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ABSTRACT

This short paper analyzes the effect of heterogeneity of markets in terms of income on the exclusion of markets under uniform price by considering linear demand curves in all markets. We show that more markets (and consumers) are excluded the higher are the inter-market income differences, and that adding markets, even with lower reservation prices than in existing ones, helps to decrease the price and thus make more markets served. The multiple market case turns out to be not an insignificant extension of the two-market case. We also address the welfare implications of price discrimination and show that discrimination could be beneficial or inefficient, crucially depending on the inter-market wealth distribution.

(*JEL Classification:* D42; L12)

Keywords: Exclusion; Heterogeneous markets; Marshallian welfare; Output effect; Third-degree price discrimination; Uniform price.

1. Introduction

The past two decades have witnessed a revival of interest in the output and welfare implication of the third degree price discrimination ever since Schmalensee's (1981) formal analysis and generalization of some ideas contained in Robinson (1933). It was shown that under quite general conditions a necessary condition for the third degree price discrimination to be Pareto improving is that the output under discrimination is strictly greater than that under simple monopoly price (uniform price), for the simple reason that under price discrimination the marginal valuations differ from one market to another (cf. Schmalensee 1981, Varian 1985, Layson 1988 and others; for an analytic survey, see Varian 1989). The output effect depends on the curvature of the demand curve in each market, which as Robinson pointed out long ago reflects intra-market income and taste differences. Shih, Mai and Liu (1988) contribute a general analysis of the implication of curvature of demand functions for the output effect.

However, as Robinson (1933, pp. 189-190) recognized many years ago, the output effect could also be due to the fact that some market is not served under uniform price since the reservation price in the said market might be below the optimal price of the monopolistic firm. Thus, the output effect due to the wealth distribution *within* a market, characterized by the curvature of the demand curve, should be conceptually distinguished from that due to the exclusion of weak markets under uniform price, which in turn results from the inter-market income (wealth) difference reflected by the different reservation prices. The output and welfare effect of the curvature of demand curves has been well understood (see Shih et al 1988 among others). Yet the output and welfare implications of the exclusion have not received much formal analysis. Although both Schmalensee (1981, pp.242-3) and Varian (1985, p.873) offer brief analysis of the exclusion, Layson (1994), presumably for the first time, comes to grips with a formal treatment of the inclusion of some market under discrimination which is not served under uniform price.[1] Layson (1994) identifies some factors favoring opening new market(s) under price discrimination by analyzing the two market case with particular emphasis on the usefulness of observable variables under the discrimination regime. A large share of the market with higher reservation price, a large difference between the profit margins in the two markets and the concavity of the demand curves contribute positively to the opening of new markets under price discrimination.

This short paper, largely drawing on the insight dating from Robinson (1933) that the curvature of the demand curve of a market reflects the wealth (income) and taste distribution among the consumers in the said market, focuses

[1] For an earlier geometric illustration, see Battalio and Ekelund (1972). Hausman and MacKie-Mason (1988) study the patent policy and argue that the third degree price discrimination allows patent holders to open new markets which are excluded under uniform price. Yet their analysis centers around the scale economies in producing innovative products by the patent holders and the dynamic incentive effect of the patent policy on innovations, and their discussion of the output and welfare implication of exclusion of weak markets under uniform price is rather brief.

on the exclusion effect of the inter-market income differences for any number of markets by abstracting from the effect of the difference in curvatures of market demand functions on the output. Namely, we assume the linearity of demand curves in all markets concerned. In doing so, we implicitly assume that each consumer in any market demands at most one unit of the good and her own reservation price of the good is determined by her income and that the intra-market income distribution is the same across all markets (uniform). We show that the heterogeneity of markets in terms of income has a significant impact on the exclusion effect. Different from the two market case, the multiple market setting provides a basis for determining how many markets would be excluded under uniform price. In expanding the set of factors favoring opening new markets under price discrimination, we show that on the demand side the heterogeneity of markets contributes positively and addition of markets with even lower incomes than already existing ones contributes negatively to the percentage of excluded markets (consumers) under uniform price, while on the supply side higher marginal cost in production favors a higher percentage of markets (consumers) being excluded. The welfare may increase or decrease when price discrimination is allowed, crucially depending on the inter-market wealth distribution.

The rest of the paper is organized as follows. Section 2 contains the main results. After a brief review of the known results about the output and welfare effect of price discrimination, we analyze the implications of income heterogeneity among markets, addition of more markets with even lower incomes than existing ones, and production costs for the uniform price and for the number and percentage of markets excluded. A brief discussion of related output and welfare effect follows. Interesting enough, it turns out to be that adding market(s) with even lower reservation price(s) may induce the firm to serve more, even all, markets than otherwise, including those that are not served without the addition of the new market(s). Our conclusions are summarized in Section 3.

2. Analysis

2.1 Preliminaries

We first briefly review some known results about the output and welfare effect of the third degree price discrimination in markets with linear demands.

As discussed in the introduction, linear aggregate demand in each market is assumed in order to address the effect of inter-market income heterogeneity on price discrimination. We assume that every consumer demands at most one unit of the good and that the consumer's reservation price is determined by her income. To simplify the analysis, further assume that for any consumer i, the reservation price (p_i) is proportional to her income (m_i), $p_i = sm_i$, where $s \in (0,1]$ is a constant across consumers. There are a lot of consumers in each market and hence the aggregate demand of the market could be seen as being represented by a continuous curve.[2] The highest individual reservation price of a market is defined as the reservation price of the said market. For any market, consumers' incomes are uniformly distributed from 0 to $\tilde{m} = \tilde{p}/s$, with $\tilde{p}$ being the reservation price of the market. That is, the density function of intra-market income distribution $f(m) = 1/\tilde{m} = s/\tilde{p}$, $m \in [0, \tilde{m}]$. Denote as a the total number of consumers of the market. Thus, for any price $p < \tilde{p}$, the demand in the said market $x(p)$ = the number of consumers whose reservation price is equal to or above p = number of consumers whose income is at least p/s. Hence $x(p)$ = $\int_{p/s}^{\tilde{p}/s} af(m)dm = a - bp$, where $b \equiv a/\tilde{p}$.

Monopolist's pricing strategy: the two-market case

Consider the simple case of two markets, each associated with a linear demand $x = a_i - b_i p$, $i = 1,2$. For each market, a_i reflects the potential demand of the market, and b_i (or more exactly a_i / b_i) characterizes the reservation price of the market.[3] The marginal cost of production is assumed constant. It is straightforward to show that under price discrimination the optimal sales of the monopolist firm in these two markets are $(a_1 - b_1 c)/2$ and $(a_2 - b_2 c)/2$ respectively. One may aggregate the demand curves of the two markets

[2] For an analysis of the continuous market demand curve aggregated from discontinuous individual demand curves, see Varian (1992, pp. 152-3).

[3] Note the demand approaches a_i when the price is extremely low and that there is no demand at all if the product is too expensive $(p \geq a_i / b_i)$ for the consumers to afford.

$x = a_1 + a_2 - (b_1 + b_2)p$ and show by similar computation that the optimal sales of the monopolist firm under the uniform price is precisely the sum of the optimal supplies in markets one and two under discriminatory pricing.

However, if the reservation prices of the two markets are not the same, say $a_1/b_1 > a_2/b_2$, then the aggregate demand curve under the uniform price turns out to be,

$$x = \begin{cases} a_1 + a_2 - (b_1 + b_2)p, & \text{if} \quad p < a_2/b_2 \\ x_1 = a_1 - b_1 p, & \text{if} \quad a_1/b_1 > p > a_2/b_2 \end{cases} \tag{1}$$

and since $p > a_2/b_2$ the demand in market two is zero. Provided the profit realized in market one under the discrimination price is higher than the profit realized in both markets under uniform pricing, the profit maximizing firm will sell to market one only with market two being excluded if discrimination pricing is not allowed. To be sure, this has been shown by Varian (1989, p.622). But in Varian's geometric presentation, that market two is small is characterized by both its small potential demand (a_2) and its reservation price (a_2/b_2). This turns out to be the case in which a small poor market is excluded.[4] We can also provide another case in which the market with a large number of potential buyers (a_2 is large) and a low reservation price (a_2/b_2 is small), is excluded. This case appears to be more empirically relevant than the example considered by Varian (1989) where a small market is excluded, given that the people in some

[4] Layson (1994) offers a very interesting analysis of the exclusion for the two market case, yet the effect of potential market sales, characterized by parameter "a" in our model, is not explicitly addressed.

large developing countries can not afford to get access to some products which they badly need. An illustration might be life saving drugs, (see Dumoulin 2000).

Let π_1 stand for the profit realized in market one under discrimination and π the profit reaped from both markets under the uniform pricing strategy. One can easily show that,

$$\pi_1 > \pi \text{ if and only if } \quad \frac{a_1}{b_1} > \frac{a_2}{b_2}(2 + \frac{a_2}{a_1}) - \frac{(b_1 + b_2)c}{a_1}(\frac{2a_2}{b_2} - c) \qquad (2)$$

It is obvious that for the above inequality to hold true does not necessarily require that both the potential sales (a_2) and the reservation price of market two (a_2/b_2) are small. Provided the reservation price in market two, *i.e.*, a_2/b_2, is relatively small enough, the second market will be excluded even if its population size (a_2) is large.[5] Indeed, to highlight the exclusion we can assume c=0 and consequently (2) is re-written as,

$$\frac{a_1 / b_1}{a_2 / b_2} > 2 + \frac{a_2}{a_1} \qquad (2')^6$$

Here the economic interpretation becomes even more straightforward. The uniform price excludes the market with a large number of people of low income provided that the relative reservation price between the two markets dominates

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[5] Note the uniform price $p^u = \dfrac{a_1 + a_2}{2(b_1 + b_2)}$. It is assumed $p^u < a_2 / b_2$, i.e., $\dfrac{a_2}{a_1} + \dfrac{2a_2 / b_2}{a_1 / b_1} > 1$, to avoid uninteresting analyses centering on corner solutions. Similar assumptions apply to the n-market case below.

[6] Note (2') implies that $a_1 / b_1 > 2a_2 / b_2$ which in turn suffices that when the monopoly charges the price $a_1 / 2b_1$ the demand in the second market at this price is zero and hence excluded.

the relative population size even if the population size of the market with a lower reservation price is much larger.

Monopolist's pricing strategy: the n-market case

We now turn to the case with n heterogeneous markets for some product X. The demand curve in market i is $x_i = a_i - b_i p_i$, i = 1, ...n. All the reservation prices are above the marginal cost, that is, $a_i / b_i > c$, $i = 1, 2, \cdots, n$. Without loss of generality, suppose $a_1 / b_1 \geq a_2 / b_2 \geq \cdots \geq a_n / b_n$. In the case of perfect price discrimination, the firm's optimal price, sales and profit for each market respectively are,

$$p_i^* = \tfrac{1}{2}(a_i/b_i + c); \quad x_i^* = \tfrac{1}{2}(a_i - b_i c); \quad \pi_i^* = \tfrac{b_i}{4}(a_i/b_i - c)^2 \tag{3}$$

Hence, the total output and profit of the firm are

$$X^* = \sum_{i=1}^{n} x_i^* = \tfrac{1}{2}\sum_{i=1}^{n} a_i - \tfrac{c}{2}\sum_{i=1}^{n} b_i = n(\bar{a} - \bar{b}c)/2 \tag{4}$$

$$\Pi^* = \sum_{i=1}^{n} \pi_i^* = \sum_{i=1}^{n} [b_i(\frac{a_i}{b_i} - c)^2]/4 \tag{5}$$

where $\bar{a}$ and $\bar{b}$ are the mean values of "a" and of "b" of the n markets respectively. If the monopolist firm is not allowed to discriminate and instead has to apply a uniform price to all markets, the monopolist firm may deliberately choose some uniform price to maximize its profit with some markets being excluded. But it immediately follows from $a_1 / b_1 \geq a_2 / b_2 \geq \cdots \geq a_n / b_n$ that for any market to be excluded from the uniform price those markets with lower reservation prices must be also excluded. We can therefore proceed in two steps to solve for the uniform price which maximizes the profit of the firm. First, assume the firm targets markets 1, 2, ..., m only with markets (m+1), .., n being excluded. One can accordingly derive the optimal price (p_m^0) and profit (Π_m^0),

$$p_m^0 = \frac{1}{2}\left(\sum_{i=1}^{m} a_i / \sum_{i=1}^{m} b_i + c\right) \tag{6}$$

$$\Pi_m^0 = \frac{1}{4}\sum_{i=1}^{m} b_i \left(\sum_{i=1}^{m} a_i / \sum_{i=1}^{m} b_i - c\right)^2 \tag{7}$$

where m=1, 2, ...,n. Note it follows from Eq. (6) and the fact that $a_1/b_1 \geq a_2/b_2 \geq \cdots \geq a_n/b_n$ that the higher the price the more markets of low reservation prices are excluded, namely, $p_1^0 \geq p_2^0 \geq \cdots \geq p_n^0$. Secondly, the firm charges a uniform price among { $p_1^0, p_2^0, \cdots, p_n^0$ }, denoted as p_{m^*}, which leads to the maximal profit. As a result, the uniform price p_{m^*} applies to markets 1, 2, ...,m*, but markets (m*+1), ..., n are excluded.[7]

Two observations immediately follow from the above analysis. First, the firm's profit under the perfect discriminatory price is always higher than or equal to that under uniform pricing. Note that for any uniform price strategy, the firm can always set some discriminatory prices applying to markets to make a profit no worse than that under the uniform price. Secondly, the sales level

[7] Using mathematical induction over the number of markets n, one can easily prove that the profit maximization price must be among { $p_1^0, p_2^0, \cdots, p_n^0$ } and the optimal profit is accordingly $\Pi_{m^*}^0$.

under discrimination pricing is the same as the sales level under uniform pricing when there is no exclusion. But it is another case if some markets are excluded under uniform pricing. It is easy to see that output under uniform pricing when markets (m+1), ..., n are excluded is $X_m^0 = (\sum_{i=1}^{m} a_i - c\sum_{i=1}^{m} b_i)/2$. Thus, it follows from (4) that the difference between output under the perfect discrimination price X* and that under the uniform price X_m^0, $X^* - X_m^0 = \sum_{s=m+1}^{n}(a_s - b_s c)/2$ increases with the number of excluded markets.

2.2. Exclusion

Similar to what is done in subsection 2.1 for the two market case, one can derive from the analysis in the preceding subsection that the aggregation demand of the n linear demand markets under the uniform price is,

$$X^\circ = \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} a_i - p^\circ \sum_{i=1}^{n} b_i \qquad \text{if } p^\circ < a_n / b_n$$

$$X^\circ = \sum_{i=1}^{n-1} x_i = \sum_{i=1}^{n-1} a_i - p^\circ \sum_{i=1}^{n-1} b_i \qquad \text{if } a_{n-1}/b_{n-1} > p^\circ \geq a_n / b_n \tag{8}$$

....

$$X^\circ = \sum_{i=1}^{2} x_i = \sum_{i=1}^{2} a_i - p^\circ \sum_{i=1}^{2} bi \text{ if } a_2/b_2 > p^\circ \geq a_3/b_3$$

$$X^\circ = x_1 = a_1 - b_1 p^\circ \qquad \text{if } a_1/b_1 \geq p^\circ \geq a_2/b_2;$$

We have shown how profit maximization leads the monopolist firm to exclude some low income markets. We examine the exclusion more closely in this subsection. Specifically, we analyze the effect of income heterogeneity among markets on the exclusion and show that more consumers are excluded under uniform pricing the more heterogeneous are the markets.

To address implications of income heterogeneity of markets on the exclusion effect under a non-discriminatory price regime, we abstract from the issue of differences in potential sales among markets (characterized by parameters a_i in the linear demand function $x_i(p) = a_i - b_i p$ as mentioned earlier), and allow the reservation prices (characterized by the parameter b_i, or more exactly, by a_i / b_i in the linear demand $x_i(p) = a_i - b_i p$) to vary. Namely, we fix a for each market i, i.e., $x_i(p) = a - b_i p$, $i = 1,2,...,n$ and let b range among $[b, B]$, $b = b_1 < b_2 < \cdots < b_n = B$. It turns out that some further specification of the discrete market structure is needed to make the model analytically solvable. Assume $b_{i+1} / b_i = \sqrt[n-1]{b_n / b_1} = \sqrt[n-1]{B / b} \equiv q$, $i = 1,2,...,n-1$. Note q measures the extent to which markets are heterogeneous, or in short, the degree of market heterogeneity.

In order to highlight the effect of demand-related factors on the exclusion, the marginal production cost is assumed to be zero to simplify the algebra without loss of insights. It can be shown that if the monopolist firm targets the top k markets, then the optimal price and profit respectively are,

$$p_k = \frac{ka}{2\sum_{i=1}^{k} b_i} = \frac{ka(q-1)}{2b(q^k - 1)}, \text{ and } \pi_k = \frac{k^2 a^2 (q-1)}{4b(q^k - 1)}$$

Abstracting from the integer problem, the optimal pricing strategy of the firm must be to charge the price from $\{p_1, p_2, \cdots, p_n\}$ that yields the maximal value among $\{\pi_1, \pi_2, \cdots, \pi_n\}$. Maximizing π_k with respect to k yields (see Appendix 1 for analysis of the second order condition and the uniqueness of the optimal solution),

$$2(q^k - 1) = q^k \ln(q^k) \tag{9}$$

It could be easily shown from the above equation that,

$$\frac{dk^*}{dq} = -\frac{k^*}{\ln q} < 0 \tag{10}$$

But recall that q characterizes the extent of heterogeneity of incomes among markets. That is, the income gap between rich consumer groups (countries) and poor ones contributes positively to the number of excluded markets under uniform pricing. Intuitively, if incomes (and reservation prices) differ significantly from one country to another, as happens in reality, it pays for the monopolist firm to serve only those markets having consumers high levels of income and the poor markets are consequently excluded.

To analyze the effect of the market number on the price level, fix q and let n vary. It follows that (see Appendix 2 for the mathematical details)

$$\frac{dp}{dn} < 0 \tag{11}$$

The price decreases with the number of markets: when more markets are added, it would pay better if the firm charges a lower price than otherwise to serve more consumers including those in low-income markets. We present a numerical example in the next subsection to further illustrate this point.

Regarding the percentage of markets excluded under the uniform price, described by

$\rho^e = \dfrac{n-k^*}{n}$, it can be derived from Eq. (9) that,

$$\frac{\partial(\rho^e)}{\partial n} < 0 \text{ and } \frac{\partial(\rho^e)}{\partial a} = 0 \tag{12}$$

The mechanism behind the proposition that the number of markets contributes negatively to the exclusion effect is precisely the same as that for the negative effect of market numbers on the uniform price. Again, a simple numerical example of 3 markets is used in subsection 2.3 as a further illustration. But also note that an increase in parameter a without a change in parameters "b" implies an increase in both the potential sales and reservation prices in all markets, and the percentage change in each of these two parameters is the same across all markets in our case of linear demand curves. The positive effect on exclusion of the increase in potential sales and the negative effect of the increase in the reservation prices cancel each other, resulting in neutrality of change in a to the change in *percentage* of excluded markets.

All the above analysis centers around the demand side. For the supply side, it could be easily shown for constant marginal production costs that the production cost exerts a positive effect on the exclusion (we omit the algebraic analysis to save space). The more costly it is to produce, the less output will be produced by the monopoly (with the other things being the same) and as a result more markets are excluded.

To sum up,

Proposition: *The more heterogeneous the markets (in terms of income and reservation price), the more markets are excluded with uniform pricing. Adding more markets helps to decrease the price and thus more markets are served. Marginal cost contributes positively but the number of markets contributes negatively to the percentage of excluded markets.*

2.3 Output and welfare: does the number of markets matter?

The output under discrimination is obviously greater than under uniform pricing if some markets are excluded in the latter. As is well understood (see, e.g., Schmalensee 1981, Varian 1985 and Shih et al 1988), a necessary, but not sufficient, condition for discrimination to be beneficial is that it increases output, due to the fact that under price discrimination the marginal valuations differ from one market to another. But the sufficient condition for discrimination to increase the welfare is yet to be known. To be sure, for the 2-market case, discrimination increases both output and welfare if only one market is served under the uniform pricing, because the uniform price is the same as that applied to the stronger market under discrimination, but both output and welfare in the weaker market under discrimination pricing is positive. Indeed, the two market case has received much attention in the literature on exclusion and discrimination (Robinson 1933, Hausman and MacKie-Mason 1988, Varian 1989 and Layson 1994).

But the above statement for the two market case can not be generalized to multiple markets. Consider the following example in which there are 3 markets with demand functions $q_1 = 1 - p/4$, $q_2 = 1 - p/2$, and $q_3 = 1 - 8p$ respectively. For the sake of illustration, the marginal cost is assumed zero. It can be shown that the optimal uniform price of the monopoly is p=4/3, with markets 1 and 2 served. The output is 1, 2/3 of which is sold in the first market and 1/3 in the second. Market 3 is excluded. The profit is 4/3. The Marshallian welfare (consumers' surplus plus producers' surplus) is 4/3+1=7/3. If price discrimination is allowed, all the three markets will be served under price 2, 1, and 1/16 respectively, and the amount sold of the product in each market is ½ and hence the output 3/2, greater than that under uniform pricing. Monopoly's profit is 49/32. But Marshallian welfare is 147/64 <7/3. Welfare decreases when the monopoly changes from uniform pricing to discriminatary pricing, though both profit and output increases. The increase in output under discrimination does not imply that it is beneficial.

If we substitute the demand function in market 3 with $q_3 = 1 - 4p$ with all other things remaining the same, then the situation under uniform pricing is the same as above. But the discrimination price in the 3rd market will be 1/8. The amount sold of the product in each market is still ½ and hence the output 3/2. The monopoly's profit is 25/16, which is greater than that under uniform pricing. Marshallian welfare is 75/32 >7/3. Thus welfare increases. In this slightly modified example, output and welfare change in the same direction from one regime to the other.

Certainly one can always compute Marshallian welfare in both regimes for specified cases. What is made clear is that whether an increase in output is brought about by an increase in discrimination or a decrease in Marshallian welfare crucially depends on the income distribution among markets. Yet, some general results regarding the condition under which discrimination will be beneficial when some markets are excluded under uniform price still remains unexplored.

More interestingly, even if some relatively weak markets are excluded under the uniform price, to add market(s) with even lower reservation price(s), which would seem to be irrelevant though, may induce the firm to serve more and even all markets. The multiple market case is therefore not an insignificant extention from Layson's (1994) two market case. Consider two markets with demand curves: $q_1 = 1 - p$ and $q_2 = 1 - 3.5p$ respectively. Again, assume the marginal cost to be zero for the sake of illustration. The optimal uniform price of the monopolist firm is 1/2, with only market one served. The output is 1/2. The profit equals 1/4 and Marshallian welfare is 3/8=0.375. Now suppose another market with an even lower reservation price, denoted as market 3, $q_3 = 1 - 4p$, is added. In this 3 market structure, the firm will take into account the potential sales in market 3 as well as those in markets 1 and 2 when deciding upon its pricing strategy. It could be shown that the optimal uniform price for the firm is 3/17, with *all* the three markets included, and the output 3/2. Note the price decreases significantly from 1/2 to 3/17; furthermore, output increases even more remarkably (tripled) due to the introduction of market three. Welfare is 0.371 + 0.265 =0.636, far above that in the case of markets one and two (as well as that when the discriminatory prices apply to each of these three markets, which is equal to 0.576). Introducing new potential markets, even those with reservation prices below that of *any* existing market, may increase *both* the monopolist firm's output and profit and consumers' surplus, resulting in an even more remarkable improvement in social welfare.

3. Concluding remarks

Drawing on the insight which dates from Robinson (1933) that the curvature of the demand curve of a market reflects the distribution of wealth (income) and taste among the consumers in the said market, we address the effect of heterogeneous demand of markets on the exclusion of markets and consumers under uniform price by considering the situation in which the curvatures of demand in all markets concerned are the same (linear demand). The linearity of demand functions in all markets allows us to focus on inter-market income inequality and its implication for exclusion. Our analysis covers both the simple two market case and the many market case. We argue that more markets (and consumers) are excluded under uniform pricing the higher are the inter-market income differences, and that addition of new markets even of relatively low income may induce the monopolist firm to charge a lower uniform price and hence serve more markets.

It should go without saying that the implication of heterogeneity of markets for the exclusion effect is of great practical relevance, given that we do live in a world of very unequal income distribution among the countries. A good example is the markets for innovative essential drugs, which are produced generally by some big monopolistic firms based in few developed countries. The practice of parallel imports as a counter-strategy in developing countries tends to result in uniform pricing and many population of poor countries are thus excluded (see Ballance, Pogany and Forstner 1992 and Dumoulin 2000).[8]

We also address the welfare implications of the price discrimination when the uniform pricing strategy does exclude some markets, and show that discrimination could be beneficial or inefficient, crucially depending on the inter-market wealth distribution. But a general criterion by which the direction of change in welfare could be directly determined by the observable variables under price discrimination regime remains to be further explored.

[8] To be sure, this short paper focuses on the exclusion for monopoly market structure. We leave the analysis of the exclusion for oligopoly, probably a more realistic setting at least for drug markets, to a sequel to this paper.

Mathematical Appendices

Appendix 1

Differentiation of the monopolist firm's profit $\pi_k = \dfrac{k^2a^2(q-1)}{4b(q^k-1)}$ with respect to k,

$$\frac{\partial \pi_k}{\partial k} = \frac{(q-1)a^2}{4b} \cdot \frac{2k\left(q^k-1\right)-q^k k^2 \ln q}{(q^k-1)^2} \qquad \text{(A1.1)}$$

hence Eq. (9) in the text. Let $f(x) \equiv 2(x-1) - x\ln x$. Then $f(1)=0$, $f(e)=e-2>0$, and

$f'(x) = 1 - \ln x = \begin{cases} +, x < e \\ -, x > e \end{cases}$. Thus, if the interior solution exists, it must be unique and such

that $q^{k^*} > e$, i.e., $k^* \ln q > 1$. It follows from Eq. (A1.1) that $\dfrac{\partial^2 \pi_k}{\partial k^2}\Big|_{k=k^*} =$

$\dfrac{(q-1)a^2}{4b} \cdot \dfrac{k^* q^{k^*} \ln q (1 - k^* \ln q)}{(q^{k^*}-1)^4} < 0$.

Appendix 2

We derive Eq. (11) in this Appendix. Note $q = \sqrt[n-1]{B/b}$, $p_k = \dfrac{ka(q-1)}{2b(q^k-1)}$ and that the first

order condition of maximizing the profit requires $2(q^k-1) = q^k \ln(q^k)$ at the optimal solution

$k = k^*$. To derive $\dfrac{dp}{dq}$, we can proceed with $\dfrac{dp}{dn} = \dfrac{\partial p}{\partial k} \cdot \dfrac{\partial k}{\partial n} + \dfrac{\partial p}{\partial n}$. Let $w = (B/b)^{k/(n-1)}$, then

differentiation of the first order condition leads to $\dfrac{\partial k}{\partial n} = -\dfrac{w_n}{w_k} = \dfrac{k}{n-1}$. But

$p = \dfrac{ka}{2b} \cdot \dfrac{(\frac{B}{b})^{\frac{1}{n-1}} - 1}{(\frac{B}{b})^{\frac{k}{n-1}} - 1}$, from which we can derive $\dfrac{\partial p}{\partial k}$ and $\dfrac{\partial p}{\partial n}$. After some algebraic

manipulation,

$$\frac{dp}{dn}=\frac{ka}{2b}\cdot\frac{[(B/b)^{k/(n-1)}-1]\{(B/b)^{1/(n-1)}-1-(B/b)^{1/(n-1)}[\ln(B/b)^{1/(n-1)}]\}}{[(B/b)^{k/(n-1)}-1]^2}\cdot\frac{1}{n-1}$$

Let $g(x)=x-1-x\ln x$, then $g(1)=0$ and $g'(x)=-\ln x<0$ for $x>1$. Hence, $\frac{dp}{dn}<0$.

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