2}{4p}k$] and she trades $\frac{1-\alpha}{2}$ [$\frac{1-\beta}{2}$]amount of X [Y] for $\frac{(1-\alpha)p}{2}$ [$\frac{1-\beta}{2p}$] of Y [X], but she can only receive $\frac{(1-\alpha)kp}{2}$ [$\frac{(1-\beta)k}{2p}$] of Y [X] due to the transaction costs incurred. Values in boxed brackets represent cases for agents trading good Y for X. Denoting the maximised utility under autarky as $u_A$, $u_A = \frac{(1-\alpha-\beta)^2}{4}$. Under the equilibrium price, either (i) some trade occurs and thus there exists the division of labour; or (ii) no trade occurs and hence everyone is in autarky. In the case of (i), it is required that

$u_X = u_Y > u_A$. It follows that the equilibrium price $p = p^* \equiv \frac{1-\beta}{1-\alpha}$ and the trading efficiency has to be high enough to support the division of labour, $k > k^* \equiv \frac{(1-\alpha-\beta)^2}{(1-\alpha)(1-\beta)}$. In case (ii), $u_A > u_X$ and $u_A > u_Y$, hence, $\frac{(1-\alpha-\beta)^2}{4} > \frac{(1-\alpha)^2}{4}kp$ and $\frac{(1-\alpha-\beta)^2}{4} > \frac{(1-\beta)^2}{4p}k$. Thus,

$$\frac{k}{k^*}p^* < p < \frac{k^*}{k}p^* \qquad (1)$$

For the price set satisfying (1) to be non-empty, it follows $k < k^*$. Any price $p \in (\frac{k}{k^*}p^*, \frac{k^*}{k}p^*)$ is an equilibrium price that clears the market (demand equals supply, both being equal to zero). Table 1 summarises the above analyses.

**Table 1. Parameter subspaces and equilibria for Example 1**

| | $k < k^*$ | $k > k^*$ |
|---|---|---|
| Equilibrium prices | $(\frac{k}{k^*}p^*, \frac{k^*}{k}p^*)$ (shadow equilibrium price set) | $p^*$ |
| Structure of the division of labour | Autarky (no trade) | Division of labour (half population specialize in producing X(Y)) |

Note p* is referred to as MUEPV (Maximal Utility Equalization Price Vector) in Sun, Yang and Yao (1999), under which the utility is the same across experts producing and selling different goods or services. It can be seen that for any values of parameters, p* is an equilibrium. What's especially interesting, however, is that when transaction costs are too high to allow for trade and the division of labour (k<k*), the equilibrium (shadow) price still exists. But the approach taken in Sun, Yang and Yao (1999) yields only one equilibrium price, namely, p*, among infinitely many others in this case. (refer to Example 1 considered in Sun, Yang and Yao (1999) wherein only one equilibrium price

is identified). The approach we develop in this paper identifies all the equilibrium prices and structures for *any* possible values of parameters. In other words, the equilibrium analysis and comparative statics could be done in one step, as shown by Table 1.

*Example 2. Ex ante identical agents with producer goods and possibly with firms*

In this example we consider an economy with one intermediate product X and one final product Y. Each agent is endowed with one unit of labour. For each agent, the production function of X is $f(l_X) = Max\{l_X - b, 0\}$ and that of Y is $h(x_Y, l_Y) = x_Y{}^{\alpha} l_Y{}^{\beta}$, $\alpha, \beta \in (0,1)$, $\alpha + \beta > 1$, where $l_X (l_Y)$ is the labour input in the production of good X (Y) and $x_Y$ is the amount of X used in the production of Y. Note that $\alpha + \beta > 1$ implies increasing returns to scale in the production of Y. The utility function is taken as $u(y) = y$ for simplicity, where y is the amount consumed of Y. As in Example 1, the transaction function for goods X and Y is specified as $g(z) = kz, Z \in \{X, Y\}$, where $k \in (0,1)$ characterises the trading efficiency in the product market. Differing from Example 1, there is an intermediate product in this model. By the indirect pricing theory of the firm (Yang and Ng 1995), there may exist trade of labour and hence firms, which could replace the market exchange of intermediate products. The firm owner may hire some agents, called workers, direct their labour effort to produce goods, part of which she may sell in the market, and claim the residual rights. The worker sells his labour for a salary (in terms of consumption good Y in this example). But note what is really traded in the labour market is the worker's production function (rather than labour *per se*) since the labour of heterogeneity by its nature cannot be aggregated (and put into anyone's or the firm's *ex ante* "production function" (for more on this point, refer to Sun 2000). The transaction function of labour is assumed as $g_L(l) = sl$, where $s \in (0,1)$ characterises the trading efficiency of the labour market.

For any given price of good X, p, and the wage rate w, both in terms of good Y, it is clear that any agent may choose among (i) selling nothing (in autarky); (ii) producing and selling X (and buying Y); (iii) selling Y; or (iv) selling labour and buying Y (being an employee). No rational person would sell two or three among X, Y and labour (refer to Appendix 1 for analysis). We denote by $u_A$($u_X$ or $u_L$) the maximum utility if the agent makes choice (i) (choice (ii) or (iv)). It is easy to see that $u_A = \frac{\alpha^\alpha \beta^\beta (1-b)^{\alpha+\beta}}{(\alpha+\beta)^{\alpha+\beta}}$, $u_X = kp(1-b)$, and $u_L = kw$.

What's intriguing is the decision problem if the agent chooses (iii). She may buy part or all of the intermediate inputs used to produce Y from the markets, produce Y and sell Y. In this case, no labour is traded. However, she may also hire some workers and direct their labour to producing part or all intermediate goods that she uses to produce Y.[2] In general, the decision problem could be stated as

$$u_Y = \underset{l,x,N}{Max}\{[Max\{l-b,0\} + kx + N(s\cdot 1-b)]^\alpha (1-l)^\beta - px - wN \qquad (2)$$

where decision variables $l$ is the labour input to producing intermediate product X, $x$ is the amount purchased of X from the market and $N$ the number of employees hired to produce X. Due to transaction costs in both labour market and product markets, only $s\cdot 1$ units of labour will be actually put into production of X by each worker and $kx$ of X is actually received.

[2] We purposely ignore the possibility that the agent may hire some workers to produce Y, mainly to simplify the algebra which would otherwise be much more complicated. We do this for the sake of illustrating how the new method can be used to solve models that necessitate the substitution between the market of labor and the market of the (intermediate) product.

The last two terms in Equation (2) refer to the balanced budget. The integer problem for N is ignored throughout to simplify the analysis. One could show that if $p < \frac{k}{s-b} w$, then $N = 0$, $l = 0$ and that if $p \geq \frac{k}{s-b} w$, then $x = 0$, $l = 0$ by noting that in any case ($p < \frac{k}{s-b} w$ or $p \geq \frac{k}{s-b} w$), the first-order conditions of interior solutions of $x$ and $N$ cannot both hold and that the interior solution for $l$ and $x$ (or $l$ and $N$), if any, is a saddle point, as could be seen from the negative-ness of determinants of the Hessian matrix (to save space, we omit the calculation details), and hence $l = 0$ as $l$ cannot be 1. It follows that for any $(p, w)$ satisfying $p < \frac{k}{s-b} w$, problem (2) turns out to be $\underset{l,x}{Max}(kx)^{\alpha} - px$, of which the solution is $x^* = (\alpha k^{\alpha} / p)^{\frac{1}{1-\alpha}}$ and the indirect utility, denoted as $u_{YX}$, $u_{YX}(p, w) = [(1-\alpha)^{1-\alpha} \alpha^{\alpha} (\frac{k}{p})^{\alpha}]^{\frac{1}{1-\alpha}}$. For any $(p, w)$ satisfying $p \geq \frac{k}{s-b} w$, problem (2) turns out to be $\underset{l,x}{Max}(s-b)^{\alpha} N^{\alpha} - wN$, of which the solution is $N^* = [\alpha(s-b)^{\alpha} / w]^{\frac{1}{1-\alpha}}$ and the indirect utility, denoted as $u_{YL}$, $u_{YL}(p, w) = [(1-\alpha)^{1-\alpha} \alpha^{\alpha} (\frac{s-b}{w})^{\alpha}]^{\frac{1}{1-\alpha}}$.

We are now ready to identify the subspaces of parameters, in which different structures of the division of labour are in equilibrium. Should $u_{YX}$ be achieved in equilibrium, there must be some other agents selling X for the demand of X to be met. In addition, the utilities of the buyers and sellers of X are equalized in equilibrium because the agents are *ex ante* identical and freely enter and exit any profession. It is required that $p < \frac{k}{s-b} w$, $u_{YX}(p, w) = u_X(p, w)$, $u_{YX}(p, w) > u_L(p, w)$ and $u_{YX}(p, w) > u_A(p, w)$, from which we can obtain, after some algebraic manipulation, that $k > \frac{s-b}{1-b}$ and $k > k^* \equiv [\frac{\beta^\beta (1-b)^\beta}{(\alpha+\beta)^{\alpha+\beta}(1-\alpha)^{1-\alpha}}]^{\frac{1}{2\alpha}}$, with $p^* = k^{2\alpha-1}(1-b)^{\alpha-1}(1-\alpha)^{1-\alpha}\alpha^\alpha$, with the shadow wage rate set at $(\frac{s-b}{k}p^*,(1-b)p^*)$ and utility given by $u_{YX} \equiv u_{YX}(p^*,w) = k^{2\alpha}(1-b)^\alpha(1-\alpha)^{1-\alpha}\alpha^\alpha$. Should $u_{YL}$ be obtained in equilibrium, there must be some potential employees in the labour market (for the demand of labour to be met). In addition, the utilities of both buyers and sellers of labour are equalized in equilibrium. It is required that $p > \frac{k}{s-b}w$, $u_{YL}(p,w) = u_L(p,w)$, $u_{YL}(p,w) > u_X(p,w)$ and $u_{YL}(p,w) > u_A(p,w)$. It then follows that $k < \frac{s-b}{1-b}$ and $k(s-b) > KS \equiv [\frac{\beta^\beta(1-b)^{\alpha+\beta}}{(\alpha+\beta)^{\alpha+\beta}(1-\alpha)^{1-\alpha}}]^{\frac{1}{\alpha}}$ with $w^* = k^{\alpha-1}(s-b)^\alpha(1-\alpha)^{1-\alpha}\alpha^\alpha$, shadow price set at $(\frac{k}{s-b}w^*,\frac{1}{1-b}w^*)$ and $u_{YL} \equiv u_{YL}(p,w^*) = k^\alpha(s-b)^\alpha(1-\alpha)^{1-\alpha}\alpha^\alpha$. If neither $u_{YX}$ nor $u_{YL}$ is the utility in equilibrium, the equilibrium utility cannot be greater than in autarky, thus we should have $u_A(p,w) > u_Y(p,w) \equiv Max\{u_{YX}(p,w), u_{YL}(p,w)\}$, $u_A(p,w) > u_X(p,w)$ and $u_A(p,w) > u_L(p,w)$. It is in turn required that $k < k^*$ and $k > \frac{s-b}{1-b}$ or $k(s-b) < KS$ and $k < \frac{s-b}{1-b}$.

A complete characterisation of equilibria and the corresponding parameter subspaces is summarised in Table 2.

**Table 2. Parameter subspaces and equilibria for Example 2**

| | |
|---|---|
| $k > \frac{s-b}{1-b}$ | **When** $k < k^*$**:**<br>Equilibrium prices and wage rates (both being shadow prices):<br>any $p \in (\alpha k(\frac{1-\alpha}{u_A})^{\frac{1}{\alpha}-1}, \frac{u_A}{k(1-b)})$, any $w \in (\alpha(s-b)(\frac{1-\alpha}{u_A})^{\frac{1}{\alpha}-1}, \frac{u_A}{k})$<br>Autarky (no trade)<br><br>**When** $k > k^*$**:**<br>Equilibrium price: $p^* = k^{2\alpha-1}(1-b)^{\alpha-1}(1-\alpha)^{1-\alpha}\alpha^{\alpha}$<br>Equilibrium wage rates (shadow prices): any $w \in (\frac{s-b}{k}p^*, (1-b)p^*)$<br>Division of labour between experts producing X and experts producing Y (without firm), with the population ratio of X experts and Y experts being $\frac{\alpha}{1-\alpha}k^{\frac{1}{1-\alpha}}$. |
| $k < \frac{s-b}{1-b}$ | **When** $k(s-b) < KS$**:**<br>Equilibrium prices and wage rates (both being shadow prices):<br>any $p \in (\alpha k(\frac{1-\alpha}{u_A})^{\frac{1}{\alpha}-1}, \frac{u_A}{k(1-b)})$, any $w \in (\alpha(s-b)(\frac{1-\alpha}{u_A})^{\frac{1}{\alpha}-1}, \frac{u_A}{k})$<br>Autarky (no trade)<br><br>**When** $k(s-b) > KS$**:**<br>Equilibrium wage rate: $w^* = k^{\alpha-1}(s-b)^{\alpha}(1-\alpha)^{1-\alpha}\alpha^{\alpha}$<br>Equilibrium prices (shadow prices): any $p \in (\frac{k}{s-b}w^*, \frac{1}{1-b}w^*)$<br>Division of labour with firms, the population ratio of employees (hired to produce X) and employers (producing and selling Y) being $\frac{\alpha}{1-\alpha}k$. |

Indeed, we may put Table 2 even more neatly, as Table 2' below.

**Table 2'. A simplified Table 2.**

| | |
|---|---|
| $k > \frac{s-b}{1-b}$ | $k < k^*$: Shadow p and w; Autarky.<br>$k > k^*$: Equilibrium $p^*$ and shadow w; Division of labour, no firm |
| $k < \frac{s-b}{1-b}$ | $k(s-b) < KS$: Shadow p and w; Autarky.<br>$k(s-b) > KS$: Equilibrium $w^*$ and shadow p; Division of labour with firms |

*Example 3. Ex ante heterogeneous agents with only consumption goods*

We reconsider Example 3.2 in Sun, Yang and Zhou (1999) in which there are infinitely many agents of two distinct types and the population ratio between the two types is 1:1. Each agent is endowed with one unit of labour, which can be used to produce two consumption goods, $X_1$ and $X_2$. The utility function of both Type 1 and Type 2 agents is $u(x_1, x_2) = x_1 x_2$, where $x_1(x_2)$ is the amount consumed of $X_1(X_2)$. The production functions are $f_{1X_1}(l_{1X_1}) = (l_{1X_1})^2$ and $f_{1X_2}(l_{1X_2}) = l_{1X_2}$ for Type 1 agents and $f_{2X_1}(l_{2X_1}) = al_{2X_1}$, $a < 1$ and $f_{2X_2}(l_{2X_2}) = (l_{2X_2})^2$ for Type 2 agents. Type 1 agents have a Ricardian advantage in producing $X_1$ while Type 2 agents have comparative advantage in producing $X_2$. The transaction technologies are $g_{1X_i}(x_i) = k_1 x_i$ and $g_{2X_i}(x_i) = k_2 x_i$ $(i = 1,2)$ for Type 1 and Type 2 agents respectively. Sun, Yang and Zhou (1999) solved this model by the benchmark price method. We re-solve the same model by our new method, which not only simplifies the algebra but more importantly, also identifies more equilibrium (shadow) prices for some structures of the division of labour.

Obviously, for any price p, $p \equiv p_{X_2} / p_{X_1}$, each agent chooses one of the following: (i) autarky, with utility $u_{1A} = 4/27$ for Type 1 agents and $u_{2A} = 4a/27$ for Type 2 agents; (ii) producing and selling $X_1$ and buying $X_2$, the indirect utilities denoted as $u_{1X_1}(p)$ and $u_{2X_1}(p)$ for Type 1 and Type 2 agents respectively; and (iii) producing and selling $X_2$ and buying $X_1$, the indirect utility denoted as $u_{1X_2}(p)$ and $u_{2X_2}(p)$ for Type 1 and Type 2 agents respectively. In choosing (ii), a Type 1 agent trades 1/2 amount of $X_1$ for

1/(2p) of $X_2$, $u_{1X_1}(p)=\frac{k_1}{4p}$, while a Type 2 agent trades a/2 amount of $X_1$ for a/(2p) of $X_2$, $u_{2X_1}(p)=\frac{k_2a^2}{4p}$. In choosing (iii), both a Type 1 agent and a Type 2 agent trade 1/2 amount of $X_2$ for *p/2* of $X_1$, $u_{1X_2}(p)=\frac{k_1p}{4}$, $u_{2X_2}(p)=\frac{k_2p}{4}$. Note in each of (ii) and (iii), the agent can only receive a fraction of what she purchases from the market due to the transaction costs.

We begin with analysing the decision made by the Type 1 agent. Given a price signal *p*, it is required that for (i) to be actually chosen that $u_A(p)>u_{1X_1}(p)$ and $u_A(p)>u_{1X_2}(p)$, i.e., $\frac{16}{27k_1}>p>\frac{27k_1}{16}$, from which it follows $k_1<k_0\equiv\frac{16}{27}$. For (ii) to be chosen, $u_{1X_1}(p)>u_A(p)$ and $u_{1X_1}(p)>u_{1X_2}(p)$, i.e., $p<1$ and $p<\frac{k_1}{k_0}$. For (iii) to be chosen, $u_{1X_2}(p)>u_A(p)$ and $u_{1X_2}(p)>u_{1X_1}(p)$, i.e., $p>1$ and $p>\frac{k_0}{k_1}$. Similar analyses could be made with regard to the decision problem that Type 2 agent faces: autarky is chosen when $\frac{ak_2}{k_0}<p<\frac{ak_0}{k_2}$ (from which it follows $k_2<k_0$); choice (ii) is made when $p<a$ and $p<\frac{ak_2}{k_0}$; and choice (iii) is made when $p>a$ and $p>\frac{ak_0}{k_2}$. Since for some subspaces of parameters, it may be that trade occurs between the two types of agents, it is necessary to simultaneously take account of the decisions made by Type 1 agents and those by Type 2 agents. It might be thought that an analysis of all possible combinations of choices by two types of agents is an algebraically complicated one. Actually, it could be rather straightforward as shown in the following analysis. If no trade occurs for price p, autarky (choice (i)) must be preferred by both types of agents, or, $\frac{k_0}{k_1} < p < \frac{k_1}{k_0}$ and $\frac{ak_2}{k_0} < p < \frac{ak_0}{k_2}$, from which it follows $k_1 < k_0$, $k_2 < k_0$, $k_1 k_2 < ak_0^2$ and the equilibrium (shadow) price set is $(Max\{k_1/k_0, ak_2/k_0\}, Min\{k_0/k_1, ak_0/k_2\})$. If trade occurs under a price $p$, the sellers of $X_1$ are types 1 agents and/or Type 2 agents. We first consider the situation in which some or all agents of Type 1 choose to be $X_1$ sellers and no Type 2 agent sells $X_1$. It follows that $p \le 1$ and $p \le k_1/k_0$. In this case, for the market to clear, some (or all) agents of Type 2 or some other Type 1 agents buy $X_1$ and sell $X_2$.

Namely,

Either (A) (some or all) Type 2 agents supply $X_2$; or

(B) no Type 2 agents supply $X_2$ ($X_2$ is supplied only by Type 1 agents).

Case (A) requires that $Max\{a, ak_0/k_2\} \le p \le Min\{1, k_1/k_0\}$, hence $k_1 \ge ak_0$, $k_2 \ge ak_0$ and $k_1 k_2 \ge ak_0^2$. Each $X_1$ seller from Type 1 agents supplies half unit of $X_1$ and each $X_2$ seller from Type 2 agents demands (p/2) unit of $X_1$. But note the population ratio between the two types is 1:1. Therefore, when $k_1 < k_0$, which implies $p \le k_1/k_0 < 1$, there are only a fraction of the population of Type 1 supplying $X_1$ in equilibrium. But the price cannot be strictly less than $k_1/k_0$, since otherwise every agent of Type 1 would prefer to be a $X_1$ supplier. Thus, the equilibrium price $p = k_1/k_0$, $(\frac{k_1}{k_0})100\%$ of Type 1 agents supply $X_1$ and $(1-\frac{k_1}{k_0})100\%$ of Type 1 agents in autarky, and all agents of Type 2 supply $X_2$. That is, Type 2 agents reap all the benefits from the division of labour. When $k_1 \geq k_0$, $p=1$ (note each Type 1 agent would sell half unit of $X_1$ and the demand of $X_1$ by Type 2 agents would be less than the total supply if $p<1$) and all agents of Type 1 (2) supply (demand) $X_1$. Case (B) requires $p=1$, $k_1 \geq k_0$ and $k_2 < ak_0$. The half population of Type 1 supply (demand) $X_1$ and demand (supply) $X_2$, and no agent of Type 2 gets involved in the division of labour and trade. We now consider the situation in which some or all agents of Type 2 choose to be $X_1$ suppliers. It follows that $p \leq a$ and $p \leq \frac{ak_2}{k_0}$. For the market to clear, some agents of Type 1 or Type 2 must demand $X_1$. But it is impossible for Type 1 agents to do so in equilibrium, otherwise, we have $p \geq 1$, contradicting $p \leq a$. Therefore, the demanders of $X_1$ must be of Type 2. It follows that $p \geq a$ and $p \geq \frac{ak_0}{k_2}$. Hence, $p=a$ and $k_2 \geq k_0$. But note that for agents of Type 1 not to be $X_1$ suppliers, it requires that $p \geq 1$ or $p \geq \frac{k_1}{k_0}$, that is, $k_1 \leq ak_0$ (due to $p=a$). Thus, when $k_2 \geq k_0$ and $k_1 \leq ak_0$, half population of Type 2 supply (demand) $X_1$ and demand (supply) $X_2$ at price $p=a$ and no agent of Type 1 gets involved in the trade.

We summarise the above analysis in Table 3.[3]

[3]It may be noted that Table 3 is slightly different from the results in Sun, Yang and Zhou (1999) in that we ignore the equilibrium analysis for the very "threshold" values of parameters $k_1$ and $k_2$ to keep Table 3 less complicated, though it is rather easy to incorporate these cases into our analysis by the new method.

**Table 3. Parameter subspaces and equilibria of Example 3**

| | $k_1 < k_0$ | $k_1 > k_0$ |
|---|---|---|
| $k_2 < k_0$ | **When** $k_1 k_2 < a k_0^2$<br>Equilibrium (shadow) price set<br>$(_{(Max\{k_1/k_0, ak_2/k_0\}, Min\{k_0/k_1, ak_0/k_2\})}$<br>Structure: all agents in autarky.<br>**When** $k_1 k_2 > a k_0^2$<br>Equilibrium price $p = k_1 / k_0$<br>Structure<br>Type 1: $(\frac{k_1}{k_0})100\%$ sell $X_1$<br>$(1 - \frac{k_1}{k_0})100\%$ in autarky<br>Type 2: all sell $X_2$ | **When** $k_2 < a k_0$<br>Equilibrium price p=1<br>Structure<br>Type 1: half sell $X_1$ and half sell $X_2$<br>Type 2: autarky<br>**When** $k_2 > a k_0$<br>Equilibrium p=1<br>Structure<br>Type 1: all sell $X_1$<br>Type 2: all sell $X_2$ |
| $k_2 > k_0$ | **When** $k_1 < a k_0$<br>Equilibrium price p=a<br>Structure<br>Type 1: autarky.<br>Type2: half sell $X_1$, half sell $X_2$<br>**When** $k_1 > a k_0$<br>Equilibrium price $p = k_1 / k_0$<br>Structure<br>Type 1: $(\frac{k_1}{k_0})100\%$ sell $X_1$<br>$(1 - \frac{k_1}{k_0})100\%$ in autarky<br>Type 2: all sell $X_2$ | Equilibrium price $p = 1$<br>Structure<br>Type 1: all sell $X_1$<br>Type 2: all sell $X_2$ |

When transaction technologies are less effective, autarky is the only equilibrium. As transaction technologies improve, labour specialisation starts to emerge. Eventually, the more effective transaction technologies will lead to the division of labour, in which all Type 1 agents produce $X_1$ and all Type 2 agents produce $X_2$. As discussed in more details in Sun, Yang and Zhou (1999), this example could be interpreted as a Ricardian story of two regions with Type 1(2) agents from Region 1(2). Note that continuous changes in transaction conditions cause discontinuous shift in the trade structure and levels of labour specialisation. If the transaction condition for both regions is high enough, the inter-regional trade will emerge from domestic trade. What's intriguing is the endogenous dual structure in one region for some range of the trading efficiency parameters. If the trading efficiency for Region 1 (characterised by $k_1$) is not that high, there will emerge two distinct sub-regions in that region, one sub-region producing $X_1$ and trading with Region 2 for $X_2$ and the other one autarkic, though the *per capita* real income is the same across the two sub-regions. But the gains from inter-regional trade are distributed between the two regions quite asymmetrically such that all the gains from trade go to Region 2. The income inequality between regions would be enlarged as the trading efficiency in Region 2 improves provided that the dual structure in Region 1 remains qualitatively unchanged. It may be worthwhile to emphasize that the existence of a dual structure within the region is robust to parameter changes in some subspaces of parameters $(k_1, k_2)$.

*Example 4. Ex ante heterogeneous agents with producer goods and possibly with firms*

In this example we consider an economy with two types of agents and one intermediate good X and one final good Y. This example is the same as Example 2 except that the trading efficiencies in the labour market for the two types of agents are allowed to be different. Namely, it is allowed that $s_1 \neq s_2$, where $s_1(s_2)$ is the trading efficiency coefficient in the labour market for Type 1 (2) agents. Without loss of generality, we assume $s_1 > s_2 > 0$ (the case $s_1 = s_2$ is precisely Example 2). As is done in the analysis of Example 2, we exclude the possibility that the agent may hire some workers to produce Y to simplify the algebra. Thus, for a given price signal (*p, w*) where *p* is the price of good X and *w* the wage rate both in terms of good Y, the agent of each type may choose from (i) selling nothing (in autarky); (ii) producing and selling X (and buying Y); (iii) selling Y; or (iv) selling labour and buying Y (being an employee). As analysed earlier, no rational person would sell two or three among X, Y and labour, and no Y seller will both buy X and hire workers to produce X. We denote by $u_A$($u_{iX}$ or $u_{iL}$ for Type $i$ agents, $i = 1,2$) the maximum utility if the agent makes a choice of (i) (choice (ii) or (iv)), and by $u_{iYX}$ ($u_{iYL}$) for a Type $i$ agent ($i = 1,2$) the maximum utility if she chooses option (iii) such that she buys intermediate good X from the market (hires workers to produce X in the firm). We have,

$u_A(p,w) = \dfrac{\alpha^\alpha \beta^\beta (1-b)^{\alpha+\beta}}{(\alpha+\beta)^{\alpha+\beta}}$, $u_{iX}(p,w) = kp(1-b)$, $u_{iL}(p,w) = kw$, $u_{iYX}(p,w)$ $= [(1-\alpha)^{1-\alpha}\alpha^\alpha (\dfrac{k}{p})^\alpha]^{\frac{1}{1-\alpha}}$ and $u_{iYX}(p,w) =$ $[(1-\alpha)^{1-\alpha}\alpha^\alpha (\dfrac{s_i - b}{w})^\alpha]^{\frac{1}{1-\alpha}}$, $i = 1,2$. For any given (*p,w*), every agent of each type chooses her specialisation pattern among the above five options to obtain the highest utility.

If no trade occurs under prices *p* and *w*, then $u_A > Max\{u_{iX}, u_{iL}, u_{iYX}, u_{iYL}\}$, $i = 1,2$ (we suppress *p* and *w* in the indirect utilities throughout this paragraph), from which we obtain $k < k^* \equiv$ $[\dfrac{\beta^\beta (1-b)^\beta}{(\alpha+\beta)^{\alpha+\beta}(1-\alpha)^{1-\alpha}}]^{\frac{1}{2\alpha}}$, $k(s_1 - b) > KS$ $\equiv [\dfrac{\beta^\beta (1-b)^{\alpha+\beta}}{(\alpha+\beta)^{\alpha+\beta}(1-\alpha)^{1-\alpha}}]^{\frac{1}{\alpha}}$ and the equilibrium (shadow) prices are any $p \in (\alpha k (\dfrac{1-\alpha}{u_A})^{\frac{1}{\alpha}-1}, \dfrac{u_A}{k(1-b)})$ and any $w \in (\alpha(s_1 - b)(\dfrac{1-\alpha}{u_A})^{\frac{1}{\alpha}-1}, \dfrac{u_A}{k})$.

If only products are traded (no labour trade) in equilibrium, the utility level for both X sellers and Y sellers must be the same across agents of the same type, $u_{iX} = u_{iYX} > Max\{u_A, u_{iL}, u_{iYL})$, $i = 1,2$, which in turn implies $k > k^*$, $k > \frac{s_1 - b}{1-b}$ and the equilibrium price $p = p^* \equiv k^{2\alpha-1}(1-b)^{\alpha-1}(1-\alpha)^{1-\alpha}\alpha^\alpha$ and the (shadow) price set of the wage rate is $(\frac{s_1 - b}{k}p^*, (1-b)p^*)$. All agents in the economy freely choose between X sellers and Y sellers, but the number of agents from each type who are X (or Y) sellers is indeterminate. If X is not traded but both labour and final good Y are traded in equilibrium, then at least some Type 1 agents are Y suppliers. Note that the real income of a Type 1 agent who sells labour cannot be higher than that of the Type 2 agent who sells Y, but the Type 1 agent (who is a Y seller) hiring workers to produce X can always have a higher income level than a Type 2 agent doing the same thing. This is due to the fact that the Type 1 agent has an advantage over a Type 2 agent in trading labour. There may or may not exist Y sellers who are Type 2 agents. In the case of coexistence of Type 1 and Type 2 Y sellers, the workers can only be Type 2 agents. Thus, $u_{1YL} > Max\{u_A, u_{1L}, u_{1YX}, u_{1X})$ and $u_{2L} = u_{2YL} >$ $Max\{u_A, u_{2X}, u_{2YX})$, from which follows $k < \frac{s_2 - b}{1-b}$, $k(s_2 - b) > KS$ and the equilibrium wage rate $w = w_2^* \equiv k^{\alpha-1}(s_2 - b)^\alpha(1-\alpha)^{1-\alpha}\alpha^\alpha$ and the equilibrium (shadow) price set is $(\frac{k}{s_2 - b}w_2^*, \frac{1}{1-b}w_2^*)$. A careful reader may be concerned with the fact that the labour market might not clear since that the population ratio between Type 1 and Type 2 is 1:1 and that all Type 1 agents choose to hire workers but only part of Type 2 agents choose to be workers. It is indeed problematic for a finite economy. But note that the existence of equilibrium usually requires the economy to be a large one (Sun, Yang and Zhou, 1999), and that the population ratio is not a problem for the equilibrium if the population of each type is a continuum, as is assumed in this example. In the case that there are only Type 1 Y sellers, both Type 1 and Type 2 agents might be workers. Hence, $u_{1YL} > Max\{u_A, u_{1YX}, u_{1X})$, $u_{1YL} \geq u_{1L}$ and $u_{2L} >$ $Max\{u_A, u_{2X}, u_{2YX}, u_{2YL})$, from which follows $k < \frac{s_1 - b}{1-b}$, $k(s_1 - b) > KS$ and the equilibrium wage rates are a continuum $(Max\{w_2*, u_A / k, k^{2\alpha-1}(1-b)^\alpha (1-\alpha)^{1-\alpha} \alpha^\alpha\}, w_1*]$ and the (shadow) price set (for any equilibrium wage rate w) is $(\frac{k}{s_1 - b} w, \frac{1}{1-b} w)$. In particular, when $w = w_1*$, some Type 1 agents might be workers while all Type 2 agents are workers. But for any equilibrium $w < w_1*$, only (all) Type 2 agents are workers. That there are infinitely many equilibrium wage rates and (shadow) prices is dependent on the assumption of continuum agents of both agents. If the population is finite, then the labour market clearing condition leads to a unique wage rate which is determined by the population ratio of the two types, provided that the integer problem (of employee numbers) that the Y seller faces does not result in the non-existence of equilibrium.

If both good X and labour (as well as Y) are traded in equilibrium, all Type 1 agents are Y sellers who hire workers to produce X. The Y sellers who buy X from the market, the workers, as well as X suppliers are all of Type 2: $u_{1YL} > Max\{u_A, u_{1YX}, u_{1X}, u_{1L}\}$, and $u_{2YX} = u_{2Y} = u_{2L} >$ $Max\{u_A, u_{2YL}\}$, from which follows $k > k*$, $\frac{s_2 - b}{1-b} < k < \frac{s_1 - b}{1-b}$, and the equilibrium price $p = p*$ and wage rate $w = (1-b)p*$. Note $w_2* < w < w_1*$. Should this example be interpreted as a two region (country) model with Type 1(2) agents from

Region 1(2) with different trading conditions in the factor markets, the workers from Region 2 (of the poorer factor market) enjoy a higher wage from the inter-regional(-national) division of labour than in the absence of inter-regional division of labour even if they work in the firms owned by Type 2 agents. Agents from Region 1 also benefit from a lower wage than if the wage was paid to workers from Region 1. Thus, this model, once elaborated, can be well developed into a story of endogenous emergence of multiple national firms and FDI.

Table 4 summarises the above analyses.

**Table 4. Parameter subspaces and equilibria for Example 4**

| | $k(s_1-b)<KS$ | $k(s_1-b)<KS$ |
|---|---|---|
| $k<k^*$ | Equilibrium prices and wage rates (both being shadow prices)<br>any $p\in(\alpha k(\frac{1-\alpha}{u_A})^{\frac{1}{\alpha}-1},\frac{u_A}{k(1-b)})$<br>any $w\in(\alpha(s_1-b)(\frac{1-\alpha}{u_A})^{\frac{1}{\alpha}-1},\frac{u_A}{k})$<br>Autarky (no trade) | Equilibrium wage rates: any<br>$w\in(Max\{w_2^*,u_A/k,k^{2\alpha-1}(1-b)^{\alpha}(1-\alpha)^{1-\alpha}\alpha^{\alpha}\},w_1^*]$<br>Shadow prices (for any equilibrium w)<br>$(\frac{k}{s_1-b}w,\frac{1}{1-b}w)$.<br>Labour and Y are traded (firms exist).<br>Type 1: firm owners and workers<br>(for $w=w_1^*$,. some firm owners some workers; for $w<w_1^*$, all firm owners)<br>Type 2: workers |
| $k>k^*$ | Equilibrium price<br>$p=p^*$<br>Shadow price of wage rate<br>any $w\in(\frac{s_1-b}{k}p^*,(1-b)p^*)$<br><br>X and Y are traded (no firms).<br>Agents of both types freely choose between X sellers and Y sellers, but how many agents from each type are X (or Y) sellers is indeterminate. | **When** $k>\frac{s_1-b}{1-b}$, $p=p^*$<br>(shadow) wages $(\frac{s_1-b}{k}p^*,(1-b)p^*)$<br>X and Y are traded (no firms).<br>Agents of both types freely choose between X sellers and Y sellers, but how many agents from each type are X (or Y) sellers is indeterminate.<br>**When** $\frac{s_2-b}{1-b}>k>\frac{s_1-b}{1-b}$,<br>$p=p^*$, $w=(1-b)p^*$ |

| | | |
|---|---|---|
| | | X, Y and labour are traded (firms exist).<br>Type 1: Firm owners (labour buyers, Y sellers)<br>Type 2: Y suppliers (no employment), X suppliers and workers.<br>**When** $k < \frac{s_2 - b}{1-b}$<br>Shadow prices $(\frac{k}{s_2 - b} w_2{*}, \frac{1}{1-b} w_2{*})$<br>wage rate $w = w_2{*}$<br>Labour and Y are traded (firms exist).<br>Type 1: firm owners.<br>Type 2: firm owners and workers. |

We may simplify Table 4 highlighting the structures of specialisation, as in Table 4'.

**Table 4'. A simplified Table 4.**

| | $k(s_1 - b) < KS$ | $k(s_1 - b) < KS$ |
|---|---|---|
| $k < k*$ | Shadow $p$ and shadow $w$<br><br>Autarky | Shadow $p$ (dependent on wage w).<br>A continuum of equilibrium $w$.<br>Division of labour<br>(with firms, no intermediate goods traded) |
| $k > k*$ | Equilibrium $p$ and shadow $w$<br>Division of labour (no firms). | **When** $k > \frac{s_1 - b}{1-b}$,<br>Equilibrium $p$ and shadow $w$<br>Division of labour (no firms).<br>**When** $\frac{s_2 - b}{1-b} > k > \frac{s_1 - b}{1-b}$,<br>Equilibrium $p$ and equilibrium $w$<br>Division of labour (firms exist, X traded).<br>**When** $k < \frac{s_2 - b}{1-b}$,<br>Shadow $p$ and equilibrium $w$<br>Division of labour (with firms). |

## 4. Concluding remarks

The new approach introduced in this paper is in some sense an elaboration on both the two-step approach and the benchmark price approach. The major breakthrough, however, is that the new approach starts by partitioning the price space and then goes to a corresponding partition of the parameter space. By doing so, the equilibrium computation and comparative statics is integrated together in a natural manner. As such, models involving possible substitution between markets across different structures of the division of labour, *ex ante* heterogeneous agents, and intra- and inter-firm division of labour can be rather easily solved, without overlooking any equilibrium price or structures.

While the major motivation for developing this analytical tool is mainly of technical interest, it may nonetheless help to clarify some theoretical issues, particularly those regarding the efficiency of the "invisible hand" price system in coordinating the division of labour, and the characterisation of equilibrium with firms. For instance, some may be concerned, after reading papers based on the two-step analyses, about how agents coordinate with each other to switch from one "bad" structure to a "good" one. The role played by the Walrasian price system in coordinating the decentralized decisions of many price-takers in our approach is much more transparent: agents simply don't bother, nor do they need to care about structures, as their decisions on labour-trade plans are guided only by the price signal.[4] In addition, the new approach may help to analyse some

[4] Note that the process of how a new market or/and the equilibrium price is created/discovered presumably by entrepreneurs goes beyond the scope of this paper. As is well known, it has long been one major concern of the Austrian School (see, for

theoretical issues in the new classical microeconomic framework such as the conditions under which the equilibrium is (locally) unique, comparative statics is monotonic, the general equilibrium with firms exists for a general class of models and so on. Some extensions to this paper may advance towards this direction.

**Appendix 1**

We establish in this appendix that in Example 2 no rational person would sell two or three "goods" among X, Y and labour. The increasing returns to specialisation in producing X immediately implies that no one sells X and labour simultaneously. Suppose someone sells both Y and labour (or X), then the MVP (marginal value product) of labour in producing Y equals the wage w (or marginal productivity in producing X in terms of good Y, namely, p). But the diminishing marginal productivity of labour in Y and the given w and p implies that it will pay better if she puts a bit less (more) labour into the production of Y (sold labour or production of X). Thus, no one sells more than one among X, Y and labour.

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example, Kirzner 1997). Also note the coordination in a non-traditional Walrian price system (say, with information asymmetry) is rather a *conceptually different* problem, which should be dealt with elsewhere as well.

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