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Based on Generalized Distributional forms**

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# PRICING AUSTRALIAN S&P200 OPTIONS : A BAYESIAN APPROACH BASED ON GENERALIZED DISTRIBUTIONAL FORMS<sup>1</sup>

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## Abstract

A new class of option price models is developed and applied to options on the Australian S&P200 Index. The class of models generalizes the traditional Black-Scholes framework by accommodating time-varying conditional volatility, skewness and excess kurtosis in the underlying returns process. An important property of the more general pricing models is that the computational requirements are practically the same as those associated with the Black-Scholes model, with both methods being based on one-dimensional integrals. Bayesian inferential methods are used to evaluate a range of models nested in the general framework, using observed market option prices. The evaluation is based on posterior distributions estimated for the parameters of the alternative models, as well as posterior model probabilities, out-of-sample predictive performance and implied volatility smiles. The empirical results provide strong evidence that time-varying volatility, leptokurtosis and skewness are priced in Australian stock market options.

*Keywords: Bayesian Option Pricing; Leptokurtosis; Skewness; Time-Varying Volatility; Option Price Prediction; Implied Volatility Smiles*

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# 1 Introduction

The Black and Scholes (1973) model for pricing options is founded on two important assumptions: first, that the distribution of returns on the asset on which the option is written is normal, and second, that the volatility of returns is constant. In practice, research on a variety of financial returns has shown that at least one, if not both, assumptions are usually violated. For reviews of the relevant literature see Bollerslev, Chou and Kroner (1992) and Pagan (1996). Associated with this misspecification of the Black-Scholes (BS) model is the occurrence of implied volatility smiles, or smirks, whereby the implied volatility backed out of observed option prices via the BS model varies across the degree of moneyness of the option contract. In Bakshi, Chao and Chen (1997), Corrado and Su (1997), Hafner and Herwartz (2001) and Lim, Martin and Martin (2002a and b), amongst others, these implied volatility patterns are linked directly to deviations in the underlying returns process from the BS specifications.

In this paper we develop a more general framework for pricing options that accommodates the empirical features of the underlying returns process, namely time-varying conditional volatility as well as conditional skewness and excess kurtosis. We use a combination of the distributional frameworks of Lye and Martin (1993, 1994) and Fernandez and Steele (1998), augmented with the time-varying volatility specification of Rosenberg and Engle (1997) and Rosenberg (1998). As in Lim, Martin and Martin (2002a and b) a distribution is specified for the return over the life of the option, with the option then priced by evaluating the expected payoff using simple univariate numerical quadrature. In this way, the computational burden is comparable to that associated with the BS price, which involves evaluation of a one-dimensional normal integral. It is also a viable alternative to the approaches that are based on the assumption of stochastic volatility and/or random jumps in returns, which produce closed form solutions for the option price only under the assumption of conditional normality and only up to a one-dimensional integral in the complex plane; see, for example Heston (1993), Bates (2000) and Pan (2002). Most notably, the approach has distinct computational advantages over the traditional Monte Carlo methods used to price non-BS options, which involve evaluating the expected payoff as an average over many simulation paths; see Hafner and Herwartz (2001), Bauwens and Lubrano (2002) and Martin, Forbes and Martin (2002), amongst others.

With the general distributional framework nesting a series of special cases, including the BS model itself, observed option prices are used to estimate the parameters of the alternative models. That is, ‘implicit’ estimation of the underlying returns models is conducted. This compares with the alternative method of estimating the models ‘directly’ using historical returns data and pricing options off the returns-based parameter estimates. Since observed option prices factor in risk premia, the implicit approach produces estimates of the parameters of the risk-neutral distributions. A Bayesian inferential approach is adopted, with both posterior parameter distributions and posterior model probabilities backed out from the option price data. The alternative models are ranked according to both the model probabilities and out-of-sample predictive performance. The extent to which the non-BS models eradicate the implied volatility smiles is also illustrated. For other recent applications of the Bayesian paradigm to option pricing, see Jacquier and Jarrow (2000), Eraker (2001), Forbes, Martin and Wright (2002), Martin, Forbes and Martin (2002) and Polson and Stroud (2002).

The methodology is applied to options written on the Australian S&P200 stock index, the dataset comprising intraday transactions data on all option trades from February 14th, 2001 until May 31st, 2002. The empirical results thus provide insight into the distributional assumptions which option market participants have factored into their pricing regarding returns on the Australian stock market.

The structure of the rest of the paper is as follows. In Section 2 the framework adopted for pricing options is introduced, with the set of alternative models for returns outlined. The simplicity of the evaluation method in the non-BS cases is highlighted. In Section 3 the Bayesian inferential approach is detailed, including the computational details associated with estimation of the marginal posteriors, model probabilities, predictive distributions and implied volatility smiles. The empirical application of the methodology is described in Section 4. The results provide strong evidence that the option market has factored in the assumption of non-constant volatility in returns, plus excess kurtosis in the conditional distribution. Negative skewness in returns is given some support by the data. There is also clear evidence that the non-BS models are more accurate predictors of future market prices and that the parameterization of higher order moments in returns does serve to reduce the extent of the volatility

smiles.

## 2 General Option Pricing Framework

An option is a derivative asset which gives one the right to either buy or sell one unit of the underlying asset at some time in the future, at a prespecified strike or exercise price,  $K$ . In this paper we focus on the European call option, which gives one the right to buy one unit of the underlying asset at price  $K$  when the option matures at time  $T$ . As such, the value of European call is a direct function of the price expected to prevail in the spot market for the asset at time  $T$ . Formally, for a non-dividend paying asset the option price,  $q$ , is the expected value of the discounted payoff of the option; see Hull (2000),

$$q = E_t [e^{-r\tau} \max(S_T - K, 0)], \quad (1)$$

where

$T$  is the time at which the option is to be exercised;

$\tau$  is the length of the option contract, expressed as a proportion of a year;

$K$  is the exercise or “strike” price;

$S_T$  is the spot price of the underlying asset at the time of maturity;

$r$  is the risk-free interest rate assumed to hold over the life of the option; and

$E_t[\cdot]$  is the conditional expectation, based on information at time  $t$ , taken with respect to the risk-neutral probability distribution for  $S_T$ .

From (1), the value of an option at time  $t$  is dependent on the known quantities  $r$ ,  $K$  and  $\tau$ , and the observed level of the spot price prevailing at time  $t$ ,  $S_t$ . The evaluation of the expectation in (1) is based on the risk-neutral probability distribution for  $S_T$ . This is the pertinent distribution to use in evaluating an option under the assumption of a replicating risk-free portfolio based on the option and the underlying asset; see Hull (2000). To make this explicit, rewrite equation (1) as

$$q = e^{-r\tau} \int_K^{\infty} (S_T - K)g(S_T|S_t)dS_T, \quad (2)$$

where the function  $g(S_T|S_t)$  is the risk-neutral probability density of the spot price at the time of maturity of the option, conditional on the current price  $S_t$ . In deriving the form of  $g(S_T|S_t)$ , the continuously compounded return over the life of the option,  $\ln(S_T/S_t)$ , is assumed to be generated according to

$$\ln(S_T/S_t) = (r - \frac{1}{2}\sigma_{T/t}^2)\tau + \sigma_{T/t}\sqrt{\tau}e_T, \quad (3)$$

where

$$e_T \sim (0, 1) \quad (4)$$

and  $\sigma_{T/t}$  is the annualized conditional volatility of the return. The specification of a mean rate of return equal to the risk free rate  $r$ , follows from the adoption of the risk-neutral probability distribution to evaluate the option.

The conditional volatility is based on the specification of Rosenberg and Engle (1997) and Rosenberg (1998), whereby volatility is a function of the net return over the life of the option,  $\ln(S_T/S_t)$ ,

$$\sigma_{T/t} = \exp(\delta_1 + \delta_2 \ln(S_T/S_t)). \quad (5)$$

This particular representation for  $\sigma_{T/t}$  renders the latter both a time-varying function, via the dependence on  $S_t$ , and a random function, via the dependence on  $S_T$ . The exponential form in (5) ensures that the conditional volatility is positive.

Given a particular distributional assumption for  $e_T$  in (4), the conditional density of  $S_T$  is given by

$$g(S_T|S_t) = |J| p(e_T), \quad (6)$$

where  $J$  is the Jacobian of the transformation from  $e_T$  to  $S_T$ , given by

$$\begin{aligned} J &= \frac{\partial e_T}{\partial S_T} \\ &= \frac{1}{S_T \sigma_{T/t} \sqrt{\tau}} \left[ 1 + \delta_2 \sigma_{T/t}^2 \tau - \delta_2 \left( \ln(S_T/S_t) - \left( r - \frac{\sigma_{T/t}^2}{2} \right) \tau \right) \right], \end{aligned}$$

and  $p(e_T)$  is the density of  $e_T$ . Several alternative distributional assumptions are adopted for  $e_T$ , implying, via (6), several alternative assumptions about the form of

$g(S_T|S_t)$  and, hence, about the value of the theoretical option price in (2). We define  $e_T$  to be the standardized version of a random variable  $w_T$  with mean  $\mu_w$  and variance  $\sigma_w^2$ ,

$$\omega_T = \sigma_w e_T + \mu_w.$$

Adopting the approach of Fernandez and Steel (1998) we then define the density of  $e_T$  as

$$p(e_T) = \frac{2}{\gamma + \frac{1}{\gamma}} \sigma_w \left\{ f\left(\frac{w_T}{\gamma}\right) I_{[0,\infty)}(w_T) + f(\gamma w_T) I_{(-\infty,0)}(w_T) \right\}, \quad (7)$$

where  $f(\cdot)$  is defined as a symmetric density function with a single mode at zero and  $I_A(w_T)$  denotes the indicator function for the set  $A$ . The mean and variance of  $w_T$  are defined respectively as

$$\mu_w = \left( \frac{\gamma^2 - 1/\gamma^2}{\gamma + 1/\gamma} \right) \int_0^\infty 2x f(x) dx$$

and

$$\sigma_w^2 = \left( \frac{\gamma^3 + 1/\gamma^3}{\gamma + 1/\gamma} \right) \left( \int_0^\infty 2x^2 f(x) dx \right) - \mu_w^2.$$

The parameter  $\gamma$  introduces skewness into the distribution of  $e_T$ , with  $\gamma > 1$  producing positive skewness,  $\gamma < 1$  negative skewness and  $\gamma = 1$  producing symmetry.

If  $f(\cdot)$  in (7) is defined as a density function with excess kurtosis, (7) produces a distribution for  $e_T$  with both leptokurtosis and skewness. By setting  $\gamma = 1$ , a symmetric leptokurtic distribution for  $e_T$  is retrieved. We adopt two leptokurtic specifications for  $f(\cdot)$ . First, defining a further random variable  $\eta_T$  with mean  $\mu_\eta$  and variance  $\sigma_\eta^2$ , where

$$\eta_T = \sigma_\eta e_T + \mu_\eta,$$

a Student t ( $ST$ ) density for  $e_T$  is defined as

$$f(e_T) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)} \sigma_\eta \left( 1 + \frac{(\sigma_\eta e_T + \mu_\eta)^2}{\nu} \right)^{-0.5(\nu+1)/2}, \quad (8)$$

where  $\mu_\eta = 0$  and  $\sigma_\eta = \sqrt{\frac{\nu}{\nu-2}}$ . Secondly, using the distributional family introduced in Lye and Martin (1993, 1994), a Generalized Student t ( $GST$ ) density for  $e_T$  is defined



Table 1:

Parameterization of Alternative Models Based on Equations (3) to (9)

$p(e_T)$	Constant volatility	Time-varying volatility
Normal	$M_1$ ( $\delta_1, \delta_2 = 0, \gamma = 1$ )	$M_7$ ( $\delta_1, \delta_2, \gamma = 1$ )
Skewed Normal	$M_2$ ( $\delta_1, \delta_2 = 0, \gamma$ )	$M_8$ ( $\delta_1, \delta_2, \gamma$ )
<i>GST</i>	$M_3$ ( $\delta_1, \delta_2 = 0, \gamma = 1, \nu$ )	$M_9$ ( $\delta_1, \delta_2, \gamma = 1, \nu$ )
Skewed <i>GST</i>	$M_4$ ( $\delta_1, \delta_2 = 0, \gamma, \nu$ )	$M_{10}$ ( $\delta_1, \delta_2, \gamma, \nu$ )
<i>ST</i>	$M_5$ ( $\delta_1, \delta_2 = 0, \gamma = 1, \nu$ )	$M_{11}$ ( $\delta_1, \delta_2, \gamma = 1, \nu$ )
Skewed <i>ST</i>	$M_6$ ( $\delta_1, \delta_2 = 0, \gamma, \nu$ )	$M_{12}$ ( $\delta_1, \delta_2, \gamma, \nu$ )

as

$$f(e_T) = k^* \sigma_\eta \left( 1 + \frac{(\sigma_\eta e_T + \mu_\eta)^2}{\nu} \right)^{-0.5(\nu+1)/2} \exp \{ -0.5(\sigma_\eta e_T + \mu_\eta)^2 \}, \quad (9)$$

where  $\mu_\eta = 0$  and both  $k^*$  and  $\sigma_\eta$  need to be computed numerically. The degree of kurtosis in (8), as measured by the fourth moment, is given by

$$E(e_T^4) = \frac{3(\nu - 2)}{(\nu - 4)}. \quad (10)$$

The degree of kurtosis in (9) has no closed form solution and needs to be computed numerically. However, the exponential component ensures more rapid decline in the tails and, as a consequence, less kurtosis for a given value of  $\nu$ , than that associated with (8). In particular, all moments of this distribution exist as a result of the exponential term in (9) dominating the Student t kernel; see Lim et. al. (1998).

In summary, the general framework nests 12 alternative models for the standardized variate  $e_T$ , denoted respectively by  $M_1$  to  $M_{12}$ . Each model, including its associated parameter set, is listed in Table 1. The models for which  $\delta_2$  in (5) is set to zero (the first panel of models in Table 1) are models for which constant volatility is imposed, but with skewness and/or leptokurtosis in returns accommodated. The models

for which  $\delta_2$  is estimated freely (listed in the second panel in Table 1) allow for both time-varying volatility and conditional skewness and leptokurtosis. The (conditional) distribution of  $e_T$  is defined by (7), with  $f(\cdot)$  therein specified respectively as normal, *GST* (equation (9)) and *ST* (equation (8)), corresponding to increasing degrees of allowable kurtosis. When  $\gamma = 1$ , the distribution of  $e_T$  is symmetric; otherwise it is skewed. Model  $M_1$  corresponds to the BS specifications, in which case  $g(S_T|S_t)$  is log-normal and the integral in (2) has the well-known solution for the BS option price,

$$q = S_t N(d_1) - K^{-r\tau} N(d_2),$$

where

$$\begin{aligned} d_1 &= \frac{\ln(S_t/K) + \left(r + \frac{\exp(2\delta_1)}{2}\right) \tau}{\exp(\delta_1)\sqrt{\tau}} \\ d_2 &= \frac{\ln(S_t/K) + \left(r - \frac{\exp(2\delta_1)}{2}\right) \tau}{\exp(\delta_1)\sqrt{\tau}}, \end{aligned} \tag{11}$$

with  $\exp(\delta_1)$  denoting the constant BS volatility parameter and  $N(\cdot)$  the cumulative normal distribution. For all other specifications, the theoretical price in (2) is calculated by evaluating the relevant integral using one-dimensional numerical quadrature.

### 3 Bayesian Inference in an Option Pricing Framework

#### 3.1 Posterior Density Functions

Assuming that the theoretical option pricing function (2) is an unbiased representation of the true data generation process, then we can specify a model for the  $i^{th}$  observed option price,  $C_i$ , as

$$C_i = q_i(z_i, \theta_k) + u_i, \quad i = 1, \dots, N, \tag{12}$$

where  $q_i(\cdot, \cdot)$  is the  $i^{th}$  theoretical price function as per (2),  $z_i = (r_i, K_i, \tau_i, S_i)'$  represents the known set of factors needed to calculate  $q_i(\cdot, \cdot)$ ,  $u_i$  is the random pricing error, and  $\theta_k$  is the vector of unknown parameters which characterize the  $k^{th}$  model for the underlying returns distribution,  $M_k$ ,  $k = 1, 2, \dots, 12$ . The index  $i$  represents

variation over time as well as variation across different option contracts at any given point in time, with  $N$  denoting the number of option prices in the sample.

The inclusion of a stochastic error term in (12) serves as recognition of the fact that option pricing models are only approximations of the true underlying process driving observed prices. The inevitable pricing error derives at least in part from “model” error; that is, the model being used to calculate the theoretical price is incorrect either in its specification or in the values assumed for its parameters. It may also arise via the non-synchronous recording of spot and option prices, transaction costs and other market frictions. The possibility of a systemic component in the pricing error can be accommodated by extending (12) to include a “regression” component. In its simplest form this implies

$$C_i = \beta_1 + \beta_2 q_i(z_i, \theta_k) + u_i. \quad (13)$$

The model in (13) is the one that we adopt in the empirical section, along with the following distributional assumption for  $u_i$ ,

$$u_i \sim iidN(0, \sigma_u^2). \quad (14)$$

As demonstrated in Bates (2000), Eraker (2001) and Forbes, Martin and Martin (2002), more general specifications are possible for both (13) and (14). Strictly speaking  $u_i$  should be truncated from below according to the no-arbitrage lower bound for  $C_i$ ; see Hull (2000). However, such truncation adds to the computational burden, whilst having only a negligible impact on the parameter estimates. Instead the data is simply filtered according to the lower bound, as well as the lower bound being imposed when producing predictive densities for out-of-sample prices.

The distributional assumptions in (13) and (14) define the likelihood function,

$$\ell(\theta_k, \beta, \sigma_u | c) \propto \sigma_u^{-N} \exp \left\{ -\frac{1}{2} \sum_{i=1}^N \left( \frac{C_i - x_i'(\theta_k)\beta}{\sigma_u} \right)^2 \right\}, \quad (15)$$

where

$$\beta = (\beta_1, \beta_2)',$$

$$x_i(\theta_k) = (1, q_i(z_i, \theta_k))'$$

and  $c = (C_1, C_2, \dots, C_N)'$  is the  $(N \times 1)$  vector of observed option prices. In order to simplify the notation, we emphasize the dependence of  $x_i(\theta_k)$  on the unknown

parameter vector  $\theta_k$ , whilst not making explicit its dependence on the known factors in  $z_i$ . We also choose not to make explicit the dependence of the likelihood function on  $z_i$ ,  $i = 1, 2, \dots, N$ . Defining the full set of unknown parameters for model  $M_k$  as

$$\Phi_k = \{\beta, \sigma_u, \theta_k\}, \quad (16)$$

and assuming a-priori independence between  $\beta$ ,  $\sigma_u$  and  $\theta_k$  respectively, we define a prior for  $\Phi_k$  of the form

$$p(\Phi_k) \propto \frac{1}{\sigma_u} \times p(\theta_k). \quad (17)$$

Details of  $p(\theta_k)$  are provided in the empirical section. The prior on  $\{\beta, \sigma_u\}$  is the standard noninformative prior for the location and scale parameters in a regression model.

Given (15) and (17), the joint posterior density for  $\Phi_k$  is given by

$$p(\Phi_k|c) \propto \sigma_u^{-(N+1)} \exp \left\{ -\frac{1}{2} \sum_{i=1}^N \left( \frac{C_i - x'_i(\theta_k)\beta}{\sigma_u} \right)^2 \right\} \times p(\theta_k). \quad (18)$$

The parameters  $\{\beta, \sigma_u\}$  can be integrated out of  $p(\Phi_k|c)$  using standard analytical results, resulting in a marginal posterior for  $\theta_k$  of the form,

$$p(\theta_k|c) \propto \hat{s}(\theta_k)^{-(N-2)} |X(\theta_k)'X(\theta_k)|^{-1/2} \times p(\theta_k), \quad (19)$$

where

$$\hat{s}(\theta_k)^2 = \frac{(C_i - x'_i(\theta_k)\hat{\beta})'(C_i - x'_i(\theta_k)\hat{\beta})}{N - 2},$$

$$\hat{\beta} = (X(\theta_k)'X(\theta_k))^{-1}X(\theta_k)'c$$

and

$$X(\theta_k) = (1, Q(\theta_k)),$$

with  $Q(\theta_k) = (q_1(z_1, \theta_k), q_2(z_2, \theta_k), \dots, q_N(z_N, \theta_k))'$ . The marginals for  $\beta$  and  $\sigma_u$  follow as

$$\begin{aligned} p(\beta|c) &= \int p(\beta|\theta_k, c)p(\theta_k|c)d\theta_k \\ &\propto \int_{\theta_k} \left( 1 + \frac{1}{N-2} \frac{(\beta - \hat{\beta})'X(\theta_k)'X(\theta_k)(\beta - \hat{\beta})}{\hat{s}^2(\theta_k)} \right)^{-N/2} p(\theta_k|c)d\theta_k \quad (20) \end{aligned}$$

and

$$\begin{aligned} p(\sigma_u|c) &= \int_{\theta_k} p(\sigma_u|\theta_k, c)p(\theta_k|c)d\theta_k \\ &\propto \int (\sigma_u)^{-(N-1)} \exp\left(-\frac{\hat{s}^2(\theta_k)}{2\sigma_u^2}\right) p(\theta_k|c)d\theta_k. \end{aligned} \quad (21)$$

As  $\theta_k$  is of low dimension for all models considered, the marginal posterior densities for the individual elements of  $\theta_k$  are produced by applying deterministic numerical integration methods to the density in (19).

### 3.2 Posterior Model Probabilities

To determine the model that is most probable given the information in the option prices, we construct implicit model probabilities for each of the models  $M_1, M_2, \dots, M_{12}$  and rank them in order of highest to lowest value. The model probabilities are constructed via the estimation of posterior odds ratios for the models  $M_2, \dots, M_{12}$ , relative to reference model  $M_1$ . The posterior odds ratio for model  $M_k$  relative to  $M_1$ ,  $PO_{k,1}$ , is given by the product of the *prior* odds ratio,  $P(M_k)/P(M_1)$ , and the Bayes Factor,  $p(c|M_k)/p(c|M_1)$ ,

$$PO_{k,1} = \frac{P(M_k|c)}{P(M_1|c)} = \frac{P(M_k)}{P(M_1)} \times \frac{p(c|M_k)}{p(c|M_1)}, \quad k = 2, \dots, 12, \quad (22)$$

where  $P(M_k|c)$  is the posterior probability of model  $M_k$ , and

$$\begin{aligned} p(c|M_k) &= \int_{\Phi_k} p(c|\Phi_k, M_k)p(\Phi_k|M_k)d\Phi_k \\ &= \int_{\Phi_k} \ell(\Phi_k|M_k)p(\Phi_k|M_k)d\Phi_k \end{aligned} \quad (23)$$

is the marginal likelihood of model  $M_k$ . With  $\beta$  and  $\sigma_u$  able to be integrated out from (18) analytically, it follows that

$$p(c|M_k) = h \int_{\theta_k} \ell(\theta_k|M_k)p(\theta_k|M_k)d\theta_k, \quad (24)$$

where  $h$  is independent of  $M_k$ . Note that the integral in (24) is the integrating constant required for the normalization of (19) above. Hence, computation of the marginal likelihood for each model requires no computation in addition to that required to produce the marginal posterior densities for each  $\theta_k$ . (*cf.* Chib, 1995). The posterior model probability for model  $k$  then follows as

$$P(M_k|c) = \frac{p(c|M_k)P(M_k)}{\sum_{j=1}^{12} p(c|M_j)P(M_j)} \equiv \frac{PO_{k,1}}{\sum_{j=1}^{12} PO_{j,1}} \quad \forall k. \quad (25)$$

### 3.3 Predictive Density Functions

Whilst the posterior model probabilities are a measure of the performance of the alternative models within-sample, the relative out-of-sample performance of the models can be assessed via the ability of each model to predict future option prices accurately. Having specified a returns model  $M_k$  indexed on  $\theta_k$ , the predictive density for an option price  $C_f$  associated with some future time period  $f$ , given model  $M_k$  and the data, is

$$\begin{aligned} p(C_f|c) &= \int_{\sigma_u} \int_{\beta} \int_{\theta_k} p(C_f, \beta, \sigma_u, \theta_k|c) d\theta_k d\beta d\sigma_u \\ &= \int_{\theta_k} p(C_f|\theta_k, c) p(\theta_k|c) d\theta_k, \end{aligned} \quad (26)$$

where  $p(C_f|\theta_k, c)$  is univariate Student  $t$  with

$$E(C_f|\theta_k, c) = x_f(\theta_k)' \hat{\beta},$$

$$var(C_f|\theta_k, c) = \hat{\sigma}_u^2 [1 + x_f(\theta_k)' (X(\theta_k)' X(\theta_k))^{-1} x_f(\theta_k)]$$

and  $x_f(\theta_k) = (1, q_f(z_f, \theta_k))'$ . The vector  $z_f = (r_f, K_f, \tau_f, S_f)$  encompasses the known set of factors needed to calculate  $q_f(\cdot, \cdot)$ . The predictive density in (26) can be estimated by computing a weighted average of the Student  $t$  conditional densities for  $C_f$ ,  $p(C_f|\theta_k, c)$ , with the weights being the probability “mass” assigned to each grid-point in the numerically evaluated posterior,  $p(\theta_k|c) d\theta_k$ . That is, the predictive is estimated as

$$\hat{p}(C_f|c) = w \sum_{j=1}^{N_{\theta_k}} p(C_f|\theta_k^{(j)}, c) p(\theta_k^{(j)}|c), \quad (27)$$

where  $N_{\theta_k}$  is the number of gridpoints used in defining the density  $p(\theta_k|c)$ ,  $w$  is the grid width and  $p(\theta_k^{(j)}|c)$  denotes the ordinate of  $p(\theta_k|c)$  at grid value  $\theta_k^{(j)}$ . Prior to computing the weighted sum in (27),  $p(C_f|\theta_k^{(j)}, c)$  is truncated at the no-arbitrage lower bound,

$$lb_f = \max\{0, S_f - e^{-r_f \tau_f} K_f\}, \quad (28)$$

and renormalized; see Hull (2000).

### 3.4 Implied Volatility Smiles

With the “moneyness” of the  $i$ th option contract denoted by  $S_i/K_i$ , the existence of a “smile”, or “smirk” in the plot of implied volatilities against moneyness is generally taken to indicate some degree of misspecification of the option pricing model, given that volatility is a feature of the underlying asset returns and *not* a function of the degree of moneyness of an option written on that asset. Accordingly, yet another criterion for testing a model’s ability to characterize option prices is the relative flatness of its implied volatility graph. That is, the volatility *implied* by each model is calculated by setting the theoretical price equal to the observed price, then solving for the volatility parameter. In the case of the Black-Scholes model this is straightforward. All other models have additional distributional parameters whose values are most conveniently set equal to their estimated marginal posterior modes. The model deemed to best characterize option price behaviour is the one that generates the flattest curve of implied volatility against moneyness.

## 4 Pricing Options on the S&P200

### 4.1 Data

The methodology outlined above is applied to the intraday transaction prices of call options written on the Australian S&P200 Index, observed over the period February 14th, 2001 to May 31st, 2002. Options on the Australian S&P200 Index are European style options, expiring at 3-monthly intervals. Settlement at exercise is in cash. The S&P200 Index represents the price of a market portfolio comprising 200 of the largest companies trading on the Australian Stock Exchange (ASX). As of August 31st, 2000, the Index comprised approximately 89% of the total market capitalization of the Australian stock market.

Each data record includes the contract date and time, the option price ( $C_i$ ), exercise price ( $K_i$ ) and the expiry date ( $\tau_i$ ), but not the value of the underlying Index at the time the contract was written. The latter information is extracted from data on the S&P200 Index itself, recorded at one minute intervals up to January 18th, 2002, and at approximately 30 second intervals thereafter. Each option trade is matched with an Index value by selecting the most closely synchronized recorded value. This

Index value we denote as the “spot” price,  $S_i$ . As the Index pays dividends, the current Index associated with each observational point,  $S_i$ , is replaced by the discounted Index,  $S_i e^{-d\tau_i}$ , where  $d = 0.033$  is the average (annualized) dividend rate on the Index over 2001; see Hull (2000).

After excluding option contracts with very long times to maturity (so-called LEPO’s), several observations which appear to be in error, and those prices that do not satisfy the arbitrage restriction (28), the final dataset comprises 5471 trades. Time to maturity,  $\tau_i$ , ranges from  $1/365$  to  $281/365$  across the sample of observations. The moneyness ( $S_i/K_i$ ) of the contracts in the sample ranges from 0.81 to 1.18, with the exception of a handful of trades with moneyness in excess of 1.18. The risk-free rate,  $r_i$ , prevailing at the time of the contract is deemed to be that day’s interest charged on 90-day Bank Accepted Bills. To assess the out-of-sample performance of the models we use data for the last 8 days, that is, trades occurring between the 22nd and the 31st of May, 2002 inclusive. This division of the final dataset into estimation and validation subsample leaves 5356 in-sample and 115 out-of-sample observations. All option price, Index and dividend data has been obtained from the ASX. Data on interest rates has been extracted from the Reserve Bank of Australia Bulletin.

Table 2 provides a summary of the main characteristics of the option price data, with observations divided into three categories according to their level of moneyness, as defined by Bakshi, Cao and Chen (1997). Specifically, an option is defined as *at-the-money* (ATM) if  $S_i/K_i \in (0.97, 1.03)$ , *out-of-the-money* (OTM) if  $S_i/K_i < 0.97$ , and *in-the-money* (ITM) if  $S_i/K_i > 1.03$ . We note that the number of in-the-money trades comprises less than 8% of the total overall; with there being none at all in our validation period. In fact there are *no* in-the-money options traded after May 9th, 2002 that are not excluded by filtering according to the lower bound in (28).

## 4.2 Priors

As noted in (17) above, the prior on the full set of unknowns for model  $M_k$  is defined as proportional to the product of the standard noninformative prior for  $(\beta, \sigma_u)$  and the prior on the remaining model parameters,  $p(\theta_k)$ . We assume *a priori* independence between all the elements of  $\theta_k$ , as well as specifying noninformative priors for  $\delta_1$  and  $\delta_2$ . That is, we assume a uniform prior for  $\delta_1$ , which implies a prior for  $\exp(\delta_1)$  in



Table 2:  
Summary of S&P200 Option Price Data

No. of Trades				
Moneyness ( $S_i/K_i$ )	Estimation period	Validation period	Overall	
OTM ( $< 0.97$ )	2002	29	2031	
ATM ( $0.97 - 1.03$ )	2943	86	3029	
ITM ( $> 1.03$ )	411	0	411	
Total	5356	115	5471	
Average Market Price (cents/share)				
Moneyness ( $S_i/K_i$ )	Estimation period	Validation period	Overall	
OTM ( $< 0.97$ )	36.77	42.97	36.86	
ATM ( $0.97 - 1.03$ )	63.00	46.21	62.52	
ITM ( $> 1.03$ )	234.30	-	234.30	
Total	66.34	45.39	65.90	

the constant volatility models ( $\delta_2 = 0$ ) proportional to  $1/\exp(\delta_1)$ . We also specify a uniform prior for  $\delta_2$ . In a strict sense, the latter is an invalid prior for use in the posterior odds ratio in (22) when  $M_k$  corresponds to any of the models for which  $\delta_2 \neq 0$ , since the marginal likelihood calculation in (24) has an arbitrariness associated with it, depending on the range over which  $\delta_2$  is integrated. However, in practice, with a sample size as large as that used in the empirical application, it is straight forward to determine a range of integration over which the likelihood function has virtually all mass, thereby eliminating the arbitrary aspect of the calculation in (24).

The prior for the degrees of freedom parameter (if present) is taken to be exponential with a hazard rate  $\lambda$  of 10%; that is,

$$p(\nu) = \lambda e^{-\lambda\nu}; \quad \lambda = 0.1.$$

This implies a prior mean equal to 10 and prior variance equal to 100. This essentially produces a prior distribution that is half-way between one having normal tails and one having fat tails (see Fernandez and Steel, 1998). For the skewness parameter we again follow Fernandez and Steel by specifying a gamma prior on  $\gamma^2$  with values of the scale and shape parameters which imply that  $E(\gamma) = 1$ ,  $var(\gamma) = 0.57$ , and  $P(\gamma < 1) = 0.58$ .

## 4.3 Empirical Results

### 4.3.1 Posterior Parameter Estimates

The joint posterior density for the parameters of each of the option pricing models is estimated by setting up a grid of parameter values, and evaluating the kernel of the posterior at each gridpoint. In order to avoid numerical overflow problems, we compute the log-kernel at each of the gridpoints, from which the largest ordinate value is subtracted. When exponentiated, these scaled posterior ordinates are then rescaled using numerical quadrature so as to produce a normalized joint density function, as described in Section 3.1. Each marginal posterior is produced by further applications of numerical integration.

Summary measures of the marginal posterior densities for the constant volatility ( $\delta_2 = 0$ ) and time-varying volatility ( $\delta_2 \neq 0$ ) models respectively are reported in Tables 3 and 4. The measures comprise marginal posterior means and modes, plus

95% Highest Posterior Density (HPD) intervals. Considering the results in Table 4, it is clear that the option prices have factored in the assumption of time varying volatility, with both point and interval estimates of  $\delta_2$  indicating a non-zero value for this parameter. The point estimates of  $\delta_1$  indicate an estimate of volatility at the time of maturity (at which point  $S_t = S_T$ ) of approximately 12% in annualized terms, which tallies closely with the point estimates of  $\exp(\delta_1)$  in the constant volatility models reported in Table 3. All but one of the point estimates of  $\gamma$  in Tables 3 and 4 indicate that a small amount of negative skewness in returns has been factored into the option prices. The negative skewness is further confirmed by the interval estimates of  $\gamma$  for  $M_6$ ,  $M_{10}$  and  $M_{12}$ , all of which cover values less than unity. The interval estimates of  $\gamma$  for  $M_2$ ,  $M_4$  and  $M_8$  suggest some uncertainty as to the existence of skewness, covering as they do values for  $\gamma$  which are associated with symmetry ( $\gamma = 1$ ), negative skewness ( $\gamma < 1$ ) and positive skewness ( $\gamma > 1$ ).

Most notably, the results in both tables provide clear evidence of option-implied excess kurtosis, both for the conditional returns distributions associated with the models in Table 4 and for the unconditional distributions associated with the constant volatility models in Table 3. For the *ST* models, in the bottom panel of each of the two tables, the point estimates of the degrees of freedom parameter  $\nu$  range from 4.48 ( $M_6$ ) to 7.28 ( $M_{11}$ ), indicating kurtosis, as measured by (10), ranging from 15.50 to 4.83. For the *GST* models, in the middle panel of each table, the corresponding point estimates of  $\nu$  range from 0.66 ( $M_4$ ) to 1.60 ( $M_9$ ), implying a range of kurtosis estimates of 3.75 to 3.49. The smaller degree of kurtosis associated with the *GST* models is a direct reflection of the fact that the *GST* specification restricts the fatness allowable in the tails of the returns distribution, relative to that allowed by the *ST* specification. Despite the smaller degree of excess kurtosis however, the interval estimates of  $\nu$  in all of the *GST* models indicate values of kurtosis which represent a departure from the kurtosis of 3 associated with the normal distribution. For the time-varying volatility models which allow for excess kurtosis, namely  $M_9$ ,  $M_{10}$ ,  $M_{11}$  and  $M_{12}$ , the kurtosis estimated is for the *conditional* distribution of  $e_T$ . Hence, we would anticipate the smaller degree of kurtosis which is estimated for these models in comparison with the kurtosis estimated for the corresponding constant volatility models,  $M_3$ ,  $M_4$ ,  $M_5$  and  $M_6$ , since the time-varying volatility specification itself is

expected to capture some of the kurtosis in the data.

Finally, the estimates of the constant volatility parameter  $\exp(\delta_1)$  range from a low of 11.92% for the BS model ( $M_1$ ) to a high of 13.47% for the skewed  $ST$  model ( $M_6$ ). The interval estimates of  $\exp(\delta_1)$  for all models other than  $M_6$  are quite narrow, being less than one percentage point in width. The interval for  $M_6$  assigns 95% probability to  $\exp(\delta_1)$  lying between 12.63% and 14.43%.

Table 3:  
Marginal Posterior Means and Modes,  
Plus 95% HPD Intervals:  
Constant Volatility Models ( $\delta_2 = 0$ ).

Model	$p(e_T)$	Parameter	Marginal Mode	Marginal Mean	95% HPD Interval
$M_1$	Normal	$\exp(\delta_1)$	0.1192	0.1192	(0.1171, 0.1213)
$M_2$	Skewed Normal	$\exp(\delta_1)$ $\gamma$	0.1192 0.9980	0.1192 1.0001	(0.1170, 0.1214) (0.9474, 1.0544)
$M_3$	$GST$	$\exp(\delta_1)$ $\nu$	0.1242 0.7581	0.1242 0.9480	(0.1214, 0.1270) (0.4292, 1.6428)
$M_4$	Skewed $GST$	$\exp(\delta_1)$ $\gamma$ $\nu$	0.1252 0.9656 0.6606	0.1253 0.9669 0.8094	(0.1220, 0.1287) (0.9225, 1.0125) (0.3691, 1.3926)
$M_5$	$ST$	$\exp(\delta_1)$ $\nu$	0.1279 5.9016	0.1281 6.5097	(0.1235, 0.1330) (4.5203, 8.9377)
$M_6$	Skewed $ST$	$\exp(\delta_1)$ $\gamma$ $\nu$	0.1323 0.9433 4.4794	0.1347 0.9412 4.7916	(0.1263, 0.1443) (0.8883, 0.9921) (3.1079, 6.8236)

In Figures 1 and 2 the marginal posterior densities for the two most highly para-

Table 4:  
Marginal Posterior Means and Modes,  
Plus 95% HPD Intervals:  
Time-Varying Volatility Models.

Model	$p(e_T)$	Parameter	Marginal Mode	Marginal Mean	95% HPD Interval
$M_7$	Normal	$\delta_1$	-2.1717	-2.1718	(-2.1898, -2.1539)
		$\delta_2$	0.3826	0.3681	(0.3267, 0.3975)
$M_8$	Skewed Normal	$\delta_1$	-2.1682	-2.1684	(-2.1871, -2.1499)
		$\delta_2$	0.3901	0.3786	(0.3342, 0.4116)
		$\gamma$	0.9550	0.9551	(0.9052, 1.0061)
$M_9$	$GST$	$\delta_1$	-2.1376	-2.1374	(-2.1613, -2.1135)
		$\delta_2$	0.3367	0.3187	(0.2608, 0.3589)
		$\nu$	1.1256	1.6032	(0.5892, 3.1652)
$M_{10}$	Skewed $GST$	$\delta_1$	-2.1228	-2.1221	(-2.1487, -2.0953)
		$\delta_2$	0.3453	0.3306	(0.2810, 0.3666)
		$\gamma$	0.9359	0.9379	(0.8936, 0.9832)
		$\nu$	0.8493	1.1380	(0.4493, 2.1173)
$M_{11}$	$ST$	$\delta_1$	-2.1259	-2.1245	(-2.1526, -2.0963)
		$\delta_2$	0.2552	0.2550	(0.2208, 0.2906)
		$\nu$	6.8264	7.2824	(5.5055, 9.3590)
$M_{12}$	Skewed $ST$	$\delta_1$	-2.0917	-2.0908	(-2.1224, -2.0584)
		$\delta_2$	0.2416	0.2434	(0.2153, 0.2724)
		$\gamma$	0.8629	0.8667	(0.8287, 0.9057)
		$\nu$	5.1635	5.3205	(4.3890, 6.3629)

meterized models,  $M_{10}$  and  $M_{12}$ , are presented. Both models allow for time-varying volatility, plus conditional skewness and leptokurtosis,  $M_{10}$  adopting the *GST* specification to accommodate excess kurtosis and  $M_{12}$  the *ST* specification. The marginals for  $\nu$  in both models exhibit positive skewness, in particular in the case of the *GST*-based model. Most notably, the smaller degree of excess kurtosis able to be captured by  $\nu$  in the *GST*-based model  $M_{10}$ , is associated with a posterior distribution for the time-varying volatility parameter  $\delta_2$  which is shifted to the right relative to the corresponding distribution for  $\delta_2$  in the *ST*-based distribution  $M_{12}$ . That is, the volatility specification assumes a larger role in capturing excess kurtosis in the data when the conditional distribution of  $e_T$  has more restricted tail behaviour. A similar relationship holds between the parameter estimates of the non-skewed *GST* and *ST* distributions with time-varying volatility ( $M_9$  and  $M_{11}$  respectively). The marginals for the skewness parameter  $\gamma$  are quite symmetric in both Figures 1 and 2, with virtually all probability mass concentrated in the region associated with negative skewness in both cases.

#### 4.3.2 Posterior Model Probabilities

The (log-) Bayes Factors and model probabilities reported in Table 5 substantiate the estimation results presented in Tables 3 and 4. The models which dominate are clearly those which allow for time-varying volatility, with the models which augment the volatility specification in (5) with a conditional *ST* distribution ( $M_{11}$  and  $M_{12}$ ), having the highest posterior probability within the time-varying volatility class. The *GST*-based conditional distribution models ( $M_9$  and  $M_{10}$ ) have the next highest posterior probability weights, followed by the conditionally normal model ( $M_7$ ) and the skewed normal conditional model ( $M_8$ ). All constant volatility models have substantially lower posterior probability weight than the corresponding time-varying volatility models, with the ranking within the constant volatility class being identical to that with the former class, namely: *ST* models first, followed by *GST*, normal then skewed normal. It is notable that, apart from  $M_{12}$ , no “skewed” model has a higher Bayes Factor than its symmetric counterpart, further emphasizing the fact that although the option price data has factored in some negative skewness, the departure from symmetry is not particularly marked.

When the Bayes Factors are transformed into posterior probabilities according to

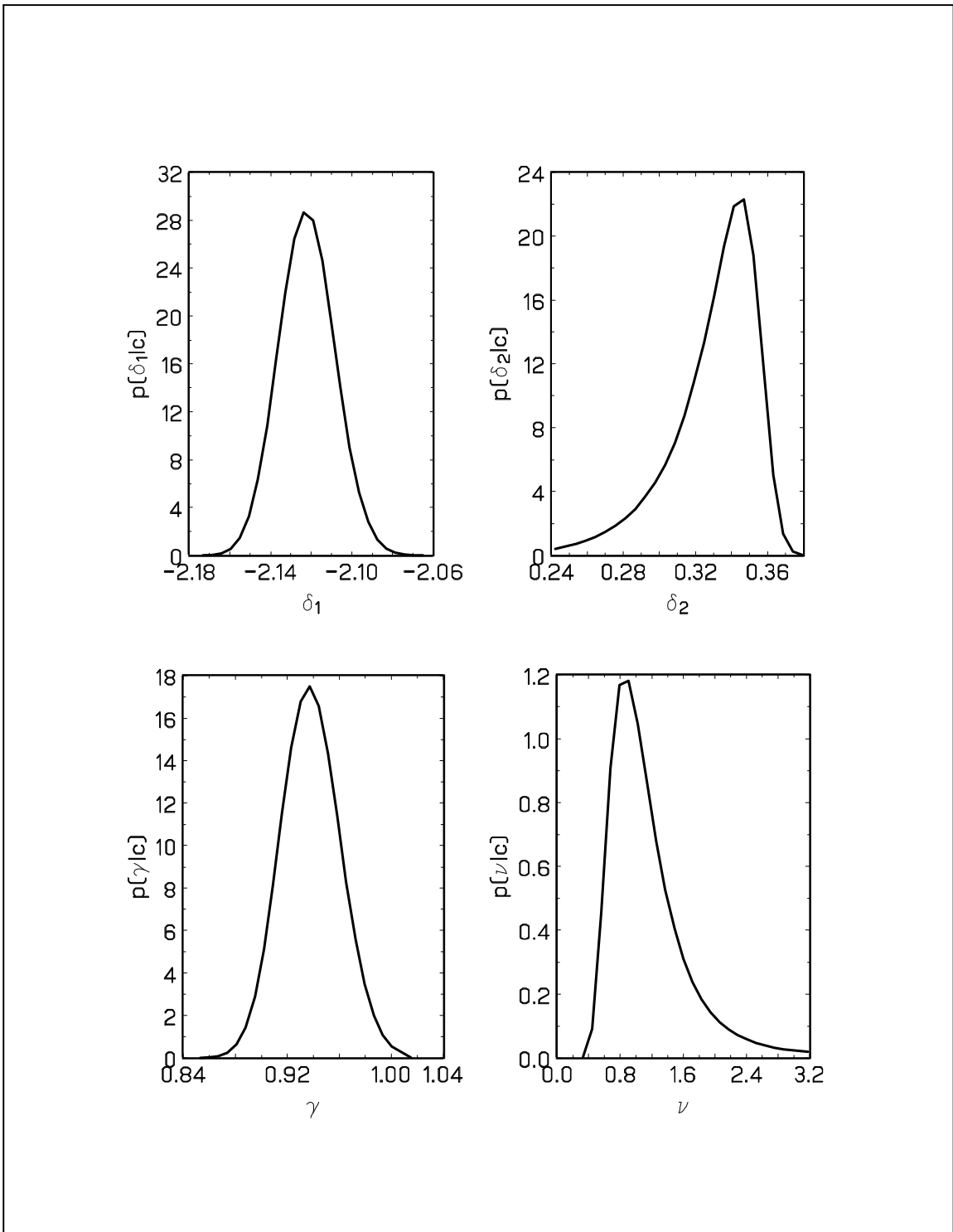


Figure 1: Marginal Posterior Densities for  $M_{10}$  (Skewed  $GST$  with Time-Varying Volatility)

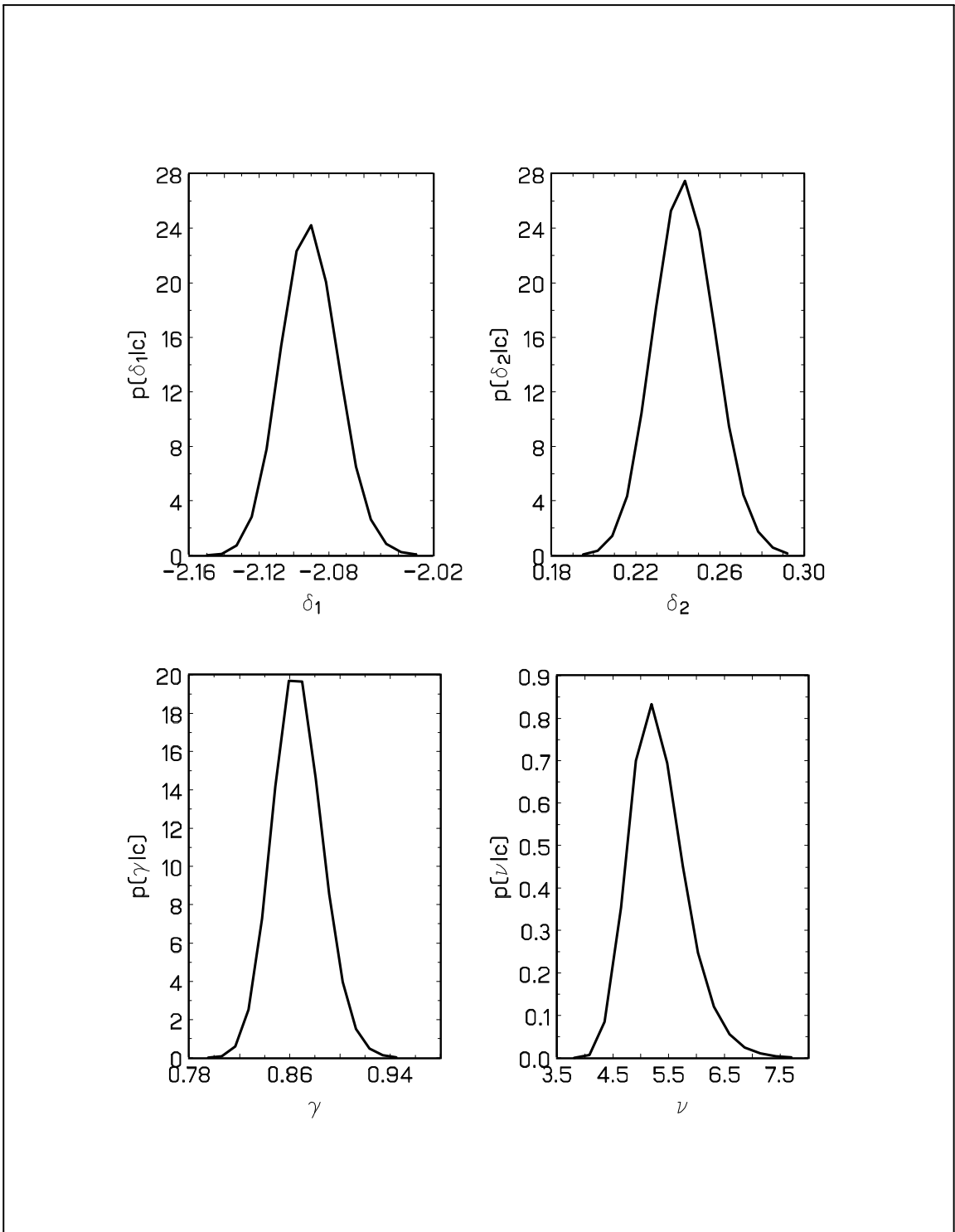


Figure 2: Marginal Posterior Densities for  $M_{12}$  (Skewed  $ST$  with Time-Varying Volatility)



Table 5:

(Log-) Bayes Factors ( $BF$ ) and Model Probabilities

$M_1$  (BS) as Reference Model; Equal Prior Probabilities for all Models

Constant Volatility Models						
	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$	$M_6$
$\log(BF)$	0.00	-3.45	11.48	8.88	13.74	12.78
$P(M_k c)$	4.53E-18	1.44E-19	4.39E-13	3.25E-14	4.18E-12	1.60E-12
Time-Varying Volatility Models						
	$M_7$	$M_8$	$M_9$	$M_{10}$	$M_{11}$	$M_{12}$
$\log(BF)$	16.56	14.56	22.40	22.12	27.86	39.94
$P(M_k c)$	7.07E-11	9.54E-12	2.40E-08	1.83E-08	5.71E-06	1.00

(25),  $M_{12}$  is essentially assigned *all* posterior weight in the entire set of 12 models.

### 4.3.3 Predictive Performance

The performance measures applied to assess the predictive performance of the competing models are the proportion of market prices contained within the interquartile interval (IQI) of the derived predictive density, as well as the proportion of market prices contained within the 95% interval (95I) which assigns 2.5% probability to each tail of the predictive. These proportions are reported in Table 6, again with the models divided into the constant volatility and time-varying volatility categories respectively. Each figure is a proportion of the total number of 115 out-of-sample prices. The IQI coverage statistics in Table 6 tend to confirm the model rankings based on the implicit posterior probabilities. The time-varying volatility models perform best overall, although their superiority over the constant volatility models is not absolute. The *ST/GST*-based models tend to out-perform the models which do not allow for excess kurtosis, whilst the addition of skewness to the *ST/GST* specifications causes a deterioration in predictive performance in 3 out of 4 cases. There is nothing to choose between the coverage performance of the normal and skewed-normal models. Using the 95I coverage as a predictive criterion, all models perform equally well, with 100% coverage in all cases!

### 4.3.4 Implied Volatility Smiles

For each model, implied volatilities are backed out from all option prices in the estimation sample associated with one particular time to maturity, namely 7 days. The value which is produced for implied volatility corresponds to  $exp(\delta_1)$ , with  $\delta_2$ ,  $\nu$  and  $\gamma$  set to their respective marginal posterior modes in the case of all models other than  $M_1$ . A quadratic function in the inverse of moneyness (denoted by  $K/S$ ) is then fitted to the implied volatility data for each model. In each graph in Figure 3 the fitted curve associated with the BS model ( $M_1$ ) is reproduced, with the curve associated with various of the other models superimposed. In this way, the impact on the shape of the implied volatility curve of modelling different distributional features, can be ascertained. If a model is adequate in capturing the distributional features implicit in the option price data, the smoothed graph of implied volatilities should be reasonably constant with respect to  $K/S$ . On the other hand, a pattern across

Table 6:

Proportions of Out-of-Sample Prices Contained in the IQI and 95I Predictive Intervals

% coverage	Constant Volatility Models					
	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$	$M_6$
IQI	0.513	0.513	0.565	0.557	0.583	0.591
95I	1.000	1.000	1.000	1.000	1.000	1.000
Time-Varying Volatility Models						
	$M_7$	$M_8$	$M_9$	$M_{10}$	$M_{11}$	$M_{12}$
IQI	0.548	0.548	0.565	0.557	0.591	0.539
95I	1.000	1.000	1.000	1.000	1.000	1.000

$K/S$  suggests that option prices have factored in distributional assumptions which are not adequately captured by the model in question; for more details see Hull (2000), Hafner and Herwartz (2001) and Lim, Martin and Martin, 2002a.

As is clear from Figure 3, the ‘smirk’ which is typically associated with post-1987 BS implied volatilities for options on equities (see, for example, Corrado and Su, 1997, Bates (2000) and Lim, Martin and Martin, 2002a) is indeed a feature of the S&P200 option data, with the BS model essentially underpricing *ITM* options (low  $K/S$ ) and overpricing *OTM* options (high  $K/S$ ). In the first panel of Figure 3, the implied volatility graph for the skewed normal model with constant volatility is compared with the BS curve, illustrating that the modelling of skewness alone is insufficient in terms of reducing the smirk. In contrast, in the panel below, the specification of a skewed *GST* model for returns produce an implied volatility graph which is virtually constant, at least for  $K/S < 1$ . Interestingly, as illustrated in the third panel on the left-hand side of Figure 3, the incorporation of more marked excess kurtosis via an *ST*-based specification, produces an overadjustment of the smirk for the *OTM* options in particular, creating the effect of a ‘frown’. The frown effect is also a feature of all

non-BS models represented in the right-hand panels, in which time-varying volatility is added to the specification for returns. In summary then, according to this criterion the skewed *GST* model with constant volatility is the best-performing model.

## 5 Concluding Remarks

The paper presents a general option pricing framework which accommodates the main empirical features of financial returns. In contrast to other attempts to generalize option pricing beyond the BS model, the approach adopted here has computational requirements equivalent to those of BS pricing, thereby rendering it a clear contender for use by practitioners. A Bayesian approach to conducting inference on the range of models accommodated within the general framework is outlined. When the methodology is applied to the prices of options on the S&P200 Index, there is clear evidence of option-implied time-varying volatility and excess kurtosis in Index returns, with slightly weaker evidence in favour of negative skewness. Whilst there is not complete consistency across all performance criteria, the results suggest that models which explicitly allow for all of these departures from the BS assumptions provide a better within-sample and out-of-sample fit to observed prices.

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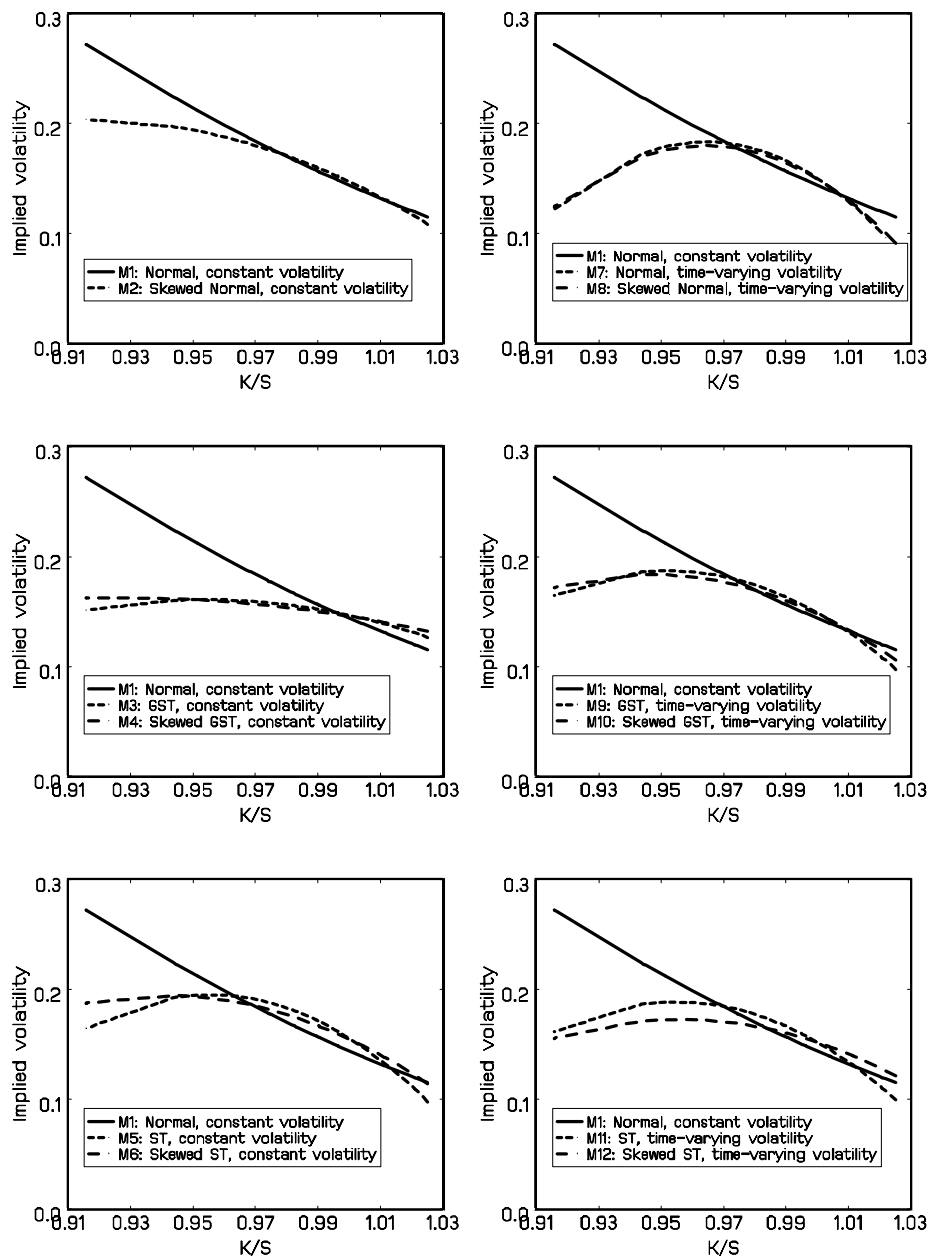


Figure 3: Implied Volatility Smiles for Alternative Models: 7 Days to Maturity.

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