Particle Swarm Optimization for MEG Source Localization*

Chenwei Jiang[1], Bin Wang[1,2], Liming Zhang[1,2]

[1]Department of Electronics Engineering, Fudan University, Shanghai 200433, China
{jiangchenwei, wangbin, lmzhang}@fudan.edu.cn
[2]The Research Center for Brain Science, Fudan University, Shanghai 200433, China

Abstract. The estimation of three-dimension neural active sources from the magnetoencephalography (MEG) record is a very critical issue for both clinical neurology and brain functions research. Nowadays multiple signal classification (MUSIC) algorithm and recursive MUSIC algorithm are widely used to locate dipolar sources from MEG data. The drawback of these algorithms is that they need excessive calculation and is quite time-consuming when scanning a three-dimensional space. In order to solve this problem, we propose a MEG sources localization scheme based on an improved Particle Swarm Optimization (PSO). This scheme uses the advantage of global searching ability of PSO to estimate the rough source location. Then combining with grids search in small area, the accurate dipolar source localization is performed. In addition, we compare the results of our method with those based on Genetic Algorithm (GA). Computer simulation results show that our PSO strategy is an effective and precise approach to dipole localization which can improve the speed greatly and localize the sources accurately.

1 Introduction

Magnetoencephalography is a noninvasive brain-measuring technique with the ability of estimating the neural active sources in a millisecond-level definition. The temporal resolution of MEG is far superior to that achieved by other brain imaging techniques such as MRI, CT, SPECT and PET. So MEG has been regarded as a powerful tool in clinic diagnosis and brain science study.

One major problem of MEG is the inverse problem [1], which concerned with the localization of the brain neural active sources from the magnetic field data measured outside a human head. Because the neural active sources distribution is inherently three-dimensional, the estimation problem is generally ill-posed. Nowadays multiple signal classification (MUSIC) [2] and recursive multiple signal classification (R-MUSIC) [3] are two widely-used methods for such MEG sources localization problem, but unfortunately they are both quite time-consuming.

To solve this problem, various optimization techniques have been proposed. One kind of these algorithms is gradient-based Newton-type methods such as Levenberg–

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Marquardt [4] and gradient-free search methods such as the Nelder–Mead downhill simplex method [5]. However, the gradient-based methods are problematic. They are easily trapped in the local optima, since the active source localization model is so complex and undetermined. Though gradient-free methods are better than the gradient technique in escaping local optima, Khosla et al. [6] demonstrated that it is sensitive to initial parameter and will also converge to a suboptimal location. Another kind of algorithms is heuristic-based method, such as Genetic Algorithm (GA) [7], Simulated Annealing (SA) [8]. Though some new features have been added in its implementation for MEG source localization, it has been shown that they still have not an enough localization performance [9].

In our scheme, we present a two-step grid-scanning procedure. First, we divide the whole three-dimension space to large grids for coarse scan, and use PSO to pick out the grid, where objective function has its optimal answer for the next step's scanning. Second, we scan the selected area in small grids to locate the MEG sources positions more precisely. Furthermore, we introduce adaptive rule to select global best particle in every generation of the swarm to overcome the disadvantage of PSO which falls into local optimum occasionally.

This paper is organized as follows. We briefly review the forward and inverse localization models in Section 2 and describe the proposed scheme in Section 3. In Section 4, we present some simulated results to show the excellent performance of the proposed scheme. Conclusion is given in Section 5.

2 MEG Localization Model

2.1 Forward Model for MEG

By using a complete MEG model comprising primary current sources of dipole-in-a-sphere head model and magnetoconductivity, *etc* [10]-[12], we can regard a neural active source as a current dipole and simplify forward model into a convenient discrete matrix according to the Biot-Savart law [2]

$$\mathrm{B}(i)=\mathrm{k}\frac{(\vec{R}_i-\vec{L}_j)\times s_i}{\left|\vec{R}_i-\vec{L}_j\right|^3}\vec{Q}_j^T=\vec{g}(i)\vec{Q}_j^T, \tag{1}$$

where $k=\mu_0/4\pi$ is a constant, in which μ_0 is the permeability constant, $\vec{Q}_j^T\in R^{3\times1}$ is the j th dipole moment, $\vec{L}_j\in R^{1\times3}$ is the j th dipole location, $\vec{\mathrm{R}}_i\in R^{1\times3}$ is the i th measurement sensor location, s_i denotes the unit orientation of the i th sensor, and $\mathrm{B}(i)$ is the magnetic field at $\vec{R}_i$.

The vector $\vec{g}(i)$ can be considered as a gain vector relating the moment intensity of the dipole to the measurement at position $\vec{R}_i$. Considering there are total q dipoles , we can easily extend equation (1) to (2) for m sensors

$$\begin{bmatrix} b(\vec{\mathrm{R}}_1) \\ \vdots \\ b(\vec{\mathrm{R}}_{\mathrm{m}}) \end{bmatrix} = \begin{bmatrix} g(\vec{\mathrm{R}}_1,\vec{\mathrm{L}}_1)^T \cdots g(\vec{\mathrm{R}}_1,\vec{\mathrm{L}}_{\mathrm{q}})^T \\ \vdots \qquad\qquad \vdots \\ g(\vec{\mathrm{R}}_{\mathrm{m}},\vec{\mathrm{L}}_1)^T \cdots g(\vec{\mathrm{R}}_{\mathrm{m}},\vec{\mathrm{L}}_{\mathrm{q}})^T \end{bmatrix} Q^T \text{ ,} \tag{2}$$

where $Q^T = [Q_1, \cdots, Q_i, \cdots, Q_q]^T$, and $Q_i^T \in R^{3\times1}$ is the moment of the i th dipole. By substituting equation (1) for the $g(\vec{\mathrm{R}}_i, \vec{\mathrm{L}}_i) \in R^{1\times3}$ in equation (2) and decomposing Q_i^T as $Q_i^T = T_i[\theta_i]^T$, B_i can be re-expressed as a function of source location p_i:

$$B_i = G(p_i)Q_i^T = G(p_i)T_i[\theta_i]^T = A(p_i,\theta_i)T_i, \tag{3}$$

where p_i represents the location of the dipole, $\theta_i \in R^{3\times1}$ does the direction of the dipole, and T_i does the current strength of the dipole. Now the magnetic field detected at each sensor can be expressed by a function of dipoles position $p_i \in \{x, y, z\}$ as shown ed in (3).

2.2 Cost Function

Based on the forward model above, the location estimation of MEG sources can be casted by a least-square problem as follows [2]

$$\{p_i,\theta_i\}_{\mathrm{LS}} = \arg\min \|F - B\|_{\mathrm{F}}^2 = \arg\min \left\|F - A(p,\theta)T^T\right\|_{\mathrm{F}}^2 \text{ ,} \tag{4}$$

where F is the magnetic field detected by Superconducting Quantum Interference Device (SQUID) sensors outside the head. $B = A(p,\theta)T^T$ is the theoretical magnetic field derived from the estimated parameter $\{p_i,\theta_i\}$ through the equation proposed in [3].

In order to reduce the computational cost for the equation (4), signal subspace method is introduced by Mosher J.C. et al [3]. By decomposing the covariance matrix of F, we can separate the useful signal from the noise by

$$R_F = FF^T / n = \widehat{\boldsymbol{\Phi}}_{\mathrm{S}}\widehat{\Lambda}_s\widehat{\boldsymbol{\Phi}}_{\mathrm{S}} + \widehat{\boldsymbol{\Phi}}_{\mathrm{n}}\widehat{\Lambda}_n\widehat{\boldsymbol{\Phi}}_{\mathrm{n}}, \tag{5}$$

where the $\widehat{\boldsymbol{\Phi}}_{\mathrm{S}}$ with large eigenvalue as our estimate of a set of vectors which span the signal subspace; similarly $\widehat{\boldsymbol{\Phi}}_{\mathrm{n}}$ with small eigenvalue refer to the noise subspace. Then we can recast cost function (4) based on equation signal subspace in equation (6)

$$\{c_1, c_2, \cdots, c_k\} = \mathrm{subcorr}(A(p,\theta), \widehat{\boldsymbol{\Phi}}_{\mathrm{S}}), \tag{6}$$

where $\widehat{\boldsymbol{\Phi}}_{\mathrm{S}}$ is the signal subspace we get from equation (5) and $A(p,\theta)$ from equation (3). { $c_1, c_2, \cdots, c_k$ } are subspace correlations between $A(p,\theta)$ and $\widehat{\boldsymbol{\Phi}}_{\mathrm{S}}$ which subject to $1 \geq c_1 \geq c_2 \cdots \geq c_k \geq 0$, and k is the number of independent dipoles. As mentioned in [3], the parameter vector θ represents a linear combination of the columns of the gain matrix, such that $A(p_i,\theta_i)=\mathrm{G}(p_i)\theta_i$, so finding the parameter p_i which maximam the $\mathrm{subcorr}(G(p),\hat{\boldsymbol{\Phi}}_{\mathrm{S}})$ can reduce the complexity of the nonlinear search. The operation symbol "subcorr" meaning subspace correlations can be computed by the following procedure

Step 1 Perform a Singular Value Decomposition (SVD) of G, such that $G=U_G\Sigma_G V_G^T$, Similarly decompose $\hat{\boldsymbol{\Phi}}_{\mathrm{S}}$. Retain the only components U_G and $U_{\hat{\boldsymbol{\Phi}}_{\mathrm{S}}}$ that correspond to nonzero singular value.

Step 2 Form $C=U_G^T U_{\hat{\boldsymbol{\Phi}}_{\mathrm{S}}}$.

Step 3 Compute the singular values of C by SVD. The maximal k ordered singular value $1\geq c_1\geq c_2\cdots\geq c_k\geq 0$ are the subspace correlations corresponding to the k dipoles.

Based on the above subspace conception, we can use a recursive method [3] to calculate the k th dipole location by

$$c_{\mathrm{k}}=\arg\max \mathrm{subcurr}([\hat{A}_{k-1},G(p_{\mathrm{k}})],\hat{\boldsymbol{\Phi}}_{\mathrm{S}})_{\mathrm{k}}, \tag{7}$$

$$\hat{A}_{k-1}=[G(p_1),\cdots,G(p_{\mathrm{k-1}})]\ and\ \hat{A}_0=[\phi], \tag{8}$$

$G(p)$ can be considered as the gain matrix or relationship between an active dipole and the column vector of measurement locations as showed in equation (2).

First we let $\hat{A}_0=[\phi]$ and form the matrix $[\hat{A}_0,G(p_1)]$, and then maximize the subspace correlation c_1 in equation (7) to find the location $p_1\in\{x,y,z\}$ as the first dipole location. After that we can designate $\hat{A}_1=G(p_1)$ by equation (8) for the second dipole search. Similarly, to search for the i th dipole, we form the matrix $[\hat{A}_{i-1},G(p_i)]$ in advance. Then we calculate equation (7) and pick out $p_i\in\{x,y,z\}$ as the i th dipole location, which maximize the subspace correlation c_i. Following this recursive rule, we can find out all k dipoles via cost function equation (7) which means our objective is to find a set of $P_i\in\{x,y,z\}$ where c_i gets its maximum value.

The enumerative method is adopted in [3] in which we have to design a sufficiently dense grid in the volume of the head, if we want to obtain high computational precision. But it is quite time-consuming to calculate at each grid.

3 The proposed Scheme

3.1 Method Based on PSO

The solution of the nonlinear multidimensional optimization for the least-squares method mentioned in equation (7) requires scanning the whole three-dimension space.

Here, we present a PSO-based R-MUSIC approach to localize the MEG sources, in which we use PSO before performing the refined grid scan. We choose a set of solutions $X_i \in R^{1\times 3}$ (the position of dipoles) for equation (7) which is homogeneously distributed in the whole head volume as initial particle swarm, $V_i \in R^{1\times 3}$ as its current velocity (position's change), p_{id} as its past optimal position, p_{gd} as the optimal position of the particle swarm and c_i in equation (7) as PSO's fitness f_i. A standard PSO method can be expressed as

$$V_{id}(m) = WV_{id}(m-1) + d_1M_1(p_{id} - x_{id}(m-1)) + d_2M_2(p_{gd} - x_{id}(m-1)), \quad (9)$$

$$X_{id}(m) = X_{id}(m-1) + V_{id}(m-1), \quad (10)$$

where $i = 1, \cdots, N$, N is the number of the particles, d_1 and d_2 are the constants of acceleration, and M_1 and M_2 are random numbers in [0, 1], and W is the weight of inertia, which is used to maintain the momentum of the particle.

As above mentioned, we utilize the PSO algorithm to find out the parameter $P_k \in \{x, y, z\}$ which maximize c_k of dipole k. In our scheme, $X_{id}(m)$ and $V_{id}(m)$ are both three-dimension vectors which stand for MEG source location $P\{x, y, z\}$ and its moving velocity. The detailed procedure of performing PSO is as follows

Step 1 Initialize $i = 1$ to search for the first dipole.

Step 2 Generate the initial population which has total N particles and is homogenously distributed in the whole three-dimension brain space.

Step 3 Calculate the object function (7), and get each particle's fitness f_i.

Step 4 Find out the best particle of all the swarm X_i as p_{gd}.

Step 5 Find out the best particle from the historical track of each particle as p_{id}.

Step 6 Judge whether the optimal answer's fitness $f_{best} > f_{req}$ or not.

If $f_{best} > f_{req}$, go to step 8.

Step 7 Use formula (9) to update X_i for the next generation particles.

Step 8 Judge whether it is the last generation. If not, go to step 2.

Step 9 Judge whether i equals dipole number k. If not, $i = i + 1$ and then go to step 2.

Step 10 End the PSO operation, and begin the grid scan.

By the use of PSO's excellent global searching ability and its high convergent speed in coarse grid scan process, the proposed method can find the area where the equation (4) has its optimal answer for the next step scanning in a comparatively short time. After that, we use the enumerative grid-scanning method to localize the more accurate position of the MEG sources. This approach assures that the final results have the same definition as the ordinary grid-scanning method in [3]. As a result, the whole procedure can improve the speed of source localization greatly and its result is also accurate. However, seldom PSO-based MUSIC approach sometimes converged to the local minimum. In order to solve this problem, we improve the PSO by the adaptive selection of p_{gd}, which guarantees diversity in the early stage of PSO and converges to the best particle at later evolution.

3.2 Improved PSO Method

In this section, we propose an additional probability pro_i corresponding to every particle X_i to make the selection of p_{gd} adaptive which can maintain the diversity of swarm at early evolution stage, since p_{gd} is no longer always chosen from the best p_{id} according to the probability pro_i. This selection strategy can avoid the local optimum effectively and enhance the searching ability of PSO algorithm. So the above universal PSO step 3 can be substituted by the procedure of the selection strategy as follows.

Step 1 Define temperature $T = T_0(1-(t-1)/G_{\max})$, where T_0 stands for the initial temperature, t is present evolution generation and $G_{\max}$ is the maximum evolution generation.

Step 2 Calculate pro_i for particle X_i by equation (11).

$$pro_i = \exp(f(X_N)/T) \Big/ \sum \exp(f(X_j)/T), \quad (j = 1, \cdots N), \tag{11}$$

Step 3 Use roulette wheel selection operator to choose p_{gd} by pro_i of each X_i.

Step 4 Go to next step of PSO.

When iterative number t accumulates to $G_{\max}$, $f(X_j) - f(X_N)$ will be a negative value, meanwhile $T = T_0(1-(t-1)/G_{\max})$ will approach to 0, so $\exp(f(X_j) - f(X_N)) \big/ T_0(1-(t-1)/G_{\max})$ will approach to 0. This makes the choice probability pro_{N} of X_N which corresponds to the best fitness value will equal 1 at later stage. This property makes sure that the improved PSO will converge to the best solution in the end. The verification is given in (12).

$$\begin{aligned} \lim_{t \to G_{\max}} pro_{\mathrm{N}} &= \exp(f(X_N)/T) \Big/ \sum \exp(f(X_j)/T), \qquad (j = 1, \cdots N) \\ &= \frac{1}{\left(\sum\limits_{j=1 to N} \exp(f(X_j) - f(X_N)) \big/ T_0(1-(t-1)/G_{\max}) + 1\right)} \quad , \\ &= 1 \end{aligned} \tag{12}$$

4 Experimental Results

The simulation experiments are used to evaluate the performance of the proposed method. We implement the program via Matlab 7 on a PC with a Pentium(R) 4 1.7G

CPU. And a standard arrangement of 37 radial SQUID sensors is adopted here. The array has a sensor at $\phi = 0$, a ring of six sensors at $\phi = \pi/8$, $\varphi = k\pi/3$, $k = 0, \ldots, 5$, a ring of twelve sensors at $\phi = \pi/4$, $\varphi = k\pi/6$, $k = 0, \ldots, 11$, and a ring of eighteen sensors at $\phi = 3\pi/8$, $\varphi = k\pi/9$, $k = 0, \ldots, 17$. They are distributed on the upper region of a 9 cm single-shell sphere as shown in Fig. 1.

In the simulation, the dipoles are assumed to have the fixed locations and orientations, whereas the current strengths are allowed to change in time according to a parametric model. Based on this model, we can obtain a 37×500 simulative spatio-temporal MEG data set. Our method based on PSO is used in experiments to localize the MEG active dipoles. In order to give a comparison with other algorithms, the method based on GA and enumerative R-MUSIC method are also tested.

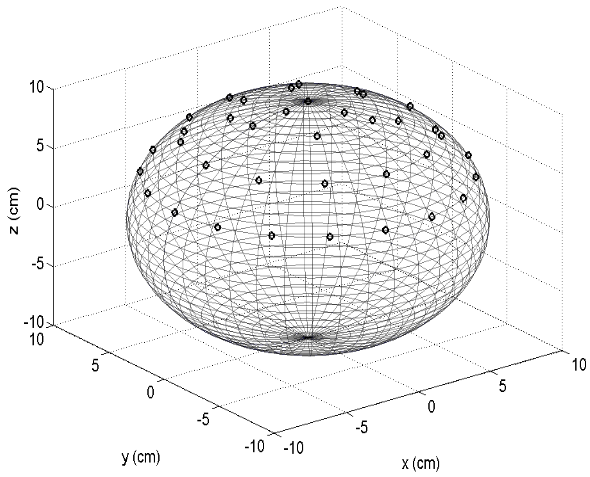

Fig.1. The location distribution of 37 SQUID sensors.

Table 1. Localization results by R-MUSIC algorithm

	True Location (cm)			Estimated Location (cm)			Time
	x	y	z	x	y	z	(s)
Dipole1	0.004	0.022	1.229	0.000	0.000	1.300	
Dipole2	0.411	0.699	1.599	0.400	0.700	1.600	11142
Dipole3	0.893	2.041	0.440	0.900	2.000	0.500	

Table 2. Rough localization results using GA

	True Location (cm)			Estimated Location (cm)			Error (cm)			Time
	x	y	z	x	y	z	x	y	z	(s)
Dipole1	0.004	0.022	1.229	0.035	0.035	1.160	0.031	0.013	0.069	
Dipole2	0.411	0.699	1.599	0.281	0.633	1.723	0.130	0.066	0.140	383
Dipole3	0.893	2.041	0.440	0.844	2.285	0.563	0.049	0.244	0.123	

Table 3. Localization results using GA combining with grids

	True Location (cm)			Estimated Location (cm)			Time
	X	y	z	x	y	z	(s)
Dipole1	0.004	0.022	1.229	0.000	0.000	1.300	
Dipole2	0.411	0.699	1.599	0.400	0.700	1.600	25
Dipole3	0.893	2.041	0.440	0.900	2.000	0.500	

Table 4. Rough localization results using PSO

	True Location (cm)			Estimated Location (cm)			Error (cm)			Time
	x	y	z	x	y	z	x	y	z	(s)
Dipole1	0.004	0.022	1.229	0.008	0.227	1.111	0.004	0.205	0.118	
Dipole2	0.411	0.699	1.599	0.369	0.674	1.576	0.042	0.025	0.023	61
Dipole3	0.893	2.041	0.440	0.943	2.126	0.583	0.050	0.085	0.143	

Table 5. Localization results using PSO combining with grids

	True Location (cm)			Estimated Location (cm)			Time
	X	y	z	x	y	z	(s)
Dipole1	0.004	0.022	1.229	0.000	0.000	1.300	
Dipole2	0.411	0.699	1.599	0.400	0.700	1.600	24
Dipole3	0.893	2.041	0.440	0.900	2.000	0.500	

Table 6. Comparison between Three Methods (3 dipoles)

Method	Generation Number	Population Number	Root Mean Square Error (cm)	Total cost Time (s)
R-MUSIC	/	/	/	11142
GA	200	100	0.013	408
PSO	200	100	0.010	85

We pick out 10 groups of 3-dipole MEG data for the simulation experiment and select 0.1cm as measurement accuracy according to the requirement of practical clinic use in the grids search. Each group was tested by all the three methods for 20 times and one of average results is listed in Table 1~5. The population is set as 100 and the evolution number is 200 for both PSO and GA algorithms. Meanwhile, a procedure to optimize the parameter value is executed before the algorithm is used to MEG active dipole localization.

Observing Tables 4 and 5, we can find that our two-step method based on PSO is very efficient and effective to localize the MEG active dipoles. The root mean square error (RMSE) of the rough localization result is 0.010 as showed in Table 6. Moreover, it cost only 85 seconds at all, while the method based on GA takes 408 seconds and enumerative R-MUSIC method takes almost 11142 seconds at all.

Compared with the method based on GA, our method based on PSO is much faster. Their total cost times are showed in Table 6. Because in the first step, PSO has taken all the particle swarm's information to speed up the source localizing which is not included in GA. And meanwhile, PSO need not code the three-dimension locations into binary genes for calculating the cost function which means it need not transform the genes back to the decimal numbers either. So the average execution time of PSO is only 1/5 of GA method, though both methods have a very high precision of the rough localization as the RMSEs. Furthermore, when the number of source increasing, the proposed method will have more superiority to other heuristic-based algorithm.

Besides the comparison with the other heuristic algorithm, there are several reasons why our proposed method has such good effect on MEG sources localization. First, the excellent global search ability of PSO has made makes a quick convergence possible. Second, our adaptive method enhances the diversity of particle swarm, and as a result, it avoids the algorithm from trapped in the local optimum. Third, the enumerative grid-scanning of second step guarantees the high definition and accuracy of MEG source localization.

5 Conclusions

In this paper, we proposed a MEG source localization scheme based on PSO. The experimental results from the simulation show that the proposed method can speed up the dipole localization operation, and it is faster than the method based on GA and much faster than enumerative R-MUSIC method. Further, combining with grid scan in small areas, we can obtain accurate results. The precise and quick localization of MEG sources based on PSO will contribute to its further applications.

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